Chapter 1

Beginning at the Very Beginning: Pre-Pre-Calculus

In This Chapter

- Brushing up on order of operations
- Solving equalities
- Graphing equalities and inequalities
- Finding distance, midpoint, and slope

Pre-calculus is the stepping stone for Calculus. It's the final hurdle after all those years of math: Pre-algebra, Algebra, Geometry, and Algebra II. Now all you need is Pre-calculus to get to that ultimate goal — Calculus. And as you may recall from your Algebra II class, you were subjected to much of the same material you saw in Algebra and even Pre-algebra (just a couple steps up in terms of complexity — but really the same stuff). As the stepping stone, pre-calculus begins with certain concepts that you're expected to solidly understand.

Therefore, we're starting here, at the very beginning, reviewing those concepts. If you feel you're already an expert at everything algebra, feel free to skip past this chapter and get the full swing of pre-calc going. If, however, you need to review, then read on.

TIP

If you don't remember some of the concepts we discuss in this chapter, or even in this book, you can pick up another *For Dummies* math book for review. The fundamentals are important. That's why they're called fundamentals. Take the time now to review — it will save you countless hours of frustration in the future!

COPYRIGHTED MATERIAL

Reviewing Order of Operations: The Fun in Fundamentals

You can't put on your sock after you put on your shoe, can you? The same concept applies to mathematical operations. There's a specific order to which operation you perform first, second, third, and so on. At this point, it should be second nature. However, because the concept is so important as we continue into more complex calculations, we review it here.

Please excuse who? Oh, yeah, you remember this one — my dear Aunt Sally! The old mnemonic still stands, even as you get into more complicated problems. Please Excuse My Dear Aunt Sally is a mnemonic for the acronym PEMDAS, which stands for:

- **P**arentheses (including absolute value, brackets, and radicals)
- **E**xponents
- **M**ultiplication and **D**ivision (from left to right)
- **A**ddition and **S**ubtraction (from left to right)

The order in which you solve algebraic problems is very important. Always work what's in the parentheses first, then move on to the exponents, followed by the multiplication and division (from left to right), and finally, the addition and subtraction (from left to right). Because we're reviewing fundamentals, now is also a good time to do a quick review of properties of equality.

When simplifying expressions, it's helpful to recall the properties of numbers:

- **Reflexive property: $a = a$.** For example, $4 = 4$.
- **Symmetric property: If $a = b$, then $b = a$.** For example, if $2 + 8 = 10$, then $10 = 2 + 8$.
- **Transitive property: If $a = b$ and $b = c$, then $a = c$.** For example, if $2 + 8 = 10$ and $10 = 5 \cdot 2$, then $2 + 8 = 5 \cdot 2$.
- **Commutative property of addition (and of multiplication): $a + b = b + a$.** For example, $3 + 4 = 4 + 3$.
- **Commutative property of multiplication: $a \cdot b = b \cdot a$.** For example, $3 \cdot 4 = 4 \cdot 3$.
- **Associative property of addition (and of multiplication): $a + (b + c) = (a + b) + c$.** For example, $3 + (4 + 5) = (3 + 4) + 5$.
- **Associative property of multiplication: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.** For example, $3 \cdot (4 \cdot 5) = (3 \cdot 4) \cdot 5$.
- **Additive identity: $a + 0 = a$.** For example, $4 + 0 = 4$.
- **Multiplicative identity: $a \cdot 1 = a$.** For example, $-18 \cdot 1 = -18$.
- **Additive inverse property: $a + (-a) = 0$.** For example, $5 + -5 = 0$.
- **Multiplicative inverse property: $a \cdot \frac{1}{a} = 1$.** For example, $-2 \cdot (-\frac{1}{2}) = 1$.
- **Distributive property: $a(b + c) = a \cdot b + a \cdot c$.** For example, $5(3 + 4) = 5 \cdot 3 + 5 \cdot 4$.
- **Multiplicative property of zero: $a \cdot 0 = 0$.** For example. $4 \cdot 0 = 0$.
- **Zero product property: If $a \cdot b = 0$, then $a = 0$ or $b = 0$.** For example, if $x(2x - 3) = 0$, then $x = 0$ or $2x - 3 = 0$.

Q. Simplify: $\dfrac{6^2-4\left(3-\sqrt{20+5}\right)^2}{|4-8|}$.

A. **The answer is 5.** Following our rules of order of operations, simplify everything in parentheses first.

REMEMBER: Radicals and absolute value marks act like parentheses. Therefore, if any of the operations are under radicals or within absolute value marks, do those first before simplifying the radicals or taking the absolute value.

Simplify the parentheses by taking the square root of 25 and the absolute value of –4: $\dfrac{6^2-4\left(3-\sqrt{25}\right)^2}{|-4|} = \dfrac{6^2-4\left(3-5\right)^2}{4} =$

$\dfrac{6^2-4\left(-2\right)^2}{4}$. Now that the parentheses are simplified, you can deal with the exponents.

