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Decision-Making in System Project, Planning, Operation, and Control: Motivation, Objectives, and Basic Concepts

The intent of this introductory chapter is to offer the reader a broad perspective on the fundamentals of decision-making problems, provide their general taxonomy in terms of criteria, objectives, and attributes involved, stress the relevance and omnipresence of the uncertainty factor, and highlight the aspects of rationality of decision-making processes. We also highlight the fundamental differences between optimization and decision-making problems. The main objectives, concepts, and characteristics of group decision-making are presented. The role of fuzzy sets is stressed in the general framework of decision-making processes. The main advantages of their application to individual and group decision-making processes are briefly discussed. The chapter also clarifies necessary notations and terminology (such as $\langle \mathbf{X}, \mathbf{M} \rangle$ models and $\langle \mathbf{X}, \mathbf{R} \rangle$ models) used throughout the book.

1.1 Decision-Making and its Support

The life of each person is filled with alternatives. From the moment of conscious thought to a venerable age, from morning awakening to nightly sleeping, a person meets the need to make a decision of some sort. This necessity is associated with the fact that any situation may have two or more mutually exclusive alternatives and it is necessary to choose one among them. The process of decision-making, in the majority of cases, consists of the evaluation of alternatives and the choice of the most preferable from them.

Making the "correct" decision means choosing such an alternative from a possible set of alternatives, in which, by considering all the diversified factors and contradictory requirements, an overall value will be optimized (Pospelov and Pushkin, 1972); that is, it will be favorable to achieving the goal sought to the maximal degree possible.

If the diverse alternatives, met by a person, are considered as some set, then this set usually includes at least three intersecting subsets of alternatives related to personal life, social life, and professional life. The examples include, for instance, deciding where to study, where to work, how to spend time on a weekend, who to elect, and so on.

At the same time, if we speak about any organization, it encounters a number of goals and achieves these goals through the use of diverse types of resources (material, energy, financial, human, etc.) and the performance of managerial functions such as organizing, planning, operating, controlling, and so on (Lu *et al.*, 2007). To carry out these functions, managers engage in a continuous decision-making process. Since each decision implies a reasonable and justifiable choice made among diverse alternatives, the manager can be called a decision-maker (DM). DMs can be managers at various levels, from a technological process manager to a chief executive officer of a large company, and their decision problems can vary in nature. Furthermore, decisions can be made by individuals or groups (individual decisions are usually made at lower managerial levels and in small organizations, and group decisions are usually made at high managerial levels and in large organizations). The examples include, for instance, deciding what to buy, when to buy, when to visit a place, who to employ, and so on. These problems can concern logistics management, customer relationship management, marketing, and production planning.

A person makes simple, habitual decisions easily, frequently in an automatic and subconscious way, not leaving much to intensive thinking. However, in many cases, alternatives are related to complex situations which are characterized by a discrepancy of requirements and multiple criteria, ambiguity in evaluating situations, errors in the choice of priorities, and others. All these factors substantially complicate the process of taking decisions.

Furthermore, various facets of uncertainty are commonly encountered in a wide range of decision-making problems, which are inherently present in the project, planning, operation, and control of complex systems (engineering, economical, ecological, etc.). In particular, diverse manifestations of the uncertainty factor are associated, for instance, with:

- the impossibility or inexpediency of obtaining sufficient amounts of reliable information;
- the lack of reliable predictions of the characteristics, properties, and behavior of complex systems that reflect their response to external (the surroundings) and internal actions;
- poorly defined goals and constraints in the project, planning, operation, and control tasks;
- the impossibility of formalizing a number of factors and criteria.

This situation should be considered as being natural and unavoidable in the context of complex systems. It is not difficult to understand that it is impossible, in principle, to reduce these problems to exact and well-formulated mathematical problems; to do this, it is necessary, in one way or another, "to take away" the uncertainty and position some hypothesis. However, the construction of a hypothesis is a prerogative of the substantial analysis; this is the formalization of informal situations. One of the ways to address the problem is the formation of subjective estimates carried out by experts, managers, and DMs in general, and the definition of the corresponding preferences.

