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The Most Valuable Chapter You Will Ever Read

Are service contracts for electronics and appliances just a scam?

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How likely are you to win at roulette?

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Is it worth going to college?

What constitutes value? On a philosophical level, I'm not sure; what's valuable for one person may not be for others. The most philosophically valuable thing I've ever learned is that bad times are always followed by good times and vice versa, but that may simply be a lesson specific to yours truly. On the other hand, if this lesson helps you, that's value added to this chapter. And if this chapter helps you financially, even better—because there is one universal common denominator of value that everyone accepts: money.

That's why this chapter is valuable, because I'm going to discuss a few basic concepts that will be worth tens of thousands—maybe even hundreds of thousands—of dollars to you. So let's get started.

Service Contracts: This Is Worth Thousands of Dollars

A penny saved is *still* a penny earned, but nowadays you can't even slip a penny into a parking meter—so let me make this book a worthwhile investment by saving you a few thousand dollars. The next time you go to buy an appliance and the salesperson offers you a service contract, *don't even consider purchasing it.* A simple table and a little sixth-grade math should convince you.

Suppose you are interested in buying a refrigerator. A basic model costs in the vicinity of $400, and you'll be offered the opportunity to buy a service contract for around $100. If anything happens to the refrigerator during the first three years, the store will send a repairman to your apartment to fix it. The salesperson will try to convince you that it's cheap insurance in case anything goes wrong, but it's not. Let's figure out why. Here is a table of how frequently various appliances need to be repaired. I found this table by typing "refrigerator repair rates" into a search engine; it's the 2006 product reliability survey from Consumer Reports National Research Center.[1] It's very easy to read: the top line tells you that 43 percent of laptop computers need to be repaired in the first three years after they are purchased.

Repair Rates for Products Three to Four Years Old

Product	*Repair Rate (Percentage of Products Needing Repair)*
Laptop computer	43
Refrigerator: side-by-side, with icemaker and dispenser	37
Rider mower	32
Lawn tractor	31
Desktop computer	31
Washing machine (front-loading)	29
Self-propelled mower	28
Vacuum cleaner (canister)	23
Washing machine (top-loading)	22
Dishwasher	21
Refrigerator: top-and-bottom freezer, with icemaker	20
Range (gas)	20
Wall oven (electric)	19
Push mower (gas)	18
Cooktop (gas)	17
Microwave oven (over-the-range)	17
Clothes dryer	15
Camcorder (digital)	13
Vacuum cleaner (upright)	13
Refrigerator: top-and-bottom freezer, no icemaker	12
Range (electric)	11
Cooktop (electric)	11
Digital camera	10
TV: 30- to 36-inch direct view	8
TV: 25- to 27-inch direct view	6

Use this chart, do some sixth-grade arithmetic, and you can save thousands of dollars during the course of a lifetime. For instance, with the refrigerator service contract, a refrigerator with a top-and-bottom freezer and no icemaker needs to be repaired in the first three years approximately 12 percent of the time; that's about one time in eight. So if you were to buy eight refrigerators and eight service contracts, the cost of the service contracts would be $8 \times \$100 = \$800$. Yet you'd need to make only a single repair call, on average, which would cost you \$200. So, if you had to buy eight refrigerators, you'd save $\$800 - \$200 = \$600$ by *not* buying the service contracts: an average saving of $\$600/8 = \$75$ per refrigerator. Admittedly, you're not going to buy eight refrigerators—at least, not all at once. Even if you buy fewer than eight refrigerators over the course of a lifetime, you'll probably buy a hundred or so items listed in the table. Play the averages, and just like the casinos in Las Vegas, you'll show a big profit in the long run.

You can save a considerable amount of money by using the chart. There are basically two ways to do it. The first is to do the computation as I did above, estimating the cost of a service call (I always figure \$200—that's \$100 to get the repairman to show up and \$100 for parts). The other is a highly conservative approach, in which you figure that if something goes wrong, you've bought a lemon, and you'll have to replace the appliance. If the cost of the service contract is more than the average replacement cost, purchasing a service contract is a sucker play.

For instance, suppose you buy a microwave oven for \$300. The chart says this appliance breaks down 17 percent of the time—one in six. To compute the average replacement cost, simply multiply \$300 by 17/100 (or 1/6 for simplicity)—the answer is about \$50. If the service contract costs \$50 or more, they're ripping you off big-time. Incidentally, note that a side-by-side refrigerator with icemaker and dispenser will break down three times as often as the basic model. How can you buy something that breaks down 37 percent of the time in a three-year period? I'd save myself the aggravation and do things the old-fashioned way, by pouring water into ice trays.

