Chapter 1

SAT Math Basics

In This Chapter

- Overviewing the three SAT math sections
- Knowing what's covered and what's not covered on the SAT
- Understanding some basic SAT problem-solving skills

SAT math — what joy, what utter bliss! Well, all right — back on Earth you probably have some work to do before you reach that stage. I promise to do everything in my power to make your study time as painless and productive as possible. All I ask is that you trust in yourself: You already know more than you think you do.

If you've taken algebra in school, much of this book may seem like review. The task at hand is to focus your work on the skills you need to get the best SAT score you can. So in this chapter, I give you a road map to rediscovering the math you know, getting clear on the math you're sketchy on, and preparing to take on some new and useful skills in time for the test.

I start off with an overview of the SAT math sections. I then go over the specific math skills you need to focus on, which I cover in detail in Part II. Then I set your mind at ease by mentioning a few areas of math that you don't have to worry about because they're *not* on the test. Finally, I talk a bit about problem-solving and applying all those math skills.

Getting an Overview of the SAT Math Sections

Your total SAT *composite score* is a number from a lowest possible score of 600 to a highest possible score of 2,400. Out of that, your mathematics score ranges from 200 to 800, based on your performance on the three mathematics sections of the test.

Here's an overview of the three math sections of the SAT:

- A 25-minute section containing 20 multiple-choice questions, which require you to choose the right answer among five choices, (A) through (E)
- A 25-minute section containing 18 questions: 8 multiple-choice questions and 10 grid-in questions (also called *student-produced response questions*), which require you to record the right answer into a special grid
- A 20-minute section containing 16 multiple-choice questions

Generally speaking, questions within each section of the SAT get progressively more difficult. Early questions usually test you on a single basic skill. In the middle of the section, the questions get a bit more complicated. By the end of the test, you usually need a variety of math skills to answer a question.

In Chapter 2, I discuss the two types of questions (multiple-choice and grid-in) in more detail. I also give you some guidelines on writing your answers for grid-in questions. Later, each of the three practice tests in Part IV (Chapters 10 through 15) gives you three math sections that mirror the ones you'll face when you sit for your SAT.

Knowing What's In: The Math You Need for the SAT

The SAT covers math up to and including the first semester of Algebra II. A good rule of thumb is that SAT math

- Includes the quadratic *equation* ($ax^2 + bx + c = 0$) and everything covered before it
- Excludes the quadratic *formula* ($x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$) and everything covered after it

In this section, I give you an overview of some important math topics that are part of the SAT, in each case focusing on the specific skills I cover in each chapter.

Calculating with arithmetic questions

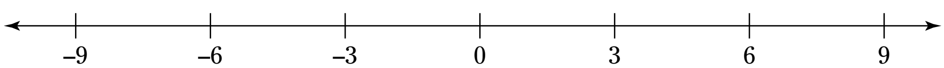
In this section, I cover the arithmetic skills you need most on the SAT. You can flip to Chapter 3 for more detail.

Digital computing

The number system uses ten digits — 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 — from which all other numbers are built. Some SAT questions require you to figure out the value of a number based on the values of its digits. For example, you may be asked to find the value of four-digit number *ABCD* based on clues about its individual digits, *A*, *B*, *C*, and *D*.

Number lines

A *number line* is a visual representation of a set of numbers. For example,



The number line here is labeled with seven *tick marks,* each labeled with a number. On this number line, the *interval* between each pair of tick marks is 3. An SAT question may ask you to figure out the value at a given point or the distance between two points on a number line. In some cases, drawing your own number line can help you solve word problems, as I show you in Chapter 8.

Divisibility, factors, and multiples

When one number is *divisible* by another, you can divide the first number by the second number without leaving a remainder. For example, 10 is divisible by 5, because $10 \div 5 = 2$. Two other important words to describe divisibility are *factor* and *multiple*. Here's how you use these words to describe the fact that 10 is divisible by 5:

5 is a *factor* of 10

10 is a *multiple* of 5

Some SAT questions ask you directly about divisibility, factors, and multiples. Other times, knowing about divisibility can help you cross off wrong answers if, for example, you're dividing and looking for an answer that's an integer (a positive or negative whole number).

