Chapter 1

Looking Closely at Linear First Order Differential Equations

In This Chapter

- Knowing what a first order linear differential equation looks like
- Finding solutions to first order differential equations with and without *y* terms
- Employing the trick of integrating factors

One important way that you can classify differential equations is as linear or nonlinear. A differential equation is considered *linear* if it involves only *linear terms* (that is, terms to the power 1) of y, y', y'', and so on. The following equation is an example of a linear differential equation:

$$L\frac{d^2Q}{dx^2} + R\frac{dQ}{dx} + \frac{1Q}{C} = E(x)$$

Nonlinear differential equations simply include nonlinear terms in y, y', y'', and so on. This next equation, which describes the angle of a pendulum, is considered a nonlinear differential equation because it involves the term sin θ (not just θ):

$$\frac{d^2\theta}{dx^2} + \frac{g}{L}\sin\theta = 0$$

This chapter focuses on linear first order differential equations. Here you have the chance to sharpen your linear-equation-spotting eye. You also get to practice solving linear first order differential equations when *y* is and isn't involved. Finally, I clue you in to a little (yet extremely useful!) trick o' the trade called integrating factors.

Identifying Linear First Order Differential Equations

Here's the general form of a linear differential equation, where $p(x)$ and $q(x)$ are functions (which can just be constants):

$$\frac{dy}{dx} + p(x)y = q(x)$$

Following are some examples of linear differential equations:

$$\frac{dy}{dx} = 5$$

$$\frac{dy}{dx} = y + 1$$

$$\frac{dy}{dx} = 3y + 1$$

For a little practice, try to figure out whether each of the following equations is linear or nonlinear.

EXAMPLE

Q. Is this equation a linear first order differential equation?

$$\frac{dy}{dx} = 17y + 4$$

A. Yes.

This equation is a linear first order differential equation because it involves solely first order terms in y and y'.

1. Is this equation a linear first order differential equation?

$$\frac{dy}{dx} = 9y + 1$$

Solve It

2. Is the following a linear first order differential equation?

$$\frac{dy}{dx} = 17y^3 + 4$$

Solve It

3. Is this equation a linear first order differential equation?

$$\frac{dy}{dx} = y\cos(x)$$

Solve It

4. Is the following a linear first order differential equation?

$$\frac{dy}{dx} = x\cos(y)$$

Solve It

Solving Linear First Order Differential Equations That Don't Involve Terms in y

REMEMBER

The simplest type of linear first order differential equation doesn't have a term in y at all; instead, it involves just the first derivative of y, y', y'', and so on. These differential equations are simple to solve because the first derivatives are easy to integrate. Here's the general form of such equations (note that $q(x)$ is a function, which may be a constant):

$$\frac{dy}{dx} = q(x)$$

Take a look at this linear first order differential equation:

$$\frac{dy}{dx} = 3$$

Note that there's no term in just y. So how do you solve this kind of equation? Just move the dx over to the right:

$$dy = 3dx$$

Then integrate to get

$$y = 3x + c$$

where c is a constant of integration.

To figure out what c is, simply take a look at the initial conditions. For example, say that $y(0)$ — that is, the value of y when $x = 0$ — is equal to

$$y(0) = 15$$

Plugging $y(0) = 15$ into $y = 3x + c$ gives you

$$y(0) = c = 15$$

So $c = 15$ and $y = 3x + 15$. That's the complete solution!

To deal with constants of integration like c, look for the specified initial conditions. For example, the problem you just solved is usually presented as

$$\frac{dy}{dx} = 3$$

where

$$y(0) = 15$$

Time for a more advanced problem! (Note that this one still doesn't involve any simple terms in y.)

$$\frac{dy}{dx} = x^3 - 3x^2 + x$$

where

$$y(0) = 3$$

Because this equation doesn't involve any terms in y, you can move the dx to the right, like this:

$$dy = x^3\,dx - 3x^2\,dx + x\,dx$$

Then just integrate to get

$$y = \frac{x^4}{4} - x^3 + \frac{x^2}{2} + c$$

To evaluate c, use the initial condition, which is

$$y(0) = 3$$

Plugging $x = 0 \rightarrow y = 3$ into the equation for y gives you

$$y(0) = 3 = c$$

So the full solution is

$$y = \frac{x^4}{4} - x^3 + \frac{x^2}{2} + 3$$

REMEMBER

As you can see, the way to deal with linear first order differential equations that don't involve a term in just y is simply to

1. **Move the *dx* to the right and integrate.**
2. **Apply the initial conditions to solve for the constant of integration.**

Following are some practice problems to make sure you have the hang of it.