Thus DMs are forced to rely on their own subjective ideas of the efficiency of possible alternatives and importance of diverse criteria. Sometimes, this subjective estimation is the only possible basis for combining the heterogeneous physical parameters of a problem to be solved into a unique model, which permits decision alternatives to be evaluated (Larichev,

1987). At the same time, there is nothing unusual and unacceptable in the subjectivity itself. For instance, experienced managers perceive, in a broad and well-informed manner, how many personal and subjective considerations they have to bring into the decision-making process. On the other hand, successes and failures of the majority of decisions can be judged by people on the basis of their subjective preferences.

However, the most complicated aspect is associated with the fact that a realm of problems solved by humans in diverse areas has been changed (Trachtengerts, 1998). New, more complicated, and unusual problems have emerged. For many centuries, people made decisions by considering one or two main factors, while ignoring others that were perceived to be marginal to the essence of the problem. They lived in a world where changes in the surroundings were few and new phenomena arose "in turn" but not simultaneously.

At the present time, this situation has changed. A considerable number of problems, or probably the majority of them, are multicriteria in nature, where it is necessary to take into account many factors. In these problems, a DM has to evaluate a set of influences, interests, and consequences which characterizes decision alternatives. For example, in decision-making dealing with the formation of an enterprise, it becomes necessary to consider not only the expected profits and necessary investment, but also market dynamics, the actions of competitors, and ecological, political, and social factors, etc.

Taking into account all the aspects listed above, it is necessary to stress that recognition of the factor of subjectivity of a DM in the process of decision-making conflicts with the fundamental methodological principle of operational research: the search for an *objectively* optimal solution. Recognition of the ultimate right of a DM in the subjectivity of decisions is a sign of the appearance of a new paradigm of multicriteria decision-making (Kuhn, 1962). However, in decision-making with multiple criteria, an objective component always exists. Usually, this component includes diverse types of constraints imposed by the environment on possible decisions (availability of resources, temporal constraints, ecological requirements, social situations, etc.).

A large number of psychological investigations demonstrate that DMs, not being provided with additional analytical support, use simplified and, sometimes, contradictory decision rules (Slovic, Fischhoff, and Lichtenstein, 1977).

Further, Lu *et al.* (2007) share the opinion given above (Trachtengerts, 1998) and indicate that decision-making in the activities of organizations is more complicated and difficult because the number of available alternatives is much larger today than ever before. Due to the availability of information technology and communication systems, especially the Internet and its search engines, we can find more information quickly and therefore more alternatives can be generated. Second, the cost of making errors can be great because of the complexity of operations, automation, and the chain reaction that an error can cause in many parts, in both the vertical and horizontal levels, of the organization. Third, there are continuous changes in the fluctuating environment and more uncertainties in the impacting elements, including information sources and information itself. More importantly, the rapid change of the decision environment requires decisions to be made quickly. These reasons cause organizational DMs to require increasing technical support to help make high-quality decisions. A high-quality decision, such as in bank management, is expected to bring greater profitability, lower costs, shorter distribution times, and increased shareholder value, attracting more new customers, or resulting in a certain percentage of customers responding positively to a direct mail campaign.

Decision support consists of assisting a DM in the process of decision-making. For instance, this support may include (Trachtengerts, 1998):

- assisting a DM in the analysis of an objective component, that is, in the understanding and evaluation of the existing situation and constraints imposed by the surroundings;
- revealing DM preferences, that is, revealing and ranking priorities, considering the uncertainty in DM estimates, and shaping the corresponding preferences;
- generating possible solutions, that is, shaping a list of available alternatives;
- evaluating possible alternatives, considering DM preferences and constraints imposed by the environment;
- analyzing the consequences of decision-making;
- choosing the best alternative, from the DM's point of view.

Computerized decision support, in any case, is based on the formalization of methods for obtaining initial and intermediate estimates given by a DM and on the algorithm for a proper decision process.

The formalization of methods for generating alternatives, their evaluation, comparison, choice, prioritization, and/or ordering, and, if necessary, concordance is a very complicated processes. One of the main complexities and challenges is associated with the fact that a DM, as a rule, is not ready to provide quantitative estimates in the decision process, is not accustomed to the evaluation of proper decisions on the basis of applying formal mathematical methods, and analyzes the consequences of decisions with difficulty.