Percents

A *percent* is a fractional portion of a whole amount. For example, 50% of 22 is 11, because half of 22 is 11. In this example, you start with the whole amount 22 and then take half of it (because 50% means *half*), which gives you 11. In Chapter 3, I show you some useful ways to work with percents, including problems in percent increase and percent decrease.

Ratios and proportions

A *ratio* is a mathematical comparison of two quantities, based on the operation of division. For example, if a family has 3 girls and 4 boys, you can express the ratio of girls to boys in any of the following ways:

$$3{:}4 \qquad 3 \text{ to } 4 \qquad \frac{3}{4}$$

A *proportion* is an equation based on two ratios set equal to each other. For example, you can set up the following equation, which pairs words and numbers:

$$\frac{\text{Girls}}{\text{Boys}} = \frac{3}{4}$$

SAT questions may give you ratios outright, or you may find that setting up a proportion is a useful way to think of a problem that deals with comparisons. For example, a problem may tell you that a club has the same ratio of girls to boys and ask you to figure out how many boys are in the club, given the number of girls. You can set the ratios equal to each other and find the number of boys by using cross-multiplication, as I show you in Chapter 3.

Powers and roots (radicals)

Raising a number to a *power* means multiplying it by itself a specified number of times. For example, $3^4 = 3 \times 3 \times 3 \times 3 = 81$. In the expression 3^4, the number 3 is the *base* — the number being multiplied — and the number 4 is the *exponent* — the number of times the base is multiplied by itself.

The most common exponent is 2, and raising a number to a power of 2 is called *squaring* that number. When you find the *square root* of a number (also called a *radical*), you reverse this process by discovering a value that, when multiplied by itself, gives the number you started with. For example, $\sqrt{49} = 7$, because $7^2 = 49$.

Doing the algebra shuffle

This section begins with a review of basic algebra concepts and terminology. In Chapter 4, I discuss the basic algebra concepts you need for the SAT.

Evaluating, simplifying, and factoring expressions

An *algebraic expression* is any string of mathematical symbols that makes sense and has at least one variable (such as x). For example,

$$3x + 2 + x$$

You can *evaluate* this expression by substituting a number for x and then finding the resulting value. For example, here's how you evaluate the expression if $x = 5$:

$$3(5) + 2 + 5 = 15 + 2 + 5 = 22$$

You can *simplify* an expression by combining *like terms,* which are parts of the expression that have the same variables. For example,

$$3x + 2 + x = 4x + 2$$

And you can *factor* an expression by separating out a common factor in the terms. For example, in the expression $4x + 2$, both terms ($4x$ and 2) are divisible by 2, so you can factor out a 2:

$$4x + 2 = 2(2x + 1)$$

Evaluating, simplifying, and factoring are important tools that give you the flexibility you need to solve equations using algebra. In turn, solving equations (which I discuss in the next section) is the central skill that makes algebra vital for answering questions on the SAT.

Solving an equation for a variable

The main event in algebra is solving an equation that has one variable (such as x) to discover the value of that variable. The most common way to do this is to *isolate the variable* — that is, get the variable alone on one side of the equal sign and a number on the other side.

Each step along the way, you must keep the equation *balanced* — that is, you have to perform the same operation on both sides of the equation. For example, you solve the following equation by subtracting 7 from both sides of the equation and then dividing both sides by 3:

$$\begin{aligned} 3x + 7 &= 13 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

