Chapter 1

Soil Structure

Soils consist of solid particles, enclosing voids or pores. The voids may be filled with air or water or both. These three soil states (or phases) can be visualized by the enlargement of three small samples of soil.

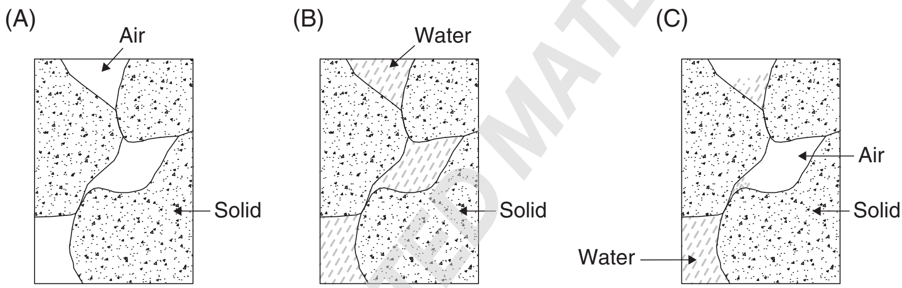

Figure 1.1

Sample A: The soil is oven-dry, that is there is only air in the voids.
Sample B: The soil is saturated, that is the voids are full of water.
Sample C: The soil is partially saturated, that is the voids are partially filled with water.

The above three soil states can be described mathematically by considering:

1. Volume occupied by each constituent.
2. Mass (or weight) of the constituents.

1.1 Volume relationships

The expressions derived in this section will answer two questions:

1. How much voids and solids are contained in the soil sample?
2. How much water is contained in the voids?

In order to obtain these answers, the partially saturated sample (C) is examined. It is assumed, for the purpose of analysis, that the soil particles are lumped together into a homogeneous mass. Similarly, the voids are combined into a single volume, which is

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partly occupied by a volume of water. The idealisation of the sample, indicating the volumes occupied by the constituents, is shown diagrammatically in Figure 1.2b.

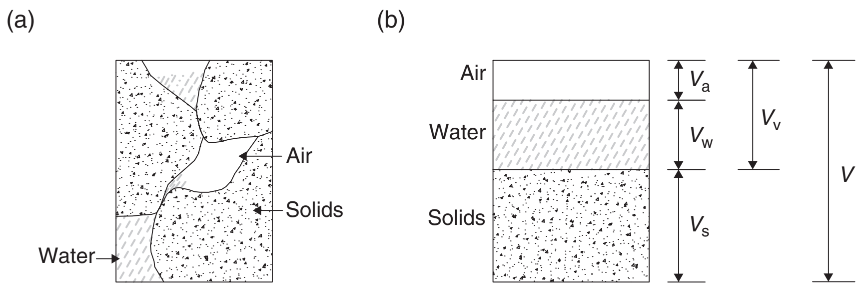


Figure 1.2

Idealized representation of sample C.

Where: V = Total volume of the sample
V_v = Volume of voids in the sample
V_s = Volume of soil in the sample
V_w = Volume of water in the sample
V_a = Volume of air in the sample

The basic relationships between the volumes can be seen in the diagram.

Total volume: $$V = V_s + V_v \quad (1.1)$$

Volume of voids: $$V_v = V_w + V_a \quad (1.2)$$

Hence: $$V = V_s + V_w + V_a \quad (1.3)$$

Three important relationships are derived from the basic ones. These are:

e = voids ratio (or void ratio)
n = porosity
S_r = degree of saturation

1.1.1 Voids ratio (*e*)

This shows the percentage of voids present in the sample, compared to the volume of solids. Thus, if V_s is considered to be 100%, then V_v is e%.

Hence: $$\boxed{e = 100\frac{V_v}{V_s}\,\%} \quad (1.4)$$

For example: if $V_s = 60\,\text{cm}^3$
and $V_v = 15\,\text{cm}^3$

then $$e = 100\frac{15}{60} = 25\%$$

That is, the volume of voids is 25% of the volume of solids, in this particular sample. Alternatively, the voids ratio maybe expressed as a decimal e.g. $e = 0.25$.

Formula (1.4) now becomes: $$\boxed{e = \frac{V_v}{V_s}} \quad (1.5)$$

The ratio of voids to solids in a sample is represented by Figure 1.3.

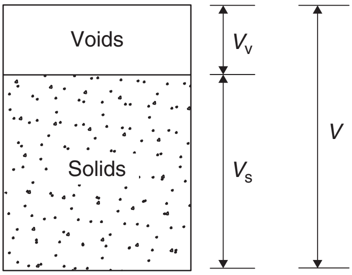


Figure 1.3

1.1.2 Porosity (n)

This shows how many percent of voids are present in the sample, compared to the total volume V. Thus, if V is considered to be 100%, then V_v is n%.

$$\boxed{n = 100\frac{V_v}{V} \,\%} \tag{1.6}$$

For example: if $V = 75\,\text{cm}^3$
and $V_v = 15\,\text{cm}^3$
then $n = 100\frac{15}{75} = 20\%$

That is, the volume of voids is 20% of the total volume of the sample of soil.

Again, n maybe expressed as a decimal number $n = 0.2$.

Formula (1.6) now becomes:

$$n = \frac{V_v}{V} \tag{1.7}$$

The diagrammatic representation of porosity is:

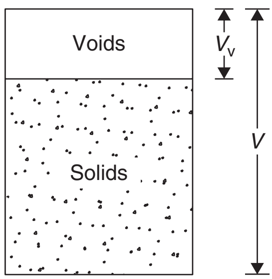


Figure 1.4

1.1.3 Degree of saturation (S_r)

This shows the percentage of voids filled with water. Thus, if V_v is considered to be 100%, then V_w is S_r%.

$$\boxed{S_r = 100\frac{V_w}{V_v} \,\%} \tag{1.8}$$

For example, if $V_w = 6\,\text{cm}^3$

and $V_v = 15\,\text{cm}^3$

then $S_r = 100\dfrac{6}{15} = 40\%$

That is, water fills 40% of the volume of voids. In decimal form $S_r = 0.4$ and formula (1.8) becomes:

$$\boxed{S_r = \frac{V_w}{V_v}} \tag{1.9}$$

Diagrammatically,

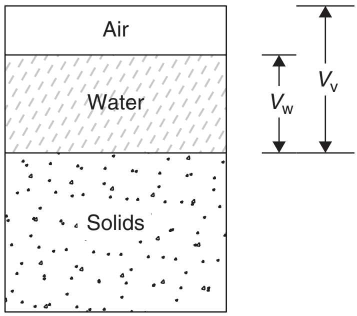


Figure 1.5

Note: For oven-dry soil (Sample A, Figure 1.1):

$V_w = 0$, hence $S_r = 0$

For fully saturated soil (Sample B, Figure 1.1):

$V_w = V_v$, hence $S_r = 1$

For partially saturated soil therefore: $0 < S_r < 1$

Combined formulae

The quantities defined by formulae (1.1) to (1.9) can be interrelated:

From (1.1): $V = V_s + V_v$

From (1.5): $V_v = eV_s$

either $V = V_s + eV_s \quad \therefore \boxed{V = (1+e)V_s}$ (1.10)

or $V = \dfrac{V_v}{e} + V_v$

$$= \left(\frac{1}{e} + 1\right)V_v \quad \therefore \boxed{V_v = \left(\frac{1+e}{e}\right)V_v} \tag{1.11}$$

From (1.7): $n = \dfrac{V_v}{V}$

From (1.10): $V = (1+e)V_s$

$$n = \frac{eV_s}{(1+e)V_s} \quad \therefore \boxed{n = \frac{e}{1+e}} \tag{1.12}$$

From (1.12): $n = \dfrac{e}{1+e}$

$n + ne = e$

$$n = e(1-n) \quad \boxed{e = \frac{n}{1-n}} \tag{1.13}$$

From (1.9): $S_r = \frac{V_w}{V_v}$

From (1.11): $V_v = \frac{eV}{1+e}$

$$S_r = \frac{V_w}{\frac{eV}{1+e}} \quad \therefore \quad \boxed{S_r = \left(\frac{1+e}{e}\right)\frac{V_w}{V}} \qquad (1.14)$$

From (1.12): $n = \frac{e}{1+e}$ or

$$\boxed{S_r = \frac{V_w}{nV}} \qquad (1.15)$$

Example 1.1

Given: $V = 946\,\text{cm}^3$ Calculate: V_v, V_a, e, n and S_r

$V_s = 533\,\text{cm}^3$

$V_w = 303\,\text{cm}^3$

From (1.1): $V_v = V - V_s = 946 - 533 = 413\,\text{cm}^3$

From (1.2): $V_a = V_v - V_w = 413 - 303 = 110\,\text{cm}^3$

From (1.5): $e = \frac{V_v}{V_s} = \frac{413}{533} = 0.775$, that is the volume of voids is 77.5% that of solids.

From (1.7): $n = \frac{V_v}{V} = \frac{413}{946} = 0.437$

or From (1.12): $n = \frac{e}{1+e} = \frac{0.775}{1.775} = 0.437$

That is, the volume of voids is 43.7% of the sample.

From (1.9): $S_r = \frac{V_w}{V_v} = \frac{303}{413} = 0.73$

or From (1.15): $S_r = \frac{V_w}{nV} = \frac{303}{(0.437 \times 946)} = 0.73$

That is, water fills 73% of voids. The sample is partially saturated.

Example 1.2

A sample of sand was taken from below the ground water table. The volumes measured were:

$V = 1000\,\text{cm}^3$ Calculate: V_v, V_a, V_s, e and n

$V_w = 400\,\text{cm}^3$

Note: Assume sand samples taken from above the water table as partially saturated ($S_r < 1$) and saturated ($S_r = 1$) if taken from below.

In this example, therefore, $S_r = 1 \quad \therefore \quad V_a = 0$.

From (1.8) $$S_r = \frac{V_w}{V_V} = 1 \quad \therefore \quad \boxed{V_w = V_v} \qquad (1.16)$$

$$V_v = 400\,\text{cm}^3$$

From (1.2): $V_a = V_v - V_w = 400 - 400 = 0$ The voids are full of water

From (1.1): $V_s = V - V_v = 1000 - 400 = 600\,\text{cm}^3$

From (1.5): $e = \dfrac{V_v}{V_s} = \dfrac{400}{600} = 0.67$ | V_v is 67% of V_s

From (1.7): $n = \dfrac{V_v}{V} = \dfrac{400}{1000} = 0.4$ | V_v is 40% of V

1.2 Weight–volume relations

As the title implies, the formulae derived in this section take into account the weights of V_s and V_w. It is assumed that air is weightless. The weight volume relations are shown diagrammatically:

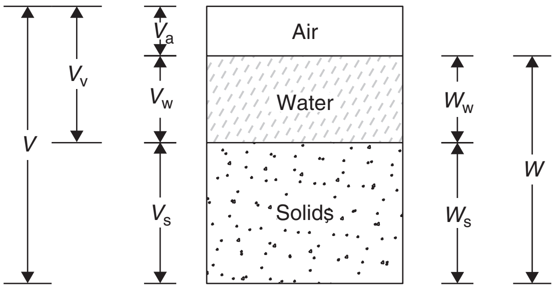


Figure 1.6

Where: W_s = Weight of solids; W_w = Weight of water; W = Total weight | From Figure 1.6 $\boxed{W = W_s + W_w}$ (1.17)

Note: The concepts of mass and weight are defined in Appendix A. Suffice to say here, that if mass (M) is given in kilograms, then weight (W) is calculated from:

$$W = 9.81 \times \text{mass } (M)\,N \quad \therefore \boxed{W = 9.81 \times 10^{-3} \times M\ \text{kN}} \tag{1.18}$$

Several important relationships are derived below in terms of mass, weight and volume. These are:

ρ = bulk mass density
γ = bulk weight density (unit weight)
ρ_d = dry mass density
γ_d = dry weight density
ρ_{sat} = saturated mass density
γ_{sat} = saturated weight density

ρ' = submerged mass density
γ' = submerged weight density
ρ_s = mass density of solids
γ_s = weight density of solids.

Note: Normally, the mass density of materials is expressed in kg/m^3. For instance, the average mass of reinforced concrete is quoted in tables as ρ=2400 kg/m^3. Sometimes, especially in laboratory work, it is more convenient to use gram as the unit of mass. Possibly for this reason ρ is often expressed in g/cm^3 or Mg/m^3 (Mg/m^3=g/cm^3).

