1

INTRODUCTION

1.1 DEFINITION OF THERMAL STRESS

Thermal stresses are stresses that result when a temperature change of the material occurs in the presence of constraints. Thermal stresses are actually mechanical stresses resulting from forces caused by a part attempting to expand or contract when it is constrained.

Without constraints, there would be no thermal stresses. For example, consider the bar shown in Figure 1-1. If the bar were subjected to a temperature change ΔT of 20°C and the ends were free to move, the stress in the bar would be zero. On the other hand, if the same bar were subjected to the same temperature change and the ends were rigidly fixed (no displacement at the ends of the bar), stresses would be developed in the bar as a result of the forces (tensile or compressive) on the ends of the bar. These stresses are called thermal stresses.

There are two types of constraints as far as thermal stresses are concerned: (a) external constraints and (b) internal constraints. *External constraints* are restraints on the entire system that prevent expansion or contraction of the system when temperature changes occur. For example, if a length of pipe were fixed at two places by pipe support brackets, this constraint would be an external one.

Internal constraints are restraints present within the material because the material expands or contracts by different amounts in various locations, yet the material must remain continuous. Suppose the pipe in the previous example were simply supported on hangers, and the inner portion of the pipe were suddenly heated 10°C warmer than the outer surface by the introduction of a hot liquid into the pipe, as shown in Figure 1-2. If the outer surface remains at the initial

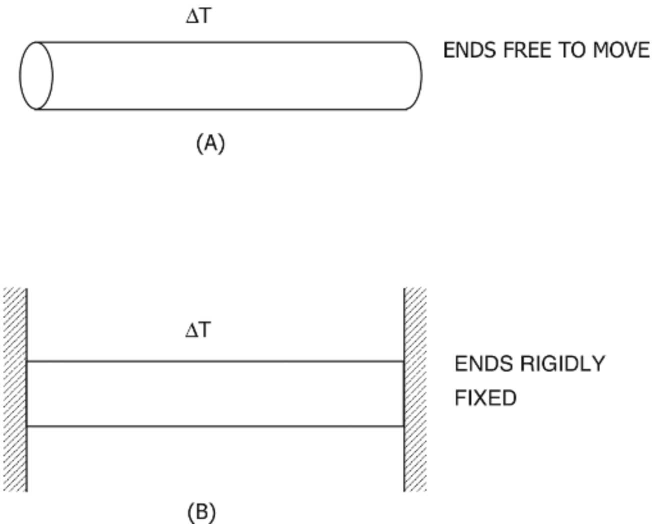

Figure 1-1. Illustration of external constraints. (A) No constraint—the bar is free to expand or contract. Thermal stresses are not present. (B) External constraint—the bar has both ends rigidly fixed and no motion is possible. Thermal stresses are induced when the bar experiences a change in temperature.

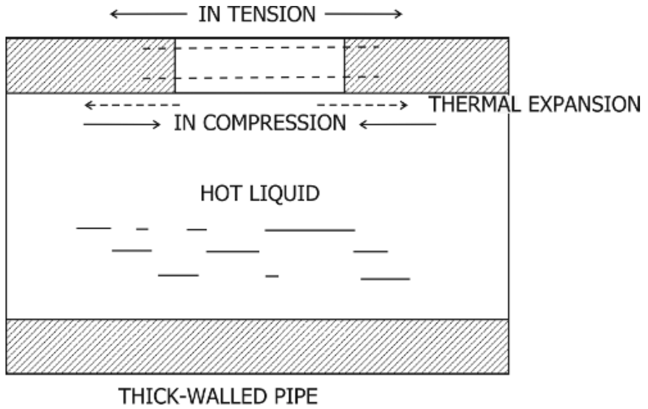

Figure 1-2. Internal constraints. The inner surface is heated by the fluid and tends to expand, but the outer (cool) surface constrains the free motion. Thermal stresses are induced by this constraint.

temperature, the outer layers would not expand, because the outer temperature did not change, whereas the inner layers would tend to expand due to a temperature change. Thermal stresses will arise in this case because the inner layer of material and outer layer of material are not free to move independently. This type of constraint is an internal one.

1.2 THERMAL–MECHANICAL DESIGN

The design process involves more than "solving the problem" in a mathematical manner [Shigley and Mischke, 1989]. Ideally, there would be no design limitations other than safety. However, usually multiple factors must be considered when designing a product. A general design flowchart is shown in Figure 1-3.

Initially, there is usually a perceived need for a product, process, or system. The specifications for the item required to meet this need must be defined. Often this specification process is called *preliminary design*. The input and output quantities, operating environment, and reliability and economic considerations must be determined. For example, anticipated forces that would be applied to the system must be specified.

After the design problem has been defined, the next step involves an interaction between synthesis, analysis, and optimization. Generally, there are many possible design solutions for a given set of specifications. (Not everyone drives the same model of car, for example, although all car models provide a solution to the problem of transportation from one place to another.) Various components for a

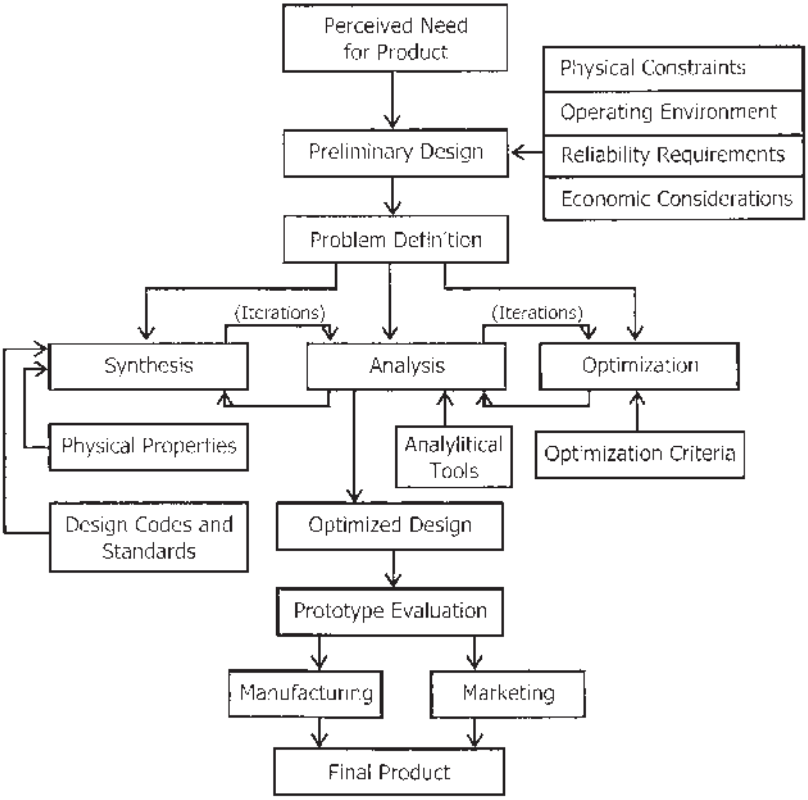

Figure 1-3. General design flowchart.

system may be proposed or synthesized. An abstract or mathematical model is developed for the analysis of the system. The results of the analysis may be used to synthesize an improved approach to the design solution. Based on specific criteria defining what is meant by the “best” system, the optimum or best system is selected to meet the design criteria.