Solving an equation in terms of other variables

When an equation has more than one variable, finding the value of any variable may be impossible. You can, however, find the value of any variable *in terms of* the other variables in the equation. For example, suppose you want to solve the following equation for b in terms of the variables a, c, and d:

$$a + bc = d$$

To do this, use algebra to isolate b on one side of the equation. Begin by subtracting a from both sides; then divide both sides by c:

$$\begin{aligned} bc &= d - a \\ b &= \frac{d-a}{c} \end{aligned}$$

Solving an equation for an expression

Sometimes, you can solve an equation that has more than one variable to find the numerical value of an expression that contains both variables. For example, look at the following:

$$7p = 3q$$

Suppose you want to solve this equation for the value of p/q. To do this, use algebra to isolate p/q on one side of the equation. Begin by dividing both sides by 7 and then divide both sides by q:

$$p = \frac{3q}{7}$$
$$\frac{p}{q} = \frac{3}{7}$$

Solving a system of equations

A *system of equations* is a set of algebraic equations that are simultaneously true. Because a system of equations contains the same number of equations as variables, you can find the value of both (or all) variables. You first solve for one variable; then you plug that value into one of the original equations and solve for the other variable. For example, suppose you have these equations:

$$x + y = 3$$
$$x - y = 1$$

To begin, first add the two equations. Because the y values cancel each other out, you're left with an equation that you can solve easily:

$$2x = 4$$
$$x = 2$$

Now substitute 2 for x back into either equation and solve for y:

$$2 + y = 3$$
$$y = 1$$

Thus, in the original system of equations, $x = 2$ and $y = 1$. I show you how to apply this skill to SAT questions in Chapter 4.

Solving an inequality

An *inequality* is a math statement that uses a symbol other than an equal sign — most commonly $<$, $>$, $\leq$, or $\geq$. Solving an inequality is similar to solving an equation, with one key difference: When you multiply or divide an inequality by a negative number, you have to reverse the direction of the sign. For example, to solve the inequality $-4x < 12$, isolate x by dividing both sides by -4 *and* changing the $<$ to a $>$:

$$\frac{-4x}{-4} > \frac{12}{-4}$$

Now simplify both sides of the equation:

$$x > -3$$

You get to practice this skill on SAT questions in Chapter 4.

Working with new notations

A common SAT question presents you with the definition of a new mathematical notation and then requires you to use it to solve a problem. For example,

Let $x@y = x^2 - y^2$

Now you can use this definition to evaluate an expression that uses the new notation. For example, here's how you find 5@3 (which tells you that $x = 5$ and $y = 3$):

$5@3 = 5^2 - 3^2 = 25 - 9 = 16$

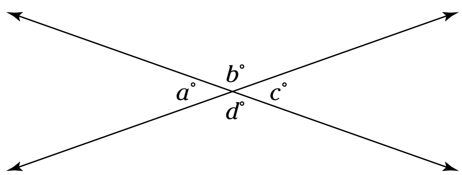
Therefore, $5@3 = 16$.

Go figure: Doing geometry

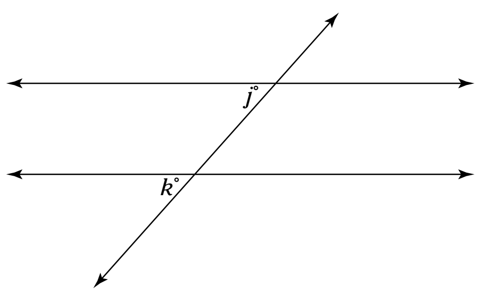
If you've taken a geometry class, you probably spent a lot of time on geometric proofs. Although the SAT doesn't test proofs directly, it does include lots of questions where a strong knowledge of geometric theorems is indispensable. In this section, I outline a few of the main topics that are covered in greater depth in Chapter 5.

Measuring angles

Geometry provides some important theorems for measuring angles. You're virtually guaranteed to see one or more questions on the SAT that require you to know these basic theorems. For instance, when two lines cross each other, any two adjacent angles are *supplementary angles*, which means that they add up to 180°. Furthermore, angles opposite each other are *vertical angles*, which means that they're equal to each other. For example, in the following figure, $a + b = 180°$ and $a = c$.

