1 Concepts, Notation, and Principles

Given a computer program, how can we determine whether or not it will do exactly what it is intended to do? This question is not only intellectually challenging, but also of primary importance in practice. An ideal solution to this problem would be to develop certain techniques that can be used to construct a formal proof (or disproof) of the correctness of a program systematically. There has been considerable effort to develop such techniques, and many different techniques for proving program correctness have been reported. However, none of them has been developed to the point where it can be used readily in practice.

There are several technical hurdles that prevent formal proof of correctness from becoming practical; chief among them is the need for a mechanical theorem prover. The basic approach taken in the development of these techniques is to translate the problem of proving program correctness into that of proving a certain statement to be a theorem (i.e., always true) in a formal system. The difficulty is that all known automatic theorem-proving techniques require an inordinate amount of computation to construct a proof. Furthermore, theorem proving is a computationally unsolvable problem. Therefore, like any other program written to solve such a problem, a theorem prover may halt if a solution is found. It may also continue to run without giving any clue as to whether it will take one more moment to find the solution, or whether it will take forever. The lack of a definitive upper bound of time required to complete a job severely limits its usefulness in practice.

Until there is a major breakthrough in the field of mechanical theorem proving, which is not foreseen by the experts any time soon, verification of program correctness through formal proof will remain impractical. The technique is too costly to deploy, and the size of programs to which it is applicable is too small (relative to that of programs in common use). At present, a practical and more intuitive solution would be to test-execute the program with a number of test cases (input data) to see if it will do what it is intended to do.

How do we go about testing a computer program for correctness? Perhaps the most direct and intuitive answer to this question is to perform an *exhaustive test*: that is, to test-execute the program for all possible input data (for which the program is expected to work correctly). If the program produces a correct result for every possible input, it obviously constitutes a direct proof that the program is correct. Unfortunately, it is in general impractical to do the exhaustive test for any nontrivial program simply because the number of possible inputs is prohibitively large.

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To illustrate, consider the following C++ program.

Program 1.1

```
main ()
{
int i, j, k, match;

   cin  >> i  >> j  >> k;
   cout << i << j << k;
   if (i <= 0 || j <= 0 || k <= 0
         || i+j <= k || j+k <= i || k+i <= j)
      match = 4;
   else if !(i == j || j == k || k == i)
      match = 3;
   else if (i != j || j != k || k != i)
      match = 2;
   else match = 1;
   cout << match << endl;
}
```

If, for an assignment of values to the input variables `i`, `j`, and `k`, the output variable `match` will assume a correct value upon execution of the program, we can assert that the program is correct for this particular test case. And if we can test the program for all possible assignments to `i`, `j`, and `k`, we will be able to determine its correctness. The difficulty here is that even for a small program like this, with only three input variables, the number of possible assignments to the values of those variables is prohibitively large. To see why this is so, recall that an ordinary integer variable in C++ can assume a value in the range −32,768 to +32,767 (i.e., 2^{16} different values). Hence, there are $2^{16} \times 2^{16} \times 2^{16} = 2^{48} \approx 256 \times 10^{12}$ possible assignments to the input triple (`i`, `j`, `k`). Now suppose that this program can be test-executed at the rate of one test per microsecond on average, and suppose further that we do testing 24 hours a day, 7 days a week. It will take more than eight years for us to complete an exhaustive test for this program. Spending eight years to test a program like this is an unacceptably high expenditure under any circumstance!

This example clearly indicates that an exhaustive test (i.e., a test using all possible input data) is impractical. It may be technically doable for some small programs, but it would never be economically justifiable for a real-world program. That being the case, we will have to settle for testing a program with a manageably small subset of its input domain.

Given a program, then, how do we construct such a subset; that is, how do we select test cases? The answer would be different depending on why we are doing the test. For software developers, the primary reason for doing the test is to find errors so that they can be removed to improve the reliability of the program. In that case we say that the tester is doing *debug testing*. Since the main goal of debug testing is to find programming errors, or *faults* in the Institute of Electrical and Electronics

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Engineers (IEEE) terminology, the desired test cases would be those that have a high probability of revealing faults.

