PART One

The Basics of the Time Value of Money

CHAPTER 1

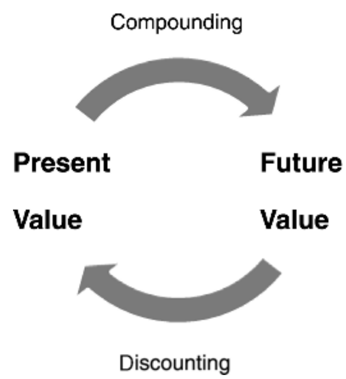
The Value of Compounding

Remember that time is money.
—Benjamin Franklin
Advice to a Young Tradesman (1748)

Most people are familiar with the Seven Wonders of the World: the Great Pyramid of Giza, the Hanging Gardens of Babylon, the Statue of Zeus at Olympia, the Temple of Artemis at Ephesus, the Mausoleum of Maussollos at Halicarnassus, the Colossus of Rhodes, and the Lighthouse of Alexandria. Supposedly, when Baron von Rothschild was asked if he could list the Seven Wonders, he said he could not. However, he did respond by saying that he could name the Eighth Wonder of the World: compound interest. Actually, labeling compound interest as the Eighth Wonder of the World has been attributed to other notable figures: Benjamin Franklin, Bernard Baruch, and Albert Einstein. Regardless of to whom we attribute this label, as you will see in this chapter, the label is appropriate.

One of the most important tools in personal finance and investing is the time value of money. Evaluating financial transactions requires valuing uncertain future cash flows; that is, determining what uncertain cash flows are worth at different points in time. We are often concerned about what a future cash flow or a set of future cash flows are worth today, though there are applications in which we are concerned about the value of a cash flow at a future point in time.

One complication is the *time value of money*: a dollar today is not worth a dollar tomorrow or next year. Another complication is that any amount of money promised in the future is uncertain, some riskier than others.



Moving money through time—that is, finding the equivalent value to money at different points in time—involves translating values from one period to another. Translating money from one period involves interest, which is how the time value of money and risk enter into the process.

Interest is the compensation for the opportunity cost of funds and the uncertainty of repayment of the amount borrowed; that is, it represents both the price of time and the price of risk. The price of time is compensation for the opportunity cost of funds—what someone could have done with the money elsewhere—and the price of risk is compensation for bearing risk. That is, the riskier the investment, the higher the interest rate.

Interest is *compound interest* if interest is paid on both the principal—the amount borrowed—and any accumulated interest. In other words, if you borrow $1,000 today for two years and the interest is 5% compound interest, at the end of two years you must repay the $1,000, plus interest on the $1,000 for two years and interest on the interest. The amount you repay at the end of two years is $1,102.50:

Repayment of principal		$1,000.00
Payment of interest on the principal—first year	5% of $1,000	50.00
Payment of interest on the principal—second year	5% of $1,000	50.00
Payment of interest in the second year on the interest from the first year	5% of $50	2.50
Total amount repaid at the end of the second year		$1,102.50

You can see the accumulation of values in Exhibit 1.1. The $2.50 in the second year is the interest on the first period's interest.

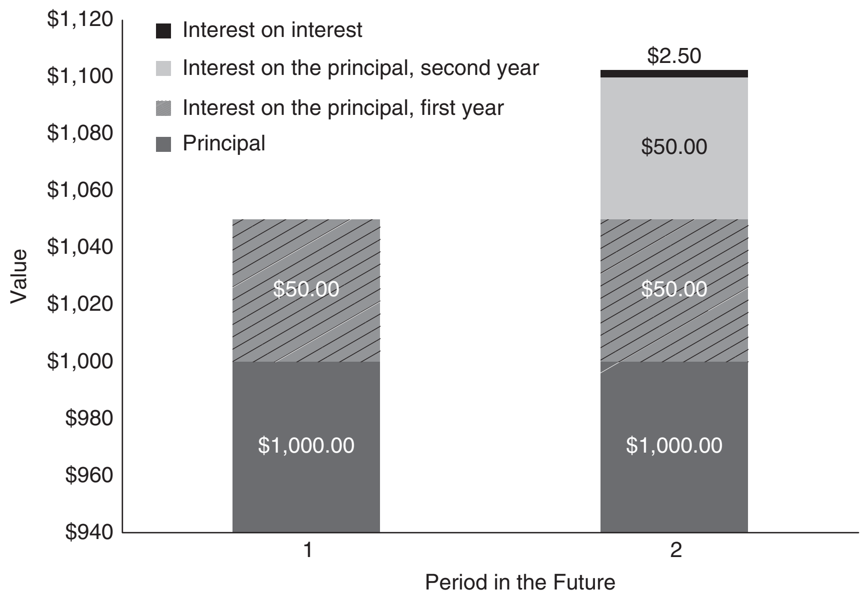


EXHIBIT 1.1 Components of the Future Value of $1,000 Invested at 5% for Two Years