Finally, notice that TVs almost *never* break down. I had a 25-inch model I bought in the mid-eighties that lasted seventeen years. Admittedly, I did have to replace the picture tube once. Digital cameras are pretty reliable, too.

The long-term average resulting from a course of action is called the expected value of that action. In my opinion, expected value is the single most bottom-line useful idea in mathematics, and I intend to devote a lot of time to exploring what you can do with it. In deciding whether to purchase the refrigerator service contract, we looked at the expected value of two actions. The first, buying the contract, has an expected value of −$100; the minus sign occurs because it is natural to think of expected value in terms of how it affects *your* bottom line, and in this case your bottom line shows a loss of $100. The second, passing it up, has an expected value of −$25; remember, if you bought eight refrigerators, only one would need a repair costing $200, and $200/8 = $25. In many situations, we are confronted with a choice between alternatives that can be resolved by an expected-value calculation. Over the course of a lifetime, such calculations are worth a minimum of tens of thousands of dollars to you—and, as you'll see, they can be worth hundreds of thousands of dollars, or more, to you. This type of cost-effective mathematical projection can be worth millions of dollars to small organizations and billions to large ones, such as nations. It can even be used in preventing catastrophes that threaten all of humanity. That's why this type of math is valuable.

Averages: The Most Important Concept in Mathematics

Now you know my opinion, but I'm not the only math teacher who believes this: averages play a significant role in all of the basic mathematical subjects and in many of the advanced ones. You just saw a simple example of an average regarding service contracts. Averages play a significant role in our everyday use of and exposure to mathematics. Simply scanning through a few sections of today's paper, I found references to the average household income, the average per-screen revenue of current motion pictures, the scoring averages of various basketball players, the average age of individuals when they first became president, and on and on.

So, what is an average? When one has a collection of numbers, such as the income of each household in America, one simply adds up all of those numbers and divides by the number of numbers. In short, an average is the sum of all of the data divided by the number of pieces of data.

Why are averages so important? Because they convey a lot of information about the past (what the average is), and because they are a good indicator of the future. This leads us to the law of averages.

The Law of Averages

The law of averages is not really a law but is more of a reasonably substantiated belief that future averages will be roughly the same as past averages. The law of averages sometimes leads people to arrive at erroneous conclusions, such as the well-known fallacy that if a coin has come up heads on ten consecutive flips, it is more likely to come up tails on the next flip in order to "get back to the average." There are actually two possibilities here: the coin is a fair coin that really does come up tails as often as it does heads (in the long run), in which case the coin is just as likely to come up heads as tails on the next flip; or the flips are somehow rigged and the coin comes up heads much more often than tails. If somebody asks me which way a coin will land that has come up heads ten consecutive times, I'll bet on heads the next time—for all I know, it's a two-headed coin.

Risk-Reward Ratios and Playing the Percentages

The phrases *risk-reward ratio* and *playing the percentages* are so much a part of the common vocabulary that we have a good intuitive idea of what they mean. The risk-reward ratio is an estimate of the size of the gain compared with the size of the loss, and playing the percentages means to select the alternative that has the most likely chance of occurring.

In common usage, however, these phrases are used qualitatively, rather than quantitatively. Flu shots are advised for the elderly because the risk associated with getting the flu is great compared with the reward of not getting it; that is, the risk-reward ratio of not getting a flu shot is high, even though we may not be able to see exactly how to quantify it. Similarly, on third down and seven, a football team will usually pass the ball because it is the percentage play: a pass is more likely than a run to pick up seven yards. There are two types of percentages: those that arise from mathematical models, such as flipping a fair coin, and those that arise from the compilation of data, such as the percentage of times a pass succeeds on third down and seven. When we flip a fair coin, we need not assume that in the long run, half of the flips will land heads and the other half tails, because that's what is meant by "a fair coin." If, however, we find out that 60 percent of the time, a pass succeeds on third down and seven, we will assume that in the long run this will continue to be the case, because we have no reason to believe otherwise unless the structure of football undergoes a radical change.