In many cases, the optimal design emerging from the synthesis/analysis design phase is evaluated or tested. A prototype may be constructed and subjected to conditions given in the initial specifications for the system. After the evaluation phase has been completed successfully, the design then moves into the manufacturing and marketing arena.

When including consideration of thermal stresses in the design process, there are many cases in which the stresses are weakly dependent or even independent of the dimensions of the part. In these cases, the designer has at least three alternatives to consider: (a) materials selection, (b) limitation of temperature changes, and (c) relaxation of constraints.

For identical loading and environmental conditions, different materials will experience different thermal stresses. For example, a bar of 304 stainless steel, rigidly fixed at both ends, will experience a thermal stress that is about eight times that for Invar under the same conditions. Many factors in addition to thermal stresses dictate the final choice of materials in most design situations. Cost, ease of fabrication, and corrosion resistance are some of these factors. The designer may not have complete freedom to select a material based on thermal stress considerations alone.

A reduction of the temperature change will generally reduce thermal stresses. For a bar with rigidly fixed ends, if the temperature change is 50°C instead of 100°C, the thermal stress will be reduced to one-half of the thermal stress value for the larger temperature difference. In some steady-state thermal conditions, the temperature change of the part may be reduced by using thermal insulation. The design temperature change is often determined by factors that cannot be changed by the designer, however.

In many cases, the most effect approach to limit thermal stresses in the design stage is to reduce or relax the constraints on the system. The system may be made less constrained by introducing more flexible elements. This approach will be illustrated in the following chapters.

1.3 FACTOR OF SAFETY IN DESIGN

In general, a part is designed such that it does not fail, except under desired conditions. For example, fuses must fail when a specified electric current is applied so that the electrical system may be protected. On the other hand, the wall thickness for a transfer line carrying liquid oxygen is selected such that the pipe does not rupture during operation of the system.

One issue in the design process is the level at which the part would tend to fail. This issue is addressed in the *factor of safety* f_s. It is defined as the ratio of the failure parameter of the part to the design value of the same parameter. The first decision that the designer must make is to define what constitutes “failure” for the component or system under consideration. There are several failure criteria, including

(a) breaking (rupture) of the part
(b) excessive permanent deformation (yielding) of the part
(c) breaking after fluctuating loads have been applied for a period of time (fatigue)
(d) buckling (elastic instability)
(e) excessive displacement or vibration
(f) intolerable wear of the part
(g) excessive noise generation by the part

The selection of the proper failure criteria is often the key to evaluating and planning for safety considerations.

If the failure criterion is the breaking or rupture of the part when stress is applied and the temperature is not high enough for creep effects to be significant, the failure parameter would be the *ultimate strength* S_u for the material. On the other hand, if the failure criterion is yielding, then the *yield strength* S_y would be the failure parameter selected. In either case, the design parameter would be the maximum applied stress σ for the part. The factor of safety may be written as follows, for these cases:

$$f_s = \frac{S_u}{\sigma} \quad \text{or} \quad f_s = \frac{S_y}{\sigma} \tag{1-1}$$

The factor of safety may be defined in a similar manner for the other failure criteria.

The factor of safety may be prescribed, as is the case for such codes as the *ASME Code for Unfired Pressure Vessels, Section VIII, Division 1*, in which the factor of safety for design of cylindrical pressure vessels is set at 3.5. When the factor of safety is not prescribed, the designer must select it during the early stages of the design process. It is generally not economical to use a factor of safety that assures that absolutely no failure will occur under the worst possible combination of conditions. As a result, the selection of the factor is often based on the experience of the designer in related design situations.

In general, the value of the factor of safety reflects uncertainties in many factors involved in the design. Some of these uncertainties are as follows:

(a) Scatter (uncertainty) in the material property data
(b) Uncertainty in the maximum applied loading
(c) Validity of simplifications (assumptions) in the model used to estimate the stresses or displacements for the system
(d) The type of environment (corrosive, etc.) to which the part will be exposed
(e) The extent to which initial stresses or deformations may be introduced during fabrication and assembly of the system

One of the more important factors in selection of the factor of safety is the extent to which human life and limb would be endangered if a failure of the system did occur or the possibility that failure would result in costly or unfavorable litigation.

The probabilistic or reliability-based design method [Shigley and Mischke, 1989] attempts to reduce the uncertainty in the design process; however, the disadvantage of this method lies in the fact that there is uncertainty in the "uncertainty" (probabilistic) data and the data is not extensive.

The uncertainty in the value of the strength parameter (ultimate or yield strength) may be alleviated somewhat by understanding the causes of the scatter in the data for the strength parameter. The values of the ultimate and yields strengths reported in the literature are generally average or mean values. In this case, 50 percent of the data lies above the mean value and 50 percent of the data lies below the reported value. A 1-in-2 chance would be excellent odds for a horse race, but this is not what one would likely employ in the design of a mechanical part. The value for the strength for which the probability of encountering a strength less than this value may be found from the normal probability distribution tables, if the standard deviation $\hat{\sigma}_S$ is known from the strength data. The ultimate strength for this case is given by the following expression:

$$S_u = k_S \overline{S}_u \tag{1-2}$$

The quantity $\overline{S}_u$ is the average ultimate strength, and the factor k_S is defined by

$$k_S = 1 - F_p(\hat{\sigma}_S/\overline{S}_u) \tag{1-3}$$

Values for the probability factor F_p are given in Table 1-1. Similar expressions may be used for the yield strength and fatigue strength.

Information on the standard deviation for the strength data is not readily available for all materials. If no specific standard deviation data are available, the following approximation may be used for the ratio $(\hat{\sigma}_S/\overline{S})$: 0.05 for ultimate

TABLE 1-1. Probability Factor F_p for Various Probabilities of Survival

Survival Rate[a]	Failure Rate[b]	Probability Factor[c], F_p
0.900	0.100	1.282
0.950	0.050	1.645
0.975	0.025	1.960
0.990	0.010	2.33
0.999	0.001	3.09
0.9999	0.0001	3.72

[a] The *survival rate* is the probability that the actual strength value is not less than the S value given by eq. (1-2).

[b] The *failure rate* is (1 − survival rate) or the probability that the actual strength is less than the S value given by eq. (1-2).

[c] F_p is used in eq. (1-3).

strength; 0.075 for yield strength; and 0.10 for fatigue strength or endurance limit. The designer has the task of deciding what risk is acceptable for the minimum strength used in the design.

The reliability of the maximum anticipated loading (either mechanical or thermal) used in the design affects the value of the factor of safety selected. If there are safeguards (pressure relief valves, for example) on the system to prevent the loading from exceeding a selected level, then the factor of safety may be smaller than for the case in which the loading is more uncertain.