Square the 6 and the –2: $= \dfrac{36-4\left(4\right)}{4}$.

WARNING! Although they're not written, parentheses are implied around the terms above and below a fraction bar. In other words, the expression $\dfrac{36-4\left(4\right)}{4}$ can also be written as $\dfrac{\left[36-4\left(4\right)\right]}{4}$. Therefore, you must simplify the numerator and denominator before dividing the terms following the order of operations:

$= \dfrac{36-4\left(4\right)}{4} = \dfrac{36-16}{4} = \dfrac{20}{4} = 5.$

Q. Simplify: $\dfrac{\left(\frac{1}{8}+\frac{1}{3}\right)+\frac{3}{8}}{\frac{3}{18}+\frac{1}{9}}$.

A. **The answer is 3.** Using the associative property of addition, rewrite the expression to make the fractions easier to add:

$\dfrac{\left(\frac{1}{8}+\frac{3}{8}\right)+\frac{1}{3}}{\frac{3}{18}+\frac{1}{9}}$. Add the fractions with common denominators, $\dfrac{\frac{4}{8}+\frac{1}{3}}{\frac{3}{18}+\frac{1}{9}}$, and reduce the resulting fraction: $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{18}+\frac{1}{9}}$. Next, find a common denominator for the fractions in the numerator and denominator:

$\dfrac{\frac{3}{6}+\frac{2}{6}}{\frac{3}{18}+\frac{2}{18}}$. Add these: $\dfrac{5/6}{5/18}$. Recognizing that this expression is a division problem,

$\frac{5}{6} \div \frac{5}{18}$, multiply by the inverse and simplify: $\frac{5}{6}\cdot\frac{18}{5} = \frac{5\cdot 18}{6\cdot 5} = \frac{\cancel{5}\cdot 18}{6\cdot\cancel{5}} = \frac{3}{1} = 3.$

1. Simplify: $\dfrac{3\sqrt{(4-6)^2+(2-(-1))^2}}{|-3-(-1)|}$.

Solve It

2. Simplify: $\dfrac{|-3|-|2|+(-1)}{|-7+2|}$.

Solve It

3. Simplify: $(2^3-3^2)^4(-5)$.

Solve It

4. Simplify: $\dfrac{|5\ \ 1-4\ \ +6|}{3\left(-\frac{1}{6}+\frac{1}{3}\right)-\frac{1}{2}}$.

Solve It

Keeping Your Balance While Solving Equalities

Just as simplifying expressions is the basics of pre-algebra, solving for variables is the basics of algebra. Both are essential to more complex concepts in pre-calculus. Solving basic algebraic equations should be easy for you; however, it's so fundamental to pre-calculus, we give you a brief review here.

Solving linear equations with the general format of $ax + b = c$, where a, b, and c are constants, is relatively easy using properties of numbers. The goal, of course, is to isolate the variable, x.

REMEMBER

One type of equation you can't forget is absolute value equations. The *absolute value* is defined as the distance from 0. In other words, $|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$. As such, an absolute value has two possible solutions: one where the quantity inside the absolute value bars is positive and another where it's negative. To solve these equations, it's important to isolate the absolute value term and then set the quantity to the positive and negative values.

EXAMPLE

Q. Solve for x: $3(2x - 4) = x - 2(-2x + 3)$.

A. $\boldsymbol{x = 6}$**.** First, using the distributive property, distribute the 3 and the –2: $6x - 12 = x + 4x - 6$. Combine like terms and solve using algebra: $6x - 12 = 5x - 6$; $x - 12 = -6$; $x = 6$.

Q. Solve for x: $\left|x-3\right| + \left(-16\right) = -12$.

A. $\boldsymbol{x = 7, -1}$**.** First, isolate the absolute value: $\left|x-3\right| = 4$. Next, set the quantity inside the absolute value bars to the positive solution: $x - 3 = 4$. Then, set the quantity inside the absolute value bars to the negative solution: $-(x - 3) = 4$. Solve both equations to find two possible solutions: $x - 3 = 4$, $x = 7$; and $-(x - 3) = 4$, $x - 3 = -4$, $x = -1$.

5. Solve: $3 - 6[2 - 4x(x + 3)] = 3x(8x + 12) + 27$.

Solve It

6. Solve $\frac{x}{2} + \frac{x-2}{4} = \frac{x+4}{2}$.

Solve It

7. Solve $\left|x-3\right| + \left|3x+2\right| = 4$.

Solve It

8. Solve $3 - 4(2 - 3x) = 2(6x + 2)$.

Solve It

9. Solve $2|x-3|+12=6$.

Solve It

10. Solve $3(2x+5)+10=2(x+10)+4x+5$.

Solve It

A Picture Is Worth a Thousand Words: Graphing Equalities and Inequalities

Graphs are visual representations of mathematical equations. In pre-calculus, you'll be introduced to many new mathematical equations and then be expected to graph them. We give you lots of practice graphing these equations when we cover the more complex equations. In the meantime, it's important to practice the basics: graphing linear equalities and inequalities.

