# 1

# Introduction



## 1.1 Scattering and Diffraction Theory

Defined broadly, scattering of electromagnetic waves is the process of re-radiation of an incident wave by a material body. In the course of scattering, the incident wave may change its type (e.g., from plane to spherical or cylindrical), the direction of propagation, amplitude, phase, and polarization state. The other name of scattering is *diffraction*. In this book we use these terms as synonyms, although in the optical literature a distinction between the two names is sometimes made. Understanding of the scattering of electromagnetic fields by material objects is essential to the design of optical, microwave, and radio devices and systems.

Electromagnetic fields scattered by an obstacle include versatile information about geometric and material properties of the scatterer, its position in space, orientation, speed, etc. This fact is the basis for remote sensing technologies that use electromagnetic waves to survey, probe, and study remotely and non-destructively the space and material objects surrounding an observer. To extract the information encoded in the scattered field, it is important to understand the physics of scattering (how electromagnetic waves interact with the target and get scattered from the target) and be able to efficiently simulate the scattered fields. Modern radar remote sensing systems predominantly operate at microwave frequencies between 100 MHz and 100 GHz, which corresponds to the wavelength from 3 m to 3 mm. Such wavelengths are comparable, smaller or larger, to the size of a typical scatterer in our everyday life. Even in the visible spectrum, with frequencies between 430 and 770 THz and corresponding wavelengths between 390 and 700 nm, the wave effects cannot be neglected since from the viewpoint of applications of nanoscience or even in atmospheric optics these wavelengths may be smaller, comparable, or larger than the size of a nanoparticle or a water droplet. Under these circumstances the wave nature of electromagnetic fields becomes noticeable via various diffraction effects such as penetration of electromagnetic energy into regions shadowed by obstacles, blurred shadow boundaries and focal points, internal resonances, etc. These wave phenomena cannot be adequately described and understood in the framework of simple ray optical constructions. Similar observations can be made with respect to radio wave propagation, where the effects of the ground, buildings, trees (or furniture in the microcell mobile communications scenario) cannot be understood without solid knowledge of scattering phenomena.

Electromagnetic scattering theory is a branch of electromagnetics that describes, explains, and predicts electromagnetic field behavior in the presence of material obstacles by fully accounting for the wave nature of the electromagnetic field.

Electromagnetic scattering theory lies on the intersection of physics (particularly, electromagnetics and optics) and mathematics. During several centuries of its history, many outstanding researchers, including René Descartes (1596–1650), Francesco Maria Grimaldi (1618–1663), Christiaan Huygens (1629–1695), Thomas Young (1773–1829), François Arago (1786–1853), Joseph von Fraunhofer (1787–1826), Augustin-Jean Fresnel (1788–1827), Hermann von Helmholtz (1821–1894), Gustav Kirchhoff (1824–1887), James Clerk Maxwell (1831–1879), Heinrich Hertz (1857–1894), Hector Macdonald (1865–1935), and Arnold Sommerfeld (1868–1951), have contributed to the establishment of the wave theory of electromagnetic diffraction. The originators of the diffraction theory were inspired by remarkable optical phenomena, like blurring shadow boundaries, lit spots behind impenetrable obstacles (Poisson's spot or the spot of

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Arago), atmospheric rainbows, etc. The inability of simple ray-based concepts to explain these phenomena has led to the creation of electromagnetic scattering and diffraction theory. The need to solve the wave equations stimulated the development of various areas of mathematics (boundary value problems, special functions, analytical, asymptotic, and numerical methods of solution of integral and differential equations). The theory is still in the process of development motivated by new application fields, from stealthy aircraft and ships to metamaterials in photonics and nanoscience.

The physics of electromagnetic scattering can be explained in terms of a few basic scattering mechanisms. They can be studied by looking at solutions of Maxwell's equation for a limited number of canonical shapes, which are a planar interface (reflected and transmitted waves, surface waves), a circular cylinder and a sphere (reflection from a curved surface, creeping waves, interior resonances), a half-plane and a wedge (edge-diffracted waves), and a circular disc (multiply diffracted waves). The respective boundary value problems can be rigorously solved in a symbolic form, typically in terms of elementary or special functions, integrals or infinite sums, and the basic wave types and the associated scattering mechanisms are extracted from the solutions by asymptotic analysis.

Exact symbolic solutions of Maxwell's equations are rare. A solution is only possible for a simply shaped scatterer (planar infinite interfaces of homogeneous materials, wedges, circular cylinders, spheres, cones) with a simple material constitution (perfect electric conductor, homogeneous or layered materials) illuminated by a canonical source (plane, cylindrical, spherical waves). However, such solutions are extremely important for the modern system of knowledge about radiation, propagation, diffraction, and scattering of electromagnetic waves, which is explained by the following circumstances:

- In many cases, even solutions of these idealized problems adequately describe solutions of real-world problems, e.g. canonical targets, like metal spheres, are used for calibration of measurement facilities.
- Exact symbolic solutions can be typically calculated with a high numerical precision, which allows their use for validation of numerical solutions and procedures. Furthermore, solutions for idealized configurations may be used in the follow-up numerical iteration or optimization procedures.
- Symbolic solutions can be often evaluated approximately or asymptotically to give simple analytic or even closed-form expressions for the fields. Such expressions provide a physical insight and can spot simple qualitative and quantitative relations between various parameters of the problem, which enables explaining and forecasting the behavior of fields in more general and complex situations.
- Asymptotic solutions for idealized geometries serve as building blocks in engineering techniques (GTD/UTD, PO, PTD) for simulation of high-frequency electromagnetic fields in realistic environment.