Other than software developers, expert users of a software system may also have the need to do testing. For a user, the main purpose is to assess the reliability so that the responsible party can decide, among other things, whether or not to accept the software system and pay the vendor, or whether or not there is enough confidence in the correctness of the software system to start using it for a production run. In that case the test cases have to be selected based on what is available to the user, which often does not include the source code or program specification. Test-case selection therefore has to be done based on something else.

Information available to the user for test-case selection includes the probability distribution of inputs being used in production runs (known as the *operational profile*) and the identity of inputs that may incur a high cost or result in a catastrophe if the program fails. Because it provides an important alternative to debug testing, possible use of an operational profile in test-case selection is explained further in Section 4.2. We discuss debug testing in Chapters 2 and 3. Chapter 4 is devoted to other aspects of testing that deserve our attention. Other than testing as discussed in Chapters 2 and 3, software faults can also be detected by means of analysis, as discussed in Chapters 5 through 7.

When we test-execute a program with an input, the test result will be either correct or incorrect. If it is incorrect, we can unequivocally conclude that there is a fault in the program. If the result is correct, however, all that we can say with certainty is that the program will execute correctly for that particular input, which is not especially significant in that the program has so many possible inputs. The significance of a correct test result can be enhanced by analyzing the execution path traversed to determine the condition under which that path will be traversed and the exact nature of computation performed in the process. This is discussed in Chapter 5.

We can also detect faults in a program by examining the source code systematically as discussed in Chapter 6. The analysis methods described therein are said to be static, in that no execution of the program is involved. Analysis can also be done dynamically, while the program is being executed, to facilitate detection of faults that become more obvious during execution time. In Chapter 7 we show how dynamic analysis can be done through the use of software instruments.

For the benefit of those who are not theoretically oriented, some helpful logico-mathematical background material is presented in Appendix A. Like many others used in software engineering, many technical terms used in this book have more than one possible interpretation. To avoid possible misunderstanding, a glossary is included as Appendix B. For those who are serious about the material presented in this book, you may wish to work on the self-assessment questions posed in Appendix C.

There are many known test-case selection methods. Understanding and comparison of those methods can be facilitated significantly by presenting all methods in a unified conceptual framework so that each method can be viewed as a particular instantiation of a generalized method. We develop such a conceptual framework in the remainder of the chapter.

1.1 CONCEPTS, TERMINOLOGY, AND NOTATION

The *input domain* of a program is the set of all possible inputs for which the program is expected to work correctly. It is constrained by the hardware on which the program is to be executed, the operating system that controls the hardware, the programming environment in which the program is developed, and the intent of the creator of the program. If none of these constraints are given, the default will be assumed.

For example, consider Program 1.1. The only constraint that we can derive from what is given is the fact that all variables in the program are of the type "short integer" in C++. The prevailing standard is to use 16 bits to represent such data in 2's-complement notation, resulting in the permissible range −32,768 to 32,767 in decimal. The input domain therefore consists of all triples of 16-bit integers; that is,

$$D = \{< x, y, z > | x, y, \text{ and } z \text{ are 16-bit integers}\}$$

Input (*data*) are elements of the input domain, and a *test case* is an input used to perform a test execution. Thus, every test case is an input, but an input is not necessarily a test case in a particular test. The set of all test cases used in testing is called a *test set*. For example, <3, 5, 2> is a possible input (or test case) in Program 1.1, and in a particular test we might choose {<1, 2, 3>, <4, 5, 6>, <0, 0, 5>, <5, 0, 1>, <3, 3, 3>} as the test set.

This notational convention for representing program inputs remains valid even if the program accepts an input repeatedly when run in an interactive mode (i.e., sequence of inputs instead of a single input). All we need to do is to say that the input domain is a product set instead of a simple set. For example, consider a program that reads the value of input variable x, which can assume a value from set X. If the function performed by the program is not influenced by the previous value of x, we can simply say that X is the input domain of the program. If the function performed by the program is dependent on the immediately preceding input, we can make the product set $X \cdot X = \{< x_1, x_2 > | x_1 \in X \textit{ and } x_2 \in X\}$ the input domain. In general, if the function performed by the program is dependent on n immediately preceding inputs, we can make the product set $X^{n+1} = \{< x_1, x_2, \ldots, x_n, x_{n+1} > | x_i \in X \text{ for all } 1 \leq i \leq n + 1\}$ the input domain. This is the property of a program with memory, often resulting from implementing the program with a finite-state machine model. The value of n is usually small and is related to the number of states in the finite-state machine.