These graphs are graphed on the *Cartesian coordinate system*. This system is made up of two axes: the horizontal, or x-axis, and the vertical, or y-axis. Each point on the coordinate plane is called a *Cartesian coordinate pair* (x, y). A set of these ordered pairs that can be graphed on a coordinate plane is called a *relation*. The x values of a relation are its *domain*, and the y values are its *range*. For example, the domain of the relation R={(2, 4), (–5, 3), (1, –2)} is {2, –5, 1}, and the range is {4, 3, –2}.

You can graph a linear equation in two ways: *plug and chug* or use *slope-intercept form:* $y = mx + b$. At this point in math, you should definitely know how to use the slope-intercept form, but we give you a quick review of the plug and chug method, because as the equations become more complex, you can use this old standby method to get some key pieces of information.

Graphing using the plug and chug method

Start by picking domain (x) values. Plug them into the equation to solve for the range (y) values. For linear equations, after you plot these points (x, y) on the coordinate plane, you can connect the dots to make a line. The process also works if you choose range values first, then plug in to find the corresponding domain values. This is a helpful method to find *intercepts,* the points that fall on the x or y axes. To find the x-intercept (x, 0), plug in 0 for y and solve for x. To find the y-intercept (0, y), plug in 0 for x and solve for y. For example, to find the intercepts of the linear equation $2x + 3y = 12$, start by plugging in 0 for y: $2x + 3(0) = 12$. Then, using properties of numbers, solve for x: $2x + 0 = 12$, $2x = 12$, $x = 6$. So the x-intercept is (6, 0). For the y-intercept, plug in 0 for x and solve for y: $2(0) + 3y = 12$, $0 + 3y = 12$, $3y = 12$, $y = 4$. Therefore, the y-intercept is (0, 4). At this point, you can plot those two points and connect them to graph the line ($2x + 3y = 12$), because, as you learned in geometry, two points make a line. See the resulting graph in Figure 1-1.

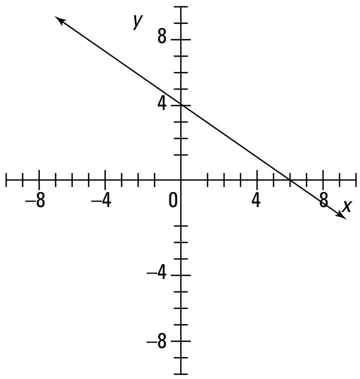


Figure 1-1: Graph of $2x + 3y = 12$.

Graphing using the slope-intercept form

The slope-intercept form of a linear equation gives a great deal of helpful information in a cute little package. The equation $y = mx + b$ immediately gives you the y-intercept (b) that you worked to find in the plug and chug method; it also gives you the slope (m). Slope is a fraction that gives you the rise over the run. To change equations that aren't written in slope-intercept form, you simply solve for y. For example, if you use the same linear equation as before, $2x + 3y = 12$, you start by subtracting $2x$ from each side: $3y = -2x + 12$. Next, you divide all the terms by 3: $y = -\frac{2}{3}x + 4$. Now that the equation is in slope-intercept form, you know that the y-intercept is 4. You can graph this point on the coordinate plane. Then, you can use the slope to plot the second point. From the slope-intercept equation, you know that the slope is $-\frac{2}{3}$. This tells you that the rise is –2 and the run is 3. From the point (0, 4), plot the point 2 down and 3 to the right. In other words, (3, 2). Lastly, connect the two points to graph the line. Note that this is the exact same graph, just plotted a different way — the resulting graph in Figure 1-2 is identical to Figure 1-1.

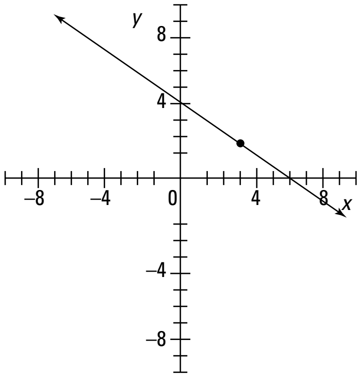
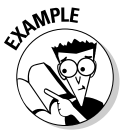


Figure 1-2: Graph of $y = -\frac{2}{3}x + 4$.

Similar to graphing equalities, graphing inequalities begins with plotting two points by either method. However, because *inequalities* are used for comparisons — greater than, less than, or equal to — you have two more questions to answer after two points are found:

- Is the line *dashed*: < or > or *solid*: ≤ or ≥?
- Do you shade under the line: $y <$ or $y \leq$ or above the line: $y >$ or $y \geq$?



Q. Sketch the graph of the inequality: $3x - 2y > 4$.

A. Begin by putting the equation into slope-intercept form. To do this, subtract $3x$ from each side of the equation: $-2y > -3x + 4$. Then divide each term by –2: $y < \frac{3}{2}x - 2$.