Solutions of electromagnetic scattering and diffraction problems even for simple geometries and even when an exact solution is available can be very complex. Recent advances in computation technologies have enabled direct numerical simulation of electromagnetic phenomena for objects of moderate size compared with the wavelength, and computer-based visualization tools bring new ways of representing the behavior of scattered fields. However, it is important to understand that pure numerical approaches are subject to severe fundamental limitations. First, a direct application of discretization-based methods (BEM, FEM, FDTD) to objects larger than a dozen wavelengths leads to systems of equations with millions of unknowns, which are hardly amenable to a numerical solution because of unrealistic execution times and heavy memory requirements. Second, a numerical solution cannot provide a qualitative, physical insight into the basic mechanisms of scattering and diffraction.

Hence, there is a continuing need for the development of approximate and physically justified approaches that supplement the direct numerical methods by providing fast and acceptably accurate results for objects that are much larger than the wavelength. Several approximations are available for describing the scattering and diffraction of high-frequency electromagnetic fields.

- Geometrical optics (GO) is a classical ray-based technique for describing optical and microwave fields. The ray fields can be calculated by using simple laws of their propagation in free space and their reflection and transmission at material interfaces. However, the applicability of GO is limited to very short waves. The ray optics does not describe the penetration of fields in the regions shadowed by the scattering body and the excitation of many types of waves at geometric and material singularities on the scattering surface. Furthermore, GO fails at caustics, which is a major deficiency since the field intensity is typically at maximum there and many important phenomena (e.g., focusing of electromagnetic energy by dielectric lenses and parabolic reflectors, atmospheric glories and rainbows) take place at caustics.

- The geometrical theory of diffraction (GTD) extends GO through the addition of new types of rays which are generated at edges (edge-diffracted waves), conical points (tip-diffracted waves), and shadow boundaries on the scattering surface (creeping waves). However, GTD is a ray-based technique which fails at caustics. Extension of GTD to caustic regions is the subject of the uniform geometrical theory of diffraction (UTD).
- Physical optics (PO) and its extension, the physical theory of diffraction (PTD), are current-based approaches. The field scattered by a body is considered as radiated by secondary currents induced by the incident wave on the scatterer. Every PO/PTD solution uses Huygens' principle, which is a rigorous integral relation between the fields at every point in a volume and their values on a given (physical or mathematical) surface enclosing the volume. Since the surface fields are not known in advance, PO substitutes them with their GO approximation, thus reducing the problem to evaluation of a surface integral. When the observation point is located far from the body, the kernel of the integral can be simplified, leading to a fundamental formula known as the radiation integral. This formula is widely used in a great many of applications, from antenna design and prediction of radio wave propagation to simulation of scattering cross sections of radar targets. When an exact solution is impractical or not available, the PO method is unique in its ability to provide accurate numerical estimations of scattered fields in the situation (electrically large scatterers, observer in the far field) when neither discretization-based numerical techniques (BEM, FDTD, FEM) nor ray optical approximations (GO, GTD, UTD) apply.
- The GO approximation for the surface currents may be insufficiently accurate when the surface has geometric and/or material singularities (edges, tips, jumps in the values of material parameters). PTD corrects the GO surface currents in the vicinity of those singularities by using solutions of canonical problems for locally conformal geometries. This procedure may significantly improve the accuracy of the current-based approximations.

These approximate solution methods need justification. GO and PO were known well before Maxwell's theory of electromagnetism as empirical techniques, based on pre-Maxwell theories of light. GTD was also formulated as a set of postulates. It is therefore important to relate these methods and concepts to Maxwell's equations, validate their correctness, associated assumptions and simplifications, and estimate their accuracy. The only way to do this is to examine exact solutions to several characteristic scattering problems, that is, to look at solutions of canonical scattering problems.

## 1.2 Books on Related Subjects

The amount of literature on diffraction and scattering theory is overwhelming as the history of the theory, which can be counted from Fresnel's *Mémoire sur la diffraction de la lumière*, is almost two centuries old. It is therefore simply impossible to give a complete bibliography of all the books and papers that have been published over the years around the world, and we have not even attempted that. Every chapter in the main body of the book is accompanied with its own list of references, and we did our best to reference the most relevant publications. If the reader feels that we have overlooked an important contribution, please let us know!

Here, we briefly review books related to scattering and diffraction theory. They fall into the following categories:

- Historical texts (Descartes 1637; Drude 1902; Fresnel 1866; Green 1828; Grimaldi 1665; Heaviside 1893; Huygens 1690; Kirchhoff 1891; Macdonald 1902; Maxwell 1873; Sommerfeld 1927; Thomson 1893; Helmholtz 1897; Young 1845), see also English translations of classical texts (Crew 1900; Sommerfeld 2004). These and many other historical texts are now freely available on the Internet, thanks to ongoing digitizing efforts. The works by Descartes, Grimaldi, Huygens, Young, Fresnel, and Green appeared before the formulation of Maxwell's theory and present physical (Descartes, Grimaldi, Huygens, Young) and mathematical (Fresnel, Green) concepts that are used in the modern scattering theory. Descartes (1637) described the laws of reflection and refraction, and used them to explain atmospheric rainbows. Grimaldi (1665) observed the fringes in the shadow of an object placed in a beam of light and named the deviation of light from straight propagation path *diffraction*. Huygens (1690) was a proponent of the wave theory of light and formulated the principle of propagation of wavefronts (Huygens' principle). Young (1845) presented a number of theoretical reasons supporting the wave theory of light, and performed and analyzed demonstrations to support this viewpoint (interference in a double-slit experiment and upon reflection from thin films). Fresnel (1866) developed a first quantitative wave theory of light by building on experimental work by Young, and showed by mathematical methods that polarization could be explained only if light was entirely transverse. Green (1828) derived the integral relation that relates the value of a wave function in a volume to its values on a surface enclosing the volume (Green's formula) and applied it to electro- and magnetostatic problems.

