1

Review of Numerical Computation

OBJECTIVES

When you have completed this chapter, you should be able to

- Perform basic arithmetic operations on signed numbers.
- Perform basic arithmetic operations on approximate numbers.
- Take powers, roots, and reciprocals of signed and approximate numbers.
- Perform combined arithmetic operations to obtain a numerical result.
- Convert numbers between decimal, scientific, and engineering notation.
- Perform basic arithmetic operations on numbers in scientific and engineering notation.
- Convert units of measurement.
- Substitute given values into formulas.
- Solve common percentage problems.

We start this first chapter with some definitions to refresh your memory of terms you probably already know. We will point out the difference between exact and *approximate* numbers, a distinction you may not have made in earlier mathematics classes. Then we will perform the ordinary arithmetic operations—addition and subtraction, multiplication and division—but here it may be a bit different from what you are used to. We will use the calculator extensively, which is probably not new to you, but now we will take great care to decide how many digits of the calculator display to keep. Why not keep them all? We will show that when working with approximate numbers keeping too many digits is misleading to anyone who must use the result of your calculation. As a further complication, we will combine both exact and approximate numbers, as well as positive and *negative* numbers, or *signed numbers*. As we proceed, we will point out some rules that will help get us ready for our next chapter on *algebra*, which is a generalization of arithmetic.

Next we will show compact ways to write a very long or very short number, in *scientific notation* or in *engineering notation*. These are important for you to know, not only for your own use but for you to understand them when you come across such numbers in reading technical material.

In technical work, we usually deal with numbers that indicate some measured quantity. Here we show how to convert a number from one unit of measurement to another, say feet to meters, how to use numbers with units of measure in computations, and how to substitute numbers with units into technical formulas. All are vital skills for technical work. Finally we will cover *percentage*. Of all the mathematical topics we cover in this text, probably the one most used in everyday life is percentage.

This is a long chapter. With its many different topics, it may appear choppy and disconnected. The good news is that most of the material should be familiar to you, with perhaps a few new twists. Throughout the chapter, as elsewhere in the book, we will give some help with the use of the calculator. But with so many types of calculators available, we are limited in what we can do, and you will really have to consult the manual that came with your calculator. We urge you to do this now, so by the time you reach Chapter 2 you will be able to calculate with speed and accuracy for the operations shown here. Computations for trigonometry and for logarithms will be covered as we get to them.

1–1 The Real Numbers

In mathematics, as in many other fields, we must learn many new terms. These definitions will make it easier to talk about mathematical ideas later.

Integers

The *integers*

$$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$$

are the *whole numbers*, also called the *natural numbers* or *counting numbers*, including zero and negative values. The three dots on the ends indicate that the sequence of numbers continues indefinitely in both directions.

Rational and Irrational Numbers

The *rational numbers* include the integers and all other numbers that can be expressed as the quotient of two integers. Some rational numbers are

$$\frac{1}{2}, \quad -\frac{3}{5}, \quad \frac{57}{23}, \quad -\frac{98}{99}, \quad \text{and} \quad 7 \left(\text{or } \frac{7}{1}\right)$$

Numbers that cannot be expressed as the quotient of two integers are called *irrational*. Some irrational numbers are

$$\sqrt{2}, \quad \sqrt[3]{5}, \quad -\sqrt{7}, \quad \pi, \quad \text{and} \quad e$$

where π is approximately equal to 3.1416 and e is approximately equal to 2.7182. We will have much more to say about the irrational numbers π and e later in the book.

Real and Imaginary Numbers

The rational and irrational numbers together make up the *real numbers*. Numbers such as $\sqrt{-4}$ do not belong to the real number system. They are called *imaginary* *numbers* and are discussed in a later chapter. Except when otherwise noted, all the numbers we will work with are real numbers.

Decimal Numbers

Most of our computations are with numbers written in the familiar *decimal* system. The names of the places relative to the *decimal point* are shown in Fig. 1–1. We say that the decimal system uses a *base of 10* because it takes 10 units in any place to equal 1 unit in the next-higher place. For example, 10 units in the hundreds position equals 1 unit in the thousands position.

The numbers 10^2, 10^3, etc., are called *powers of 10.* Don't worry if they are unfamiliar to you. We will explain them later.

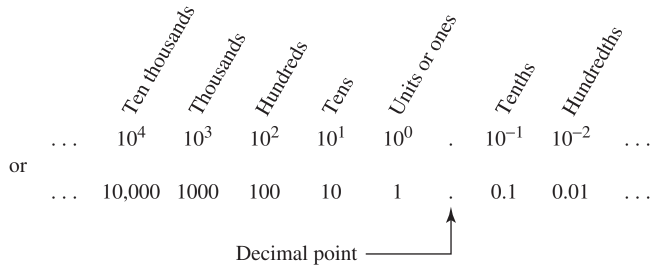


FIGURE 1–1 Values of the positions in a decimal number.

Positional Number Systems and Place Value

A *positional* number system is one in which the *position* of a digit determines its value. Our decimal system is positional.

Each position in a number has a *place value* equal to the base of the number system raised to the position number. The place values in the decimal number system, as well as the place names, are shown in Fig. 1–1.

The Number Line

A mathematical idea is much easier to grasp if shown as a *picture*; thus we will try to picture ideas whenever possible. Such a picture will often be in the form of a *graph*, and the simplest graph is the *number line* (Fig. 1–2). We draw a line on which we mark a zero point, and indicate the direction of increasing values. The line is usually drawn horizontal with increasing values taken to the right, marked with an arrowhead. We next indicate a *scale,* with consecutive numbers equally spaced along the line.

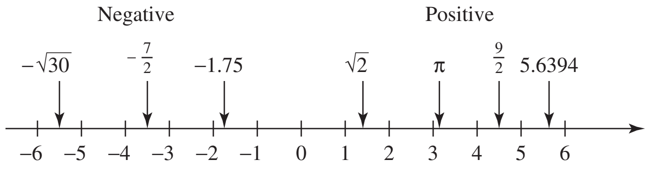


FIGURE 1–2 The number line.

Signed Numbers

A *positive* number is a number that is *greater* than zero, and a *negative* number is *less* than zero. On the number line it is customary to show the positive numbers to the right of zero and the negative numbers to the left of zero. These numbers may be integers, fractions, rational numbers, or irrational numbers.

To distinguish negative numbers from positive numbers, we always place a negative sign $(-)$ in front of a negative number. We usually omit writing the positive sign $(+)$ in front of a positive number. Thus a number without a sign is always assumed to be positive. In this chapter we will often write in a $+$ sign for emphasis.

The Opposite of a Number

The *opposite* of a number n is the number which, when added to n, gives a sum of zero.

◆◆◆ **Example 1:** The opposite of 2 is -2, because $2 + (-2) = 0$. The opposite of -6 is $+6$. ◆◆◆

On the number line, the opposite of a number lies on the opposite side of, and at an equal distance from, the zero point.

Infinity

If we place a number, however large, on the number line, it is always possible to find a *larger* one. We say that numbers, as we proceed to the right on the number line, *approach infinity*. Here we are thinking of infinity (given the symbol ∞) as some value greater than any real number. We will use the notion of infinity again in later chapters.

Symbols of Equality and Inequality

Several symbols are used to show the relative positions of two quantities on the number line.

$a = b$ means that *a equals b,* and that a and b occupy the same position on the number line.

$a \neq b$ means that a and b are *not equal* and have different locations on the number line.

$a > b$ means that a is *greater than b,* and a lies to the right of b on the number line.

$a < b$ means that a is *less than b,* and a lies to the left of b on the number line.

$a \approx b$ means that a is *approximately equal to* b and that a and b are *near* each other on the number line. Other symbols sometimes used for approximately equal to are $\cong$ and $\simeq$.

◆◆◆ **Example 2:** Here are examples of the use of symbols of equality and inequality.

(a) $4 < 8$ (b) $47 > 24$

(c) $29 > -37$ (d) $-77 < -48$

(e) ${}^1/_2 \neq 0.6$ (f) $1/3 \approx 0.33$ ◆◆◆

Symbols of Grouping

Symbols of grouping, or signs of aggregation, are the parentheses ( ), brackets [ ], braces { }, and the bar —, also called a *vinculum*. Each symbol means that the terms enclosed are to be treated as a single term.

◆◆◆ **Example 3:** In each of these expressions, the quantity $a + b$ is to be treated collectively.

$(a + b)$ $[a + b]$ $\{a + b\}$

$\dfrac{c}{a + b}$ $\dfrac{a + b}{c}$ $\sqrt{a + b}$ ◆◆◆

Absolute Value

The *absolute value* of a number n is its *magnitude* regardless of its algebraic sign. It is written $|n|$. It is the distance between n and zero on the number line, without regard to direction.

◆◆◆ **Example 4:** Here is the evaluation of some expressions containing absolute value signs. See if you get the same results.

(a) $|5| = 5$

(b) $|-9| = 9$

(c) $|3 - 7| = |-4| = 4$

(d) $-|-4| = -4$

(e) $-|7 - 21| - |13 - 19| = -|-14| - |-6| = -14 - 6 = -20$ ◆◆◆

Some calculators have a key for evaluating absolute values. Use it to evaluate any of these expressions.

Approximate Numbers

Most of the numbers we deal with in technology are *approximate*.

◆◆◆ **Example 5:**

(a) All numbers that represent *measured* quantities are approximate. A certain shaft, for example, is approximately 1.75 inches in diameter.
(b) Many *fractions* can be expressed only approximately in decimal form. Thus $\frac{2}{3}$ is approximately equal to 0.6667.
(c) *Irrational numbers* can be written only approximately in decimal form. The number $\sqrt{3}$ is approximately equal to 1.732. ◆◆◆

Exact Numbers

An approximate number always has some *uncertainty* in the rightmost digit. That is, we cannot be sure of its exact value. On the other hand, an *exact number* is one that has no uncertainty.

◆◆◆ **Example 6:**

(a) There are exactly 24 hours in a day, no more, no less.
(b) Most automobiles have exactly four wheels.
(c) Exact numbers are usually integers, but not always. For example, there are *exactly* 2.54 cm in an inch, by definition.
(d) On the other hand, not all integers are exact. For example, a certain town has a population of *approximately* 3500 people. ◆◆◆

Significant Digits and Accuracy

In a decimal number, zeros are sometimes used just to locate the decimal point. When zeros are used in that way, we say that they are *not significant*. The remaining digits in the number, including zeros, are called *significant digits*.

◆◆◆ **Example 7:**

(a) The numbers 497.3, 39.05, 8003, 140.3, and 2.008 each have *four* significant digits.
(b) The numbers 1570, 24,900, 0.0583, and 0.000583 each have *three* significant digits. The zeros in these numbers serve only to locate the decimal point.

An overscore is sometimes placed over the last trailing zero that is significant. Thus the numbers $395\bar{0}$ and $735{,}\bar{0}00$ each have four significant digits.

(c) The numbers 18.50, 1.490, and 2.000 each have *four* significant digits. The zeros here are not needed to locate the decimal point. They are placed there to show that those digits are in fact zeros, and not some other digit. ◆◆◆

The number of significant digits in a number is often called the *accuracy* of that number. Thus the numbers in Example 7(a) are said to be *accurate to* four significant digits. Knowing the number of significant digits in a number is important for multiplication and division, as we will see in the next section.

◆◆◆ **Example 8:** Verify the number of significant digits in each approximate number.

(a)	39.3	has 3	(b)	274.2	has 4
(c)	3700	has 2	(d)	3.000	has 4
(e)	0.0486	has 3	(f)	3700.0	has 5

◆◆◆

Decimal Places and Precision

We will see that to add or subtract a number properly, we need to know its *number of decimal places*. To find it, we simply count the number of digits to the right of the decimal point. The number of decimal places is often called the *precision* of the decimal number.

Keep in mind that we are talking about accuracy and precision of *numbers,* not of *measurements.* The *accuracy of a measurement* of some quantity refers to the nearness of the measured value to the "true," correct, or accepted value of that quantity. The *precision of measurements* is a measure of the *repeatability* of a group of measurements, that is, how close together a group of measurements are to *each other.*

◆◆◆ **Example 9:**

(a) The number 395.2 has one decimal place. We say it is precise to one decimal place, or precise to the nearest tenth.
(b) The number 7.284 is precise to three decimal places or precise to the nearest thousandth.
(c) The number 23,800 has no decimal places but is accurate to three significant digits and precise to the nearest hundred.
(d) In the number 18.30, the trailing zero is significant. Therefore it is accurate to four significant digits and precise to the nearest hundredth. ◆◆◆

Thus when using an approximate number, we need to be clear about its number of (a) significant digits and (b) decimal places. These will govern how we treat that number in a calculation. Which of these we call accuracy and which we call precision is not as important, especially as the two words are often confused, even in technical work.

◆◆◆ **Example 10:** Verify the number of decimal places in each approximate number.

(a)	39.3	has 1	(b)	274.2	has 1
(c)	3700	has 0	(d)	3.000	has 3
(e)	0.0486	has 4	(f)	3700.0	has 1

◆◆◆

Rounding

In the next few sections, we will see that the numbers we get from a computation often contain *worthless digits* that must be *thrown away*. Whenever we do this, we must *round* our answer.

Round down (do not change the last retained digit) when the first discarded digit is 4 or less. *Round up* (increase the last retained digit by 1) when the first discarded digit is 6 or more, or a 5 followed by a nonzero digit in any of the decimal places to the right.

Sometimes we must round to a certain number of decimal places, and other times we must round to a certain number of significant digits. The procedure is no different.

◆◆◆ **Example 11:** Here are some numbers rounded to four significant digits.

Number	Rounded to Four Significant Digits
395.67	395.7
1.09356	1.094
0.0057284	0.005728

◆◆◆

We have seen that the rightmost digit in an approximate number has some uncertainty, but how much? If that last digit is the result of rounding in a previous step, it could be off by as much as *half a unit, either greater or smaller*. This is its *uncertainty*.

◆◆◆ **Example 12:** Here are some examples of rounding to three decimal places.

Number	Rounded to Three Decimal Places
4.3654	4.365
4.3656	4.366
4.365501	4.366
1.764999	1.765
1.927499	1.927

◆◆◆

When the discarded portion is 5 *exactly,* it usually does not matter whether you round up or down. The exception is when you are adding or subtracting a long column of figures, as in statistical computations. If, when discarding a 5, you always rounded up, you could bias the result in that direction. To avoid that, you want to round up about as many times as you round down, and a simple way to do that is to always *round to the nearest even number*. This is just a convention. We could just as well round to the nearest odd number.

◆◆◆ **Example 13:**

Number	Rounded to Two Decimal Places
4.365	4.36
4.355	4.36
7.76500	7.76
7.75500	7.76

◆◆◆

◆◆◆ **Example 14:** The approximate number 35.85, rounded to one decimal place, is 35.8. The number 35.75, rounded to one decimal place, is also 35.8. Thus the number 35.8 could be the rounded value of any number between 35.75 and 35.85. There is simply no way to tell what value may have been in the second decimal place. Now if there is uncertainty in a particular decimal place, it is clear that the values in any places to its right are completely unknown. ◆◆◆

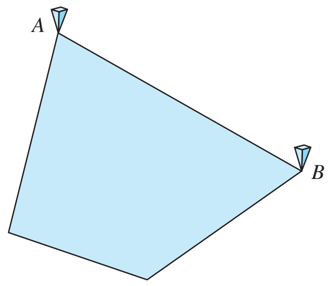


FIGURE 1–3

◆◆◆ **Example 15:** *An Application*. In laying out a ground plan, Fig. 1–3, the distance *AB* is calculated to be 35.8368 ft. Knowing that the surveyors can only measure to a hundredth of a foot, how would you give this dimension on the site plan?

Solution: We would round to two decimal places, getting

$$35.84 \text{ ft}$$ ◆◆◆

Exercise 1 ◆ The Real Numbers

Symbols of Equality and Inequality Insert the proper symbol of equality or inequality ($=$, $\approx$, $>$, $<$) between each pair of numbers.

1. 7 and 10
2. 9 and -2
3. -3 and 4
4. -3 and -5
5. $\frac{3}{4}$ and 0.75
6. $\frac{2}{3}$ and 0.667

Absolute Value Evaluate each expression.

7. $|4|$
8. $|-3|$
9. $-|-6|$
10. $-|9 - 23| - |-7 + 3|$
11. $|12 - 5 + 8| - |-6| + |15|$
12. $-|3 - 9| - |5 - 11| + |21 + 4|$

Significant Digits and Decimal Places

Determine the number of significant digits in each approximate number.

13. 78.3
14. 9274
15. 4.008
16. 9400
17. 20,000
18. 5000.0
19. 0.9972
20. 1.0000

Determine the number of decimal places in each approximate number.

21. 39.5
22. 9.55
23. 5.882
24. 193

Rounding

Round each number to two decimal places.

25. 38.468
26. 1.996
27. 96.835001
28. 55.8650
29. 398.372
30. 2.9573

Round each number to one decimal place.

31. 13.98
32. 745.62
33. 5.6501
34. 0.482
35. 398.36
36. 34.927

Round each number to the nearest hundred.

37. 28,583
38. 7550
39. 3,845,240
40. 274,837

Round each number to three significant digits.

41. 9.284
42. 2857
43. 0.04825
44. 483,982
45. 0.08375
46. 29.555

Round each number to five significant digits.

47. 34.9274
48. 827.365
49. 4.03726
50. 0.00365286

51. 5.937254
52. 374.8264
53. Evaluate the expressions in problems 7 through 12 by calculator. On the TI-83/84 and TI-89 it is indicated by **abs(** and is located in the MATH **NUM** menu.

An Application

54. When calculating the required length of a girder, an architect gets a value of 14.8363 ft on her calculator. What dimension should she put on the plans if it is customary to specify griders to the nearest hundredth of a foot?
55. *Team Project:* Make a drawing of a cylindrical steel bar, 1 inch in diameter and 3 in. long. Label the diameter as 1.00 in. Take your drawing to a machine shop and ask for a cost estimate for each of six bars, having lengths of

3 in.	3.000 in.
3.0 in.	3.0000 in.
3.00 in.	3.00000 in.

Before you go, have each member of your team make cost estimates.

56. *Internet:* Systems of numbers having bases other than 10 are used in computer science. They are *binary numbers* (base 2), *octal numbers,* (base 8), and *hexadecimal numbers* (base 16). A complete chapter on these kinds of numbers, which you may download and print, is located on our Web site at www.wiley.com/college/calter

1–2 Addition and Subtraction

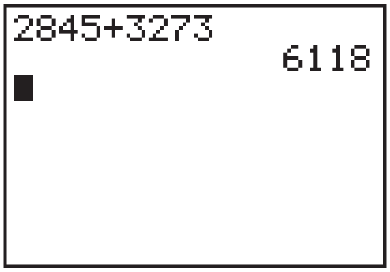
Now that we have refreshed our memory about the different kinds of numbers, let's see how they are used in the various arithmetic operations. We will start with addition and subtraction.

Adding and Subtracting Integers by Calculator

There are many types of calculators in use. In this text we will show screens for the TI-83 Plus calculator, which will usually be the same for the TI-84 Plus, and for TI-89 Titanium, which we shall indicate simply as TI-89.

To add two numbers by calculator, simply enter the first number; press +; enter the second number; and press the *enter* key ENTER or the *equals* key =, or the *execute* key EXE, depending upon your particular calculator. The number we get is called the *sum* of the two numbers.