We refer to translating a value today into a value in the future as *compounding*, whereas *discounting* is translating a future value into the present.

The future value is the sum of the present value and interest:

$$\text{Future value} = \text{Present value} + \text{Interest}$$

Most financial transactions involve compound interest, though there are a few consumer transactions that use *simple interest*. Simple interest is the financing arrangement in which the amount repaid is the principal amount and interest on the principal amount. That is, interest is paid only on the principal or amount borrowed. For example, if you borrow $10,000 at 5% simple interest and repay the loan after two years, you must repay the $10,000, plus two periods' interest at 5%:

$$\begin{aligned}\text{Repayment with simple interest} &= \$10{,}000 + [\$10{,}000 \times 2 \times 0.05]\\ &= \$11{,}000\end{aligned}$$

In the case of compound interest, the amount repaid has three components:

1. The amount borrowed
2. The interest on the amount borrowed
3. The interest on interest

The *basic valuation equation* is the foundation of all the financial mathematics that involves compounding, and if you understand this equation, you understand most everything in financial mathematics:

$$FV = PV(1 + i)^n$$

where: FV = the future value
PV = the present value
i = the rate of interest
n = is the number of compounding periods

The term $(1 + i)^n$ is the *compound factor*. When you multiply the value today—the present value—by the compound factor, you get the future value.

We can rearrange the basic valuation equation to solve for the present value, PV:

$$PV = FV\underbrace{\left[\frac{1}{(1+i)^n}\right]}_{\substack{\uparrow \\ \text{Discount} \\ \text{factor}}} = \frac{FV}{(1+i)^n},$$

where $1 \div (1 + i)^n$ is the *discount factor*. When you multiply the value in the future by the discount factor, you get the present value.

In sum,

$$\begin{array}{c}\text{Future}\\\text{value}\end{array} = \begin{array}{c}\text{Present}\\\text{value}\end{array} \times \begin{array}{c}\text{Compound}\\\text{factor}\end{array}$$

$$\begin{array}{c}\text{Present}\\\text{value}\end{array} = \begin{array}{c}\text{Future}\\\text{value}\end{array} \times \begin{array}{c}\text{Discount}\\\text{value}\end{array}$$

The focus of this chapter is on compounding—that is, determining a value in the future. We look at discounting in the next chapter.

OF INTEREST

The word *interest* is from the Latin word *intereo,* which means "to be lost." Interest developed from the concept that lending goods or money results in a loss to the lender because he or she did not have the use of the goods or money that is loaned.

In the English language, the word *usury* is associated with lending at excessive or illegal interest rates. In earlier times, however, usury (from the Latin *usura,* meaning "to use") was the price paid for the use of money or goods.

COMPOUNDING

We begin with compounding because this is the most straightforward way of demonstrating the effects of compound interest. Consider the following example: You invest $1,000 in an account today that pays 6% interest, compounded annually. How much will you have in the account at the end of one year if you make no withdrawals? Using the subscript to indicate the year the future value is associated with, after one year you will have

$$FV_1 = \$1{,}000\,(1 + 0.06) = \$1{,}060$$

After two years, the balance is

$$\begin{aligned} FV_2 &= \$1{,}000\,(1 + 0.06)\,(1 + 0.06) = \$1{,}000\,(1 + 0.06)^2 \\ &= \$1{,}000\,(1.1236) = \$1{,}123.60 \end{aligned}$$

After five years, the balance is

$$FV_5 = \$1{,}000\,(1 + 0.06)^5 = \$1{,}000\,(1.3382) = \$1{,}338.23$$

After 10 years, the balance is

$$FV_{10} = \$1{,}000\,(1 + 0.06)^{10} = \$1{,}000\,(1.7908) = \$1{,}790.85$$

You can see the accumulation of interest from interest on the principal and interest on interest over time in Exhibit 1.2.