Do not confuse a program with memory with an interactive program (i.e., a program that has to be executed interactively). Readers should have no difficulty convincing themselves that an interactive program could be memoryless and that a program with memory does not have to be executed interactively. We shall now proceed to define some terms in program testing that might, at times, have a different meaning for different people.

The composition of a test set is usually prescribed using a *test-case selection criterion*. Given such a criterion, any subset of the input domain that satisfies the criterion is a candidate. We say "any subset" because more than one subset in the input domain may satisfy the same criterion. Examples of a test-case selection criterion include $T = \{0, 1, 2, 3\}$, $T = \{< i, j, k > | i = j = k \text{ and } k > 1 \text{ and } k < 10\}$, and "$T$ is a set of inputs that cause 60% of the statements in the program to be exercised at least once during the test."

Let D be the input domain of a given program P, and let $\text{OK}(P, d)$, where $d \in D$, be a predicate that assumes the value of TRUE if an execution of program P with input d terminates and produces a correct result, and FALSE otherwise. Predicate $\text{OK}(P, d)$ can be shortened to $\text{OK}(d)$ if the omission of P would not lead to confusion.

After we test-execute the program with input d, how can we tell if $\text{OK}(d)$ is true? Two assumptions can be made in this regard. One is that the program specification is available to the tester. $\text{OK}(d)$ is true if the program produced a result that satisfies the specification. Another is the existence of an *oracle*, a device that can be used to determine if the test result is correct. The target-practice equipment used in testing the software that controls a computerized gunsight is a good example of an oracle. A "*hit*" indicates that the test is successful, and a "*miss*" indicates otherwise. The main difference between a specification and an oracle is that a specification can be studied to see how to arrive at a correct result, or the reason why the test failed. An oracle gives no clue whatsoever.

Let T be a test set: a subset of D used to test-execute a program. A test using T is said to be *successful* if the program terminates and produces a correct result for every test case in T. A successful test is to be denoted by the predicate SUCCESSFUL(T). To be more precise,

$$\text{SUCCESSFUL}\ (T) \equiv (\forall t)_T(\text{OK}(t))$$

The reader should not confuse a successful test execution with a successful program test using test set T. The test using T fails if there exists a test case in T that causes the program to produce an incorrect result [i.e., $\neg\text{SUCCESSFUL}(T) \equiv \neg(\forall t)_T(\text{OK}(t)) \equiv (\exists t)_T(\neg\text{OK}(t))$]. The test using T is successful if and only if the program executes correctly for all test cases in T.

Observe that not every component in a program is involved in program execution. For instance, if Program 1.1 is executed with input `i = j = k = 0`, the assignment statement `match = 1` will not be involved. Therefore, if this statement is faulty, it will not be reflected in the test result. This is one reason that a program can be fortuitously correct, and therefore it is insufficient to test a program with just one test case.

According to the IEEE glossary, a part of a program that causes it to produce an incorrect result is called a *fault* in that program. A fault causes the program to fail (i.e., to produce incorrect results) for certain inputs. We refer to an aggregate of such inputs as a *failure set*, usually a small subset of the input domain.

In debug testing, the goal is to find faults and remove them to improve the reliability of the program. Therefore, the test set should be constructed such that it maximizes the probability and minimizes the cost of finding at least one fault during the test. To be more precise, let us assume that we wish to test the program with a set of n test cases: $T = \{t_1, t_2, \ldots, t_n\}$. What is the reason for using multiple test cases? It is because for all practical programs, a single test case will not cause all program components to become involved in the test execution, and if there is a fault in a component, it will not be reflected in the test result unless that component is involved in the test execution.

Of course, one may argue that a single test case would suffice if the entire program were considered as a component. How we choose to define a component for test-case selection purposes, however, will affect our effectiveness in revealing faults. If the granularity of component is too coarse, part of a component may not be involved in test execution, and therefore a fault contained therein may not be reflected in the test result even if that component is involved in the test execution. On the other hand, if the granularity of the component is too fine, the number of test cases required and the effort required to find them will become excessive. For all known unit-testing methods, the granularities of the component range from a statement (finest) to an execution path (coarsest) in the source code, with one exception that we discuss in Section 7.2, where the components to be scrutinized are operands and expressions in a statement.