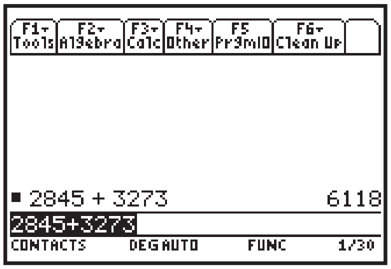
For subtraction we use the − key (*not* the (−) key). The result is called the *difference* of the two numbers.



TI-83/84 screen for Example 16. Your calculator display may differ depending upon which numerical format is chosen from the MODE menu. Here we are in Float mode.

◆◆◆ Example 16: Evaluate 2845 + 3273 by calculator.

Solution: We key in 2845, then press the + key, then 3273, and finally ENTER. The screens for the TI-83 Plus (and TI-84) as well as the TI-89 Titanium are shown.



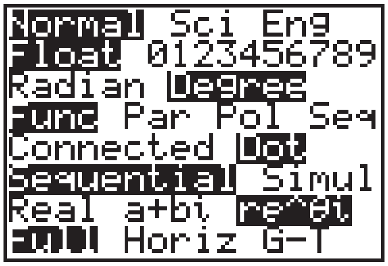
TI-89 screen for Example 16.

Changing the Calculator Display

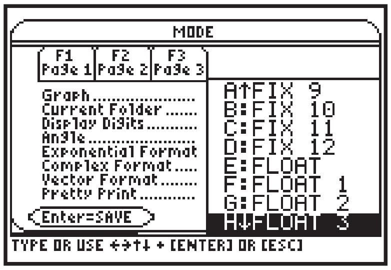
You can select the way a calculator displays numbers from the MODE menu.

Float (floating) mode on the TI-83 will give the full calculator display, up to ten digits. On the TI-89 you can select the total number of digits to be displayed, including those to the left to the decimal point.

Fix (fixed) mode on either calculator will display a result with the number of decimal places chosen.



MODE screen for the TI-83/84. You can select either the floating mode (**Float**), or the number of digits to be displayed to the right of the decimal point.



MODE screen for the TI-89. You can select the FIX mode and choose the number of decimal places to be displayed, or a FLOAT mode, and choose the total number of digits to be displayed.

Changing the display *does not* affect the accuracy of a computation, but only the way the result is displayed. However, your mode settings may make your answers look different than those given here.

Adding Signed Numbers

Let us say that we have a shoebox (Fig. 1–4) into which we toss all of our uncashed checks and unpaid bills until we have time to deal with them. Let us further assume that the total checks minus the total bills in the shoebox is \$500.

We can think of the amount of a check as a *positive* number because it increases our wealth, and the amount of a bill as a *negative* number because it decreases our wealth. We thus represent a check for \$100 as $(+100)$, and a bill for \$100 as (-100).

Now, let's *add a check* for \$100 to the box. If we had \$500 at first, we must now have \$600.

$$500 + (+100) = 600$$

or

$$500 + 100 = 600$$

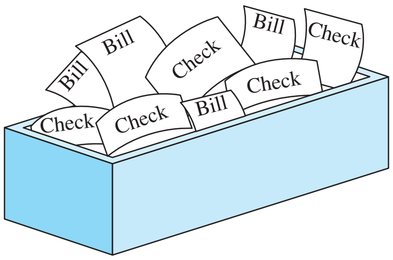


FIGURE 1–4 The shoebox.

Here we have added a positive number, and our total has increased by that amount. That is easy to understand. But what does it mean to *add a negative number*?

To find out, let us now *add a bill* for \$100 to the box. If we had \$500 at first, we must now have \$100 less, or \$400. Representing the bill by (-100), we have

$$500 + (-100) = 400$$

It seems clear that to *add a negative number is no different than subtracting the absolute value of that number.*

$$500 + (-100) = 500 - 100 = 400$$

This gives us our rule of signs for addition of signed numbers.

All boxed and numbered formulas are tabulated in numerical order in Appendix A. There they are arranged in logical order by type and are numbered consecutively. Since the formulas often appear in the text in a different order than in Appendix A, they may not be in numerical order here in the text.

Rule of Signs for Addition	$a + (-b) = a - b$	1

◆◆◆ **Example 17:** Combine as indicated.

(a) $7 + (-2) = 7 - 2 = 5$

(b) $-8 + (-3) = -8 - 3 = -11$

(c) $-9 + 4 = -5$ ◆◆◆

Subtracting Signed Numbers

Let us return to our shoebox. But now instead of adding checks or bills to the box, we will *subtract* (remove) checks or bills from the box.

First we remove (subtract) a check for \$100 from the box. If we had \$500 at first, we must now have \$400.

$$500 - (+100) = 400$$

or

$$500 - 100 = 400$$

Here we have subtracted a positive number, and our total has decreased by the amount subtracted, as expected.

Now let us see what it means to *subtract a negative number*. We will remove (subtract) a bill for \$100 from the box. If we had \$500 at first, we must now have \$100 more, or \$600, since we have removed a bill. Representing the bill by (-100), we have

$$500 - (-100) = 600$$

It seems clear that *to subtract a negative number is the same as to add the absolute value of that number*.

$$500 - (-100) = 500 + 100 = 600$$

This gives us our rule of signs for subtraction of signed numbers.

Rule of Signs for Subtraction	$a - (-b) = a + b$	2

◆◆◆ **Example 18:**

(a) $8 - (-6) = 8 + 6 = 14$ (b) $-7 - (-5) = -7 + 5 = -2$

(c) $-(-16) - 7 - (-9) = 16 - 7 + 9 = 18$ ◆◆◆

Subtracting Negative Numbers by Calculator

Note that two similar-looking calculator keys are used for two different things:

1. To subtract two quantities.
2. To enter a negative quantity.

This difference is clear on the calculator, which has separate keys for these two functions. The [−] key is used only for subtraction; the [(−)] key is used to enter a negative quantity.

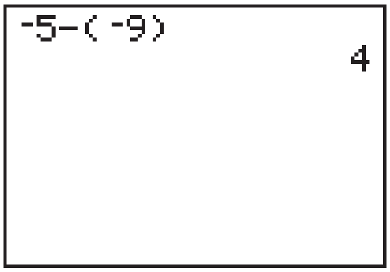
To enter a negative number on some calculators, simply press the [(−)] key and then enter in the number.

Try the following examples on your calculator and see if you get the correct answers.

◆◆◆ **Example 19:** Combine as indicated.

(a) $15 - (-3) = 15 + 3 = 18$ (b) $-5 - (-9) = -5 + 9 = 4$

(c) $-25 - (-5) = -25 + 5 = -20$ ◆◆◆



TI-83/84 screen for Example 19(b). Note the different appearances of the negative and the subtraction signs.

Common Error	The [−] key and the [(−)] key look almost alike. Be careful not to confuse them. Note that the key used to enter a negative quantity has parentheses.

Commutative and Associative Laws

These laws are surely familiar to you, even if you do not recognize their names. We will run into them again when studying algebra. The *commutative law* simply says that you can add numbers in *any order*.

Commutative Law for Addition	$a + b = b + a$	3

◆◆◆ **Example 20:** Here is an example of the commutative law with numbers.

$$2 + 3 = 3 + 2$$
$$= 5$$ ◆◆◆

The *associative law* says that you can group numbers to be added in several ways.

Associative Law for Addition	$a + (b + c) = (a + b) + c$ $= (a + c) + b$	4

◆◆◆ **Example 21:** The associative law, with numbers, is shown here.

$$2 + 3 + 4 = 2 + (3 + 4) = 2 + 7 = 9$$
$$= (2 + 3) + 4 = 5 + 4 = 9$$
$$= (2 + 4) + 3 = 6 + 3 = 9$$ ◆◆◆

Adding and Subtracting Approximate Numbers

Addition and subtraction of integers are simple enough. But now let us tackle the problem mentioned earlier: How many digits do we keep in our answer when adding or subtracting *approximate numbers?*

■ Explorations:

(a) A six-foot-tall person stands on a box. How high would you say the person plus the box are if the box is 1.14 ft high?
(b) A certain gasoline tank contains 14.5 gallons. If we siphon 2.585 gallons from a full tank, how many gallons would you say are left in the tank?
(c) A person who weighs 135 pounds picks up a laboratory weight marked 1.750 lb. What would you state as their combined weight?

Keeping in mind that the rightmost digit in an approximate number contains some uncertainty, and that those to its right are unknown, what conclusions can you draw about the addition and subtraction of approximate numbers? Can you say why it is misleading to give the height of the person plus box as 7.14 ft? Can you see the reason for the following rule? ■

Addition and Subtraction	When adding or subtracting approximate numbers, keep as many decimal places in your answer as contained in the number having the fewest decimal places.	5

◆◆◆ **Example 22:** Removing 2.585 gallons from 14.5 gallons gives

$$14.5 - 2.585 = 11.915 \text{ gallons}$$

As 14.5 has just one decimal place, we must round our answer to one decimal place. So we write

$$14.5 - 2.585 = 11.9 \text{ gallons}$$

Here it is common practice to use the equals sign rather than the ≈ sign. ◆◆◆

◆◆◆ **Example 23:** Let us add 32.4 cm and 5.825 cm.

$$32.4 \text{ cm} + 5.825 \text{ cm} = 38.2 \text{ cm } (\textit{not } 38.225 \text{ cm})$$

Here we can see the reason for our rule for rounding. In one of the original numbers (32.4 cm), we do not know what digit is to the right of the 4, in the hundredths place. We cannot assume that it is zero, for if we *knew* that it was zero, it would have been written in (as 32.40). Not knowing the digit in the hundredths place in an original number causes uncertainty in the tenths place in the answer. So we discard the hundredths digit and any digits to the right of it. ◆◆◆

◆◆◆ **Example 24:** Here is another example of adding approximate numbers.

```
25.8
18.3 125
 5.4 07
--------
49.5|195
    |  discard
    +---------->
```

◆◆◆

Common Error	Students *hate* to throw away those last digits. Remember that by keeping worthless digits, you are telling whoever reads that number that it is more precise than it really is.

◆◆◆ **Example 25:** A certain stadium contains about 3,500 people. It starts to rain, and 372 people leave. How many are left in the stadium?

Solution: Subtracting, we obtain

$$3500 - 372 = 3128$$

which we round to 3100, because 3500 here is known to only two significant digits. ◆◆◆

It is safer to round the answer *after* adding (or subtracting), rather than to round the original numbers before adding. If you do round before adding, it is prudent to round each original number to *one more* decimal place than you expect to keep in the rounded answer.

Combining Exact and Approximate Numbers

When you are combining an exact number and an approximate one, the accuracy of the result will be limited by the approximate number. Thus round the result to the number of decimal places found in the approximate number, even though the exact number may *appear* to have fewer decimal places.

◆◆◆ **Example 26:** Express 2 hr and 35.8 min in minutes.

Solution: We must add an exact number (120 minutes) and an approximate number 35.8 minutes).

$$\begin{array}{rrl} & 120 & \text{min} \\ + & 35.8 & \text{min} \\ \hline & 155.8 & \text{min} \end{array}$$

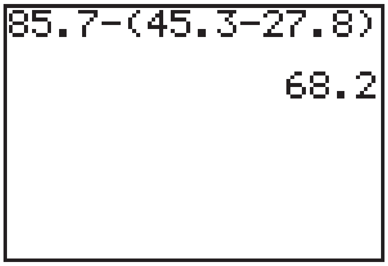
Since 120 is exact, we retain as many decimal places as in the approximate number, 35.8. Our answer is thus 155.8 min. ◆◆◆

Common Error	Be sure to recognize which numbers in a computation are exact; otherwise, you may perform drastic rounding by mistake.

Combining Approximate Numbers by Calculator

The keystrokes for combining approximate numbers are the same as for combining integers. Just be sure not to select a fixed decimal mode with so few decimal places as to cut off significant digits after the decimal point.

To evaluate an expression *containing parentheses,* use the (key and the) key.



TI-83/84 screen for Example 27.

◆◆◆ **Example 27:** The keystrokes used to evaluate

$$85.7 - (45.3 - 27.8)$$

are shown on the screen. ◆◆◆

Loss of Significant Digits During Subtraction

Subtracting two nearly equal numbers can lead to a drastic loss of significant digits.

◆◆◆ **Example 28:** When we subtract, say,

$$6{,}755 - 6{,}753 = 2$$

we get a result having one significant digit, while our original numbers each had four. While not common, you should be aware that it can happen, and can destroy the accuracy of a computation. ◆◆◆

Similarly, significant digits can be gained by addition, say,

$$8.0 + 5.0 = 13.0$$

which gives a three significant digit result from numbers having two significant digits each. ◆◆◆

Exercise 2 ◆ Addition and Subtraction

Adding and Subtracting Signed Numbers Combine as indicated.

1. $926 + 863$
2. $274 + (-412)$
3. $-576 + (-553)$
4. $-207 + (-819)$
5. $-575 - 275$
6. $-771 - (-976)$
7. $1123 - (-704)$
8. $818 - (-207) + 318$

Adding and Subtracting Approximate Numbers Combine each set of approximate numbers as indicated. Round your answer.

9. $4857 + 73.8$
10. $39.75 + 27.4$
11. $296.44 + 296.997$
12. $385.28 - 692.8$
13. $0.000583 + 0.0008372 - 0.00173$

Applications

14. Mt. Blanc is 15,572 ft high, and Pike's Peak is about 14,000 ft high. What is the difference in their heights?
15. California contains 158,933 square miles (mi^2), and Texas 237,321 mi^2. How much larger is Texas than California?

16. A man willed \$125,000 to his wife and two children. To one child he gave \$44,675, to the other \$26,380, and to his wife the remainder. What was his wife's share?

17. A circular pipe has an inside radius r of 10.6 cm and a wall thickness of 2.125 cm. It is surrounded by insulation having a thickness of 4.8 cm (Fig. 1–5). What is the outside diameter D of the insulation?

18. A batch of concrete is made by mixing 267 kg of stone, 125 kg of sand, 75.5 kg of cement, and 25.25 kg of water. Find the total weight of the mixture.

19. Three resistors, having values of 27.3 ohms (Ω), 4.0155 Ω, and 9.75 Ω, are wired in series. What is the total resistance? (See Eq. 1062 which says that the total series resistance is the sum of the individual resistances.)

20. *Writing:* Does your calculator have two separate keys marked with a negative sign? Why? Is there any difference between them, and if so, what? When would you use each? Write a paragraph or two explaining these keys and answering these questions.

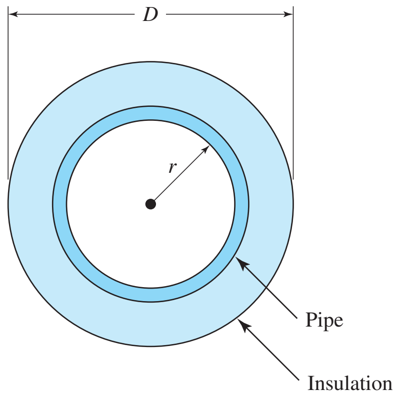


FIGURE 1–5 An insulated pipe.

1–3 Multiplication

Addition and subtraction were easy enough. Let's move on to multiplication.

Symbols and Definitions

Multiplication can be indicated in several ways: by the usual $\times$ symbol; by a dot; or by parentheses, brackets, or braces. Thus the product of b and d could be written

$$b \cdot d \qquad b \times d \qquad b(d) \qquad (b)d \qquad (b)(d)$$

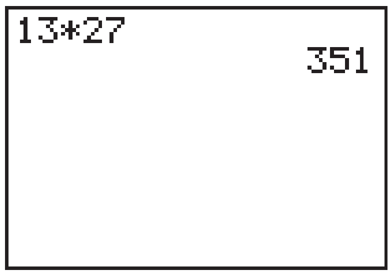
Most common of all is to use no symbol at all. The product of b and d would usually be written bd. When doing algebra avoid using the $\times$ symbol because it could get confused with the letter x.

We get a *product* when we multiply two or more *factors*.

$$\text{(factor) (factor) (factor)} = \text{product}$$

Multiplying by Calculator

Many calculators use the [×] key for multiplication and use an asterisk or star (*) on the screen to represent a multiplication dot.



This TI-83/84 screen shows the multiplication of the factors 13 and 27.

Multiplying Signed Numbers

To get our rules of signs for multiplication, we use the idea of *multiplication as repeated addition.* For example, to multiply 3 by 4 means to add four 3's (or three 4's)

$$3 \times 4 = 3 + 3 + 3 + 3$$

or

$$3 \times 4 = 4 + 4 + 4$$

Let us return to our shoebox example. Recall that it contains uncashed checks and unpaid bills. Let's first *add 5 checks* (+5), each worth \$100 (+100), to the box. The value of the contents of the box then increases by \$500. Multiplying, we have

$$\text{(number of checks)} \times \text{(value of one check)} = \text{change in value of contents}$$

$$(+5)(+100) = +500$$

Thus a positive number times a positive number gave a positive product. This is nothing new.

Now let's *add 5 bills*, each for \$100, to the box, thus decreasing its value by \$500. To show this multiplication, we use (+5) for the number of bills, taking (−100) for the value of each bill, and (−500) for the change in value of the box contents.

$$(+5)(-100) = -500$$

Here, a positive number times a negative number gives a negative product.

Next we *remove 5 checks*, each for \$100, from the box, thus decreasing its value by \$500.

$$(-5)(+100) = -500$$

Here again, the product of a positive number and a negative number is negative. Thus it doesn't matter whether the negative number is the first or the second.

Finally, we *remove 5 bills*, each for \$100, from the box, causing its value to increase by \$500.

$$(-5)(-100) = +500$$

Here, the product of two negative numbers is positive.

We summarize these findings to get our *rules of signs for multiplication.*

Rules of Signs for Multiplication	$(+a)(+b) = (-a)(-b) = +ab$ *The product of two quantities of like sign is positive.*	**6**
a and *b* are positive numbers	$(+a)(-b) = (-a)(+b) = -ab$ *The product of two quantities of unlike sign is negative.*	**7**

◆◆◆ **Example 29:** Multiply.

(a) $2(-3) = -6$ (b) $(-2)3 = -6$ (c) $(-2)(-3) = 6$ ◆◆◆

Multiplying a String of Numbers

When we multiplied two negative numbers, we got a positive product. So when we are multiplying a *string* of numbers, if an even number of them are negative, the answer will be positive, and if an odd number of them are negative, the answer will be negative.

◆◆◆ **Example 30:** Multiply.

(a) $2(-3)(-1)(-2) = -12$ (b) $2(-3)(-1)(2) = 12$ ◆◆◆

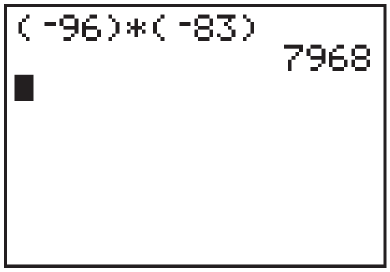
Multiplying Negative Numbers by Calculator

As we mentioned earlier, you enter negative numbers into the calculator by using a key marked [(−)], [+/−], or [CHS]. On some calculators you first enter the number and then change its sign using the proper key; on other calculators you press the [(−)] key first and then enter the number.

◆◆◆ **Example 31:** Use your calculator to multiply -96 by -83.