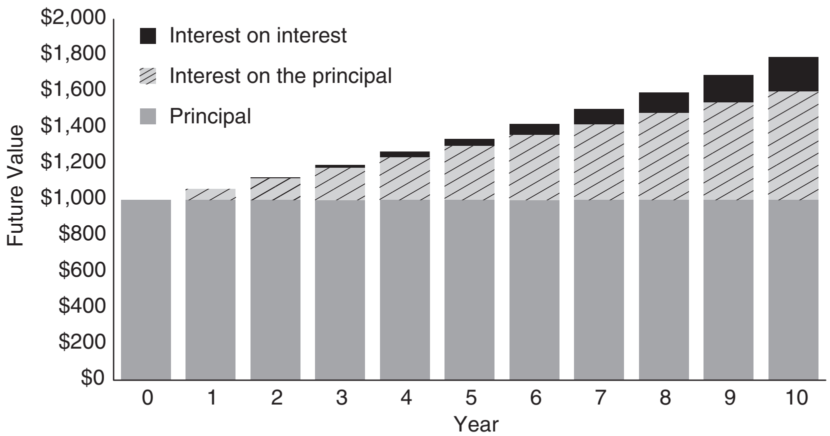


EXHIBIT 1.2 The Accumulation of Interest and Interest on Interest of a $1,000 Deposit with 6% Compound Annual Interest

If you invest $1,000 today and receive $1,790.85 at the end of 10 years, we say that you have a return of 6% on your investment. This return is an average annual return, considering compounding.

TRY IT 1.1: SAVINGS

Suppose you deposit $1,000 in an account that earns 5% interest per year. If you do not make any withdrawals, how much will you have in the account at the end of 20 years?

What if interest was not compounded interest, but rather simple interest? Then we would have a somewhat lower balance in the account after the first year. At the end of one year, with simple interest, you will have:

$$FV_1 = \$1{,}000 + [\$1{,}000\,(0.06)] = \$1{,}060$$

After two years:

$$\begin{aligned} FV_2 &= \$1{,}000 + [\$1{,}000\,(0.06)] + [\$1{,}000\,(0.06)] \\ &= \$1{,}000 + [\$1{,}000\,(0.06)\,(2)] = \$1{,}120 \end{aligned}$$

FINANCIAL MATH IN ACTION

Analysts often come up with estimates of growth in revenues and earnings for publicly traded companies. We can use these estimates to make projections.

Consider the Walt Disney Company. At the end of fiscal year 2008, analysts expected Disney's earnings to grow at a rate of 12.19% per year, in the long-term.* If Disney's earnings for fiscal year 2008 were \$2.2788 per share and if we concur with the analysts, we can estimate the earnings per share for fiscal years into the future. For example, the estimate for the earnings per share for 2009 is

$$\$2.2788\,(1 + 0.1219) = \$2.5566 \text{ per share}$$

The estimate for 2010's earnings is

$$\$2.2788\,(1 + 0.1219)^2 = \$2.8682 \text{ per share}$$

The estimate for 2011's earnings is

$$\$2.2788 \times (1 + 0.1219)^3 = \$3.2179 \text{ per share}$$

*Estimates from Reuters.com/finance, accessed December 25, 2008.

After five years:

$$FV_5 = \$1{,}000 + [\$1{,}000\,(0.06)\,(5)] = \$1{,}300$$

And after 10 years:

$$FV_{10} = \$1{,}000 + [\$1{,}000\,(0.06)\,(10)] = \$1{,}600$$

You can see the difference between compounded and simple interest in Exhibit 1.3, in which we show the growth of \$1,000 at 6% using both types of interest.

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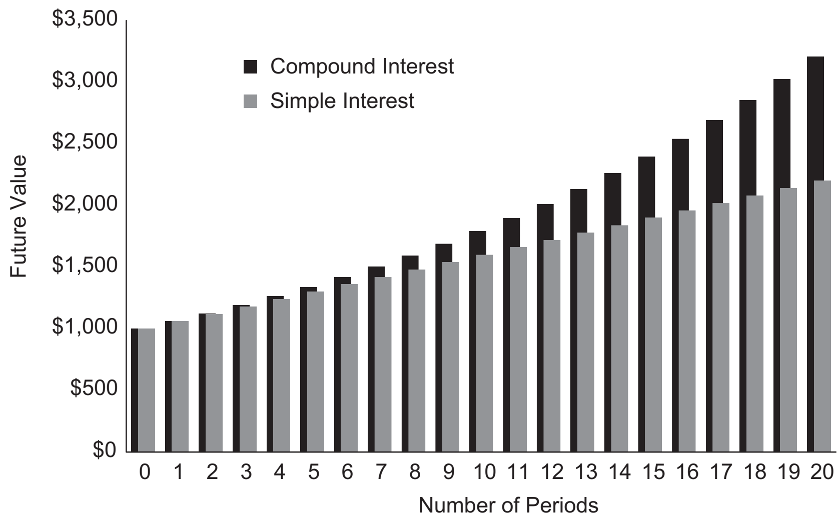