Remember that when you multiply or divide an inequality by a negative, you need to reverse the inequality.

From the resulting equation, you can find the *y*-intercept, –2, and the slope, ($\frac{3}{2}$). Using this information, you can graph two points using the slope-intercept form method. Next, you need to decide the nature of the line (solid or dashed). Because the inequality is not also an equality, the line is dashed. Graph the dashed line, and then you can decide where to shade. Because the inequality is less than, shade below the dashed line, as you see in Figure 1-3.

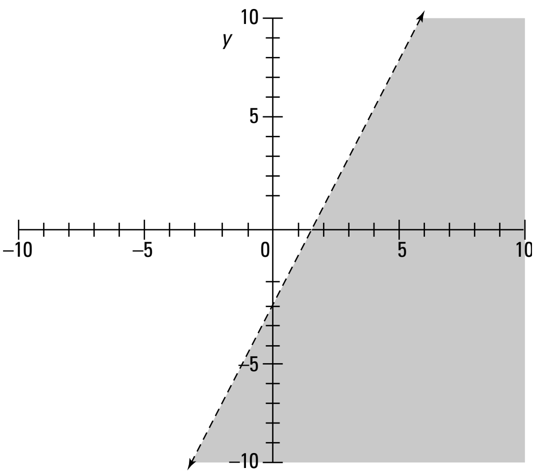


Figure 1-3: Graph of $3x - 2y > 4$.

11. Sketch the graph of $\frac{4}{3}\left(6x+2y\right)=16$.

Solve It

12. Sketch the graph of $\frac{5x+4y}{2}\geq 6$.

Solve It

13. Sketch the graph of $4x+5y\geq 2\left(3y+2x+4\right)$.

Solve It

14. Sketch the graph of $x-3y=4-2y-y$.

Solve It

Using Graphs to Find Information (Distance, Midpoint, Slope)

Graphs are more than just pretty pictures. From a graph, it's possible to determine two points. From these points, you can determine the distance between them, the midpoint of the segment connecting them, and the slope of the line connecting them. As graphs become more complex in both pre-calculus and calculus, you'll be asked to find and use all three of these pieces of information. Aren't you lucky?

Finding the distance

Distance is how far two things are apart. In this case, you're finding the distance between two points. Knowing how to calculate distance is helpful for when you get to conics (Chapter 12). To find the distance between two points (x_1, y_1) and (x_2, y_2), you can use the following formula:

$$d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$$

Calculating the midpoint

The *midpoint,* as you would think, is the middle of a segment. This concept also comes up in conics (Chapter 12) and is ever so useful for all sorts of other pre-calculus calculations. To find the midpoint of those same two points (x_1, y_1) and (x_2, y_2), you just need to average the x and y values and express them as an ordered pair:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Discovering the slope

Slope is a key concept for linear equations, but it also has applications for trigonometric functions and is essential for differential calculus. *Slope* describes the steepness of a line on the coordinate plane (think of a ski slope). To find the slope of two points (x_1, y_1) and (x_2, y_2), you can use the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Positive slopes move up and to the right $\left(\frac{+}{+}\right)$ or down and to the left $\left(\frac{-}{-}\right)$. Negative slopes move down and to the right $\left(\frac{-}{+}\right)$ or up and to the left $\left(\frac{+}{-}\right)$. Horizontal lines have a slope of 0, and vertical lines have an undefined slope.

Q. Find the distance, slope, and midpoint of $\overline{AB}$ in Figure 1-4.

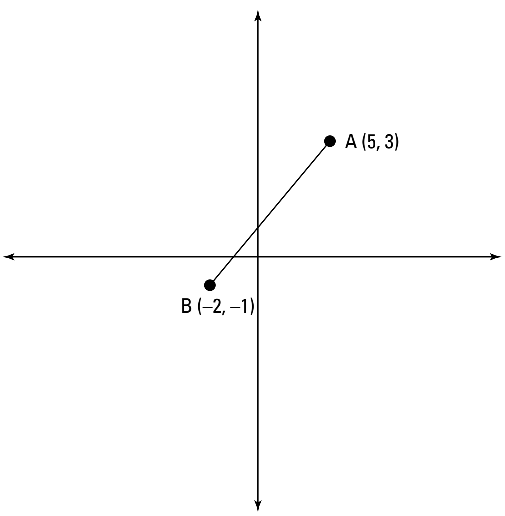


Figure 1-4: Segment AB.

A. **The distance is $\sqrt{65}$, the slope is $\frac{4}{7}$, and the midpoint is M = ($\frac{3}{2}$, 1).** First, plug the *x* and *y* values into the distance formula. Then, following the order of operations, simplify the terms under the radical. (Keep in mind those implied parentheses of the radical itself.) It should look something like this:

$$d = \sqrt{\left(5-(-2)\right)^2+\left(3-(-1)\right)^2} = \sqrt{(5+2)^2+(3+1)^2} = \sqrt{(7)^2+(4)^2} = \sqrt{49+16} = \sqrt{65}$$

Because 65 doesn't contain any perfect squares as factors, this is as simple as you can get.

