Chapter 1

Essential Mathematics and Physics

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Introduction

This chapter introduces and explores the principles of mathematics and physics that will make following chapters and the principles of radiography easier to understand. Although many of the concepts introduced in this chapter are only for revision, they are relevant to later chapters.

Matter, energy, power and heat

Matter

The entire world is made up of matter. Anything that occupies space can be termed 'matter'. Matter is a collection of atoms, the basic building blocks. All matter has mass, that is, the measure of matter in an actual object. If gravity is involved, this mass is known as the weight of an object. If an object is placed in a lesser gravitational field, such as the atmosphere on the moon, the mass will remain the same, but the weight will decrease. The weight will also change if the object changes form, but, again, the mass will remain the same. An example of this is water in its three forms – solid (ice), liquid (water) and gas (steam). In these three forms, the mass is the same throughout, but the weight changes considerably.

Matter	**A collection of atoms and molecules**
Mass	**The measure of matter in an object**
Weight	**Mass under the influence of gravity**

Energy

The process of matter altering its state or form produces energy. Any object, however large or small, that is able to do 'work' is said to have energy. Energy has a number of different forms. Energy can be neither created nor destroyed, although it can change from one form to another (Table 1.1).

> **Total energy is measured in joules (J):**
>
> **Total energy = Potential energy + Kinetic energy**

Table 1.1 Energy types, definitions and examples.

Energy type	Description	Example
Potential	The amount of work an object could do because of its position	An axe raised, ready to be brought down to chop, has potential energy
Kinetic	As an object leaves its state of potential energy, it gains kinetic energy	An apple gains kinetic energy as it falls out of a tree
Electrical	The movement of electrons inside a conductor after the application of a potential difference	The movement of electrons in a cable produces the electrical energy needed to make a bulb light up
Nuclear	Nuclear energy is the energy stored within the nucleus of an atom	This energy is formed when the nucleus of an atom is split
Thermal	The energy of a hot object. This is caused by the vibration of molecules within matter	A hot bath has faster moving molecules than a cool bath
Sound	The energy produced by sound vibrations	A musical instrument, engine noise, speech, thunder, diagnostic ultrasound, sonar
Chemical	The energy generated when a reaction occurs between two substances	Thermal energy produced when water is added to hot oil
Electromagnetic	Electric and magnetic energy moving in waves	X-ray production, radio waves, infrared light

Energy conversion

As energy cannot be created or destroyed, it changes form, and this process is known as energy conversion.

In radiography, the X-ray tube is an example where energy is converted from one form (electrical) into other forms (X-rays, heat, light). We also use ultrasound where an ultrasound transducer converts

electrical energy into sound energy, and the reflected sound energy is converted back into electrical energy.

Power

Power is the rate of doing work or the rate of transforming energy. This is measured in joules per second or watts. In radiography, due to the amount of energy transformation occurring, power is measured in thousands of watts or kilowatts (kW). A typical X-ray room will have a 50-kW generator to supply electric power to the X-ray equipment.

> **Power is measured in joules per second (J s^{-1}) or watts (W).**

Heat

Heat is the total energy of atoms and molecules moving in matter. The average speed of movement is known as temperature. Heat always flows from hot to cold until an equilibrium is reached. This movement can occur through three different methods – convection, conduction and radiation (Table 1.2). The rate of heat loss or transfer will depend on the type of surface material and the difference between the two areas of heat. This is utilised in an X-ray tube through the choice of material used for the anode and the colour of the tube head (black).

Table 1.2 Definitions of conduction, convection and radiation.

Convection	This occurs in liquids and gases. The matter moves, taking the heat with it. This occurs because of the reduction in density associated with heating. Hotter material rises and displaces cooler material above. This is the principle behind surrounding an X-ray tube in oil as a cooling technique.
Conduction	This is found in metals. The heat is transferred through contact, with heat flowing from the hot area to the cooler area. This principle is used in the anode and stem of the X-ray tube.
Radiation	Vibrating molecules on the surface of matter generate electromagnetic waves. Energy in the form of heat leaves the surface and transfers the energy to whatever it strikes. This is most effective in a vacuum.

Units and prefixes used in radiography

The use of scientific terminology in radiography is based on standardised units and prefixes to abbreviate large or very small numbers. It also provides an international language amongst radiographers. The

use of standardised units extends to the description of units of measure and the identification of units of ionising radiation.

Standard scientific notation

Radiography uses both very large units and very small units. Examples of this are the 100,000 volts necessary to radiograph a chest and the 0.004 amperes (amps) needed to demonstrate a cat's carpus. These are two of the core units used in radiography and are described as kilovolts (kV) and milliamperes (mA). Using this notation, 100,000 volts is described as 100 kV and 0.004 amperes as 4 mA. Where large numbers are used, the numbers can also be described as exponents. Exponents describe numbers as multiples of ten (the system most widely used in everyday life is the decimal system; see Table 1.3).

Table 1.3 Standard scientific notation, prefixes and symbols.