For a reason, justified in the Appendix, the unit adopted in this book is kg/m^3, unless otherwise stated.

1.2.1 Bulk densities

These are the densities of a partially saturated soil sample, taken from above ground water level.

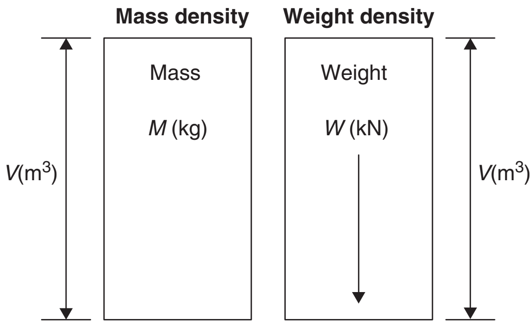


Figure 1.7

$$\boxed{\rho = \frac{M}{V} \,\Big|\, \text{kg/m}^3} \quad (1.20) \qquad \boxed{\gamma = \frac{W}{V} \,\Big|\, \text{kN/m}^3} \quad (1.19)$$

$$\gamma = \frac{9.81\times10^{-3} M}{V} = 9.81\times10^{-3}\left(\frac{M}{V}\right) \qquad \therefore \boxed{\gamma = 9.81\times10^{-3}\rho \,\big|\, \text{kN/m}^3} \quad (1.21)$$

For water: $$\boxed{\rho_w = \frac{M_w}{V_w} = 1000\ \text{kg/m}^3} \quad (1.22) \qquad \boxed{\gamma_w = \frac{W_w}{V_w} = 9.81\ \text{kN/m}^3} \quad (1.23)$$

All practical problems in soil mechanics are concerned with forces acting in one way or another. As the weight density (or unit weight) itself is a force, its application is a matter of necessity. For this reason the formulae derived in the rest of this section are mostly in terms of weight. Remember, however, that 1 kg=1000 g and 1 g/cm^3=10^3 kg/m^3. Therefore, if the mass density is given in gram and centimeter units as:

$$\boxed{\rho = \frac{M}{V} \,\Big|\, \text{g/cm}^3} \quad \text{then} \quad \boxed{\gamma = 9.81\rho \,\big|\, \text{kN/m}^3} \quad (1.24)$$

Example 1.3

Partially saturated sand was tested in a laboratory. Its volume was measured to be 75.4 cm^3 and weighed 136.2 g. Calculate the unit weight in kN/m^3.

Mass: $M = 136.2\,g = 136.2 \times 10^{-3}\,kg$

Volume: $V = 75.4\,cm^3 = 75.4 \times 10^{-6}\,m^3$

Weight: $W = 9.81 \times 10^{-3} \times 136.2 \times 10^{-3} = 1336 \times 10^{-6}\,kN$

Mass density: $\rho = \frac{M}{V} = \frac{136.2 \times 10^{-3}}{75.4 \times 10^{-6}} = 1806\,kg/m^3$

Weight density: $\gamma = \frac{M}{V} = \frac{1336 \times 10^{-6}}{75.4 \times 10^{-6}} = 17.72\,kg/m^3$

or by (1.21): $\gamma = 9.81 \times 10^{-3}\,\rho = 9.81 \times 10^{-3} \times 1806 = 17.72\,kg/m^3$

1.2.2 Dry densities

These are the densities of oven-dry soil, after the excavated sample has completely dried out.

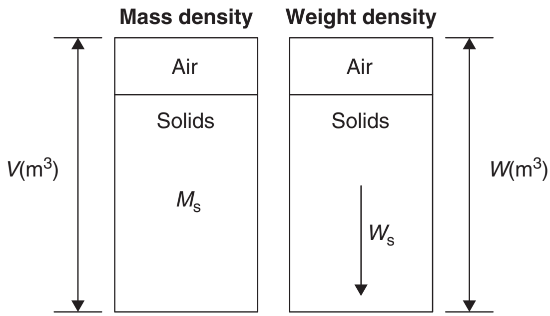


Figure 1.8

$$\rho_d = \frac{M_s}{V}\ kg/m^3 \quad (1.25) \qquad \gamma_d = \frac{M_s}{V}\ kg/m^3 \quad (1.26)$$

Therefore,

$$\gamma_d = 9.81 \times 10^{-3}\rho_d\ kN/m^3 \quad (1.27)$$

1.2.3 Saturated densities

These are the bulk densities of a sample, taken from below the ground water level (GWL), hence the sample is fully saturated and the degree of saturation is unity.

From formula (1.9): $S_r = \frac{V_w}{V_v} = 1 \quad \therefore \quad V_w = V_v$

and from (1.1): $V = V_s + V_v = V_s + V_w$

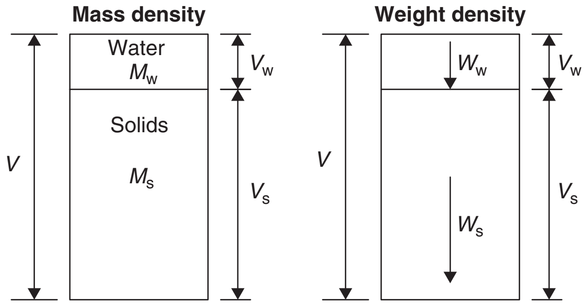


Figure 1.9

For water $\rho_w = 1\,\text{g/cm}^3$

Or $\rho_w = 1000\,\text{kg/m}^3$ and $\gamma_w = 9.81\,\text{kN/m}^3$

Total mass: $M_{sat} = M_s + \rho_w \times V_v$ and $W_{sat} = W_s + \gamma_w \times V_v$

Either $\rho_{sat} = \dfrac{M_{sat}}{V}$

Or $\rho_{sat} = \dfrac{M_s + \rho_w V_v}{V}$ kN/m³ (1.28)

Either $\gamma_{sat} = \dfrac{W_{sat}}{V}$

Or $\gamma_{sat} = \dfrac{W_s + \gamma_W V_v}{V}$ kN/m³ (1.29)

Therefore, $\gamma_{sat} = 9.81 \times 10^{-3} \rho_{sat}$ kN/m³ (1.30)

Example 1.4

The partially saturated sand in Example 1.3 was saturated by the addition of water and then dried out completely. The quantities measured were:

Dry mass: $M_s = 122.9\,\text{g} = 122.9 \times 10^{-3}\,\text{kg}$

Mass of water lost: $M_w = 29.0\,\text{g} = 29 \times 10^{-3}\,\text{kg}$

Total volume: $V = 75.4\,\text{cm}^3 = 75.4 \times 10^{-6}\,\text{m}^3$

Calculate the saturated unit weight of the sample:

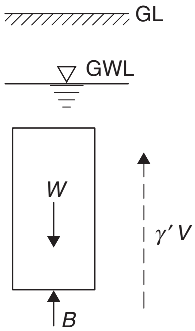
$$M_{sat} = 122.9 \times 10^{-3} + 29 \times 10^{-3} = 151.9 \times 10^{-3}\,\text{kg}$$

$$\rho_{sat} = \frac{M_{sat}}{V} = \frac{151.9 \times 10^{-3}}{75.4 \times 10^{-6}} = 2015\,\text{kg/m}^3$$

$$\therefore \gamma_{sat} = 9.81 \times 10^{-3} \times 2015 = 19.76\,\text{kN/m}^3$$

1.2.4 Submerged density (γ')

It is the saturated density of soil, taking its buoyancy into account. In other words, as long as the saturated sample remains under water, an uplift force is exerted on it in accordance with Archimedes' Principle.



Weight of sample: $W = \gamma_{sat} V \downarrow$
Buoyant force: $B = \gamma_w V \uparrow$
Resultant force on sample $= \gamma' V$

Now: $\gamma' V = W - B$
$= \gamma_{sat} \times V - \gamma_w \times V$

Figure 1.10

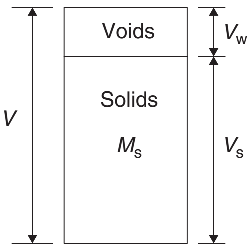
Cancelling volume V: $\boxed{\gamma' = \gamma_{sat} - \gamma_w}$ (1.31)

Note: The submerged density is to be used, when assessing the stresses induced in the soil below GWL by surface loading. This type of problem includes the determination of:

a) Effective pressure
b) Load bearing capacity of a soil.

1.2.5 Density of solids (γ_s)

It is the unit weight of the soil particles, occupying the volume V_s. Particle mass density is denoted by ρ_s.



$$\boxed{\rho_s = \frac{M_s}{V_s} \quad \text{kg/cm}^3} \quad (1.32)$$

$$\therefore \boxed{\gamma_s = 9.81 \times 10^{-3} \rho_s = \frac{W_s}{V_s} \quad \text{kN/m}^3} \quad (1.33)$$

Figure 1.11

1.2.6 Specific gravity (G_s)

It is also called "Particle specific gravity", as it shows how heavy the solids are compared to water. In other words, the weight of V_s volume of solids is compared with the same volume of water.

Weight of solids: $W_s = \gamma_s \times V_s$
Weight of water: $W_w = \gamma_w \times V_s$

and $G_s = \frac{W_s}{W_w} = \frac{\gamma_s V_s}{\gamma_w V_s}$

Cancelling V_s to obtain $\boxed{G_s = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s}{\rho_w}}$ (1.34)

Table 1.1 Average values of G_s

Soil	G_s
Clay	2.75
Silt	2.68
Sand	2.65
Gravel	2.65

1.2.7 Moisture content (m)

This expresses the mass or weight of water as a percentage of the mass or weight of solids.

$$m = 100\frac{M_w}{M_s} = 100\frac{W_w}{W_s}\Big|\% \tag{1.35}$$

Or in decimal form:

$$m = \frac{M_w}{M_s} = \frac{W_w}{W_s} \tag{1.35a}$$

Note: The quantities given in formulae (1.1) to (1.35) can be calculated from these four laboratory results:

1. Total mass of the sample M (g).
2. Mass of solids M_s (g).
3. Total volume of the sample V (cm^3).
4. Specific gravity of the soil particles G_s.

Example 1.5

Using the laboratory results of Examples 1.3 and 1.4, tabulate the calculations for all soil characteristics introduced this far, in Table 1.2. Assume G_s=2.65.

Table 1.2

Formula Number	Soil characteristic	Calculations and Results	Unit
	Total mass	M=136.2×10^{-6}	mg
	Mass of solids	M_s=122.9×10^{-6}	mg
	Total volume	V=75.4×10^{-6}	m^3
	Specific gravity	G_s=2.65	-
1.18	Total weight	$W=9.81\times10^{-3}\times 136.2\times10^{-3}=1336\times10^{-6}$	kN
1.18	Weight of solids	$W_s=9.81\times10^{-3}\times122.9\times10^{-3}=1206\times10^{-6}$	kN
1.17	Weight of water	$W_w=W-W_s=(1336-1206)\times10^{-6}=130\times10^{-6}$	kN
1.35	Water content	$m=100\times\frac{W_w}{W_s}=100\frac{130\times10^{-6}}{1206\times10^{-6}}=10.8$	%
1.20	Bulk mass density	$\rho=\frac{M}{V}=\frac{136.2\times10^{-3}}{75.4\times10^{-6}}=1806$	kg/m^3

Table 1.2 (*continued*)

	Soil characteristic	Calculations and Results	Unit
1.21	Bulk unit weight	$\gamma = 9.81\times10^{-3}\rho = 9.81\times10^{-3}\times1806 = 17.7$	kN/m³
1.25	Dry mass density	$\rho_d = \frac{M_s}{V} = \frac{122.9\times10^{-3}}{75.4\times10^{-6}} = 1630$	kg/m³
1.27	Dry unit weight	$\gamma_d = 9.81\times10^{-3}\rho_d = 9.81\times10^{-3}\times1630 = 16$	kN/m³
1.34	Unit weight of solids	$\gamma_s = G_s\times\gamma_w = 2.65\times9.81 = 26$	kN/m³
1.33	Volume of solids	$V_s = \frac{W_s}{\gamma_s} = \frac{1206\times10^{-6}}{26} = 46.4\times10^{-6}$	m³
1.32	Mass density of solids	$\rho_s = \frac{M_s}{V_s} = \frac{122.9\times10^{-3}}{46.4\times10^{-6}} = 2649$	kg/m³
1.1	Volume of voids	$V_v = V - V_s = (75.4 - 46.4)\times10^{-6} = 29\times10^{-6}$	m³
1.5	Voids ratio	$e = 100\times\frac{V_v}{V_s} = 100\times\frac{29\times10^{-6}}{46.4\times10^{-6}} = 62.5$	%
1.6	Porosity	$n = 100\times\frac{V_v}{V} = 100\times\frac{29\times10^{-6}}{75.4\times10^{-6}} = 38.5$	%
1.29	Saturated unit weight	$\gamma_{sat} = \frac{W_s + \gamma_w\times V_v}{V} = \frac{(1206 + 9.81\times29)\times10^{-6}}{75.4\times10^{-6}} = 19.8$	kN/m³
1.28	Saturated mass density	$\rho_{sat} = \frac{\gamma_{sat}}{9.81\times10^{-3}} = \frac{19.8\times10^{-3}}{9.81} = 2018$	kg/m³
1.23	Volume of water	$V_w = \frac{W_w}{\gamma_w} = \frac{130\times10^{-6}}{9.81} = 13.3\times10^{-6}$	m³
1.2	Volume of air	$V_a = V_v - V_w = (29 - 13.3)\times10^{-6} = 15.7\times10^{-6}$	m³
1.8	Degree of saturation	$S_r = 100\times\frac{V_w}{V_v} = 100\times\frac{13.3\times10^{-6}}{29\times10^{-6}} = 45.9$	%
1.31	Submerged unit weight	$\gamma' = \gamma_{sat} - \gamma_w = 19.8 - 9.81 = 10$	kN/m³

Further useful relationships can be derived by the combination of the above formulae.

