Chapter 1

Production of Net Magnetization

Magnetic resonance (MR) is a measurement technique used to examine atoms and molecules. It is based on the interaction between an applied magnetic field and a particle that possesses spin. While electrons and other subatomic particles possess spin (or more precisely, spin angular momentum) and can be examined using MR techniques, this book focuses on nuclei and the use of MR techniques for their study, formally known as nuclear magnetic resonance (NMR). Nuclear spin, or more precisely, nuclear spin angular momentum, is one of several intrinsic properties of an atom, and its value depends on the precise atomic composition. Every element in the periodic table except argon and cerium has at least one naturally occurring isotope that possesses spin. Thus, in principle, nearly every element can be examined using MR, and the basic ideas of resonance absorption and relaxation are common for all of these elements. The precise details will vary from nucleus to nucleus and from system to system.

The structure of an atom is an essential component of the MR experiment. Atoms consist of three fundamental particles: *protons*, which possess a positive charge; *neutrons*, which have no charge; and *electrons*, which have a negative charge. The protons and neutrons are located in the nucleus or core of an atom, and the electrons are located in shells or orbitals surrounding the nucleus. The characteristic chemical reactions of elements depend on the particular number of each of these particles. The properties most commonly

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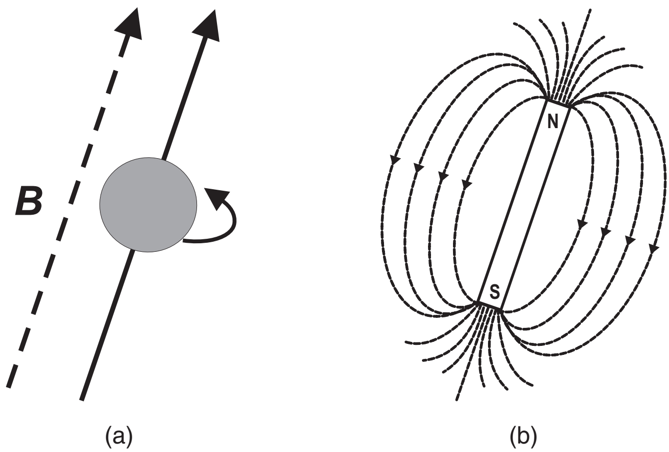

Figure 1-1. *A rotating nucleus (spin) with a positive charge produces a magnetic field known as the magnetic moment oriented parallel to the axis of rotation (a). This arrangement is analogous to a bar magnet in which the magnetic field is considered to be oriented from the south to the north pole (b).*

used to categorize elements are the atomic number and the atomic weight. The *atomic number* is the number of protons in the nucleus and is the primary index used to differentiate atoms. All atoms of an element have the same atomic number and undergo the same chemical reactions. The *atomic weight* is the sum of the number of protons and the number of neutrons. Atoms with the same atomic number but different atomic weights are called *isotopes*. Isotopes of an element will undergo the same chemical reactions but at different reaction rates.

A third property of the nucleus is *spin*, or intrinsic spin angular momentum. Nuclei with spin can be considered to be constantly rotating about an axis at a constant rate or velocity. This self-rotation axis is perpendicular to the direction of rotation (Figure 1-1). A limited number of values for the spin are found in nature; that is, the spin, I, is quantized to certain discrete values, which depend on the atomic number and atomic weight of the particular nucleus. There are three groups of values for I: zero, integral, and half-integral values. A nucleus has no spin ($I = 0$) if it has an even-number atomic weight and an even atomic number: for example, ^{12}C (6 protons and 6 neutrons) or ^{16}O (8 protons and 8 neutrons). Such a nucleus does not interact with an external magnetic field and cannot be studied using MR. A nucleus has an integral value for I (e.g., 1, 2, 3) if it has an even atomic weight and an odd atomic number: for example, ^{2}H (1 proton and 1 neutron) or ^{6}Li (3 protons and 3 neutrons). A nucleus has a half-integral value for I (e.g., 1/2, 3/2, 5/2) if it has an odd atomic weight. Table 1-1 lists the spin and isotopic composition for several elements commonly found in biological systems. The ^{1}H nucleus, consisting of a single proton, is a natural choice for probing the body using MR