The validity of the mathematical model (set of assumptions or simplifications) used in the design has a definite influence on the factor of safety selected. It may be noted that a very complicated numerical analysis (or, as is commonly stated, a "sophisticated" analysis) is not precisely accurate, despite the opinions of some overly enthusiastic novice computer analysist. The estimated uncertainty in the analysis may be used as a guide in selecting the factor of safety.

Example 1-1 304 stainless steel is to be used in a design. A factor of safety of 2.5 is selected, based on yielding as the failure criterion. It is desired that the uncertainty (failure rate) for the yield strength be 0.1%, and the standard deviation for the yield strength data is 7.5 percent of the mean yield strength. Determine the stress to be used in the design.

The average yield strength for 304 stainless steel is found in Appendix B:

$$\overline{S}_y = 232\,\text{MPa} \qquad (33{,}600\,\text{psi})$$

For a 0.1 percent failure rate or 99.9 percent survival rate, the probability factor from Table 1-1 is $F_p = 3.09$. The factor k_S may be found from eq. (1-3):

$$k_S = 1 - (0.075)(3.09) = 0.7683$$

The yield strength value that will be exceeded 99.9 percent of the time for 304 stainless steel is found from eq. (1-2):

$$S_y = k_S\overline{S}_y = (0.07683)(232) = 178.2\,\text{MPa} \qquad (25{,}850\,\text{psi})$$

The design stress is found from the definition of the factor of safety:

$$\sigma\,(\text{design}) = \frac{S_y}{f_s} = \frac{178.2}{2.5} = 71.3\,\text{MPa} \qquad (10{,}340\,\text{psi})$$

1.4 THERMAL EXPANSION COEFFICIENT

One of the important material properties related to thermal stresses is the *thermal expansion coefficient*. There are generally two thermal expansion coefficients that we will consider: (a) the *linear* thermal expansion coefficient, α, and (b) the *volumetric* thermal expansion coefficient, β_t.

The linear thermal expansion coefficient is defined as the fractional change in length (or any other linear dimension) per unit change in temperature while the stress on the material is kept constant. The following is the mathematical definition of the linear thermal expansion coefficient:

$$\alpha = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_{\sigma} \tag{1-4}$$

Usually, the linear thermal expansion coefficient is measured under conditions of zero applied stress σ. Values for the linear thermal expansion coefficient for several engineering materials are given in Appendix B. Values for the linear thermal expansion coefficient as a function of temperature for several metals are presented in Appendix C.

The volumetric thermal expansion coefficient is defined as the fractional change in volume per unit change in temperature while the pressure (all-around stress) is held constant. The following is the mathematical definition of the volumetric thermal expansion coefficient:

$$\beta_t = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p} \tag{1-5}$$

For an isotropic material (properties the same in all directions), the two thermal expansion coefficients are related by the following simple relation:

$$\beta_t = 3\alpha \tag{1-6}$$

The variation of the thermal expansion coefficient with temperature may be understood by considering the intermolecular forces of the material [Kittel, 1966]. The intermolecular potential energy curve for a pair of atoms, as shown in Figure 1-4, is not symmetrical. As the atom acquires more energy (or as the temperature is increased), the mean spacing of the two atoms becomes larger, i.e., the material expands.

If the potential energy curve were symmetric, for example, if $U = \frac{1}{2}K(r - r_0)^2$, then the positions of the two atoms at the extreme positions r_1 and r_2 for a given energy E are

$$r_1 = r_0 - \sqrt{2U/K} \qquad \text{and} \qquad r_2 = r_0 + \sqrt{2U/K} \tag{1-7}$$

The quantity r_0 is the equilibrium spacing at $T = 0$. The average spacing of the two atoms for a symmetrical energy curve is

$$r_{\text{ave}} = \tfrac{1}{2}(r_1 + r_2) = r_0 = \text{constant} \tag{1-8}$$

Because the equilibrium spacing remains constant, independent of the energy level, for a pair of atoms with a symmetrical energy curve, there would be no thermal expansion for this material, because, although the atoms would move farther apart as the temperature is increased, their average spacing would remain unchanged.

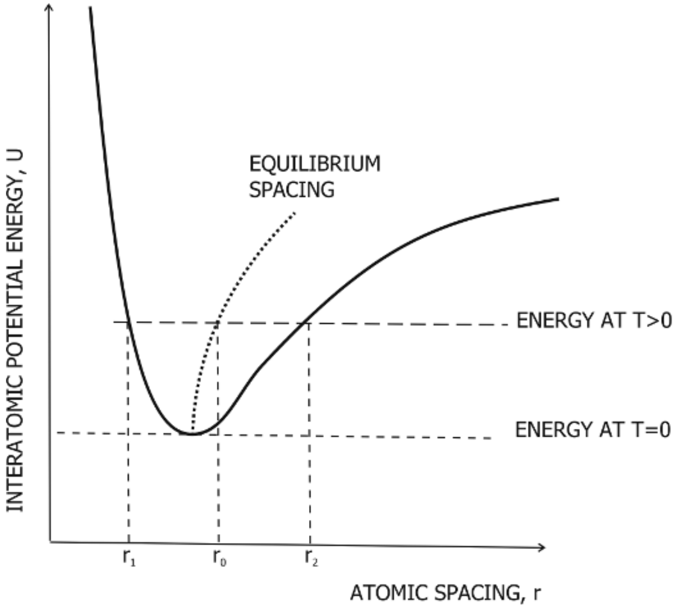

Figure 1-4. Interatomic potential energy curve for the potential energy between two atoms.

The actual potential energy curve is asymmetrical about the equilibrium spacing at absolute zero; therefore, the equilibrium spacing of the atoms increases as the temperature of the material is increased. The rate at which the mean spacing of the atoms changes increases as the energy or temperature is increased. This results in an increase of the thermal expansion coefficient as the temperature is increased. The thermal expansion coefficient approaches a value of zero as the material temperature approaches absolute zero, as required by the third law of thermodynamics [McClintock et al., 1984].

For crystalline solids, the specific heat of the material is dependent on the vibrational energy of the atoms. Since the thermal expansion coefficient is also associated with interatomic vibrational energy, one might except to find a relationship between these two properties. This interdependence is given by the Grüneisen relationship [Yates, 1972]:

$$\beta_t = \frac{\gamma_G c_v \rho}{B} \tag{1-9}$$

or

$$\alpha = \frac{\gamma_G c_v \rho (1 - 2\mu)}{E} \tag{1-10}$$

The quantity c_v is the specific heat at constant volume, ρ is the material density, and B is the isothermal bulk modulus, discussed in Section 1.6. The quantity E is Young's modulus, which is directly related to the bulk modulus by eq. (1-20).