1.2.8 Partially saturated soil

It has already been mentioned, that soil is normally partially saturated above ground water level that is the degree of saturation is less than unity. In fine-grained soil capillary action may saturate the soil somewhat above GWL. In any case, always assume partial saturation, unless proven otherwise.

From (1.9): $V_v = \dfrac{V_w}{S_r}$

From (1.23): $V_w = \dfrac{W_w}{\gamma_w}$

From (1.33): $V_s = \dfrac{W_s}{\gamma_s}$

From (1.34): $G_s = \dfrac{\gamma_s}{\gamma_w}$

From (1.35) $m = \dfrac{W_w}{W_s}$

$W_w = m \times W_s$

From (1.5): $e = \dfrac{V_v}{V_s} = \dfrac{\frac{V_w}{S_r}}{\frac{W_s}{\gamma_s}}$

$$= \frac{\frac{W_w}{S_r\gamma_w}}{\frac{W_s}{\gamma_s}} = \frac{1}{S_r} \times \frac{W_w}{W_s} \times \frac{\gamma_s}{\gamma_w} = \frac{1}{S_r} \times m \times G_s$$

$$\therefore \boxed{e = \frac{mG_s}{S_r}} \tag{1.36}$$

From (1.17) $W = W_s + W_w = W_s + mW_s$

$$\boxed{W = (1+m)W_s} \tag{1.37}$$

From (1.10): $V = (1+e) \times V_s$

From (1.36): $m = \dfrac{S_r e}{G_s}$

From (1.33): $\gamma_s = \dfrac{W_s}{V_s}$

From (1.34): $\gamma_w = \dfrac{\gamma_s}{G_s}$

From (1.19): $\gamma = \dfrac{W}{V} = \dfrac{(1+m)W_s}{(1+e)V_s}$

$$\gamma = \frac{1 + \frac{S_r e}{G_s}}{1+e} \times \gamma_s = \frac{G_s + S_r e}{1+e} \times \frac{\gamma_s}{G_s}$$

$$\boxed{\gamma = \left(\frac{G_s + S_r e}{1+e}\right)\gamma_w} \tag{1.38}$$

$$\gamma = \left(\frac{G_s + m \times G_s}{1+e}\right)\gamma_w$$

$$\boxed{\gamma = \left(\frac{1+m}{1+e}\right)G_s\gamma_w} \tag{1.39}$$

From (1.37): $W_s = \dfrac{W}{1+m}$

From (1.19) $\gamma = \dfrac{W}{V}$

From (1.26): $\gamma_d = \dfrac{W_s}{V} = \dfrac{\frac{W}{1+m}}{V} = \dfrac{\frac{W}{V}}{1+m}$

$$\boxed{\gamma_d = \frac{\gamma}{1+m}} \tag{1.40}$$

From (1.38): $\gamma = \left(\dfrac{G_s + S_r e}{1+e}\right)\gamma_w$

For dry soil $S_r = 0$ and $\gamma = \gamma_d$

$$\boxed{\gamma_d = \left(\frac{G_s}{1+e}\right)\gamma_w} \tag{1.41}$$

Note: Dry density is an important factor in the compaction of soils.

For fully saturated soil $S_r = 1$ and $\gamma = \gamma_{sat}$

From (1.38): $\gamma = \left(\dfrac{G_s + S_r e}{1+e}\right)\gamma_w$ hence $\boxed{\gamma_{sat} = \left(\dfrac{G_s + e}{1+e}\right)\gamma_w}$ (1.42)

From (1.31):

$$\gamma' = \gamma_{sat} - \gamma_w = \left(\frac{G_s + e}{1+e}\right)\gamma_w - \gamma_w$$

$$= \left(\frac{G_s + e - 1 - e}{1+e}\right)\gamma_w$$

Hence the submerged density:

$$\gamma' = \left(\frac{G_s - 1}{1+e}\right)\gamma_w \tag{1.43}$$

Table 1.3 **(Comparison of formulae)**

Partially saturated soil	Saturated soil	Dry soil
$S_r<1$	$S_r=1$	$S_r=0$
$V_w<V_v$	$V_w=V_v$	$V_w=0$
$m=\frac{S_r e}{G_s}$	$m=\frac{e}{G_s}$	$m=0$
$W=(1+m)\ W_s$	$W=(1+m)\ W_s$	$W=W_s$
$\gamma=\left(\frac{G_s+S_r e}{1+e}\right)\gamma_w$	$\gamma_{sat}=\left(\frac{G_s+e}{1+e}\right)\gamma_w$	
$\gamma_d=\frac{\gamma}{1+m}$	$\gamma_d=\frac{\gamma_{sat}}{1+m}$	$\gamma_d=\left(\frac{G_s}{1+e}\right)\gamma_w$

Example 1.6

Clay of G_s=2.8 was compacted into six standard ASTM moulds at different water contents. The internal volume of each mould was 944 cm^3. The total and dry masses of samples were found to be:

Table 1.4

Quantity	Sample 1	2	3	4	5	6
M (g)	1743	1827	1880	1890	1880	1834
M_s (g)	1449	1502	1533	1542	1510	1467

1. Calculate the quantities contained in Table 1.2 (Example 1.5) for sample No.1, in both mass and weight units. Show calculations in Table 1.5.
2. Complete Table 1.6 by evaluating for each sample the:
   a. Water content (m %)
   b. Bulk unit weight (γ kN/m^3)
   c. Dry unit weight (γ_d kN/m^3)
   d. Voids ratio (e %)
   e. Volume of air (V_a cm^3)
3. Plot γ, γ_d, e and V_a against m on Graph 1.1, indicating their variation with increasing water content.

Table 1.5 For sample No. 1

In mass units		In weight units	
$M=1743\,g=1.743$	kg	$W=9.81\times10^{-3}\times1.743=0.0171$	kN
$M_s=1449\,g=1.449$	kg	$W_s=9.81\times10^{-3}\times1.449=0.0142$	kN
$V=944$	cm³	$V=944\times10^{-6}$	m³
$G_s=2.8$	-	$G_s=2.8$	-
$M_w=M-M_s=1.743-1.449=0.294$	Kg	$W_w=W-W_s$ $=0.0171-0.0142=0.0029$	kN
$m=100\times\frac{M_w}{M_s}=\frac{100\times0.294}{1.449}=20.3$	%	$m=100\times\frac{W_w}{W_s}=\frac{100\times0.29}{14.2}=20.4$	%
$\rho=\frac{M}{V}=\frac{1.743}{944\times10^{-6}}=1846$	kg/m³	$\gamma=9.81\times10^{-3}\times1846=18.1$ $\gamma=\frac{W}{V}=\frac{0.0171}{944\times10^{-6}}=18.1$	kN/m³ kN/m³
$\rho_d=\frac{M_s}{V}=\frac{1.449}{944\times10^{-6}}=1535$	kg/m³	$\gamma_d=9.81\times10^{-3}\times1535=15.1$	kN/m³
$\rho_d=\frac{\rho}{1+\frac{m}{100}}=\frac{1846}{1.203}=1535$	kg/m³	$\gamma_d=\frac{W_s}{V}=\frac{0.0142}{944\times10^{-6}}=15.1$	kN/m³
$\rho_s=G_s\rho_w=2.8\times1000=2800$	kg/m³	$\gamma_s=9.81\times10^{-3}\times2800=27.5$ $\gamma_s=G_s\gamma_w=2.8\times9.81=27.5$	kN/m³ kN/m³
$V_s=\frac{M_s}{\rho_s}=\frac{1.449}{2800}\times10^{6}=518$	cm³	$V_s=\frac{W_s}{\gamma_s}=\frac{0.0142}{27.5}=516\times10^{-6}$	m³
$V_v=V-V_s=944-518=426$	cm³	$V_v=V-V_s=(944-516)\times10^{-6}$ $=428\times10^{-6}$	m³
$e=100\times\frac{V_v}{V_s}=\frac{100\times426}{518}=82$	%	$e=82$	%
$n=100\times\frac{e}{1+e}=\frac{100\times0.82}{1.82}=45$	%	$n=45$	%
$\rho_{sat}=\frac{M_s+\rho_w V_v}{V}$ $=\frac{1.449+10^3\times428\times10^{-6}}{944\times10^{-6}}=1981$	kg/m³	$\gamma_{sat}=9.81\times10^{-3}\times1981=19.4$ $\gamma_{sat}=\frac{W_s+\gamma_w V_v}{V}=$ $\frac{0.0142+9.81\times428\times10^{-6}}{944\times10^{-6}}=19.5$	 kN/m³ kN/m³
$V_w=\frac{M_w}{\rho_w}=\frac{0.294}{1000}\times10^{-6}=294$	cm³	$V_w=\frac{W_w}{\gamma_w}=\frac{0.0029}{9.81}=296\times10^{-6}$	m³
$V_a=V_v-V_w=426-294=132$	cm³	$V_a=V_v-V_w$ $=(428-296)\times10^{-6}=132\times10^{-6}$	m³
$S_r=100\times\frac{V_w}{V_v}=\frac{100\times294}{426}=69$	%	$S_r=\frac{100\times296\times10^{-6}}{428\times10^{-6}}=69$	%
$\rho'=\rho_{sat}-\rho_w=1981-1000=981$	kg/m³	$\gamma'=\gamma_{sat}-\gamma_w=19.5-9.81=9.69$	kN/m³

Revision

In mass units | **In weight units**

$$\rho_w = 1000\frac{\text{kg}}{\text{m}^3} = 1\,\text{g/cm}^3 = 1\,\text{Mg/m}^3 \qquad \gamma_w = 9.81\,\text{kN/m}^3$$

$$\rho = \frac{10^3\gamma}{9.81}\text{kg/m}^3 \quad (1.21) \quad \gamma = 9.81\times10^{-3}\rho\,\text{kN/m}^3$$

$$M = \frac{10^3 W}{9.81}\text{kg} \quad (1.18) \quad W = 9.81\times10^{-3}M\ \text{kN}$$

Table 1.6 could be completed by calculating all of the quantities in succession as in Table 1.5. Instead, formulae are derived, where necessary, for the determination of the five unknowns.