Solution: You should get

$$(-96)(-83) = 7968$$ ◆◆◆



TI-83/84 screen for Example 31. On some calculators the multiplication symbol may be omitted, as the parentheses themselves indicate multiplication.

A simpler way to do the last problem would be to multiply $+96$ and $+83$ and determine the sign by inspection.

Common Error	Do not try to use the [−] key to enter negative numbers into your calculator. The [−] key is only for subtraction.

Multiplication of Approximate Numbers

■ Exploration:

Try this. Multiply two approximate numbers, say 5.43 and 4.75, and write down the full calculator display.

$$(5.43)(4.75) = 25.7925$$

But each of the original numbers has some uncertainty: 4.75, for example, could have been any value between 4.746 and 4.754 before it was rounded in some previous step. So repeat the multiplication, replacing 4.75 with 4.746. How does this affect the product? Repeat again, replacing 4.75 with 4.754. Repeat again, now letting 5.43 take on some uncertainty. What can you conclude about whether all those digits in the product should be kept?

Repeat this exploration with other approximate numbers. Do you see the reason for the following rule? ■

Rule	When multiplying two or more approximate numbers, round the result to as many digits as in the factor having the fewest significant digits.	11

◆◆◆ **Example 32:** Here we multiply two numbers, each with 3 significant digits.

12.1	×	15.6	=	189
↑		↑		↑
three digits		three digits		three digits

◆◆◆

When the factors have different numbers of significant digits, keep the same number of digits in your answer as is contained in the factor that has the *fewest* significant digits.

◆◆◆ **Example 33:** Here we multiply numbers that do not have the same number of significant digits.

123.56	×	2.21	=	273
↑		↑		↑
five digits		three digits		keep three digits

◆◆◆

Common Error	Do not confuse *significant digits* with *decimal places*. The number 274.56 has *five* significant digits and *two* decimal places. Decimal places determine how we round after adding or subtracting. Significant digits determine how we round after multiplying and, as we will soon see, after dividing, raising to a power, or taking roots.

◆◆◆ **Example 34:** *An Application.* Find the weight of 3.845 cubic feet of stone that has a density of 175 lb/ft^3.

Solution: The weight equals the volume times the density, so,

$$\text{Weight} = 3.845 \times 175 = 673 \text{ lb}$$

after rounding to the three significant digits found in 175. ◆◆◆

Multiplying Exact and Approximate Numbers

When using *exact numbers* in a computation, treat them as if they had *more* significant figures than any of the approximate numbers in that computation.

◆◆◆ **Example 35:** Multiplying the exact number 3 by the approximate number 6.836 gives

$$3 \times 6.836 = 20.51$$

We kept as many significant digits (4) as found in the approximate number. ◆◆◆

◆◆◆ **Example 36:** *An Application.* If a certain car tire weighs 32.2 lb, how much will four such tires weigh?

Solution: Multiplying, we obtain

$$32.2(4) = 128.8 \text{ lb}$$

Since the 4 is an exact number, we retain as many significant figures as contained in 32.2, and round our answer to 129 lb. ◆◆◆

Exercise 3 ◆ Multiplication

Multiplying Signed Numbers

1. $4 \times (-2)$
2. $(-5) \times (3)$
3. $(-24) \times (-5)$
4. $(-41) \times (-22)$

Multiplying Approximate Numbers Multiply each approximate number and retain the proper number of digits in your answer.

5. 3.967×2.84
6. 4.900×59.3
7. 93.9×0.0055908
8. $4.97 \times 9.27 \times 5.78$
9. $69.0 \times (-258)$
10. $-385 \times (-2.2978)$
11. $2.86 \times (4.88 \times 2.97) \times 0.553$
12. $(5.93 \times 7.28) \times (8.26 \times 1.38)$

Multiplying Exact and Approximate Numbers Multiply, and keep the proper number of significant digits in your answer. Take each integer as an exact number.

13. 4×2.55
14. -1.46×3
15. $-4.273 \times (-5)$
16. $(-5) \times (-1.022)$

Applications

17. What is the cost of 52.5 tons of cement at $63.25 a ton?
18. If 108 tons of rail is needed for 1 mi of track, how many tons will be required for 476 mi, and what will be its cost at $925 a ton?
19. Three barges carry 26.0 tons of gravel each, and a fourth carries 35.0 tons. What is the value, to the nearest dollar, of the whole shipment, at $12.75 per ton?
20. What will be the cost of installing a telephone line 274 km long, at $5723 per kilometer?
21. The current to a projection lamp is measured at 4.7 A when the line voltage is 115.45 V. Using (power = voltage × current), find the power dissipated in the lamp.
22. A gear in a certain machine rotates at the speed of 1808 rev/min. How many revolutions will it make in 9.500 min?
23. How much will 1000 washers weigh if each weighs 2.375 g?
24. One inch equals exactly 2.54 cm. Convert 385.84 in. to centimeters.
25. If there are 360 degrees per revolution, how many degrees are there in 4.863 revolutions?

1–4 Division

Definitions

The *dividend*, when divided by the *divisor,* gives us the *quotient*.

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

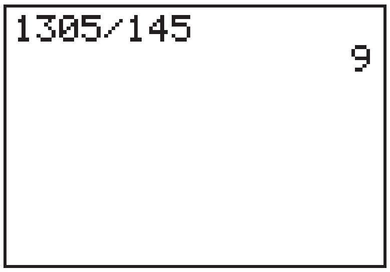
or

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

A quantity a/b is also called a *fraction*, where a is called the *numerator* and b is called the *denominator*. It can also be referred to as the *ratio* of a to b. Fractions and ratios are covered in detail in later chapters.

Dividing by Calculator

To divide by calculator, enter the dividend, then press [÷], then enter the divisor and press [ENTER], [=], or [EXE], depending on your particular calculator.



TI-83/84 screen for Example 37.

◆◆◆ **Example 37:** Here is the screen for the division of 1305 by 145.

When we multiplied two integers, we always got an integer for an answer. This is not always the case when dividing, as shown in the next example. ◆◆◆

◆◆◆ **Example 38:** When we divide 2 by 3, we get 0.666666666. . . . We must choose how many digits we wish to retain and round our answer. Rounding to, say, three significant digits, we obtain

$$2 \div 3 \approx 0.667$$

Here it is appropriate to use the $\approx$ symbol. ◆◆◆

Dividing Signed Numbers

We will now use the rules of signs for multiplication to get the rules of signs for division.

(a) We know that the product of a negative number and a positive number is negative. For example,

$$(-2)(+3) = -6$$

If we divide both sides of this equation by $(+3)$, we get

$$-2 = \frac{-6}{+3}$$

From this we see that *a negative number divided by a positive number gives a negative quotient.*

(b) Again starting with

$$(-2)(+3) = -6$$

we divide both sides by (-2) and get

$$+3 = \frac{-6}{-2}$$

Here we see that *a negative number divided by a negative number gives a positive* quotient.

(c) We also know that the product of two negative numbers is positive. Thus

$$(-2)(-3) = +6$$

Dividing both sides by (-3), we get

$$-2 = \frac{+6}{-3}$$

Thus *a positive number divided by a negative number gives a negative quotient.*

We combine these findings with the fact that the quotient of two positive numbers is positive and get our *rules of signs for division.*

Rules of Signs for Division *a* and *b* are positive numbers	$\frac{+a}{+b} = \frac{-a}{-b} = \frac{a}{b}$ *The quotient is positive when dividend and divisor have the same sign.*	**12**
	$\frac{+a}{-b} = \frac{-a}{+b} = -\frac{a}{b}$ *The quotient is negative when dividend and divisor have opposite signs.*	**13**

◆◆◆ **Example 39:**

(a) $8 \div (-4) = -2$ (b) $-8 \div 4 = -2$

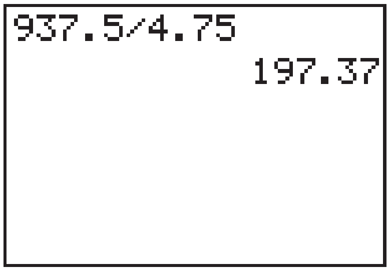
(c) $-8 \div (-4) = 2$ ◆◆◆

Dividing Approximate Numbers

The rule for rounding after division is almost the same as with multiplication, and is given for the same reason.

Rule	After dividing one approximate number by another, round the quotient to as many digits as there are in the original number having the fewest significant digits.	14

◆◆◆ **Example 40:** Divide 937.5 by 4.75, keeping the proper number of significant digits in the quotient.



TI-83/84 screen for Example 40.

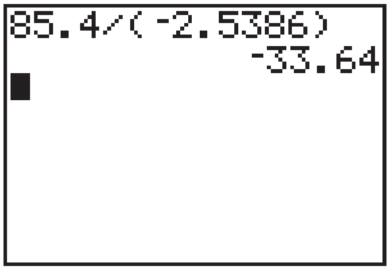
Solution: Since 4.75 has fewer significant digits (three) than 937.5, we round our answer to three significant digits, getting 197. ◆◆◆

◆◆◆ **Example 41:** Divide 846.2 into three equal parts.

Solution: We divide by the integer 3, and since we consider integers to be exact, we retain in our answer the same number of significant digits as in 846.2.

$$846.2 \div 3 = 282.1$$ ◆◆◆

◆◆◆ **Example 42:** Divide 85.4 by -2.5386 by calculator.



TI-83/84 screen for Example 42.

Solution: You should get

$$85.4 \div (-2.5386) = -33.6, \text{ rounded}$$ ◆◆◆

As with multiplication, the sign could also have been found by inspection.

◆◆◆ **Example 43:** *An Application.* How fast would an airplane have to travel to go 3895 miles in 5.25 hours? (rate = distance ÷ time)

Solution: Dividing gives

$$\text{rate} = \frac{3895 \text{ mi}}{5.25 \text{ h}} = 742 \text{ mi/h}$$

We have rounded to the three significant digits found in 5.25. ◆◆◆

Zero

■ Exploration:

Try this. Using your calculator, do the following divisions.

$0 \div 5$ $0 \div 295$ $5 \div 0$ $295 \div 0$ $0 \div 0$

From your results, can you deduce the rules for dividing zero by a number, and for dividing a number by zero? ■

Division Involving Zero	Zero divided by any quantity (except zero) is zero. Division by zero is not defined. It is an illegal operation in mathematics.	15

Reciprocals

The *reciprocal* of any nonzero number n is $1/n$. Thus the product of a quantity and its reciprocal is equal to 1.

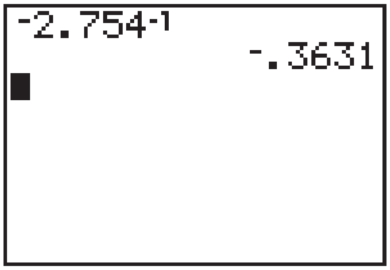
◆◆◆ **Example 44:**

(a) The reciprocal of 10 is $1/10$.

(b) The reciprocal of $1/2$ is 2.

(c) The reciprocal of $-\frac{3}{4}$ is $-\frac{4}{3}$. ◆◆◆

Reciprocals by Calculator



TI-83/84 screen for Example 45(b).

On the TI-83/84, simply enter the number and press the $\boxed{x^{-1}}$ key. On the TI-89 this operation is found in CATALOG and is indicated by **^ –1**. Keep as many digits in your result as there are significant digits in the original number.

◆◆◆ **Example 45:**

(a) The reciprocal of 6.38 is 0.157.
(b) The reciprocal of -2.754 is -0.3631. ◆◆◆

◆◆◆ **Example 46:** *An Application.* The unit of electrical conductance is the *mho* (ohm spelled backward) or *Siemens*. It is the reciprocal of resistance. Find the conductance of a circuit element having a resistance of 598 ohms.

Solution: Taking the reciprocal,

$$\frac{1}{598} = 0.00167 \text{ mhos}$$

◆◆◆

Exercise 4 ◆ Division

Dividing Signed Numbers Divide, keeping the proper sign on your answer.

1. $14 \div (-2)$
2. $(-15) \div (3)$
3. $(-24) \div (-4)$
4. $(-49) \div (-7)$

Dividing Approximate Numbers Divide, and then round your answer to the proper number of digits.

5. $947 \div 5.82$
6. $0.492 \div 0.00478$
7. $-99.4 \div 286.5$
8. $-4.8 \div -2.557$
9. $5836 \div 8264$
10. $5.284 \div 3.827$
11. $94{,}840 \div 1.33876$
12. $3.449 \div (-6.837)$

Reciprocals Find the reciprocal of each number, retaining the proper number of digits in your answer.

13. 693
14. 0.00630
15. -396
16. 39.74

17. −0.00573
18. 938.4
19. 4.992
20. −6.93

Applications

21. A stretch of roadway 1858.54 m long is to be divided into 5 equal sections. Find the length of each section.

22. At what rate must a person walk to go 24.5 km in 12.75 h? (rate = distance ÷ time)

23. If three masons can build 245 ft of wall in 4.50 days, how many feet of wall can one mason build in a day? Assume that each mason works at the same rate, and that the same length of wall is built each day.

24. If 867 shares of stock are valued at $84,099, what is the value of each share?

25. The equivalent resistance R of a 475-Ω resistor and a 928-Ω resistor connected in parallel is given by

$$\frac{1}{R} = \frac{1}{475} + \frac{1}{928}$$

Find R.

26. When an object is placed 126 cm in front of a certain thin lens having a focal length f, the image will be formed 245 cm from the lens. The distances are related by

$$\frac{1}{f} = \frac{1}{126} + \frac{1}{245}$$

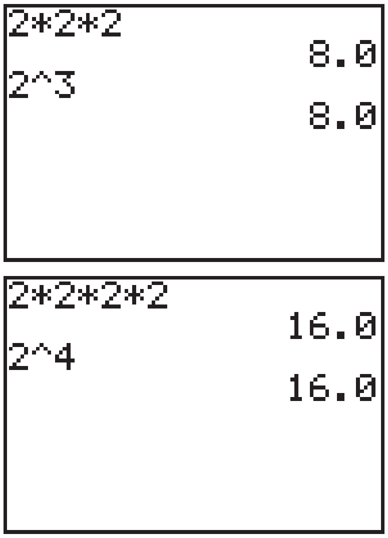
Find f.

27. The sine of an angle θ (written $\sin\theta$) is equal to the reciprocal of the cosecant of θ ($\csc\theta$). Find $\sin\theta$ if $\csc\theta = 3.58$.

28. If two straight lines are perpendicular, the slope of one line is the negative reciprocal of the slope of the other. If the slope of a line is −2.55, find the slope of a perpendicular to that line.

1–5 Powers and Roots

Now that we know how to do the four basic arithmetic operations on signed and approximate numbers, let us learn how to find powers and roots.



TI-83/84 screens for the Exploration.

Powers

■ Exploration:

Try this. Use your calculator to multiply $2 \times 2 \times 2$. Then use it to evaluate 2^3. You can do this using the [^] key on your calculater, as shown on the screen. Then evaluate $2 \times 2 \times 2 \times 2$, as well as 2^4.
Can you summarize what you have found? ■

In the expression

$$2^4$$

the number 2 is called the *base,* and the number 4 is called the *exponent*. The expression is read “two to the fourth power.” Its value is

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$

To *square* a number means to raise it to the power 2. To *cube* a number means to raise it to the power 3.

Stated as a formula,

Positive Integral Exponent	$x^n = \underbrace{x \cdot x \cdot x \ldots x}_{n \text{ factors}}$	**21**

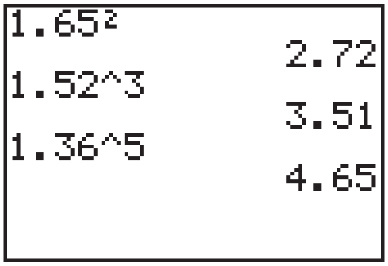
◆◆◆ Example 47:

(a) $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

(b) $5^3 = 5 \cdot 5 \cdot 5 = 125$

(c) $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$ ◆◆◆

When raising an approximate number to a power, round your answer to the number of significant digits in the base, not the exponent.



TI-83/84 screen for Example 48.

◆◆◆ Example 48: Use your calculator to verify the following:

(a) $1.65^2 = 2.72$

(b) $1.52^3 = 3.51$

(c) $1.36^5 = 4.65$ ◆◆◆

Negative Base

A negative base raised to an *even* power gives a *positive* number. A negative base raised to an *odd* power gives a *negative* number.

◆◆◆ Example 49: Here are some negative bases raised to various powers.

(a) $(-2)^2 = (-2)(-2) = 4$

(b) $(-2)^3 = (-2)(-2)(-2) = -8$

(c) $(-1)^{24} = 1$

(d) $(-1)^{25} = -1$ ◆◆◆

If you try to do these problems on your calculator, you may get an error indication. Some calculators will not work with a negative base, even though this is a valid operation.

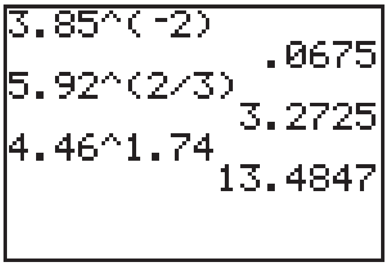
Then how do you do it? Simply enter the base as *positive,* find the power, and determine the sign by inspection.

Negative, Fractional, and Approximate Exponents

From our exploration we know the meaning of, say, 2^4. But what is the meaning of expressions like

$$2^{-4} \quad \text{or} \quad 2^{3/5} \quad \text{or} \quad 2^{1.55}$$

The explanation will have to wait for later when we have a bit more background. But for now, we can use our calculators to evaluate expressions such as these, in case you need to for, say, your physics course. The keystrokes are no different than for integer exponents.



TI-83/84 screen for Example 50.

◆◆◆ Example 50: See if you can verify the following:

(a) $3.85^{-2} = 0.0675$

(b) $5.92^{2/3} = 3.27$

(c) $4.46^{1.74} = 13.5$

Note that we have rounded our answers to the three significant digits found in each base. If the exponent is approximate, round the result to the number of significant digits in the exponent or the base, whichever has the fewest. ◆◆◆

Common Errors	When entering a negative base or negative exponent, be sure to use the key for entering a negative number, not the key for subtraction. Use parentheses when entering a negative base, negative exponent, or a fractional exponent.

◆◆◆ Example 51: *An Application.* The volume V of a sphere of radius r is given by

$$V = \frac{4}{3}\pi r^3$$

where $\pi = 3.142$. Find the volume of a sphercial weather balloon having a radius of 2.74 m.

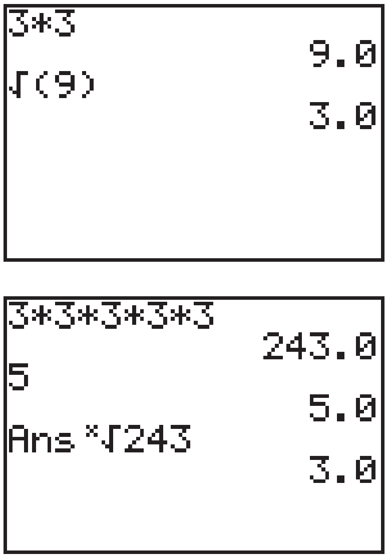
Solution:

$$V = \frac{4}{3}\pi(2.74)^3 = 86.2 \text{ m}^3$$ ◆◆◆

Roots

■ Exploration:

Try this. Calculate 3×3. Then use your calculator to evaluate $\sqrt{9}$. Then calculate $3 \times 3 \times 3 \times 3 \times 3$. Then evaluate $\sqrt[5]{243}$. On the TI-83/84 you first type 5 and press ENTER, then select the $\sqrt[x]{\ }$ operation from the MATH menu, type in 243, and press ENTER again. Can you summarize your results? ■



TI-83/84 screens for the Exploration.

