1

Introduction to Network Analysis of Microwave Circuits

ABSTRACT

Network presentation has been used as a technique in the analysis of low-frequency electrical and electronic circuits. The same technique is equally useful in the analysis of microwave circuits, although different network parameters are used. In this chapter, network parameters for microwave circuit analysis, in particular scattering parameters, are introduced together with a Smith chart for one-port networks and a new chart for two-port networks. The analyses of two-port connected networks and a circuit composed of multi-port networks are also presented.

KEYWORDS

Network analysis, Network parameters, Impedance parameters, Admittance parameters, *ABCD* parameters, Scattering parameters, Smith chart, Two-port chart, Connected networks

Network presentation has been used as a technique in the analysis of low-frequency electrical and electronic circuits (Ramo, Whinnery and van Duzer, 1984). The same technique is equally useful in the analysis of microwave circuits, although different network parameters may be used (Collin, 1966; Dobrowolski, 1991; Dobrowolski and Ostrowski, 1996; Fooks and Zakarev, 1991; Gupta, Garg and Chadha, 1981; Liao, 1990; Montgomery, Dicke and Purcell, 1948; Pozar, 1990; Rizzi, 1988; Ishii 1989; Wolff and Kaul, 1998). Using such a technique, a microwave circuit

Software VNA and Microwave Network Design and Characterisation Zhipeng Wu

can be regarded as a network or a composition of a number of networks. Each network may also be composed of many elementary components. A network may have many ports, from which microwave energy flows into or out of the network. One- and two-port networks are, however, the most common, and most commercial network analysers provide measurements for one- or two-port networks. In this chapter, the network analysis will be based on one- and two-port networks. Network parameters, in particular scattering parameters, will be introduced together with a Smith chart for one-port networks and a new chart for two-port networks. The analysis of two connected networks and a circuit composed of a number of networks will also be presented. For further reading, see references at the end of the chapter.

1.1 ONE-PORT NETWORK

A one-port network can be simply represented by load impedance Z to the external circuit. When the network is connected to a sinusoidal voltage source with an open circuit peak voltage V_s and a reference internal impedance of $Z_{0,\mathrm{ref}}$ as shown in Figure 1.1, the circuit can be analysed using the circuit theory based on total voltage and current quantities. It can also be analysed using the transmission-reflection analysis based on incident and reflected voltage and current quantities. Both analyses are described below. The reference internal impedance $Z_{0,\mathrm{ref}}$ of the source is assumed to be 50 Ω throughout this book.

1.1.1 Total Voltage and Current Analyses

Using circuit theory, the voltage V on the load impedance and the current I flowing through it as shown in Figure 1.1 are related by

$$V = ZI \tag{1.1}$$

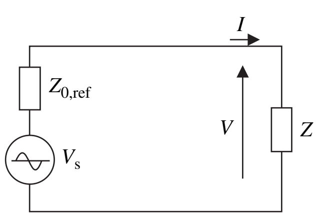


Figure 1.1 Simplified one-port network

and they can be obtained by

$$V = \frac{V_s Z}{Z_{0,\mathrm{ref}} + Z} \tag{1.2a}$$

and

$$I = \frac{V_s}{Z_{0,\mathrm{ref}} + Z}. \tag{1.2b}$$

The power delivered to the load impedance by the voltage source can be obtained by

$$P_L = \frac{1}{2}\mathrm{Re}(VI^*) = \frac{|V_s|^2}{2|Z_{0,\mathrm{ref}} + Z|^2}\mathrm{Re}(Z), \tag{1.3}$$

where * indicates the complex conjugate.

1.1.2 Transmission-Reflection Analysis

1.1.2.1 Voltage and current

Using the transmission-reflection analysis, the incident voltage V^+ is defined to be the voltage that the voltage source could provide to a matched load, i.e. when $Z = Z_{0,\mathrm{ref}}$, and the incident current I^+ to be the corresponding current flowing through the matched load. Hence

$$V^+ = \frac{V_s}{2} \tag{1.4a}$$

and

$$I^+ = \frac{V^+}{Z_{0,\mathrm{ref}}} = \frac{V_s}{2Z_{0,\mathrm{ref}}}. \tag{1.4b}$$

Therefore if $Z_L = Z_{0,\mathrm{ref}}$, then $V = V^+$ and $I = I^+$. However, in the general case that $Z \neq Z_{0,\mathrm{ref}}$, the voltage V can be taken to be the superposition of two voltages: the incident voltage V^+ and a reflected voltage V^-. Similarly the current I can be taken as the superposition of two currents: the incident current I^+ and a reflected current I^-. Since V can be written as $V = IZ_{0,\mathrm{ref}} + V_e$ with

$$V_e = \frac{V_s(Z - Z_{0,\mathrm{ref}})}{Z + Z_{0,\mathrm{ref}}}, \tag{1.5}$$

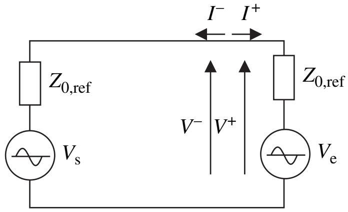


Figure 1.2 Equivalent circuit

the circuit in Figure 1.1 can be represented by an equivalent circuit shown in Figure 1.2. The load impedance Z is replaced by a 'voltage source' V_e with an 'internal impedance' $Z_{0,\mathrm{ref}}$. By using the superposition theorem, the reflected voltage can be taken to be that produced by the equivalent voltage source V_e so that

$$V^- = \frac{V_e}{2} = \frac{V_s}{2}\frac{(Z - Z_{0,\mathrm{ref}})}{(Z + Z_{0,\mathrm{ref}})} \tag{1.6a}$$

and

$$I^- = \frac{V^-}{Z_{0,\mathrm{ref}}} = \frac{V_s}{2Z_{0,\mathrm{ref}}}\frac{(Z - Z_{0,\mathrm{ref}})}{(Z - Z_{0,\mathrm{ref}})}. \tag{1.6b}$$

The total voltage and current are then

$$V = V^+ + V^- = V_s\frac{Z_{0,\mathrm{ref}}}{Z + Z_{0,\mathrm{ref}}} \tag{1.7a}$$

and

$$I = I^+ - I^- = V_s\frac{1}{Z + Z_{0,\mathrm{ref}}}, \tag{1.7b}$$

which are the same as those in Equation (1.2) obtained using circuit theory.

1.1.2.2 Reflection coefficient

Using the transmission-reflection analysis, the total voltage is expressed as the sum of the incident voltage and the reflected voltage, and the current as the difference of the incident current and the reflected current. For the convenience of analysis, a reflection coefficient can be introduced to relate the reflected and incident quantities. The reflection coefficient is defined as

$$\Gamma = \frac{V^-}{V^+} = \frac{I^-}{I^+} = \frac{Z - Z_{0,\mathrm{ref}}}{Z + Z_{0,\mathrm{ref}}} = \frac{Y_{0,\mathrm{ref}} - Y}{Y_{0,\mathrm{ref}} + Y}, \tag{1.8}$$

where $Y_{0,\text{ref}} = 1/Z_{0,\text{ref}}$ and $Y = 1/Z$ and is defined with respect to the reference impedance $Z_{0,\text{ref}}$.

The total voltage and current at the load can then be expressed as

$$V = V^+(1+\Gamma) \tag{1.9a}$$

and

$$I = I^+(1-\Gamma). \tag{1.9b}$$

Hence the total voltage and current quantities can be obtained when Γ is determined.

1.1.2.3 Power

Associated with the incident voltage V^+ and the incident current I^+ is the incident power which is given by

$$P^+ = \frac{1}{2}\text{Re}(V^+ I^{+*}) = \frac{1}{2}\frac{|V^+|^2}{Z_{0,\text{ref}}} = \frac{|V_\text{s}|^2}{8Z_{0,\text{ref}}} = P_\text{max}. \tag{1.10}$$

This power is also the maximum power available from the voltage source. Similarly the reflected power is associated with the reflected voltage V^- and the reflected current I^- and is given by

$$P^- = \frac{1}{2}\text{Re}(V^- I^{-*}) = \frac{1}{2}\frac{|V^-|^2}{Z_{0,\text{ref}}} = P_\text{max}|\Gamma|^2. \tag{1.11}$$

The power delivered to the load impedance is the difference between the incident power and the reflected power, i.e.

$$P_\text{L} = P^+ - P^- = P^+(1-|\Gamma|^2), \tag{1.12}$$

which is identical to Equation (1.3).