TABLE 1-1 Constants for Selected Nuclei of Biological Interest

Element	Nuclear Composition		Nuclear Spin, *I*	Gyromagnetic Ratio, γ ($MHz\,T^{-1}$)	% Natural Abundance	ω at 1.5 T (MHz)
	Protons	Neutrons				
^{1}H, protium	1	0	1/2	42.5774	99.985	63.8646
^{2}H, deuterium	1	1	1	6.53896	0.015	9.8036
^{3}He	2	1	1/2	32.436	0.000138	48.6540
^{6}Li	3	3	1	6.26613	7.5	9.39919
^{7}Li	3	4	3/2	16.5483	92.5	24.8224
^{12}C	6	6	0	0	98.90	0
^{13}C	6	7	1/2	10.7084	1.10	16.0621
^{14}N	7	7	1	3.07770	99.634	4.6164
^{15}N	7	8	1/2	4.3173	0.366	6.4759
^{16}O	8	8	0	0	99.762	0
^{17}O	8	9	5/2	5.7743	0.038	8.6614
^{19}F	9	10	1/2	40.0776	100	60.1164
^{23}Na	11	12	3/2	11.2686	100	16.9029
^{31}P	15	16	1/2	17.2514	100	25.8771
^{129}Xe	54	75	1/2	11.8604	26.4	17.7906

Source: Adapted from Mills (1989).

techniques, for several reasons. It has a spin of 1/2 and is the most abundant isotope for hydrogen. Its response to an applied magnetic field is one of the largest found in nature. Since the body is composed of tissues that contain primarily water and fat, both of which contain hydrogen, a significant MR signal can be produced naturally by normal tissues.

Whereas a rigorous mathematical description of a nucleus with spin and its interactions requires the use of quantum mechanical principles, most of MR can be described using the concepts of classical mechanics, particularly in describing the actions of a nucleus with spin. Subsequent discussions of MR phenomena in this book use a classical approach. In addition, although the concepts of resonance absorption and relaxation apply to all nuclei with spin, the descriptions in this book focus on ^{1}H (commonly referred to as a *proton*) since most imaging experiments visualize the ^{1}H nucleus.

Recall that the nucleus is the location of the protons. For many nuclei, the arrangement of the protons is asymmetric. When this asymmetric charge arrangement rotates due to the nuclear spin, a local magnetic field or magnetic moment is induced about the nucleus. This magnetic moment will be oriented parallel to the axis of rotation. Since the nuclear spin is constant in magnitude and orientation or direction, its associated magnetic moment will also be constant in magnitude and orientation. This magnetic moment is fundamental to MR. A bar magnet provides a useful analogy. A bar magnet has a north and a south pole, or more precisely, a magnitude and an orientation to the magnetic field can be defined. The axis of rotation for a nucleus with spin can be viewed as a vector with a definite orientation and magnitude (Figure 1-1). This orientation of the nuclear spin and the changes induced in it due to the

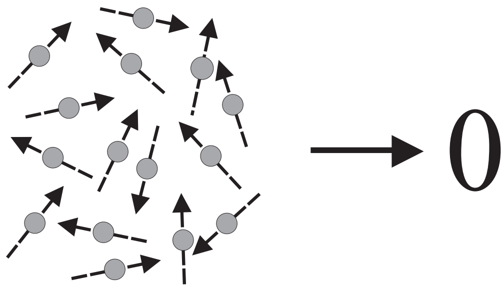

Figure 1-2. *Microscopic and macroscopic pictures of a collection of spins in the absence of an external magnetic field. In the absence of a magnetic field, the spins have their axes oriented randomly (microscopic picture, left side of figure). The vector sum of these spin vectors is zero (macroscopic picture, right side).*

experimental manipulations that the nucleus undergoes provide the basis for the MR signal.

In general, MR measurements are made on collections of similar spins rather than on an individual spin. It is convenient to consider such a collection both as individual spins acting independently (a "microscopic" picture) and as a single entity (a "macroscopic" picture). For many concepts, the two pictures provide equivalent results, even though the microscopic picture is more complete. Conversion between the two pictures requires the principles of statistical mechanics. Although necessary for a complete understanding of MR phenomena, the nature of this conversion is beyond the scope of this book. However, the macroscopic picture is sufficient for an adequate description of most concepts presented in this book. When necessary, the microscopic picture will be used.