For debug testing, we would like to reveal at least one fault in the test. To be more precise, we would like to maximize the probability that at least one test case causes the program to produce an incorrect result. Formally, we would like to maximize

$$\begin{aligned} p(\neg\mathrm{OK}(t_1) \vee \neg\mathrm{OK}(t_2) \vee \cdots \vee \neg\mathrm{OK}(t_n)) &= p((\exists t)_T(\neg\mathrm{OK}(t))) \\ &= p(\neg(\forall t)_T(\mathrm{OK}(t))) \\ &= 1 - p((\forall t)_T(\mathrm{OK}(t))) \end{aligned}$$

The question now is: What information can be used to construct such a test set?

It is well known that programmers tend to forget writing code to make sure that the program does not do division by zero, does not delete an element from an empty queue, does not traverse a linked list without checking for the end node, and so on. It may also be known that the author of the program has a tendency to commit certain types of error or the program is designed to perform certain functions that are particularly difficult to implement. Such information can be used to find test cases for which the program is particularly error-prone [i.e., the probability $p(\neg\mathrm{OK}(t_1) \vee \neg\mathrm{OK}(t_2) \cdots \vee \neg\mathrm{OK}(t_n))$ is high]. The common term for making use of such information is *error guessing*. The essence of that technique is described in Section 3.4.

Other than the nature or whereabouts of possible latent faults, which are unknown in general, the most important information that we can derive from the program and use to construct a test set is the degree of similarity to which two inputs are processed by the program. It can be exploited to enhance the effectiveness of a test set. To see why that is so, suppose that we choose some test case, t_1, to test the program first, and we wish to select another test case, t_2, to test the program further. What relationship must hold between t_1 and t_2 so that the joint fault discovery probability is arguably enhanced?

Formally, what we wish to optimize is $p(\neg\mathrm{OK}(t_1) \vee \neg\mathrm{OK}(t_2))$, the probability of fault discovery by test-executing the program with t_1 and t_2. It turns out that this probability can be expressed in terms of the conditional probability $p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1))$:

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the probability that the program will execute correctly with input t_2 given the fact that the program executed correctly with t_1. To be exact,

$$\begin{aligned} p(\neg\mathrm{OK}(t_1) \vee \neg\mathrm{OK}(t_2)) &= p(\neg(\mathrm{OK}(t_1) \wedge \mathrm{OK}(t_2))) \\ &= p(\neg(\mathrm{OK}(t_2) \wedge \mathrm{OK}(t_1))) \\ &= 1 - p(\mathrm{OK}(t_2) \wedge \mathrm{OK}(t_1)) \\ &= 1 - p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1))p(\mathrm{OK}(t_1)) \end{aligned}$$

This equation shows that if we can choose t_2 to make the conditional probability $p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1))$ smaller, we will be able to increase $p(\neg\mathrm{OK}(t_1) \vee \neg\mathrm{OK}(t_2))$, the probability of fault discovery.

The value of $p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1))$ depends on, among other factors, the degree of similarity of operations performed in execution. If the sequences of operations performed in test-executing the program using t_1 and t_2 are completely unrelated, it should be intuitively clear that $p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1)) = p(\mathrm{OK}(t_2))$, that is, the fact that the program test-executed correctly with t_1 does not influence the probability that the program will test-execute correctly with test case t_2. Therefore, $p(\mathrm{OK}(t_2) \wedge \mathrm{OK}(t_1)) = p(\mathrm{OK}(t_2))p(\mathrm{OK}(t_1))$. On the other hand, if the sequences of operations performed are similar, then $p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1)) > p(\mathrm{OK}(t_2))$, that is, the probability that the program will execute correctly will become greater given that the program test-executes correctly with input t_1. The magnitude of the difference in these two probabilities, denoted by

$$\delta(t_1, t_2) = p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1)) - p(\mathrm{OK}(t_2))$$

depends on, among other factors, the degree of commonality or similarity between the two sequences of operations performed by the program in response to inputs t_1 and t_2.