EXHIBIT 1.3 Future Value of $1,000 at a 6% Interest Rate

The difference between the future value with compounded interest and that with simple interest is the interest-on-interest. For example, at the end of 10 years the interest on interest is

Future value with compound interest	$1,790.85
Less future value with simple interest	$1,600.00
Interest on interest	$190.85

Most financial transactions involve compound interest. If the method of calculating interest is not stated, you should assume that the interest is compound interest.

Example 1.1

Suppose that you invest $100,000 today in an investment that produces a return of 5% per year. What will the investment be worth in two years? Answer: **$110,250.**

We calculate the future value at the end of the second year, FV_2, as

$$FV_2 = \$100{,}000\ (1 + 0.05)^2 = \$100{,}000\ (1.1025) = \$110{,}250$$

Example 1.2

Suppose you have a choice between two accounts, Account A and Account B. Account A provides 5% interest, compounded annually and Account B provides 5.25% simple interest. Which account provides the highest balance at the end of four years? Answer: Account A provides the higher balance at the end of four years. Consider a deposit of $10,000 today (though it really doesn't matter what the beginning balance is). What is the difference in the values of the two accounts? Answer: **$55.05.**

$$\text{Account A: } FV_4 = \$10{,}000 \times (1 + 0.05)^4 = \$12{,}155.06$$
$$\text{Account B: } FV_4 = \$10{,}000 + (\$10{,}000 \times 0.0525 \times 4) = \$12{,}100.00$$

The difference, $55.06, is the interest on interest.

TRY IT 1.2: LOAN REPAYMENT

If you borrow $10,000 and the interest on the loan is 8% per year, all payable at the end of the loan, what is the amount that you must repay if the loan is for four years?

CALCULATOR AND SPREADSHEET SOLUTIONS

The calculations are easier with the help of a financial calculator or a spreadsheet program. The calculator's financial functions assume compound interest. If you want to perform a calculation with simple interest, you must rely on the mathematical programs of your calculator.

The future value of $1,000, invested for 10 years at 6%, is $1,790.85, which we can calculate using a financial calculator or a spreadsheet with the following key strokes:

TI-83/84 Using TVM Solver	HP10B	Microsoft Excel
N = 10	1000 +/− PV	=FV(0.06,10,0,−1000)
I% = 6	10 N	
PV = −1000	6 I/YR	
PMT = 0	FV	
FV = *Solve*		

CALCULATION TIP

You will notice that we changed the sign on the PV when we put this information into the calculator. This is because of the way the calculator manufacturers program the financial function: assuming that the present value is the outflow. The changing of the sign for the present value is required in most (but not all) financial calculators and spreadsheets.

In the calculators, PV is the present value, N is the number of compound periods, I% or I/YR is the interest rate per period, and FV is the future value.

In Microsoft Excel®, the future value calculation uses the worksheet function FV:

$$= \text{FV}\,(\text{rate per period, number of periods, periodic payment, present value, type})$$

Where "type" is 0 (indicating cash flows and values occur at the end of the period).[1] Using notation similar to that found on calculators, this command becomes

=FV(i,N,PMT,PV,0)

Because there are no other cash flows in this problem, PMT (which represents periodic cash flows, such as a mortgage payment) is zero. To calculate the FV, the function requires the following inputs:

=FV(.06,10,0,−1000,0)

CALCULATION TIP

In the financial functions of your calculator, the interest rate is represented as a whole number (that is, 6 for 6%), whereas in the math functions of your calculator and in spreadsheet functions, the interest rate is input in decimal form (that is, 0.06 for 6%).

[1]If we leave off the 0, this is assumed to be an end-of-period value.