As a matter of fact, decision support systems have existed for a long time, for example, councils of war, ministry boards, various meetings, analytical centers, and so on (Trachtengerts, 1998). Although they were never called decision support systems, they executed the functions of such systems, at least partially.

The term "decision support system" appeared at the beginning of the 1970s (Eom, 1995). There are several definitions of this concept, such as that given in Larichev and Moshkovich (1996): "Decision support systems are man-machine objects, which permit a DM to use data, knowledge, objective and subjective models for the analysis and solution of semi-structured or unstructured problems".

Taking into account this definition, it is necessary to indicate that one of the important features of decision-making problems is associated with their structures. In particular, it is possible to distinguish structured, semi-structured, and unstructured problems of decision-making (Simon, 1977; Larichev and Moshkovich, 1996; Lu *et al.*, 2007). The latter two types of decision-making problems are also called *ill-structured*.

In *structured problems* (quantitatively formulated problems), essential relationships are established so convincingly that they can be expressed in numbers or symbols which receive, ultimately, numerical estimates. Such problems can be described by existing "traditional" mathematical models. Their analysis becomes possible by applying standard methods leading to the solution.

Unstructured problems (qualitatively expressed problems) include only a description of the most important resources, indicators, and characteristics. Quantitative relationships between them are not known. These problems cannot be described by existing traditional mathematical models and cannot be analyzed by applying standard methods.

Finally, *semi-structured problems* (or mixed problems) include quantitative as well as qualitative elements. As these are examined, qualitative, little-known, poorly explored, uncertain parameters have a tendency to dominate. These problems fall between structured and unstructured problems, having both structured and unstructured elements. The solutions to these problems involves a combination of both standard solution procedures and active DM participation.

According to the classification given above, typical problems in operational research can be called structured. This class of problems is widely used in the project planning, operation, and control of engineering systems. For example, it is possible to talk about the design of forms of an aircraft hull, planning of water supply systems, control of power systems, and so on.

The distinctive characteristics of unstructured problems are as follows (Larichev and Moshkovich, 1996):

- uniqueness of choice in the sense that, at any time, the problem is a new one for a DM or it has new properties in comparison to a similar problem solved in the past;
- uncertainty in the evaluation of alternative solutions;
- the qualitative character of the evaluations of problem solutions, most often formulated in verbal form;
- the evaluation of alternatives obtained only on the basis of the subjective preferences of a DM;
- the estimates of criteria obtained only from experts.

Typical unstructured problems are associated, for example, with planning new services, hiring executives, selecting a locale for a new branch, choosing a set of research and development projects, and alike.

If we speak about *semi-structured problems*, their solutions are based on applying traditional analytical models as well as models based on DM preferences. As an example, one can look at the problem (Trachtengerts, 1998) related to liquidation of the consequences of extraordinary situations associated with radioactive contamination. In the solution of this problem, analytical models can be applied to define the degree and character of radioactive contamination for given temporal intervals. At the same time, models based on DM preferences can be applied in the choice of measures for liquidation of the consequences of radioactive contamination. It is possible to qualify many problems associated with economical and political decisions, medical diagnostics, and so on, as semi-structured problems.

Returning to the issue of computerized decision support, we should note that, due to the large number of components (variables, functions, and parameters) involved in many decisions, this has become a basic requirement to assist DMs in considering and examining the implications of various courses of decision-making (Lu *et al.*, 2007). Furthermore, the impact of computer technologies, particularly the Internet, on organizational management is increasing rapidly. Interaction and cooperation between users and computers are growing to cover more and more aspects of organizational decision-making activities. Internet- or intranet-based computerized information systems have now become vital to all kinds of organizations.

Thus, computer applications in organizations are moving from transaction processing and monitoring activities to problem analysis and finding solutions (Lu *et al.*, 2007). Internet- or intranet-based online analytical processing and real-time decision support are becoming the cornerstones of modern management, in particular within the elaboration of e-commerce, e-business, and e-government. There is a trend toward providing managers with information systems that can assist them directly with their most important task, that is, making decisions.