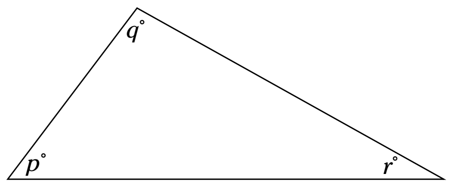


When a line crosses a pair of parallel lines, any two alternate angles on the same side of the line are called *corresponding angles*, which means that they're equal to each other. For example, in this next figure, $j = k$:



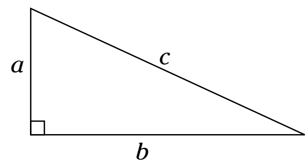
Finding angles and sides of triangles

Geometry includes many theorems about triangles, and some of these are pivotal to answering SAT questions. For instance, the three angles in a triangle always add up to 180°. In the following triangle, $p + q + r = 180°$:

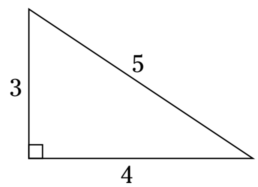


Right triangles also play a big role on the SAT. Every triangle with a right angle (90° angle) is a *right triangle*. The two short sides of a right triangle are called *legs,* and the long side is called the *hypotenuse*. As the following figure shows, the Pythagorean theorem always holds true for a right triangle with legs a and b and a hypotenuse of c:

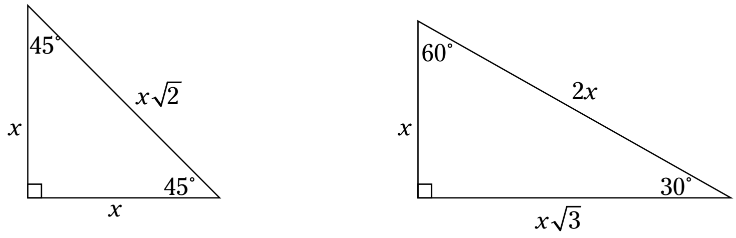
$$a^2 + b^2 = c^2$$



One common right triangle is the 3-4-5 triangle, which has legs of lengths 3 and 4 and a hypotenuse of length 5.



Two other important right triangles are the 45-45-90 triangle and the 30-60-90 triangle, which are named by their angles and have sides in set ratios:



Finding area, perimeter, and volume, and more

Geometry provides a bunch of useful formulas for measuring a variety of shapes and solids. Here are a few important formulas that you need to know how to use to do well on the SAT:

- **Triangle:** Area = $\frac{1}{2}bh$ (b = base, h = height)
- **Square:** Area = s^2 (s = side), perimeter = $4s$
- **Rectangle:** Area = lw (l = length, w = width), perimeter = $2l + 2w$
- **Parallelogram:** Area = bh (b = base, h = height)
- **Circle:** Area = πr^2 (r = radius), circumference = $2\pi r$, diameter = $2r$
- **Rectangular solid (box):** Volume = lwh (l = length, w = width, h = height)
- **Cylinder:** Volume = $\pi r^2 h$ (r = radius, h = height)

Geometric perception

Geometric perception is the ability to imagine a geometric object when it's turned around and viewed from a different perspective. SAT questions typically test geometric perception in a few different ways. In some cases, a two-dimensional shape is rotated on the plane. In others, a solid is turned around in space. And another common question type requires you to imagine folding a two-dimensional shape into a solid. You see how to handle these types of questions in Chapter 5.

Working with functions and coordinate geometry

Functions and coordinate geometry are usually the focus of the second half of Algebra I and a starting point for most of Algebra II, so they play a big role on the SAT. A *function* is an equation linking an input variable (usually x) and an output variable (usually y) so that any value of x produces no more than one value of y. *Coordinate geometry* brings together concepts from algebra and geometry by graphing equations on the xy-plane. In this section, you get an overview of what I cover in Chapter 6.

Modeling with functions on the xy-plane

A function is simply a mathematical connection between two values. For example, if you save \$5 every day, you'll have a total of \$5 on the first day, \$10 on the second day, \$15 on the third day, and so forth. You can place this information into an *input-output table*, with the input x being the day and the output y being the amount saved:

x = day	1	2	3	4	5	…	10	…	100
y = amount	\$5	\$10	\$15	\$20	\$25	…	\$50	…	\$500

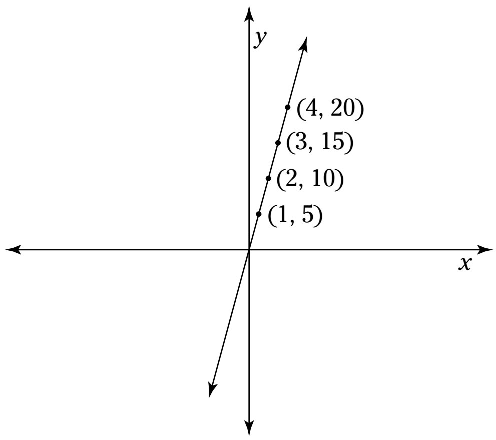
As you can see, for any day you input, the table allows you to output a dollar amount. You can make the mathematical connection between x and y more explicit by representing it as an equation:

$$y = 5x$$

In this equation, y is determined by x — that is, for any value of x, you always know the value of y. Another way of saying this is that "y is a *function of* x." You can write this statement mathematically as follows:

$$y = f(x)$$

Every point in the function corresponds to a *coordinate pair* (x, y) on the xy-plane, connecting a value of x with a value of y. The xy-plane provides a setting to connect two important branches of math — algebra and geometry — allowing you to plot algebraic equations containing x and y. For example, to plot the equation $y = 5x$, plot the points from the table, and then draw a line connecting them:



I discuss functions on the xy-plane in greater detail in Chapter 6.

Looking at common functions: Linear and quadratic functions

The most common functions on the SAT are linear and quadratic functions. The most basic function on the xy-plane is the *linear function*, which produces a straight line. The basic form of the linear function is the *slope-intercept form:*

$$y = mx + b$$

In this function, m represents the slope (steepness) of the line and b represents the *y-intercept* (the point where the line crosses the y-axis). You can find the slope of a line passing through any two points (x_1, y_1) and (x_2, y_2) using the *two-point slope formula*:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

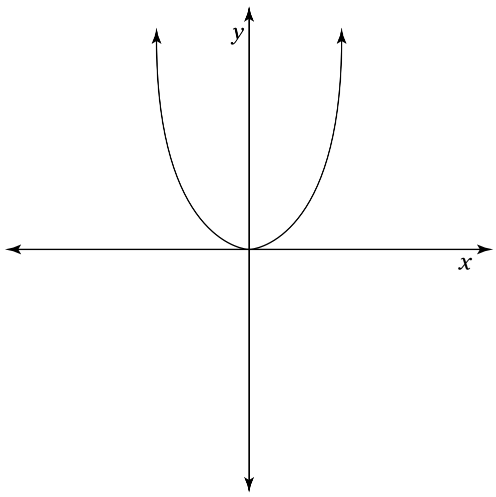
You can also find the equation of a line that has a slope m and includes point (x_1, y_1) using the *point-slope form* for a linear equation:

$$y - y_1 = m(x - x_1)$$

Quadratic functions are also common on the test. A *quadratic* function contains a term whose variable x is squared:

$$f(x) = ax^2 + bx + c$$

For graphing, the $f(x)$ is usually replaced by y, so $y = ax^2 + bx + c$. The graph of a quadratic function is a *parabola* — a bullet-shaped figure as shown here:



Often, the variable y is set to 0, resulting in the basic form of the *quadratic equation*:

$$ax^2 + bx + c = 0$$

You often have to factor this equation and solve for x. I show you how to handle linear and quadratic functions on the SAT in Chapter 6.

Transforming functions

A small change to a function can cause a predictable change in the graph of that function. The result is the *transformation* of that function. Two common transformations are

- **Reflection:** Changing a function to its mirror image along either the x-axis or y-axis
- **Shift:** Displacing a function up, down, left, or right

In Chapter 6, I discuss both of these types of transformations and how to apply them on the SAT.

Rounding up some grab-bag skills

Some SAT math questions are drawn from a variety of math sources that I collect in Chapter 7 under the loose category “grab-bag skills.” In this section, I give you a quick introduction to this variety of problems.

Number sequences

A *sequence* is a list of numbers following a pattern or rule. For example,

1, 4, 7, 10, 13, 16, …

In this sequence, adding 3 to a number gives you the next number in the list. Most SAT questions about number sequences require you to figure out the rule that generates the sequence and then apply it.

