

INTRODUCTION

My first teaching job did not start out too smoothly. I would feverishly spend my evenings preparing material that I thought would excite the students. Then the next day I would watch the expression on my students' faces as they sat through my lecture. Their expressions were similar to that on my Uncle Moe's face when he once recalled an experience on the Bataan Death March. How could the lectures that I painstakingly prepared with the hope of instilling excitement have been as tortuous as the Bataan Death March? To find an answer to this question, I went to the source. I asked the students what was going wrong. After 16 years, with the exception of one suggestion, I have vague recollection of the students' feedback. After 16 years, with the exception of one individual, I cannot remember the faces behind any of the suggestions. Concerning the one individual, not only do I have clarity concerning her face and suggestion, but I also have perfect recollection of my response.

The individual suggested that I deliver the lectures in storylike fashion and have a story behind the mathematics that was being taught. My response that I kept to myself was "you have got to be joking." My feeling was that mathematics was the story; the story cannot be changed to something else to accommodate someone's lack of appreciation for the subject. This was one suggestion that I did not oblige. And while for the most part the other students responded positively to the changes that I did make, this student sat through the entire semester with her tortured expression intact.

It is difficult to recall the specifics of something that was said over 16 years ago, the contents of a normal conversation remain in the past while we move on. Despite my reaction, there must have been some meaning that resonated and continued doing so, otherwise I would have long ago forgotten the conversation. Now I see the student's suggestion as brilliant and right on target. By not taking her suggestion, I blew the chance to get more students excited by mathematics through compelling and human stories that are at the heart of mathematics. At the time, I just did not have the vision to see what she was getting at. After 16 years, I have once more given it some thought and this book is the resulting vision. This is a mathematical story and a true one at that.

The story follows man's pursuit of the ellipse. The ellipse is the shape of a planet's path as it orbits the sun. The ellipse is special because it is a demonstration of man's successful efforts to describe his natural environment using mathematics and this mathematical revelation paved the pathway from the Counter Reformation

to the Enlightenment. Man pursued the ellipse in a dogged manner as if a mission to find it had been seeded into his genetic code. Through wars, enlightened times, book burnings, religious persecution, imprisonment, vanquished empires, centuries of ignorance, more wars, plagues, fear of being ridiculed, the Renaissance, the Reformation, the Counter Reformation, excommunication, witchcraft trials, the Inquisition, more wars, and more plagues, man leaf by leaf nurtured a mathematical beanstalk toward the ellipse. This book examines the development and fabric of the beanstalk. It describes the creation of geometry, algebra, trigonometry, and finally calculus, all targeted toward the ellipse.

What are the ingredients that make up a good story? *Heroes*: They are in this story as the book presents a glimpse of the lives of several mathematicians from Aristarchus to Leibniz who made significant contributions to the beanstalk. *Villains*: The story of men threatened by progress and doing their best to thwart—being central to the story of the ellipse. *Struggles*: The problem of planetary motion is sufficiently vexing to assure some mathematical difficulty, and as the previous paragraph indicates, additional struggles result from a tormented history. *Dedication*: The dedicated effort of the contributors is at once admirable and inspiring. *Uncertainty*: While the book reconstructs mathematical history with the certainty that man arrives at the ellipse, many contributors had absolutely no premonition of where their contributions would lead. This uncertainty is germane to our story. *Character flaws*: Our heroes were not perfect and their mistakes are part of the story. *Tragedy*: Getting speared in the back while contemplating geometry, a victim of one's own insecurity, a burning at the stake as a victim of the Inquisition—these are a small sampling of personal tragedies that unfold as we follow the ellipse. *Triumph*: After a tortuous path, this story triumphantly ends at the ellipse. What else is in a good story? I dare not get explicit, but it is in there.