For convenience we shall refer to $\delta(t_1, t_2)$ henceforth as the (*computational*) *coupling coefficient* between test cases t_1 and t_2, and simply write δ if the identities of t_1 and t_2 are understood. The very basic problem of test-case selection can now be stated in terms of this coefficient simply as follows. Given a test case, find another that is as loosely coupled to the first as possible!

Obviously, the value of this coefficient is in the range $0 \leq \delta(t_1, t_2) \leq 1 - p(\mathrm{OK}(t_2))$, because if $\mathrm{OK}(t_1)$ implies $\mathrm{OK}(t_2)$, then $p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1)) = 1$, and if the events $\mathrm{OK}(t_1)$ and $\mathrm{OK}(t_2)$ are completely independent, then $p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1)) = p(\mathrm{OK}(t_2))$. The greater the value of $\delta(t_1, t_2)$, the tighter the two inputs t_1 and t_2 are coupled, and therefore the lower the joint probability of fault discovery (through the use of test cases t_1 and t_2). Asymptotically, $\delta(t_1, t_2)$ becomes zero when the events of successful tests with t_1 and t_2 are absolutely and completely independent, and $\delta(t_1, t_2)$ becomes $1 - p(\mathrm{OK}(t_2)) = p(\neg\mathrm{OK}(t_2))$ when a successful test with t_1 surely entails a successful test with t_2.

Perhaps a more direct way to explain the significance of the coupling coefficient $\delta(t_1, t_2)$ is that

$$\begin{aligned} p(\neg\mathrm{OK}(t_1) \vee \neg\mathrm{OK}(t_2)) &= 1 - p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1))p(\mathrm{OK}(t_1)) \\ &= 1 - (p(\mathrm{OK}(t_2) \mid \mathrm{OK}(t_1)) - p(\mathrm{OK}(t_2)) \\ &\quad + p(\mathrm{OK}(t_2)))p(\mathrm{OK}(t_1)) \\ &= 1 - (\delta(t_1, t_2) + p(\mathrm{OK}(t_2)))p(\mathrm{OK}(t_1)) \\ &= 1 - \delta(t_1, t_2)p(\mathrm{OK}(t_1)) - p(\mathrm{OK}(t_2))p(\mathrm{OK}(t_1)) \end{aligned}$$

The values of $p(\mathrm{OK}(t_1))$ and $p(\mathrm{OK}(t_2))$ are intrinsic to the program to be tested; their values are generally unknown and beyond the control of the tester. The tester, however, can select t_1 and t_2 with a reduced value of the coupling coefficient $\delta(t_1, t_2)$, thereby increasing the fault-discovery probability $p(\neg\mathrm{OK}(t_1) \vee \neg\mathrm{OK}(t_2))$.

How can we reduce the coupling coefficient $\delta(t_1, t_2)$? There are a number of ways to achieve that, as discussed in this book. One obvious way is to select t_1 and t_2 from different input subdomains, as explained in more detail later.

1.2 TWO PRINCIPLES OF TEST-CASE SELECTION

Now we are in a position to state two principles. The *first principle of test-case selection* is that in choosing a new element for a test set being constructed, preference should be given to those candidates that are computationally as loosely coupled as possible to all the existing elements in the set. A fundamental problem then is: Given a program, how do we construct a test set according to this principle? An obvious answer to this question is to select test cases such that the program will perform a distinctly different sequence of operations for every element in the set.

If the test cases are to be selected based on the source code, the most obvious candidates for the new element are those that will cause a different execution path to be traversed. Since almost all practical programs have a large number of possible execution paths, the next question is when to stop adding test cases to the test set. Since the purpose of using multiple test cases is to cause every component, however that is defined, to be exercised at least once during the test, the obvious answer is to stop when there are enough elements in the test set to cause every component to be exercised at least once during the test.

Thus, the *second principle of test-case selection* is to include in the test set as many test cases as needed to cause every contributing component to be exercised at least once during the test. (*Remark: Contributing* here refers to the component that will make some difference to the computation performed by the program. For brevity henceforth, we omit this word whenever the term *component* is used in this context.)