How, and When, to Compute Expected Value

The utility of the concept of expected value is that it incorporates both risk-reward ratios and playing the percentages in a simple calculation that gives an excellent quantitative estimate of the long-term average payoff from a given decision.[2] Expected value is used to compute the long-term average result of an event that has different possible outcomes. The casinos of the world are erected on a foundation of expected value, and roulette wheels provide an easy way to compute an example of expected value. A roulette wheel has 36 numbers (1 through 36), half of which are red and half of which are black. In the United

States, the wheel also has 0 and 00, which are green. If you bet \$10 on red and a red number comes up, you win \$10; otherwise, you lose your \$10. To compute the expected value of your bet, suppose you spin the wheel so that the numbers come up in accordance with the laws of chance. One way to do this is to spin the wheel 38 times; each of the 38 numbers—1 through 36, 0, and 00—will come up once (that's what I mean by having the numbers come up in accordance with the laws of chance). Red numbers account for 18 of the 38, so when these come up, you will win \$10, a total of $18 \times \$10 = \$180$. You will lose the other 20 bets, a total of $20 \times \$10 = \$200$. That means that you lose \$20 in 38 spins of the wheel, an average loss of a little more than \$.52. Your expected value from each spin of the wheel is thus $-\$.52$, and the casinos and all of those neon lights are built on your contribution and those of your fellow gamblers.

Expected value is frequently expressed as a percentage. In the preceding example, you have an average loss of about \$.52 on a wager of \$10. Because \$.52 is 5.2% of \$10, we sometimes describe a bet on red as having an expected value of -5.2%. This enables us to compute the expected loss for bets of any size. Casinos know what the expected value of a bet on red is, and they can review their videotapes to see whether the actual expected value approximates the computed expected value. If this is not the case, maybe the wheel needs rebalancing, or some sort of skullduggery is taking place.

Expected value can be used only in situations where the probabilities and associated rewards can be quantified with some accuracy, but there are a lot of these. Many of the errands I perform require me to drive some distance; that's one of the drawbacks of living in Los Angeles. Often, I have two ways to get there: freeways or surface streets. Freeways are faster most of the time, but every so often there's an event (an accident or a car chase) that causes lengthy delays. Surface streets are slower, but one almost never encounters an event that turns a surface street into a parking lot, as can happen on the freeways.

Nonetheless, like most Angelenos, I have made an expected-value calculation: given a choice, I take the freeway because on average I save time by doing so. It is not always necessary to perform expected-value calculations; simple observation and experience give you a good estimate of what's happening, which is why most Angelenos take the freeway. You don't have to perform the calculation for the roulette wheel, either; just go to Vegas, make a bunch of bets, and watch your bankroll dwindle over the long run.

Insurance: This Is Worth Tens of Thousands of Dollars

There's a lot of money in the gaming industry, but it pales in comparison with another trillion-dollar industry that is also built on expected value. I'm talking about the insurance industry, which makes its profits in approximately the same way as the gaming industry. Every time you buy an insurance policy, you are placing a bet that you "win" if something happens that enables you to collect insurance, and that you "lose" if no such event occurs. The insurance company has computed the average value of paying off on such an event (think of a car accident) and makes certain that it charges you a large enough premium that it will show a profit, which will make your expected value a negative one.

Nonetheless, this is a game that you simply have to play. If you are a driver, you are required to carry insurance, and there are all sorts of insurance policies (life, health, home) that it is advisable to purchase, even though your expected value is negative—because you simply cannot afford the cost of a disaster. Despite that, there is a correct way to play the insurance game, and doing this is generally worth tens of thousands of dollars (maybe more) over the course of a lifetime.

Let's consider what happens when you buy an auto insurance policy, which many people do every six months. My insurance company offers me a choice of a $100 deductible policy for $300 or a $500 deductible policy for $220. If I buy the $100 deductible policy and I get into an accident, I get two estimates for the repair bill and go to the mechanic who gives the cheaper estimate (this is standard operating procedure for insurance companies). The insurance company sends me a check for the amount of the repair less $100. If I had bought the $500 deductible policy, the company would have sent me a check for the amount of the repair less $500. It's cheaper to buy the $500 deductible policy than the $100 deductible policy, because if I get in an accident, the insurance company will send me $400 less than I would receive if I'd bought the $100 deductible policy.