EXAMPLE

Q. Solve for y in this differential equation:

$$\frac{dy}{dx} = 2x$$

where

$$y(0) = 3$$

A. $y = x^2 + 3$

1. Multiply both sides by dx:

$$dy = 2x\,dx$$

2. Integrate both sides to get the following, where c is a constant of integration:

$$y = x^2 + c$$

3. Apply the initial condition to get

$$c = 3$$

4. Having solved for c, you can find the solution to the differential equation:

$$y = x^2 + 3$$

5. Solve for y in this differential equation:

$$\frac{dy}{dx} = 8x$$

where

$$y(0) = 4$$

Solve It

6. What's y in the following equation?

$$\frac{dy}{dx} = 2x + 2$$

where

$$y(0) = 2$$

Solve It

7. Solve for y in this differential equation:

$$\frac{dy}{dx} = 6x + 5$$

where

$$y(0) = 10$$

Solve It

8. What's y in the following equation?

$$\frac{dy}{dx} = 8x + 3$$

where

$$y(0) = 12$$

Solve It

Solving Linear First Order Differential Equations That Involve Terms in y

Wondering what to do if a differential equation you're facing involves both x and y?

$$\frac{dy}{dx} + p(x)\, y = q(x)$$

Start by taking a look at this representative problem:

$$\frac{dy}{dx} = ay - b$$

The preceding is a linear first order differential equation that contains both dy/dx and y. How do you handle it and find a solution? By using some algebra, you can rewrite this equation as

$$\frac{dy / dx}{y - (b / a)} = a$$

Multiplying both sides by dx gives you

$$\frac{dy}{y-(b/a)} = a\,dx$$

Congrats! You've just separated x on one side of this differential equation and y on the other, making the integration much easier. Speaking of integration, integrating both sides gives you

$$\ln|y-(b/a)| = ax + C$$

where C is a constant of integration. Raising both sides to the power e gives you this, where c is a constant defined by $c = e^C$:

$$y = (b/a) + ce^{ax}$$

Anything beyond this level of difficulty must be approached in another way, and you deal with such equations throughout the rest of the book.

If you think you have solving linear first order differential equations in terms of y all figured out, try your hand at these practice questions.

Q. Solve for y in this differential equation:

$$\frac{dy}{dx} = 2y - 4$$

where

$y(0) = 3$

A. $y = 2 + e^{2x}$

1. Use algebra to get

$$\frac{dy/dx}{y-2} = 2$$

2. Then multiply both sides by dx:

$$\frac{dy}{y-2} = 2dx$$

3. Integrate to get

$$\ln|y-2| = 2x + C$$

4. Then raise e to the power of both sides:

$$y = 2 + e^C e^{2x} = 2 + ce^{2x}$$

5. Finally, apply the initial condition to get

$$y = 2 + e^{2x}$$

9. What's y in the following equation?

$$\frac{dy}{dx} = 4y - 8$$

where

$y(0) = 5$

Solve It

10. Solve for y in this differential equation:

$$\frac{dy}{dx} = 3y - 9$$

where

$y(0) = 9$

Solve It

11. What's y in the following equation?

$$\frac{dy}{dx} = 9y - 18$$

where

$y(0) = 5$

Solve It

12. Solve for y in this differential equation:

$$\frac{dy}{dx} = 4y - 20$$

where

$y(0) = 16$

Solve It

Integrating Factors: A Trick of the Trade

Because not all differential equations are as nice and neat to work with as the ones featured earlier in this chapter, you need to have more power in your differential equation–solving arsenal. Enter *integrating factors,* which are functions of $\mu(x)$. The idea behind an integrating factor is to multiply the differential equation by it so that the resulting equation can be integrated easily.