A detailed description of the advantages generated by applying computerized decision support systems for individual as well group decision-making is given, for instance, in Lu *et al.* (2007). At the same time, these authors indicate that the important issue is that, with computerized decision support technologies, many complex decision-making problems can now be handled effectively. However, these technologies can be better used in analyzing structured problems rather than semi-structured and unstructured problems. In an unstructured problem, only part of the problem can be supported by advanced tools such as intelligent decision support systems. For semi-structured problems, the computerized decision support technologies can improve the quality of information on which the decision is based by providing not just a single, unique solution, but a range of alternative solutions from the decision uncertainty regions. Their occurrence and essence will be discussed in the next section.

1.2   Optimization and Decision-Making Problems

Is there any difference between the notions of "optimization" and "decision-making"? Are these notions synonymous or not? Partial answers to these questions have been given in the previous section. However, deeper and more detailed considerations are beneficial here.

A traditional optimization problem is associated with the search for an extremum (minimum or maximum, according to the essence of the problem) of a certain objective function, which reflects our interests, when observing diverse types of constraints (imposed on allowable resources, physical laws, standards, industrial norms, etc.). Formally, it is possible to represent an optimization problem as follows:

$$F(x) \to \underset{x \in \mathbf{L}}{\mathrm{extr}} \tag{1.1}$$

where $\mathbf{L}$ is a set of feasible solutions in $\mathbf{R}^n$ defined by the constraints indicated above.

To solve the problem (1.1) we should find a vector $\boldsymbol{x}^0$ such that

$$\boldsymbol{x}^0 = \arg \underset{x \in \mathbf{L}}{\mathrm{extr}} F(x) \tag{1.2}$$

If numerical details of the problem (1.1) have been provided and we can obtain a unique solution without any guidance or assistance from a DM, then we are concerned with an optimization problem.

Generally, an optimization problem may be complicated from the mathematical point of view, and we need a large amount of computing time to generate a solution. Can human participation in the search for a solution be useful? Definitely, such participation could be useful, because, for instance, the introduction of heuristics or a change of initial points for a search can reduce the time necessary to obtain an optimal solution. However, in principle, a unique solution to the problem can be obtained without human participation.

At the same time, the presence of any type of uncertainty can call for human participation in order to arrive at a unique solution to the problem.

For instance, the uncertainty of information gives rise to some decision uncertainty regions. As shown in Figure 1.1, the uncertainty of information $\delta F(x)$ in the estimation of an objective function $F(x)$ leads to a situation where formally the solutions coming from a region δx
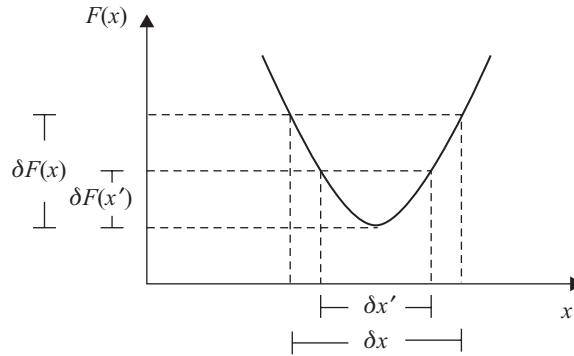
Figure 1.1 Decision uncertainty region and its reduction through the reduction of the level of uncertainty of information.

cannot be distinguished, thus giving rise to a decision uncertainty region. Taking this into consideration, the formal formulation (1.1) can be transformed to the following:

$$F(x, \theta) \to \operatorname*{extr}_{x \in \mathbf{L}(\theta)} \tag{1.3}$$

where θ is a vector of uncertain parameters, whose existence changes the essence of (1.1). In particular, we can say that the solution (1.2) is an optimal solution for a concrete realization of θ (a concrete hypothesis); however, for some other realization (another hypothesis), it is no longer optimal.

What are the ways to reduce this uncertainty region? The first way is to "buy" information (let us not forget that any information has some cost associated with it), for example, by acquiring additional measurements or examining experts to reduce the level of uncertainty. As shown in Figure 1.1, the reduction of the uncertainty $\delta F(x)$ to $\delta F(x')$ permits one to obtain a reduced decision uncertainty region with $\delta x' < \delta x$.