If we want to use the math functions instead of the financial program of a calculator, you would need to use a power key, such as y^x or ˆ and input the interest in decimal form:

TI-83/84	HP10B	Microsoft Excel
(1+.06)ˆ10 ENTER X1000 ENTER	1+.06= N y^x 10 y^x X 1000 ENTER	=1000*(1.06ˆ10)

WHY CAN'T I CALCULATE THE FUTURE VALUE WITH SIMPLE INTEREST USING MY CALCULATOR FUNCTIONS?

Calculators' time value of money programs are set up to perform calculations involving compound interest. If you want to calculate the future value using simple interest, you must resort to old-fashioned mathematics:

$$\text{Simple interest} = \text{Principal amount} \times \text{interest rate per period} \times \text{number of periods}$$

or

$$\text{Simple interest} = \text{PV}\,in$$

The future value of a lump-sum if interest is computed using simple interest is, therefore

$$\text{FV}_{\text{simple}} = \text{PV} + \text{PV}\,in = \text{PV}\,(1 + in)$$

If the present value is \$1,000 and interest is simple interest at 5% per year, the future value after four periods is

$$\text{FV}_{\text{simple}} = \$1{,}000 + \$1{,}000\,(0.05)\,(4)$$

$$\text{FV}_{\text{simple}} = \$1{,}000\,(1 + 0.2) = \$1{,}200$$

The interest paid on interest in compounding is the difference between the future values with compound and simple interest.

Why not always use the financial functions in your calculator or spreadsheet? Because not every financial math problem fits neatly in the standard program and you may have to resort to the basic financial math.

We provide additional information on using calculators for financial mathematics in Appendix A. We provide additional information on using spreadsheets for financial mathematics in Appendix B.

FREQUENCY OF COMPOUNDING

If interest compounds more frequently than once per year, you need to consider this in any valuation problem involving compounded interest. Consider the following scenario.

You deposit \$1,000 in account at the beginning of the period, and interest is 12% per year, compounded quarterly.

This means that at the end of the first quarter, the account has a balance of

$$\text{FV}_{\text{1st quarter}} = \$1{,}000\left(1+\frac{0.12}{4}\right) = \$1{,}000\ (1+0.03) = \$1{,}030$$

We calculate the quarters' balances in a like manner, with interest paid on the balance in the account:

$$\begin{aligned}
&\text{FV}_{\text{2nd quarter}} = \$1{,}030.00\,(1+0.03) = \$1{,}060.90\\
&\Downarrow\\
&\text{FV}_{\text{3rd quarter}} = \$1{,}060.90\,(1+0.03) = \$1{,}092.73\\
&\Downarrow\\
&\text{FV}_{\text{4th quarter}} = \$1{,}092.73\,(1+0.03) = \$1{,}125.51
\end{aligned}$$

Therefore, at the end of one year, there is a balance of \$1,000 $(1 + 0.03)^4 = \$1{,}125.51$.

We show the growth of the funds in Exhibit 1.4.

When an interest rate is stated in terms of a rate per year, but interest is compounded more frequently than once per year, the stated annual rate is

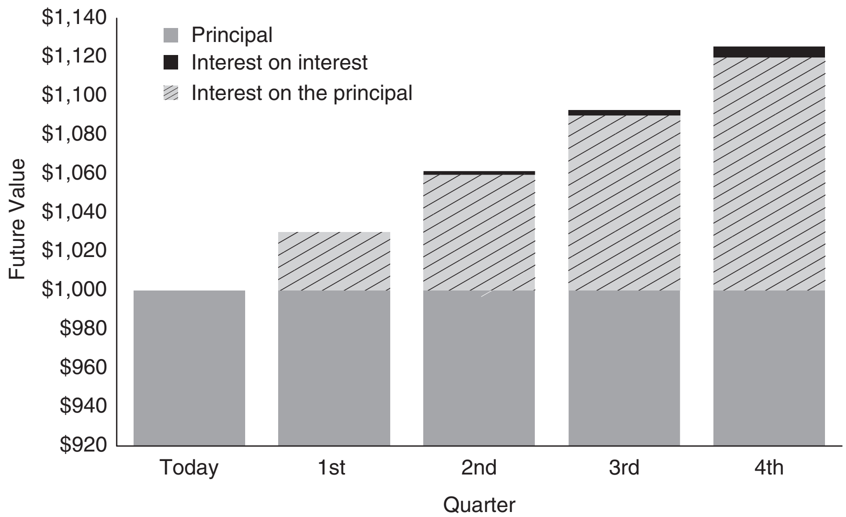

EXHIBIT 1.4 Growth of $1,000 in an Account with 12% Interest per Year, Compounding Quarterly