1.1.2.4 Introduction of a_1 and b_1

Since the incident power is related to V^+ and $Z_{0,\text{ref}}$ and the reflected power to V^- and $Z_{0,\text{ref}}$ as in Equations (1.10) and (1.11), their expressions can be simplified with the introduction of two new quantities a_1 and b_1 which are defined as (Collin, 1966; Pozar, 1990)

$$a_1 = \frac{V^+}{\sqrt{Z_{0,\text{ref}}}} \tag{1.13a}$$

and

$$b_1 = \frac{V^-}{\sqrt{Z_{0,\mathrm{ref}}}}. \tag{1.13b}$$

Using these two new quantities, the incident, reflected and total powers can then be expressed, respectively, as

$$P^+ = \frac{1}{2}|a_1|^2, \tag{1.14a}$$

$$P^- = \frac{1}{2}|b_1|^2 \tag{1.14b}$$

and

$$P_\mathrm{L} = \frac{1}{2}(|a_1|^2 - |b_1|^2). \tag{1.14c}$$

The voltage and current quantities can also be written as

$$V^+ = a_1\sqrt{Z_{0,\mathrm{ref}}}, \tag{1.15a}$$

$$V^- = b_1\sqrt{Z_{0,\mathrm{ref}}}, \tag{1.15b}$$

$$I^+ = \frac{a_1}{\sqrt{Z_{0,\mathrm{ref}}}}, \tag{1.15c}$$

$$I^- = \frac{b_1}{\sqrt{Z_{0,\mathrm{ref}}}}, \tag{1.15d}$$

$$V = (a_1 + b_1)\sqrt{Z_{0,\mathrm{ref}}} \tag{1.15e}$$

and

$$I = \frac{a_1 - b_1}{\sqrt{Z_{0,\mathrm{ref}}}}. \tag{1.15f}$$

The reflection coefficient defined in Equation (1.8) becomes

$$\Gamma = \frac{b_1}{a_1} = \frac{Z - Z_{0,\mathrm{ref}}}{Z + Z_{0,\mathrm{ref}}} = \frac{Y_{0,\mathrm{ref}} - Y}{Y_{0,\mathrm{ref}} + Y}. \tag{1.16}$$

Using a_1 and b_1, the signal reflection property of the one-port network can be described by

$$b_1 = \Gamma a_1. \tag{1.17}$$

1.1.2.5 Z in terms of Γ

The formulas derived above are useful for the analysis of one-port network when the load impedance is known. However, very often in practice, Z has to be determined from measurement. In this case, Z can be obtained from the measurement of the reflection coefficient using

$$Z = \frac{1+\Gamma}{1-\Gamma} Z_{0,\mathrm{ref}}. \tag{1.18}$$

1.1.3 Smith Chart

1.1.3.1 Impedance chart

The impedance Smith chart (Smith, 1939, 1944) is based on the expression of the reflection coefficient Γ in terms of load impedance Z. With the introduction of the normalised load impedance with respect to the reference impedance $Z_{0,\mathrm{ref}}$ as

$$z = \frac{Z}{Z_{0,\mathrm{ref}}} = r + \mathrm{j}x, \tag{1.19}$$

where r and x are the normalised resistance and reactance, respectively, the reflection coefficient Γ can be written as

$$\Gamma = \frac{z-1}{z+1} = \frac{r + \mathrm{j}x - 1}{r + \mathrm{j}x + 1} = u + \mathrm{j}v, \tag{1.20}$$

where u and v are the real and imaginary projections of Γ on the complex u–v plane. Equation (1.20) can be rearranged to give the following two separate equations:

$$\left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2} \tag{1.21a}$$

and

$$(u-1)^2+\left(v-\frac{1}{x}\right)^2=\frac{1}{x^2}. \tag{1.21b}$$

Equation (1.21a) represents a family of constant resistance circles. The centre of the circle for a normalised resistance r is at $(r/(1+r),0)$ and the radius is $1/(1+r)$. Equation (1.21b) describes a family of constant reactance circles. The centre of the circle with a normalised reactance x is at $(1,1/x)$ and the radius of the circle is $1/|x|$. A simplified impedance Smith chart is shown in Figure 1.3. On the chart, the normalised resistance and reactance values, r and x, can be read when the reflection coefficient Γ is plotted. On the other hand, the complex reflection coefficient can be determined when r and x values are known and plotted on the impedance chart.

1.1.3.2 Admittance chart

With the introduction of the normalised admittance

$$y=\frac{Y}{Y_{0,\mathrm{ref}}}=g+\mathrm{j}b, \tag{1.22}$$

where g and b are the normalised conductance and admittance, respectively. Equation (1.16) for the reflection coefficient Γ can be rearranged to

$$-\Gamma=\frac{g+\mathrm{j}b-1}{g+\mathrm{j}b+1}. \tag{1.23}$$

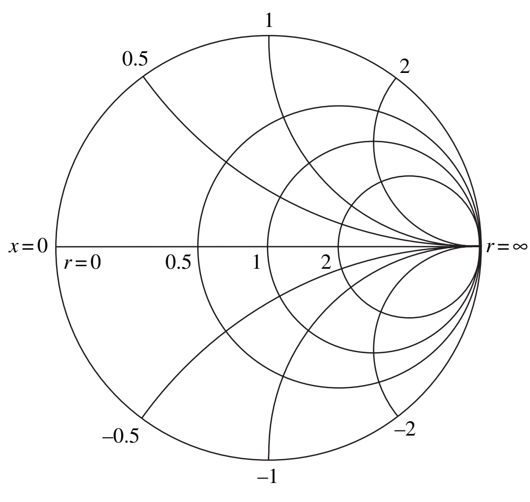


Figure 1.3 Simplified Smith chart

Comparing Equation (1.23) with Equation (1.20), it can be seen that g would be the same as r and x as b if Γ in Equation (1.20) is replaced by $-\Gamma$. Hence the admittance Smith chart can be obtained by rotating the impedance chart by 180°. Alternatively the values of g and b can be read on the impedance chart as for r and x by plotting $-\Gamma_L$ on the impedance chart.

1.1.4 Terminated Transmission Line

For a terminated transmission line, as shown in Figure 1.4, the voltage and current at any position on the transmission line can be described by the following two equations:

$$V(z) = V^{+}e^{-\gamma z} + V^{-}e^{\gamma z} = V^{+}(e^{-\gamma z} + \Gamma_L e^{\gamma z}) \tag{1.24a}$$

and

$$I(z) = I^{+}e^{-\gamma z} - I^{-}e^{\gamma z} = \frac{V^{+}}{Z_0}(e^{-\gamma z} - \Gamma_L e^{\gamma z}), \tag{1.24b}$$

where Z_0 is the characteristic impedance of the transmission line, Γ_L the reflection coefficient with respect to the transmission line impedance of Z_0 and γ the propagation constant. The reflection coefficient Γ_L is given by

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L - Z_0} \tag{1.25a}$$

and the propagation constant by

$$\gamma = \alpha + j\beta = (\alpha_c + \alpha_d + \alpha_r) + j\frac{\omega}{v_p}, \tag{1.25b}$$

where α is the attenuation constant in Nepers/m, β the phase constant in rad/m and v_p the phase velocity. The attenuation constant may consist of

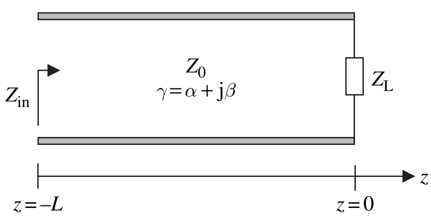


Figure 1.4 Terminated transmission line

three parts: α_c for conducting loss, α_d for dielectric loss and α_r for radiation loss. The dielectric loss α_d can be expressed as

$$\alpha_d = \frac{1}{2}\beta \tan\delta, \tag{1.26}$$

where tan δ is the loss tangent of the dielectric material used in the transmission line.

The input impedance of the transmission line of length L can be obtained by

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma L)}{Z_0 + Z_L \tanh(\gamma L)}. \tag{1.27}$$

If the terminated transmission line is connected to a sinusoidal voltage source with an internal impedance $Z_{0,ref}$, the reflection coefficient with respect to the voltage source is

$$\Gamma = \frac{Z_{in} - Z_{0,ref}}{Z_{in} - Z_{0,ref}}. \tag{1.28}$$

For a lossless transmission line, the input impedance becomes

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}. \tag{1.29}$$

The total voltage and current on the transmission line depend not only on the source, but also on the load impedance. The reflection from the load impedance will cause a standing wave on the transmission line with V_{max} at voltage maximum positions and V_{min} at voltage minimum positions. For a lossless transmission line, the voltage standing wave ratio (VSWR) can be defined as

$$\text{VSWR} = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}. \tag{1.30}$$

This equation can be used to determine the amplitude of Γ_L by measuring the VSWR.

1.2 TWO-PORT NETWORK

1.2.1 Total Quantity Network Parameters

The most commonly used total quantity network parameters for two-port networks are Z, Y and $ABCD$ parameters. Using these parameters, the

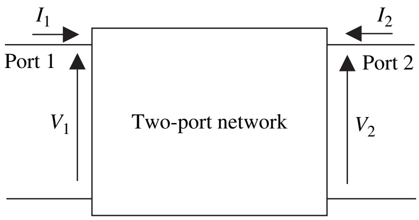


Figure 1.5 A two-port network

properties of the networks are represented by the relation between the total voltage and current quantities at the network ports. For a two-port network, as shown in Figure 1.5, the Z, Y and $ABCD$ parameters are defined, respectively, as (Ramo, Whinnery and van Duzer, 1984)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \tag{1.31a}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \tag{1.31b}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}, \tag{1.31c}$$

where V_1 and V_2 are the total voltages at Port 1 and Port 2, respectively, and I_1 and I_2 the total currents flowing into the network at Port 1 and Port 2, respectively.

1.2.2 Determination of Z, Y and $ABCD$ Parameters

The network parameters can be determined using open-circuit or short-circuit terminations at the network ports as shown.