TABLE 1-2. Values of the Grüneisen Constant for Selected Materials at Ambient Temperature

Material	Lattice Structure	Grüneisen Constant, γ_G
Aluminum, Al	FCC	2.17
Copper, Cu	FCC	1.96
Gold, Au	FCC	2.40
Lead, Pb	FCC	2.73
Nickel, Ni	FCC	1.88
Palladium, Pa	FCC	2.23
Platinum, Pt	FCC	2.54
Silver, Ag	FCC	2.40
Iron, Fe	BCC	1.60
Molybdenum, Mo	BCC	1.57
Tantalum, Ta	BCC	1.75
Tungsten, W	BCC	1.62
Cobalt, Co	HCP	1.87
Zinc, Zn	HCP	2.01
Bismuth, Bi	Rhombic	1.14
Tin, Sn	BC tetra	2.14

Note. FCC, face-centered-cubic; BCC, body-centered-cubic; HCP, hexagonal close-packed.

The parameter γ_G is the *Grüneisen constant* [Grüneisen, 1926]. Some typical values for the Grüneisen constant are given in Table 1-2.

The bulk modulus and density for a metal are not strongly dependent on temperature. If the Grüneisen constant were truly independent of temperature (it does actually depend on temperature in certain temperature ranges), then eq. (1-9) indicates that the thermal expansion coefficient would vary in the same manner with temperature as the specific heat does. The temperature variation of some metals is given in Appendix C. For a pure crystalline solid at low temperatures, the thermal expansion coefficient is proportional to T^3. At temperatures around ambient temperature and above ambient temperature, the thermal expansion coefficient is practically proportional to temperature, and is much less dependent on temperature than is the case at very low temperatures.

There are some cases, particular at cryogenic temperatures, that the thermal expansion coefficient cannot be treated as a constant, within acceptable accuracy. The cryogenic temperature range is defined [Scott, 1959] as temperatures less than 123 K or -150°C (-238°F). In this temperature region, we may use the *thermal strain parameter* e_t, defined by the following expression:

$$e_t(T) = \int_0^T \alpha \, dT \tag{1-11}$$

The average thermal expansion coefficient between two temperature limits, T_1 and T_2, is given by

$$\overline{\alpha} = \frac{e_t(T_2) - e_t(T_1)}{T_2 - T_1} \tag{1-12}$$

The thermal strain parameter e_t is tabulated in Appendix C for some metals. The parameter may be found for other materials by (a) fitting the thermal expansion coefficient to an analytical expression, using the least-squares curve-fitting technique, and then carrying out the integration analytically, or (b) carrying out the integration of the tabular or experimental thermal expansion coefficient data numerically.

Example 1-2 The density of silver at 300 K (80°F) is 10,500 kg/m^3 (0.379 lb$_\mathrm{m}$/in^3), and the bulk modulus for silver is 92.82 GPa (13.46×10^6 psi). The vibrational energy contribution to the specific heat (Debye specific heat) is 0.216 kJ/kg-K (0.0517 Btu/lb$_\mathrm{m}$-°F) [Gopal, 1966]. It may be noted that the total specific heat for silver is 0.236 kJ/kg-K (0.0564 Btu/lb$_\mathrm{m}$-°F). Determine the linear thermal expansion coefficient from the Grüneisen relationship.

The value of the Grüneisen constant is found from Table 1-2 for silver:

$$\gamma_G = 2.40$$

The volumetric thermal expansion coefficient is found from eq. (1-9):

$$\beta_t = \frac{(2.40)(0.216 \times 10^3)(10{,}500)}{(92.82 \times 10^9)} = 58.6 \times 10^{-6}\ \mathrm{K}^{-1}$$

The linear thermal expansion coefficient is found from eq. (1-6):

$$\alpha = (58.6 \times 10^{-6})/(3) = 19.5 \times 10^{-6}\ \mathrm{K}^{-1} \qquad (10.8 \times 10^{-6}\ {}^\circ\mathrm{F}^{-1})$$

The measured value for the linear thermal expansion coefficient is in excellent agreement with this calculated value [Corruccini and Gniewek, 1961]:

$$\alpha(\text{measured}) = 19.3 \times 10^{-6}\mathrm{K}^{-1}$$

1.5 YOUNG'S MODULUS

Young's modulus gives a measure of the flexibility of a material, so this is another material property of importance in determining thermal stresses. Young's modulus is usually measured under isothermal (constant temperature) conditions. The mathematical definition of Young's modulus (specifically, the *tangent* modulus) is

$$E = \left(\frac{\partial \sigma}{\partial \varepsilon}\right)_T \tag{1-13}$$

The stress level σ is below the proportional limit for the material. The quantity ε is the mechanical strain caused by the stress σ. Values for Young's modulus for several materials are given in Appendix B.

Young's modulus is related primarily to the forces between atoms in a material. A typical interatomic force curve is shown in Figure 1-5. The value of

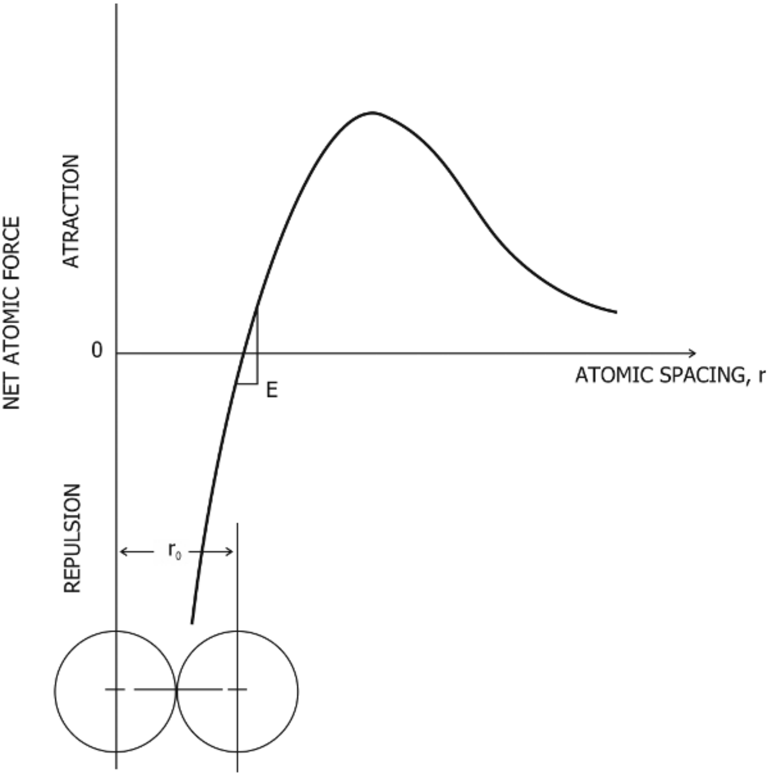

Figure 1-5. Interatomic force curve for the force between two atoms. Young's modulus is related to the slope of this curve.