The electromagnetic theory by Maxwell (1873) established the connection between light and electromagnetism. In its original formulation, the theory assumed the existence of aether and used quaternion algebra. Kirchhoff (1891) described optics based on the aether theory of light. The modern form of Maxwell's equations, in terms of the vectors of electric and magnetic fields, is attributed to Heaviside (1893). Experimental verification of Maxwell's theory and a first theoretical solution of Maxwell's equations (for the field excited by an oscillating electric dipole) are attributed to Hertz (1894). First descriptions of electromagnetic scattering, diffraction, and propagation entirely based on Maxwell's theory can be found in Drude (1902), Helmholtz (1897), and Thomson (1893). The beginning of rigorous electromagnetic scattering theory is associated with the works by Sommerfeld (1927) (formulation of scattering and diffraction problems as boundary value problems and their solution with rigorous mathematical methods, exact solutions for a half-plane and a wedge, for a dipole source over a homogeneous half-space) and Macdonald (1902) (electromagnetic formulation of Huygens' principle and conception of electromagnetic version of the PO method, solution for wedge diffraction).

- General courses in electromagnetics and electromagnetic waves (Balanis 1989; Borgnis and Papas 1955; Chew 1990; Felsen and Marcuvitz 1973; Franz 1957; Harrington 1961; Hönl *et al.* 1961; Ishimaru 1991; Jackson 1998; Jones 1964, 1979, 1989; Katsenelenbaum 2006; King and Wu 1959; Kong 1986; Plonus 1978; Schelkunoff 1943; Stratton 1941; Van Bladel 1964). These books give general theoretical knowledge about electromagnetic scattering and diffraction, and contain material on the most fundamental scattering problems, like scattering from wedges, cylinders, spheres, and apertures, mostly assuming PEC boundaries and often in scalar PO approximation.
- Handbooks, collections of formulas of diffraction and scattering theory (Bouman *et al.* 1987; Ruck *et al.* 1970), overviews of the state of the art (e.g., Langer (1962), Pike and Sabatier (2002), Ufimtsev (1999), and Uslenghi (1978)). These books offer a broad, eventually encyclopedic coverage but omit derivations of the presented results.
- Textbooks and monographs on computational methods of electromagnetics (e.g., Chew (1990), Chew *et al.* (2001), Harrington (1993), Jin (1993), Kunz and Luebbers (1993), Peterson *et al.* (1998), Silvester and Ferrari (1990), Taflove (1995), Tsang *et al.* (2001), and Volakis *et al.* (1998)). These books deal with various discretization-based methods for numerical solution of Maxwell's equations.
- Books on the wave theory of light (e.g., Akhmanov and Nikitin (1997), Born and Wolf (1959), Longhurst (1973), Newton (1966), Nieto-Vesperinas (2006), Petykiewicz (1992), Solimeno *et al.* (1986), and Sommerfeld (1964)). These books include chapters on diffraction of electromagnetic waves and address applications in optics.
- Books aimed at specific configurations or aspects of electromagnetic diffraction and scattering theory, e.g. dyadic methods and analytical modeling (Lindell 1992; Tai 1971; Tretyakov 2003), field singularities (Van Bladel 1991), stratified media (Brekhovskikh 1960; Wait 1962), wedges (Bobrovnikov and Fisanov 1988; Budaev 1995; Lyalinov and Zhu 2013), spheres, cylinders, and small particles (Bohren and Huffman 1983; Grandy 2000; Kerker 1969; Van de Hulst 1957; Wait 1959), cavities (Vinogradov *et al.* 2002), curved structures (Lewin *et al.* 1977), and rough surfaces and random media (Ishimaru 1978; Rytov *et al.* 1989; Tsang and Kong 2001).
- Books on the mathematical theory of diffraction (Babič and Buldyrev 1991; Baker and Copson 1953; Borovikov 1966; Colton and Kress 1983, 1998; Fock 1965; Kline and Kay 1965; Müller 1957; Northover 1971; Ramm 1986). Scattering and diffraction theory is presented as a branch of mathematics, mostly using the formal mathematical language and concentrating on methods rather than applications.
- Books on specific mathematical methods of diffraction and scattering theory (Babič and Kirpičnikova 1979; Babich *et al.* 2007; Bouche *et al.* 1997; Daniele and Zich 2014; Mittra and Lee 1971; Molinet *et al.* 2005; Noble 1958; Weinstein 1969).
- Books on engineering techniques such as GTD/UTD (Borovikov and Kinber 1994; James 1976; McNamara *et al.* 1990) and PO/PTD (Diaz and Milligan 1996; Rubinowicz 1957; Senior and Volakis 1995; Ufimtsev 1962, 2003, 2014).

- Application of scattering and diffraction theory to problems of RCS engineering (Crispin and Siegel 1968; Jenn 2005; Knott *et al.* 1993; Mentzer 1955; Saez de Adana *et al.* 2011; Tretyakov and Osipov 2006), remote sensing (Beckmann and Spizzichino 1963; Ulaby *et al.* 1982), radio wave propagation and wireless communication (Bertoni 2000; Bremmer 1949; Makarov *et al.* 1991; Wait 1981, 1962), and to closely related radiation problems, specifically to reflector and aperture antennas (Balanis 1997; Collin and Zucker 1969; Fradin 1961; Silver 1984; Wait, 1959).

## 1.3 Concept and Outline of the Book

The book describes deterministic scattering of electromagnetic waves from material bodies with full consideration of the polarization. Although many qualitative features of scattering can be described by using solutions of scalar problems, for example in the framework of the Fresnel–Kirchhoff approach, in practice vector Maxwell's equations always have to be solved, and formulation of a truly scalar scattering problem is possible only in exceptional cases, for example for PEC bodies with translational or spherical symmetry or for infinite planar material interfaces illuminated by plane waves. The components of the electric and magnetic fields are typically coupled through boundary conditions on non-metallic interfaces. Thus, throughout the book we study truly electromagnetic problems, which makes the analysis more involved, but it is these solutions that are really needed in practice. For the same reason the book does not address problems of acoustic scattering since acoustic fields are scalar fields, for which polarization coupling is irrelevant.