From (1.35)
$$m = 100\frac{M_w}{M_s} = \frac{100\,M - M_s}{M_s} = 100\left(\frac{M}{M_s} - 1\right)(\%)$$

From (1.21):
$$\gamma = 9.81\times10^{-3}\rho = 9.81\times10^{-3}\times\frac{M}{V}\left(\text{kN/m}^3\right)$$

From (1.40):
$$\gamma_d = \frac{\gamma}{1+m}(\text{kN/m}^3)$$

From (1.34): $\gamma_s = G_s\times\gamma_w$

From (1.33): $V_s = \dfrac{W_s}{\gamma_s}$

$$\boxed{V_s = \frac{W_s}{G_s\gamma_w}} \text{ or } \boxed{V_s = \frac{9.81\times10^{-3}M_s}{G_s\gamma_w}(\text{m}^3)} \quad (1.44)$$

From (1.5):
$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \boxed{\frac{V}{V_s} - 1} \text{ and } \boxed{V_s = \frac{V}{1+e}} \quad (1.45)$$

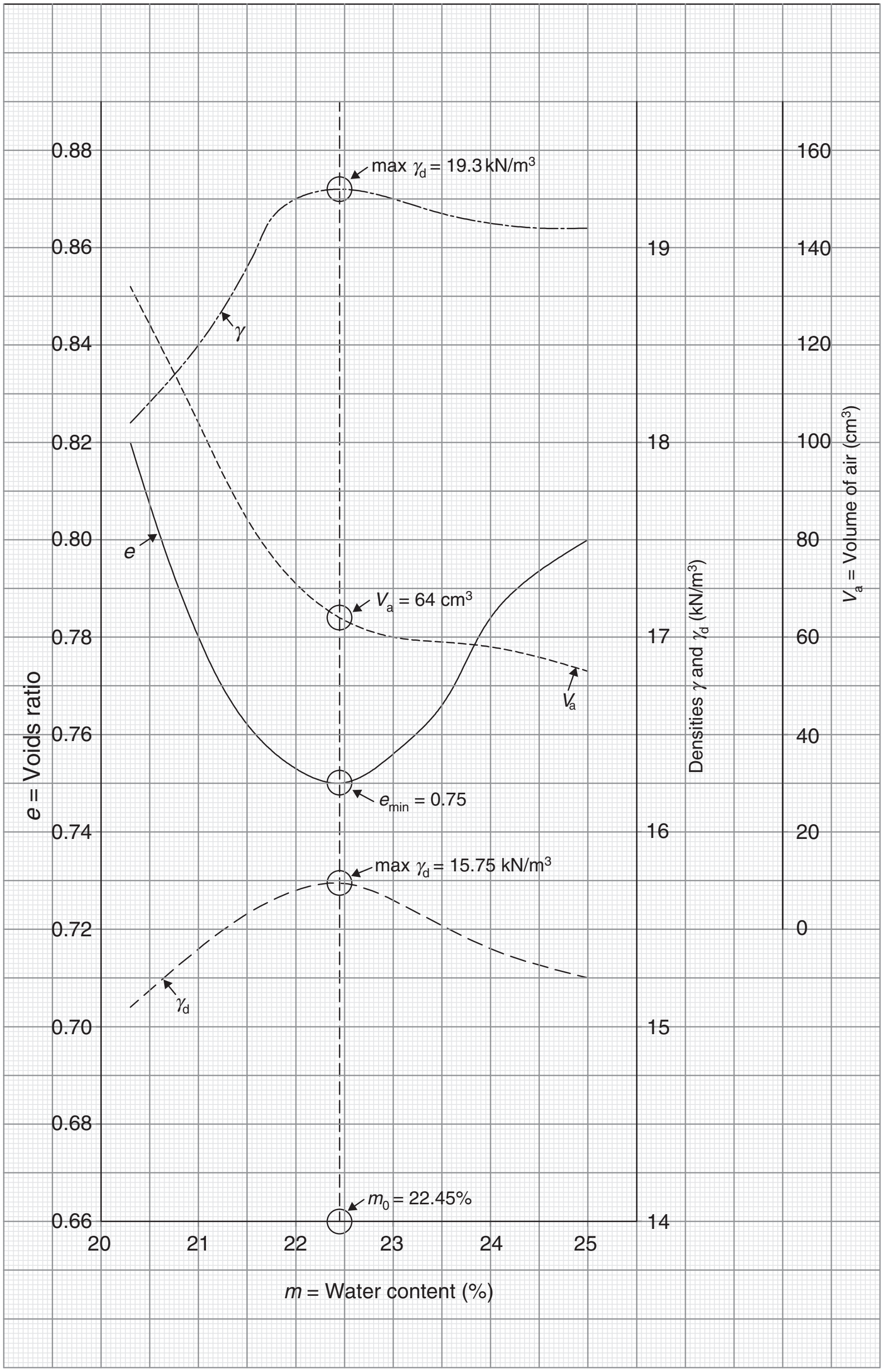
From (1.2):
$$V_a = V_v - V_w = V - V_s - V_w = V - \frac{V}{1+e} - \frac{W_w}{\gamma_w}$$

$$\boxed{V_a = \frac{eV}{1+e} - \frac{W_w}{\gamma_w}} \text{ or } \boxed{V_a = \frac{eV}{1+e} - \left(\frac{9.81\times10^{-3}M_w}{\gamma_w}\right)\ \text{m}^3} \quad (1.46)$$

But $\gamma_w = 9.81\,(\text{kg/m}^3)$, $V = 944\,(\text{cm}^3)$, M and M_s (kg)

$$\therefore \boxed{V_a = \frac{eV}{1+e} - (M - M_s)\times10^{-3}\ \text{cm}^3}$$

$$\text{or } \boxed{V_a = \frac{eV}{1+e} - 10^3\times M_w\ \text{cm}^3} \quad (1.46\text{a})$$


0.88
0.86
0.84
0.82
0.80
0.78
0.76
0.74
0.72
0.70
0.68
0.66
e = Voids ratio
20
21
22
23
24
25
m = Water content (%)
max γ_d = 19.3 kN/m^3
γ
e
V_a = 64 cm^3
V_a
e_{min} = 0.75
max γ_d = 15.75 kN/m^3
γ_d
m_0 = 22.45%
19
18
17
16
15
14
Densities γ and γ_d (kN/m^3)
160
140
120
100
80
60
40
20
0
V_a = Volume of air (cm^3)

Graph 1.1

Table 1.6 (See Graph 1.1)

Quantity		Sample					
		1	2	3	4	5	6
M	kg	1.743	1.827	1.855	1.846	1.838	1.834
M_s	kg	1.449	1.502	1.514	1.496	1.479	1.467
m	%	20.3	21.6	22.5	23.4	24.3	25.0
γ	kN/m³	18.1	19.0	19.3	19.2	19.1	19.1
γ_d	kN/m³	15.1	15.6	15.73	15.55	15.4	15.25
e	-	0.82	0.76	0.75	0.77	0.79	0.80
V_a	cm³	132.5	82.6	62.3	59.7	56.8	53.1

Notes:

1. As m increases, the air voids hence e decrease due to compaction, resulting in higher densities.
2. At the "optimum moisture content" m_o, the densities reach their maximum values, whilst e attains its minimum. The volume of air is also reduced considerably. In this example, the changes are:

Table 1.7

			Change	
	m=20%	m_o=22.45%	+2.45	%
γ	18.1	19.3	+1.2	%
γ_d	15.1	15.75	+0.65	%
e	0.82	0.75	−0.07	–
V_a	132.5	64	−68.5	cm³

3. If the water content is increased beyond the optimum value, the soil becomes less compact. This is indicated by the decreasing values of γ and $\gamma_{d.}$ The increase in the volume of water in the voids is reflected in the changed value of e.
4. It is not possible to compact partially saturated soil so, that all air is expelled (V_a=0). In this example, the minimum amount of air voids remaining beyond m=25% is about V_a=50 cm³.

1.2.9 Relative density (D_r)

Granular soil, sand in particular, is often described as either loose or dense. The relative density, alternatively called "density index" compares the voids ratio of sand, in its natural state, with those in its most loose and most dense states. It is formulated as:

$$D_r = 100\left(\frac{e_{max} - e}{e_{max} - e_{min}}\right)\% \qquad (1.47)$$

Where e = in-situ voids ratio
e_{min} = voids ratio in loosest state
e_{max} = voids ratio in densest state

The values of D_r tabulated below should be taken as indicative only, because of the uncertainties in obtaining minimum and maximum voids ratios or densities.

Table 1.8

Description of soil	D_r (%)
Very loose	0–15
Loose	15–35
Medium	35–65
Dense	65–85
Very dense	85–100

D_r can be expressed in terms of dry unit weight by means of formula (1.41) from which:

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1$$

$$\text{and } e_{max} = \frac{G_s \gamma_w}{\min \gamma_d} - 1$$

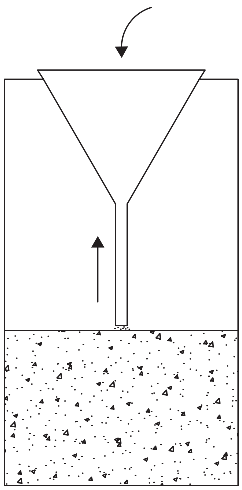
$$\text{Also, } e_{min} = \frac{G_s \gamma_w}{\max \gamma_d} - 1$$

Substituting these into formula (1.47) we get:

$$D_r = 100 \times \frac{(\gamma_d - \min \gamma_d) \max \gamma_d}{(\max \gamma_d - \min \gamma_d) \gamma_d} \% \quad (1.48)$$

Determination of e_{max} and min γ_d

Dry sand is poured slowly into a cylinder through a funnel, keeping its end near the surface of the material to prevent compaction. When the cylinder is full, measure the weight of the contained sand.



V = volume of the cylinder (m^3)

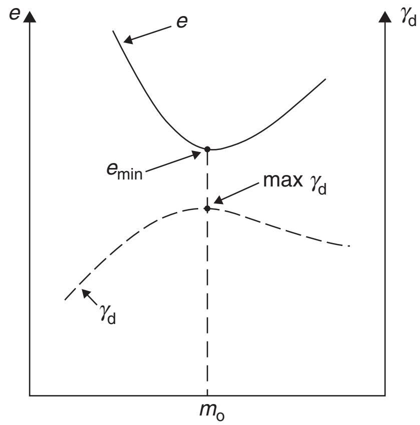
W_s = weight of soil (kN)

From formula (1.26): $\min \gamma_d = \frac{W_s}{V}$

Hence from (1.41): $e_{max} = \frac{V G_s \gamma_w}{W_s} - 1$

Figure 1.12

Determination of e_{min} and max γ_d
The sand is compacted into cylinders at different water contents. Plot the voids ratio and dry density against moisture content as in Example 1.6.



m_o = optimum moisture content, where:
Voids ratio = e_{min}
Dry density = max γ_d

Figure 1.13

Example 1.7

The results of density test conducted on sand were:

$e = 58.2\%$, $e_{max} = 62.4\%$ and $e_{min} = 41.5\%$

Calculate the relative density of the sand.

$$D_r = 100 \times \left(\frac{e_{max} - e}{e_{max} - e_{min}} \right) = 100 \times \left(\frac{62.4 - 58.2}{62.4 - 41.5} \right) = 20\%$$

The sand can be considered as loose, hence not load-bearing. It is not suitable for foundation construction.

1.3 Alteration of soil structure by compaction

It often occurs that soil has to be excavated at one place and deposited elsewhere for various reasons. Some of the reasons are:

1. Construction of embankments.
2. Construction of large horizontal areas for housing, roads, runways, etc.
3. Exchanging soil of unsuitable bearing strength with strong, compacted soil, prior to erection of structures.

The excavated soil is in loose condition; hence it has to be compacted during deposition. The purpose of compaction is to:

1. increase density by decreasing the air voids, hence the voids ratio;
2. decrease permeability by the reduction of voids;
3. increase shear strength by packing the soil particles closer.

The soil is partially saturated during compaction. The process, therefore, must not be confused with consolidation, where water is expelled from fully saturated soil, whereas in compaction, air is expelled from partially saturated one. In effect, the voids ratio in well compacted soil is low and the grains are packed so, that future consolidation settlement is minimized.

The efficiency or rather the degree of compaction is measured in terms of either dry mass density (ρ_d) or dry unit weight (γ_d) and moisture content (m%). Some amount of water added helps compaction by reducing surface tension. However, if m% is in excess of the so called "Optimum moisture content (m_o)", then the void ratio begins to increase and the soil becomes looser. The variation of γ_d, γ, e and V_a with m% is illustrated in Graph 1.1.

Soil stabilization is carried out in five stages:

1. Retrieval of soil samples from the area to be quarried.
2. The samples are compacted in a laboratory and the maximum value of γ_d at the m_o% is obtained.
3. The engineer or architect specifies these values in the earthworks contract.
4. The contractor should compact the imported soil as specified.
5. The engineer or architect initiates spot checks on site, in order to determine the in-situ dry density, hence the efficiency of the compaction.