Say you encounter this differential equation:

$$\frac{dy}{dx} + 3y = 9$$

where

$$y(0) = 7$$

To solve this equation with an integrating factor, try multiplying by $\mu(x)$, your as-yet-undetermined integrating factor:

$$\mu(x)\frac{dy}{dx} + 3\mu(x)y = 9\mu(x)$$

The trick now is to select $\mu(x)$ so you can recognize the left side as a derivative of something that can be easily integrated. If you take a closer look, you notice that the left side of this equation appears very much like differentiating the product $\mu(x)y$, because the derivative of $\mu(x)y$ with respect to x is

$$\frac{d(\mu(x)y)}{dx} = \mu(x)\frac{dy}{dx} + y\frac{d\mu(x)}{dx}$$

Comparing the right side of this differential equation to the left side of the previous one gives you

$$\frac{d\mu(x)}{dx} = 3\mu(x)$$

At last! That looks like something you can work with. Rearrange the equation to get the following:

$$\frac{d\mu(x)/dx}{\mu(x)} = 3$$

Then go ahead and multiply both sides by dx to get

$$\frac{d\mu(x)}{\mu(x)} = 3dx$$

Integrating gives you

$$\ln|\mu(x)| = 3x + b$$

where b is a constant of integration.

Raising e to the power of both sides gives you

$$\mu(x) = ce^{3x}$$

where c is another constant ($c = e^b$).

Guess what? You've just found an integrating factor, specifically $\mu(x) = ce^{3t}$.

You can use that integrating factor with the original differential equation, multiplying the equation by $\mu(x)$:

$$\mu(x)\frac{dy}{dx} + 3\mu(x)y = 9\mu(x)$$

which is equal to

$$ce^{3x}\frac{dy}{dx} + 3ce^{3x}y = 9ce^{3x}$$

As you can see, the constant c drops out, leaving you with

$$e^{3x}\frac{dy}{dx} + 3e^{3x}y = 9e^{3x}$$

TIP

Because you're only looking for a multiplicative integrating factor, you can either drop the constant of integration when you find an integrating factor or set $c = 1$.

This is where the whole genius of integrating factors comes in, because you can recognize the left side of this equation as the derivative of the product $e^{3x}y$. So the equation becomes

$$\frac{d\left(e^{3x}y\right)}{dx} = 9e^{3x}$$

That sure looks a lot easier to handle than the original version of this differential equation, doesn't it?

Now you can multiply both sides by dx to get

$$d(e^{3x}y) = 9e^{3x}\,dx$$

Then integrate both sides:

$$e^{3x}y = 3e^{3x} + c$$

and solve for y:

$$y = 3 + ce^{-3x}$$

Because the initial condition stated that $y(0) = 7$, that means $c = 4$, so

$$y = 3 + 4e^{-3x}$$

Pretty cool, huh?

Here are some practice equations to get you better acquainted with the trick of integrating factors.