The Z parameters can be obtained using open-circuit terminations as

$$\begin{array}{ll} Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} & Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \\ Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} & Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \end{array}. \tag{1.32}$$

Similarly, the Y parameters can be obtained using short-circuit terminations as

$$\begin{aligned} Y_{11} &= \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \\ Y_{21} &= \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \end{aligned}. \tag{1.33}$$

The $ABCD$ parameters can be obtained using the combination of open-circuit and short-circuit terminations as

$$\begin{aligned} A &= \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \\ C &= \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \end{aligned}. \tag{1.34}$$

1.2.3 Properties of Z, Y and $ABCD$ Parameters

For a reciprocal network,

$$Z_{12} = Z_{21} \quad Y_{12} = Y_{21} \quad AD - BC = 1. \tag{1.35}$$

For a symmetrical network,

$$Z_{11} = Z_{22} \quad Y_{11} = Y_{22} \quad A = D. \tag{1.36}$$

1.2.4 Scattering Parameters

By using Z, Y and *ABCD* parameters, the properties of a two-port network can be described in terms of total voltages and currents at input and output ports of the network. At low frequencies, the voltages and currents can be easily measured so that the Z, Y and $ABCD$ parameters can be determined. At microwave frequencies, however, the voltage and current are difficult to measure due to lack of instrument and also the fact that voltage and current are not always well defined. However, microwave power is relatively easy to measure. A more satisfactory approach is to use variables relating to the incident and reflected quantities. Scattering parameters are then introduced to describe microwave networks. The scattering parameters are easier to measure than voltage or current.

As for the one-port network discussed in Section 1.1.2, the total voltage at each port can be expressed as the sum of an incident voltage and a 'reflected' voltage, and the total current as the difference of an incident current and a 'reflected' current, i.e.

$$\begin{aligned} V_1 &= V_1^+ + V_1^- \quad I_1 = I_1^+ - I_1^- \\ V_2 &= V_2^+ + V_2^- \quad I_2 = I_2^+ - I_2^- \end{aligned} \tag{1.37}$$

as shown in Figure 1.6.

In general, the source connected to the network can have different internal impedances. However, throughout this book for all two-port networks, it is considered that the sources connected to the network have the same reference internal impedance of $Z_{0,\mathrm{ref}} = 50\ \Omega$. It should be noted that V_1^- and I_1^- include those produced by V_{s2} and V_2^- and I_2^- include those produced by V_{s1}. The incident and 'reflected' voltages and currents satisfy the relation

$$\frac{V_1^+}{I_1^+} = \frac{V_1^-}{I_1^-} = \frac{V_2^+}{I_2^+} = \frac{V_2^-}{I_2^-} = Z_{0,\mathrm{ref}}. \tag{1.38}$$

Unlike Z, Y and $ABCD$ parameters, scattering parameters do depend on the reference internal impedance chosen.

With the introduction of a_1, b_1, a_2 and b_2 quantities,

$$\begin{aligned} a_1 &= \frac{V_1^+}{\sqrt{Z_{0,\mathrm{ref}}}} = I_1^+\sqrt{Z_{0,\mathrm{ref}}}\ b_1 = \frac{V_1^-}{\sqrt{Z_{0,\mathrm{ref}}}} = I_1^-\sqrt{Z_{0,\mathrm{ref}}} \\ a_2 &= \frac{V_2^+}{\sqrt{Z_{0,\mathrm{ref}}}} = I_2^+\sqrt{Z_{0,\mathrm{ref}}}\ b_2 = \frac{V_2^-}{\sqrt{Z_{0,\mathrm{ref}}}} = I_2^-\sqrt{Z_{0,\mathrm{ref}}} \end{aligned}, \tag{1.39}$$

a new set of parameters, i.e. scattering parameters

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix},$$

can be defined. The scattering parameters or S-parameters relate b_1 and b_2 to a_1 and a_2 as (Collin, 1966; Pozar, 1990)

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}. \tag{1.40}$$

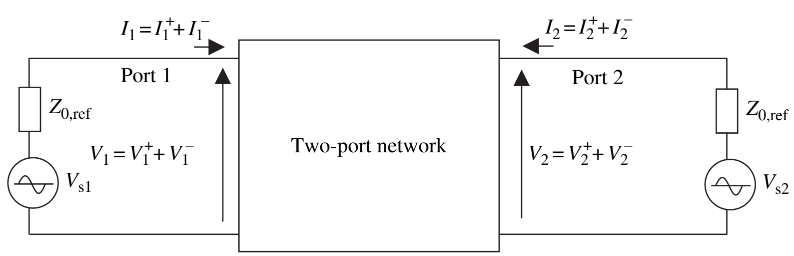


Figure 1.6 A two-port network with external sources

1.2.5 Determination of *S*-Parameters

The *S*-parameters can be determined by connecting one of the ports to a source with reference internal impedance $Z_{0,\mathrm{ref}}$ and terminating at the other port with a matched load, i.e. $Z_\mathrm{L} = Z_{0,\mathrm{ref}}$, as follows:

$$\begin{array}{ll} S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} & S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \\ S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} & S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \end{array}. \quad (1.41)$$

The *S*-parameters are generally complex parameters and are often represented in terms of amplitude and phase.

1.2.6 Total Voltages and Currents in Terms of *a* and *b* Quantities

Using Equation (1.39), the total voltages and currents of the two-port network can be expressed in terms of a_1, a_2, b_1 and b_2 as

$$\begin{array}{ll} V_1 = (a_1 + b_1)\sqrt{Z_{0,\mathrm{ref}}} & I_1 = (a_1 - b_1)/\sqrt{Z_{0,\mathrm{ref}}} \\ V_2 = (a_2 + b_2)\sqrt{Z_{0,\mathrm{ref}}} & I_2 = (a_2 - b_2)/\sqrt{Z_{0,\mathrm{ref}}} \end{array}. \quad (1.42)$$

It can be shown that a_1, a_2, b_1 and b_2 can also be written as

$$\begin{array}{ll} a_1 = \frac{1}{2}\left(\frac{V_1}{\sqrt{Z_{0,\mathrm{ref}}}} + I_1\sqrt{Z_{0,\mathrm{ref}}}\right) & b_1 = \frac{1}{2}\left(\frac{V_1}{\sqrt{Z_{0,\mathrm{ref}}}} - I_1\sqrt{Z_{0,\mathrm{ref}}}\right) \\ a_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{Z_{0,\mathrm{ref}}}} + I_2\sqrt{Z_{0,\mathrm{ref}}}\right) & b_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{Z_{0,\mathrm{ref}}}} - I_2\sqrt{Z_{0,\mathrm{ref}}}\right) \end{array}. \quad (1.43)$$

1.2.7 Power in Terms of *a* and *b* Quantities

The incident power at Port 1 and Port 2 is, respectively,

$$P_1^+ = \tfrac{1}{2}|a_1|^2 \; P_2^+ = \tfrac{1}{2}|a_2|^2 \quad (1.44\mathrm{a})$$

and the 'reflected' power from Port 1 and Port 2 is

$$P_1^- = \tfrac{1}{2}|b_1|^2 \; P_2^- = \tfrac{1}{2}|b_2|^2. \quad (1.44\mathrm{b})$$

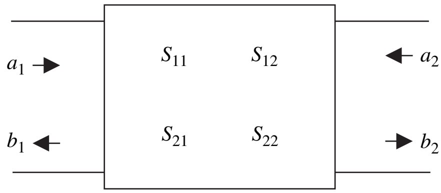


Figure 1.7 A two-port network illustrated using a_1, a_2, b_1 and b_2

The power lost at the network is therefore

$$P_{\text{Loss}} = P_1^+ + P_2^+ - P_1^- P_2^- = \frac{1}{2}(|a_1|^2 + |a_2|^2 - |b_1|^2 - |b_2|^2). \quad (1.45)$$

Using a_1, a_2, b_1 and b_2 quantities, the power flow of the two-port network can be illustrated as shown in Figure 1.7.

1.2.8 Signal Flow Chart

The power flow can also be represented graphically using a signal flow chart (Pozar, 1990) as shown in Figure 1.8. The chart has four nodes and four branches. The a nodes represent the incoming signals and b nodes the 'reflected' signals. The branches represent the signal flow along the indicated arrow directions, so that Equation (1.40) can be directly written from the graphical representation.

1.2.9 Properties of S-Parameters

The S-parameters have the following properties. $S_{11} = 0$ when Port 1 of the network is matched to the reference internal impedance of the source. Similarly $S_{22} = 0$ when Port 2 of the network is matched to the reference internal impedance of the source.

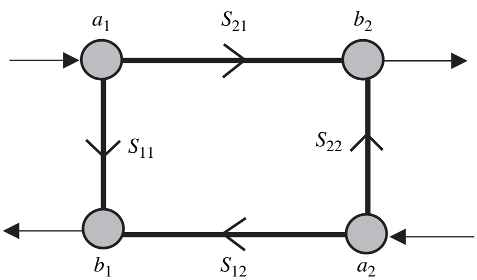


Figure 1.8 Signal flow chart

For a reciprocal two-port network,

$$S_{21} = S_{12}. \tag{1.46a}$$

For a lossless two-port network (Liao, 1990),

$$[S][S^*] = [I] \tag{1.46b}$$

or

$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 &= 1 \\ |S_{12}|^2 + |S_{22}|^2 &= 1 \\ S_{11}{}^*S_{12} + S_{21}{}^*S_{22} &= 0 \\ S_{12}{}^*S_{11} + S_{22}{}^*S_{21} &= 0 \end{aligned} \tag{1.46c}$$

where $[I]$ is a unit matrix of rank 2.