1.3.1 Laboratory compaction tests (BS 1377-4: 1990)

There are three British standard and two American tests in use:

1. B.S. 'light' –2.5 kg rammer test
2. B.S. 'heavy' – 4.5 kg rammer test
3. B.S. vibrating hammer test
4. American (ASTM) light and heavy tests.

The British standard tests are outlined below. Figure 1.14 shows the equipment used.

B.S. 'light' test

It is carried out, using either a 1000 cm^3 or a 2305 cm^3 (CBR) mould and a 2.5 kg rammer. (See Figure 1.14). The procedure is as follows:

Step 1: Compact the soil in three layers, by dropping the rammer from a height of 300 mm. The number of drops (or blows) depends on the mould used.
1000 cm^3 mould requires 27 blows/layer
2305 cm^3 mould requires 62 blows/layer

Step 2: Obtain the mass of the soil.

Step 3: Measure its moisture content.

Step 4: Mix a little water to the soil.

Step 5: Repeat the procedure from step 1 at least five times.

Step 6: Calculate the dry unit weight and the volume of air voids for each moisture content and plot the compaction curve.

B.S. 'heavy' test

In this test a 4.5 kg rammer is applied.

Step 1: Compact the soil in five layers. The number of blows again depends on the mould used.
1000 cm^3 mould requires 27 blows/layer
2305 cm^3 mould requires 62 blows/layer
The rammer is dropped from a height of 450 mm.

Steps 2–6: As for the 'light'test.

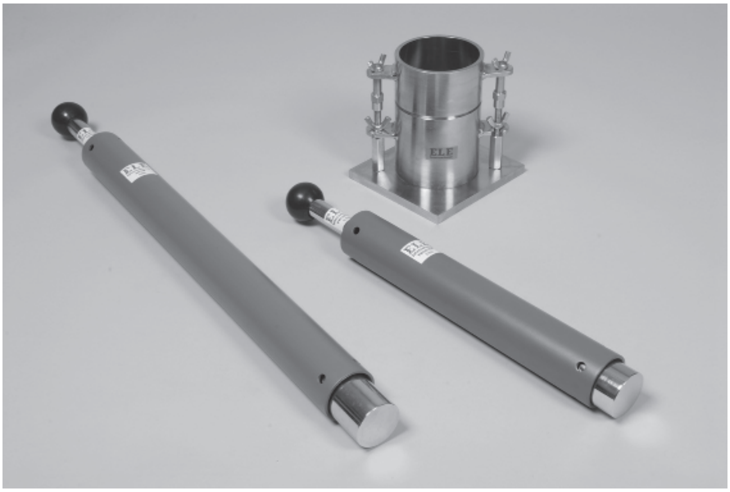


Figure 1.14 BS Compaction mould and rammers EL24-9002.
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B.S. vibrating hammer test

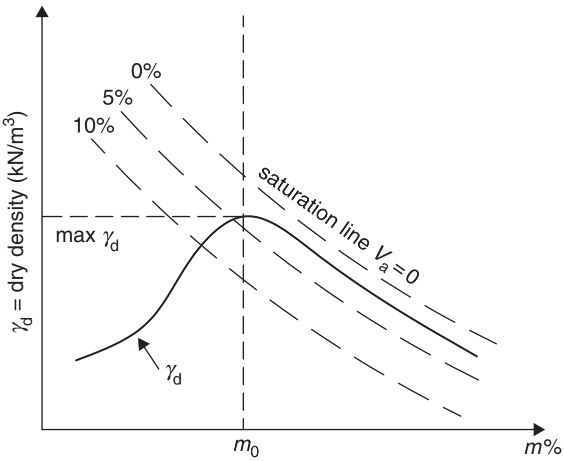
The 2305 cm^3 CBR (California Bearing Ratio) mould is used in this test, which is applicable to granular soil only. The soil is compacted in three layers. Each layer is vibrated for one minute by a 32 – 41 kg vibrating hammer.

Notes:

a) The ASTM tests are similar in principle to the B.S. ones. The difference is in the size of the moulds, number of layers, mass of the rammer and number of blows per layer.
b) The CBR mould is normally used in the CBR test, which helps in the determination of the strength of soil layers under roads and pavements.

Presentation of results

The usual way to present the outcome of a compaction test is by plotting the dry unit weight against moisture content. Also, curves indicating 0, 5 and 10% air voids are drawn to determine the efficiency of the compaction.



Where:
m_o = Optimum moisture content, corresponding to the maximum value of dry unit weight (max γ_d).

Figure 1.15

The dry unit weight/moisture graph is drawn by means of formula (1.40):

$$\gamma_d = \frac{\gamma}{1+m}$$

The dry unit weight has to be expressed in terms of m and percentage of air voids so, that the saturation lines may be drawn. The first step is to define the volume of air as a percentage of the total volume:

$$\boxed{P_a = 100 \times \frac{V_a}{V} \quad \%} \qquad (1.49)$$

Of course, P_a is expressed in decimals in formulae just like *m*, *e*, *n*, or *s*, that is $P_a = \frac{V_a}{V}$

From (1.3): $V = V_s + V_w + V_a$

From (1.33): $V_s = \frac{W_s}{\gamma_s}$

From (1.34): $\gamma_s = G_s \times \gamma_w$

$V_s = \frac{W_s}{G_s \gamma_w}$

From (1.23): $V_w = \frac{W_w}{\gamma_w}$

From (1.35a): $W_w = m \times W_s$

$$V = \frac{W_s}{G_s \gamma_w} + \frac{W_w}{\gamma_w} + V_a$$

$$I = \frac{W_s}{V G_s \gamma_w} + \frac{m W_s}{V \gamma_w} + \frac{V_a}{V}$$

$$I = \frac{\gamma_s}{G_s \gamma_w} + \frac{m \gamma_d}{\gamma_w} + \frac{V_a}{V}$$

$$I = \frac{\gamma_d}{\gamma_w} \times \left(\frac{1}{G_s} + m \right) + \frac{V_a}{V}$$

Expressing

$$\gamma_d = \frac{\left(1 - \frac{V_a}{V}\right)\gamma_w}{\frac{1}{G_s} + m}$$

Or

$$\boxed{\gamma_d = \frac{(1-P_a) G_s \gamma_w}{1+mG_s} \quad \text{kN/m}^3} \qquad (1.50)$$

For $P_a = 0\%$

$$\boxed{\gamma_d = \frac{G_s \gamma_w}{1+mG_s} \quad \text{kN/m}^3} \qquad (1.51)$$

Example 1.8

Table 1.9 contains the results of a compaction test carried out on soil to be placed in a 3 m thick layer under a heavy industrial building.

The dry unit weight (γ_d) is plotted against *m*% on Graph 1.2 and the results noted. In this example:

$$\max \gamma_d = 16.85 \text{kN} / \text{m}^3$$

$$m_o = 19.15\%$$

Table 1.9

Compaction							
Sample number		**1**	**2**	**3**	**4**	**5**	**6**
Mass of wet soil + mould	g	6141	6498	6602	6556	6441	7271
Mass of mould	g	1900	1900	1900	1900	1900	1900
M_c = mass of wet soil	g	4241	4598	4702	4656	4541	5371
V = volume of mould	cm^3	2305	2305	2305	2305	2305	2305
Mass density: $\rho = M_c/V$	g/cm^3	1.84	1.99	2.04	2.02	19.70	2.33
Unit weight: $\gamma = 9.81\,\rho$	kN/m^3	18.1	19.5	20.0	19.8	19.3	22.9
Moisture content ($G_s = 2.7$)							
Mass of wet soil + container	g	173.1	140.7	121.2	129.3	142.7	153.6
Mass of container	g	8.71	10.28	7.95	8.92	9.51	8.53
M = mass of wet soil	g	164.39	130.42	113.25	120.38	133.19	145.07
M_s = mass of dry soil	g	143.01	106.30	95.14	98.02	106.20	113.16
Mass of water: $M_w = M - M_s$	g	21.38	24.12	18.11	22.36	26.99	31.91
Water content: $m = 100\,M_w/M_s$	%	14.95	17.40	19.03	22.81	25.41	28.20
Dry unit weight: $\gamma_d = \gamma/(1+m)$	kN/m^3	15.7	16.7	16.9	16.2	15.4	17.8
Air voids lines							
$\gamma_d = \dfrac{(1-P_a)G_s\gamma_w}{1+mG_s}$ $P_a = 0\%$	kN/m^3	19.3	17.9	17.2	16.4	15.7	15.0
$P_a = 5\%$		17.9	17.0	16.6	15.6	14.9	14.8
$P_a = 10\%$		17.0	16.2	15.8	14.8	14.1	13.5

The volume of air in the soil at maximum dry density can be determined by interpolating between the 0% and 5% lines. From Graph 1.2:

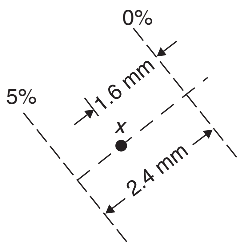


Figure 1.16

if 2.4 mm represents 5%
Then 1.6 mm represents P_a%

or $\dfrac{2.4}{5} = \dfrac{1.6}{P_a}$ $\quad\therefore\quad P_a = 3.3\%$

$$V_a = 0.033 \times V = 0.033 \times 2305 = 76.8\ cm^3$$

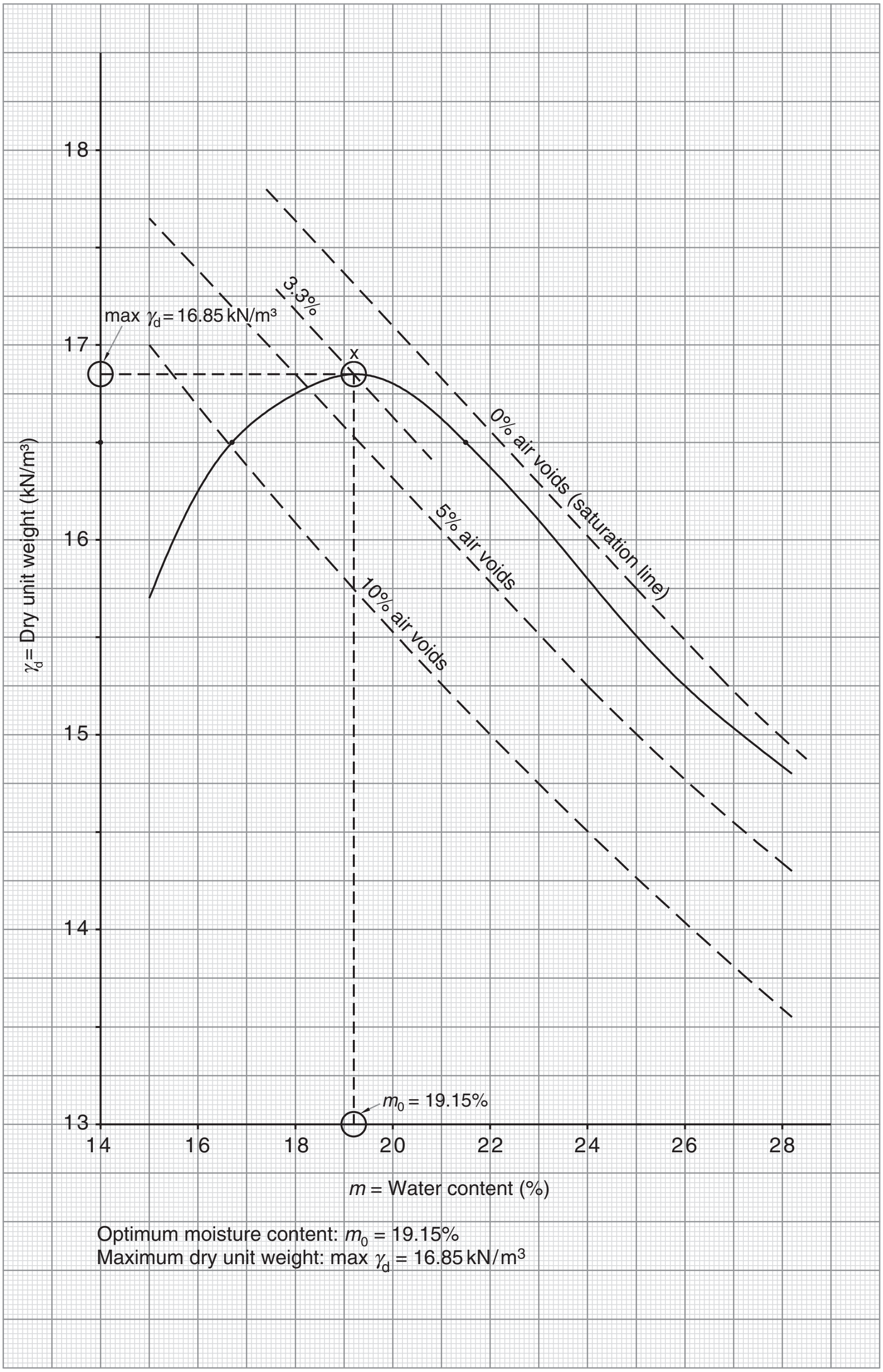
Alternatively, by formula (1.50):