If $a^n = b$, then

$$\sqrt[n]{b} = a$$

which is read "the nth root of b equals a." The symbol $\sqrt{\ }$ is a *radical sign,* b is the *radicand,* and n is the *index* of the radical.

◆◆◆ Example 52:

(a) $\sqrt{4} = 2$ because $2^2 = 4$

(b) $\sqrt[3]{8} = 2$ because $2^3 = 8$

(c) $\sqrt[4]{81} = 3$ because $3^4 = 81$ ◆◆◆

Principal Root

When we speak about the *root* of a number, we mean the *principal root*, unless otherwise specificed. The *principal root* of a positive number is defined as the *positive* root. Thus $\sqrt{4} = +2$, not ± 2.

The principal root is *negative* when we take an *odd* root of a *negative* number.

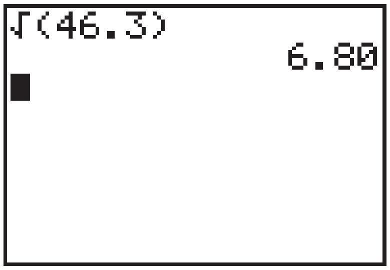
◆◆◆ Example 53:

$$\sqrt[3]{-8} = -2$$

because $(-2)(-2)(-2) = -8$. ◆◆◆

Roots by Calculator

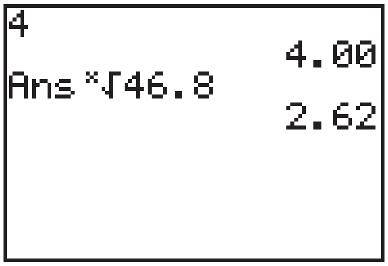
To find square roots, we simply use the $\boxed{\sqrt{x}}$ key. Retain as many significant digits as in the radicand.



TI-83/84 screen for Example 54.

◆◆◆ **Example 54:** To find $\sqrt{46.3}$ we use the keystrokes shown. ◆◆◆

For roots other than square roots, your calculator may have a special key, marked something like $\boxed{\sqrt[x]{y}}$. On the TI-83/84, first type the index and press $\boxed{\text{ENTER}}$, next select the $\sqrt[x]{\ }$ operation from the $\boxed{\text{MATH}}$ menu, then type in the radicand and press $\boxed{\text{ENTER}}$ again.



TI-83/84 screen for the Example 55.

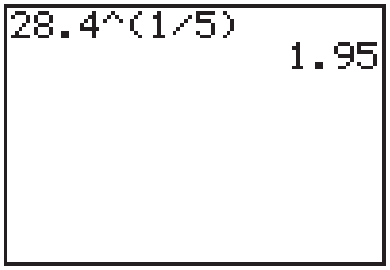
◆◆◆ **Example 55:** See if you get this result.

$$\sqrt[4]{46.8} = 2.62$$

As with the TI-89, if you have no special root key, you can still find roots using the $\boxed{\wedge}$ key. There is a relationship between powers and roots that we will prove later, but for now we will just use it for computing roots by calculator. It is,

Fractional Exponents	$a^{1/n} = \sqrt[n]{a}$	29

In other words, taking the nth root of a number is the same as raising that number to the $(1/n)$th power. So, for example, to take a fifth root of a number, we simply raise that number to the 1/5 power.



TI-83/84 screen for Example 56.

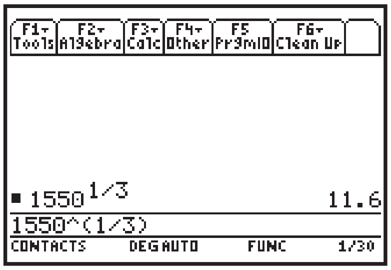
◆◆◆ **Example 56:** The keystrokes for finding $\sqrt[5]{28.4}$, the fifth root of 28.4, are shown. ◆◆◆

In the preceding example, instead of raising 28.4 to the 1/5 power, we could also have raised it to the 0.2 power, because $0.2 = 1/5$. In fact, we will show in the next chapter, that we can raise a base to any decimal or fractional power.

◆◆◆ **Example 57:** See if you can verify the following:

(a) $4.46^{1.74} = 13.5$

(b) $5.92^{2/3} = 3.27$ ◆◆◆



TI-89 screen for Example 58.

◆◆◆ **Example 58:** *An Application.* A storage bin in the shape of a cube is to have a volume of 1550 cubic feet. Find the dimension of each side.

Solution: The volume of a cube is equal to the cube of the side, so the side s is the cube root of the volume.

$$s = \sqrt[3]{V} = \sqrt[3]{1550} = 11.6 \text{ ft}$$

Odd Roots of Negative Numbers by Calculator

An *even* root of a negative number is *imaginary* (such as $\sqrt{-4}$). We will study these later. But an *odd* root of a negative number is *not* imaginary. It is a real, negative number. As with powers, some calculators will not accept a negative radicand. Fortunately, we can outsmart our calculators and take odd roots of negative numbers anyway.

◆◆◆ **Example 59:** Find $\sqrt[5]{-875}$.

Solution: We know that an odd root of a negative number is real and negative. So we take the fifth root of +875, by calculator,

$$\sqrt[5]{+875} = 3.88 \text{ (rounded)}$$

and we only have to place a minus sign before the number.

$$\sqrt[5]{-875} = -3.88$$ ◆◆◆

Exercise 5 ◆ Powers and Roots

Evaluate each expression. Retain the proper number of significant digits in your answer.

Powers

1. 2^3
2. 5^3
3. 9^2
4. 1^3
5. 10^3
6. 10^1
7. 10^2
8. 10^4

Powers by Calculator

9. $(8.55)^3$
10. $(1.07)^5$
11. $(9.55)^3$
12. $(84.2)^2$
13. $(3.95)^3$
14. $(13.9)^2$
15. $(1.65)^4$
16. $(2.98)^2$
17. $(12.5)^2$
18. $(1.35)^5$
19. $(2.26)^6$
20. $(1.94)^7$

Negative Base

21. $(-3)^3$
22. $(-2)^3$
23. $(-4)^3$
24. $(-3)^1$
25. $(-8.01)^3$
26. $(-1.71)^5$
27. $(-5.33)^3$
28. $(-12.5)^2$
29. $(-1.33)^3$
30. $(-2.34)^5$
31. $(-2.84)^3$
32. $(-5.84)^2$

Negative Exponent

33. 1^{-3}
34. 2^{-3}
35. 10^{-2}
36. 10^{-4}
37. $(3.85)^{-2}$
38. $(1.83)^{-5}$
39. $(3.84)^{-3}$
40. $(22.5)^{-2}$
41. $(-5.37)^{-3}$
42. $(-2.24)^{-5}$
43. $(-1.85)^{-3}$
44. $(-4.24)^{-3}$

Fractional and Demical Exponents

45. $(8.55)^{1/3}$
46. $(1.07)^{1/5}$
47. $(9.55)^{1/3}$
48. $(84.2)^{1/2}$
49. $(2.85)^{2/3}$
50. $(9.27)^{2/5}$
51. $(1.84)^{2/3}$
52. $(4.22)^{1/2}$
53. $(8.88)^{2.13}$
54. $(5.27)^{3.25}$
55. $(4.38)^{2.63}$
56. $(2.48)^{1.42}$

Applications Involving Powers

57. The distance traveled by a falling body, starting from rest, is equal to $16t^2$, where t is the elapsed time. In 5.448 s, the distance fallen is $16(5.448)^2$ ft. Evaluate this quantity. (Treat 16 here as an approximate number.)

58. The power P dissipated in a resistance R through which is flowing a current I is $P = I^2R$. Therefore the power in a 365-Ω resistor carrying a current of 0.5855 A is $(0.5855)^2(365)$ W. Evaluate this power.

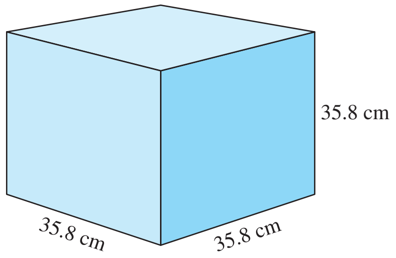


FIGURE 1–6

59. The volume of a cube of side 35.8 cm (Fig. 1–6) is $(35.8)^3$. Evaluate this volume.

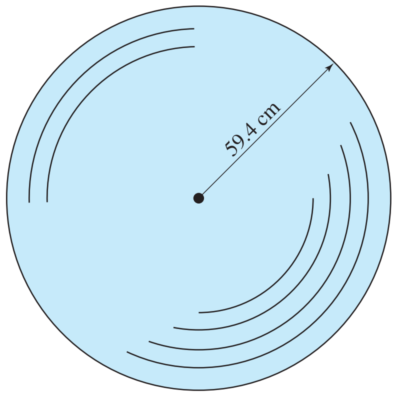


FIGURE 1–7

60. The volume of a 59.4-cm-radius sphere (Fig. 1–7) is $\frac{4}{3}\pi(59.4)^3$ cm^3. Find this volume.

61. An investment of \$2000 at a compound interest rate of $6\frac{1}{4}\%$, left on deposit for $7\frac{1}{2}$ years, will be worth $2000(1.0625)^{7.5}$ dollars. Find this amount, to the nearest cent.

Roots

Find each principal root without using your calculator.

62. $\sqrt{25}$ **63.** $\sqrt[3]{27}$

64. $\sqrt{49}$ **65.** $\sqrt[3]{-27}$

66. $\sqrt[3]{-8}$ **67.** $\sqrt[5]{-32}$

Evaluate each radical by calculator, retaining the proper number of digits in your answer:

68. $\sqrt{49.2}$ **69.** $\sqrt{1.863}$

70. $\sqrt[3]{88.3}$ **71.** $\sqrt{772}$

72. $\sqrt{3875}$ **73.** $\sqrt[3]{7295}$

74. $\sqrt[3]{-386}$ **75.** $\sqrt[5]{-18.4}$

76. $\sqrt[3]{-2.774}$

Applications of Roots

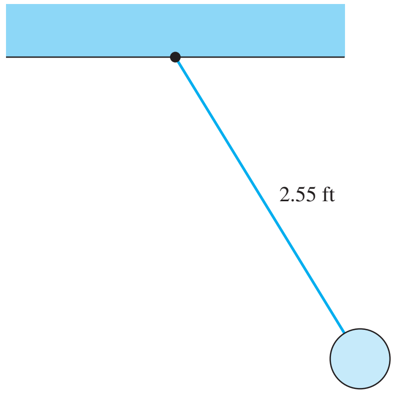


FIGURE 1–8

77. The period T (time for one swing) of a simple pendulum (Fig. 1–8) 2.55 ft long is

$$T = 2\pi\sqrt{\frac{2.55}{32.0}} \text{ seconds}$$

Evaluate T.

78. The magnitude Z of the impedance in a circuit having a resistance of 3540 Ω and a reactance of 2750 Ω is

$$Z = \sqrt{(3540)^2 + (2750)^2} \text{ ohms}$$

Find Z.

79. The geometric mean B between 3.75 and 9.83 is

$$B = \sqrt{(3.75)(9.83)}$$

Evaluate B.

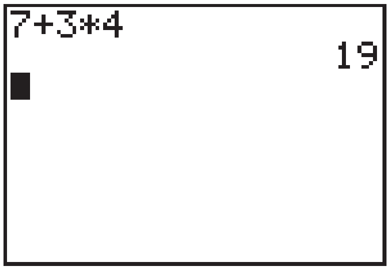
1–6 Combined Operations

Order of Operations

If the expression to be evaluated does not contain parentheses, perform the operations in the following order:

1. *Powers and roots, in any order.*
2. *Multiplications and divisions, from left to right.*
3. *Additions and subtractions, from left to right.*

Our first group of calculations will be with integers only, and later we will do some problems that require rounding. We first show a problem containing both addition and multiplication.



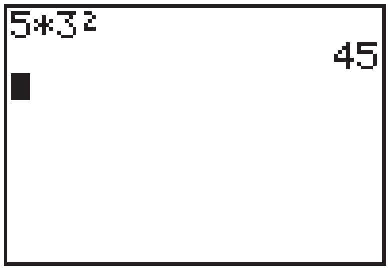
TI-83/84 screen for Example 60.

◆◆◆ **Example 60:** Evaluate $7 + 3 \times 4$.

Solution: The multiplication is done before the addition.

$$7 + 3 \times 4 = 7 + 12 = 19$$ ◆◆◆

Next we do a calculation having both a power and multiplication.



TI-83/84 screen for Example 61.

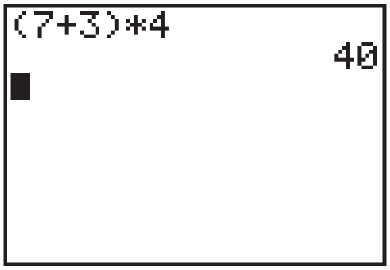
◆◆◆ **Example 61:** Evaluate 5×3^2.

Solution: We raise to the power before multiplying:

$$5 \times 3^2 = 5 \times 9 = 45$$ ◆◆◆

Parentheses

When an expression contains parentheses, first evaluate the expression within the parentheses and then the entire expression.



TI-83/84 screen for Example 62.

◆◆◆ **Example 62:** Evaluate $(7 + 3) \times 4$.

Solution:

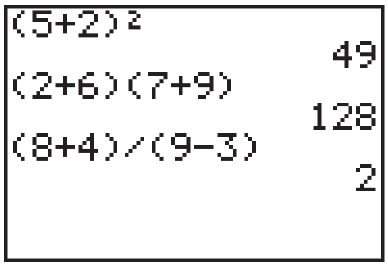
$$(7 + 3) \times 4 = 10 \times 4 = 40$$ ◆◆◆

If the sum or difference of two or more numbers is to be raised to a power, those numbers must be enclosed in parentheses.

◆◆◆ **Example 63:** Evaluate $(5 + 2)^2$.

Solution: We combine the numbers inside the parentheses before squaring.

$$(5 + 2)^2 = 7^2 = 49$$ ◆◆◆



TI-83/84 screen for the Examples 63, 64, and 65.

◆◆◆ **Example 64:** Evaluate $(2 + 6)(7 + 9)$.

Solution: Evaluate the two quantities in parentheses before multiplying.

$$(2 + 6)(7 + 9) = 8 \times 16 = 128$$ ◆◆◆

◆◆◆ **Example 65:** Evaluate $\dfrac{8 + 4}{9 - 3}$.

Solution: Here the fraction line acts like parentheses, grouping the 8 and 4, as well as the 9 and 3. Written on a single line, this problem would be

$$(8 + 4) \div (9 - 3)$$

or

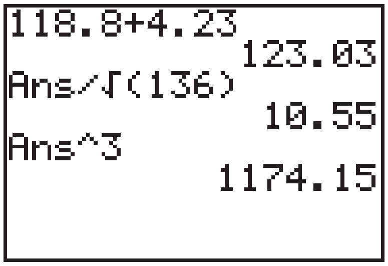
$$12 \div 6 = 2$$ ◆◆◆

Combined Operations with Approximate Numbers

Combined operations with approximate numbers are done the same way as those with exact numbers. However, we must round our answer properly using the rules given earlier in this chapter.

If you do a calculation in steps, writing down the intermediate values, it is a good practice to carry *one more* digit in those intermediate values than permitted by our rules, and round the final result.

Instead of writing down intermediate steps, you might keep the calculation entirely in the calculator, and round the final result. However, you will not have a "paper trail" to help check your work. A good compromise is to do the calculation in steps, by calculator, writing down the result of each step to provide a "trail." But instead of clearing the intermediate step from your calculator, use it for the next step. Finally, round the final result. We will show this procedure with an example.



TI-83/84 screen for Example 66.

◆◆◆ **Example 66:** Evaluate the expression

$$\left(\frac{118.8 + 4.23}{\sqrt{136}}\right)^3$$

Solution: Let's do the calculation one step at a time, and write down the result of the intermediate steps.

$$\left(\frac{118.8 + 4.23}{\sqrt{136}}\right)^3 = \left(\frac{123.03}{\sqrt{136}}\right)^3$$
$$= (10.55)^3$$
$$= 1174.15$$

Our calculator will give us as many digits as we want, but how many should we keep? In the numerator, we added a number with one decimal place to another with two decimal places, so we are allowed to keep just one. Thus the numerator, after addition, is good to one decimal place, or, in this case, four significant digits. The denominator, however, has just three significant digits, so we round our answer to three significant digits, getting 1170. ◆◆◆

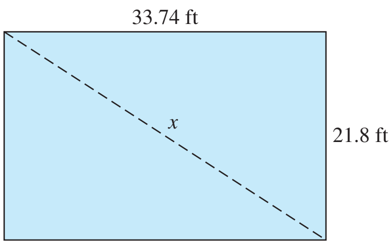
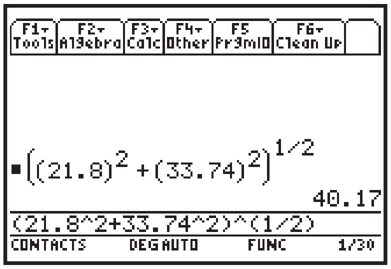


FIGURE 1–9 A rectangular courtyard.

◆◆◆ **Example 67:** *An Application.* A rectangular courtyard (Fig. 1–9) having sides of 21.8 ft and 33.74 ft has a diagonal measurement x given by the expression

$$x = \sqrt{(21.8)^2 + (33.74)^2}$$

Evaluate the expression to find x.



TI-89 screen for Example 67.

Solution: The screen shows the calculation on the TI-89 calculator. We round the answer to 3 significant digits contained in 21.8, and we get $x = 40.2$ ft. ◆◆◆

Exercise 6 ◆ Combined Operations

Combined Operations with Exact Numbers

Perform each computation by calculator.

1. $(37)(28) + (36)(64)$
2. $(22)(53) - (586)(4) + (47)(59)$
3. $(63 + 36)(37 - 97)$
4. $(89 - 74 + 95)(87 - 49)$
5. $\frac{219}{73} + \frac{194}{97}$
6. $\frac{228}{38} - \frac{78}{26} + \frac{364}{91}$

7. $\dfrac{647 + 688}{337 + 108}$

8. $\dfrac{809 - 463 + 1858}{958 - 364 + 508}$

9. $(5 + 6)^2$

10. $(422 + 113 - 533)^4$

11. $(423 - 420)^3$

12. $\left(\dfrac{853 - 229}{874 - 562}\right)^2$

13. $\left(\dfrac{141}{47}\right)^3$

14. $\sqrt{434 + 466}$

15. $\sqrt{(8)(72)}$

16. $\sqrt[3]{657 + 553 - 1085}$

17. $\sqrt[4]{(27)(768)}$

18. $\sqrt{\dfrac{2404}{601}}$

19. $\sqrt[4]{\dfrac{1136}{71}}$

20. $\sqrt{961} + \sqrt{121}$

21. $\sqrt[4]{625} + \sqrt{961} - \sqrt[3]{216}$

22. $\sqrt[4]{256} \times \sqrt{49}$

Combined Operations with Approximate Numbers

Perform each computation, keeping the proper number of digits in your answer.