referred to as the *annual percentage rate* (APR), but the actual calculation requires using the rate per compound period and the number of compound periods. For example, if a loan of $5,000 for three years has an APR of 10% and interest compounds semi-annually, the calculation of the future value at the end of three years uses:

$$\begin{aligned} i &= \text{rate per compound period} \\ &= 10\% \div 2 = 5\% \text{ per six months} \\ n &= \text{number of compounding periods} \\ &= 2 \text{ per year} \times 3 \text{ years} = 6 \text{ periods} \end{aligned}$$

and, therefore,

$$FV_3 = \$5{,}000\ (1 + 0.05)^6 = \$6{,}700.48$$

Notice that this future value is more than if we had ignored compounding of interest within a year, which would have produced a future value of $\$5{,}000\ (1 + 0.10)^3 = \$6{,}655$.

Example 1.3

Suppose you invest $20,000 in an account that pays 12% interest, compounded monthly. How much do you have in the account at the end of 5 years? Answer: **$36,333.93.**

The number of periods is 60:

$$n = 5 \text{ years} \times 12 \text{ months per year} = 60 \text{ months}$$

and the rate per period is 1%:

$$i = \text{Rate per period} = 12\% \div 12 = 1\%$$

Therefore, the future value is $36,333.93. Using the math,

$$\text{FV} = \$20{,}000\,(1 + 0.01)^{60} = \$20{,}000\,(1.8167) = \$36{,}333.93$$

Using the financial calculator or spreadsheet time value of money functions:

TI-83/84	HP10B	Excel
PV −20000 I 1 N 60 FV Solve	20000 +/− PV 1 I/YR 60 N FV	=FV(.01,60,0,−20000,0)

TRY IT 1.3: FREQUENCY

Suppose you have a choice of borrowing $1 million with the following terms, with interest paid at the end of the loan:

10% APR, quarterly interest
10.5% APR, semi-annual interest
11% APR, annual interest

Under which loan terms would you have the largest payment at the end of four years?

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FINANCIAL MATH IN ACTION

Credit card companies allow customers with balances to pay a minimum amount, instead of the full amount each month. What remains unpaid accumulates interest at sometimes quite high interest rates. Suppose you have charged \$1,000 and choose to pay the minimum balance of 2% at the end of each month. And suppose your credit card company charges 29.99% APR interest, with monthly compounding.

How much will you owe after using the strategy of paying the minimum? Interest on unpaid balances is 29.99% ÷ 12 = 2.4992% per month:

Month From Now	Starting Balance	Interest for the Month	Balance Owed	Minimum Payment	Ending Balance
1	\$1,000.00			\$20.00	\$980.00
2	\$980.00	\$24.49	\$1,004.49	\$20.09	\$984.40
3	\$984.40	\$24.60	\$1,009.00	\$20.18	\$988.82
4	\$988.82	\$24.71	\$1,013.54	\$20.27	\$993.27
5	\$993.27	\$24.82	\$1,018.09	\$20.36	\$997.73
6	\$997.73	\$24.93	\$1,022.66	\$20.45	\$1,002.21
7	\$1,002.21	\$25.05	\$1,027.26	\$20.55	\$1,006.71
8	\$1,006.71	\$25.16	\$1,031.87	\$20.64	\$1,011.23
9	\$1,011.23	\$25.27	\$1,036.50	\$20.73	\$1,015.77
10	\$1,015.77	\$25.39	\$1,041.16	\$20.82	\$1,020.34
11	\$1,020.34	\$25.50	\$1,045.84	\$20.92	\$1,024.92
12	\$1,024.92	\$25.61	\$1,050.53	\$21.01	\$1,029.52

In other words, you will end up owing more at the end of the month.