$$\max\gamma_d = \frac{(1-P_a)G_s\gamma_w}{1+m_o \times G_s} \quad \text{or} \quad 16.85 = \frac{(1-P_a)\times 2.7 \times 9.81}{1+0.1915\times 2.7}$$

$$1 - P_a = \frac{16.85\times(1+0.1915\times 2.7)}{2.7\times 9.81} = 0.965$$

$$P_a = \frac{V_a}{V} = 1 - 0.965 = 0.0359\ (3.5\%)$$

$$\therefore \quad V_a = 0.035 \times V = 0.035 \times 2305 = 80.7\,cm^3$$


18
17
16
15
14
13
max γ_d = 16.85 kN/m³
3.3%
x
0% air voids (saturation line)
5% air voids
10% air voids
m_0 = 19.15%
γ_d = Dry unit weight (kN/m³)
14
16
18
20
22
24
26
28
m = Water content (%)
Optimum moisture content: m_0 = 19.15%
Maximum dry unit weight: max γ_d = 16.85 kN/m³

Graph 1.2

Alternatively, V_a can be determined from the basic formulae.

Known: $m_o = 19.15\%$ and $\gamma_d = 16.85\ \text{kN/m}^3$

From (1.40): $\gamma = (1+m_o)\ \gamma_d = 1.1915 \times 16.85 = 20.08\ \text{kN/m}^3$

From (1.19): $W = \gamma V = 20.08 \times \left(\dfrac{2305}{10^6}\right) = 46.3 \times 10^{-3}\ \text{kN}$

From (1.37): $W_s = \dfrac{W}{1+m_o} = \dfrac{46.3 \times 10^{-6}}{1.1915} = 38.9 \times 10^{-3}\ \text{kN}$

From (1.17): $W_w = W - W_s = (46.3 - 38.9) \times 10^{-3} = 7.4 \times 10^{-3}\text{kN}$

From (1.3): $V_a = V - V_w - V_s$

But, $V_w = \dfrac{W_w}{\gamma_w} = \dfrac{7.4 \times 10^{-3}}{9.81} = 0.754 \times 10^{-3}\text{m}^3$

$= (0.754 \times 10^{-3}) \times 10^6\ \text{cm}^3$

$= 754\ \text{cm}^3$

And $V_s = \dfrac{W_s}{\gamma_s} = \dfrac{W_s}{G_s \gamma_w} = \dfrac{38.9 \times 10^{-3}}{2.7 \times 9.81} = 1.4686 \times 10^{-3}\ \text{m}^3$

$= (1.4686 \times 10^{-3}) \times 10^6\ \text{cm}^3$

$= 1469\ \text{cm}^3$

Therefore, $V_a = 2305 - 754 - 1469 = 82\ \text{cm}^3$

In percentage terms: $P_a = 100 \times \dfrac{82}{2305} = 3.6\%$

The three results, therefore, are comparable.

1.3.2 Practical considerations

It is very much unlikely, or rather impossible to achieve the same compaction in the field as predicted by the laboratory results. It is somewhat difficult to maintain the soil at optimum water content because of rain or very dry weather. It is therefore unreasonable to expect a contractor to produce the exact dry density shown on a compaction curve. For

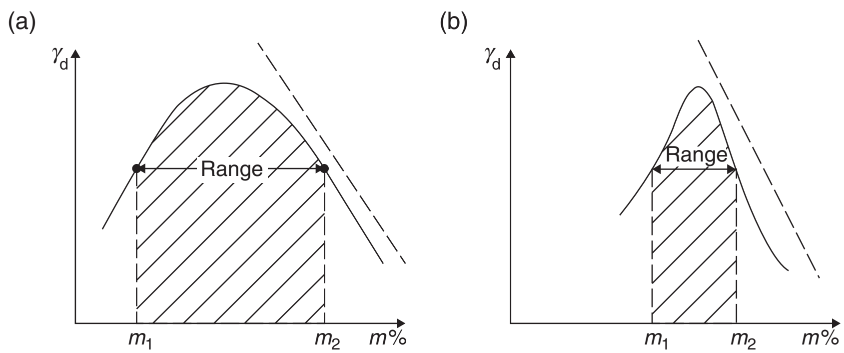


Figure 1.17

this reason, the specification should state an acceptable range of moisture content. This range is chosen by observing the variation of γ_d with m on the compaction curve. For example, the dry bulk density for m=16.7% and m=21.6% is γ_d=16.5 kN/m^3 on Graph 1.2. For this range of moisture content, the deviation from the maximum value is 16.85–16.5=0.35 kN/m^3. Should this small variation be acceptable, this range would be specified.

In general, the flatter is the compaction curve the less sensitive is the dry density to the variation in m%.

Soils with flatter curves (Figure 1.17a) need less compactive effort than those with steeper ones (Figure 1.17b). On the other hand, however higher values of dry density can be achieved by soils sensitive to moisture content variations.

1.3.3 Relative compaction (C_r)

The allowed deviation described above may be specified by the relative compaction (C_r).

$$\boxed{C_r = 100 \times \frac{\overline{\gamma}_d}{\max \gamma_d} \quad \%} \tag{1.52}$$

where $\overline{\gamma}_d$ = dry unit weight to be achieved in the field.

In example 1.7, for the range discussed above:

$$C_r = \frac{100 \times 16.5}{16.85} = 98\%$$

Therefore, for the latitude of moisture content variation only 2% of the density was lost. It is not desirable to depart too far from m_o% because:

1. If $m < m_o$ then more air voids can remain in the soil, after compaction, than intended.
2. If $m > m_o$, then the additional moisture could make the soil weaker than intended.

1.3.4 Compactive effort

There are various types of compactors used in the field, depending on the soil treated. The efficiency of their compacting effort is a function of the:

1. thickness of the compacted layer
2. number of passes over the layer
3. mass of the compactor
4. moisture content of the soil.

Types of compaction plant:

vibratory roller
smooth-wheeled roller
sheepfoot roller
pneumatic-tyred roller
grid roller
power rammer.

The thickness of each layer and the number of passes depend on the mass of the plant used. On the average, 4–5 passes are sufficient, as long as the roller is of large enough mass. Detailed information on the types and capability of compaction plants is available in the relevant literature. In general, vibrating rollers are applicable to cohesive as well as well graded granular soil, as long as their mass is over 1800 kg. The thickness of layers is within the 150–300 mm range.

1.3.5 Under- and overcompaction

Dry density increases, whilst the optimum moisture content decreases, and vice versa, with compactive effort.

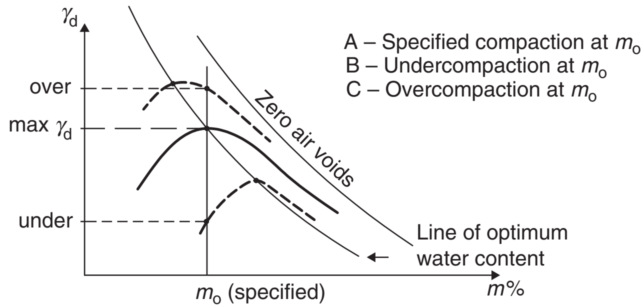


Figure 1.18

Undercompaction means that the compactive effort is less than necessary, when the soil is at the specified moisture content. It is now too dry (at point B) and not at its maximum dry density (point D) for the particular effort.

Conversely, overcompaction means unnecessary extra effort, when the soil is worked at the specified m_o%. Although the dry density is increased (point C), the soil becomes wetter than it would be at the new optimum (point E), hence it becomes weaker than intended.

1.3.6 Site tests of compaction

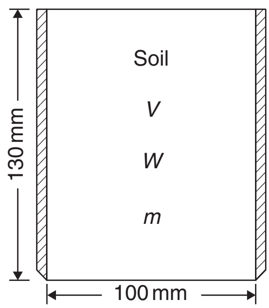
It is imperative to carry out daily checks on the dry density achieved by the compactor. There are five well known in-situ methods to do this:

1. core cutter method
2. sand replacement method
3. water displacement method
4. penetration needle measurement
5. nuclear radiation

Of these, only the first two will be outlined.

1.3.6.1 Core cutter method

It is applicable to cohesive soils. A cylindrical steel cutter of volume V is driven into the layer. The soil mass is measured and the dry density determined on site. The moisture content is obtained normally by the drying-out process. The cutter shown is pressed into the soil by a rammer-dolly assembly.



Core cutter

$$\gamma = \frac{W}{V}$$

$$\gamma_d = \frac{\gamma}{1+m}$$

Figure 1.19

1.3.6.2 Sand replacement method

It is used mainly for granular soil, as the dimensions of the hole dug for a sample are irregular and cannot be measured normally.

Calibration of the apparatus (Figure 1.20):

a) Fill the pourer with sand.
b) Place the pourer on a flat surface and release sand, filling the cone. Weigh the sand released (M_1).

The bulk mass density of the sand (ρ_s) has to be determined.

c) Fill the pourer with sand and weigh it (M_2).
d) Place the pourer on the calibrating cylinder and release sand to fill it as well as the cone.
e) Weigh the pourer (M_3).
f) Calculate $\rho_s = \dfrac{M_2 - M_3 - M_1}{V_c}$ where V_c=volume of cylinder

Measurement of soil mass density on site:

g) Excavate a round hole, approximately 100 mm in diameter.
h) Weigh the excavated soil (M_4)
i) Completely fill the pourer with sand and place it over the hole.
j) Release sand until it fills the hole.
k) Weigh the pourer (M_5).
l) Calculate the mass of sand filling the hole: $M_s = M_2 - M_1 - M_5$
m) Calculate the volume of the hole:

$$V = \frac{M_s}{\rho_s}$$

Figure 1.20 Photograph of ELE29-4000.100 calibrating container, metal tray. Reproduced by permission of ELE International.

n) Calculate the mass density of the soil sample:

$$\rho = \frac{M_4}{V}$$

o) Determine the moisture content (m) and hence the dry density ρ_d by (1.40)

1.4 California bearing ratio (CBR) test

The test is entirely empirical and the CBR value depends on the degree of compacted that is on the dry density and the moisture content of the soil to be tested. The result is used in the design of pavements, roads and air-field runways.

Definition
The CBR value of a material is the ratio of the force required to penetrate the compacted soil to a standard force, causing the same penetration. In other words, if the standard force is 100%, then the measured force is CBR% i.e.

$$\boxed{\text{CBR} = 100 \times \left(\frac{\text{Measured force}}{\text{Standard force}} \right) \%} \tag{1.53}$$

It can also be considered as an index of shear strength of a soil in known state of compaction.

Outline of the laboratory test
The sketch of the CBR apparatus is shown below:

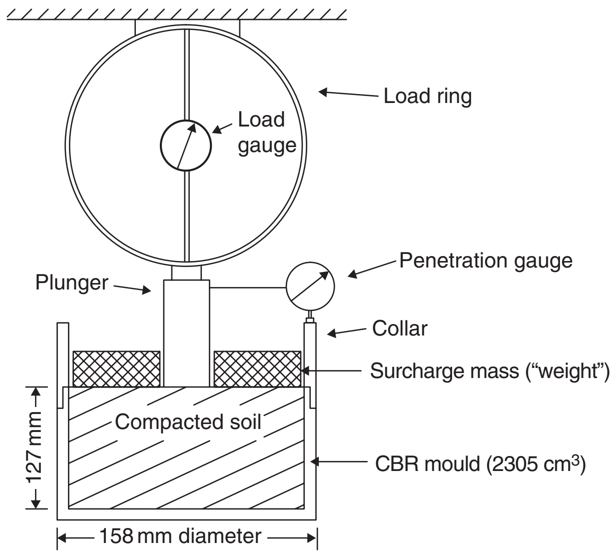


Figure 1.21

Step 1: Compact the soil in five layers into the mould by either of the 2.2 kg or 4.5 kg rammer.