Young's modulus is determined from the slope of the interatomic force curve at the equilibrium spacing r_0 of the atoms. One theoretical relationship for Young's modulus is as follows [Ruoff, 1973]:

$$E = \frac{9Z^2e^2}{16\pi\varepsilon_0 r_0^4} \tag{1-14}$$

The quantity Z is the valence of the atomic ion, e is the electron charge ($e = 0.1601 \times 10^{-18}$ C), ε_0 is the permittivity of free space ($\varepsilon_0 = 8.8542 \times 10^{-12}$ F/m), and r_0 is the equilibrium spacing of the atoms.

Example 1-3 The equilibrium spacing of the silver atoms in the metal is 0.288 nm, and the valence of silver is +1. Estimate the value of Young's modulus for silver.

Using these values in eq. (1-14), we find the following value of Young's modulus:

$$E = \frac{(9)(1)^2(0.1601 \times 10^{-18})^2}{(16\pi)(8.8542 \times 10^{-12})(0.288 \times 10^{-9})^4} = 75.3 \times 10^9 \text{ Pa} = 75.3 \text{ GPa}$$

The experimental value of Young's modulus for silver is 72.4 GPa (10.6×10^6 psi) [Bolz and Tuve, 1970].

1.6 POISSON'S RATIO

When the atoms of a material are pulled apart by a force applied in a certain direction, there is a corresponding contraction of the material in the lateral direction, perpendicular to the applied force. *Poisson's ratio* μ is the magnitude of the ratio of the lateral strain to the strain in the direction of the applied force. For a force applied in the x-direction, Poisson's ratio may be written as follows:

$$\mu = \frac{-\varepsilon_y}{\varepsilon_x} \tag{1-15}$$

The quantity ε_y is the mechanical strain in the y-direction when a force is applied in the x-direction, and ε_x is the mechanical strain in the x-direction (the direction of the applied force). The negative sign is introduced because the strain in the transverse direction will be a contraction (negative strain) if the force causes an elongation (positive strain) in the x-direction. Numerical values of Poisson's ratio for several materials are given in Appendix B.

The effect of application of a tensile force on the volume of a material may be examined. Suppose we have a bar with a length L, and cross-sectional dimensions $a \times b$. The initial volume V_0, with the bar unloaded, is

$$V_0 = Lab$$

The final volume V_1, after a tensile load has been applied in the lengthwise direction, is related to the original dimensions and Poisson's ratio:

$$V_1 = (L + \varepsilon_x L)(a + \varepsilon_y a)(b + \varepsilon_z b)$$

If we introduce Poisson's ratio from eq. (1-15) and expand the expression for the final volume, we obtain

$$V_1 = Lab(1 + \varepsilon_x)(1 - \mu\varepsilon_x)(1 - \mu\varepsilon_x) = V_0(1 + \varepsilon_x)(1 - 2\mu\varepsilon_x + \mu^2\varepsilon_x^2)$$

The fractional change in volume may be found as follows, where we have omitted the terms involving ε_x^2 and ε_x^3, because these values are negligible for small strains:

$$\frac{\Delta V}{V} = \frac{V_1 - V_0}{V_0} = \varepsilon_x(1 - 2\mu) \tag{1-16}$$

For a homogeneous isotropic material, the value of Poisson's ratio is between zero and $\frac{1}{2}$. If Poisson's ratio were greater than $\frac{1}{2}$, a pressure applied to the material would cause the volume of the material to increase, and this behavior is not observed in engineering materials. For a material with Poisson's ratio $\mu = \frac{1}{2}$, the volume does not change as a tensile or compressive force is applied.

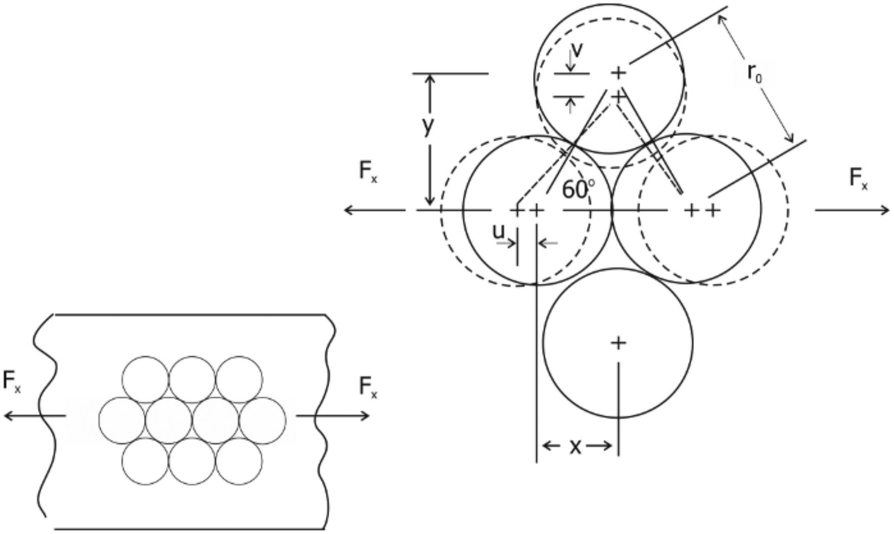

Figure 1-6. Poisson's ratio for a material with a face-centered-cubic or hexagonal close-packed lattice structure.

Poisson's ratio is a property that depends primarily on the geometry or arrangement of the atoms in the material. Because of this characteristic, Poisson's ratio is practically independent of temperature. It may be shown that Poisson's ratio for a metal having a face-centered-cubic (FCC) or hexagonal close-packed (HCP) lattice arrangement should be $\mu = \frac{1}{3}$. From Figure 1-6, we observe the following:

$$x^2 + y^2 = r_0^2 = \text{constant}$$

For small displacements u and v, the displacements may be found as follows:

$$2xu + 2yv \approx 2x\,dx + 2y\,dy = 0$$

or

$$\frac{v}{u} = \frac{y\varepsilon_y}{x\varepsilon_x} = \frac{-x}{y}$$

Poisson's ratio is defined by eq. (1-15):

$$\mu = \frac{-\varepsilon_y}{\varepsilon_x} = \left(\frac{x}{y}\right)^2 = \tan^2(30^\circ) = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

1.7 OTHER ELASTIC MODULI

In addition to Young's modulus and Poisson's ratio, several other elastic moduli have been defined. For an isotropic material, only two of the elastic moduli are independent. In this text, we will usually choose Young's modulus and Poisson's ratio as the independent properties.

The *modulus of elasticity in shear* G is defined as the ratio of the shearing stress τ to the shear strain γ for a material in the elastic region (stresses less than the proportional limit). This property is also called the *shear modulus* and the *modulus of rigidity*:

$$G = \frac{\tau}{\gamma} \tag{1-17}$$

The shear modulus is related to Young's modulus and Poisson's ratio for an isotropic material by the following relationship:

$$G = \frac{E}{2(1+\mu)} \tag{1-18}$$

For a material with Poisson's ratio $\mu = \frac{1}{3}$, the shear modulus is $G = \frac{3}{8}E$.