1.2.10 Power Flow in a Terminated Two-Port Network

Consider a two-port network connected to a source with a reference internal impedance $Z_{0,\text{ref}}$ as shown in Figure 1.9. The power flow in the terminated two-port network in general depends not only on the S-parameters of the network, but also on the load impedance Z_L. When $Z_L = Z_{0,\text{ref}}$, the power delivered to the load is fully absorbed. No reflection will take place at the termination, i.e. $a_2 = 0$. The returned power from Port 1 of the terminated network is then

$$P_{\text{return}} = P_{\text{max}}|S_{11}|^2, \tag{1.47a}$$

where $P_{\text{max}} = P_1^+$ is the maximum available power from the source, as given in Equation (1.44a), and the power received by the load is

$$P_{\text{load}} = P_{\text{max}}|S_{21}|^2. \tag{1.47b}$$

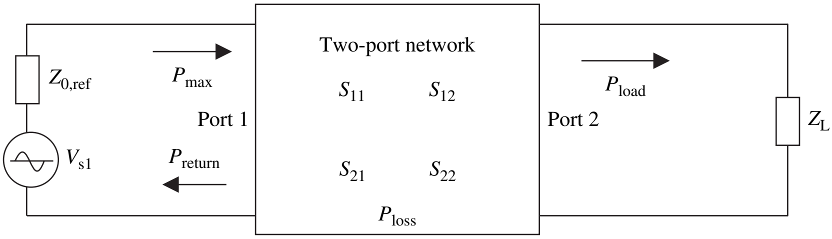


Figure 1.9 A terminated two-port network with an external source connected to Port 1

Hence the input power to the terminated network and the power lost in the network are, respectively,

$$P_{\text{in}} = P_{\text{max}}(1 - |S_{11}|^2) \tag{1.48a}$$

and

$$P_{\text{loss}} = P_{\text{max}}(1 - |S_{11}|^2 - |S_{21}|^2). \tag{1.48b}$$

The power gain of the two-port network for a matched termination can be obtained as

$$G_{\text{m}} = \frac{P_{\text{load}}}{P_{\text{in}}} = \frac{|S_{21}|^2}{(1 - |S_{11}|^2)} \tag{1.49a}$$

and the transducer power gain is given by

$$G_{\text{Tm}} = \frac{P_{\text{load}}}{P_{\text{max}}} = |S_{21}|^2. \tag{1.49b}$$

The insertion loss due to the two-port network is therefore

$$\text{IL}_{\text{m}} = \frac{P_{\text{max}}}{P_{\text{load}}} = \frac{1}{|S_{21}|^2} \quad \text{or} \quad \text{IL}_{\text{m}}(\text{dB}) = -20\log_{10}(|S_{21}|). \tag{1.50}$$

In the general case that $Z_{\text{L}} \neq Z_{0,\text{ref}}$, a reflection will take place at the termination, and the reflection coefficient with respect to $Z_{0,\text{ref}}$ is

$$\Gamma_{\text{L}} = \frac{Z_{\text{L}} - Z_{0,\text{ref}}}{Z_{\text{L}} + Z_{0,\text{ref}}}. \tag{1.51}$$

Using Equation (1.40) and applying Equation (1.17) to the terminated load with $a_2 = \Gamma_{\text{L}} b_2$,

$$b_1 = \left(S_{11} + \frac{\Gamma_{\text{L}} S_{21} S_{12}}{1 - \Gamma_{\text{L}} S_{22}} \right) a_1 \tag{1.52a}$$

and

$$b_2 = \frac{a_2}{\Gamma_{\text{L}}} = \frac{S_{21}}{1 - \Gamma_{\text{L}} S_{22}} a_1. \tag{1.52b}$$

can be obtained. Hence the returned power from the terminated network is

$$P_{\text{return}} = P_{\text{max}} \left| S_{11} + \frac{\Gamma_{\text{L}} S_{21} S_{12}}{1 - \Gamma_{\text{L}} S_{22}} \right|^2 \tag{1.53a}$$

and the power received by the load, with the consideration of reflection, is

$$P_{\text{load}} = P_{\max}\frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-\Gamma_L S_{22}|^2}. \tag{1.53b}$$

The input power to the network is then

$$P_{\text{in}} = P_{\max}\left(1-\left|S_{11}+\frac{\Gamma_L S_{21}S_{12}}{1-\Gamma_L S_{22}}\right|^2\right) \tag{1.54a}$$

and the power lost in the network is

$$P_{\text{loss}} = P_{\max}\left(1-\left|S_{11}+\frac{\Gamma_L S_{21}S_{12}}{1-\Gamma_L S_{22}}\right|^2-\left|\frac{S_{21}}{1-\Gamma_L S_{22}}\right|^2\right). \tag{1.54b}$$

With the earlier expression for power on the terminated network, the power gain of the two-port network can be obtained as

$$G = \frac{P_{\text{load}}}{P_{\text{in}}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2\left(1-\left|S_{11}+\frac{\Gamma_L S_{21}S_{12}}{1-S_{22}\Gamma_L}\right|^2\right)} \tag{1.55a}$$

and the transducer power gain is given by

$$G_T = \frac{P_{\text{load}}}{P_{\max}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2}. \tag{1.55b}$$

The insertion loss due to the two-port network is then

$$\text{IL} = \frac{P_{\text{load,direct}}}{P_{\text{load}}} = \frac{|1-\Gamma_L S_{22}|^2}{|S_{21}|^2}, \tag{1.56}$$

where $P_{\text{load,direct}}$ is the power received by the load when it is directly connected to the source and $P_{\text{load,direct}} = P_{\max}(1-|\Gamma_L|)^2$.

1.3 CONVERSIONS BETWEEN *Z*, *Y* AND *ABCD* AND *S*-PARAMETERS

S-parameters can be converted from *Z*, *Y* and *ABCD* parameters and vice versa. The conversions are shown in Tables 1.1 and 1.2, respectively.

Table 1.1 Conversions from *Z*, *Y* and *ABCD* to S-parameters

Z	***Y***	***ABCD***
$S_{11} = \frac{(Z_{11} - Z_{0,\mathrm{ref}})(Z_{22} + Z_{0,\mathrm{ref}}) - Z_{12}Z_{21}}{(Z_{11} + Z_{0,\mathrm{ref}})(Z_{22} + Z_{0,\mathrm{ref}}) - Z_{12}Z_{21}}$	$S_{11} = \frac{(Y_{0,\mathrm{ref}} - Y_{11})(Y_{0,\mathrm{ref}} + Y_{22}) + Y_{12}Y_{21}}{(Y_{0,\mathrm{ref}} + Y_{11})(Y_{0,\mathrm{ref}} + Y_{22}) - Y_{12}Y_{21}}$	$S_{11} = \frac{A + B/Z_{0,\mathrm{ref}} - CZ_{0,\mathrm{ref}} - D}{A + B/Z_{0,\mathrm{ref}} + CZ_{0,\mathrm{ref}} + D}$
$S_{12} = \frac{2Z_{12}Z_{0,\mathrm{ref}}}{(Z_{11} + Z_{0,\mathrm{ref}})(Z_{22} + Z_{0,\mathrm{ref}}) - Z_{12}Z_{21}}$	$S_{12} = \frac{-2Y_{12}Y_{0,\mathrm{ref}}}{(Y_{0,\mathrm{ref}} + Y_{11})(Y_{0,\mathrm{ref}} + Y_{22}) - Y_{12}Y_{21}}$	$S_{12} = \frac{2(AD - BC)}{A + B/Z_{0,\mathrm{ref}} + CZ_{0,\mathrm{ref}} + D}$
$S_{21} = \frac{2Z_{21}Z_{0,\mathrm{ref}}}{(Z_{11} + Z_{0,\mathrm{ref}})(Z_{22} + Z_{0,\mathrm{ref}}) - Z_{12}Z_{21}}$	$S_{21} = \frac{-2Y_{21}Y_{0,\mathrm{ref}}}{(Y_{0,\mathrm{ref}} + Y_{11})(Y_{0,\mathrm{ref}} + Y_{22}) - Y_{12}Y_{21}}$	$S_{12} = \frac{2}{A + B/Z_{0,\mathrm{ref}} + CZ_{0,\mathrm{ref}} + D}$
$S_{22} = \frac{(Z_{11} + Z_{0,\mathrm{ref}})(Z_{22} - Z_{0,\mathrm{ref}}) - Z_{12}Z_{21}}{(Z_{11} + Z_{0,\mathrm{ref}})(Z_{22} + Z_{0,\mathrm{ref}}) - Z_{12}Z_{21}}$	$S_{22} = \frac{(Y_{0,\mathrm{ref}} + Y_{11})(Y_{0,\mathrm{ref}} - Y_{22}) + Y_{12}Y_{21}}{(Y_{0,\mathrm{ref}} + Y_{11})(Y_{0,\mathrm{ref}} + Y_{22}) - Y_{12}Y_{21}}$	$S_{22} = \frac{-A + B/Z_{0,\mathrm{ref}} - CZ_{0,\mathrm{ref}} + D}{A + B/Z_{0,\mathrm{ref}} + CZ_{0,\mathrm{ref}} + D}$

Table 1.2 Conversions from S-parameters to Z, Y and $ABCD$

Z	**Y**	**ABCD**
$Z_{11} = Z_{0,\mathrm{ref}} \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	$Y_{11} = Y_{0,\mathrm{ref}} \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$A = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$
$Z_{12} = Z_{0,\mathrm{ref}} \frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	$Y_{12} = Y_{0,\mathrm{ref}} \frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$B = Z_{0,\mathrm{ref}} \frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$
$Z_{21} = Z_{0,\mathrm{ref}} \frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	$Y_{21} = Y_{0,\mathrm{ref}} \frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$C = \frac{1}{Z_{0,\mathrm{ref}}} \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$
$Z_{22} = Z_{0,\mathrm{ref}} \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	$Y_{22} = Y_{0,\mathrm{ref}} \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$D = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$

1.4 SINGLE IMPEDANCE TWO-PORT NETWORK

A single impedance network in one-port connection has been dealt with in Section 1.1. The consideration here is made for two-port connections. There are two possible two-port configurations as discussed below.