Step 2: Place surcharge rings on the top of the soil, if necessary, to simulate possible overburden pressure.

Step 3: Seat the 49.6 mm diameter plunger on the surface of the soil and apply seating load according to the expected CBR value:

Table 1.10

CBR %	Seating load (N)
≤30	50
>30	250

Step 4: Start motor and read the load gauge (Q) at every 0.25 mm indicated on the penetration gauge up to 7.5 mm maximum penetration.

Step 5: Remove the soil from the mould. Obtain its moisture content and dry density.

Step 6: Calculate the applied force (P) from:

$$\boxed{P = \frac{Qk}{1000}\,\text{kN}} \tag{1.54}$$

where k = load ring factor (N / division).

Step 7: Plot the value of (P) against penetration (δ). The curve can have either of the two shapes shown:

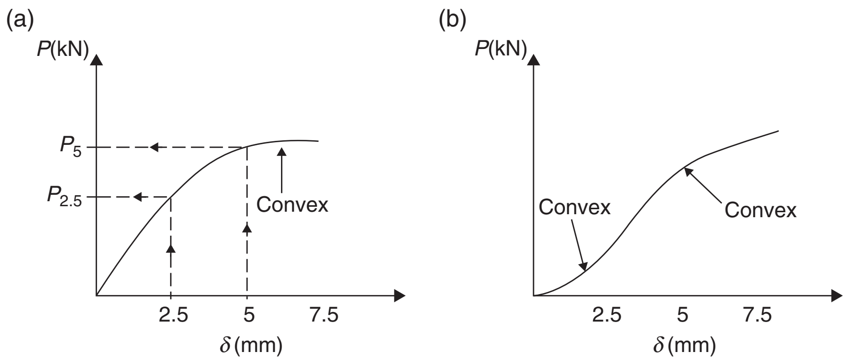


Figure 1.22

Step 8: Calculate the CBR values by comparing the loads at δ = 2.5 mm and δ = 5 mm to the loads on the standard 100% CBR curve at the same penetrations. The standard curve is given by:

Table 1.11

δ (mm)	2	2.5	4	5	6	8	10	12
Standard P_{100}	11.5	13.24	17.6	19.96	22.2	26.3	30.3	33.5

The results for a curve type Figure 1.22(a) can be calculated directly from formula (1.53).

$$\text{CBR1} = 100 \times \left(\frac{\text{Measured force at 2.5 mm}}{\text{Standard force at 2.5 mm}}\right) = \boxed{100 \times \frac{P_{2.5}}{13.24}\,\%}$$
$$\text{CBR2} = 100 \times \left(\frac{\text{Measured force at 5 mm}}{\text{Standard force at 5 mm}}\right) = \boxed{100 \times \frac{P_{5}}{19.96}\,\%} \tag{1.55}$$

Alternative graphical procedure: Plot the experimental curve on standard charts as shown in the following example.

Construction of the charts

In order to draw the curve for a particular CBR% (say CBR=60%), the standard force in Table 1.11 has to be multiplied by 0.6 at each penetration. Table 1.12 contains the figures for CBR=12% and 60%

Table 1.12

δ (mm)	2	2.5	4	5	6	8	10	12
Standard P_{100}	11.5	13.24	17.6	19.96	22.2	26.3	30.3	33.5
12% CBR=0.12 P_{100}	1.38	1.59	2.11	2.40	2.66	3.16	3.64	4.02
60% CBR=0.6 P_{100}	6.90	7.94	10.56	11.98	13.32	15.78	18.18	20.10

The table can be completed this way for any CBR value and either one or several charts drawn. In this case, Chart 1.1a has been drawn for easier interpretation under CBR=12%.

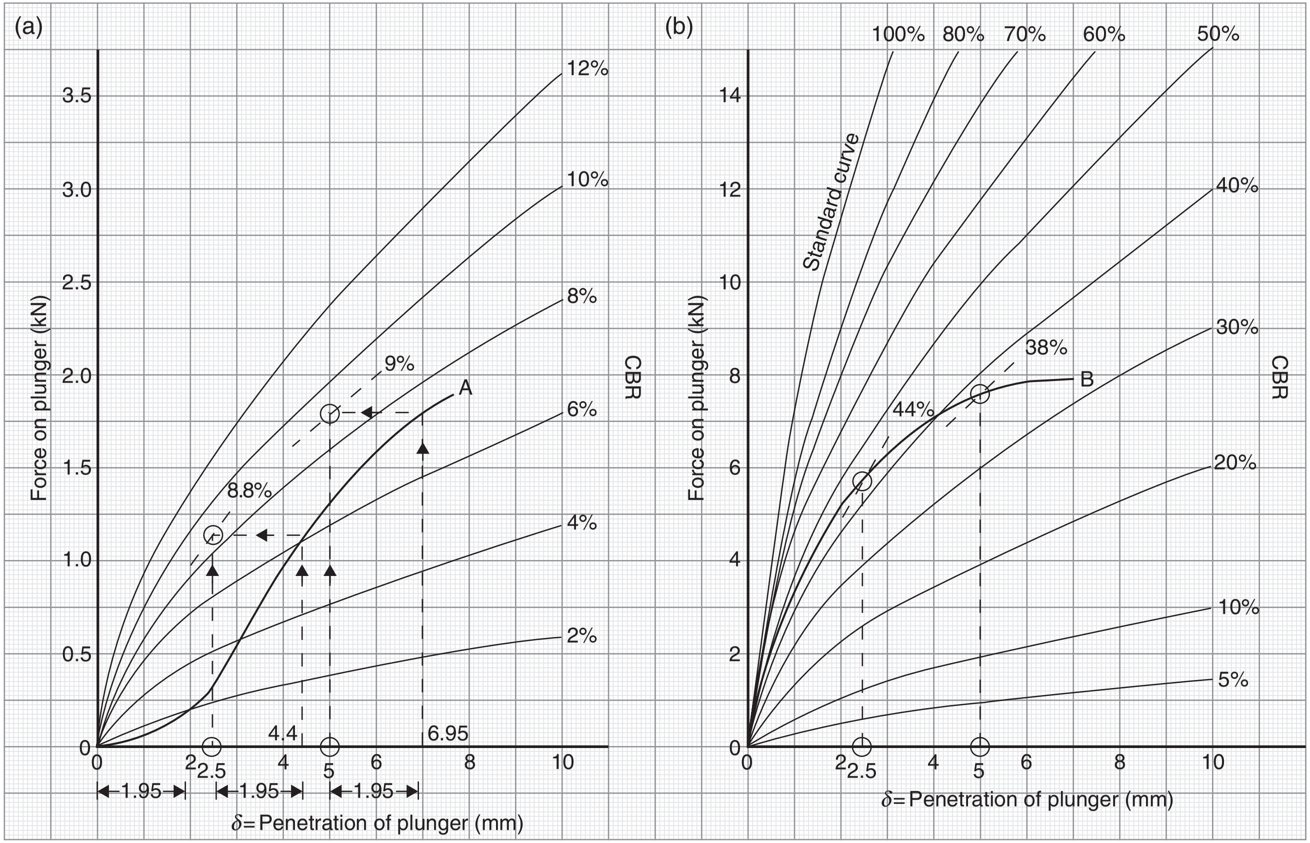
Example 1.9

Two soils A and B were tested in the CBR mould and the results tabulated. Determine the CBR value for each, analytically and graphically.

Table 1.13

δ (mm)	P_A(kN)	P_B(kN)	δ(mm)	P_A(kN)	P_B(kN)
0.00	0.00	0.00	4.00	0.98	7.14
0.25	0.01	1.10	4.25	1.07	7.28
0.5	0.03	2.00	4.50	1.16	7.42
0.75	0.05	2.82	4.75	1.26	7.58
1.00	0.07	3.51	**5.00**	1.34	**7.65**
1.25	0.09	3.95	5.25	1.39	7.74
1.50	0.13	4.22	5.50	1.47	7.80
1.75	0.17	4.80	5.75	1.54	7.81
2.00	0.23	5.21	6.00	1.10	7.85
2.25	0.26	5.53	6.25	1.64	7.88
2.50	0.34	**5.84**	6.50	1.72	7.92
2.75	0.43	6.17	6.75	1.76	7.95
3.00	0.54	6.35	7.00	1.79	7.98
3.25	0.66	6.57	7.25	1.83	7.99
3.50	0.77	6.76	7.50	1.87	8.00
3.75	0.87	6.98	-	-	-

Note: It is prudent to draw the load-penetration curves in order to ascertain their shapes. A correction has to be made if the shape is as indicated in Figure 1.22(b). In this example, P_A is plotted on Chart 1.1a. Its curve is convex downwards near the origin, hence it has to be corrected as shown.



(a)
Force on plunger (kN)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
12%
10%
8%
6%
4%
2%
9%
8.8%
A
CBR
4.4
6.95
0
2
2.5
4
5
6
8
10
1.95
1.95
1.95
δ= Penetration of plunger (mm)
(b)
Force on plunger (kN)
14
12
10
8
6
4
2
0
100%
80%
70%
60%
50%
40%
30%
20%
10%
5%
Standard curve
38%
44%
B
CBR
0
2
2.5
4
5
6
8
10
δ= Penetration of plunger (mm)

Chart 1.1

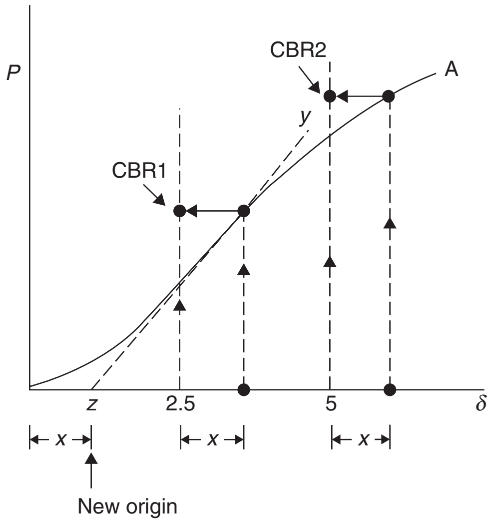


Figure 1.23

Draw line y–z. Point z is the new origin for δ. Interpolate for the two CBR values in Chart 1.1a.

From Chart 1.1a: $x = 1.95$ $\quad \therefore \quad 2.5 + x = 4.45$ mm

And $\quad 5 + x = 6.95$ mm

Interpolating between curves 8% and 10%.

$$\text{CBR 1} = 8.8\%$$
$$\text{CBR 2} = 9\%$$

Accept the larger figure as the CBR value for material A, that is 9%.

Curve B for P_B is convex upwards along its entire length, hence no correction is necessary. In this case, it is easier to calculate the CBR values by formula (1.55), then by interpolation on Chart 1.1b.

From Table 1.13: For $\delta = 2.5$ mm, $P_B = 5.84$ kN $\quad \therefore \text{CBR1} = 100 \times \dfrac{5.84}{13.24} = 44.1\%$

For $\delta = 5$ mm, $P_B = 7.65$ kN $\quad \therefore \text{CBR2} = 100 \times \dfrac{7.65}{19.96} = 38.3\%$

The larger answer is taken as the CBR value rounded to 44%. The graphical solution is given on Chart 1.1b.

Note: The shape of curve A near the origin is assumed to be due to inadequate compaction of the surface layer compared to the rest of the material.

Comparative CBR values of various soils are tabulated below.

Table 1.14

Soil type	Plasticity index (PI, %)	CBR (%)	Strength
Heavy clay	70	1–2	Weak
	60	1.5–2.0	
	50	2.0–2.5	
	40	2–3	
Silty clay	30	3–5	Normal
Sandy clay	20	4–6	
	10	5–7	
Sand	Non-plastic		Stable
Poorly graded	Non-plastic	10–20	
Well graded	Non-plastic	15–40	
Sandy gravel	Non-plastic	20–60	

Guidance is given in Road Note 29 of the Road Research Laboratory as to the design of flexible and concrete roads and pavements in terms of CBR values and estimated traffic intensities.