An expected-value calculation using your own driving record is a good way to decide which option to choose. I've been driving fifty years and bought a hundred six-month policies. During that period, I've had three accidents. One was my fault—I wasn't paying attention. The other two both occurred during a three-day period in 1983: in each case, I was *not even moving* and a car rammed into me and totaled my vehicle. I am getting older, however, and am probably not as good a driver as my record shows, so I estimate that having one accident every five years is probably a little more accurate than having three in fifty years. This means that if I buy ten policies (two every year for five years) and choose the $100 deductible, rather than the $500 dollar deductible, I'll save $80 the nine times out of ten that I don't have an accident and lose $400 the one time that I do. So, by buying the $100 deductible, I save an average of $32, because (9 × $80 − $400)/10 = $32. It actually figures to be somewhat more than that for two reasons. I think that the estimate of one accident every five years is a little conservative, but, more important, if I have an accident that doesn't have to be reported (for instance, if I accidentally back up too far and hit the wall of my garage), I just might pay for the repair myself, because I know my insurance rates will skyrocket once I file a claim.

This calculation occurs countless times, as the deductible option is presented to you every time you buy health insurance or any kind of property insurance as well—and you and your family will purchase an extraordinary amount of insurance during the course of a lifetime. For some people, the savings from making the correct decisions will be in the hundreds of thousands of dollars, but for everyone it's at least in the tens of thousands—unless you're a Luddite who has rejected modern technology.

Because a crucial factor of the calculation is an estimate of the likelihood of certain events occurring, it's important to have a plan to figure this out. When purchasing auto insurance, I use my own driving record, but if you are just starting out, a reasonable approach is to use the accident statistics of people in a group similar to yours. If you are a twenty-five-year-old woman, look for accident statistics for women between twenty and thirty years old; numerous Web sites exist that contain this or similar information. If you are considering buying earthquake insurance, find out something about the frequency of earthquakes where you live. If you live in an area that has never experienced an earthquake, why would you want to buy earthquake insurance?

Let's Take a Break

You might be a little weary from all of these calculations. Fortunately, today is the day that you will go to a taping of your favorite game show. Like many game shows, it has a preliminary round in which the contestant wins some money. The host then tries to persuade the contestant to risk that money in an attempt to win even more. Incredibly, you have been selected from the studio audience to be a contestant on such a game show, you have successfully managed to answer who was buried in Grant's tomb, and you have won $100,000. The host congratulates you on the depth of your knowledge, and a curtain is drawn back onstage, revealing three doors. The host informs you that behind one of these doors is a check for $1,000,000, and behind the other two is a year's supply of the sponsor's product, which happens to be toothpaste. The host tells you that in addition to the $100,000 that you have already won, you get to pick a door, and you will receive whatever lies behind that door.

Three has always been your lucky number, so you go with door three. The host walks over to door three, hesitates—and turns the handle on door two. Tubes of toothpaste cascade all over the stage. The host, now knee-deep in toothpaste, turns and says, "Have I got a deal for you! You can either keep the $100,000 and whatever lies behind door three, or you can give me back the $100,000 and take what lies behind door one instead." Well, what do you do?

I give this question to every class in which I teach probability and ask the students what they would do. To a man (or a woman), they keep the $100,000 and whatever lies behind door three. After all, a bird (or $100,000) in the hand is not something most people are comfortable letting get away.

The correct answer to this problem actually involves a consideration of external factors. For instance, if you have a child who needs a critical operation that costs exactly $100,000 and this is your only way of getting the money, of course you would keep the $100,000. This $100,000 is worth far more to you than the $1,000,000 you might receive in addition; economists have devised a concept called *marginal utility* to describe the fact that each extra dollar beyond the $100,000 needed for the operation has significantly less value to you than the dollars that make up the $100,000 for the operation.

Let's say, however, that you regard all dollars as having equal value and, having been placed in a game situation, feel that you are obliged to play the game to earn the most dollars in the long run. In other words, when situations such as this are presented to you, you want to make the play that gives you the greatest expected value. In this case, you should relinquish the $100,000 (albeit with regret) and take what lies behind door one—because the probability that the big prize lies behind door one is twice as great as the probability that it lies behind door three!

The first time most people encounter a situation like this, they see it as highly counterintuitive. How can it be twice as likely to be behind one door as another? Isn't it equally likely to be behind either door? Yes, but the tricky point here (occasionally, tricky points really do show up in math problems) is that you are not being asked to choose between door three and door one, you are asked to choose between door three and *the other two doors*. And it just happens that you have seen the toothpaste behind one of the other two doors. To make this a little clearer, suppose that there were a thousand doors rather than three doors, and only one of them contained a $1,000,000 check. As before, the host opens all of the doors except door three (your choice) and door one, and (this time up to his neck in toothpaste) he asks you if you want to switch. Your chance of guessing the correct door was originally 1 in 1,000, and nothing has happened to change those odds: there are 999 chances out of 1,000 that the million-dollar check is behind door one.