This book is intended to serve as a bridge between textbooks and handbooks. Textbooks describe the basic facts and ideas, simplifying the material as much as possible for better understanding, presenting the results that can rarely be used in applications. Handbooks provide broad collections of ready-to-use results but often do not explain how and under which assumptions these results have been obtained. In this book we formulate the rigorous boundary value problems, explain solution methods, and let the reader follow the derivations, showing how to arrive at the handbook formulas. In particular, through the high-frequency analysis of solutions for canonical bodies, the ray structure of fields is extracted, thus validating the postulates of GO and GTD.

An important feature of the book is its focus on scattering from non-metallic (imperfectly conducting) and impedance bodies. This is important because of the increasing use of advanced materials in electromagnetic engineering; also natural surfaces (ground, see, buildings, etc.) are non-metallic. Scattering from non-metallic scatterers is addressed in the book by using either exact solutions (for simply shaped scatterers, like wedges, cylinders, spheres) or approximate PO solutions (for generally shaped bodies). Reflection and scattering from impedance-matched bodies (when the surface impedance is equal to the intrinsic impedance of the surrounding medium) show a number of extremal features, so special attention is devoted to the impedance-matched plane (section 4.6.3), wedge (section 5.4.3), and sphere (section 7.5.4). We also discuss metamaterial scatterers, which includes the superlens (section 4.5.3) and the perfect cloak (section 7.6.2).

The book presents novel and less known results. We believe that the complete theory of low-frequency scattering from spheres (sections 7.4.2, 7.5.2, and 7.5.3) and a theory of backscattering from electrically large low-absorption spheres in terms of interior creeping waves (section 7.5.5) have not been described in the literature so far.

The less-known results include a simplified derivation of the electromagnetic Huygens principle in two and three dimensions (section 2.6), formulation of the forward-scattering theorem for cylindrical objects under oblique illumination (section 3.4.4), solutions for planar material layers excited by dipoles arbitrarily oriented with respect to the interface of the media (sections 2.5.4 and 4.4.2), a proof of the minimum reflection property for the planar impedance-matched boundary (section 4.6.3), a complete collection of the available exact solutions for impedance wedges (section 5.4), a discussion of the analytical structure of Meixner's series, including logarithmic terms (section 5.6.2), an exact solution for an obliquely illuminated impedance cylinder (section 6.5), a systematic exposition of a fully electromagnetic version of the PO method which is applicable to non-metallic bodies and describes fields not only in the far-field region but also at finite distances from the scatterer, up to the scattering surface, including caustics (Chapter 8), PO solutions for a broad variety of non-metallic simply shaped scatterers and a new derivation of Gordon's formulas by integrating by parts in the PO solution for plates (Chapter 9), and complete specification of Debye's asymptotic approximations of Bessel functions on the complex plane of the order and derivation of the Debye approximations by the method of phase integrals (Appendix E).

Solution and analysis of solutions of scattering problems require the use of various quite sophisticated mathematical methods. We present the methods not in an abstract way but rather by showing how they work in solving and analyzing the scattering problems. The book describes a variety of methods, including

- exact solution methods: separation of variables (Chapters 5, 6, and 7), the Wiener–Hopf (factorization) method (section 5.3), and the Sommerfeld–Maliuzhinets method (section 5.4);
- methods of approximate solution of scattering problems: GO, GTD, UTD (Chapters 5, 6, and 7), and PO (Chapters 8 and 9);
- methods of asymptotic analysis of solutions: the steepest descent method (section C.3), the stationary point method in one and two dimensions (Appendix D), and the WKB method (Appendix E).

Special functions play a key role in solutions of canonical scattering problems. Addressed are Legendre functions (section 7.2.2), Bessel and Riccati–Bessel functions (sections 7.2.2 and E.1.1), Maliuzhinets and Bobrovnikov–Fisanov functions (sections 5.4.2 and 5.4.3), Fresnel integrals and related functions (Appendix B), and Airy and Fock functions (sections D.2.3 and E.3).

In order to keep the size of the book reasonable, we had to restrict our attention to the time-harmonic deterministic linear direct electromagnetic scattering theory in open regions. Electromagnetics of waveguides, resonators, and periodic arrays is not addressed (e.g., see Collin (1960)), nor is the theory of inverse scattering (e.g., see Colton and Kress (1998)). A number of geometries, solutions, and solution methods are not included, for example the rigorous solutions for PEC circular discs and cones (Bouman *et al.* 1987), mainly because of missing generalizations to imperfectly reflecting boundaries (scattering from a conical point on an impedance boundary is addressed in the PO approximation in section 9.2). Such methods as PTD, the parabolic equation method, the boundary-layer method, Maslov's method, etc. are also not discussed, and the reader is referred to the many good sources on the approximate methods of scattering, diffraction, and propagation theory, for example Babič and Buldyrev (1991), Bouche *et al.* (1997), Fock (1965), Ufimtsev (2014), Ziolkowski and Deschamps (1984), Babič and Kirpičnikova (1979), and Levy (2000). No anisotropic materials are addressed in the book, except for the case of the spherical perfect cloak.

Organization of the book is seen from the table of contents, so here we only briefly outline the content of the chapters. Chapter 2 presents the fundamentals of the scattering theory. This includes Maxwell's equations and constitutive relations, exact and approximate boundary conditions, vector potentials and Green's functions, and basic solutions of Maxwell's equations (plane, cylindrical, and spherical waves). The fundamental properties of the electromagnetic fields are also addressed, including energy conservation, reciprocity, and the electromagnetic version of Huygens' principle for compact and cylindrical scatterers.