To find the midpoint, plug the points into the midpoint equation. Again, simplify using order of operations.

$$M = \left(\frac{5+(-2)}{2}, \frac{3+(-1)}{2}\right) = \left(\frac{3}{2}, \frac{2}{2}\right) = \left(\frac{3}{2}, 1\right)$$

To find the slope, use the formula and plug in your *x* and *y* values. Using order of operations, simplify:

$$m = \frac{-1-3}{-2-5} = \frac{-4}{-7} = \frac{4}{7}$$

---

15. Find the distance of segment CD, where C is (–2, 4) and D is (3, –1).

Solve It

16. Find the midpoint of segment EF, where E is (3, –5) and F is (7, 5).

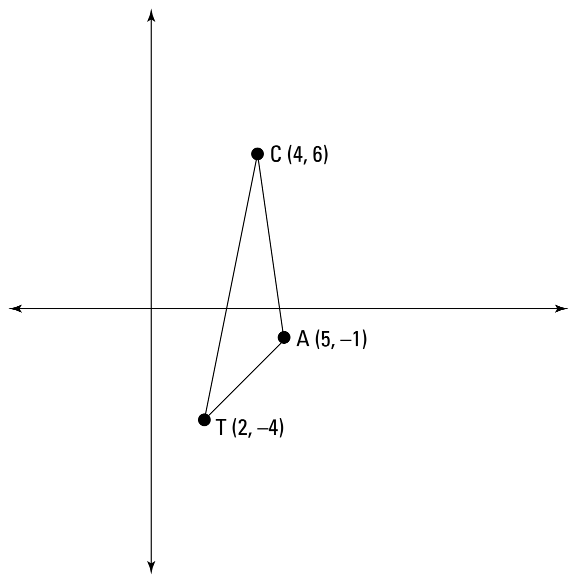
Solve It

17. Find the slope of line GH, where G is (–3, –5) and H is (–3, 4).

Solve It

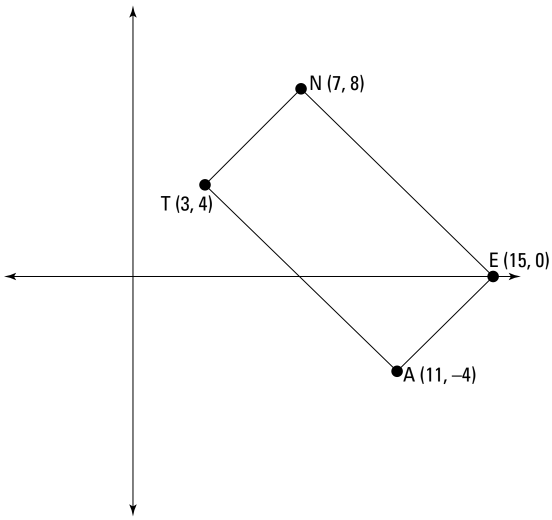
18. Find the perimeter of triangle CAT.

Solve It



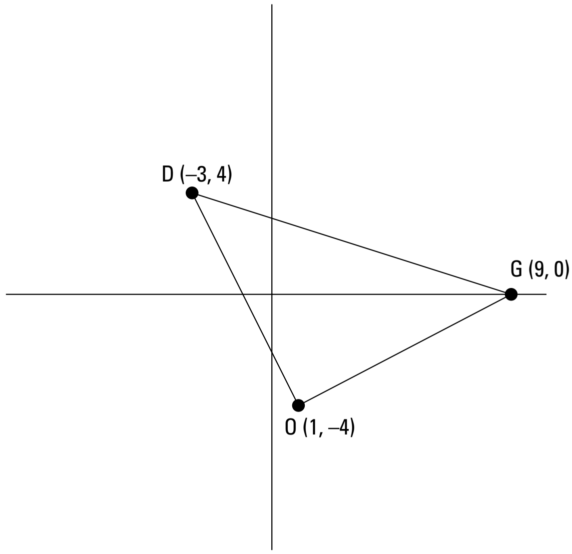
19. Find the center of the rectangle NEAT.