Notation	Decimal number	Prefix	Symbol
10^9	1,000,000,000	giga-	G
10^6	1,000,000	mega-	M
10^3	1000	kilo-	k
10^2	100	hecto-	h
10^1	10	deka-	da
10^{-1}	0.1	deci-	d
10^{-2}	0.01	centi-	c
10^{-3}	0.001	milli-	m
10^{-6}	0.000001	micro-	μ

SI base units

In order to maintain a common radiographic language, the units used as a baseline for measurements and discussions need to be standardised. Radiography uses the International System of Units or 'SI'. Problems would occur if the focus-to-film distance was given in metres on the practice exposure chart and the veterinary nurse carrying out examinations worked in inches. The base units in Table 1.4 are the units used to calculate more complicated measures such as speed ($m \ s^{-1}$) or force ($kg \ m \ s^{-2}$). There are seven base units from which all other units are derived.

Radiological units

Radiology has a number of units specific to the field that are in common use (see Table 1.5). These are all related to the measurement of the production of X-rays and the effect of the energy produced, and

Table 1.4 SI units used in radiography.

Term	SI unit	Definition	Application to radiography
Energy	Joule J	Ability to do work	Production of X-rays
Mass	Kilogram kg	A measure of the number of atoms and molecules in a body	Important when determining the radiation dose to a patient
Gray	Joules per kilogram Gy	Energy imparted to a body by ionising radiation	Unit of radiation dose measurement
Power	Joules per second W	Rate of doing work	Output of X-ray generator
Electric current	Ampere A	Movement of electrons flowing per unit time	Quantity of electrons flowing per unit time
Electric charge	Coulomb C	One ampere flowing per second	Quantity of electrons flowing per second
Electrical potential	Volt V	The force that moves electrons within a conductive material	Potential difference across an X-ray tube, acceleration of electrons and quality of X-ray beam
Frequency	Hertz Hz	Number of cycles per second	Electromagnetic radiation

Table 1.5 Radiological units.

Unit	Description	Symbol
kV_p	Maximum energy of X-ray photons	kV_p
mA (mAs)	Electron production in the X-ray tube	mA
keV	Kinetic energy of electrons in X-ray tube	keV
Heat unit	Heat produced at anode ($kV_p \times$ mAs)	HU
Gray	Dose absorbed by a medium	Gy
Sievert	Dose equivalent	Sv
Coulomb/kilogram	Measure of atmospheric exposure	C/kg
Becquerel	Radioactive disintegrations per second	Bq

used in diagnostic imaging. The units are mainly used in assessing and maintaining radiation safety or when discussing the use of the X-ray tube.

kV_p

The potential difference between the cathode and anode in an X-ray tube is measured in kilovolts. This value determines the maximum energy of the X-ray photons emitted that will give the quality and

intensity of the beam. In many machines, this value may fluctuate and so the peak value is given (kV_p).

mA/mAs

In the production of X-rays, fast-moving electrons must strike the anode within the X-ray tube. To produce these electrons, an electrical current must be applied to the cathode. This is measured in milliamperes (mA). These electrons could be produced continuously, but this would cause damage to the tube and so the production of electrons is limited to a period of time (exposure time). The exposure time is expressed in mAs or milliamperes per second.

keV

As an electron is accelerated across the tube from the cathode to the anode, it gains kinetic energy. This is measured in keV. The keV will be the same as the kV_p.

Heat units

The production of X-rays produces heat at the anode. The amount of heat is specific to each exposure and can be calculated by multiplying kV_p and mAs together. This is correct only if the voltage and current remain constant throughout the exposure.

Absorbed dose

The dose absorbed by the patient is measured in gray (Gy). This is specific to the patient dose received and will vary according to the exposure used and the region being examined. The absorbed dose is the measurement of the energy absorbed by a medium.

> **1 gray = 1 joule per kilogram**

Dose equivalent

The dose received by designated people working with radiation (dose equivalent) is measured in sieverts (Sv). This measurement is calculated by multiplying the grays received by a quality factor. The quality factor will take into account the different levels of damage caused by radiation and will alter depending on the type of ionising radiations and the energy of the ionising radiation. The dose equivalent is

calculated from monitoring devices worn by personnel working with radiation.

Exposure in air

The amount of radiation in the atmosphere can be measured in coulomb/kilogram (C/kg). This measure of radiation can only be used for air and for X-rays or gamma rays within this air. The measure gives the total electric charge formed by ionisation in air. This can be used for X-rays emerging from the tube or the intensity of gamma rays during a scintigraphic examination.

Activity

The final radiographic unit is the becquerel (Bq). Radioactive substances have unstable nuclei and try to change the structure of the nucleus to a more stable form. Each change in structure is called disintegration. The becquerel measures the number of changes per second.

Useful mathematics

Day-to-day radiography involves mathematics. This may be simple addition or multiplication, but can also involve fractions and ratios. As a simple 'aide memoir', this section demonstrates the basic mathematics essential to radiography in Table 1.6, where a and b denote any number and x is any number that you wish to calculate.