Consider an arbitrary volume of tissue containing hydrogen atoms (protons). Each proton has a spin vector of equal magnitude. However, the spin vectors for the entire collection of protons within the tissue are oriented randomly in all directions; there is a continuous distribution of the spin orientations. Performing a vector addition (head to toe) of these spin vectors produces a zero sum; that is, no net magnetization is observed in the tissue (Figure 1-2). If the tissue is placed inside a magnetic field $\boldsymbol{B_0}$,[1] the individual protons begin to rotate perpendicular to, or precess about, the magnetic field. The spin vectors for the protons are tilted slightly away from the axis of the magnetic field, but their axis of precession is parallel to $\boldsymbol{B_0}$. This precession is at a constant rate and occurs because of the interaction of the magnetic field with the spinning positive charge of the nucleus. By convention, $\boldsymbol{B_0}$ and the axis of precession are defined to be oriented in the z direction of a Cartesian coordinate system. (This convention is not followed universally, but it is the

[1] In this book, vector quantities with direction and magnitude are indicated by boldface italic type, while scalar quantities that are magnitude only are indicated by italic type.

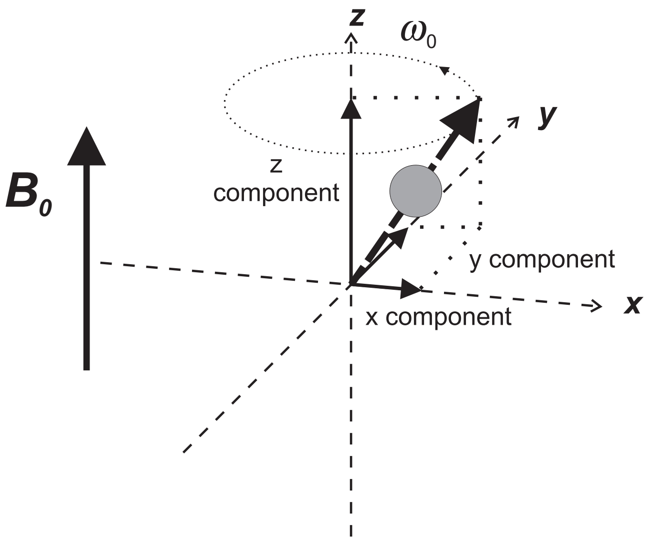

Figure 1-3. *Inside a magnetic field, a proton precesses or revolves about the magnetic field. The precessional axis is parallel to the main magnetic field,* $\mathbf{B_0}$. *The* z *component of the spin vector (projection of the spin onto the* z *axis) is the component of interest because it does not change in magnitude or direction as the proton precesses. The* x *and* y *components vary with time at a frequency* ω_0 *proportional to* B_0 *as expressed by equation (1-1).*

prevailing convention.) The motion of each proton can be described by a unique set of coordinates perpendicular (x and y) and parallel (z) to $\boldsymbol{B_0}$. The perpendicular or transverse coordinates are nonzero but vary with time as the proton precesses, while the parallel or longitudinal coordinate is constant with time (Figure 1-3). The rate or frequency of precession is proportional to the strength of the magnetic field and is expressed by the *Larmor equation*:

$$\omega_0 = \frac{\gamma B_0}{2\pi} \tag{1-1}$$

where ω_0 is the Larmor frequency in megahertz (MHz),[2] B_0 is the magnetic field strength in tesla (T) that the proton experiences, and γ is a constant for each nucleus in $s^{-1}T^{-1}$, known as the *gyromagnetic* ratio. Values for γ and ω at 1.5T for several nuclei are tabulated in Table 1-1.

[2]The factor 2π in equation (1-1) is necessary to convert from angular to cyclical frequency. In many discussions of MR, ω (Greek letter omega) is used to represent angular frequency, with units of s^{-1}, while cyclical frequency, in units of hertz, is represented by either ν (Greek letter nu) or f. The Larmor equation is more properly written as $\nu_0 = \gamma B_0/2\pi$. In imaging derivations, the Larmor equation is usually expressed as equation (1-1) using ω but with units of hertz. To minimize confusion, we follow the imaging tradition throughout this book.