The *isothermal bulk modulus* B is defined as the change in pressure per unit volumetric strain (change in volume per unit volume) of a material under constant-temperature conditions. The bulk modulus has also been called the *volume modulus of elasticity*:

$$B = V\left(\frac{\partial p}{\partial V}\right)_T \tag{1-19}$$

The bulk modulus is related to Young's modulus and Poisson's ratio for an isotropic material by the following relationship:

$$B = \frac{E}{3(1-2\mu)} \tag{1-20}$$

For a material with Poisson's ratio $\mu = \frac{1}{3}$, the bulk modulus is $B = E$. Note that the bulk modulus is infinite for a material having a Poisson's ratio $\mu = \frac{1}{2}$. As mentioned in Section 1.6, materials having a Poisson's ratio of $\frac{1}{2}$ experience no volume change (zero volumetric strain) when a pressure is applied.

To obtain a relationship between B, G, and E, let us combine eqs. (1-18) and (1-20) as follows:

$$\frac{1}{B} + \frac{3}{G} = \frac{3(1-2\mu)}{E} + \frac{(3)(2)(1+\mu)}{E} = \frac{9}{E} \tag{1-21}$$

The *Lamé elastic constant* λ_L is defined by the following relationship:

$$\lambda_L = \frac{\mu E}{(1+\mu)(1-2\mu)} = \frac{3\mu B}{1+\mu} \tag{1-22}$$

For a material with Poisson's ratio $\mu = \frac{1}{3}$, the Lamé constant is $\lambda_L = \frac{3}{4}E$. Using eq. (1-18), we may write the following relationship for the Lamé constant in terms of the shear modulus and the bulk modulus for any value of Poisson's ratio:

$$\lambda_L + \tfrac{2}{3}G = B \tag{1-23}$$

Values for the elastic moduli may be found from the data in Appendix B and the relationships given in this section.

Example 1-4 Determine the elastic moduli for 304 stainless steel at 300 K (80°F). Young's modulus and Poisson's ratio are found from Appendix B for 304 stainless steel: $E = 193$ MPa (28.0×10^6 psi) and $\mu = 0.305$.

The shear modulus is found from eq. (1-18):

$$G = \frac{(193)}{(2)(1+0.305)} = 73.9 \text{ GPa} \qquad (10.7 \times 10^6 \text{ psi})$$

The bulk modulus is found from eq. (1-20):

$$B = \frac{(193)}{(3)[1-(2)(0.305)]} = 165 \text{ GPa} \qquad (23.9 \times 10^6 \text{ psi})$$

Finally, the Lamé constant is found from eq. (1-22):

$$\lambda_L = \frac{(0.305)(193)}{(1+0.305)[1-(2)(0.305)]} = 115.7 \text{ GPa} \qquad (16.8 \times 10^6 \text{ psi})$$

1.8 THERMAL DIFFUSIVITY

In many situations involving thermal stresses, transient or time-dependent temperature distributions are involved. In these cases, the temperature distribution and the thermal stress distribution are dependent on a material property called the *thermal diffusivity* κ. The thermal diffusivity is defined in terms of the material thermal conductivity k_t, density ρ, and specific heat c:

$$\kappa \equiv \frac{k_t}{\rho c} \tag{1-24}$$

The units for the thermal diffusivity in the SI system are {m^2/s}, and typical units in the conventional system are {ft^2/hr}.

The value of the thermal diffusivity gives a measure of how rapidly energy may be conducted into a solid material. A large value of thermal diffusivity means that energy may diffuse rapidly into the material, and steep temperature gradients (large temperature changes over small distances) will not be developed. This behavior tends to result in lower thermal stresses in the transient situation than the case of a material with a small thermal diffusivity.

One relationship for the thermal diffusivity of a solid material is as follows [Berman, 1976]:

$$\kappa = \tfrac{1}{3}\overline{v}\lambda_m \tag{1-25}$$

The quantity $\overline{v}$ is the velocity of the "energy carriers" (electrons, lattice vibrational waves or phonons, etc.), and λ_m is the average distance traveled by the carriers between collisions, or the *mean free path* for the energy carriers. For metals at ambient temperature and higher, the thermal diffusivity is relatively constant with temperature change. At very low temperatures, the thermal diffusivity of metals is strongly dependent on temperature and varies as T^{-3} to T^{-4}. The temperature dependence of the thermal diffusivity of some selected materials is displayed in Appendix C.

1.9 THERMAL SHOCK PARAMETERS

Thermal shock occurs when a material is subjected to rapidly changing temperatures in the environment around the material. Some examples of thermal shock situations include space vehicle reentry into the atmosphere, start-up of a cold automobile engine, and quenching of a metal part. Under identical environmental conditions, some materials are more resistant to thermal shock than others. Brittle materials exhibit small mechanical strains before rupture, so thermal shock can be a serious problem for such materials. Ductile materials can withstand larger mechanical strains before rupture; however, thermal shock may cause yielding for ductile materials. In addition, repeated thermal shock can result in a thermal fatigue failure for ductile materials.

The *strength–weight ratio* S_y/ρ is an important parameter in selection of materials to withstand a specified tensile load for minimum weight of the part. Similarly, a thermal shock parameter would be a convenient material property to assist the designer in selection of materials that would resist thermal shock for a given temperature change. Schott and Winkelmann suggested one of the original thermal shock parameters in 1894 [Richards, 1961]:

$$\mathrm{TSP} = \frac{S_u\sqrt{\kappa}}{\alpha E} \tag{1-26}$$

The quantity S_u is the ultimate tensile strength of the material, and κ is the thermal diffusivity for the material.

A material with a high value of ultimate tensile strength would be able to withstand a higher stress level than a material with a low ultimate tensile strength. A material with a low thermal expansion coefficient α would develop smaller thermal strains (and correspondingly lower thermal stresses) than a material that expands by a large amount when the material temperature is changed. A material with a small Young's modulus E would be more flexible and able to accommodate thermal strains better than a material with a large Young's modulus. Finally, a material with a large thermal diffusivity κ would tend to develop smaller temperature gradients than a material with small κ, because thermal energy can be spread out throughout the high-κ material more rapidly.