With such a great story, one would think that someone had told it before. Indeed, the story has been told; the most comprehensive historical presentation is Arthur Koestler's distinguished book, *The Sleepwalkers*. In addition, there are history books and excellent biographies of the main contributors, mathematical history books, and books covering the various mathematical topics that are contained in this book. So what is different about this book? Simply put, the history books only address the history, the math books only address the math, and the mathematical history books only address the mathematical history. This book is a math book covering the topics of geometry, algebra, trigonometry, and calculus which contains a historical narrative that sets the context for the mathematical developments. Following my belief that separating the disciplines of the history of mathematics and science from general history is an unnatural amputation, the narrative weaves the mathematical history into the broader history of the times while focusing along the main thread of uncovering the ellipse.

There is a final category of book that readers of this book may be interested in, popular books that explain mathematical and scientific theory—books explaining general relativity, quantum mechanics, chaos theory, and string theory abound for those without the requisite mathematical background. Of necessity, the core is missing in these books, the mathematics. Just as love binds two humans in true intimacy,

mathematics binds the theorist with evidence. It is difficult to have a true appreciation of the theory without the mathematics, which is unfortunate because it keeps the general public at a distance from theory. This book takes the reader through all the mathematical developments needed to uncover the ellipse, and the reader will become truly intimate with the theory. The book delves into the subjects of geometry, algebra, trigonometry, and calculus, and once the mathematical machinery is finally assembled, we stock the ellipse.

Mathematicians are explorers. They follow their imagination into new territory and map out their findings. Then their discoveries become gateways for other mathematicians who can push the path into further unexplored territories. Unlike the great sea-going explorers of the fifteenth and sixteenth centuries who were exploring the surface of a finite earth, the domain of the mathematician is infinite. The subject will never be exhausted, mathematical knowledge will continue to expand, and the beanstalk will keep growing. However, like the great explorers of the fifteenth and sixteenth centuries, mathematical journeys may target a specific objective (akin to Magellan's circumnavigation of the world) or the consequences of mathematical journeys may be fully unrelated to their intentions (akin to Columbus' accidental discovery of a new continent). We can even go one step further; it is possible that some mathematical journeys have no intent whatsoever other than to amuse the journeying mathematician.

This book presents mathematics as a journey. There is the intended pathway toward the ellipse and there are sojourns along bifurcating branches of the beanstalk that are unrelated to the ellipse. The journey passes through the normal high school curriculum and calculus. By placing all the subject matter together, it is possible to demonstrate relations between what are normally taught as separate disciplines. For example, the area of an ellipse, a geometric concept, is finally arrived at only after developing concepts in linear algebra and trigonometry; the approach highlights the interplay of all the disciplines toward an applied problem. In addition, setting the objective of uncovering the ellipse motivates the mathematics. For example, studies of motion motivate the presentation of calculus and the fundamental theorem of calculus is presented as a statement of the relation between displacement and velocity. The sojourns with no apparent relation to the ellipse are undertaken solely because they are irresistible.

The book allows you as a reader to plot your own course in accordance with your own purpose. Readers with excellent proficiency in calculus will certainly plot their way through the book differently from those who may be a little out of touch with their high school mathematics and calculus. And those entirely unfamiliar with one or more of the subjects will plot another course altogether. The first section of Chapter 2 hosts the main narrative and tells the story of man's pursuit of the ellipse beginning with Aristarchus, the first known heliocentrist, and ending with Newton's successful unveiling nearly two millennia after Aristarchus. Each subsequent chapter begins with a narrative that is pertinent to the mathematical material in the chapter. By and large the mathematical material is included for one or more of the following reasons. The material is necessary to understand the topic of the chapter and will be used in subsequent chapters, or the material presents concrete examples of relevant

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concepts, or I have just indulged my own fancy and included material that I find fun. Sections containing material that falls solely within the last category are clearly marked as excursions and may be skipped without compromising your understanding of the remaining material. As for the remaining material, plot your course in accordance with your own purpose. You may grasp the high-level concepts and move on, or for those who want to go through the nitty-gritty, it is in there. Enjoy your journey.