23. $(7.37)(3.28) + (8.36)(2.64)$

24. $(522)(9.53) - (586)(4.70) + (847)(7.59)$

25. $(63.5 + 83.6)(8.37 - 1.72)$

26. $(8.93 - 3.74 + 9.05)(68.70 - 64.90)$

27. $\dfrac{583}{473} + \dfrac{946}{907}$

28. $\dfrac{6.73}{8.38} - \dfrac{5.97}{8.06} + \dfrac{8.63}{1.91}$

29. $\dfrac{6.47 + 8.604}{3.37 + 90.8}$

30. $\dfrac{809 - 463 + 744}{758 - 964 + 508}$

31. $(5.37 + 2.36)^2$

32. $(4.25 + 4.36 - 5.24)^4$

33. $(6.423 + 1.05)^2$

34. $\left(\dfrac{45.3 - 8.34}{8.74 - 5.62}\right)^{2.5}$

35. $\left(\dfrac{8.90}{4.75}\right)^2$

36. $\sqrt{4.34 + 4.66}$

37. $\sqrt[3]{657 + 553 - 842}$

38. $\sqrt{(28.1)(5.94)}$

39. $\sqrt[5]{(9.06)(4.86)(7.93)}$

40. $\sqrt{\dfrac{653}{601}}$

41. $\sqrt[4]{\dfrac{4.50}{7.81}}$

42. $\sqrt{9.74} + \sqrt{12.5}$

43. $\sqrt[4]{528} + \sqrt{94.2} - \sqrt[3]{284}$

44. $\sqrt[4]{653} + \sqrt{55.3}$

45. *Writing:* Suppose you have submitted a report that contains calculations in which you have rounded the answers according to the rules given in this chapter. Jones, your company rival, has sharply attacked your work, calling it "inaccurate" because you did not keep enough digits, and your boss seems to agree.

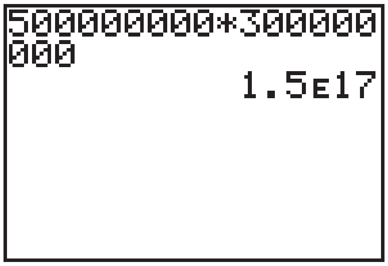
Write a memo to your boss defending your rounding practices. Point out why it is misleading to retain too many digits. Do not write more than one page. You may use numerical examples to prove your point.

1–7 Scientific Notation and Engineering Notation

■ Exploration:

Try this. On your calculator, multiply

$$500{,}000{,}000 \times 300{,}000{,}000$$



Your display might be different, partly depending on your MODE settings.

What did your calculator show for this calculation? Can you explain the meaning of your display? Try multiplying some other very large or very small numbers and try to explain the display. ■

If our answers in the exploration have too many zeros for the display, the calculator will automatically switch into the kind of notation that we will study in this section. The display

1.5 E17

contains two parts: a decimal number, here 1.5, and an integer, 17. We read this as the decimal number times 10 raised to the value of the integer, or

$$1.5 \times 10^{17}$$

Here, 10^{17} is called a *power of ten.*

Powers of 10

We did some work with powers in Sec. 1–5. We saw, for example, that 2^3 meant

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

Here, the power 3 tells how many 2's are to be multiplied to give the product. For powers of 10, the power tells how many 10's are to be multiplied to give the product.

◆◆◆ **Example 68:** Here are some powers of ten expressed as decimal numbers.

(a) $10^2 = 10 \times 10 = 100$

(b) $10^3 = 10 \times 10 \times 10 = 1000$ ◆◆◆

◆◆◆ **Example 69:** Here are some demical numbers expressed as powers of ten.

(a) $10{,}000 = 10^4$ (b) $1{,}000{,}000 = 10^6$

We can evaluate 10 raised to a *negative* power using a formula that we will derive in the next chapter.

Negative Exponent	$x^{-a} = \dfrac{1}{x^a} \quad (x \neq 0)$	28

◆◆◆ **Example 70:** Here are some examples of 10 raised to a negative power.

(a) $10^{-2} = \dfrac{1}{10^2} = \dfrac{1}{100} = 0.01$

(b) $10^{-5} = \dfrac{1}{10^5} = \dfrac{1}{100{,}000} = 0.00001$ ◆◆◆

Some powers of 10 are summarized in the following table:

Positive Powers	Negative Powers
$1{,}000{,}000 = 10^6$	$0.1 = 1/10 = 10^{-1}$
$100{,}000 = 10^5$	$0.01 = 1/10^2 = 10^{-2}$
$10{,}000 = 10^4$	$0.001 = 1/10^3 = 10^{-3}$
$1{,}000 = 10^3$	$0.0001 = 1/10^4 = 10^{-4}$
$100 = 10^2$	$0.00001 = 1/10^5 = 10^{-5}$
$10 = 10^1$	$0.000001 = 1/10^6 = 10^{-6}$
$1 = 10^0$	

Converting Numbers to Scientific Notation

A number is said to be in *scientific notation* when it is written as a number whose absolute value is between 1 and 10, multiplied by a power of 10.

◆◆◆ **Example 71:** The following numbers are written in scientific notation:

(a) 2.74×10^4 (b) 8.84×10^8

(c) 5.4×10^{-7} (d) -1.2×10^{-5} ◆◆◆

To convert a decimal number to scientific notation, first rewrite the given number with a single digit to the left of the demical point, discarding any nonsignificant zeros. Then multiply this number by the power of 10 that will make it equal to the original number.

◆◆◆ **Example 72:** Here we convert to scientific notation.

$$\begin{aligned} 346 &= 3.46 \times 100 \\ &= 3.46 \times 10^2 \end{aligned}$$ ◆◆◆

◆◆◆ **Example 73:** Another example of conversion to scientific notation.

$$\begin{aligned} 2700 &= 2.7 \times 1000 \\ &= 2.7 \times 10^3 \end{aligned}$$

Note that we have discarded the two nonsignificant zeros. ◆◆◆

When we are converting a number whose absolute value is less than 1, our power of 10 will be negative, as in Example 74.

◆◆◆ **Example 74:** Here is an example resulting in a negative power.

$$\begin{aligned} 0.00000950 &= 9.50 \times 0.000001 \\ &= 9.50 \times 10^{-6} \end{aligned}$$

Since the trailing zero is significant in our original number, it is retained in our answer. ◆◆◆

The sign of the exponent has nothing to do with the sign of the original number. You can convert a negative number to scientific notation just as you would a positive number.

◆◆◆ **Example 75:** Convert $-34{,}720$ to scientific notation.

Solution: Converting to scientific notation, we obtain

$$\begin{aligned} -34{,}720 &= -3.472 \times 10{,}000 \\ &= -3.472 \times 10^4 \end{aligned}$$ ◆◆◆

◆◆◆ **Example 76:** *An Application.* A certain tract of land contains 39,700,000 ft^2 of land. Convert this to scientific notation.

Solution: $39{,}700{,}000 \text{ ft}^2 = 3.97 \times 10{,}000{,}000 \text{ ft}^2$
$= 3.97 \times 10^7 \text{ ft}^2$ ◆◆◆

Converting Numbers to Scientific Notation

To convert *from* scientific notation, simply reverse the process.

◆◆◆ **Example 77:** Here we convert from scientific notation to decimal form.

$$4.82 \times 10^5 = 4.82 \times 100{,}000$$
$$= 482{,}000$$ ◆◆◆

◆◆◆ **Example 78:** Another example of converting to decimal form.

$$8.25 \times 10^{-3} = 8.25 \times 0.001$$
$$= 0.00825$$ ◆◆◆

◆◆◆ **Example 79:** *An Application.* The resistance of a certain transmission line is $5.85 \times 10^{-4}\ \Omega$ (ohms). Write this resistance in decimal notation.

Solution: $5.85 \times 10^{-4}\ \Omega = 0.000585\ \Omega$ ◆◆◆

Converting Numbers to Engineering Notation

Engineering notation is similar to scientific notation. The difference is that

- the exponent is *a multiple of three;* and
- there can be one, two, or three digits to the left of the decimal point, rather than just one digit.

Having an exponent that is a multiple of 3 makes it easier to use the *metric prefixes* we will introduce later in this chapter.

◆◆◆ **Example 80:** Some examples of numbers written in engineering notation are as follows:

(a) 66.3×10^3 (b) 8.14×10^9
(c) 725×10^{-6} (d) 28.72×10^{-12} ◆◆◆

Converting to engineering notation is simple if the digits of the decimal number are grouped by commas into sets of three, in the usual way.

◆◆◆ **Example 81:** Here are some conversions to engineering notations.

(a) $21{,}840 = 21.84 \times 10^3$
(b) $548{,}000 = 548 \times 10^3$
(c) $72{,}560{,}000 = 72.56 \times 10^6$ ◆◆◆

For numbers less than 1, it helps to first separate the digits following the decimal point into groups of three.

◆◆◆ **Example 82:** Try to follow these conversions to engineering notation.

(a) $0.87217 = 0.872\ 17 = 872.17 \times 10^{-3}$
(b) $0.000736492 = 0.000\ 736\ 492 = 736.492 \times 10^{-6}$
(c) $0.0000000472 = 0.000\ 000\ 047\ 2 = 47.2 \times 10^{-9}$ ◆◆◆

◆◆◆ **Example 83:** *An Application.* A certain heating furnace is rated at 2.85×10^5 Btu/h (British thermal units per hour). Express this rating in engineering notation.

Solution: $2.85 \times 10^5 = 285{,}000 = 285 \times 10^3$ Btu/h ◆◆◆

Converting Numbers from Engineering Notation

As with converting from scientific notation, we simply reverse the process.

◆◆◆ **Example 84:** Here are some conversions from engineering to decimal notation.

(a) $48.342 \times 10^3 = 48.342 \times 1000 = 48{,}342$
(b) $8.559 \times 10^6 = 8.599 \times 1{,}000{,}000 = 8{,}559{,}000$
(c) $8.352 \times 10^{-3} = 8.352 \times 0.001 = 0.008352$
(d) $736 \times 10^{-6} = 736 \times 0.000001 = 0.000736$ ◆◆◆

◆◆◆ **Example 85:** *An Application*. The weight of a certain punch press is 28.56×10^3 lb. Express this weight in decimal notation.

Solution: $28.56 \times 10^3 = 28{,}560$ lb ◆◆◆

Addition and Subtraction

Let us turn now to *computations* using scientific and engineering notation. We will first do some simple problems by hand, to show how the powers of ten are combined, and then we will do similar problems by calculator.

If two or more numbers to be added or subtracted have the *same power of 10,* simply combine the numbers and keep the same power of 10.

◆◆◆ **Example 86:** Here we combine numbers that have the same power of 10.

(a) $(2 \times 10^5) + (3 \times 10^5) = 5 \times 10^5$
(b) $(8 \times 10^3) - (5 \times 10^3) + (3 \times 10^3) = 6 \times 10^3$ ◆◆◆

If the sum is greater than 10 or less than 1, it is no longer, strictly speaking, in scientific notation. We change it to scientific notation as we did earlier in this section.

◆◆◆ **Example 87:** These show more examples with numbers having the same power of 10.

(a) $(8.4 \times 10^4) + (7.2 \times 10^4) = 15.6 \times 10^4$
$= 1.56 \times 10^5$
(b) $(5.822 \times 10^3) - (5.000 \times 10^3) = 0.822 \times 10^3$
$= 8.22 \times 10^2$ ◆◆◆

If the powers of 10 are different, *they must be made equal* before the numbers can be combined. A shift of the decimal point of one place to the *left* will *increase* the exponent by 1. Conversely, a shift of the decimal point one place to the *right* will *decrease* the exponent by 1.

◆◆◆ **Example 88:** Here are examples of combining numbers having different powers of 10.

(a) $(1.5 \times 10^4) + (3 \times 10^3) = (1.5 \times 10^4) + (0.3 \times 10^4)$
$= 1.8 \times 10^4$

(b) $(1.25 \times 10^5) - (2.0 \times 10^4) + (4 \times 10^3)$
$$= (1.25 \times 10^5) - (0.20 \times 10^5) + (0.04 \times 10^5)$$
$$= 1.09 \times 10^5$$

(c) $(8.84 \times 10^3) + (9.92 \times 10^4) = (0.884 \times 10^4) + (9.92 \times 10^4)$
$$= 10.80 \times 10^4$$
$$= 1.080 \times 10^5$$

Note that in (c) we started with numbers each having three significant digits, and that our result has four significant digits. This is an example of significant digits being gained during addition. Similarly, significant digits can be lost during subtraction. ◆◆◆

◆◆◆ **Example 89:** *An Application.* Two resistors having resistances of $3.74 \times 10^3\ \Omega$ and $9.37 \times 10^4\ \Omega$ are wired in series. Find their combined resistance, which is the sum of the two, in engineering notation.

Solution:
$$(3.74 \times 10^3) + (9.37 \times 10^4) = (3.74 \times 10^3) + (93.7 \times 10^3)$$
$$= 97.4 \times 10^3\ \Omega$$
◆◆◆

Multiplication

We are really using $x^a \cdot x^b = x^{a+b}$. This equation is one of the *laws of exponents* that we will study later.

We multiply powers of 10 by *adding their exponents.*

◆◆◆ **Example 90:**

(a) $10^3 \cdot 10^4 = 10^{3+4} = 10^7$

(b) $10^{-2} \cdot 10^5 = 10^{-2+5} = 10^3$ ◆◆◆

To multiply two numbers in scientific notation, multiply the decimal parts and the powers of 10 *separately.*

◆◆◆ **Example 91:**

(a) $(2 \times 10^5)(3 \times 10^2) = (2 \times 3)(10^5 \times 10^2)$
$$= 6 \times 10^{5+2} = 6 \times 10^7$$

(b) $(2.84 \times 10^3)(7.21 \times 10^4) = (2.84 \times 7.21)(10^3 \times 10^4)$
$$= 20.5 \times 10^{3+4}$$
$$= 20.5 \times 10^7$$
$$= 2.05 \times 10^8$$
◆◆◆

◆◆◆ **Example 92:** *An Application.* The dimensions of a rectangular tract are 5.983×10^3 by 1.395×10^4 feet. Find the area of the field.

Solution: Multiplying the two dimensions gives
$$(5.983 \times 10^3)(1.395 \times 10^4) = 8.346 \times 10^{3+4}$$
$$= 8.346 \times 10^7\ \text{ft}^2$$
◆◆◆

Division

We are using a law of exponents for quotients that we will study in Chapter 2.

We divide powers of 10 by subtracting the exponent of the denominator from the exponent of the numerator.

◆◆◆ **Example 93:** Here are some quotients of powers of 10.

(a) $\dfrac{10^5}{10^3} = 10^{5-3} = 10^2$

(b) $\dfrac{10^{-4}}{10^{-2}} = 10^{-4-(-2)} = 10^{-2}$

As with multiplication, we divide the decimal parts and the powers of 10 separately. ◆◆◆

◆◆◆ **Example 94:** These examples show how to divide numbers in scientific notation.

(a) $\dfrac{8 \times 10^5}{4 \times 10^2} = \dfrac{8}{4} \times \dfrac{10^5}{10^2} = 2 \times 10^{5-2} = 2 \times 10^3$

(b) $\dfrac{12 \times 10^3}{4 \times 10^5} = 3 \times 10^{3-5} = 3 \times 10^{-2}$

(c) $(1.97 \times 10^3) \div (2.52 \times 10^4) = 0.782 \times 10^{-1}$
$= 7.82 \times 10^{-2}$ ◆◆◆

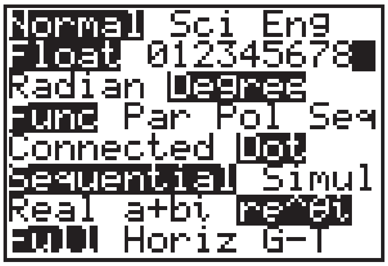
◆◆◆ **Example 95:** *An Application*. A truck with a capacity of 3.24×10^2 ft^3 contains a load of gravel which weighs 3.77×10^4 lb. Find the density of the sand by dividing the weight by the volume.

Solution:

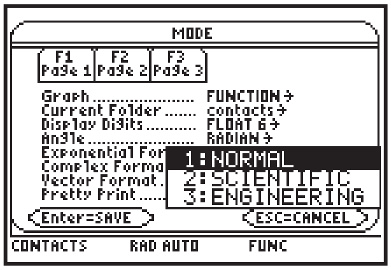
$$\text{density} = \frac{\text{weight}}{\text{volume}} = \frac{3.77 \times 10^4}{3.24 \times 10^2}$$
$$= 1.16 \times 10^2$$
$$= 116 \text{ lb/ft}^3$$
◆◆◆

Scientific and Engineering Notation on the Calculator

Displaying Numbers: We can choose the way a number is *displayed* on a calculator, either in decimal, scientific, or engineering notation. This choice is usually made from a menu.

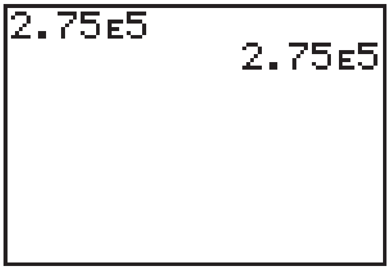


MODE screen for the TI-83/84, showing the Normal, Sci, and Eng modes.



MODE screen for the TI-89 showing the NORMAL, SCIENTIFIC, and ENGINEERING modes.

Entering Numbers: You can *enter* a number in any of these notations, regardless of the mode the calculator is in. However, the entered number will be displayed according to the chosen mode. The power of ten is entered using the *enter exponent* key, usually marked EE, EXP, or EEX.



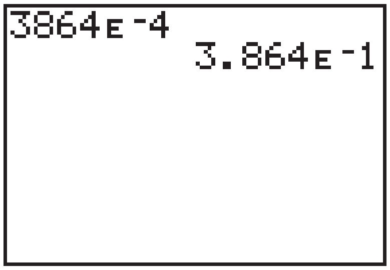
TI-83/84 screen for Example 96. Calculator is in Sci mode.

◆◆◆ **Example 96:** The keystrokes to enter the number 2.75×10^5 are shown. The number displayed will depend on the mode in which the calculator is set,

275000	if in decimal mode
2.75 E5	if in scientific notation mode
275 E3	if in engineering notation mode

◆◆◆

The calculator will even accept numbers that are not strictly in scientific or engineering notation.



TI-83/84 screen for Example 97. To enter a negative exponent, we must be careful not to use the subtraction key, but the key used for entering a negative number. Calculator is in Sci mode.

◆◆◆ **Example 97:** The keystrokes to enter 3864×10^{-4} are shown. ◆◆◆

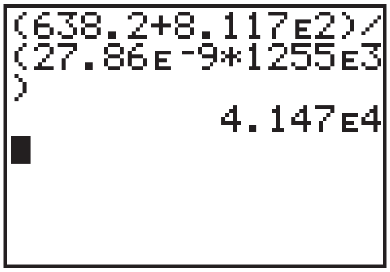
Calculating in Scientific Notation: The numbers you enter can be in any notation, and can even be mixed in the same calculation.