You can now see that in the original three-door problem, there is one chance in three that the million-dollar check lies behind your choice of door three, and two chances in three that it lies behind door one. If you stick with your original choice of door three, thinking of the toothpaste as valueless, you have two out of three chances to win $100,000 and one out of three chances to win $1,100,000, for an average win of a little more than $433,000—so $433,000 is the expected value of choosing door three. If you switch doors and pick door one, you will have one chance to win $0 (ouch) but two chances to win $1,000,000, for an average win of a few hundred short of $667,000—so $667,000 is the expected value of choosing door one.

I mentioned earlier that external considerations have to be taken into account. If you are married, switch doors and give up the $100,000, and emerge with nothing but toothpaste to show for your efforts, be prepared to listen to your spouse bring it up until the end of time.[3]

Going to College: A Decision Worth Hundreds of Thousands of Dollars

So far, we've looked at a couple of very ordinary events: buying a refrigerator and selecting an insurance policy. Now let's look at an extraordinary event: deciding whether to go to college. Although many of us go to college, the use of the word *extraordinary* is justified by the dictionary, for going to college is a one-time experience for most of us and is highly exceptional or unusual within the context of our own lives.

Back in the early 1990s, I worked on a project that involved high school teachers. One of them taught math at a high school in the San Fernando Valley and told me that he had tried to persuade one of his better students to go to college. At the last moment, the student told the teacher that he had been offered a good job in the construction industry and had decided to take that instead.

Many of the readers of this book will have faced this or a similar decision: Should I take my B.A. and get a job, or should I go to graduate school, med school, or law school? It is one of the most financially important decisions you will ever make, and there are lots of factors to take into account. It will cost money to go to college, and you may not complete it. It will take you out of the job market for several years. As against that, college graduates make considerably more than high school graduates do. What's the right thing to do?

Almost invariably, the right thing is to seek more schooling. Yes, lots of people will tell you this, but here we will do the math. In 2004, a high school graduate earned an average of about $28,000 a year, whereas a college graduate earned about $51,000 per year.[4] Even if you assume you have only a fifty-fifty chance of graduating from a public college and it costs you $50,000 to attend school for five years and graduate (the time needed by a typical student where I teach), let's look at what it's worth to you. If you are eighteen years old with a high school degree and planning on working until you are sixty-five (that's forty-seven years), the cost to you (compared with the high school graduate who goes straight into the job market) of failing to graduate after five years in college is $50,000 plus five years of earning $28,000 a year, for a total of $190,000. If, however, you graduate after five years of college, compared to the high school graduate who went straight to work, you will have lost the five years of earning $28,000 a year and the $50,000 tuition, but you will gain $23,000 per year for the forty-two years you will be in the workforce. That's a net gain of $776,000. If you were to flip a coin (analogous to the fifty-fifty chance of graduating from college) and if the coin lands heads you win $776,000, and tails you lose $190,000, your expected value is $293,000. This computation is highly conservative: the college graduation rate is generally much higher than 50 percent. If your chances of graduating are 75 percent—three out of four—you rate to win $776,000 three times and lose $190,000 once, for an average gain of (3 × $776,000 − 1 × $190,000)/4 = $534,500! (It may be somewhat self-serving of me to make this remark, but my guess is that if you are reading this book, your chances of graduating from college are considerably better than fifty-fifty.) If you do the same calculation for the decision as to whether to pursue an advanced degree, the results are similar.

One Long Season

A friend of mine once had a conversation with a sports gambler who made a successful living betting the Big Three: baseball, football, and basketball. Each of these three sports has a season, and even though they overlap slightly, essentially the year consists of a baseball season, a football season, and a basketball season. The gambler told my friend that even though he liked to show a profit at the end of each season, he recognized that you win some and you lose some. The key was to regard life as one long season—you're in it to show a profit over the long haul.

The same is true with playing the percentages. Certain situations will recur, such as buying auto insurance or service contracts, and it is easy to see that the law of averages will work for you in this type of situation. Other things, however, such as deciding to go to college, are essentially one-shot affairs: although people do drop out of school and return thirty years later to pick up the sheepskin, most people who drop out for several years never come back. Nonetheless, every time you play the percentages in the long season of life, you are giving yourself the best chance of showing a profit, and over that long season this is the best strategy.