Chapter 1

Arming Yourself with Math Basics

In This Chapter

▶ Identifying four important sets of numbers

▶ Reviewing addition, subtraction, multiplication, and division

▶ Examining commutative, associative, and distributive operations

▶ Knowing exponents, roots, and absolute values

▶ Understanding how factors and multiples are related

*Y*ou already know more about math than you think you know. In this chapter, you review and gain perspective on basic math ideas such as sets of numbers and concepts related to the Big Four operations (adding, subtracting, multiplying, and dividing). I introduce you (or reintroduce you) to properties and operations that will assist with solving problems. Finally, I explain the relationship between factors and multiples, taking you from what you may have missed to what you need to succeed as you move onward and upward in math.

Understanding Sets of Numbers

You can use the number line to deal with four important *sets* (or groups) of numbers. Each set builds on the one before it:

✔ **Counting numbers (also called natural numbers):** The set of numbers beginning 1, 2, 3, 4, ... and going on infinitely

✔ **Integers:** The set of counting numbers, zero, and negative counting numbers

    ✔ **Rational numbers:** The set of integers and fractions
    ✔ **Real numbers:** The set of rational and irrational numbers

Even if you filled in all the rational numbers, you'd still have points left unlabeled on the number line. These points are the irrational numbers.

An *irrational number* is a number that's neither a whole number nor a fraction. In fact, an irrational number can only be approximated as a *non-repeating decimal.* In other words, no matter how many decimal places you write down, you can always write down more; furthermore, the digits in this decimal never become repetitive or fall into any pattern. (For more on repeating decimals, see Chapter 5.)

The most famous irrational number is π (you find out more about π when I discuss the geometry of circles in Chapter 11):

π = 3.14159265358979323846264338327950288419716939937510...

Together, the rational and irrational numbers make up the *real numbers,* which comprise every point on the number line.

The Big Four Operations

When most folks think of math, the first thing that comes to mind is four little (or not-so-little) words: addition, subtraction, multiplication, and division. I call these operations the *Big Four* all through the book.

Adding things up

Addition is the first operation you find out about, and it's almost everybody's favorite. Addition is all about bringing things together, which is a positive thing. This operation uses only one sign — the plus sign (+).

REMEMBER

When you add two numbers together, those two numbers are called *addends,* and the result is called the *sum.*

Adding a negative number is the same as subtracting, so 7 + –3 is the same as 7 – 3.

Take it away: Subtracting

Subtraction is usually the second operation you discover, and it's not much harder than addition. As with addition, subtraction has only one sign: the minus sign (–).

When you subtract one number from another, the result is called the *difference.* This term makes sense when you think about it: When you subtract, you find the difference between a higher number and a lower one.

Subtracting a negative number is the same as adding a positive number, so 2 – (–3) is the same as 2 + 3. When you're subtracting, you can think of the two minus signs canceling each other out to create a positive.

Multiplying

Multiplication is often described as a sort of shorthand for repeated addition. For example,

4×3 means *add 4 to itself 3 times:* 4 + 4 + 4 = 12

9×6 means *add 9 to itself 6 times:* 9 + 9 + 9 + 9 + 9 + 9 = 54

When you multiply two numbers, the two numbers that you're multiplying are called *factors,* and the result is the *product.* In the preceding example, 4 and 3 are the factors and 12 is the product.

When you're first introduced to multiplication, you use the *times* sign ($\times$). However, algebra uses the letter x a lot, which looks similar to the times sign, so people often choose to use other multiplication symbols for clarity.

Arriving on the dot

In math beyond arithmetic, the symbol $\cdot$ replaces $\times$. For example,

$6 \cdot 7 = 42$ means $6 \times 7 = 42$

$53 \cdot 11 = 583$ means $53 \times 11 = 583$

That's all there is to it: Just use the · symbol anywhere you would've used the standard times sign (×).

Speaking parenthetically

In math beyond arithmetic, using parentheses *without* another operator stands for multiplication. The parentheses can enclose the first number, the second number, or both numbers. For example,

3(5) = 15 means $3 \times 5 = 15$

(8)7 = 56 means $8 \times 7 = 56$

(9)(10) = 90 means $9 \times 10 = 90$

However, notice that when you place another operator between a number and a parenthesis, that operator takes over. For example,

3 + (5) = 8 means $3 + 5 = 8$

(8) − 7 = 1 means $8 - 7 = 1$

Doing division lickety-split

The last of the Big Four operations is division. *Division* literally means splitting things up. For example, suppose you're a parent on a picnic with your three children. You've brought along 12 pretzel sticks as snacks and want to split them fairly so that each child gets the same number (don't want to cause a fight, right?).

Each child gets four pretzel sticks. This problem tells you that

$$12 \div 3 = 4$$

As with multiplication, division also has more than one sign: the *division sign* (÷) and the *fraction slash* (/) or fraction bar (—). So some other ways to write the same information are

$^{12}\!/_3 = 4$ and $\dfrac{12}{3} = 4$

When you divide one number by another, the first number is called the *dividend,* the second is called the *divisor,* and the result is the *quotient.* For example, in the division from the

earlier example, the dividend is 12, the divisor is 3, and the quotient is 4.