◆◆◆ **Example 98:** Evaluate

$$\frac{638.2 + (8.117 \times 10^2)}{(27.86 \times 10^{-9})(1255 \times 10^3)}$$

Solution: Here we have some numbers in decimal, scientific, and engineering notation, and another that is in none of these. We key them in just as they appear. Note that the numerator must be enclosed in parentheses. Also be careful not to use the subtraction key to enter the negative exponent. We get, after rounding,

41470 in normal mode
4.147 E4 in scientific notation mode
41.47 E3 in engineering notation mode ◆◆◆



TI-83/84 screen for Example 98, with the calculator in scientific notation mode.

Common Errors

Students often enter powers of 10 (such as 10^4) incorrectly into their calculators, forgetting that 10^4 is really 1×10^4.

Thus to enter 10^4, we press

1 [EE] 4

and not

10 [EE] 4

Do not press the times key before the *enter exponent* key. Thus to enter 10^4, we *do not* enter

1 [×] [EE] 4

but *do* enter

1 [EE] 4

TI-83/84 screen for Example 99.

◆◆◆ **Example 99:** *An Application.* The current in a resistor equals the voltage across the resistor divided by the resistance of that resistor. Find the current in a $4.55 \times 10^4\ \Omega$ resistor having a voltage across it of 8.25×10^{-2} volts.

Solution: Dividing the voltage by the resistance we get,

$$\text{Current} = \frac{\text{voltage}}{\text{resistance}} = \frac{8.25 \times 10^{-2}}{4.55 \times 10^4}$$

$$= 1.81 \times 10^{-6} \text{ amperes}$$ ◆◆◆

Exercise 7 ◆ Scientific and Engineering Notation

Powers of 10

Write each power of 10 as a decimal number.

1. 10^5
2. 10^{-2}
3. 10^{-5}
4. 10^{-1}
5. 10^4

Write each number as a power of 10.

6. 100
7. 1,000,000
8. 0.0001
9. 0.001
10. 100,000,000

Converting Numbers to Scientific Notation

Write each number in scientific notation.

11. 186,000
12. 0.0035
13. 25,742
14. 8020
15. 98.3×10^3
16. 0.0775×10^{-2}

Converting Numbers from Scientific Notation

Convert each number from scientific notation to decimal notation.

17. 2.85×10^3
18. 1.75×10^{-5}
19. 9×10^4
20. 9.05×10^4
21. 3.667×10^{-3}

Converting Numbers to Engineering Notation

Convert each number to engineering notation.

22. 34,382
23. 3.58×10^2
24. 26,940
25. 0.134
26. 23.48×10^{-2}
27. 0.00374

Converting Numbers from Engineering Notation

Convert each number from engineering notation to decimal notation.

28. 385×10^3
29. $18{,}640 \times 10^{-3}$
30. 8488×10^{-3}
31. 7739×10^{-3}
32. 6.37×10^3
33. 2.66×10^6

Addition and Subtraction

Combine without using a calculator. Give your answer in scientific notation.

34. $(3.0 \times 10^4) + (2.1 \times 10^5)$
35. $(75.0 \times 10^2) + 3210$
36. $(1.557 \times 10^2) + (9.000 \times 10^{-1})$
37. $0.037 - (6.0 \times 10^{-3})$
38. $(7.2 \times 10^4) + (1.1 \times 10^4)$

Multiplication

Multiply the following powers of 10.

39. $10^5 \cdot 10^2$ **40.** $10^4 \cdot 10^{-3}$ **41.** $10^{-5} \cdot 10^{-4}$
42. $10^{-2} \cdot 10^5$ **43.** $10^{-1} \cdot 10^{-4}$

Multiply without using a calculator. Give your answer in scientific notation.

44. $(3.0 \times 10^3)(5.0 \times 10^2)$ **45.** $(5 \times 10^4)(8 \times 10^{-3})$
46. $(2 \times 10^{-2})(4 \times 10^{-5})$ **47.** $(2 \times 10^4)(30{,}000)$

Division

Divide the following powers of 10.

48. $10^8 \div 10^5$ **49.** $10^4 \div 10^6$ **50.** $10^5 \div 10^{-2}$
51. $10^{-3} \div 10^5$ **52.** $10^{-2} \div 10^{-4}$

Divide without using a calculator. Give your answer in scientific notation.

53. $(8 \times 10^4) \div (2 \times 10^2)$ **54.** $(6 \times 10^4) \div 0.03$
55. $(3 \times 10^3) \div (6 \times 10^5)$ **56.** $(8 \times 10^{-4}) \div 400{,}000$
57. $(9 \times 10^4) \div (3 \times 10^{-2})$ **58.** $49{,}000 \div (7.0 \times 10^{-2})$

Scientific and Engineering Notation on the Calculator

Perform the following computations. Display your answer in scientific notation.

59. $(1.58 \times 10^2)(9.82 \times 10^3)$
60. $(9.83 \times 10^5) \div (2.77 \times 10^3)$
61. $(3.87 \times 10^{-2})(5.44 \times 10^5)$
62. $(2.74 \times 10^3) \div (9.13 \times 10^5)$
63. $(5.6 \times 10^2)(3.1 \times 10^{-1})$
64. $(7.72 \times 10^8) \div (3.75 \times 10^{-9})$

Applications

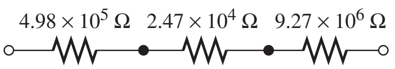


FIGURE 1–10 Resistors in series.

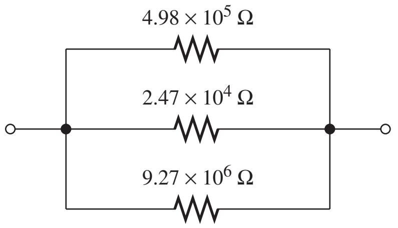


FIGURE 1–11 Resistors in parallel.

65. Three resistors, having resistances of $4.98 \times 10^5\ \Omega$, $2.47 \times 10^4\ \Omega$, and $9.27 \times 10^6\ \Omega$, are wired in series (Fig. 1–10). Find the total resistance, using the formula $R = R_1 + R_2 + R_3$.

66. Find the equivalent resistance if the three resistors of problem 65 are wired in parallel (Fig. 1–11). Use the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

67. Find the power in watts dissipated in a resistor if a current I of 3.75×10^{-3} A produces a voltage drop V of 7.24×10^{-4} V across the resistor. Use the formula $P = VI$.

68. The voltage across an $8.35 \times 10^5\ \Omega$ resistor is 2.95×10^{-3} V. Find the power dissipated in the resistor, using the formula $P = V^2/R$.

69. Three capacitors, 8.26×10^{-6} farad (F), 1.38×10^{-7} F, and 5.93×10^{-5} F, are wired in parallel. Find the equivalent capacitance using the formula $C = C_1 + C_2 + C_3$

70. A wire 4.75×10^3 cm long when loaded is seen to stretch 9.55×10^{-2} cm. Find the strain in the wire, using the formula strain = elongation ÷ length.

71. *Writing:* Study your calculator and its manual specifically on the different display formats (normal, scientific notation, and so forth). List the different formats available to you and explain the diferences between them. Write a few lines explaining how to switch from one format to another.

72. *Project:* In a technical magazine or journal find several uses of scientific or engineering notation. Show why the author has chosen it over decimal notation.

73. *Internet:* Surf the Web for data on the various planets. Find the mean distance of each planet from the sun. Tabulate this distance in decimal, scientific, and engineering notation, in both miles and kilometers. Make a number line, with the sun at zero, and mark the distance of each planet on that line.

1–8 Units of Measurement

Systems of Units

A *unit* is a standard of measurement, such as the meter, inch, hour, or pound. The two main systems in use are the U.S. customary units (feet, pounds, gallons, etc.) and the SI or *metric* system (meters, kilograms, liters, etc.). SI stands for Le Système International d'Unites, or the *International System of Units*. In addition, some special units, such as a *square* of roofing material, and some obsolete units, such as *rods* and *chains,* must occasionally be dealt with.

Most units have an abbreviated form or a symbol, so that we do not have to write the full word. Thus the abbreviation for millimeters is mm, and the symbol for ohms is Ω (capital Greek omega). In this section we will usually give the full word *and* the abbreviation.

Conversion of Units

We convert from one unit of measurement to another by means of a *conversion factor*. Conversion factors for units of measurement, as well as the abbreviations for those units, are given in Appendix B.

◆◆◆ **Example 100:** Convert 1.530 miles (mi) to feet (ft).

Solution: From Appendix B we find the relation between miles and feet.

$$5280 \text{ ft} = 1 \text{ mi}$$

Dividing both sides by 1 mile, we get the conversion factor.

$$\frac{5280 \text{ ft}}{1 \text{ mi}} = 1$$

We know that we can multiply any quantity by 1 without changing the value of that quantity. Thus if we multiply our original quantity (1.530 mi) by the conversion factor (5280 ft/mi), we do not change the value of the original quantity. We will, however, change the units. Multiplying yields

$$1.530 \text{ mi} = 1.530 \cancel{\text{mi}} \times \frac{5280 \text{ ft}}{1 \cancel{\text{mi}}} = 8078 \text{ ft}$$

Note that we have rounded our answer to four significant digits, because all numbers used in the calculation have at least four significant digits (the 5280 is exact). ◆◆◆

Suppose that in the first step of Example 100, we had divided both sides by 5280 ft instead of by 1 mi. We could have gotten another conversion factor:

$$\frac{1 \text{ mi}}{5280 \text{ ft}} = 1$$

Thus each relation between two units of measurement gives us *two* conversion factors. But suppose, in the precending example, we had written

$$1.530 \text{ mi} \times \frac{1 \text{ mi}}{5280 \text{ ft}} = ??$$

This is not incorrect but does us no good because *miles* does not cancel.

Common Errors	
	Choose the conversion factor that will cancel the units you wish to eliminate. Write the units in the equation and make sure they cancel properly.
	Be sure to write the original quantity as a *built-up* fraction, such as $\frac{a}{b}$, rather than on a single line, *a/b*. This will greatly reduce your chances of making an error.

The use of conversion factors and making sure that the units in an expression are compatible and cancel properly is part of what is called *dimensional analysis*.

Significant Digits

You should try to use a conversion factor that is exact, or one that contains at least as many significant digits as in your original number. Then you should round your answer to as many significant digits as in the original number.

◆◆◆ **Example 101:** Convert 934 acres to square miles (mi^2).

Solution: From Appendix B we find

$$1 \text{ mi}^2 = 640 \text{ acres}$$

where 640 is an exact number. We must write our conversion factor so that the unwanted unit (acres) is in the denominator, so the acres will cancel. Our conversion factor is thus

$$\frac{1 \text{ mi}^2}{640 \text{ acres}} = 1$$

Multiplying, we obtain

$$934 \text{ acres} = 934 \cancel{\text{acres}} \times \frac{1 \text{ mi}^2}{640 \cancel{\text{acres}}}$$
$$= 1.46 \text{ mi}^2$$

The conversion factor used here is exact, so we have rounded our answer to the three significant digits of the given number. ◆◆◆

Converting Areas and Volumes

Length may be given in, say, centimeters (cm), but an *area* may be given in *square* centimeters (cm^2). Similarly, a *volume* may be in *cubic* centimeters (cm^3). So to get a conversion factor for area or volume, if not found in Appendix B, simply square or cube the conversion factor for length. For example, if we take the equation

$$2.54 \text{ cm} = 1 \text{ in.}$$

this conversion is *exact*, by international agreement. Squaring both sides we get

$$(2.54 \text{ cm})^2 = (1 \text{ in.})^2$$

or

$$6.4516 \text{ cm}^2 = 1 \text{ in.}^2$$

This gives us a conversion between square centimeters and square inches. Since 2.54 is an exact number, we may keep all the significant digits in its square.

◆◆◆ **Example 102:** *An Application.* A buliding lot is seen to contain 864 square yards. The deed requires that it be given in acres. Convert this quantity.

Solution: Appendix B has no conversion for square yards. However,

$$1 \text{ yd} = 3 \text{ ft}$$

Squaring yields

$$1 \text{ yd}^2 = (3 \text{ ft})^2 = 9 \text{ ft}^2$$

Also from the table,

$$1 \text{ acre} = 43{,}560 \text{ ft}^2$$

So

$$864 \text{ yd}^2 = 864 \cancel{\text{yd}^2} \times \frac{9 \cancel{\text{ft}^2}}{1 \cancel{\text{yd}^2}} \times \frac{1 \text{ acre}}{43{,}560 \cancel{\text{ft}^2}} = 0.179 \text{ acre}$$ ◆◆◆

Converting Rates to Other Units

A *rate* is the amount of one quantity expressed *per unit of some other quantity*. Some rates, with typical units, are

rate of travel (mi/h) or (km/h) *flow rate (gal/min) or* (m^3/s)
application rate (lb/acre) *unit price (dollars/lb)*

Each rate contains *two* units of measure; miles per hour, for example, has *miles* in the numerator and *hours* in the denominator. It may be necessary to convert *either* or *both* of those units to other units. Sometimes a single conversion factor can be found (such as 1 m/h = 1.466 ft/s), but more often you will have to convert each unit with a *separate* conversion factor.

◆◆◆ **Example 103:** A certain chemical is to be added to a pool at the rate of 3.74 oz per gallon of water. Convert this to pounds of chemical per cubic foot of water.

Solution: We write the original quantity as a fraction and multiply by the appropriate factors, themselves written as fractions.

$$3.74 \text{ oz/gal} = \frac{3.74 \cancel{\text{oz}}}{\cancel{\text{gal}}} \times \frac{1 \text{ lb}}{16 \cancel{\text{oz}}} \times \frac{7.481 \cancel{\text{gal}}}{\text{ft}^3} = 1.75 \text{ lb/ft}^3$$ ◆◆◆

Using More Than One Conversion Factor

Sometimes you may not be able to find a *single* conversion factor linking the units you want to convert. You may have to use *more than one*.

◆◆◆ **Example 104:** *An Application*. A map shows that the distance to a lighthouse is 7375 yards. We want to lay out this distance on another chart, marked in nautical miles. Convert this quantity.

Solution: In Appendix B we find no conversion factor between nautical miles and yards, but we see that.

$$1 \text{ nau mi} = 6076 \text{ ft} \quad \text{and} \quad 3 \text{ ft} = 1 \text{ yd}$$

So

$$7375 \text{ yd} = 7375 \cancel{\text{yd}} \times \frac{3 \cancel{\text{ft}}}{1 \cancel{\text{yd}}} \times \frac{1 \text{ nau mi}}{6076 \cancel{\text{ft}}} = 3.641 \text{ nautical mi}$$ ◆◆◆

Metric Units

The *metric system* is a system of weights and measures that was developed in France in 1793 and that has since been abopted by most countries of the world. It is widely used in scientific work in the United States.

The basic unit of length in the metric system is the *meter* (m). The unit of area is the *are*, or 100 square meters $(m)^2$. The unit of volume is the *liter* (L), the volume of a cube one-tenth of a meter on a side. The unit of mass is the *gram* (g), the theoretical weight of a cube of distilled water measuring $\frac{1}{100}$ of a meter on a side.

Metric Prefixes

Converting between metric units is made easy because larger and smaller metric units are related to the basic units by *factors of 10*. These larger or smaller units are indicated by placing a *prefix* before the basic unit. A prefix is a group of letters placed at the beginning of a word to modify the meaning of that word. For example, the prefix *kilo* means 1000, or 10^3. Thus a *kilogram* is 1000 grams. Other metric prefixes are given in Table 1–1.

TABLE 1–1 Metric prefixes.

Amount	Multiples and Submultiples	Prefix	Symbol	Meaning
1 000 000 000 000	10^{12}	tera	T	One trillion times
1 000 000 000	10^{9}	giga	G	One billion times
1 000 000	10^{6}	mega	M*	One million times
1 000	10^{3}	kilo	k*	One thousand times
100	10^{2}	hecto	h	One hundred times
10	10	deka	da	Ten times
0.1	10^{-1}	deci	d	One tenth of
0.01	10^{-2}	centi	c*	One hundredth of
0.001	10^{-3}	milli	m*	One thousandth of
0.000 001	10^{-6}	micro	μ*	One millionth of
0.000 000 001	10^{-9}	nano	n	One billionth of
0.000 000 000 001	10^{-12}	pico	p	One trillionth of
0.000 000 000 000 001	10^{-15}	femto	f	One quadrillionth of
0.000 000 000 000 000 001	10^{-18}	atto	a	One quintillionth of

*Most commonly used.

◆◆◆ **Example 105:** Here are some uses of metric prefixes.

(a) A *kilo*meter (km) is a thousand meters, because *kilo* means one thousand.

$$1 \text{ km} = 1000 \text{ m}$$

(b) A *centi*meter (cm) is one-hundredth of a meter, because *centi* means one hundredth.

$$1 \text{ cm} = 1/100 \text{ m}$$

(c) A *milli*meter (mm) is one-thousandth of a meter, because *milli* means one thousandth.

$$1 \text{ mm} = 1/1000 \text{ m}$$

◆◆◆

Converting Between Metric Units

Converting from one metric unit to another is usually a matter of multiplying or dividing by a power of 10. Most of the time the names of the units will tell how they are related, so we do not even have to look them up.

◆◆◆◆ **Example 106:** Convert 72,925 meters (m) to kilometers (km).

Solution: A *kilometer* is a thousand meters, so our conversion factor is

$$\frac{1\text{ km}}{1000\text{ m}} = 1$$

So, as before,

$$72{,}925\text{ m} = 72{,}925\,\cancel{\text{m}} \times \frac{1\text{ km}}{1000\,\cancel{\text{m}}} = 72.925\text{ km}$$ ◆◆◆

For more unusual metric units, simply look up the conversion factor in a table.

◆◆◆ **Example 107:** Convert 2.75 newtons (N) to dynes.

Solution: These two metric units of force do not have any basic units in their names, or any prefixes. Thus we cannot tell just from their names how they are related to each other. However, from Appendix B we find that

$$1\text{ newton} = 10^5\text{ dynes}$$

Converting in the usual way, we obtain

$$2.75\text{ newtons} = 2.75\,\cancel{\text{newtons}} \times \frac{10^5\text{ dynes}}{1\,\cancel{\text{newton}}} = 2.75 \times 10^5\text{ dynes}$$

or 275,000 dynes. ◆◆◆

Converting Between Customary and Metric Units

We convert between customary and metric units in the same way that we converted within each system.

◆◆◆ **Example 108:** Convert 2.84 U.S. gallons (gal) to liters (L).

Solution: From Appendix B we find

$$1\text{ gal (U.S.)} = 3.785\text{ L}$$

Converting gives

$$2.84\text{ gal} = 2.84\,\cancel{\text{gal}} \times \frac{3.785\text{ L}}{1\,\cancel{\text{gal}}} = 10.7\text{ L}$$

rounded to three significant digits. ◆◆◆

Angle Conversions

The degree (°) is a unit of angular measure equal to 1/360 of a revolution; thus 360° = one revolution. A fractional part of a degree may be expressed as a common fraction (such as $36\frac{1}{2}^\circ$), as a decimal (28.74°), or as *minutes* and *seconds*.

A *minute* (′) is equal to 1/60 of a degree; a *second* (″) is equal to 1/60 of a minute, or 1/3600 of a degree.

Another important unit of angular measure is the *radian*. We will learn about radians later.

◆◆◆ **Example 109:** Some examples of angles written in degrees, minutes, and seconds are

$$85°18'42'' \qquad 62°12' \qquad 75°06'03''$$ ◆◆◆

Note that minutes or seconds less than 10 are written with an initial zero. Thus 6′ is written 06′.