1.5 The pycnometer

It is a glass jar, fitted with a conical screw-top with a 6 m circular orifice at the apex, as shown schematically below. The pycnometer is used to determine:

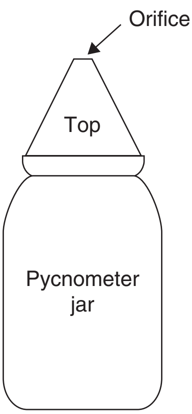


Figure 1.24

1. Specific gravity (G_s)
2. Moisture content (m)

The test is based on formula (1.34).

$$G_s = \frac{\rho_s}{\rho_w} = \frac{\frac{M_s}{V_s}}{\frac{M_w}{\gamma_s}} = \frac{M_s}{M_w} = \frac{\text{Mass of soil}}{\text{Mass of equal volume of water}}$$

It is necessary to find that mass of water, which is displaced by the soil.

Notes: The units used are:

a) volume in cm^3
b) mass in grams.

Therefore:

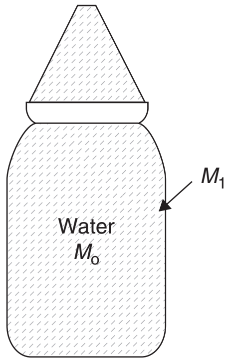
$$\rho_w = 1\,\text{g/cm}^3$$

$$G_s = \frac{\rho_s}{1} = \rho_s$$

$$M_w = \rho_w \times V_w = V_w\ \text{cm}^3$$

Outline of the test

Step 1: Weigh the empty jar + top (M_p). Fill the pycnometer to the orifice with water. Weigh the pycnometer + water (M_1)



$$M_1 = M_p + M_o = M_p + V_o \times \rho_w = M_p + V_o$$

where

M_p = mass of the pycnometer

M_o = mass of water filling the pycnometer

V_o = volume of the pycnometer

Figure 1.25

Step 2: Place about 200 g oven-dried fine-grained or 400 g coarse-grained soil (M_s) into the dry, empty pycnometer. Add water at room temperature. Stir the mixture to remove air bubbles.

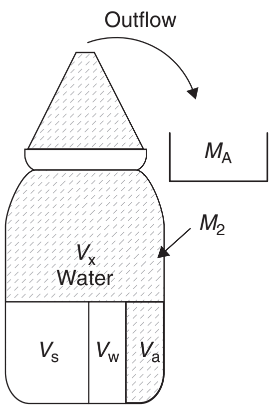
Step 3: Screw on the conical top and fill the pycnometer completely. Cover the orifice with a finger and shake the jar to remove any air from the soil and water.

Step 4: Weigh the pycnometer + soil + water (M_2).

Step 5: Apply the formulae derived below.

1. **Derivation of G_s** (in terms of dry mass M_s)

   In order to determine the mass (M_A) of the displaced water, imagine that **wet** soil of mass M is dropped into the full pycnometer and the discharge is collected.



Volume of partially saturated soil:

$$V = V_s + V_w + V_a$$

Mass of partially saturated soil:

$$M = M_s + M_w$$
$$= V_s \times \rho_s + V_w \times \rho_w = V_s \times G_s + V_w$$

Volume of water displaced from the jar by the volume of solids and water in the soil:

$$V_A = V_x + V_a = V_o - (V_s + V_w)$$

Figure 1.26

The mass of the displaced water is given by:

$$M_A = V_A \times \rho_w = V_A = V_o - V_s - V_w = M_o - \frac{M_s}{G_s} - M_w$$

The formula in terms of the dry mass (M_s) and the measurements made (M_1 and M_2) during the test:

From step 1: $M_1 = M_p + M_o$

From step 2:
$$M_2 = M_p + M + (V_x + V_a) \times \rho_w$$
$$= M_p + M + M_A$$

Change in the mass of pycnometer in steps 1 and 2.

$$\Delta M = M_2 - M_1 = M_p + M + M_A - M_p - M_o$$
$$= M_p + (M_s + M_w) + \left(M_o - \frac{M_s}{G_s} - M_w \right) - M_p - M_o$$
$$= M_s + M_w + M_o - \frac{M_s}{G_s} - M_w - M_o$$

$$M_2 - M_1 = M_s - \frac{M_s}{G_s} = \left(1 - \frac{1}{G_s}\right) \times M_s = \left(\frac{G_s - 1}{G_s}\right) \times M_s$$

Expressing dry mass:
$$\boxed{M_s = (M_2 - M_1)\left(\frac{G_s}{G_s - 1}\right)} \tag{1.56}$$

Therefore, should G_s of a partially or fully saturated soil be known from another source, then its dry mass can be determined by a pycnometer.

Expressing G_s from:
$$M_2 - M_1 = \left(1 - \frac{1}{G_s}\right) \times M_s$$

$$\frac{1}{G_s} = 1 - \frac{M_2 - M_1}{M_s} = \frac{M_s + M_1 - M_2}{M_s}$$

Hence,
$$\boxed{G_s = \frac{M_s}{M_s + M_1 - M_2}} \tag{1.57}$$

2. **Derivation of *m*** (in terms of total mass *M*)

From (1.35a):
$$m = \frac{M_w}{M_s} = \frac{M - M_s}{M_s} = \frac{M}{M_s} - 1$$
$$= \frac{M}{(M_2 - M_1) \times \left(\frac{G_s}{G_s - 1}\right)} - 1$$

Hence,
$$m = \left(\frac{M}{M_2 - M_1}\right) \times \left(\frac{G_s - 1}{G_s}\right) - 1 \tag{1.58}$$

Therefore, the moisture content can be found by a pycnometer if G_s is known.

Example 1.10

A partially saturated soil specimen, weighing 1743 g was tested by placing its oven-dried mass of 1449 g in a pycnometer. The following results were obtained:

Step 1: $M_p = 610$ g

$M_1 = 1923$ g

Step 4: $M_2 = 2854$ g

Calculate:

a) volume of the pycnometer
b) specific gravity
c) moisture content of the soil
d) volume of water
e) volume of solids

Check: Total and dry mass of the specimen

a) Volume of pyconmeter = volume of water to fill it

$$M_o = M_1 - M_p = 1923 - 610 = 1313 \text{ g}$$

b) From (1.57): $G_s = \dfrac{M_s}{M_s + M_1 - M_2} = \dfrac{1449}{1449 + 1923 - 2854} = 2.8$

c) From (1.58):

$$m = \left(\frac{M}{M_2 - M_1}\right) \times \left(\frac{G_s - 1}{G_s}\right) - 1 = \left(\frac{1743}{2854 - 1923}\right) \times \left(\frac{2.8 - 1}{2.8}\right) - 1 = 0.203$$

d) From (1.35): $m = \dfrac{M_w}{M_s}$

But, $M_w = V_w$

$\therefore \quad V_w = m \times M_s = 0.203 \times 1449 = 294 \text{ cm}^3$

e) From (1.32): $V_s = \dfrac{M_s}{\rho_s}$

But, $\rho_s = G_s$

$\therefore \quad V_s = \dfrac{M_s}{G_s} = \dfrac{1449}{2.8} = 518 \text{ cm}^3$

Check: (1.56): $M_s = (M_2 - M_1) \times \left(\dfrac{G_s}{G_s - 1}\right) = (2854 - 1923) \times \left(\dfrac{2.8}{1.8}\right) = 1448 \text{ g}$

And $M = M_s + M_w = V_s \times G_s + V_w = 518 \times 2.8 + 294 = 1744$ g

Note: See also Supplementary problem 1.11.

Problem 1.1

A site test was carried out in order to check the compacting efficiency of a contractor. 1500 cm^3 soil was removed and tested. The available results are:

Volume of sample:	$V=1500\,cm^3$
Dry Density:	$\gamma_d=17\,kN/m^3$
Degree of saturation:	$S_r=53\%$
Specific gravity:	$G_s=2.7$

Calculate:

V_a = Volume of air in the sample
W_w = weight of water in the sample
M_w = mass of water in the sample

Problem 1.2

A compacted, partially saturated sand sample has to be fully saturated by the addition of water. Calculate, in the light of the following information, the weight of water to be added.

Volume of sample:	$V=5260\,cm^3$
Water content:	$m_1=15\%$
Porosity:	$n=35\%$
Specific gravity:	$G_s=2.67$

Problem 1.3

The following information is known about a sample of soil:

Volume:	$V=3000\,cm^3$
Water content:	$m=15\%$
Specific gravity:	$G_s=2.65$
Submerged density:	$\gamma'=8.69\,kN/m^3$

Calculate:

a) How many percent of voids are filled with water?
b) Weight of the pore water.
c) Mass of the pore water.

Problem 1.4

Starting from formula (1.38), expressing the bulk unit weight of partially saturated soil, derive the formulae:

1. $\gamma = \left(\frac{1+m}{m}\right)\left(\frac{e}{1+e}\right) S_r \gamma_w$	Partially saturated soil
2. $\gamma = (1+m)(1-n) G_s \gamma_w$	
3. $\gamma_{sat} = [(1-n) G_s - n] \gamma_w$	Saturated soil
4. $\gamma_{sat} = \left(\frac{1+m}{m}\right)\left(\frac{e}{1+e}\right) \gamma_w$	
5. $\gamma_d = (1-n) G_s \gamma_w$	Dry soil
6. $\gamma_d = \gamma_{sat} - n\gamma_w$	
7. $\gamma' = \frac{(e-m)\gamma_w}{(1+e)m}$	Submerged soil
8. $\gamma' = \gamma_d - (1-n) \gamma_w$	

Problem 1.5

Test of site compaction was carried out by means of sand pourer equipment. The apparatus was calibrated prior to its application.

The results were:

Calibration stage

Mass of pourer and sand = 4.991 kg
Mass of sand released into the cone = 0.58 kg

Final mass of pourer after filling cylinder and cone = 1.19 kg
Volume of cylinder = 2000 cm^3

Testing stage

Mass of excavated soil = 2.574 kg
Water content of excavated soil = 19%
Mass of pourer after filling the hole = 2.321 kg

Estimate the dry density of the compacted soil.

Problem 1.6

Dry sand weighing 100 kg, is compacted to a voids ratio of 52%. The available information on the sand is:

Specific gravity = 2.66
Minimum voids ratio = 31%
Density Index = 40%

Estimate:

a) the Volume of sand in its most loose, most dense as well as compacted state.
b) the moisture content in the above three states, given that the degree of saturation is 80% in each case.
c) The saturated density in all three states in kN/m^3.

Problem 1.7

The weight of 2.5 m^3 saturated soil is 48.5 kN. Given that the specific gravity of the material is 2.7, calculate the volume of water in the voids.

Problem 1.8

An embankment of 12 m^2 cross-sectional area is to be constructed. Site survey indicates that 40,000 m^3 suitable soil of $G_s = 2.66$ can be excavated near the site. Tests carried out on 1 m^3 of the material yielded the following average values:

$$W = 18.1\,kN$$
$$W_s = 16\,kN$$

If the soil is compacted at in-situ moisture content to dry density = 18.2 kN/m^3, then:

a) compare the voids ratios as well as the percentages of air in the excavated and compacted soil.
b) determine the length embankment, that can be built with the available material, in kilometres.

Problem 1.9

Partially saturated clay was tested and its characteristics calculated. Most of the results were lost however, except the following four:

Volume:	$V = 0.15\ \text{m}^3$
Moisture content:	$m = 12\%$
Degree of saturation:	$S_r = 49\%$
Dry unit weight:	$\gamma_d = 16\ \text{kN/m}^3$

Find:
a) Bulk unit weight
b) Voids ratio
c) Specific gravity
d) Saturated density

Problem 1.10

Suppose the available results in Problem 1.9 are:

$m = 12\%$
$e = 66\%$
$\gamma = 17.9\ \text{kN/m}^3$
$V = 0.15\ \text{m}^3$

Calculate:
1. Total weight
2. Weights of solids and water
3. Volumes of solids, water and voids.

Problem 1.11

The results of pycnometer test, carried out on a saturated specimen are:

$M = 519\ \text{g}$
$M_s = 412\ \text{g}$
$M_1 = 1923\ \text{g}$
$M_2 = 2185\ \text{g}$

Show that, for a saturated soil, the entire list of soil characteristics (see Table 1.2 and 1.5) can be determined by the pycnometer test.