Fun and Useful Properties of the Big Four Operations

When you know how to do the Big Four operations — add, subtract, multiply, and divide — you're ready to grasp a few important properties of these important operations. *Properties* are features of the Big Four operations that always apply no matter which numbers you're working with.

Inverse operations

Each of the Big Four operations has an *inverse* — an operation that undoes it. Addition and subtraction are inverse operations because addition undoes subtraction, and vice versa. In the same way, multiplication and division are inverse operations. Here are two inverse equation examples:

$$184 - 10 = 174 \qquad 4 \cdot 5 = 20$$
$$174 + 10 = 184 \qquad 20 \div 5 = 4$$

In the example on the left, when you subtract a number and then add the same number, the addition undoes the subtraction and you end up back at 184.

In the example on the right, you start with the number 4 and multiply it by 5 to get 20. And then you divide 20 by 5 to return to where you started at 4. So division is the inverse operation of multiplication.

Commutative operations

Addition and multiplication are both commutative operations. *Commutative* means that you can switch around the order of the numbers without changing the result. This property of

addition and multiplication is called the *commutative property*. For example,

$$3 + 5 = 8 \text{ is the same as } 5 + 3 = 8$$

$$2 \cdot 7 = 14 \text{ is the same as } 7 \cdot 2 = 14$$

TIP

In contrast, subtraction and division are *noncommutative* operations. When you switch around the order of the numbers, the result changes. For example,

$$6 - 4 = 2, \text{ but } 4 - 6 = -2$$

$$5 \div 2 = \frac{5}{2} \text{ but } 2 \div 5 = \frac{2}{5}$$

Associative operations

Addition and multiplication are both *associative operations,* which means that you can group them differently without changing the result. This property of addition and multiplication is also called the *associative property.* Here's an example of how addition is associative. Suppose you want to add 3 + 6 + 2. You can solve this problem in two ways:

(3 + 6) + 2	3 + (6 + 2)
= (9) + 2	= 3 + (8)
= 11	= 11

And here's an example of how multiplication is associative. Suppose you want to multiply 5 · 2 · 4. You can solve this problem in two ways:

(5 · 2) · 4	5 · (2 · 4)
= 10 · 4	= 5 · 8
= 40	= 40

In contrast, subtraction and division are *nonassociative* operations. This means that grouping them in different ways changes the result.

Distributing to lighten the load

In math, distribution (also called the *distributive property* of multiplication over addition) allows you to split a large multiplication problem into two smaller ones and add the results to get the answer.

For example, suppose you want to multiply $17 \cdot 101$. You can multiply them out, but distribution provides a different way to think about the problem that you may find easier. Because $101 = 100 + 1$, you can split this problem into two easier problems as follows:

$$= 17 \cdot (100 + 1)$$
$$= (17 \cdot 100) + (17 \cdot 1)$$

You take the number outside the parentheses, multiply it by each number inside the parentheses one at a time, then add the products. At this point, you may be able to solve the two multiplications in your head and then add them up easily:

$$= 1{,}700 + 17 = 1{,}717$$

Other Operations: Exponents, Square Roots, and Absolute Values

In this section, I introduce you to three new operations that you need as you move on with math: exponents, square roots, and absolute values. As with the Big Four operations, these three operations take numbers and tweak them in various ways.

Understanding exponents

Exponents (also called *powers*) are shorthand for repeated multiplication. For example, 2^3 means to multiply 2 by itself 3 times. To do that, use the following notation:

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

In this example, 2 is the *base number* and 3 is the *exponent*. You can read 2^3 as "two to the third power" or "two to the power of 3" (or even "two cubed," which has to do with the formula for finding the volume of a cube — see Chapter 11 for details).

When the base number is 10, figuring out any exponent is easy. Just write down a 1 and that many 0s after it:

$10^2 = 100$ (1 with two 0s)

$10^7 = 10,000,000$ (1 with seven 0s)

$10^{20} = 100,000,000,000,000,000,000$ (1 with twenty 0s)

The most common exponent is the number 2. When you take any whole number to the power of 2, the result is a square number. For this reason, taking a number to the power of 2 is called *squaring* that number. You can read 3^2 as "three squared," 4^2 as "four squared," and so forth.

Any number raised to the 0 power equals 1. So 1^0, 37^0, and $999,999^0$ are equivalent, or equal.

Discovering your roots

Earlier in this chapter, in "Fun and Useful Properties of the Big Four Operations," I show you how addition and subtraction are inverse operations. I also show you how multiplication and division are inverse operations. In a similar way, roots are the inverse operation of exponents.

The most common root is the square root. A *square root* undoes an exponent of 2. For example,

$$4^2 = 4 \cdot 4 = 16, \text{ so } \sqrt{16} = 4$$

You can read the symbol $\sqrt{}$ either as "the square root of" or as "radical." So read $\sqrt{16}$ as either "the square root of 16" or "radical 16."