Set theory and Venn diagrams

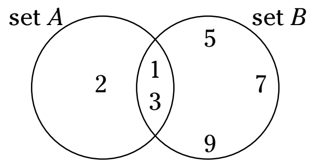
A *set* is a collection of things, typically listed inside a pair of braces. For example,

set $A = \{1, 2, 3\}$

set $B = \{1, 3, 5, 7, 9\}$

The things in a set are called *elements* of the set. For example, set A has three elements: the numbers 1, 2, and 3. The *union* of two sets is the set of every element that appears in either set. For example, the union of set A and set B is $\{1, 2, 3, 5, 7, 9\}$. The *intersection* of two sets is the set of every element that appears in both sets. For example, the intersection of set A and set B is $\{1, 3\}$.

A *Venn diagram* is a visual representation of two or more sets as a group of interlocking circles, as you see here:



In Chapter 7, you discover how to answer SAT questions that focus on set theory and Venn diagrams.

Logic

A *logic question* provides you with a collection of statements and requires you to make logical deductions to answer the question. In some questions, you may need to place a group of people or events in order from first to last. In others, you may be asked to deduce which statement must be true, given a set of facts. Chapter 7 gives you a good look at how to answer logic questions.

Statistics

Statistics is the mathematical analysis of data — that is, making sense of numbers compiled through measuring real-world phenomena. On the SAT, you need to know the formula for the *average* (*arithmetic mean*) of a set of numbers:

$$\text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}}$$

You also need to know how to find the *median* of a set of numbers — that is, the middle number in the set (or the arithmetic mean of the two middle numbers). You may have to identify the *mode* of a set of numbers, which is the most frequently repeated number in the set. SAT questions may also ask you to determine a *weighted average,* which is the mean average of a set of mean values. You get solid on these skills in Chapter 7.

Probability

Probability is the mathematical likelihood that a specified outcome will occur. Probability questions may focus on flipping a coin, rolling dice, or selecting items at random. The formula for the probability is

$$\text{Probability} = \frac{\text{Target outcomes}}{\text{Total outcomes}}$$

In this formula, *target outcomes* means the number of ways in which the outcome you're measuring can happen, and *total outcomes* means the total number of outcomes that can occur. For example, suppose you want to measure the probability of rolling the number 5 on a six-sided die. The number of target outcomes is 1 (rolling a 5), and the total number of outcomes is 6 (rolling 1, 2, 3, 4, 5, or 6). Thus, you can calculate the probability of this outcome as follows:

$$\text{Probability} = \frac{\text{Target outcomes}}{\text{Total outcomes}} = \frac{1}{6}$$

Therefore, the probability of rolling a 5 on a six-sided die is $\frac{1}{6}$. You discover more about calculating probability and its close cousin, geometric probability, in Chapter 7.

Graphs of data

A *graph* provides visual representations of data. The most common type of graph is the *xy*-graph, which I cover in Chapter 6. Additionally, the SAT may include a variety of other types of graphs, including bar graphs, pie charts, line graphs, pictograms, and scatterplots. In Chapter 7, you get practice working with these types of graphs.

Knowing What's Out: A Few Topics Not Covered on the SAT

Almost as important as knowing what math topics are covered on the SAT (which I discuss in the preceding sections) is knowing the topics you can safely avoid. Here, I put your mind at rest with a list of math skills that you don't need to do well on the SAT:

- **No big number crunching:** SAT math questions are designed to be relatively quick to answer if you approach them right. Although you can use a calculator on the SAT, you don't need to worry about big, unwieldy numbers or endless calculations. In fact, if you find that your calculations for a problem are resulting in surprisingly long numbers, take a step back and look again: You may find that you've made a mistake and that the numbers don't turn out to be as awkward as you thought.
- **Nothing to prove (geometrically speaking):** A typical geometry course focuses a vast quantity of time on Euclidean proofs: beginning with five assumptions called *postulates*, showing how more-complex *theorems* follow logically, and then using these theorems to prove even more-complex theorems. On the SAT, you can forget everything you know (or don't know) about doing proofs.