Solve It



20. Determine whether triangle DOG is a right triangle.

Solve It



Answers to Problems on Fundamentals

1 Simplify $\frac{3\sqrt{(4-6)^2+(2-(-1))^2}}{|-3-(-1)|}$. **The answer is $\frac{3\sqrt{13}}{2}$.**

Start by simplifying everything in the parentheses. Next, simplify the exponents. Finally, add the remaining terms. It should look something like this:

$$\frac{3\sqrt{(4-6)^2+(2-(-1))^2}}{|-3-(-1)|}=\frac{3\sqrt{(-2)^2+(2+1)^2}}{|-3+1|}=\frac{3\sqrt{4+(3)^2}}{|-2|}=\frac{3\sqrt{4+9}}{2}=\frac{3\sqrt{13}}{2}$$

2 Simplify $\frac{|-3|-|2|+(-1)}{|-7+2|}$. **The answer is 0.**

Recognizing that the absolute value in the denominator acts as parentheses, add the –7 and 2 inside there first. Then, you can rewrite the absolute value of each. Next, add the terms in the numerator. Finally, recognize that $\frac{0}{5}$ equals zero.

$$\frac{|-3|-|2|+(-1)}{|-7+2|}=\frac{|-3|-|2|+(-1)}{|-5|}=\frac{3-2+(-1)}{5}=\frac{0}{5}=0$$

3 Simplify $(2^3-3^2)^4(-5)$. **The answer is –5.**

Begin by simplifying the exponents in the parentheses. Next, simplify the parentheses by subtracting 9 from 8. Then, simplify the –1 to the 4th power. Finally, multiply the resulting 1 by –5.

$(2^3-3^2)^4(-5)=(8-9)^4(-5)=(-1)^4(-5)=1(-5)=-5$

4 Simplify $\frac{|5\ \ 1-4\ \ +6|}{3\left(-\frac{1}{6}+\frac{1}{3}\right)-\frac{1}{2}}$. **The answer is undefined.**

Start by simplifying the parentheses. To do this, subtract 4 from 1 in the numerator and find a common denominator for the fractions in the denominator in order to add them. Next, multiply the terms in the numerator and denominator. Then, add the terms in the absolute value bars in the numerator and subtract the terms in the denominator. Take the absolute value of –9 to simplify the numerator. Finally, remember that you can't have 0 in the denominator; therefore, the resulting fraction % is undefined.

$$\frac{|5\ \ 1-4\ \ +6|}{3\left(-\frac{1}{6}+\frac{1}{3}\right)-\frac{1}{2}}=\frac{|5(-3)+6|}{3\left(\frac{1}{6}\right)-\frac{1}{2}}=\frac{|-15+6|}{\frac{1}{2}-\frac{1}{2}}=\frac{|-9|}{0}=\text{Undefined}$$

5 Solve $3-6[2-4x(x+3)]=3x(8x+12)+27$. **The answer is $x = 1$.**

Lots of parentheses in this one! Get rid of them by distributing terms. Start by distributing the $-4x$ on the left side over $(x+3)$ and, on the right side, $3x$ over $(8x+12)$. This gives you $3-6[2-4x^2-12x]=24x^2+36x+27$. Then distribute the –6 over the remaining parentheses on the left side of the equation: $3-12+24x^2+72x=24x^2+36x+27$. Combine like terms on the left side: $-9+24x^2+72x=24x^2+36x+27$. To isolate x onto one side, subtract $24x^2$ from each side to get $-9+72x=36x+27$. Subtracting $36x$ from each side gives you $-9+36x=27$. Adding 9 to both sides results in $36x=36$. Finally, dividing both sides by 36 leaves you with your solution: $x=1$.

6 Solve $\frac{x}{2}+\frac{x-2}{4}=\frac{x+4}{2}$. **The answer is x = 10.**

Don't let those fractions intimidate you! Start by multiplying through by the common denominator, 4. This eliminates the fractions altogether. Now, just solve like normal, combining like terms, and isolating x. It should look something like this:

$\frac{x}{2}+\frac{x-2}{4}=\frac{x+4}{2}$; $4\left[\frac{x}{2}+\frac{x-2}{4}=\frac{x+4}{2}\right]$; $2x + x - 2 = 2x + 8$; $3x - 2 = 2x + 8$; $3x = 2x + 10$; $x = 10$

7 Solve $|x-3|+|3x+2|=4$. **The answer is $x = -\frac{3}{4}, -\frac{1}{2}$.**

Okay, this one is really tricky! Two absolute value terms, oh my! Relax. Just remember that absolute value means distance from 0, so you have to consider all the possibilities to solve this problem. In other words, you have to consider and try four different possibilities: both absolute values are positive, both are negative, the first is positive and the second is negative, and the first is negative and the second is positive.

TIP: Not all these possibilities are going to work. As you calculate these possibilities, you may create what math people call *extraneous solutions*. These aren't solutions at all — they're false solutions that don't work in the original equation. You create extraneous solutions when you change the format of an equation, as you're going to do here. So to be sure a solution is real and not extraneous, you need to plug your answer into the original equation to check.

Now, try each of the possibilities:

Positive/positive: $(x - 3) + (3x + 2) = 4$, $4x - 1 = 4$, $4x = 5$, $x = \frac{5}{4}$. Plugging this back into the original equation, you get $\frac{30}{4} = 4$. Nope! You have an extraneous solution.

Negative/negative: $-(x - 3) + -(3x + 2) = 4$, $-x + 3 - 3x - 2 = 4$, $-4x + 1 = 4$, $-4x = 3$, $x = -\frac{3}{4}$. Plug it back into the original equation and you get $4 = 4$. *Voilà!* Your first solution.