Note that the first principle guides us as to what to choose, and the second, as to when to stop choosing. These two principles are easy to understand and easy to apply,

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and therefore become handy under situations when no existing method is applicable. For example, when a new software system is procured, the user organization often needs to test it to see if it is reliable enough to pay the vendor and release the system for production runs. If an operational profile is available, the obvious choice is to perform operational testing as described in Section 4.2. Otherwise, test-case selection becomes a problem, especially if the system is fairly large. Source code is generally not available to the user to make use of the methods presented in Chapter 2, and detailed design documents or specifications are not available to use the methods presented in Chapter 3. Even if they are available, a typical user organization simply does not have the time, resources, and technical capability to deploy the methods. In that event, the two principles can be utilized to select test cases. The components to be exercised could be the constituent subsystems, which can be recognized by reading the system user manual. Two inputs are weakly coupled computationally if they cause different subsystems to be executed in different sequences. Expert users should be able to apply the two principles readily to achieve a high probability of fault detection. In short, if one finds it difficult to use any existing method, use the two principles instead.

Next, in practical application, we would like to be able to compare the cost-effectiveness of test sets. In the literature, the effectiveness of a test set is measured by the probability of discovering at least one fault in the test (see, e.g., [FHLS98]). It is intuitively clear that we can increase the fault-discovery capability of a test set simply by adding elements to it. If we carry this idea to the extreme, the test set eventually would contain all possible inputs. At that point, a successful test constitutes a direct proof of correctness, and the probability of fault discovery is 100%. The cost of performing the test, however, will become unacceptably high. Therefore, the number of elements in a test set must figure prominently when we compare the cost-effectiveness of a test set. We define the *cost-effectiveness of a test set* to be the probability of revealing a fault during the test, divided by the number of elements in the test set.

A test set is said to be *optimal* if it is constructed in accordance with the first and second principles for test-case selection and if its size (i.e., the number of elements contained therein) is minimal. The concept of path testing (i.e., to choose the execution path as the *component* to be exercised during the test) is of particular interest in this connection because every feasible execution path defines a subdomain in the input domain, and the set of all subdomains so defined constitutes a partition of the input domain (in a set-theoretical sense; i.e., each and every element in the domain belongs to one and only one subdomain). Therefore, a test set consisting of one element from each such subdomain is a good one because it will not only cause every component to be exercised at least once during the test, but its constituent elements will be loosely coupled as well. Unfortunately, path testing is impractical in general because most programs in the real world contain loop constructs, and a loop construct expands into a prohibitively large number of execution paths. Nevertheless, the idea of doing path testing remains of special interest because many known test-case selection methods can be viewed as an approximation of path testing, as we demonstrate later.

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1.3 CLASSIFICATION OF FAULTS

In the preceding section we said that a test case should be selected from a subdomain or a subset of inputs that causes a component to be exercised during the test. Is there a better choice if there is more than one? Is there any way to improve the fault-detection probability by using more than one test case from each subdomain? The answer depends on the types of faults the test is designed to reveal. What follows is a fault classification scheme that we use throughout the book.

In the abstract, the intended function of a program can be viewed as a function f of the nature $f : X \to Y$. The definition of f is usually expressed as a set of subfunctions $f_1, f_2, \ldots, f_m$, where $f_i : X_i \to Y$ (i.e., f_i is a function f restricted to X_i for all $1 \leq i \leq m$), $X = X_1 \cup X_2 \cup \cdots \cup X_m$, and $f_i \neq f_j$ if $i \neq j$. We shall use $f(x)$ to denote the value of f evaluated at $x \in X$, and suppose that each X_i can be described in the standard subset notation $X_i = \{x \mid x \in X \wedge C_i(x)\}$.

Note that, above, we require the specification of f to be *compact* (i.e., $f_i \neq f_j$ if $i \neq j$). This requirement makes it easier to construct the definition of a type of programming fault in the following. In practice, the specification of a program may not be compact (i.e., f_i may be identical to f_j for some i and j). Such a specification, however, can be made compact by merging X_i and X_j.