Q. Solve for y by using an integrating factor:

$$\frac{dy}{dx} + 5y = 10$$

where

$$y(0) = 6$$

A. $y = 2 + 4e^{-5x}$

1. Multiply both sides of the differential equation by $\mu(x)$ to get

$$\mu(x)\frac{dy}{dx} + 5\mu(x)y = 10\mu(x)$$

2. Identify the left side with a derivative (in this case, the derivative of a product):

$$\frac{d\left(\mu(x)y\right)}{dx} = \mu(x)\frac{dy}{dx} + \frac{d\mu(x)}{dx}y$$

3. Then identify the right side of the equation in Step 2 with the left side of the equation in Step 1:

$$\frac{d\mu(x)}{dx} = 5\mu(x)$$

4. Rearrange terms to get

$$\frac{d\mu(x)}{\mu(x)} = 5dx$$

5. Then integrate:

$$\ln|\mu(x)| = 5x + b$$

6. Next up, raise e to the power of both sides(where $c = e^b$) to get

$$\mu(x) = ce^{5x}$$

7. Multiply the original differential equation by the integrating factor (canceling out c) to get

$$e^{5x}\frac{dy}{dx} + 5e^{5x}y = 10e^{5x}$$

8. Combine the terms on the left side of this equation:

$$\frac{d\left(e^{5x}y\right)}{dx} = 10e^{5x}$$

9. Then multiply by dx:

$$d(e^{5x}y) = 10e^{5x}\,dx$$

10. Integrate:

$$e^{5x}y = 2e^{5x} + c$$

11. Divide both sides by e^{5x} to get

$$y = 2 + ce^{-5x}$$

12. Finally, apply the initial condition:

$$y = 2 + 4e^{-5x}$$

13. Solve for y by using an integrating factor:

$$\frac{dy}{dx} + 2y = 4$$

where

$y(0) = 3$

Solve It

14. In the following differential equation, find y by using an integrating factor:

$$\frac{dy}{dx} + 3y = 9$$

where

$y(0) = 8$

Solve It

15. Solve for y by using an integrating factor:

$$\frac{dy}{dx} + 2y = 14$$

where

$y(0) = 9$

Solve It

16. In the following differential equation, find y by using an integrating factor:

$$\frac{dy}{dx} + 9y = 63$$

where

$y(0) = 8$

Solve It

Answers to Linear First Order Differential Equation Problems

Following are the answers to the practice questions presented throughout this chapter. Each one is worked out step by step so that if you messed one up along the way, you can more easily see where you took a wrong turn.

1 **Is this equation a linear first order differential equation?**

$$\frac{dy}{dx} = 9y + 1$$

Yes. This equation is a linear first order differential equation because it involves solely first order terms in y and y'.

2 **Is the following a linear first order differential equation?**

$$\frac{dy}{dx} = 17y^3 + 4$$

No. This equation is *not* a linear first order differential equation because it doesn't involve solely first order terms in y and y'.

3 **Is this equation a linear first order differential equation?**

$$\frac{dy}{dx} = y\cos(x)$$

No. This equation is *not* a linear first order differential equation because it doesn't involve solely first order terms in y and y'.

4 **Is the following a linear first order differential equation?**

$$\frac{dy}{dx} = x\cos(y)$$

No. This equation is *not* a linear first order differential equation because it doesn't involve solely first order terms in y and y'.