Chapter 3 describes scattered fields in the far (Fraunhofer) zone where they approach a spherical wave (cylindrical or conical for cylindrical scatterers), which is particularly important for radar and optical applications. We define the relevant quantities, including the far-field coefficient, the scattering matrix, and various scattering cross sections and scattering widths. The optical theorem is addressed both in two and three dimensions, and classification of far-field scattering according to the scattering regimes (low-frequency, resonant, and high-frequency) is given. The chapter concludes with a theory of Rayleigh scattering for simply shaped PEC or magneto-dielectric scatterers smaller than the wavelength.

Chapter 4 addresses reflection and transmission of electromagnetic waves at planar interfaces. We define reflection and transmission coefficients for various polarization frames (parallel–perpendicular, TE-TM, *E-H*) and present a vector solution for reflection and transmission of a plane wave at the interface between two homogeneous magneto-dielectric half-spaces (section 4.2). A general solution for multilayered slabs is derived in section 4.3. Fields excited by localized sources are superpositions of plane and inhomogeneous waves, which complicates the problem by requiring complex integration of reflected/transmitted components (section 4.4). In section 4.5, the general solution for the multilayered slab is specialized to the important particular case of a homogeneous layer between two homogeneous eventually different half-spaces. Finally, the accuracy of impedance formulations for planar interfaces is studied, and the extremal properties of the impedance-matched interface are addressed (section 4.6).

A wedge is a canonical body to model the presence of an edge in a material boundary. In Chapter 5 we derive and study solutions for various wedge-shaped configurations with PEC and impedance boundary conditions. A variety of solution methods is used, including separation of variables (section 5.2), and the Wiener–Hopf (section 5.3) and Maliuzhinets (section 5.4) methods. The general solution for an arbitrarily angled obliquely illuminated impedance wedge is not available, so section 5.4 addresses all the special cases that have an exact solution. In section 5.5, the solutions are analyzed far from the edge, and the GO, surface, and edge-diffracted waves are extracted, showing that the result has the structure postulated by GO/GTD. The high-frequency approximations have to be corrected at the shadow boundaries, and we show how to do that, presenting uniform approximations in terms of Fresnel integrals and UTD transition functions. The presence of an edge leads to singularities of field components at the edge, and Meixner's method permits determining the order of singularity of the fields. In section 5.6 we show how this method works and discuss the problem of construction of higher-order terms in the Meixner expansions.

Similarly to the wedge, an infinite circular cylinder is a configuration with translational symmetry. On the other hand, its boundary is smooth and of finite diameter in the plane perpendicular to the cylinder axis, which leads to a different scattering picture. In Chapter 6 we solve (by separation of variables) scattering problems and study solutions for PEC (section 6.2), magneto-dielectric homogeneous (section 6.3), coated (section 6.5.1), and impedance (section 6.5.2) circular cylinders. To construct high-frequency approximations for the fields, Watson's transformation is introduced and applied in section 6.4. In addition to the GO reflected wave, the scattered field includes creeping waves, gliding along the surface on their way from the source to the observer, which justifies the GO/GTD postulates about the ray structure in the presence of a smoothly curved boundary. In the transition zone across the shadow boundary (penumbra region) on the surface of the cylinder, GTD is not applicable, and a special treatment in terms of Fock functions is necessary (section 6.4.4). Finally, the high-frequency solution for the circular cylinder is extended to electrically large generally shaped convex impedance bodies (section 6.6).

A sphere is one of the most important canonical bodies. In contrast to infinite cylinders, which are idealizations, the sphere is compact and serves as a model of a basic scatterer. Chapter 7 is devoted to scattering from spheres of various material compositions (PEC, magneto-dielectric, multilayered). An exact solution is derived by separation of variables, and the high-frequency approximations for homogeneous spheres are obtained by Watson's transformation. The homogeneous sphere allows new types of waves, not only the interior resonances and GO rays multiply reflected in the interior of the sphere but also waves propagating on the concave side of the spherical boundary. We present the full GO solution for the homogeneous magneto-dielectric sphere and apply it to the theory of atmospheric rainbows. It is shown that the backscattering from electrically large low-absorption spheres is dominated by the interior creeping waves, which leads to a new explanation of the atmospheric glory. Electrically small spheres are also an extremely important canonical case, which has gained importance because of the developments in nanoscience, photonics, and plasmonics. The classical theory of Rayleigh scattering is insufficient when multipole and radiation corrections are required. Furthermore, in spheres with large values of permittivity/permeability internal resonances are possible, which is not covered by the Rayleigh theory. In the chapter we derive a complete theory of low-frequency scattering that closes this gap. The chapter concludes with the solution for the spherical perfect cloak and with a discussion of new applications of small spheres in the design of metamaterials.

Chapter 8 describes the PO method. We derive a fully electromagnetic version of the method by combining the electromagnetic version of Huygens' principle with the vector solution for reflection of a plane wave at material interfaces (section 8.2). In section 8.3 the PO solution is applied to the case of an aperture in an impenetrable screen, and it is shown that high-frequency analysis extracts a wave diffracted at the rim of the opening. The wave has the form of the edge-diffracted wave studied in the chapter on wedges, generated at the points on the rim where the incidence and scattering directions make the same angle with the edge, which is in agreement with the GTD concept of the edge-diffracted wave and the Keller cone. Next, in section 8.4, reflection from curved surfaces is studied in the PO approximation, and we arrive at the classical Fock reflection formula. Finally, the PO solution for an imperfectly reflecting surface with an edge is studied, and the edge-diffracted wave component is extracted (section 8.4.4). The PO approximation leads to the edge-diffracted wave with the correct phase but an inaccurate amplitude, the latter being determined by the PO diffraction coefficient. Advantages and limitations of the PO method are discussed in section 8.5.