Positive/negative: $(x - 3) + -(3x + 2) = 4$, $x - 3 - 3x - 2 = 4$, $-2x - 5 = 4$, $-2x = 9$, $x = -\frac{9}{2}$. Put it back into the original equation and you get $12 = 4$. Nope, again — another extraneous solution.

Negative/positive: $-(x - 3) + (3x + 2) = 4$, $-x + 3 + 3x + 2 = 4$, $2x + 5 = 4$, $2x = -1$, $x = -\frac{1}{2}$. Into the original equation it goes, and you get $4 = 4$. Your second solution.

8 Solve $3 - 4(2 - 3x) = 2(6x + 2)$. **The answer is no solution.**

To solve, distribute over the parentheses on each side: $3 - 8 + 12x = 12x + 4$. Combine like terms: $-5 + 12x = 12x + 4$. Subtract $12x$ from each side and you get $-5 = 4$, which is false. So there is no solution.

9 Solve $2|x-3|+12=6$. **The answer is no solution.**

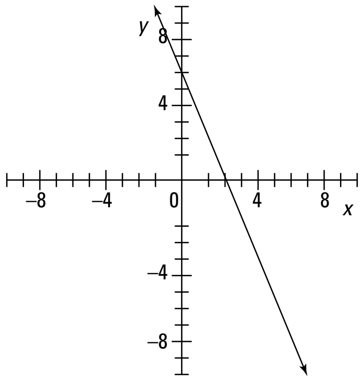
Start by isolating the absolute value: $2|x-3|+12=6$, $2|x-3|=-6$, $|x-3|=-3$. Because an absolute value must be positive, there is no solution that would satisfy this equation.

10 Solve $3(2x + 5) + 10 = 2(x + 10) + 4x + 5$. **The answer is all real numbers.**

Begin by distributing over the parentheses on each side: $3(2x + 5) + 10 = 2(x + 10) + 4x + 5$, $6x + 15 + 10 = 2x + 20 + 4x + 5$. Next, combine like terms on each side: $6x + 25 = 6x + 25$. Subtracting $6x$ from each side gives you $25 = 25$. This is a true statement, indicating that all real numbers would satisfy this equation.

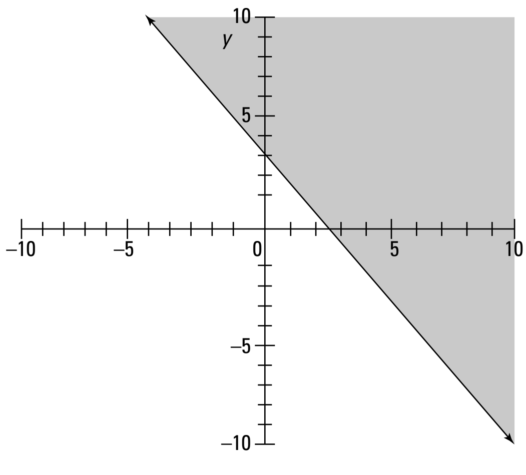
11 Sketch the graph of $\frac{4}{3}(6x+2y)=16$. **See the graph for the answer.**

Using slope-intercept form, you start by multiplying both sides of the equation by the inverse of $\frac{4}{3}$, which is $\frac{3}{4}$: $\frac{3}{4}\cdot\frac{4}{3}(6x+2y)=\frac{3}{4}\cdot 16$. This leaves you with $6x + 2y = 12$. Next, solve for y by subtracting $6x$ from each side and dividing by 2: $2y = -6x + 12$, $y = -3x + 6$. Now, because it's in slope-intercept form, you can identify the slope (-3) and y intercept (6). Use these to graph the equation. Start at the y intercept (0, 6) and move down 3 units and to the right 1 unit. Connect the two points to graph the line.



12 Sketch the graph of $\frac{5x+4y}{2} \geq 6$. **See the graph for the answer.**

Start by multiplying both sides of the equation by 2: $5x + 4y \geq 12$. Next, isolate y by subtracting $5x$ from each side and dividing by 4: $4y \geq -5x + 12$, $y \geq -\frac{5}{4}x + 3$. Now that it's in slope-intercept form, you can graph the inequality. Because it's greater than or equal to, draw a solid line and shade above the line.