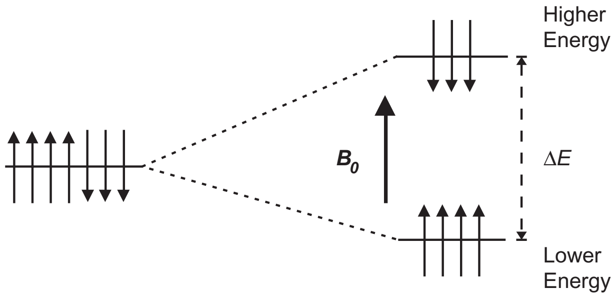

Figure 1-4. *Zeeman diagram. In the absence of a magnetic field (left side of figure), a collection of spins will have the configurations of* z *components equal in energy so that there is no preferential alignment between the spin-up and spin-down orientations. In the presence of a magnetic field (right side), the spin-up orientation (parallel to* $\mathbf{B_0}$*) is of lower energy and its configuration contains more spins than does the higher-energy spin-down configuration. The difference in energy* ΔE *between the two levels is proportional to* $\mathbf{B_0}$.

If a vector addition is performed, as before, for the spin vectors inside the magnetic field, the results will be slightly different than those for the sum outside the field. In the direction perpendicular to $\boldsymbol{B_0}$, the spin orientations are still randomly distributed just as they were outside the magnetic field, despite the time-varying nature of each transverse component. There is still no net magnetization perpendicular to $\boldsymbol{B_0}$. However, in the direction parallel to $\boldsymbol{B_0}$, there is a different result. Because there is an orientation to the precessional axis of the proton that is constant with time, there is a constant, nonzero interaction or coupling between the proton and $\boldsymbol{B_0}$, known as the *Zeeman interaction*. Instead of a continuous range of values, this coupling causes the z component to be quantized to a limited or discrete number of values. For the ^{1}H nucleus, there are only two possible values for the z component: parallel or along $\boldsymbol{B_0}$ and antiparallel or against $\boldsymbol{B_0}$. This coupling also causes a difference in energy ΔE between these two orientations that is proportional to B_0 (Figure 1-4).

The result of the Zeeman interaction is that spins in the two orientations, parallel (also known as *spin up*) and antiparallel (*spin down*), have different energies. Those spins oriented parallel to $\boldsymbol{B_0}$ are of lower energy than those oriented antiparallel. For a collection of protons, more will be oriented parallel to $\boldsymbol{B_0}$ than antiparallel; that is, there is residual polarization of the spins induced parallel to the magnetic field (Figure 1-5a). The exact number of protons in each energy level can be predicted by a distribution function known as the *Boltzmann distribution*:

$$\frac{N_{\text{UPPER}}}{N_{\text{LOWER}}} = e^{-\Delta E/kT} \tag{1-2}$$

where k is Boltzmann's constant, $1.381 \times 10^{-23}\,\text{J}\,\text{K}^{-1}$ and N_{UPPER} and N_{LOWER} are the number of protons in the upper and lower energy levels, respectively.

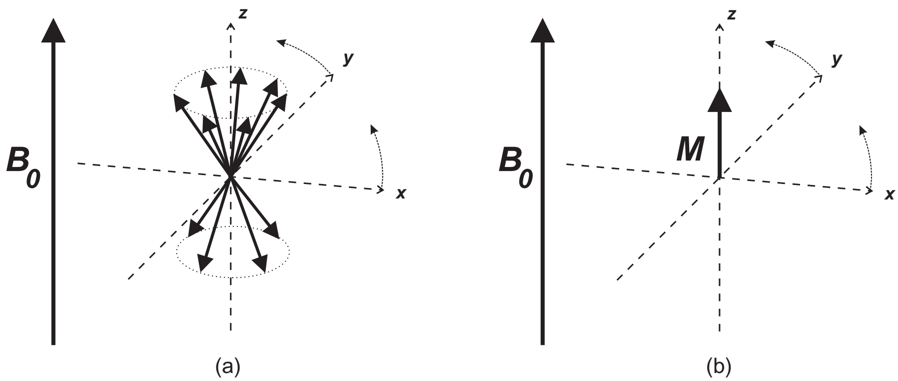

Figure 1-5. *Microscopic (a) and macroscopic (b) pictures of a collection of spins in the presence of an external magnetic field. Each spin precesses about the magnetic field. If a rotating frame of reference is used with a rotation rate of* ω_0*, the collection of protons appears stationary. Whereas the* z *components are one of two values (one positive and one negative), the* x *and* y *components can be any value, positive or negative. The spins will appear to track along two cones, one with a positive* z *component and one with a negative* z *component. Because there are more spins in the upper cone, there will be a nonzero vector sum* $\mathbf{M_0}$*, the net magnetization. It will be of constant magnitude and parallel to* $\mathbf{B_0}$*.*