However, if there is no possibility of reducing the uncertainty of information, we can resort to some alternative approach. This way is associated with introducing additional criteria to try to reduce the decision uncertainty regions. As demonstrated in Figure 1.2, introduction of the objective function $F'(x)$ allows us to reduce the decision uncertainty region as well, arriving at $\delta x' < \delta x$.

On the other hand, the existence of more than one objective function may be considered as uncertainty as well. This comes in as the uncertainty of goals. Although the nature of this type of uncertainty is not the same as the uncertainty of available information, it also leads to the generation of decision uncertainty regions.

To focus our attention, let us consider the simple problem of minimizing two objective functions $F_1(x) = F_1(x_1, x_2)$ and $F_2(x) = F_2(x_1, x_2)$, considering a set of feasible solutions $\mathbf{L}$. We can transform $\mathbf{L}$ from the decision space to some region $\mathbf{L}_F$ of the space of objective functions $F_1(x)$ and $F_2(x)$ (or, simply, the objective space). In Figure 1.3, we can see that point a corresponds to the best solution ($\min_{x \in \mathbf{L}} F_1(x)$) from the point of view of the first objective
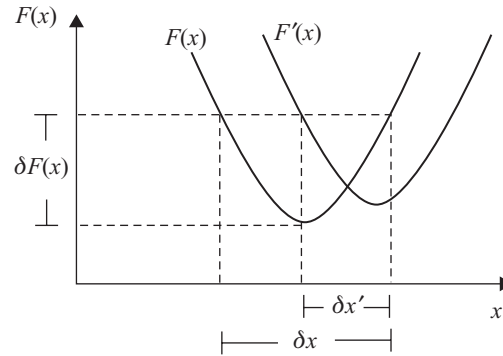
Figure 1.2 Decision uncertainty region and its reduction through the introduction of additional criteria.

function. On the other hand, point b corresponds to the best solution ($\min_{x \in \mathbf{L}} F_2(x)$) when considered from the viewpoint of the second objective function.

Is point c a solution to the problem? Yes, it is. Can we improve this solution? Yes, we can do that by passing to point d. Can we improve this solution? Yes, this is possible by passing to point e. Can we improve this solution? This is possible by passing to point f. Can we improve this solution? We cannot advance here. It is possible to pass to point g, but this step does not make the resulting solution any better: we can improve it from the point of view of $F_1(x)$ but deteriorate its quality from the point of view of $F_2(x)$. In a similar way, by passing to point h, we can improve the solution from the point of view of $F_2(x)$ but deteriorate it from the point of view of $F_1(x)$.

Thus, formally, the solution to the problem presented in the objective space is a boundary Ω_F^P of $\mathbf{L}_F$ located between points a and b. The set $\Omega^P \subseteq \mathbf{L}$ corresponding to Ω_F^P is the problem solution, which is called a Pareto-optimal solution set. This concept of optimality was proposed by Edgeworth (1881) and was further generalized by Pareto (1886). Although we say that Ω^P is the problem solution, from a formal point of view this is not a solution that can be implemented. In reality, it is the decision uncertainty region. The choice of a particular Pareto-optimal solution is based on the DM's involvement.
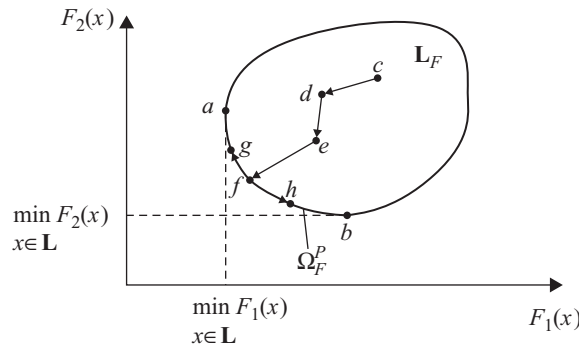

Figure 1.3 The concept of Pareto-optimal solutions.

The more difficult situations are associated with problems where there exists an uncertainty of information as well as an uncertainty of goals.