In summary, a material that would have good thermal shock resistance should have a large ultimate tensile strength S_u, a small thermal expansion coefficient α, a small Young's modulus E, and a large thermal diffusivity κ. These characteristics are brought together in the thermal shock parameter TSP. A material having a

TABLE 1-3. Values of the Thermal Shock Parameter TSP and Thermal Stress Ratio TSR for Several Materials at 300 K (27°C or 80°F)

Material	αE, MPa/K	TSR $= S_u/\alpha E$, K	TSP $= S_u\kappa^{1/2}/\alpha E$, K-m/s$^{1/2}$
Aluminum, 2024-T3	1.652	280	1.94
Aluminum, 3003-H12	1.553	83.7	0.68
Aluminum, 6061-T6	1.615	192	1.57
Beryllium copper	2.208	521	2.59
Brass, 70/30	1.210	291	1.70
Bronze, UNS-22000	1.833	202	0.60
Copper/10% Ni	2.009	179	0.67
Inconel, 600	2.795	229	0.68
Invar	0.387	1660	2.81
Monel, K-500	2.516	380	0.84
Gray cast iron, Class 20	0.880	160	0.65
Gray cast iron, Class 40	1.339	218	0.88
Steel, C1020, annealed	2.440	180	0.77
Steel, 4340	2.397	422	1.19
Steel, 9% Ni	2.230	384	1.03
Stainless steel, 304	3.088	167	0.34
Stainless steel, 416	1.980	258	0.69
Titanium, Ti–5Al–2.5Sn	1.066	819	1.57
Concrete, 1:2½:3¼	0.224	7.7[a]	0.006[a]
Glass, silicate 7740	0.282	35.4[a]	0.02[a]
Glass, Pyrex	0.205	134[a]	0.09[a]
Nylon	0.252	241	0.33
Teflon	0.050	366	0.13

[a] Strength values in tension.

large value of TSP would have good thermal shock resistance. The values for the thermal shock parameter for several materials are listed in Table 1-3.

Under steady-state conditions, the transient thermal properties do not influence the thermal stresses. In these cases, the *thermal stress ratio TSR* is an important material property for use in assessing the material resistance to thermal stresses [Gatewood, 1957]:

$$\mathrm{TSR} = \frac{S_u}{\alpha E} \tag{1-27}$$

Values for the thermal stress ratio for several materials are also tabulated in Table 1-3. Generally, a material with a large thermal stress ratio will have good resistance to thermal stresses.

Example 1-5 Tubes made of red brass (UNS-C2300, 85% Cu, 15% Zn) having a 05105 temper are to be used in a steam condenser. The ultimate tensile strength for the material is 305 MPa (44,200 psi), and Young's modulus for the red brass is 90 GPa (13×10^6 psi). The thermal expansion coefficient for the material is 18×10^{-6} K^{-1} (10×10^{-6} °F^{-1}), and the thermal diffusivity is 18.0 mm^2/s.

Determine the thermal stress ratio and thermal shock parameter for the red brass tubing.

The thermal stress ratio is found from eq. (1-27):

$$\mathrm{TSR} = \frac{(305)(10^6)}{(18)(10^{-6})(90)(10^9)} = 188.3\ \mathrm{K}$$

The thermal shock parameter is found from eq. (1-26):

$$\mathrm{TSP} = (188.3)(18.0 \times 10^{-6})^{1/2} = 0.799\ \mathrm{K\text{-}m/s}^{1/2}$$

It is noted from Table 1-3 that these values are slightly lower than the corresponding values for 70/30 brass.

Example 1-6 In the design of a heat exchanger, the engineer has a choice of the following materials for use as the heat exchanger tubing: red brass, copper (Cu/10% Ni), and aluminum (2024-T3). Which material should be selected from a thermal stress resistance standpoint?

From Table 1-3 and Example 1.5, we find the following values for the thermal stress ratio and thermal shock parameter:

Material	TSR, K	TSP, K-m/s$^{1/2}$
Aluminum, 2024-T3	280	1.94
Red brass	188	0.80
Copper (Cu/10% Ni)	179	0.67

When the fluid is suddenly introduced into the heat exchanger, originally at ambient temperature, the tubing may experience thermal shock. The aluminum has the largest TSP, so aluminum would be the best material for thermal shock resistance. In addition, the TSR for aluminum is largest of the three materials, so aluminum would also be best for steady-state thermal stress resistance.

We may conclude that the engineer should select aluminum (2024-T3) as the best of the three materials from a thermal stress standpoint.

1.10 HISTORICAL NOTE

People have known about thermal stresses from the time that the first person broke a clay vessel by heating the vessel too rapidly. It wasn't until the 1800s, however, that the first analytical analysis was made for thermal stresses [Timoshenko, 1983].

Robert Hooke (1635–1703) worked with Robert Boyle on perfecting an air pump at Oxford. Boyle recommended Hooke as the curator of the experiments of the Royal Society in England, of which Hooke was a charter member. In the

1670s Hooke conducted experiments with elastic bodies, and in 1678 he published the first technical paper in which elastic properties of materials were examined. Based on his experiments with springs and other elastic bodies, Hooke concluded in his paper "De Potentiâ Restitutiva" ("Of Springs") in 1678: "It is very evident that the Rule or Law of Nature in every springing body is, that the force or power thereof to restore itself to its natural position is always proportional to the distance or space it is removed therefrom, whether it be by rarefaction, or the separation of the parts the one from the other, or by Condensation, or crowding of those parts together." In less formal words, Hooke's law may be stated in the form: "There is a linear relationship between the force and deformation for bodies at stresses below the proportional limit." This principle is the beginning point for all elastic analyses, including thermoelastic analysis.

Thomas Young (1778–1829) (Figure 1-7) originally studied medicine and received his doctor's degree from Göttingen University in 1796. A few years later while at Cambridge (in 1796) he became interested in the physical sciences, including acoustics and optics. In 1802 he was appointed a professor of natural philosophy (the forerunner of today's physics and other scientific areas) by the Royal Institution. Many of his main contributions to mechanics of materials were presented in his course on natural philosophy during the year he taught at the Royal Institution. He introduced the concept of the modulus of elasticity, which is called *Young's modulus* today (although Young's definition was somewhat different from that used now). In his lecture notes entitled *A Course of Lectures on Natural Philosophy and the Mechanical Arts*, published in 1807,Young stated: "The modulus of elasticity of any substance is a column of the same substance, capable of producing a pressure on its base which is to the weight causing a

Figure 1-7. Thomas Young (From S. P. Timoshenko, 1983. Used by permission of Dover Publications, Inc.)

certain degree of compression as the length of the substance is to the diminution of its length." The modulus defined by Young was essentially the product of Young's modulus and the cross-sectional area in present-day terminology.

C.L.M.H. Navier (1785–1836) published a book on strength of materials in 1826, in which he defined the modulus of elasticity for tension or compression as the ratio of the force per unit cross-sectional area to the elongation per unit length. This modulus is the property that is denoted as Young's modulus today. Navier actually measured the Young's modulus for the iron that was used in the construction of the Pont des Invalides in Paris.

S. D. Poisson (1781–1840) (Figure 1-8) taught mathematics at the École Polytechnique, and he applied his mathematical skills in solving several problems involving the theoretical strength of materials. He was interested in the theory of elasticity based on molecular force considerations. In his memoir, *Mémoire sur l'équilibre et le mouvement des corps élastiques*, published in 1829, Poisson applies general elasticity equations that he had developed to isotropic materials. He found that, for simple tension of a rod or bar, the axial elongation produces a lateral contraction. Poisson's relationships yielded a value of the ratio of lateral strain to axial strain, called *Poisson's ratio* today, to have a universal value of $\mu = \frac{1}{4}$. As discussed in Section 1.6, Poisson's ratio for isotropic materials may have values between 0 and $\frac{1}{2}$.