Since the separation between the energy levels ΔE depends on the field strength B_0, the exact number of spins in each level also depends on B_0 and increases with increasing B_0. For a collection of protons at body temperature (310 K) at 1.5T, there will typically be an excess of about 1×10^6 protons in the lower level out of the approximately 10^{25} protons within the tissue. This unequal number of protons in each energy level means that the vector sum of spins will be nonzero and will point along the magnetic field. In other words, the tissue will become polarized or magnetized in the presence of $\boldsymbol{B_0}$ with a value $\boldsymbol{M_0}$, known as the *net magnetization*. The orientation of this net magnetization will be in the same direction as $\boldsymbol{B_0}$ and will be constant with respect to time (Figure 1-5b). For tissues in the body, the magnitude of $\boldsymbol{M_0}$ is proportional to $\boldsymbol{B_0}$:

$$\boldsymbol{M_0} = \chi \boldsymbol{B_0} \tag{1-3}$$

where χ is known as the bulk magnetic susceptibility or simply the *magnetic susceptibility*. This arrangement with $\boldsymbol{M_0}$ aligned along the magnetic field with no transverse component is the normal, or equilibrium, configuration for the protons. This configuration of spins has the lowest energy and is the arrangement to which the protons will naturally try to return following any perturbations, such as energy absorption. This induced magnetization, $\boldsymbol{M_0}$, is the source of signal for all of the MR experiments. Consequently, all other things being equal, the greater the field strength, the greater the value of M_0 and the greater the potential MR signal.

The magnetic susceptibility describes the response of a substance to the applied magnetic field. Although a complete analysis of the origin of this response is beyond the scope of the book, it is useful to describe the three levels of response that are encountered in MR. A *diamagnetic* response is found in all materials, arising from the electrons surrounding the nuclei. It is a very weak response (χ is very small) and is the only response for most materials, including tissue. A *paramagnetic* response is larger in magnitude than a diamagnetic response but is still relatively weak (χ is small). It is found in molecules where there are "unpaired" electrons. The final response level is known as *ferromagnetic*, typically found in certain metals, in which χ is very large. Whereas diamagnetic and paramagnetic materials are safe to be in or near MR scanners, ferromagnetic materials should remain outside the scan room.

An alternative picture known as a *rotating frame of reference* or *rotating coordinate system* is often used in MR. It is a convenient view when describing objects that undergo rotational motion. When viewed in a rotating frame of reference, the coordinate system rotates about one axis while the other two axes vary with time. By choosing a suitable axis and rate of rotation for the coordinate system, the moving object appears stationary.

For MR experiments, a convenient rotating frame uses the z axis, parallel to $\boldsymbol{B_0}$, as the axis of rotation, while the x and y axes rotate at the Larmor frequency, ω_0. When viewed in this fashion, the precessing spin appears stationary in space with a fixed set of x, y, and z coordinates. If the entire collection of protons in the tissue volume is examined, a complete range of x and y values will be found, both positive and negative, but only two z values. There will be an equal number of positive and negative x and y values but a slight excess of positive z values, as just described. If a vector sum is performed on this collection of protons, the x and y components sum to zero but a nonzero, positive z component will be left, the net magnetization $\boldsymbol{M_0}$. In addition, since the z axis is the axis of rotation, $\boldsymbol{M_0}$ does not vary with time. Regardless of whether a stationary or fixed coordinate system is used, $\boldsymbol{M_0}$ is of fixed amplitude and is parallel to the main magnetic field. For all subsequent discussions in this book, a rotating frame of reference with the rotation axis parallel to $\boldsymbol{B_0}$ is used when describing the motion of the protons.