1.4.1 *S*-Parameters for Single Series Impedance

For a single series impedance two-port network as shown in Figure 1.10, the *S*-parameters can be obtained as

$$S_{11} = S_{22} = \frac{1}{1 + 2Z_{0,\mathrm{ref}}/Z} = \frac{1}{1 + 2Z_{0,\mathrm{ref}}Y} \tag{1.57a}$$

and

$$S_{21} = S_{12} = \frac{1}{1 + Z/(2Z_{0,\mathrm{ref}})}, \tag{1.57b}$$

where Z is the impedance of the series component and $Y = 1/Z$.

1.4.2 *S*-Parameters for Single Shunt Impedance

For a single shunt impedance two-port network as shown in Figure 1.11, the *S*-parameters can be written as

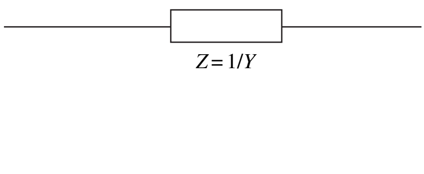


Figure 1.10 Single series impedance two-port network

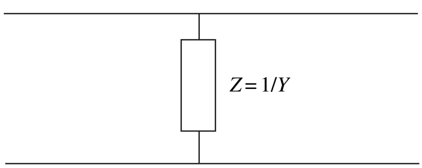


Figure 1.11 Single shunt impedance two-port network

$$-S_{11} = -S_{22} = \frac{1}{1 + 2Y_{0,\mathrm{ref}}Z} = \frac{1}{1 + 2Y_{0,\mathrm{ref}}/Y} \tag{1.58a}$$

and

$$S_{21} = S_{12} = \frac{1}{1 + Y/(2Y_{0,\mathrm{ref}})}, \tag{1.58b}$$

where Z is the impedance of the shunt component and $Y = 1/Z$.

1.4.3 Two-Port Chart

The Smith chart described in Section 1.1.3 is useful for one-port networks where constant resistance and reactance circles can be identified. A similar chart for two-port networks would be useful when a two-port network can be equivalently described by a single impedance network in either series or shunt connections as shown in Figure 1.10 or 1.11. A two-port chart for single impedance network is now introduced (Wu, 2001).

1.4.3.1 Single series impedance network

By introducing S and $(a + jb)$ as defined in Table 1.3, Equations (1.57a) and (1.57b) can be written as

$$S = u + jv = \frac{1}{1 + a + jb} = \frac{1}{A + jB}, \tag{1.59a}$$

where

$$A = 1 + a \text{ and } B = b. \tag{1.59b}$$

Equation (1.59a) can be rearranged to give the following two independent equations:

$$\left(u - \frac{1}{2A}\right)^2 + v^2 = \frac{1}{(2A)^2} \tag{1.60a}$$

Table 1.3 Definition of S and $(a + jb)$ for a single series impedance network

$S = u + jv = \lvert S \rvert e^{j\phi}$	$a + jb$
S_{11} or S_{22}	$(2Z_{0,\mathrm{ref}})Y$
S_{21} or S_{12}	$Z/(2Z_{0,\mathrm{ref}})$

and

$$u^2 + \left(v + \frac{1}{2B}\right)^2 = \frac{1}{(2B)^2}. \qquad (1.60b)$$

Equation (1.60a) represents a family of circles, on each of which $A = 1 + a$ is a constant, with a centre at $(1/(2A), 0)$ and a radius $1/(2A)$. Equation (1.60b) describes a family of circles, on each of which $B = b$ is a constant, with a centre at $(0, 1/(2B))$ and a radius $1/(2B)$. Plotting Equations (1.60a) and (1.60b) gives the two-port chart as shown in Figure 1.12.

For a single series impedance two-port network, the two-port charts for S_{11} and S_{21} are identical. For a passive impedance with $\mathrm{Re}(Z) > 0$, A is greater than or equal to 1. The impedance point on the chart will only be on the right-hand side of the v axis. If S_{11} or S_{21} is plotted on the chart so that the A and B values can be read or determined, the impedance Z or admittance Y can be found from the expressions given in Table 1.4. It can be seen from Table 1.4 that the S_{11} chart is most suitable for finding the admittance Y, but the S_{21} chart is most suitable for obtaining the impedance Z.

1.4.3.2 Single shunt impedance network

With the definition of S and $(a + jb)$ for a single shunt impedance two-port network, as given in Table 1.5, Equation (1.58a) can be written as

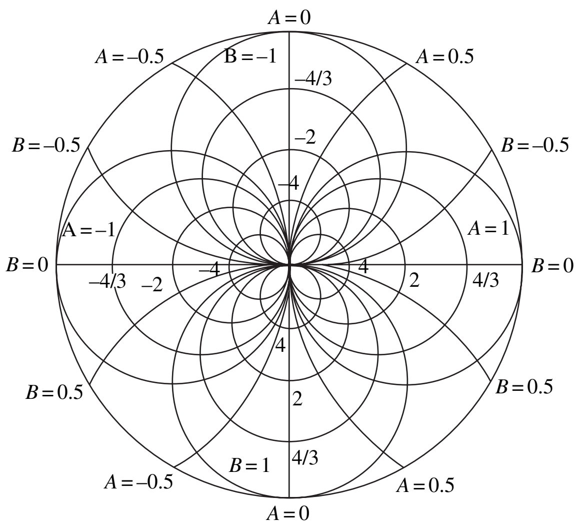


Figure 1.12 The two-port chart for a single impedance network

Table 1.4 Relation between $A+\mathrm{j}B$ and Z or Y

$S=\vert S\vert e^{\mathrm{j}\phi}$	**Z or Y**
S_{11} or S_{22} chart	$Y=[(A-1)+\mathrm{j}B)/(2Z_{0,\mathrm{ref}})$
S_{21} or S_{12} chart	$Z=[(A-1)+\mathrm{j}B](2Z_{0,\mathrm{ref}})$

Table 1.5 Definition of S and $(a+\mathrm{j}b)$ for a single shunt impedance network

$S=u+\mathrm{j}v=\vert S\vert e^{\mathrm{j}\phi}$	$a+\mathrm{j}b$
S_{11} or S_{22}	$(2Y_{0,\mathrm{ref}})Z$
S_{21} or S_{12}	$Y/(2Y_{0,\mathrm{ref}})$

$$-S_{11}=-S_{22}=-(u+\mathrm{j}v)=\frac{1}{1+a+\mathrm{j}b}=\frac{1}{A+\mathrm{j}B} \tag{1.61a}$$

and Equation (1.58b) as

$$S_{21}=S_{12}=(u+\mathrm{j}v)=\frac{1}{1+a+\mathrm{j}b}=\frac{1}{A+\mathrm{j}B}, \tag{1.61b}$$

where $A=1+a$ and $B=b$ as defined in Equation (1.61b). Since Equation (1.61b) is identical to Equation (1.59a), the chart shown in Figure 1.12 is applicable to S_{21} and S_{12}, but with a different definition of $a+\mathrm{j}b$ as given in Table 1.5. Equation (1.61.a) is, however, different from Equation (1.59a). It can be rearranged to give the following two independent equations:

$$\left(u+\frac{1}{2A}\right)^2+v^2=\frac{1}{(2A)^2} \tag{1.62a}$$

and

$$u^2+\left(v-\frac{1}{2B}\right)^2=\frac{1}{(2B)^2}. \tag{1.62b}$$

Equations (1.62a) and (1.62b), respectively, represent the same families of circles as Equations (1.60a) and (1.60b) except that the constant A circles now have a centre at $(-1/(2A),0)$ and constant B circles a centre at $(0,-1/(2B))$ and a radius $1/2B$. Hence the chart shown in Figure 1.12 is generally valid for a single shunt impedance two-port network, but with the understanding that A and B change signs for the S_{11} or S_{22} chart, and $(a+\mathrm{j}b)$ is defined in Table 1.5 rather than in Table 1.3.

For a single shunt impedance two-port network, the two-port charts for S_{11} and S_{21} have 180° rotational symmetry. This is different from that for a single series impedance two-port network where the S_{11} and S_{21} charts are identical. This property can be used to identify whether the single impedance in two-port network is in series or shunt connection. For a passive impedance with $\mathrm{Re}(Z) > 0$, A is greater than or equal to 1. The impedance point on the S_{11} chart will only be on the left-hand side of the v axis. If S_{11} or S_{21} is plotted on the chart so that the A and B values can be read or determined, the impedance Z or admittance Y can be found from the expression in Table 1.6. It can be seen from Table 1.6 that the S_{11} chart is most suitable for finding the impedance Z, but the S_{21} chart is most suitable for obtaining the admittance Y.

1.4.3.3 Scaling property

In the two-port chart shown in Figure 1.12, which can be used as either transmission or reflection chart, the outermost circle corresponds to $|S| = 1$, i.e. the chart has a unity scale. Hence for small values of S-parameters, the points plotted on the chart will be concentrated at the centre of the chart. On the other hand, the amplitude of the S-parameter may be greater than unity. It is therefore useful to be able to change the scale of the chart so that the S-parameter of interest can be better displayed. This can be done using the scaling property of the two-port chart as described below.