Gabriel Lamé (1795–1870) (Figure 1-9) graduated from the École Polytechnique in 1818 and worked with a then-new Russian engineering school, the Institute of Engineers of Ways of Communication in St. Petersburg. Lamé taught applied mathematics and physics at the school and helped with the design of

Figure 1-8. S. D. Poisson (From S. P. Timoshenko, 1983. Used by permission of Dover Publications, Inc.)

Figure 1-9. Gabriel Lamé (From S. P. Timoshenko, 1983. Used by permission of Dover Publications, Inc.)

several suspension bridges built in the St. Petersburg area. In 1852, Lamé produced the first book on the theory of elasticity, entitled *Leçons sur la Théorie Mathématique de l'Élasticité des Corps Solides.* He concluded that to define the elastic properties of an isotropic material, only two different elastic constants were required. In Lamé's general elasticity equations, he selected the two constants as Lamé's elastic modulus λ_L and the modulus of elasticity in shear G.

J. M. Constant Duhamel (1797–1872) also graduated from Éçole Polytechnique in 1816 and, after studying for law and teaching mathematics in some other schools, he joined the faculty at Éçole Polytechnique in 1830. After publishing several papers in the area of conduction heat transfer, Duhamel made some basic contributions to the theory of elasticity. In 1835 he published the paper "Mémoir sur le calcul des actions moléculaires développées par les changements du température dans les corps solids." In this paper he developed the basic partial differential equations for stress equilibrium conditions, including stresses produced by temperature variation. This paper was the first to give an analytical treatment of thermal stresses.

Duhamel applied the general equations to several problems of practical interest. He obtained a solution for the stress distribution in the wall of a hollow spherical vessel and a hollow cylindrical vessel in which the temperature varies across the wall of the vessel. Duhamel was one of the first investigators to use the principle of superposition in thermal stress analysis.

Franz Neumann (1798–1895) (Figure 1-10) began service in the German Army when he was only 17 years old. After a year in the military service, he returned Berlin to complete his high school education and to study at Berlin University. He studied mineralogy and was awarded a faculty position to teach

Figure 1-10. Franz Neumann (From S. P. Timoshenko, 1983. Used by permission of Dover Publications, Inc.)

mineralogy at the University of Königsberg after receiving his doctorate. In 1843, Neumann published an extensive memoir dealing with double refraction of light, in which he presents the basic principles used in experimental photoelastic stress analysis. Based on stress equilibrium equations similar to those developed by Duhamel, Neuman solved the stress distribution problem for a sphere with radially varying temperature. He also conducted photoelastic tests and experimentally measured the thermal stresses in the sphere, and found that the experimental and theoretical data were in satisfactory agreement. This is the first time that thermal stresses were measured in the laboratory. Neumann analyzed the problem of thermal stresses in circular plates in which the temperature varies in the radial direction in the plate, but is constant in the thickness direction. He also solved the plate thermal bending problem for a circular plate in which the temperature was a function of the axial coordinate only.

PROBLEMS

1-1. A part is to be constructed using 9-percent nickel steel, for which the average ultimate strength is 856 mPa (124,200 psi) and the standard deviation of the ultimate strength data is 42.9 MPa (6220 psi). If an ultimate strength of 756 MPa (109,600 psi) is used in the design, determine the probability that the actual ultimate strength of the material used is less than 756 MPa.

1-2. In a certain design, 6061-T6 aluminum is used, for which the average yield strength is 275 MPa (39,890 psi) and the standard deviation for the yield strength data is 20 MPa (2900 psi). A factor of safety of 1.50, based on the yield strength for a 99.99 percent survival rate, is to be used in the design. Determine the design stress that should be used.

1-3. At room temperature, Young's modulus and Poisson's ratio for constantan (cupronickel, 55% Cu/45% Ni) are $E = 165$ GPa (23.9×10^6 psi) and $\mu = 0.325$. The specific heat and density for constantan are $c_v = 0.409$ kJ/kg-K (0.0977 Btu/lb_m-°F) and $\rho = 8922\ \text{kg/m}^3$ ($0.322\ \text{lb}_\text{m}/\text{in}^3$) at room temperature. Determine the linear thermal expansion coefficient α for constantan at room temperature, if the Grüneisen constant for constantan is $\gamma_\text{G} = 1.91$.

1-4. Niobium is a material used in superconducting magnets, which operate at cryogenic temperatures. The properties of niobium at 4.2 K (−269°C or 7.6°R or −452°F) are density, $8580\ \text{kg/m}^3$ ($0.310\ \text{lb}_\text{m}/\text{in}^3$); Young's modulus, 68.9 GPa (10.0×10^6 psi); Poisson's ratio, 0.270; and Grüneisen constant, 1.57. The specific heat of niobium for temperatures below 22 K is given by

$$c_v = C_0(T/\theta_D)^3$$

The factor $C_0 = 20.921$ kJ/kg-K, T is the absolute temperature, and $\theta_\text{D} = 265$ K (477°R) = the *Debye temperature* for niobium. Determine the linear thermal expansion coefficient for niobium at 4.2 K.

1-5. Using the data from Problem 1-4, determine the total change in length of a niobium wire having an initial length of 800 m (2625 ft) when the wire is cooled from 20.3 to 4.2 K. The properties, except the thermal expansion coefficient, may be treated as constant.

1-6. Platinum has a valence of +2, and the equilibrium spacing of the atoms in platinum is 0.28 nm. Determine the value of Young's modulus for platinum predicted by the theoretical expression, eq. (1-14). How does this value compare with the measured value of Young's modulus for platinum, $E = 146.9$ GPa?

1-7. Determine the shear modulus, bulk modulus, and Lamé constant for (a) 2024-T3 aluminum and (b) C1020 steel.

1-8. The shear modulus and Lamé constant for platinum are $G = 55.5$ GPa (8.05×10^6 psi) and $\lambda_L = 103.0$ GPa (14.94×10^6 psi). Determine the value of Young's modulus and Poisson's ratio for platinum.

1-9. In a particular application involving transient thermal stresses, it is desired to select the best material for thermal shock resistance from the following: (a) 3003-H12 aluminum, (b) copper/10% nickel, and (c) 304 stainless steel. Determine the thermal shock parameter for each of these materials and select the one that would have the best shock resistance characteristics.

1-10. In a steady-state situation involving thermal stresses, the designer has a choice of the following three materials: C1020 carbon steel, annealed;

6061-T6 aluminum; and a nickel alloy ($\alpha = 11.3 \times 10^{-6}\,\mathrm{K}^{-1} = 6.28 \times 10^{-6}\,{}^\circ\mathrm{F}^{-1}$; $E = 220\,\mathrm{GPa} = 31.9 \times 10^{6}\,\mathrm{psi}$; $S_u = 955\,\mathrm{MPa} = 138{,}500\,\mathrm{psi}$). Determine the thermal stress ratio for each material and select the one that would have the best thermal stress resistance.

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