Let (P, S) denote a program written to implement the function f described above, where P is the condition imposed on the input and S is the sequence of statements to be executed. Furthermore, let D be the set of all possible inputs for the program. Set D is the computer-implemented version of set X mentioned above. No other constraints are imposed. The definition of set D, on the hand, will be constrained by programming language used and by the hardware on which the program will be executed. For example, if it is implemented as the short integers in C++, then D is a set of all integers representable by using 16 bits in 2's-complement notation. The valid input domain (i.e., the set of all inputs for which the program is expected to work correctly) is seen to be the set $\{d \mid d \in D \, and \, P(d)\}$. The program should be composed of n paths:

$$(P, S) = (P_1, S_1) + (P_2, S_2) + \cdots + (P_n, S_n)$$

Here (P_i, S_i) is a subprogram designed to compute some subfunction f_j. $P \equiv P_1 \vee P_2 \vee \cdots \vee P_n$, and P is in general equal to T (true) unless the programmer places additional restrictions on the inputs. We shall use $S(x)$ to denote the computation performed by an execution of S with x as the input.

Two basic types of fault may be committed in constructing the program (P, S). The program created to satisfy a specification must partition its input domain into at least as many subdomains as that required by the specification, each of which must be contained completely in some subdomain prescribed by the specification. Otherwise, there is a domain fault. If there is an element in the input domain for which the program produces a result different from that prescribed by the specification, and the input is in a subdomain that is contained completely in a subdomain prescribed by the specification, there is a computational fault. To be more precise, we can restate the definitions as follow.

1. *Computational fault*. The program has a computational fault if $(\exists i)(\exists j)((P_i \supset C_j \wedge S_i(x) \neq f_j(x))$.
2. *Domain fault*. The program has a domain fault if $\neg(\forall i)(\exists j)(P_i \supset C_j)$.

In words, if the program specification says that the input domain should consist of m subdomains $X_1, X_2, \ldots, X_m$, the program should partition the input domain into n subdomains $D_1, D_2, \ldots, D_n$, where n must be greater than or equal to m if the partition prescribed by the specification is compact. The partition created by the program must satisfy the condition that every $D_i = \{d \mid d \in D \text{ and } P_i(d)\}$ be contained completely in some X_j, $X_1 \cup X_2 \cup \ldots \cup X_m = X$, and $D_1 \cup D_2 \cup \ldots \cup D_n = D$. Otherwise, there is a domain fault.

If there is a subdomain D_i created by the program that is contained completely in some X_j prescribed by the specification, then for every input in D_i, the value computed by S_i must be equal to that computed by f_j. Otherwise, there is a computation fault.

It is possible that a program contains both domain and computational faults at the same time. Nevertheless, the same element in the input domain cannot be involved in both kinds of fault. If the program is faulty at a particular input, it is either of domain or computational type, but not both.

Previously published methods of program-fault classification include that of Goodenough and Gerhart [GOGE77], Howden [HOWD76], and White and Cohen [WHCO80]. All three include one more type of fault, called a *subcase fault* or *missing-path fault*, which occurs when the programmer fails to create a subdomain required by the specification [i.e., if $\neg(\forall i)(\exists j)(C_i \subset P_j)$]. Since such a fault also manifests as a computational fault, we chose, for simplicity, not to identify it as a fault of separate type.

In Chapters 2 and 3 we discuss test-case selection methods that are designed particularly for revealing the domain faults. In such methods, the components to be exercised are the boundaries of subdomains embodied by the predicates found in the source code or program specification.

1.4 CLASSIFICATION OF TEST-CASE SELECTION METHODS

It was observed previously that when a program is being test-executed, not all constituent components would be involved. If a faulty component is not involved, the fault will not be reflected in the test result. A necessary condition, therefore, for revealing all faults in a test is to construct the test set in such a way that every contributing component in the program is involved (exercised) in at least one test execution!

What is a *component* in the statement above? It can be defined in many different ways. For example, it can be a statement in the source code, a branch in the

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control-flow diagram, or a predicate in the program specification. The use of different components leads to the development of different test-case selection methods. As shown in Chapters 2 and 3, many test-case selection methods have been developed.

If the component used is to be identified from the source code, the resulting test-case selection method is said to be *code-based*. The most familiar examples of such a method are the statement test, in which the program is to be tested to the extent that every statement in its source code is exercised at least during the test, and the branch test, in which the program is to be tested to the extent that every branch in its control-flow diagram is traversed at least once during the test. There are several others that cannot be explained as simply. All the methods are discussed in detail in Chapter 2.