5 **Solve for y in this differential equation:**

$$\frac{dy}{dx} = 8x$$

where

$$y(0) = 4$$

Solution: $y = 4x^2 + 4$

1. Multiply both sides by dx:

$$dy = 8x\,dx$$

2. Then integrate both sides to find the following, where c is a constant of integration:

   $y = 4x^2 + c$

3. Apply the initial condition to get

   $c = 4$

4. Having solved for c, you can now find the solution, which is

   $y = 4x^2 + 4$

6 **What's y in the following equation?**

$$\frac{dy}{dx} = 2x + 2$$

where

$y(0) = 2$

Solution: $y = x^2 + 2x + 2$

1. Start by multiplying both sides by dx:

   $dy = 2x\,dx + 2dx$

2. Integrate both sides (noting that c is a constant of integration):

   $y = x^2 + 2x + c$

3. Apply the initial condition:

   $c = 2$

4. Having solved for c, obtain the solution to the equation:

   $y = x^2 + 2x + 2$

7 **Solve for y in this differential equation:**

$$\frac{dy}{dx} = 6x + 5$$

where

$y(0) = 10$

Solution: $y = 3x^2 + 5x + 10$

1. Multiply both sides by dx:

   $dy = 6x\,dx + 5dx$

2. Integrate both sides to find the following, where c is a constant of integration:

   $y = 3x^2 + 5x + c$

3. Apply the initial condition to get

   $c = 10$

4. Having solved for c, you can now find the solution, which is

   $y = 3x^2 + 5x + 10$

8 **What's y in the following equation?**

$$\frac{dy}{dx} = 8x + 3$$

where

$$y(0) = 12$$

Solution: $y = 4x^2 + 3x + 12$

1. Start by multiplying both sides by dx:

   $$dy = 8x\,dx + 3dx$$

2. Integrate both sides (noting that c is a constant of integration):

   $$y = 4x^2 + 3x + c$$

3. Apply the initial condition:

   $$c = 12$$

4. Having solved for c, obtain the solution to the equation:

   $$y = 4x^2 + 3x + 12$$

9 **What's y in the following equation?**

$$\frac{dy}{dx} = 4y - 8$$

where

$$y(0) = 5$$

Solution: $y = 2 + 3e^{4x}$

1. First, use algebra to get

   $$\frac{dy/dx}{y-2} = 4$$

2. Then multiply both sides by dx:

   $$\frac{dy}{y-2} = 4dx$$

3. Integrate:

   $$\ln|y-2| = 4x + c$$

4. Raise e to the power of both sides:

   $$y = 2 + ce^{4x}$$

5. Finally, apply the initial condition:

   $$y = 2 + 3e^{4x}$$

10 **Solve for y in this differential equation:**

$$\frac{dy}{dx} = 3y - 9$$

where

$$y(0) = 9$$

Solution: $y = 3 + 6e^{3x}$

1. Use algebra to change the equation to

$$\frac{dy/dx}{y-3} = 3$$

2. Multiply both sides by dx:

$$\frac{dy}{y-3} = 3dx$$

3. Then integrate to get

$$\ln|y - 3| = 3x + c$$

4. Raise e to the power of both sides:

$$y = 3 + ce^{3x}$$

5. Last but not least, apply the initial condition to get

$$y = 3 + 6e^{3x}$$

11 **What's y in the following equation?**

$$\frac{dy}{dx} = 9y - 18$$

where

$$y(0) = 5$$

Solution: $y = 2 + 3e^{9x}$

1. First, use algebra to get

$$\frac{dy/dx}{y-2} = 9$$

2. Then multiply both sides by dx:

$$\frac{dy}{y-2} = 9dx$$

3. Integrate:

$$\ln|y - 2| = 9x + c$$

4. Raise e to the power of both sides:

$$y = 2 + ce^{9x}$$

5. Finally, apply the initial condition:

$$y = 2 + 3e^{9x}$$

12 **Solve for y in this differential equation:**

$$\frac{dy}{dx} = 4y - 20$$

where

$$y(0) = 16$$

Solution: $y = 5 + 11e^{4x}$

1. Use algebra to change the equation to

$$\frac{dy/dx}{y-5} = 4$$

2. Multiply both sides by dx:

$$\frac{dy}{y-5} = 4dx$$

3. Then integrate to get

$$\ln |y-5| = 4x + c$$

4. Raise e to the power of both sides:

$$y = 5 + ce^{4x}$$

5. Last but not least, apply the initial condition to get

$$y = 5 + 11e^{4x}$$

13 **Solve for y by using an integrating factor:**

$$\frac{dy}{dx} + 2y = 4$$

where

$$y(0) = 3$$

Solution: $y = 2 + e^{-2x}$

1. Multiply both sides of the equation by $\mu(x)$ to get

$$\mu(x)\frac{dy}{dx} + 2\mu(x)y = 4\mu(x)$$

2. Identify the left side with a derivative (in this case, the derivative of a product):

$$\frac{d(\mu(x)y)}{dx} = \mu(x)\frac{dy}{dx} + \frac{d\mu(x)}{dx}y$$

3. Then identify the right side of the equation in Step 2 with the left side of the equation in Step 1:

$$\frac{d\mu(x)}{dx} = 2\mu(x)$$

4. Rearrange the terms to get

$$\frac{d\mu(x)}{\mu(x)} = 2dx$$

5. Then integrate:

   $\ln |\mu(x)| = 2x + b$

6. Raise e to the power of both sides (where $c = e^b$) to get

   $\mu(x) = ce^{2x}$

7. Multiply the original differential equation by the integrating factor (canceling out c):

   $$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 4e^{2x}$$

8. Then combine the terms on the left side of this equation to get

   $$\frac{d\left(e^{2x}y\right)}{dx} = 4e^{2x}$$

9. Next, multiply by dx:

   $d(e^{2x}y) = 4e^{2x}\,dx$

10. Integrate:

    $e^{2x}y = 2e^{2x} + c$

11. Then divide both sides by e^{2x} to get

    $y = 2 + ce^{-2x}$

12. Finally, apply the initial condition to achieve your answer:

    $y = 2 + e^{-2x}$

14 **In the following differential equation, find y by using an integrating factor:**