  Even though you don't have to know how to write proofs, you're not completely off the hook. You still need to know some basic theorems — that is, the bottom-line results of proofs, such as the idea that two angles are equal — and how to apply them. You just don't have to *prove* them on the SAT.
- **Avoiding the quadratic quagmire:** At some point in Algebra II, most students commit the quadratic formula to memory. And here it is:

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  Isn't that just a sight to behold? Truly a work of art. Now forget about it — at least for the SAT — because you don't need to know it. You can solve any quadratic equation on the SAT by gentler means, such as factoring (see Chapter 6).

- **Getting real (numbers):** The SAT includes only the set of real numbers — that is, numbers that you can find on the number line. The number line includes positive and negative whole numbers (and of course, zero) and rational numbers (that is, fractions). It also includes irrational numbers like π and $\sqrt{2}$.

  In contrast, imaginary numbers are *not* found on the number line. (In fact, they have their own number line — but that's neither here nor there.) Imaginary numbers are numbers that contain a multiple of $\sqrt{-1}$, which is represented by the symbol i (for *imaginary*). You may have studied imaginary numbers and *complex numbers* — which are the sum of an imaginary number and a real number — in one of your math classes. They're very interesting and useful (or from another perspective, totally boring and useless). But for the purposes of the SAT, you don't have to worry about them.

  The square root of any negative number is imaginary, so if you find that any question leads to the square root of a negative number, there must be a mistake somewhere. Step back and look for where you went wrong in the calculations.

- **No sines of trigonometry:** *Trigonometry* is the study of triangles, specifically right triangles. You can usually spot trig problems because they contain notation not found in other problems: sine (sin), cosine (cos), tangent (tan), and so forth. Although right triangles are important on the SAT (see Chapter 5 for details), you can safely skip the trig.

Furthermore, any math topic introduced in a trigonometry, pre-calculus, or calculus class is excluded from the SAT.

Building Your Problem-Solving Skills

In Part III of this book, the focus is on important *problem-solving skills* — the application of what you know about math to a specific problem. In this section, I give you an overview of what awaits.

Solving word problems

Word problems (also called *story problems*) require you to apply your math skills to a problem expressed in words rather than symbols. To solve a word problem, you usually need to translate the statements in the problem into one or more equations and then solve for a variable.

Students often find word problems tricky, but they're sometimes easier to solve than other types of problems. After you translate the words into an equation, you may find that the equation is relatively simple. In Chapter 8, I show you how to approach a variety of common SAT word problems.

Figuring out which tools to use

In a typical math class, you practice one set of skills before moving on to the next unit. So even a final exam gives you a big advantage you may not be aware of: The test contains relatively few types of problems, so you don't have to spend a lot of time figuring out what you need to remember before answering each question.

On the SAT, however, each question has no relationship to the previous question, so you have to be able to identify the type of math you need to answer the question as quickly as possible. Instead of testing you on a specific math topic, the SAT has the more general goal of testing your ability to *solve problems* — that is, how well you apply the math you know in new ways.

That's why practice focusing specifically on SAT problems is so important. Frequently, SAT questions (especially the tough ones) respond well to a variety of approaches. Depending upon your strengths, the way you like to think, and the stuff you remember from your classes, you and a friend could arrive at the right answer in two completely different ways. You need to practice finding your own smart ways to cut through to the heart of a problem and arrive at the solution.

TIP

Just as in sports, the most important thing for the SAT is to play game after game after game to find your own unique rhythm. What works best for you? Does drawing a picture help you see better or just confuse you? Should you try to solve an equation or instead try to plug in numbers until one works? Do you do your best work when you spend a few moments thinking about a question until you know which direction to go? Or do you tend to get the right answer when you dive in and start calculating, knowing that the numbers will take you where you need to go? The only way to find out what kind of SAT player you are and improve your performance is to get in the game and play.

In Part IV, I give you three practice SAT math tests. Each test contains three sections with a total of 54 questions — 44 multiple-choice and 10 grid-in — just like the real SAT. Doing practice tests with the clock running is the best way to hammer down the individual math skills you practice in Parts II and III.