The PO method permits an immediate solution, in the form of a surface integral, for the field scattered by an arbitrarily shaped metallic or non-metallic scatterer. In Chapter 9, PO solutions are constructed for polygonal plates, cones, and bodies of revolution of various cross sections.

Appendices A to E importantly supplement the main body of the book by providing the background mathematical information about the special functions and methods that are used. Appendix A is a short collection of necessary formulas from vector analysis. Appendix B describes the Fresnel integral, error function for complex argument, and UTD transition function. The principles of the complex analysis and complex integration, including the steepest descent method, are addressed in Appendix C, while Appendix D describes in a relatively detailed manner the stationary phase method in one and two dimensions. Appendix E contains a collection of formulas for Bessel (and related) functions and a study on Debye's asymptotic approximations, performed with a new method (WKB method on the complex plane of the order of the Bessel functions). Explicit Debye's approximations in every part of the complex plane of the order are provided.

Bibliographic references are collected at the end of every chapter. A significant part of the bibliography comes from German and Russian technical literature, less known to the English-speaking community. Problem sections at the end of every chapter are rich collections of some 250 original problems for practice, most with detailed solutions. Some of the problems are easy, but some are difficult. Many are of a technical nature, which is due to the fact that electromagnetic theory in general and scattering theory in particular use a lot of mathematics as a language to formulate the results and as a tool to derive solutions from Maxwell's equations.

## References

Akhmanov A and Nikitin SY 1997 *Physical Optics*, Oxford University Press.
Babič VM and Kirpičnikova NY 1979 *The Boundary-Layer Method in Diffraction Problems*, Springer-Verlag.
Babič VM and Buldyrev VS 1991 *Short-Wavelength Diffraction Theory*, Springer-Verlag.
Babich VM, Lyalinov MA, and Grikurov VG 2007 *Sommerfeld-Malyuzhinets Technique in Diffraction Theory.* Alpha Science International Ltd.
Baker BB and Copson ET 1953 *The Mathematical Theory of Huygens' Principle*, Clarendon Press.
Balanis CA 1989 *Advanced Engineering Electromagnetics*, John Wiley & Sons.
Balanis CA 1997 *Antenna Theory: Analysis and Design*, Second edition, John Wiley & Sons.
Beckmann P and Spizzichino A 1963 *The Scattering of Electromagnetic Waves from Rough Surfaces*, Pergamon.
Bertoni HL 2000 *Radio Propagation for Modern Wireless Systems*, Prentice Hall.
Bobrovnikov MS and Fisanov VV 1988 *Diffraction of Waves in Angular Regions*, Tomsk University Press, Tomsk (in Russian).
Bohren CF and Huffman DR 1983 *Absorption and Scattering of Light by Small Particles*, Wiley-Interscience.
Borgnis FE and Papas CH 1955 *Randwertprobleme der Mikrowellenphysik*, Springer.
Born M and Wolf E (eds) 1959 *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, Pergamon Press.
Borovikov VA 1966 *Diffraction by Polygons and Polyhedra*, Nauka, Moscow (in Russian).
Borovikov VA and Kinber BY 1994 *Geometrical Theory of Diffraction*, IEE Electromagnetic Waves Series 37, The Institution of Electrical Engineers, London.
Bouche D, Molinet F, and Mittra R 1997 *Asymptotic Methods in Electromagnetics*, Springer-Verlag.
Bouman JJ, Senior TBA, and Uslenghi PLE (eds) 1987 *Electromagnetic and Acoustic Scattering by Simple Shapes*, Hemisphere Publishing Corporation.
Brekhovskikh LM 1960 *Waves in Layered Media*, Academic Press.
Bremmer H 1949 *Terrestrial Radio Waves*, Elsevier.
Budaev BV 1995 *Diffraction by Wedges*, Longman Scientific & Technical.
Chew WC 1990 *Waves and Fields in Inhomogeneous Media*, Van Nostrand Reinhold.
Chew WC, Jin JM, Michielssen E, and Song J (eds) 2001 *Fast and Efficient Algorithms in Computational Electromagnetics*, Artech House.
Collin RE 1960 *Field Theory of Guided Waves*, McGraw-Hill.
Collin RE and Zucker FJ 1969 *Antenna Theory: Part 2*, McGraw-Hill.
Colton D and Kress R 1983 *Integral Equation Methods in Scattering Theory*, John Wiley & Sons.
Colton D and Kress R 1998 *Inverse Acoustic and Electromagnetic Scattering Theory*, Springer.
Crew H (ed) 1900 *The Wave Theory of Light. Memoirs by Huygens, Young and Fresnel*, American Book Company.
Crispin JW and Siegel KM (eds) 1968 *Methods of Radar Cross-Section Analysis*, Academic Press.
Daniele V and Zich R 2014 *The Wiener–Hopf Method in Electromagnetics*, SciTech Publishing, Rayleigh, NC.
Descartes R 1637 *Discours de la Méthode, Pour Conduire la Raison et Chercher la Vérité des Sciences, Plus la Dioptrique et les Météores Qui Sont des Essais sur Cette Méthode*, Leiden.
Diaz L and Milligan T 1996 *Antenna Engineering Using Physical Optics: Practical CAD Techniques and Software*, Artech House.
Drude P 1902 *The Theory of Optics*, Longmans Green and Co.
Felsen LB and Marcuvitz N 1973 *Radiation and Scattering of Waves*, Prentice-Hall.
Fock VA 1965 *Electromagnetic Diffraction and Propagation Problems*, Pergamon Press.
Fradin AZ 1961 *Microwave Antennas*, Pergamon Press.