If the scale of the outermost circle is increased or decreased from unity to M so that on the circle $|S| = M$, the length of the displayed S-parameter, $S = u + \mathrm{j}v$, on the chart will decrease or increase accordingly. The S-parameter is thus scaled to $S' = (1/M) = U + \mathrm{j}V$ with $U = (1/M)u$ and $V = (1/M)v$. It can be shown using Equations (1.59a), (1.61a) and (1.61b) that U and V satisfy the following equations:

$$\left(U \pm \frac{1}{2A'}\right)^2 + V^2 = \frac{1}{(2A')^2} \tag{1.63a}$$

and

$$U^2 + \left(V \pm \frac{1}{2B'}\right)^2 = \frac{1}{(2B')^2}, \tag{1.63b}$$

Table 1.6 Relation between $A + \mathrm{j}B$ and Z or Y

$\boldsymbol{S = \lvert S\rvert e^{j\phi}}$	**Z or Y**
S_{11} or S_{22} chart	$Z = [(A-1) + \mathrm{j}B]/(2Y_{0,\mathrm{ref}})$
S_{21} or S_{12} chart	$Y = [(A-1) + \mathrm{j}B](2Y_{0,\mathrm{ref}})$

where

$$A' = AM, \ B' = BM \tag{1.64a}$$

and

$$A = \frac{A'}{M}, \ B = \frac{B'}{M}. \tag{1.64b}$$

Equation (1.63a) is identical to Equation (1.60a) or (1.62a) and Equation (1.63b) to Equation (1.60b) or (1.62b). Hence, the same chart can be used for the scaled S-parameter, with the scale of the chart changed to the amplitude of S-parameter represented at the outermost circle. The A' and B' values for the scaled S-parameter can be read on the unity chart as if there were no scaling. The actual values of A and B can be obtained using Equation (1.64b). Alternatively when the scale is changed from unity, the A and B values shown on the chart are updated to the corresponding A' and B' values as given in Equation (1.64a). The values read on the chart will then be the A and B values directly. The impedance or admittance values, i.e. Z or Y, can be found using the expressions in Table 1.4 or 1.6.

1.4.4 Applications of Two-Port Chart

1.4.4.1 Identification of pure resonance

For a pure RLC resonance, $(a + \mathrm{j}b)$ is related to the resonance frequency f_0 and the unloaded quality factor Q_0. Assuming the following equation can be established from Table 1.3 or 1.5 for a given resonance,

$$a + \mathrm{j}b = a\left(1 + \mathrm{j}Q_0\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right) = a(1 + \mathrm{j}Q_0\delta(f)), \tag{1.65}$$

Equations (1.59a), (1.61a) and (1.61b) can be written as

$$\pm S(f) = \pm(u + \mathrm{j}v) = \frac{1}{A + \mathrm{j}B} = \frac{1/A}{1 + \mathrm{j}B/A} = \frac{S_0}{1 + \mathrm{j}Q_L\delta(f)}, \tag{1.66a}$$

where the '–' sign applies to Equation (1.59a),

$$S_0 = \pm S(f_0) = \frac{1}{1 + a} = \frac{1}{A} \tag{1.66b}$$

and Q_L is the loaded quality factor and

$$Q_L = (1 - |S_0|)Q_0 \text{ or } Q_0 = \frac{Q_L}{(1 - |S_0|)}. \tag{1.66c}$$

For a pure resonance, A is a constant and B changes with frequency. Hence the trace of the resonance on a two-port chart discussed in Section 1.4.3 would be a pure circle on which $A = 1 + a$ as the frequency changes from 0 to ∞. This property can be used to identify whether the resonance is a pure RLC resonance or not.

1.4.4.2 *Q*-factor measurements

For a pure resonance, as indicated in Equation (1.66a), the loaded quality factor Q_L can be measured at frequencies f_1 and f_2 on the $A = \pm B$ lines. At these frequencies,

$$Q_L \delta(f_{1,2}) = 1, \tag{1.67}$$

i.e. $|S(f_1)|^2$ and $|S(f_2)|^2$ fall to one-half of the $|S_0|^2$ value at the resonance frequency f_0, and $S(f_1)$ and $S(f_2)$ have a phase shift of 45° from the phase of $S(f_0)$ at the resonance frequency. With a further measurement of $|S_0|$, the unloaded quality factor Q_0 can be obtained using Equation (1.66c). The only parameter that remains to be defined is the suitable S-parameter. The S-parameter valid for the assumption in Equation (1.65) leading to Equation (1.66) is listed in Table 1.7.

1.4.4.3 Resonance with power-dependent losses

When the resistance or its equivalence in the RLC series/parallel circuit is power dependent or nonlinear with respect to the power loss on the resonance circuit which happens, for example, in a high-temperature

Table 1.7 Suitable S-parameter for Q-factor measurement

	Single series impedance two-port network		**Single shunt impedance two-port network**	
Resonance type	RLC series	RLC parallel	RLC series	RLC parallel
S_{21} response	Bandpass	Bandstop	Bandstop	Bandpass
S-parameter for Q-factor measurement	S_{21} or S_{12}	S_{11} or S_{22}	S_{11} or S_{22}	S_{21} or S_{12}

superconducting resonator, the power-dependent resonance can be observed from the two-port chart. The trace of the resonance will be symmetrical about the $v$axis at frequencies around f_0, but will divert from the constant A circle. Such a property can be used to identify power-dependent resonance of a resonator or a resonant circuit.

1.4.4.4 Impedance or admittance measurement using the two-port chart

The S_{21} chart may be used as an alternative to the Smith chart for the load impedance or admittance measurement of a one-port network. In this case, the transmitted signal is measured, rather than the reflected signal, which may give practical advantages.

The series configuration shown in Figure 1.10 can be used for the impedance measurement using the S_{21} chart. The impedance $Z = R_Z + \mathrm{j}X_Z$ can be obtained from the chart using

$$R_Z = (A-1)(2Z_{0,\mathrm{ref}}) \text{ and } X_Z = 2Z_{0,\mathrm{ref}}B. \qquad (1.68)$$

The shunt configuration shown in Figure 1.11 can be used for the admittance measurement using the S_{21} chart. The admittance $Y = G_Y + \mathrm{j}B_Y$ can be obtained from the chart using

$$G_Y = (A-1)(2Y_{0,\mathrm{ref}}) \text{ and } B_Y = B(2Y_{0,\mathrm{ref}}). \qquad (1.69)$$

1.5 *S*-PARAMETERS OF COMMON ONE- AND TWO-PORT NETWORKS

For convenience, the *S*-parameter for a number of common one- and two-port networks are given in Table 1.8.

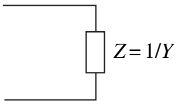
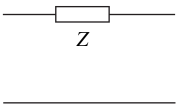
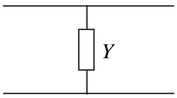
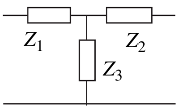
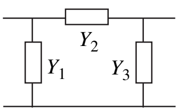
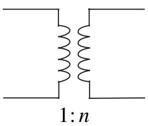
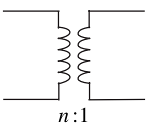
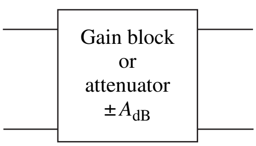
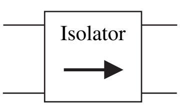
1.6 CONNECTED TWO-PORT NETWORKS

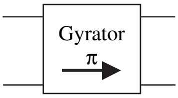
1.6.1 T-Junction

When one arm of the T-junction is connected to a shunt impedance or a network with a reflection coefficient Γ_T with respect to $Z_{0,\mathrm{ref}}$ forming a two-port network as shown in Figure 1.13, the resultant *S*-parameters are given by

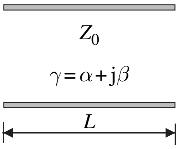
$$[S] = \frac{\begin{bmatrix} \Gamma_\mathrm{T}-1 & 2(1+\Gamma_\mathrm{T}) \\ 2(1+\Gamma_\mathrm{T}) & \Gamma_\mathrm{T}-1 \end{bmatrix}}{3+\Gamma_\mathrm{T}}. \qquad (1.70)$$