What we have seen so far with respect to compounding is discrete or periodic compounding. However, many financial transactions, including credit card financing, involve *continuous compounding*. This is the extreme of the frequency of compounding, because interest compounds instantaneously. If interest compounds continuously, the compound factor uses the exponential function, e, which is the inverse of the natural

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logarithm.[2] The compound factor for continuous compounding requires the stated rate per year (that is, the APR) and the number of years:

$$e^{(\text{Annual interest rate}) \times (\text{Number of years})} = e^{\text{APR}\, n}$$

If annual interest is 10%, continuously compounded, the compound factor for one year is

$$e^{0.10} = 1.1052$$

For two years, the factor is

$$e^{0.10 \times 2} = e^{0.20} = 1.2214$$

For 10 years, the factor is

$$e^{0.10 \times 10} = e^{1} = 2.7183$$

The formula for the future value of an amount with continuous compounding is:

$$\text{FV} = \text{PV}\left[e^{\text{APR}\, n}\right]$$

The compound factor is $e^{\text{APR}\, n}$.

You can view continuous compounding as the limit of compounding frequency. Consider a $1 million deposited in an account for five years, where this accounts pays 10% interest. With annual compounding, this deposit grows to

$$\text{FV}_{5,\ \text{annual compounding}} = \$1{,}610{,}510$$

With continuous compounding, this deposit grows to

$$\text{FV}_{5,\ \text{continuous compounding}} = \$1{,}648{,}721$$

You can see the difference between annual compounding and continuous compounding in Exhibit 1.5. At the end of 40 years, the difference between continuous compounding and annual compounding is over $9 million.

[2] The "*e*" in the exponential function is also referred to as Euler's *e*, or the base of the natural logarithm. The numerical value of *e* truncated to 10 decimal places is 2.7182818284. We will see more of Euler's *e* in Chapter 4.

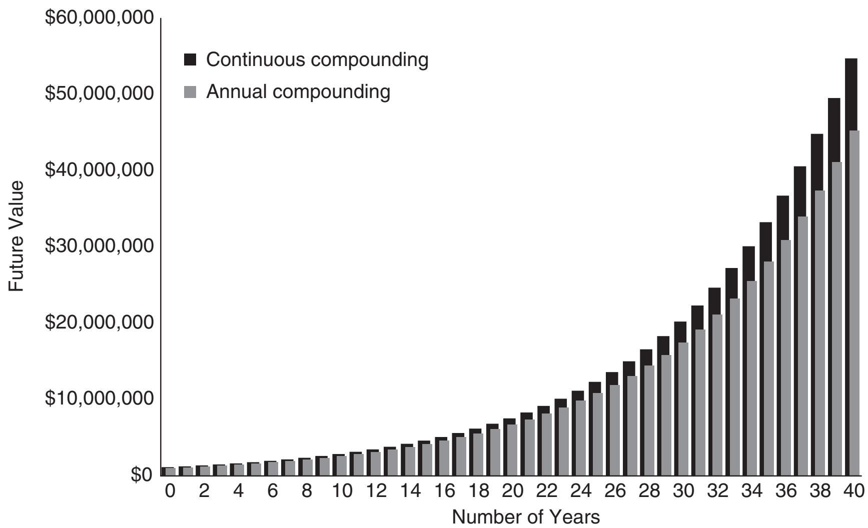

EXHIBIT 1.5 The Value of $1 Million at 10% Interest, Compounded Annually vs. Continuously

Using a calculator, you can find e^{x} in the math functions. For example, suppose you want to calculate the future value of $1,000 invested five years at 4%, with interest compounded continuously. The future value is:

$$FV_5 = \$1{,}000\ e^{0.04\times 5} = \$1{,}000\ e^{0.2} = \$1{,}000\ (1.2214) = \$1{,}221.40$$

Using Microsoft Excel, you use the exponential worksheet function, EXP:

=1,000*EXP(0.04*5)

where the value in parentheses is the exponent.

CALCULATION TIP

The programmers of financial calculator and spreadsheet financial functions set these up for discrete compounding. You will need to use the math functions of the calculator or the spreadsheet to perform calculations for continuous compounding.