We will sometimes abbreviate "degrees, minutes, and seconds" as DMS.

Conversions involving degrees, minutes, and seconds require several steps. Further, to know how many digits to retain, we note the following:

$$1 \text{ min} = 1/60 \text{ degree} \approx 0.02°$$

and

$$1 \text{ sec} = 1/3600 \text{ degree} \approx 0.0003°$$

Thus an angle known to the nearest minute is about as accurate as a decimal angle known to two decimal places. Also, an angle known to the nearest second is about as accurate as a decimal angle known to the fourth decimal place. Therefore we would treat an angle such as 28°17′37″ as if it had four decimal places and six significant digits.

◆◆◆ **Example 110:** Convert 28°17′37″ to decimal degrees.

Solution: We separately convert the minutes and the seconds to degrees and add them. Since the given angle is known to the nearest second, we will work to four decimal places.

$$37 \text{ sec}\left(\frac{1 \text{ deg}}{3600 \text{ sec}}\right) = 0.0103°$$

$$17 \text{ min}\left(\frac{1 \text{ deg}}{60 \text{ min}}\right) = 0.2833°$$

$$28° = \underline{28.0000°}$$

$$\text{Add:} \qquad 28.2936°$$ ◆◆◆

◆◆◆ **Example 111:** Convert 105.2821° to degrees, minutes, and seconds.

Solution: We first convert the decimal part (0.2821°) to minutes,

$$0.2821°\left(\frac{60'}{1°}\right) = 16.93'$$

and convert the decimal part of 16.93′ to seconds,

$$0.93'\left(\frac{60''}{1 \text{ min}}\right) = 56''$$

So

$$105.2821° = 105°16'56''$$ ◆◆◆

Common Error

When you are reading an angle quickly, it is easy to mistake decimal degrees for degrees and minutes. Don't mistake

$$28°50' \quad \text{for} \quad 25.50°$$

28.50° is really 28°30′, and 28°50′ is

$$28°50' = 28° + \left(\frac{50}{60}\right)^{\circ} \approx 28.83°$$

Angle Conversions by Calculator

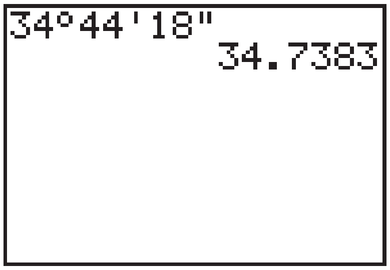
We can convert between decimal degrees and degrees, minutes, seconds by calculator.

◆◆◆ **Example 112:** Convert 34°44′18″ to decimal degrees.

Solution:

- Put the calculator into degree mode.
- Enter the angle, including the degree, minute, and second symbols. On the TI-83/84, the degree symbol and the minute symbol are found in the ANGLE menu. The ″ second symbol is an alpha character on the + key.
- Press ENTER.

The angle will be displayed in decimal form, as shown. ◆◆◆



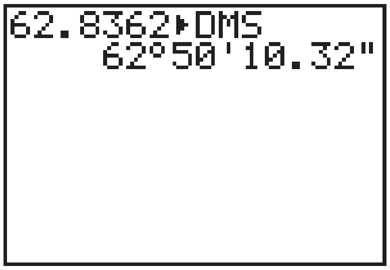
TI-83/84 screen for Example 112. On the TI-89, the degree, minute, and second symbols are alpha characters on the |, =, and 1 keys, respectively.

◆◆◆ **Example 113:** *An Application.* A surveyor calculates that the angle between two sides of a building site must be 62.8362º. Convert this to degrees, minutes, and seconds so that it can be laid out using a transit or theodolite.

Solution:

- Put the calculator into degree mode.
- Enter the angle.
- Choose ▶ DMS from the ANGLE menu.
- Press ENTER. ◆◆◆

The angle will be displayed in DMS form, as shown.



TI-83/84 screen for Example 113. On the TI-89, ▶ DMS is the MATH **Angle** menu.

Substituting into Formulas

A *formula* is an equation expressing some general mathematical or physical fact, such as the formula for the area of a circle of radius r.

Area of a Circle	$A = \pi r^2$	75

To *substitute into* a formula or other equation means to replace the letter (or literal) quantities in the formula with their numerical values, and evaluate. With formulas, we carry the *unit of measure* along with each numerical value. We must often convert units so that they cancel properly, leaving the answer in the desired units. If the units to be used in a certain formula are specified, convert all quantities to those specified before substituting into the formula.

Further, we usually substitute approximate values, so we must round our answer properly.

◆◆◆ **Example 114:** *An Application.* A tensile load P of 4510 lb is applied to a bar, Fig. 1–12. It is seen to stretch or elongate by 0.390 mm. Find the modulus of elasticity E, in pounds per square inch, using

$$E = \frac{PL}{ae}$$

where P is the applied load, L is the length of the bar, a is the cross-sectional area of the bar, and e is the elongation.

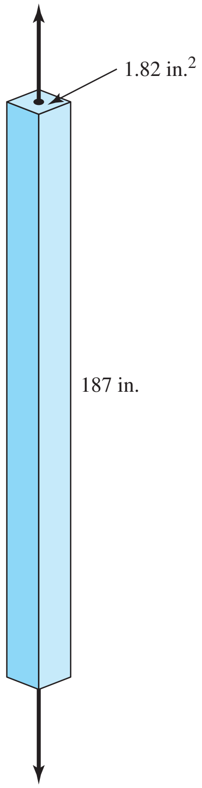


FIGURE 1-12

Solution: Let us first convert the elongation e so that all units of length will be in inches.

$$e = 0.390 \text{ mm}\left(\frac{1 \text{ in.}}{25.4 \text{ mm}}\right) = 0.01535 \text{ in.}$$

Then substituting, with $P = 4510$ lb, $L = 187$ in. and $a = 1.82$ in.^{2},

$$E = \frac{4510 \text{ lb} \times 187 \text{ in.}}{1.82 \text{ in.}^2 \times 0.01535 \text{ in.}} = 30{,}200{,}000 \text{ lb/in.}^2$$
$$= 30.2 \times 10^6 \text{ lb/in.}^2$$

in engineering notation. ◆◆◆

Common Error	Students often neglect to include *units* when substituting into a formula, with the result that the units often do not cancel properly.

Exercise 8 ◆ Units of Measurement

Conversion of Units

Convert the following customary units.

1. 152 inches to feet
2. 0.153 mile to yards
3. 762.0 feet to inches
4. 627 feet to yards
5. 29 tons to pounds
6. 88.90 pounds to ounces
7. 89,600 pounds to tons
8. 8552 ounces to pounds

Converting Between Metric Units

Convert the following metric units. Write your answer in scientific notation if the numerical value is greater than 1000 or less than 0.1.

9. 364,000 meters to kilometers
10. 0.000473 volt to millivolts
11. 735,900 grams to kilograms
12. 7.68×10^{-5} kilowatts to watts
13. 6.2×10^9 ohms to megohms
14. 825×10^4 newtons to kilonewtons
15. 9348 picofarads to microfarads
16. 84,398 nanoseconds to milliseconds

Converting Between Customary and Metric Units

Convert between the given customary and metric units.

17. 364.0 meters to feet
18. 6.83 inches to millimeters
19. 7.35 pounds to newtons
20. 2.55 horsepower to kilowatts
21. 4.66 U.S. gallons to liters
22. 825×10^4 dynes to pounds
23. 3.94 yards to meters
24. 834 cubic centimeters to gallons

Converting Areas and Volumes

Convert the following areas and volumes.

25. 2840 square yards to acres
26. 1636 square meters to ares

27. 24.8 square feet to square meters
28. 3.72 square meters to square feet
29. 0.982 square kilometer to acres
30. 5.93 acres to square meters
31. 7.360 cubic feet to cubic inches
32. 4.83 cubic meters to cubic yards

Converting Rates to Other Units

Convert units on the following time rates.

33. 4.86 feet per second to miles per hour
34. 777 gallons per minute to cubic meters per hour
35. 66.2 miles per hour to kilometers per hour
36. 52.0 knots to miles per minute
37. 953 births per year to births per week

Convert units on the following unit prices.

38. \$1.25 per gram to dollars per kilogram
39. \$800 per acre to cents per square meter
40. \$3.54 per pound to cents per ounce
41. \$4720 per ton to cents per pound

Angle Conversions

Convert to degrees (decimal).

42. 52°17′ 43. 87°25′ 44. 118°33′
45. 72°12′22″ 46. 29°27′41″ 47. 275°18′35″

Convert to degrees, minutes, and seconds. Round to the nearest second.

48. 45.257° 49. 61.339° 50. 27.129°
51. 177.344° 52. 185.972° 53. 128.259°

Substituting into Formulas

In the following exercises, substitute the given quantities into the indicated formula from technology and finance.

54. Use the formula for simple interest $y = a(1 + nt)$, to find, to the nearest dollar, the amount y to which a principal a of \$3000 will accumulate in $t = 5$ years at a simple interest rate n of 6.5%.

55. Using the formula for uniformly accelerated motion,

$$s = v_0 t + \frac{at^2}{2}$$

find the displacement s after $t = 1.30$ s, of a body thrown downward with a speed v_0 of 12.0 ft/s. Take the acceleration a as 32.2 ft/s^2.

56. Use the formula $C = \frac{5}{9}(F - 32)$ to convert 128ºF to degrees Celsius

57. A bar (Fig. 1–13) whose length L is 15.2 m has a cross-sectional area a of 12.7 cm^2. It has an elongation e of 2.75 mm when it is subjected to a tensile load of 22,500 N. Use the equation $E = \dfrac{PL}{ae}$ to find the modulus of elasticity E, in newtons per square centimeter.

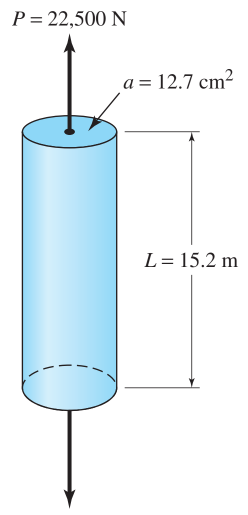


FIGURE 1–13 A bar in tension.

58. Use the formula for compound interest, $y = a(1 + n)^t$, to find, to the nearest dollar, the amount y to which a principal a of \$9570 will accumulate in $t = 5$ years at a compound interest rate n of 6.75%.

59. The resistance R_1 of a copper coil is 775 Ω at $t_1 = 20.0°$ C. The temperature coefficient of resistance α is 0.00393 at 20.0° C. Use the formula $R = R_1[1 + \alpha(t - t_1)]$ to find the resistance at 80.0° C.

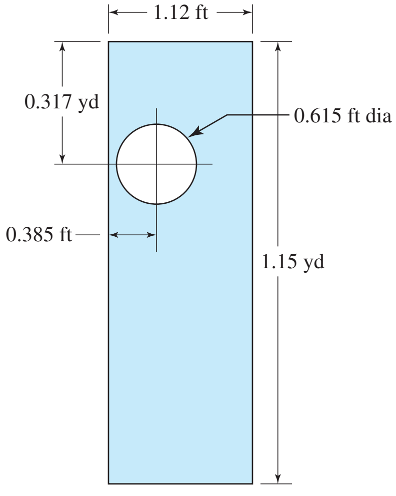


FIGURE 1–14

Applications

60. Convert all of the dimensions for the parts in Fig. 1–14 to inches.

61. A jet fuel tank, in Fig. 1–15, has a volume of 15.7 cubic feet. How many gallons of jet fuel will it hold?

62. A certain circuit board weighs 0.176 pound. Find its weight in ounces.

63. A certain laptop computer weighs 6.35 kilograms. What is its weight in pounds?

64. A generator has an output of 5.34×10^6 millivolts. What is the output in kilovolts?

65. Convert all of the dimension in Fig. 1–16 to centimeters.

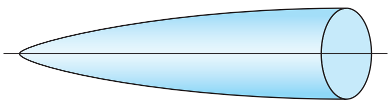


FIGURE 1–15

66. The surface area of a certain lake, Fig. 1–17, is 7361 square yards. Convert this to square meters.

67. A solar collector, Fig. 1–18, has an area of 8834 square inches. Convert this to square meters.

68. The volume of a balloon, Fig. 1–19, is 8360 cubic feet. Convert this to cubic inches.

69. The volume of a certain gasoline tank, Fig. 1–20, is 9274 cubic centimeters. Convert this to gallons.

70. An airplane is cruising at a speed of 785 miles per hour. Convert this speed to kilometers per hour.

71. *Internet:* On the Web, find today's currency exchange rates. Use them to convert \$100 to
(a) Euros (b) Japanese yen (c) Canadian dollars

72. *Project:* Scan a newspaper or magazine, noting whether measurements are given in metric units, customary units, or both. Estimate the percent given in metric units. Repeat for a technical journal in your chosen field.

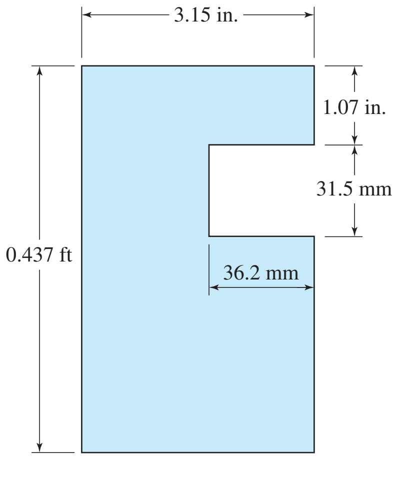


FIGURE 1–16

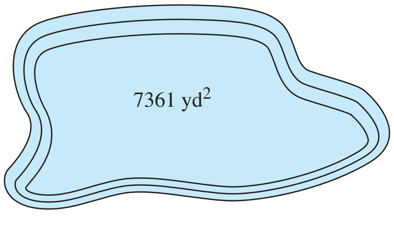


FIGURE 1–17

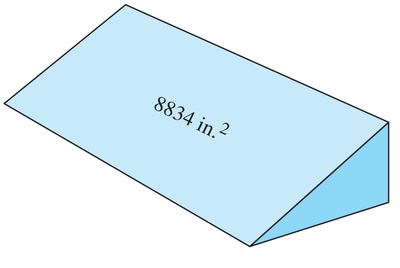


FIGURE 1–18

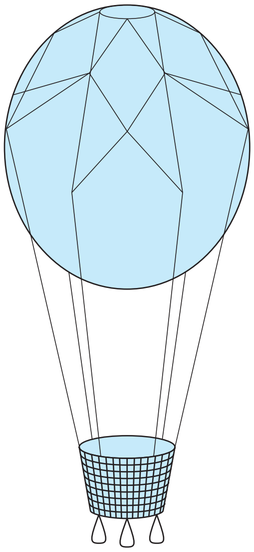


FIGURE 1–19

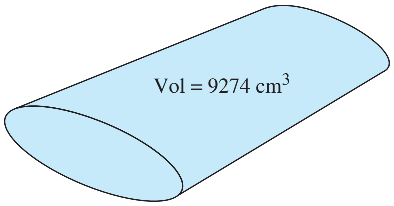


FIGURE 1–20

1–9 Percentage

Our final topic in this long chapter is the very useful subject of percentage.

Definition of Percent

The word *percent* means *by the hundred,* or *per hundred*. A percent thus gives the number of parts in every hundred.

◆◆◆ **Example 115:** If we say that a certain concrete mix is 12% cement by weight, we mean that 12 lb out of every 100 lb of mix will be cement. ◆◆◆

The word *rate* is often used to indicate a percent, or percentage rate, as in “rejection rate,” “rate of inflation,” or “growth rate.”

◆◆◆ **Example 116:** A failure rate of 2% means that, on average, 2 parts out of every 100 would be expected to fail. ◆◆◆

Percent is another way of expressing a *fraction* having 100 as the denominator.

◆◆◆ **Example 117:** If we say that a builder has finished 75% of a house, we mean that he has finished $\frac{75}{100}$ (or $\frac{3}{4}$) of the house. ◆◆◆

Converting to Percent

Before working some percentage problems, let us first get some practice in converting decimals and fractions to percents, and vice versa. To convert decimals to percent, simply move the decimal point two places to the right and affix the percent symbol (%).

◆◆◆ **Example 118:** Here we convert decimals to percent.

(a) $0.75 = 75\%$ (b) $3.65 = 365\%$

(c) $0.003 = 0.3\%$ (d) $1.05 = 105\%$ ◆◆◆

To convert fractions or mixed numbers to percent, first write the fraction or mixed number as a decimal, and then proceed as above.

◆◆◆ **Example 119:**

(a) $\frac{1}{4} = 0.25 = 25\%$ (b) $\frac{5}{2} = 2.5 = 250\%$

(c) $1\frac{1}{4} = 1.25 = 125\%$ ◆◆◆

Converting from Percent

To convert percent to decimals, move the decimal point two places to the left and remove the percent sign.

◆◆◆ **Example 120:**

(a) $13\% = 0.13$ (b) $4.5\% = 0.045$

(c) $155\% = 1.55$ (d) $27\frac{3}{4}\% = 0.2775$

(e) $200\% = 2$ ◆◆◆

To convert percent to a fraction, write a fraction with 100 in the denominator and the percent in the numerator. Remove the percent sign and reduce the fraction to lowest terms.

◆◆◆ **Example 121:** Here we show some percents converted to fractions.

(a) $75\% = \frac{75}{100} = \frac{3}{4}$

(b) $87.5\% = \frac{87.5}{100} = \frac{875}{1000} = \frac{7}{8}$

(c) $125\% = \frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$ ◆◆◆

Solving Percentage Problems

Percentage problems always involve three quantities:

1. The *percent rate*, P.
2. The *base*, B, the quantity we are taking the percent of.
3. The *amount*, A, that we get when we take the percent of the base, also called the *percentage*.

In a percentage problem, you will know two of these three quantities (amount, base, or rate), and you will be required to find the third. This is easily done, for the rate, base, and amount are related by the following equation:

Percentage	amount = base × rate $A = BP$ where P is expressed as a decimal	**16**

Finding the Amount When the Base and the Rate Are Known

We substitute the given base and rate into Eq. 16 and solve for the amount.

◆◆◆ **Example 122:** What is 35.0 percent of 80.0?

Solution: In this problem the rate is 35.0%, so

$$P = 0.350$$

But is 80.0 the amount or the base?

Tip	In a particular problem, if you have trouble telling which number is the base and which is the amount, look for the key phrase *percent of*. The quantity following this phrase is *always the base*.

Thus we look for the key phrase "percent of."

What is 35.0 percent of [80.0]?

Since 80.0 immediately follows *percent of,* it is the base. So

$$B = 80.0$$

From Eq. 16,

$$A = PB = (0.350)80.0 = 28.0$$ ◆◆◆

Common Error	Do not forget to convert the percent rate to a *decimal* when using Eq. 16.

◆◆◆ **Example 123:** Find 3.74% of 5710.

Solution: We substitute into Eq. 16 with

$$P = 0.0374 \text{ and } B = 5710$$

So

$$A = PB = (0.0374)(5710) = 214$$

after rounding to three significant digits. ◆◆◆

◆◆◆ **Example 124:** *An Application*. A proposed beam having a width of 8.50 in. is to have its width increased by 15%. How much width must be added?