Proportions and the inverse square law

Proportions

Measurements can be either directly or indirectly proportional. If two measurements are directly proportional, the ratio of one to the other is constant:
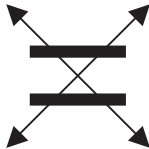
$$\frac{a}{b} = \text{constant}$$

If something is described as being inversely proportional, the factors will be inverted. As one factor increases, the other will decrease, or vice versa:

$$a \times b = \text{constant}$$

Table 1.6 Useful mathematics.

Description	How to do it
Percentage change	$100 \times (b - a/a)$
Percentage of b compared to a	$100b/a$
x% of a	$(x/100) \times a$
Parts of a fraction	$\dfrac{\text{Numerator}}{\text{Denominator}} \quad \dfrac{a}{b}$
Adding and subtracting fractions	Find a common denominator and then add or subtract the numerators
Multiplying fractions	Multiply numerators and denominators
Dividing fractions	Turn the second fraction upside down and then multiply
Ratio	Demonstrates the relationship between two related measures
	kV : X-rays produced
Decimal	A fraction that has a denominator that can be divided by ten can be shown as a decimal: $5/10 = 0.5$
To calculate x when a and b are known: divide both known numbers by the multiple of x	$ax = b$ $ax/a = b/a$ $x = b/a$
When a known number is added to x: subtract the known number from both sides	$x + a = b$ $x + a - a = b - a$ $x = b - a$
When x is part of a fraction: cross multiply	$x/a = b/c$ $xc = ab$ $x = ab/c$
Cross multiplication	

Inverse square law

The intensity of radiation from a given point is inversely proportional to the square of the distance between that point and the source. This means that the greater the distance between the two points, the weaker the intensity. This plays an important role in radiation safety. The greater the distance between you and the source of radiation, the lower the dose you will receive:

$$I \propto 1/d^2$$

The effect distance has on the exposure is determined by the inverse square law. As the distance of the object from the source increases, the intensity of the radiation will decrease. If you double the distance, the exposure intensity decreases by 4. This can be seen in a similar way using a torch beam. The closer the wall is to the torch beam, the stronger the intensity of the beam against the wall. As you move away from the wall, the beam will be weaker when it hits the wall (Figure 1.1).
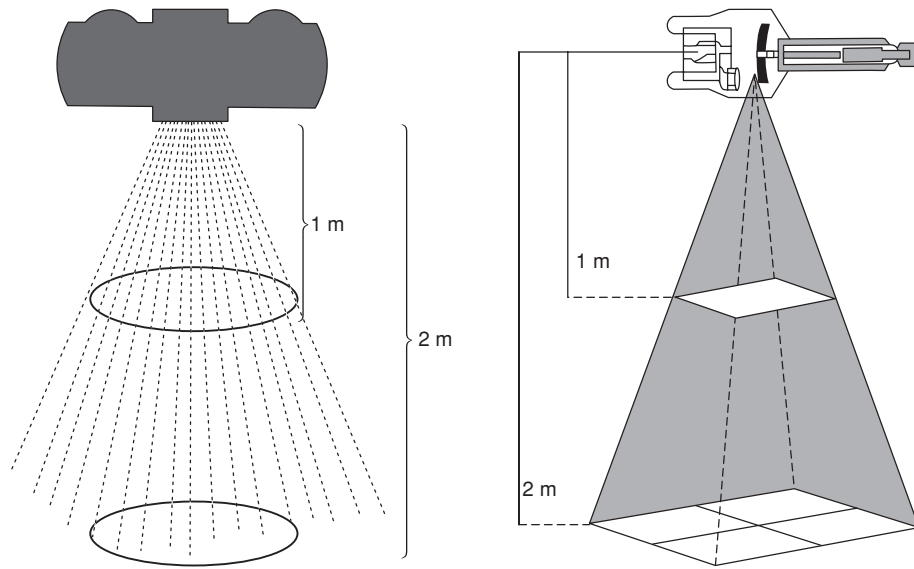

Figure 1.1   The inverse square law.

Revision questions

1 How are mass and matter related?

2 What is weight?

3 List and give examples of three types of energy.

4 What is 30,000 volts in kilovolts?

5 What is 4.5 mA in decimal notation?

6 What name is given to the unit of measure for time?

7 Give the measure of absorbed dose and describe what the measurement demonstrates.

8 Describe the calculation of a becquerel.

9 What is the symbol for a sievert?

(continued)

10 What is a ratio?

11 Add 4/5 to 3/7.

12 Divide 3/10 by 1/8.

13 Work out the following equations:

$$x+10 = 37$$
$$4x = 24$$
$$x/8 = 3/4$$

14 If a is inversely proportional to b, what will happen to a if b doubles?

15 Using the inverse square law and thinking about an X-ray beam, if you double your object–source distance, what will happen to the intensity and size of the beam when it reaches the object?