Table 1.8 *S*-parameters of common one- and two-port networks

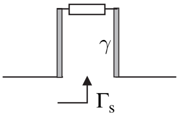
Network	S-parameters
	$\Gamma_L = \frac{Z_L - Z_{0,ref}}{Z_L + Z_{0,ref}} = \frac{Y_{0,ref} - Y_L}{Y_{0,ref} + Y_L}$
	$[S] = \frac{\begin{bmatrix} Z & 2Z_{0,ref} \\ 2Z_{0,ref} & Z \end{bmatrix}}{Z + 2Z_{0,ref}}$
	$[S] = \frac{\begin{bmatrix} -Y & 2Y_{0,ref} \\ 2Y_{0,ref} & -Y \end{bmatrix}}{Y + 2Y_{0,ref}}$
	$[S] = \frac{\begin{bmatrix} -Z_{0,ref}^2 + (Z_1 - Z_2)Z_{0,ref} + Z_1Z_2 + Z_2Z_3 + Z_3Z_1 & 2Z_{0,ref}Z_3 \\ 2Z_{0,ref}Z_3 & -Z_{0,ref}^2 + (Z_2 - Z_1)Z_{0,ref} + Z_1Z_2 + Z_2Z_3 + Z_3Z_1 \end{bmatrix}}{Z_{0,ref}^2 + (Z_1 + Z_2 + 2Z_3)Z_{0,ref} + Z_1Z_2 + Z_2Z_3 + Z_3Z_1}$
	$[S] = \frac{\begin{bmatrix} -Y_{0,ref}^2 - (Y_1 - Y_2)Y_{0,ref} - (Y_1Y_2 + Y_2Y_3 + Y_3Y_1) & 2Y_{0,ref}Y_3 \\ 2Y_{0,ref}Y_3 & Y_{0,ref}^2 - (Y_2 - Y_1)Y_{0,ref} - (Y_1Y_2 + Y_2Y_3 + Y_3Y) \end{bmatrix}}{Y_{0,ref}^2 + (Y_1 + Y_2 + 2Y_3)Y_{0,ref} + Y_1Y_2 + Y_2Y_3 + Y_3Y_1}$
	$[S] = \frac{\begin{bmatrix} 1 - n^2 & 2n \\ 2n & n^2 - 1 \end{bmatrix}}{1 + n^2}$
	$[S] = \frac{\begin{bmatrix} n^2 - 1 & 2n \\ 2n & 1 - n^2 \end{bmatrix}}{1 + n^2}$
	$[S] = \begin{bmatrix} 10^{\pm A_{dB}/20} & 0 \\ 0 & 10^{\pm A_{dB}/20} \end{bmatrix}$
	$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$



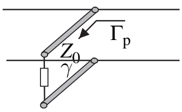
$$[S] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



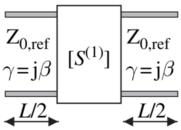
$$[S] = \frac{\begin{bmatrix} (Z_0^2 - Z_{0,\mathrm{ref}}^2)\sinh(\gamma L) & 2Z_{0,\mathrm{ref}}Z_0 \\ 2Z_{0,\mathrm{ref}}Z_0 & (Z_0^2 - Z_{0,\mathrm{ref}}^2)\sinh(\gamma L) \end{bmatrix}}{2Z_{0,\mathrm{ref}}Z_0\cosh(\gamma L) + (Z_0^2 + Z_{0,\mathrm{ref}}^2)\sinh(\gamma L)}$$



$$[S] = \frac{\begin{bmatrix} 1+\Gamma_s & 2(1-\Gamma_s) \\ 2(1-\Gamma_s) & 1+\Gamma_s \end{bmatrix}}{3-\Gamma_s}$$



$$[S] = \frac{\begin{bmatrix} \Gamma_p - 1 & 2(1+\Gamma_p) \\ 2(1+\Gamma_p) & \Gamma_s - 1 \end{bmatrix}}{3+\Gamma_p}$$



$$[S] = \begin{bmatrix} S_{11}^{(1)}e^{-j\theta} & S_{12}^{(1)}e^{-j\theta} \\ S_{21}^{(1)}e^{-j\theta} & S_{22}^{(1)}e^{-j\theta} \end{bmatrix} = [S^{(1)}]e^{-j\theta},\ \theta = \beta L = \frac{\omega}{v_p}L$$

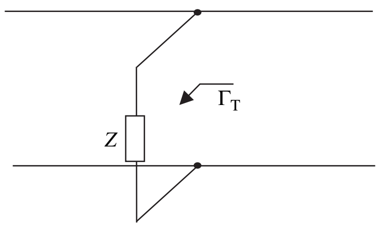


Figure 1.13 T-junction connection

1.6.2 Cascaded Two-Port Networks

For two two-port networks in cascade as shown in Figure 1.14, the resultant S-parameters are

$$[S] = \begin{bmatrix} S_{11}^{(1)} + \dfrac{S_{12}^{(1)}S_{21}^{(1)}S_{11}^{(2)}}{1 - S_{22}^{(1)}S_{11}^{(2)}} & \dfrac{S_{12}^{(1)}S_{12}^{(2)}}{1 - S_{22}^{(1)}S_{11}^{(2)}} \\ \dfrac{S_{21}^{(1)}S_{21}^{(2)}}{1 - S_{22}^{(1)}S_{11}^{(2)}} & S_{22}^{(2)} + \dfrac{S_{12}^{(2)}S_{21}^{(2)}S_{11}^{(1)}}{1 - S_{22}^{(1)}S_{11}^{(2)}} \end{bmatrix}. \tag{1.71}$$

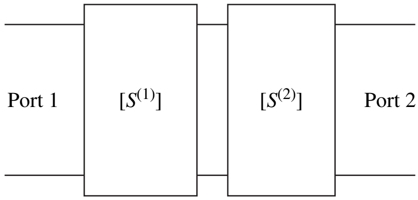


Figure 1.14 Two two-port networks in cascade

When more than two two-port networks are connected in cascade, Equation (1.71) can be used repeatedly to obtain the resultant S-parameters of the cascaded networks.

1.6.3 Two-Port Networks in Series and Parallel Connections

In addition to the cascade connection, two-port networks can also be connected in series or parallel or their combinations, forming a new two-port network. Four configurations are shown in Figure 1.15.

It has been shown that the resultant S-parameters for the above four configurations can be expressed as (Bodharamik, Besser and Newcomb, 1971)

$$[S] = [E]h([E][S^{(1)}], [E][S^{(2)}]), \tag{1.72a}$$

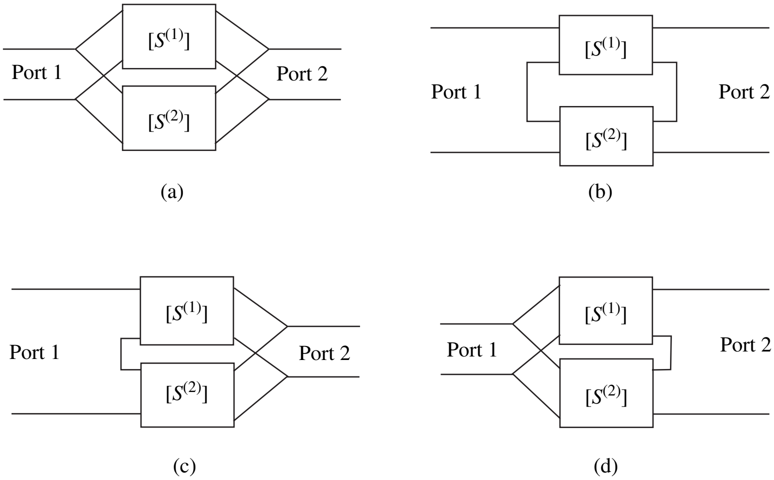


Figure 1.15 Two-port network connection in different configurations: (a) parallel to parallel, (b) series to series, (c) series to parallel and (d) parallel to series

Table 1.9 Matrix $[E]$ for different connections

Port 1	Port 2	[E]
Parallel	Parallel	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Series	Series	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
Series	Parallel	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Parallel	Series	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

where

$$h([S_1],[S_2]) = [A]^{-1}\{[B] + 4[C][S_2]([A] - [B][S_2])^{-1}[C]\} \tag{1.72b}$$

with

$$[A] = 3[I] + [S_1], \tag{1.72c}$$

$$[B] = [S_1] - [I], \tag{1.72d}$$

$$[C] = [S_1] + [I], \tag{1.72e}$$

$$[I] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{1.72f}$$

and $[E]$ given in Table 1.9.

1.7 SCATTERING MATRIX OF MICROWAVE CIRCUITS COMPOSED OF ONE-PORT AND MULTI-PORT DEVICES

1.7.1 *S*-Parameters of a Multi-Port Device

The *S*-parameters defined in Section 1.2.4 is for two-port networks. The analysis can be extended to three or more multi-port devices. The total voltage at the *n*th port can be expressed as the sum of an incident voltage and a 'reflected' voltage and the total current as the difference of an incident current and a 'reflected' current, i.e.

$$V_n = V_n^+ + V_n^- \quad \text{and} \quad I_n = I_n^+ - I_n^-. \tag{1.73}$$

Consider that the sources connected to the multi-port network have the same reference internal impedance of $Z_{0,\mathrm{ref}} = 50\,\Omega$. The incident and 'reflected' voltages and currents satisfy the relation

$$\frac{V_n^+}{I_n^+} = \frac{V_n^-}{I_n^-} = Z_{0,\mathrm{ref}}. \tag{1.74}$$

In the same way as a_1, b_1, a_2 and $b_2$are introduced for two-port networks, a_n and b_n can be introduced to the nth port as

$$a_n = \frac{V_n^+}{\sqrt{Z_{0,\mathrm{ref}}}} = I_n^+\sqrt{Z_{0,\mathrm{ref}}} \quad \text{and} \quad b_n = \frac{V_n^-}{\sqrt{Z_{0,\mathrm{ref}}}} = I_n^-\sqrt{Z_{0,\mathrm{ref}}} \tag{1.75}$$

so that a new set of scattering parameters can be defined, i.e.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{1n} & S_{1N} \\ S_{21} & S_{22} & S_{2n} & S_{2N} \\ S_{n1} & S_{n2} & S_{nn} & S_{nN} \\ S_{N1} & S_{N2} & S_{Nn} & S_{NN} \end{bmatrix} \tag{1.76}$$

for $n = 1$ to N. The multi-port S-parameters relate b_n to a_n by (Collin, 1966; Pozar, 1990)

$$\begin{bmatrix} b_1 \\ b_2 \\ b_n \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{1n} & S_{1N} \\ S_{21} & S_{22} & S_{2n} & S_{2N} \\ S_{n1} & S_{n1} & S_{nn} & S_{nN} \\ S_{N1} & S_{N1} & S_{N1} & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_n \\ a_N \end{bmatrix}. \tag{1.77}$$

1.7.2 *S*-Parameters of a Microwave Circuit

In general, a microwave circuit can be composed of a number of one-port, two-port or multi-port devices, as shown in Figure 1.16. Among the ports, one is connected to an external source with internal impedance $Z_\mathrm{s} = Z_{0,\mathrm{ref}}$, one may be connected to an external load impedance $Z_\mathrm{L} = Z_{0,\mathrm{ref}}$ and the rest are internally connected. Hence the ports that are connected to external source or load impedance are referred to as the external ports and the rest as the internal ports.