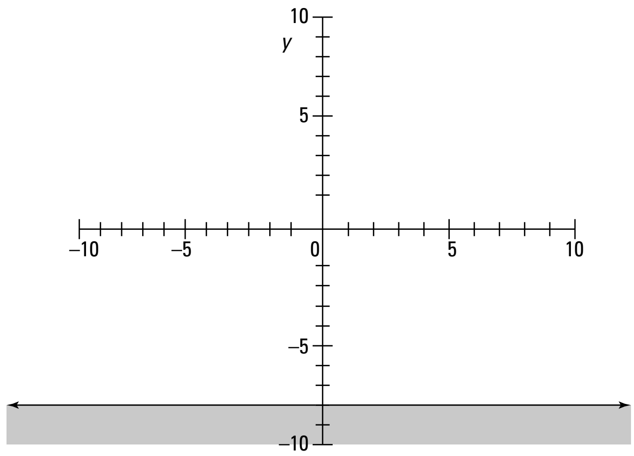


13 Sketch the graph of $4x + 5y \geq 2(3y + 2x + 4)$. **See the graph for the answer.**

Again, start by getting the equation into slope-intercept form. To do this, distribute the 2 on the left side. Next, isolate y by subtracting $4x$ from each side, subtracting y from each side, and then dividing by –1 (don't forget to switch your inequality sign!):

$4x + 5y \geq 2(3y + 2x + 4)$; $4x + 5y \geq 6y + 4x + 8$; $5y \geq 6y + 8$; $-y \geq 8$; $y \leq -8$

Because there's no x term, this indicates that the slope is 0 ($0x$). Therefore, the resulting line is a horizontal line at –8. Because the inequality is less than, you shade below the line.

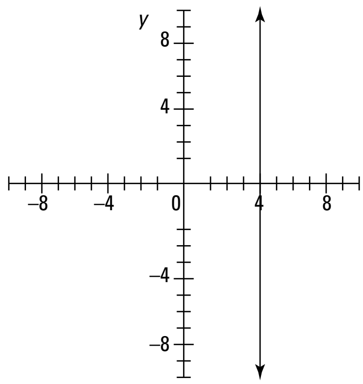


14 Sketch the graph of $x - 3y = 4 - 2y - y$. **See the graph for the answer.**

Again, simplify to put in slope-intercept form. Combine like terms and add $3y$ to each side.

$x - 3y = 4 - 2y - y$; $x - 3y = 4 - 3y$; $x = 4$

Here, the resulting line is a vertical line at 4.



15 Find the distance of segment CD, where C is (–2, 4) and D is (3, –1). **The answer is $d = 5\sqrt{2}$.**

Using the distance formula, plug in the x and y values: $d = \sqrt{(-2-3)^2 + [4-(-1)]^2}$. Then, simplify using order of operations: $d = \sqrt{(-5)^2 + (5)^2}$, $d = \sqrt{25+25}$, $d = \sqrt{50}$, $d = 5\sqrt{2}$.

16 Find the midpoint of segment EF, where E is (3, –5) and F is (7, 5). **The answer is *M* = (5, 0).**

Using the midpoint formula, you get $M = \left(\frac{3+7}{2}, \frac{-5+5}{2}\right)$. Simplify from there: $M = (10/2, 0/2)$, $M = (5, 0)$.

17 Find the slope of line GH, where G is (–3, –5) and H is (–3, 4). **The answer is *m* = undefined.**

Using the formula for slope, plug in the *x* and *y* values for the two points: $m = \frac{-5-4}{-3-(-3)}$. This simplifies to $m = \frac{-9}{0}$, which is undefined.

18 Find the perimeter of triangle CAT. **The answer is $8\sqrt{2} + 2\sqrt{26}$.**

To find the perimeter, you need to calculate the distance on each side, which means you have to find CA, AT, and TC. Plugging the values of *x* and *y* for each point into the distance formula, you find that the distances are as follows: CA = $5\sqrt{2}$, AT = $3\sqrt{2}$, and TC = $2\sqrt{26}$. Adding like terms gives you the perimeter of $8\sqrt{2} + 2\sqrt{26}$.

19 Find the center of the rectangle NEAT. **The answer is (9, 2).**

Ah! Think we're being tricky here? Well, the trick is to realize that if you find the midpoint of one of the rectangle's diagonals, you will have identified the center of it. Easy, huh? So, by using the diagonal NA, you can find the midpoint and thus the center: $M = \left(\frac{7+11}{2}, \frac{8+(-4)}{2}\right)$. This simplifies to $m = (9, 2)$.

20 Determine whether triangle DOG is a right triangle. **The answer is yes.**

We had to end it with another challenging one. Here you need to remember that right triangles have one set of perpendicular lines (forming that right angle). Also, you need to remember that perpendicular lines have negative reciprocal slopes. In other words, if you multiply their slopes together, you get –1. So, all you have to do to answer this question is find the slopes of the lines that appear to be perpendicular, and if they multiply to equal –1, then you know you have a right triangle. Okay? Then let's go!

Start by finding the slope of DO: $m = \frac{4-(-4)}{-3-1}$, $m = -8/4$, $m = -\frac{2}{1}$, or –2. Next, find the slope of OG: $m = \frac{-4-0}{1-9}$, $m = \frac{-4}{-8}$, $m = 1/2$. Multiplying the two slopes together, you find that, indeed, it does equal –1, indicating that you have perpendicular lines: $(-2)(1/2) = -1$. Therefore, triangle DOG is a right triangle.