If the component used is to be identified from the program specification, the resulting test-case selection method is said to be *specification-based*. Examples of the components identifiable from a program specification include predicates, boundaries of input/output variables, and subfunctions. Chapter 3 is devoted to a discussion of such methods.

It is possible that certain components can be identified from either the source code or the program specification. The component defined in the subfunction testing method discussed in Chapter 3 is an example.

Since a component can be also a subdomain consisting of those and only those inputs that cause that component to be exercised during the test, a test-case selection method that implicitly or explicitly requires execution of certain components in the program during the test can also be characterized as being *subdomain-based* [FHLS98]. The test methods and all of their derivatives, discussed in Chapters 2 and 3, are therefore all subdomain-based.

Are there any test-case selection methods that are not subdomain-based? There are at least two: random testing [DUNT84, CHYU94, BOSC03] and operational testing [MUSA93, FHLS98]. The first, although interesting, is not discussed in this book because its value has yet to be widely recognized. The second is important in that it is frequently used in practice. Because it is neither code- nor specification-based, we choose to discuss it in Section 4.2.

1.5 THE COST OF PROGRAM TESTING

Software testing involves the following tasks: (1) test-case selection; (2) test execution; (3) test-result analysis; and if it is debug testing, (4) fault removal and regression testing.

For *test-case selection*, the tester first has to study a program to identify all input variables (parameters) involved. Then, depending on the test-case selection method used, the tester has to analyze the source code or program specification to identify all the components to be exercised during the test. The result is often stated as a condition or predicate, called the *test-case selection criterion*, such that any set of inputs that satisfies the criterion is an acceptable test set. The tester then constructs the test cases by finding a set of assignments to the input variables (parameters) that satisfies the test-case selection criterion. This component of the cost is determined by the complexity of the analysis required and the number of test cases needed to satisfy the criterion.

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A commonly used test-case selection criterion is the statement test: testing the program to the extent that every statement in the source code is exercised at least once during the test. The critics say that this selection criterion is far too simplistic and ineffectual, yet it is still commonly used in practice, partly because the analysis required for test-case selection is relatively simple and can be automated to a great extent.

The process of test-case selection is tedious, time consuming, and error prone. The most obvious way to reduce its cost is through automation. Unfortunately, some parts of that process are difficult to automate. If it is specification based, it requires analysis of text written in a natural language. If test cases satisfying a selection criterion are to be found automatically, it requires computational power close to that of a mechanical theorem prover.

For operational testing, which we discuss in Section 4.2, the cost of test-case selection is minimal if the operational profile (i.e., the probability distribution of inputs to be used in production runs) is known. Even if the operational profile had to be constructed from scratch, the skill needed to do so is much less than that for debug testing. That is one of the reasons that many practitioners prefer operational testing. It should be pointed out, however, that a real operational profile may change in time, and the effort required to validate or to update an existing profile is nontrivial.

Test execution is perhaps the part of the testing process that is most amenable to automation. In addition to the machine time and labor required to run the test, the cost of test execution includes that of writing the test harness (i.e., the additional nondeliverable code needed to produce an executable image of the program).

The cost of *test-result analysis* depends largely on the availability of an oracle. If the correctness of test results has to be deduced from the specification, it may become tedious, time consuming, and error prone. It may also become difficult to describe the correctness of a test result if it consists of a large aggregate of data points, such as a graph of a photographic image. For that reason, the correctness of a program is not always unequivocally definable.

A class of computer programs called *real-time programs* have hard time constraints; that is, they not only have to produce results of correct values but have to produce them within prescribed time limits. It often requires an elaborate test harness to feed the test cases at the right time and to determine if correct results are produced in time. For that reason, a thorough test of a real-time software system is usually done under the control of an environment simulator. As the timing aspect of program execution is not addressed in this work, testing of real-time programs is beyond the scope of this book.

Finally, it should be pointed out that, in practice, the ultimate value of a test method is not determined solely by the number of faults it is able to reveal or the probability that it will reveal at least one fault in its application. This is so because the possible economical consequence of a fault could range from nil to catastrophic, and the value of a program often starts to diminish beyond a certain point in time. A test method is therefore of little value in practice if the faults it is capable of revealing are mostly inconsequential, or if the amount of time it takes to complete the test is excessive.