You probably won't use square roots too much until you get to algebra, but at that point they become very handy.

Figuring out absolute value

The *absolute value* of a number is the positive value of that number. It tells you how far away from 0 a number is on the number line. The symbol for absolute value is a set of vertical bars.

Taking the absolute value of a positive number doesn't change that number's value. For example,

$|12| = 12$

$|145| = 145$

However, taking the absolute value of a negative number changes it to a positive number:

$|-5| = 5$

$|-212| = 212$

Finding Factors

In this section, I show you the relationship between factors and multiples. When one number is a *factor* of a second number, the second number is a *multiple* of the first number. For example, 20 is divisible by 5, so 5 is a factor of 20 and 20 is a multiple of 5.

Generating factors

You can easily tell whether a number is a factor of a second number: Just divide the second number by the first. If it divides evenly (with no remainder), the number is a factor; otherwise, it's not a factor.

For example, suppose you want to know whether 7 is a factor of 56. Because 7 divides 56 without leaving a remainder, 7 is a factor of 56. This method works no matter how large the numbers are.

The *greatest factor* of any number is the number itself, so you can always list all the factors of any number because you have a stopping point. Here's how to list all the factors of a number:

1. **Begin the list with 1, leave some space for other numbers, and end the list with the number itself.**

   Suppose you want to list all the factors of the number 18. Following these steps, you begin your list with 1 and end it with 18.

2. **Test whether 2 is a factor — that is, see whether the number is divisible by 2.**

   If it is, add 2 to the list, along with the original number divided by 2 as the second-to-last number on the list. For instance, 18 ÷ 2 = 9, so add 2 and 9 to the list of factors of 18.

3. **Test the number 3 in the same way.**

   You see that 18 ÷ 3 = 6, so add 3 and 6 to the list.

4. **Continue testing numbers until the beginning of the list meets the end of the list.**

   Check every number between to see whether it's evenly divisible. If it is, that number is also a factor. You get remainders when you divide 18 by 4 or 5, so the complete list of factors of 18 is 1, 2, 3, 6, 9, and 18.

A *prime number* is divisible only by 1 and itself — for example, the number 7 is divisible only by 1 and 7. On the other hand, a *composite number* is divisible by at least one number other than 1 and itself — for example, the number 9 is divisible not only by 1 and 9 but also by 3. A number's *prime factors* are the set of prime numbers (including repeats) that equal that number when multiplied together.

Finding the greatest common factor (GCF)

The *greatest common factor* (GCF) of a set of numbers is the largest number that's a factor of all those numbers. For example, the GCF of the numbers 4 and 6 is 2, because 2 is the greatest number that's a factor of both 4 and 6.

To find the GCF of a set of numbers, list all the factors of each number, as I show you in "Generating factors." The greatest factor appearing on every list is the GCF.

For example, suppose you want to find the GCF of 28, 42, and 70. Start by listing the factors of each:

> ✔ **Factors of 28:** 1, 2, 4, 7, 14, 28
>
> ✔ **Factors of 42:** 1, 2, 3, 6, 7, 14, 21, 42
>
> ✔ **Factors of 70:** 1, 2, 5, 7, 10, 14, 35, 70

The largest factor that appears on all three lists is 14; thus, the GCF of 28, 42, and 70 is 14.

Finding Multiples

Even though multiples tend to be larger numbers than factors, most students find them easier to work with. Read on for info on finding multiples and identifying the least common multiple of a set of numbers.

Generating multiples

The earlier section "Finding Factors" tells you how to find *all* the factors of a number. Finding all the factors is possible because a number's factors are always less than or equal to the number itself. So no matter how large a number is, it always has a *finite* (limited) number of factors.

Unlike factors, multiples of a number are greater than or equal to the number itself. (The only exception to this is 0, which is a multiple of every number.) Because of this, the multiples of a number go on forever — that is, they're *infinite*. Nevertheless, generating a partial list of multiples for any number is simple.

REMEMBER

To list multiples of any number, write down that number and then multiply it by 2, 3, 4, and so forth.

For example, here are the first few positive multiples of 7:

> 7   14   21   28   35   42

As you can see, this list of multiples is simply part of the multiplication table for the number 7.

Finding the least common multiple (LCM)

The *least common multiple* (LCM) of a set of numbers is the lowest positive number that's a multiple of every number in that set.

To find the LCM of a set of numbers, take each number in the set and jot down a list of the first several multiples in order. The LCM is the first number that appears on every list.

When looking for the LCM of two numbers, start by listing multiples of the higher number, but stop this list when the number of multiples you've written down equals the lower number. Then start listing multiples of the lower number until one of them matches a number in the first list.

For example, suppose you want to find the LCM of 4 and 6. Begin by listing multiples of the higher number, which is 6. In this case, list only four of these multiples, because the lower number is 4.

>**Multiples of 6:** 6, 12, 18, 24, ...

Now start listing multiples of 4:

>**Multiples of 4:** 4, 8, 12, ...

Because 12 is the first number to appear on both lists of multiples, 12 is the LCM of 4 and 6.