Franz W 1957 *Theorie der Beugung elektromagnetischer Wellen*, Springer-Verlag.
Fresnel A 1866 *Œuvres Complètes d'Augustin Fresnel, Tome Premier: Théorie de la Lumière*. De Sénarmont H, Verdet É, and Fresnel L (eds), Imprimerie impériale, Paris.
Grandy WT 2000 *Scattering of Waves from Large Spheres*, Cambridge University Press.
Green G 1828 *An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*, printed for the author by T. Wheelhouse, Nottingham.
Grimaldi FM 1665 *Physico-Mathesis de Lumine, Coloribus, et Iride*, Bologna.
Harrington RF 1961 *Time-Harmonic Electromagnetic Fields*, McGraw-Hill.
Harrington RF 1993 *Field Computation by Moment Methods*, IEEE Press.
Heaviside O 1893 *Electromagnetic Theory. Volume 1*, The Electrician Printing and Publishing Co, London.
Hertz H 1894 *Untersuchungen über die Ausbreitung der elektrischen Kraft*, Johann Ambrosius Barth, Leipzig.
Hönl H, Maue AW, and Westpfahl K 1961 *Theorie der Beugung*, in *Encyclopedia of Physics, vol. XXV/1*, Springer-Verlag.
Huygens C 1690 *Traite de la Lumiere*, Leyden.
Ishimaru A 1978 *Wave Propagation and Scattering in Random Media*, Academic Press.
Ishimaru A 1991 *Electromagnetic Wave Propagation, Radiation, and Scattering*, Prentice Hall.
Jackson JD 1998 *Classical Electrodynamics*, Third edition, Wiley.
James GL 1976 *Geometrical Theory of Diffraction for Electromagnetic Waves*, Peter Peregrinus, London.
Jenn DC 2005 *Radar and Laser Cross Section Engineering*, American Institute of Aeronautics and Astronautics, Reston.
Jin JM 1993 *The Finite Element Method in Electromagnetics*, John Wiley & Sons.
Jones DS 1964 *The Theory of Electromagnetism*, Pergamon Press.
Jones DS 1979 *Methods in Electromagnetic Wave Propagation*, Clarendon Press.
Jones DS 1989 *Acoustic and Electromagnetic Waves*, Clarendon Press.
Katsenelenbaum BZ 2006 *High-frequency Electrodynamics*, Wiley-VCH.
Kerker M 1969 *The Scattering of Light and Other Electromagnetic Radiation*, Academic Press.
King RWP and Wu TT 1959 *The Scattering and Diffraction of Waves*, Harvard University Press, Cambridge, MA.
Kirchhoff G 1891 *Mathematische Optik (Vorlesungen über mathematische Physik, 2. Band)*, Teubner Verlag, Leipzig.
Kline M and Kay IW 1965 *Electromagnetic Theory and Geometrical Optics*, John Wiley & Sons.
Knott EF, Shaeffer JF, and Tuley MT 1993 *Radar Cross Section*, Artech House.
Kong JA 1986 *Electromagnetic Wave Theory*, John Wiley & Sons.
Kouyoumjian RG and Pathak PH 1974 A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface. *Proc. IEEE*, **62**, 1448–1461.
Kunz KS and Luebbers RJ 1993 *The Finite Difference Time Domain Method for Electromagnetics*, CRC Press.
Langer RE (ed.) 1962 *Electromagnetic Waves*, The University of Wisconsin Press.
Levy M 2000 *Parabolic Equation Methods for Electromagnetic Wave Propagation*, The Institution of Electrical Engineers, London.
Lewin L, Chang DC, and Kuester EF 1977 *Electromagnetics Waves and Curved Structures*, Peter Peregrinus.
Lindell IV 1992 *Methods for Electromagnetic Field Analysis*, Oxford Science Publications.
Longhurst RS 1973 *Geometrical and Physical Optics*, Longman.
Lyalinov MA and Zhu NY 2013 *Scattering of Waves by Wedges and Cones with Impedance Boundary Conditions*, SciTech Publishing.
Macdonald HM 1902 *Electric Waves*, Cambridge University Press.
Makarov GI, Novikov VV, and Rybachek ST 1991 *Electromagnetic Waves Propagation over Earth's Surface*, Nauka, Moscow (in Russian).
Maxwell JC 1873 *A Treatise on Electricity and Magnetism*, Clarendon Press, Oxford.
McNamara DA, Pistorius CWI, and Malherbe JAG 1990 *Introduction to the Uniform Geometrical Theory of Diffraction*, Artech House.
Mentzer JR 1955 *Scattering and Diffraction of Radio Waves*, Pergamon Press.
Mittra R and Lee SW 1971 *Analytical Techniques in the Theory of Guided Waves*, The Macmillan Company.
Molinet F, Andronov I, and Bouche D 2005 *Asymptotic and Hybrid Methods in Electromagnetics*, IEE Electromagnetic Waves Series 51, The Institution of Electrical Engineers, London.
Müller C 1957 *Grundprobleme der mathematischen Theorie elektromagnetischer Schwingungen*, Springer-Verlag.
Newton RG 1966 *Scattering Theory of Waves and Particles*, McGraw-Hill.
Nieto-Vesperinas M 2006 *Scattering and Diffraction in Physical Optics*, World Scientific.
Noble B 1958 *Methods Based on the Wiener–Hopf Technique*, Pergamon Press.
Northover FH 1971 *Applied Diffraction Theory*, American Elsevier.
Peterson AF, Ray SL, and Mittra R 1998 *Computational Methods for Electromagnetics*, IEEE Press.
Petykiewicz J 1992 *Wave Optics*, Kluwer.
Pike R and Sabatier P (eds) 2002 *Scattering: Scattering and Inverse Scattering in Pure and Applied Science*, Academic Press.
Plonus MA 1978 *Applied Electromagnetics*, McGraw-Hill.