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Example 1.4

Suppose you invest \$1,000 today in an account that pays 9% interest, compounded continuously. What will be the value in this account at the end of ten years? Answer: **\$2,459.60.**

The future value is \$2,459.60:

$$\begin{aligned} FV &= \$1{,}000\ e^{0.09\times 10} = \$1{,}000\ e^{0.9} \\ &= \$1{,}000\ (2.4596) = \$2{,}459.60 \end{aligned}$$

Example 1.5

Suppose you invest \$5,000 in an account that earns 10% interest. How much more would you have after 20 years if interest compounds continuously instead of compounded semi-annually? Answer: **\$1,745.34.**

You would have \$1,745.34 more:

$$\begin{aligned} FV_{\text{continuously}} &= \$5{,}000\ e^{0.1\times 20} \\ &= \$5{,}000\ (7.3891) = \$36{,}945.28 \\ FV_{\text{semiannually}} &= \$5{,}000\ (1 + 0.05)^{40} \\ &= \$5{,}000\ (7.0400) = \$35{,}199.94 \\ \text{Difference} &= \$36{,}945.28 - 35{,}199.94 = \$1{,}745.34 \end{aligned}$$

TRY IT 1.4: CONTINUOUS COMPOUNDING

If you borrow \$10,000 and the interest on the loan is 8% per year, compounded continuously, what is the amount that you must repay at the end of four years?

SUMMARY

Understanding how compounding works helps you understand financial transactions, including how investments grow in value over time. The basic valuation equation, $FV = PV\ (1 + i)^n$, is the foundation of all of the financial math that you'll encounter in finance. What we will do in the following chapters is build upon this basic valuation equation, and help you to understand how to translate future values to the present, calculate the yield or return on investments, calculate effective interest rates, amortize a loan, and value stocks and bonds.

Throughout this book, we show you how to use the fundamental math behind the calculations, as well as the financial calculator and spreadsheet functions. While calculators and spreadsheets are very helpful, it is also important to understand the underlying math—because you might just encounter a financial transaction that doesn't fit neatly into one of these functions.

"TRY IT" SOLUTIONS

1.1. Savings

$$\text{Amount on deposit} = \$1{,}000\ (1 + 0.05)^{20} = \$2{,}653.30$$

1.2. Loan Repayment

$$\text{Amount of repayment} = \$10,000(1 + 0.08)^4 = \$13,604.89$$

Using a financial calculator: PV $= -10000$; $i\% = 8$, $n = 4$.Solve for FV

1.3. Frequency of Compounding
The 10.5% with semiannual compounding requires the largest repayment:

$$\begin{aligned}
FV_{\text{10\% APR, quarterly compounding}} &= \$1{,}000{,}000(1 + 0.025)^{16} \\
&= \$1{,}484,505 \\
FV_{\text{10.5\% APR, semiannual compounding}} &= \$1{,}000{,}000(1 + 0.0525)^{8} \\
&= \$1{,}505,833 \\
FV_{\text{10.75\% APR, annual compounding}} &= \$1{,}000{,}000(1 + 0.1075)^{4} \\
&= \$1{,}504{,}440
\end{aligned}$$

1.4. Continuous Compounding

$$\begin{aligned}
\text{Amount of repayment} &= \$10{,}000\, e^{(0.08)(4)} = \$10{,}000\, e^{0.32} \\
&= \$13{,}771.28
\end{aligned}$$

PROBLEMS

1.1. If you invest $10,000 in an account that pays 4% interest, compounded quarterly, how much will be in the account at the end of five years if you make no withdrawals?

1.2. If you invest $2,000 in an account that pays 12% per year, compounded monthly, how much will be in the account at the end of six years if you do not make any withdrawals?

1.3. Suppose you invest $3,000 in an account that pays interest at the rate of 8% per year, compounded semi-annually. How much will you have in the account at the end of five years if you do not make any withdrawals?

1.4. Suppose you invest $100 for 20 years in an account that pays 2% per year, compounded quarterly.

a. How much will you have in the account at the end of 20 years?

b. How much interest on interest will be in the account at the end of 20 years?

1.5. If you deposit $100 in an account that pays 4% interest, compounded annually, what is the balance in the account at the end of three years if you withdraw only the interest on the interest each year?

1.6. Suppose you invest €100 today in an investment that yields 5% per year, compounded annually. How much will you have in the account at the end of six years?

1.7. Which investment of $10,000 will provide the larger value after four years:

a. Investment A earns 5% interest, compounded semiannually.

b. Investment B earns 4.8% interest, compounded continuously.

1.8. What will be the value in an account at the end of 12 years if you deposit $100 today and the account earns 6% interest, compounded annually?

1.9. What will be the value in an account at the end of six years if you deposit $100 today and the account earns 12% interest, compounded annually?

1.10. What will be the value in an account at the end of 10 years if you deposit $1,000 today and the account earns 7% interest, compounded continuously?

For solutions to these problems, see Appendix E.