Solution:

$$\text{width added} = 0.15 \times 8.50 = 1.28 \text{ in.}$$ ◆◆◆

Finding the Base When a Percent of It Is Known

We see from Eq. 16 that the base equals the amount divided by the rate (expressed as a decimal), or $B = A/P$.

◆◆◆ **Example 125:** 12% of what number is 78?

Solution: First find the key phrase.

12 percent of [what number] (base) is 78?

It is clear that we are looking for the base. So

$$A = 78 \quad \text{and} \quad P = 0.12$$

By Eq. 16,

$$B = \frac{A}{P} = \frac{78}{0.12} = 650$$ ◆◆◆

◆◆◆ **Example 126:** 140 is 25% of what number?

Solution: From Eq. 16,

$$B = \frac{A}{P} = \frac{140}{0.25} = 560$$ ◆◆◆

◆◆◆ **Example 127:** *An Application.* How much gravel must we start with if we remove 2.50 cubic yards, which is 35% of the original load?

Solution: The original load is the base B, the quantity removed is the amount A, and the rate P is 0.35. So,

$$\text{Original amount } B = \frac{A}{P} = \frac{2.50}{0.35} = 7.14 \text{ cubic yards}$$ ◆◆◆

Finding the Percent That One Number Is of Another Number

From Eq. 16, the rate equals the amount divided by the base, or $P = A/B$.

◆◆◆ **Example 128:** 42.0 is what percent of 405?

Solution: By Eq. 16, with $A = 42.0$ and $B = 405$,

$$P = \frac{A}{B} = \frac{42.0}{405} = 0.104 = 10.4\%$$ ◆◆◆

◆◆◆ **Example 129:** What percent of 1.45 is 0.357?

Solution: From Eq. 16,

$$P = \frac{A}{B} = \frac{0.357}{1.45} = 0.246 = 24.6\%$$ ◆◆◆

◆◆◆ **Example 130:** *An Application*. A steel beam that used to cost \$885 now costs \$65 more. By what percent did the cost increase?

Solution: The base B is the old cost of the beam; the amount A is the amount of increase. The percent increase is then

$$P = \frac{A}{B} = \frac{65}{885} = 0.0734$$

or an increase of 7.34%. ◆◆◆

Percent Change

Percentages are often used to compare two quantities. You often hear statements such as the following:

The price of steel rose 3% over last year's price.

The weights of two cars differed by 20%.

Production dropped 5% from last year.

When the two numbers being compared involve a *change* from one to the other, the *original value* is usually taken as the base.

Percent Change	$$\text{percent change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100$$	17

◆◆◆ **Example 131:** A quantity changed from 521 to 835. What was the percent change?

Solution:

$$\text{percent change} = \frac{835 - 521}{521} \times 100 = 60.3\% \text{ increase}$$ ◆◆◆

◆◆◆ **Example 132:** *An Application*. A certain price rose from \$1.55 to \$1.75. Find the percentage change in price.

Solution: We use the original value, \$1.55, as the base. From Eq. 17,

$$\text{percent change} = \frac{1.75 - 1.55}{1.55} \times 100 = 12.9\% \text{ increase}$$ ◆◆◆

Be sure to show the *direction* of change with a plus or a minus sign, or with words such as *increase* or *decrease*.

A common type of problem is to *find the new value* when the original value is changed by a given percent. We see from Eq. 17 that

$$\text{new value} = \text{original value} + (\text{original value}) \times (\text{percent change})$$

◆◆◆ **Example 133:** *An Application.* The voltage across a certain power line dropped from 220 V to 215 V. What was the percent change in voltage?

Solution: Using the original value, 220 V, as the base, we get,

$$\text{percent change} = \frac{220 - 215}{220} \times 100 = 2.27\% \text{ decrease}$$ ◆◆◆

◆◆◆ **Example 134:** *An Application.* Find the cost of a \$356.00 suit after the price increases by $2\frac{1}{2}\%$.

Solution: The original value is 356.00, and the percent change, expressed as a decimal, is 0.025. So

$$\text{new value} = 356.00 + 356.00(0.025) = \$364.90$$ ◆◆◆

Percent Efficiency

The power output of any machine or device is always *less* than the power input, because of inevitable power losses within the device. The *efficiency* of the device is a measure of those losses.

Percent Efficiency	$\text{percent efficiency} = \frac{\text{output}}{\text{input}} \times 100$	**20**

◆◆◆ **Example 135:** *An Application.* A certain electric motor consumes 865 W and has an output of 1.12 hp (Fig. 1–21). Find the efficiency of the motor. (1 hp = 746 W.)

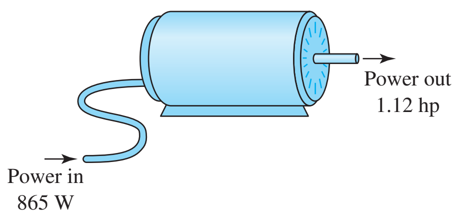


FIGURE 1–21

Solution: Since output and input must be in the same units, we must convert either to horsepower or to watts. Converting the output to watts, we obtain

$$\text{output} = 1.12 \text{ hp}\left(\frac{746 \text{ W}}{\text{hp}}\right) = 836 \text{ W}$$

By Eq. 20,

$$\text{percent efficiency} = \frac{836}{865} \times 100 = 96.6\%$$ ◆◆◆

Percent Error

The accuracy of measurements is often specified by the *percent error*. The percent error is the difference between the measured value and the known or "true" value, expressed as a percent of the known value.

Percent Error	$$\text{percent error} = \frac{\text{measured value} - \text{known value}}{\text{known value}} \times 100$$	**18**

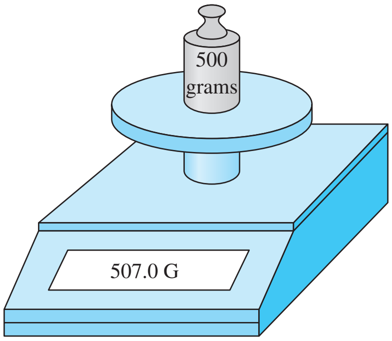


FIGURE 1–22

◆◆◆ **Example 136:** *An Application.* A laboratory weight that is certified to be 500.0 g is placed on a scale (Fig. 1–22). The scale reading is 507.0 g. What is the percent error in the reading?

Solution: From Eq. 18,

$$\text{percent error} = \frac{507.0 - 500.0}{500.0} \times 100 = 1.4\% \text{ high}$$ ◆◆◆

As with percent change, be sure to specify the *direction* of the error.

Percent Concentration

The following equation applies to a mixture of two or more ingredients:

Percent Concentration	$$\text{percent concentration of ingredient } A = \frac{\text{amount of } A}{\text{amount of mixture}} \times 100$$	**19**

◆◆◆ **Example 137:** *An Application:* A certain fuel mixture contains 18.9 liters of alcohol and 84.7 liters of gasoline. Find the percentage of gasoline in the mixture.

Solution: The total amount of mixture is

$$18.9 + 84.7 = 103.6 \text{ liters}$$

So by Eq. 19,

$$\text{percent gasoline} = \frac{84.7}{103.6} \times 100 = 81.8\%$$ ◆◆◆

Common Error	The denominator in Eq. 19 must be the *total amount* of mixture, or the sum of *all* of the ingredients. Do not use just one of the ingredients.

Exercise 9 ◆ Percentage

Converting to Percent Convert each decimal to a percent.

1. 3.72 **2.** 0.877 **3.** 0.0055 **4.** 0.563

Convert each fraction to a percent. Round to three significant digits.

5. $\frac{2}{5}$ **6.** $\frac{3}{4}$ **7.** $\frac{7}{10}$ **8.** $\frac{3}{7}$

Converting from Percent

Convert each percent to a decimal.

9. 23% **10.** 2.97% **11.** $287\frac{1}{2}\%$ **12.** $6\frac{1}{4}\%$

Convert each percent to a fraction.

13. 37.5% **14.** $12\frac{1}{2}\%$ **15.** 150% **16.** 3%

Finding the Amount Find:

17. 41.1% of 255 tons.

18. 15.3% of 326 mi.

19. 33.3% of 662 kg.

20. 12.5% of 72.0 gal.

21. 35.0% of 343 liters.

22. 50.8% of $245.

23. A resistance, now 7250 Ω, is to be increased by 15.0%. How much resistance should be added?

24. It is estimated that $\frac{1}{2}\%$ of the earth's surface receives more energy than the total projected yearly needs. Assuming the earth's surface area to be 1.97×10^8 mi^2, find the required area in acres.

25. As an incentive to install solar equipment, a tax credit of 42% of the first $1100 and 25% of the next $6400 spent on solar equipment is proposed. How much credit, to the nearest dollar, would a homeowner get when installing $5500 worth of solar equipment?

26. How much metal will be obtained from 375 tons of ore if the metal is 10.5% of the ore?

Finding the Base Find the number of which

27. 86.5 is 16.7%.

28. 45.8 is 1.46%.

29. 1.22 is 1.86%.

30. 55.7 is 25.2%.

31. 66.6 is 66.6%.

32. 58.2 is 75.4%.

33. A Department of Energy report on an experimental electric car gives the range of the car as 161 km and states that this is "49.5% better than on earlier electric vehicles." What was the range of earlier electric vehicles?

34. A man withdrew 25.0% of his bank deposits and spent 45.0% of the amount withdrawn on a car costing $31,100. How much money was originally in the bank?

35. Solar panels provide 65.0% of the heat for a certain building. If $1560 per year is now spent for heating oil, what would have been spent if the solar panels were not used?

36. If the United States imports 9.14 billion barrels (bbl) of oil per day, and if this is 48.2% of its needs, how much oil is needed per day?

Finding the Percentage Rate What percent of

37. 26.8 is 12.3?

38. 36.3 is 12.7?

39. 44.8 is 8.27?

40. 844 is 428?

41. 455 h is 152 h?

42. 483 tons is 287 tons?

43. A 50,500-liter-capacity tank contains 5840 liters of water (Fig. 1–23). Express the amount of water in the tank as a percentage of the total capacity.

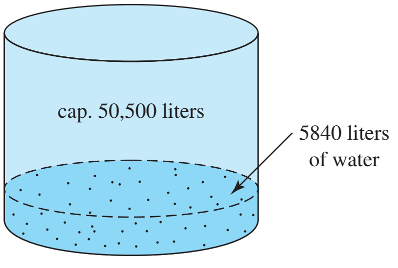


FIGURE 1–23

44. In a journey of 1560 km, a person traveled 195 km by car and the rest of the distance by rail. What percent of the distance was traveled by rail?

45. A power supply has a dc output of 51 V with a ripple of 0.75 V peak to peak. Express the ripple as a percentage of the dc output voltage.

46. The construction of a building costs $136,000 for materials and $157,000 for labor. What percentage of the total is the labor cost?

Percent Change Find the percent change when a quantity changes

47. from 29.3 to 57.6.
48. from 107 to 23.75.
49. from 227 to 298.
50. from 0.774 to 0.638.

51. The temperature in a building rose from 19.0°C to 21.0°C during the day. Find the percent change in temperature.

52. A casting initially weighing 115 lb has 22.0% of its material machined off. What is its final weight?

53. A certain common stock rose from a value of $$35\frac{1}{2}$ per share to $$37\frac{5}{8}$ per share. Find the percent change in value.

54. A house that costs $635 per year to heat has insulation installed in the attic, causing the fuel bill to drop to $518 per year. Find the percent change in fuel cost.

Percent Efficiency

55. A certain device (Fig. 1–24) consumes 18.5 hp and delivers 12.4 hp. Find its efficiency.

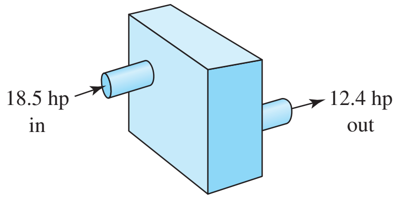


FIGURE 1–24

56. An electric motor consumes 1250 W. Find the horsepower it can deliver if it is 85.0% efficient. (1 hp = 746 W.)

57. A water pump requires an input of 0.50 hp and delivers 10,100 lb of water per hour to a house 72 ft above the pump. Find its efficiency. (1 hp = 550 ft lb/s.)

58. A certain speed reducer delivers 1.7 hp with a power input of 2.2 hp. Find the percent efficiency of the speed reducer.

Percent Error

59. A certain quantity is measured at 125.0 units but is known to be actually 128.0 units. Find the percent error in the measurement.

60. A shaft is known to have a diameter of 35.000 mm. You measure it and get a reading of 34.725 mm. What is the percent error of your reading?

61. A certain capacitor has a working voltage of 125.0 V dc, −10%, +150%. Between what two voltages would the actual working voltage lie?

62. A resistor is labeled as 5500 Ω with a tolerance of ±5%. Between what two values is the actual resistance expected to lie?

Percent Concentration

63. A solution is made by mixing 75.0 liters of alcohol with 125 liters of water. Find the percent concentration of alcohol.

64. 8.0 cubic feet of cement is contained in a concrete mixture that is 12% cement by volume. What is the volume of the total mixture?

65. How many liters of alcohol are contained in 455 liters of a gasohol mixture that is 5.5% alcohol by volume?

66. How many liters of gasoline are there in 155 gal of a methanol–gasoline blend that is 10.0% methanol by volume?

67. *Writing:* We said, "Of all the mathematical topics we cover in this text, probably the one most used in everyday life is percentage." Do you agree? Write a few paragraphs saying if you agree or not, and back your reasons up with specific examples from personal experience.

CHAPTER 1 REVIEW PROBLEMS

1. Combine: $1.435 - 7.21 + 93.24 - 4.1116$
2. Give the number of significant digits in:
   (a) 9.886 (b) 1.002 (c) 0.3500 (d) 15,000
3. Multiply: $21.8(3.775 \times 1.07)$
4. Divide: $88.25 \div 9.15$
5. Find the reciprocal of 2.89.
6. Evaluate: $-|-4 + 2| - |-9 - 7| + 5$
7. Evaluate: $(9.73)^2$
8. Evaluate: $(7.75)^{-2}$
9. Evaluate: $\sqrt{29.8}$
10. Evaluate: $(123)(2.75) - (81.2)(3.24)$
11. Evaluate: $(91.2 - 88.6)^2$
12. Evaluate: $\left(\dfrac{77.2 - 51.4}{21.6 - 11.3}\right)^2$
13. Evaluate: $y = 3x^2 - 2x$ when $x = -2.88$
14. Evaluate: $y = 2ab - 3bc + 4ac$ when $a = 5$, $b = 2$, and $c = -6$
15. Evaluate: $y = 2x - 3w + 5z$ when $x = 7.72$, $w = 3.14$, and $z = 2.27$
16. Round to two decimal places.
   (a) 7.977 (b) 4.655 (c) 11.845 (d) 1.004
17. Round to three significant digits.
   (a) 179.2 (b) 1.076 (c) 4.8550 (d) 45,725
18. A news report states that a new hydroelectric generating station will produce 47 million kWh/yr and that this power, for 20 years of operation, is equivalent to 2.0 million barrels of oil. Using these figures, how many kilowatt-hours is each barrel of oil equivalent to?
19. A certain generator has a power input of 2.50 hp and delivers 1310 W. Find its percent efficiency.
20. Use the equation, $\sigma = \dfrac{P}{a}$, find the stress σ, in pounds per square inch, for a force P of 1.17×10^3 N, distributed over an area a of 3.14×10^3 mm^2.
21. Combine: $(8.34 \times 10^5) + (2.85 \times 10^6) - (5.29 \times 10^5)$
22. A train running at 25 mi/h increases its speed by $12\frac{1}{2}\%$. How fast does it then go?

23. The average solar radiation in the continental United States is about 0.206 kW per square meter. How many kilowatts would be collected by 15.0 acres of solar panels?

24. An item rose in price from \$29.35 to \$31.59. Find the percent increase.

25. Find the percent concentration of alcohol if 2.0 liters of alcohol is added to 15 gal of gasoline.

26. A bar, known to be 2.0000 inches in diameter, is measured at 2.0064 in. Find the percent error in the measurement.

27. The Department of Energy estimates that there are 700 billion barrels (bbl) of oil in the oil shale deposits of Colorado, Wyoming, and Utah. Express this amount in scientific notation.

28. Multiply: $(7.23 \times 10^5) \times (1.84 \times 10^{-3})$

29. Divide: -39.2 by -0.003826

30. Convert 6930 Btu/h to foot-pounds per minute.

31. Divide: 8.24×10^{-3} by 1.98×10^7

32. What percent of 40.8 is 11.3?

33. Evaluate: $\sqrt[5]{82.8}$

34. Multiply: $(4.92 \times 10^6) \times (9.13 \times 10^{-3})$

35. Insert the proper sign of equality or inequality between $-\frac{2}{3}$ and -0.660.

36. Convert 0.000426 mA to microamperes.

37. Find 49.2% of 4827.

38. Combine: $-385 - (227 - 499) - (-102) + (-284)$

39. Find the reciprocal of -0.582.

40. Find the percent change in a voltage that increased from 111 V to 118 V.

41. A homeowner added insulation, and her yearly fuel consumption dropped from 628 gal to 405 gal. Her present oil consumption is what percent of the former?

42. Write in decimal notation: 5.28×10^4

43. Convert 49.3 pounds to newtons.

44. Evaluate: $(45.2)^{-0.45}$

45. Using the equation $s = v_0 t + \dfrac{at^2}{2}$, find the distance s (in feet) traveled by a falling object in $t = 5.25$ s, when thrown downward with an initial velocity v_o of 284 m/min. Here a is the acceleration due to gravity, 32.2 ft/s^2.

46. Write in scientific notation: 0.000374

47. 8460 is what percent of 38,400?

48. The U.S. energy consumption of 37 million barrels of oil equivalent per day is expected to climb to 48 million in 6 years. Find the percent increase in consumption.

49. The population of a certain town is 8118, which is $12\frac{1}{2}\%$ more than it was 3 years ago. What was the population then?

50. The temperature of a room rose from 68.0°F to 73.0°F. Find the percent increase.

51. Combine: $4.928 + 2.847 - 2.836$

52. Give the number of significant digits in 2003.0.

53. Multiply: 2.84(38.4)

54. Divide: $48.3 \div 2.841$
55. Find the reciprocal of 4.82.
56. Evaluate: $|-2| - |3 - 5|$
57. Evaluate: $(3.84)^2$
58. Evaluate: $(7.62)^{-2}$
59. Evaluate: $\sqrt{38.4}$
60. Evaluate: $(49.3 - 82.4)(2.84)$
61. Evaluate: $x^2 - 3x + 2$ when $x = 3$
62. Round 45.836 to one decimal place.
63. Round 83.43 to three significant digits.
64. Multiply: $(7.23 \times 10^5) \times (1.84 \times 10^{-3})$
65. Convert 36.82 in. to centimeters.
66. What percent of 847 is 364?
67. Evaluate: $\sqrt[3]{746}$
68. Find 35.8% of 847.
69. 746 is what percent of 992?
70. Write 0.00274 in scientific notation.
71. Write 73.7×10^{-3} in decimal notation.
72. Evaluate: $(47.3)^{-0.26}$
73. Combine: $6.128 + 8.3470 - 7.23612$
74. Give the number of significant digits in 6013.00.
75. Multiply: 7.184(16.8)
76. Divide: $78.7 \div 8.251$
77. Find the reciprocal of 0.825.
78. Evaluate: $|-5| - |2 - 7| + |-6|$