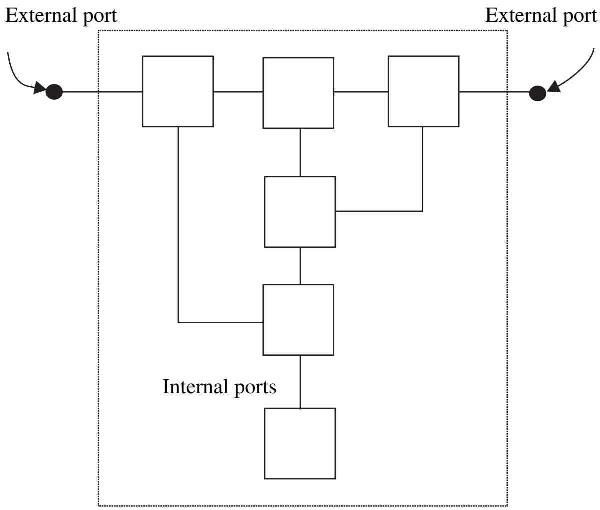


Figure 1.16 Microwave circuit composed of multi-port devices

The signal flow of the circuit can be expressed in terms of S-parameters as $b = Sa$, which can be arranged with the grouping of external and internal ports to

$$\begin{bmatrix} b_e \\ b_i \end{bmatrix} = \begin{bmatrix} S_{ee} & S_{ei} \\ S_{ie} & S_{ii} \end{bmatrix} \begin{bmatrix} a_e \\ a_i \end{bmatrix}, \tag{1.78}$$

where e represents the external ports and i the internal ports. The connections between the ports can be expressed as $b = \Gamma_c a$, where Γ_c is the connection matrix with $\Gamma_{mm} = 0$. The external and internal ports can also be regrouped so that for internally connected ports,

$$b_i = \Gamma_{ii} a_i, \tag{1.79}$$

where $\Gamma_{mn} = 1$, $\Gamma_{nm} = 1$, $a_m = b_n$ and $b_m = a_n$ if ports m and n are connected. Hence

$$a_i = (\Gamma_{ii} - S_{ii})^{-1} S_{ie} a_e \tag{1.80}$$

and

$$b_e = [S_{ee} + S_{ei} (\Gamma_{ii} - S_{ii})^{-1} S_{ie}] a_e \tag{1.81}$$

can be obtained. The new scattering matrix with respect to the external ports is (Dobrowolski, 1991)

$$S_e = S_{ee} + S_{ei} (\Gamma_{ii} - S_{ii})^{-1} S_{ie}. \tag{1.82}$$

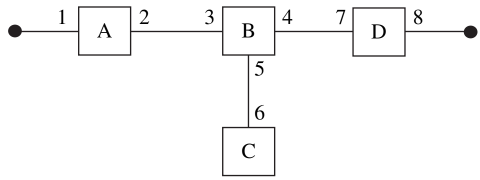


Figure 1.17 Example of a microwave circuit

For example, the circuit shown in Figure 1.17 consists of four circuit blocks or networks A, B, C and D. Among the numbered ports, 1 and 8 are the external ports and 2–7 are the internal ports. The networks in the circuit have the following *S*-parameters:

$$[S_\mathrm{A}]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix};\ [S_\mathrm{B}]=\begin{bmatrix} S_{33} & S_{34} & S_{35} \\ S_{43} & S_{44} & S_{45} \\ S_{53} & S_{54} & S_{55} \end{bmatrix};\ [S_\mathrm{C}]=[S_{66}];\ [S_\mathrm{D}]=\begin{bmatrix} S_{77} & S_{78} \\ S_{87} & S_{88} \end{bmatrix}. \tag{1.83}$$

The signal flow of the circuit can be described by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_8 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ S_{21} & S_{22} & 0 & 0 & 0 & 0 & 0 & S_{87} \\ 0 & 0 & S_{33} & S_{34} & S_{35} & 0 & 0 & 0 \\ 0 & 0 & S_{43} & S_{44} & S_{45} & 0 & 0 & 0 \\ 0 & 0 & S_{53} & S_{54} & S_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{77} & S_{78} \\ 0 & S_{78} & 0 & 0 & 0 & 0 & S_{87} & S_{88} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \tag{1.84}$$

and the connection matrix of the circuit is

$$\Gamma_\mathrm{c} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}. \tag{1.85}$$

Rearranging the external and internal ports gives

$$\begin{bmatrix} b_1 \\ b_8 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} = \begin{bmatrix} S_{11} & 0 & S_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{88} & 0 & 0 & 0 & 0 & 0 & S_{87} \\ S_{21} & 0 & S_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{33} & S_{34} & S_{35} & 0 & 0 \\ 0 & 0 & 0 & S_{43} & S_{44} & S_{45} & 0 & 0 \\ 0 & 0 & 0 & S_{53} & S_{54} & S_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{66} & 0 \\ 0 & S_{78} & 0 & 0 & 0 & 0 & 0 & S_{77} \end{bmatrix} \begin{bmatrix} a_1 \\ a_8 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \tag{1.86}$$

and

$$\Gamma_c = \begin{matrix} & \begin{matrix} 1 & 8 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 8 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix} = \begin{bmatrix} \Gamma_{ee} & \Gamma_{ei} \\ \Gamma_{ie} & \Gamma_{ii} \end{bmatrix} \tag{1.87}$$

so that

$$S_{ee} = \begin{bmatrix} S_{11} & 0 \\ 0 & S_{88} \end{bmatrix}; \; S_{ei} = \begin{bmatrix} S_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{87} \end{bmatrix}; \; S_{ie} = \begin{bmatrix} S_{21} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & S_{78} \end{bmatrix};$$

$$S_{ii} = \begin{bmatrix} S_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{33} & S_{34} & S_{35} & 0 & 0 \\ 0 & S_{43} & S_{44} & S_{45} & 0 & 0 \\ 0 & S_{53} & S_{54} & S_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{77} \end{bmatrix} \tag{1.88}$$

and

$$\Gamma_{ii} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \tag{1.89}$$

With the above matrixes, the S-parameters of the circuit, which is a two-port network, can be obtained using

$$S_e = S_{ee} + S_{ei} (\Gamma_{ii} - S_{ii})^{-1} S_{ie}. \tag{1.90}$$

REFERENCES

Bodharamik, P., Besser, L. and Newcomb, R.W. (1971) 'Two scattering matrix programs for active circuit analysis', *IEEE Transactions on Circuit Theory*, **C-18** (6), 610–9.

Collin, R.E. (1966) *Foundations for Microwave Engineering*, McGraw-Hill, New York.

Dobrowolski, J.A. (1991) *Introduction to Computer Methods for Microwave Circuit Analysis and Design*, Artech House, Boston.

Dobrowolski, J.A. and Ostrowski, W. (1996) *Computer Aided Analysis, Modelling, and Design of Microwave Networks: The Wave Approach, Boston*, Artech House, Boston.

Fooks, H.E. and Zakarev, R.A. (1991) *Microwave Engineering Using Circuits*, Prentice Hall, London.

Gupta, K.C., Garg, R. and Chadha, R. (1981) *Computer Aided Design of Microwave Circuits*, Artech House, Boston.

Ishii, T.K. (1989) *Microwave Engineering*, Harcourt Brace Jovanovich, London.

Liao, S.Y. (1990) *Microwave Devices and Circuits*, 3rd edn, Prentice Hall, London.

Montgomery, C.G., Dicke, R.H. and Purcell, E.M. (1948) *Principles of Microwave Circuits*, Vol. 8 of MITRad. Lab. Series, McGraw-Hill, New York.

Pozar, D.M. (1990) *Microwave Engineering*, Addison-Wesley, New York.

Ramo, S., Whinnery, T.R. and van Duzer, T. (1984) *Fields and Waves in Communication Electronics*, 2nd edn, John Wiley & Sons, Ltd, New York.

Rizzi, P.A. (1988) *Microwave Engineering: Passive Circuits*, Prentice Hall, Englewood Cliffs, NJ.

Smith, P.H. (1939) 'Transmission line calculator', *Electronics*, **12** (1), 29–31.

Smith, P.H. (1944) 'An improved transmission line calculator', *Electronics*, **17** (1), 130–3. See also 318–25.

Wolff, E.A. and Kaul, R. (1988) *Microwave Engineering and Systems Applications*, John Wiley & Sons, Ltd, New York.

Wu, Z. (2001) Transmission and reflection charts for two-port single impedance networks. *IEE Proceedings: Microwaves, Antennas and Propagation*, **146** (6), 351–6.