$$\frac{dy}{dx} + 3y = 9$$

where

$y(0) = 8$

Solution: $y = 3 + 5e^{-3x}$

1. Multiply both sides by $\mu(x)$:

   $$\mu(x)\frac{dy}{dx} + 3\mu(x)y = 9\mu(x)$$

2. Identify the left side of the equation with a derivative (in this case, the derivative of a product):

   $$\frac{d\left(\mu(x)y\right)}{dx} = \mu(x)\frac{dy}{dx} + \frac{d\mu(x)}{dx}y$$

3. Identify the right side of the equation in Step 2 with the left side of the equation in Step 1:

   $$\frac{d\mu(x)}{dx} = 3\mu(x)$$

4. Next, rearrange the terms:

$$\frac{d\mu(x)}{\mu(x)} = 3dx$$

5. Integrate to get

$$\ln |\mu(x)| = 3x + b$$

6. Raise e to the power of both sides(where $c = e^b$):

$$\mu(x) = ce^{3x}$$

7. Multiply the original equation by the integrating factor (canceling out c) to get

$$e^{3x}\frac{dy}{dx} + 3e^{3x}y = 9e^{3x}$$

8. Combine terms on the left side of the equation:

$$\frac{d\left(e^{3x}y\right)}{dx} = 3e^{3x}$$

9. Multiply by dx:

$$d(e^{3x}y) = 3e^{3x}\,dx$$

10. Then integrate to get

$$e^{3x}y = 3e^{3x} + c$$

11. Next, divide both sides by e^{3x}:

$$y = 3 + ce^{-3x}$$

12. After applying the initial condition, you should have

$$y = 3 + 5e^{-3x}$$

15 **Solve for y by using an integrating factor:**

$$\frac{dy}{dx} + 2y = 14$$

where

$$y(0) = 9$$

Solution: $y = 7 + 2e^{-2x}$

1. Multiply both sides of the equation by $\mu(x)$ to get

$$\mu(x)\frac{dy}{dx} + 2\mu(x)y = 14\mu(x)$$

2. Identify the left side with a derivative (in this case, the derivative of a product):

$$\frac{d\left(\mu(x)y\right)}{dx} = \mu(x)\frac{dy}{dx} + \frac{d\mu(x)}{dx}y$$

3. Then identify the right side of the equation in Step 2 with the left side of the equation in Step 1:

$$\frac{d\mu(x)}{dx} = 2\mu(x)$$

4. Rearrange the terms to get

$$\frac{d\mu(x)}{\mu(x)} = 2dx$$

5. Then integrate:

$\ln|\mu(x)| = 2x + b$

6. Raise e to the power of both sides (where $c = e^b$) to get

$\mu(x) = ce^{2x}$

7. Multiply the original differential equation by the integrating factor (canceling out c):

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 14e^{2x}$$

8. Then combine the terms on the left side of this equation to get

$$\frac{d\left(e^{2x}y\right)}{dx} = 14e^{2x}$$

9. Next, multiply by dx:

$d(e^{2x}y) = 14e^{2x}\,dx$

10. Integrate:

$e^{2x}y = 7e^{2x} + c$

11. Then divide both sides by e^{2x} to get

$y = 7 + ce^{-2x}$

12. Finally, apply the initial condition to achieve your answer:

$y = 7 + 2e^{-2x}$

16 **In the following differential equation, find y by using an integrating factor:**

$$\frac{dy}{dx} + 9y = 63$$

where

$y(0) = 8$

Solution: $y = 7 + e^{-9x}$

1. Multiply both sides by $\mu(x)$:

$$\mu(x)\frac{dy}{dx} + 9\mu(x)y = 63\mu(x)$$

2. Identify the left side of the equation with a derivative (in this case, the derivative of a product):

$$\frac{d\left(\mu(x)y\right)}{dx} = \mu(x)\frac{dy}{dx} + \frac{d\mu(x)}{dx}y$$

3. Identify the right side of the equation in Step 2 with the left side of the equation in Step 1:

$$\frac{d\mu(x)}{dx} = 9\mu(x)$$

4. Next, rearrange the terms:

$$\frac{d\mu(x)}{\mu(x)} = 9dx$$

5. Integrate to get

$$\ln|\mu(x)| = 9x + b$$

6. Raise e to the power of both sides (where $c = e^b$)

$$\mu(x) = ce^{9x}$$

7. Multiply the original equation by the integrating factor (canceling out c) to get

$$e^{9x}\frac{dy}{dx} + 9e^{9x}y = 63e^{9x}$$

8. Combine terms on the left side of the equation:

$$\frac{d\left(e^{9x}y\right)}{dx} = 63e^{9x}$$

9. Multiply by dx:

$$d(e^{9x}y) = 63e^{9x}\,dx$$

10. Then integrate to get

$$e^{9x}y = 7e^{9x} + c$$

11. Next, divide both sides by e^{9x}:

$$y = 7 + ce^{-9x}$$

12. After applying the initial condition, you should have

$$y = 7 + e^{-9x}$$