Ramm AG 1986 *Scattering by Obstacles*, D. Reidel Publishing Company.

Rubinowicz A 1957 *Die Beugungswelle in der Kirchhoffschen Theorie der Beugung*, Panstwowe Wydawnictwo Naukowe, Warsaw.

Ruck GT, Barrick DE, Stuart WD, and Krichbaum CK 1970 *Radar Cross Section Handbook*, Plenum Press.

Rytov SM, Kravtsov YA, and Tatarskii VI 1989 *Principles of Statistical Radiophysics*, Springer.

Saez de Adana F, Gutierrez O, Gonzalez I, Catedra MF, and Lozano L 2011 *Practical Applications of Asymptotic Techniques in Electromagnetics*, Artech House.

Schelkunoff SA 1943 *Electromagnetic Waves*, D. Van Nostrand Company.

Senior TBA and Volakis JL 1995 *Approximate Boundary Conditions in Electromagnetics*, IEE Electromagnetic Waves Series 41, The Institution of Electrical Engineers, London.

Silver S (ed) 1984 *Microwave Antenna Theory and Design*, IEE Electromagnetic Waves Series 19, The Institution of Electrical Engineers, London.

Silvester PP and Ferrari RL 1990 *Finite Elements for Electrical Engineers*, Second edition, Cambridge University Press, Cambridge.

Solimeno S, Crosignani B, and DiPorto P 1986 *Guiding, Diffraction, and Confinement of Optical Radiation*, Academic Press.

Sommerfeld A 1927 Elektromagnetische Schwingungen, in *Die Differential- und Integralgleichungen der Mechanik und Physik. 2. Physikalischer Teil*, Frank P and von Mises R (eds), chapters 19–23, Vieweg & Sohn, Braunschweig.

Sommerfeld A 1964 *Lectures on Theoretical Physics: Optics*, Academic Press.

Sommerfeld A 2004 *Mathematical Theory of Diffraction*, Birkhäuser.

Stratton JA 1941 *Electromagnetic Theory*, McGraw-Hill.

Taflove A 1995 *Computational Electrodynamics: the Finite-difference Time-domain Method*, Artech House.

Tai C-T 1971 *Dyadic Green's Functions in Electromagnetic Theory*, Intext Educational Publishers.

Thomson JJ 1893 *Notes on Recent Researches in Electricity and Magnetism*, Clarendon Press, Oxford.

Tretyakov SA 2003 *Analytical Modeling in Applied Electromagnetics*, Artech House.

Tretyakov SA and Osipov AV 2006 *Applied Theory of Electromagnetic Scattering and Diffraction*, Helsinki University of Technology Radio Laboratory Publications, Espoo.

Tsang L and Kong JA 2001 *Scattering of Electromagnetic Waves: Advanced Topics*, John Wiley & Sons.

Tsang L, Kong JA, Ding K-H, and Ao CO 2001 *Scattering of Electromagnetic Waves: Numerical Simulations*, John Wiley & Sons.

Ufimtsev PY 1962 *Method of Edge Waves in the Physical Theory of Diffraction*, Sovetskoe Radio, Moscow (in Russian).

Ufimtsev PY 1999 Backscatter. In *Wiley Encyclopedia of Electrical and Electronics Engineering, vol. 2* (ed. Webster JG), pp. 175–187, John Wiley & Sons.

Ufimtsev PY 2003 *Theory of Edge Diffraction in Electromagnetics*, Tech Science Press.

Ufimtsev PY 2014 *Fundamentals of the Physical Theory of Diffraction*, Wiley.

Ulaby FT, Moore RK, and Fung AK 1982 *Microwave Remote Sensing: Active and Passive, vol.2: Radar Remote Sensing and Surface Scattering and Emission Theory*, Addison-Wesley.

Uslenghi PLE (ed) 1978 *Electromagnetic Scattering*, Academic Press.

van Bladel J 1964 *Electromagnetic Fields*, McGraw-Hill.

van Bladel J 1991 *Singular Electromagnetic Fields and Sources*, Oxford University Press, New York.

van de Hulst HC 1957 *Light Scattering by Small Particles*, John Wiley & Sons.

Vinogradov SS, Smith PD, and Vinogradova ED 2002 *Canonical Problems in Scattering and Potential Theory. Part II: Acoustic and Electromagnetic Diffraction by Canonical Structures*, CRC Press.

Volakis JL, Chatterjee A, and Kempel LC 1998 *Finite Element Method for Electromagnetics: Antennas, Microwave Circuits, and Scattering Applications*, IEEE Press.

von Helmholtz H 1897 *Vorlesungen über elektromagnetische Theorie des Lichtes*, Voss Verlag, Hamburg und Leipzig.

Wait JR 1959 *Electromagnetic Radiation from Cylindrical Structures*, Pergamon Press.

Wait JR 1962 *Electromagnetic Waves in Stratified Media*, Pergamon Press.

Wait JR 1981 *Lectures on Wave Propagation Theory*, Pergamon Press.

Weinstein LA 1969 *The Theory of Diffraction and the Factorization Method (Generalized Wiener-Hopf Technique)*, Golem Press.

Weston VH 1963 Theory of absorbers in scattering. *IEEE Trans. Antennas Propag.*, **11**, 578–584.

Young Th 1845 *A Course of Lectures on Natural Philosophy and Mechanical Arts*, Second edition, Taylor and Walton, London, lecture XXXIX.

Ziolkowski RW and Deschamps GA 1984 Asymptotic evaluation of high-frequency fields near a caustic: An introduction to Maslov's method. *Radio Sci.*, **19**, 1001–1025.