The problems of an optimization character, which include the uncertainty of information and/or the uncertainty of goals and demand human participation in their solution, are inherent problems in decision-making. Taking this into consideration, it is necessary to make some additional observations.

One of the most important criteria (Larichev, 1984) for classifying decision-making problems is the existence or lack of an objective model for the corresponding problem. Note here that it is not uncommon to encounter situations where it is impossible to talk about the existence of objective functions in decision-making problems. The models which can be used for analyzing these problems reflect "a point of view" and, in a more general sense, the "world outlook" of a DM. In these cases, an obvious question is how to choose actions which correspond, in the best way, to the preferences of a DM (Keeney and Raifa, 1976) and are based on his/her knowledge, experience, and intuition. Taking this into account, semi-structured and unstructured problems, classified in the previous section, are subjects of decision-making.

In conclusion, the following general tendency is visible. If we solve an optimization problem, we generally look for the *best* solution. If we talk about a decision-making problem, the methodology used to solve it is quite distinct: we do not look for the best solution, but apply information arriving from different sources and try to eliminate some alternatives, which are dominated by other alternatives, in order to reduce the decision uncertainty regions.

1.3   Multicriteria Decision-Making

The uncertainty of goals in decision-making is an important manifestation of uncertainty that relates to the multicriteria character of many problems encountered in the project, planning, operation, and control of complex systems of different nature. Some professionals in the field of decision-making and systems analysis (for example, Lyapunov, 1972) agree that, from the general point of view, this type of uncertainty is the most difficult to treat and overcome because "we simply do not know what we want". In reality, this type of uncertainty cannot be effectively captured on the basis of applying formal models and methods, because sometimes the unique information sources are the individuals who make decisions.

Multicriteria decision-making is related to making decisions in the presence of multiple and conflicting criteria. Multicriteria decision-making problems may range from everyday decision problems, such as the purchase of a car, to those affecting entire nations, as in the judicious use of money to preserve national security (Lu *et al.*, 2007).

However, even with this existing diversity, all multicriteria decision-making problems share the following common characteristics (Hwang and Yoon, 1981):

- **multiple criteria**: each problem has multiple criteria, which can be objectives or attributes;
- **conflicting criteria**: multiple criteria conflict with each other;
- **incommensurable units**: criteria may have different units of measurement;
- **design/selection**: solutions to multicriteria decision-making problems are either to design the best alternative(s) or to select the best one among previously specified finite alternatives.

Taking the above into account, we distinguish two types of *criteria*: *objectives* and *attributes*. In such a manner, multicriteria decision-making problems can be classified into two wide classes:

- *multiobjective* decision-making;
- *multiattribute* decision-making.

The main difference between these two classes is that the first concentrates on continuous decision spaces and the second focuses on problems with discrete decision spaces.

To elaborate further, some basic concepts and terminology are given below. They are in line with the notation presented in the literature (Hwang and Masud, 1979; Hwang and Yoon, 1981; Lu *et al.*, 2007).

Criteria form the standard of judgment or rules to test acceptability. In the multicriteria decision-making literature, they indicate objectives and/or attributes.

Objectives are the reflection of the desire of DMs and indicate the direction on which DMs want to concentrate. Multiobjective decision-making problems, as a result, involve the design of alternatives that optimize or at least satisfy the objectives of DMs.

Goals are entities desired by DMs and expressed in terms of a specific state in space and time. Thus, while objectives give the desired direction, goals give a desired (or target) level to achieve.

Attributes are the characteristics, qualities, or performance parameters of alternatives. Multiattribute decision-making problems involve the selection of the "best" alternative from a pool of preselected alternatives described in terms of their attributes.

Multiobjective decision-making is known as the continuous type of multicriteria decision-making and its main characteristics are that DMs need to achieve multiple objectives while these objectives are noncommensurable and conflict with each other. A multiobjective decision-making model includes a vector of decision variables, objective functions that describe the objectives, and constraints. DMs attempt to maximize or minimize the objective functions.

Multiattribute decision-making is related to making a preference decision (that is, comparison, choice, prioritization, and/or ordering) over the available alternatives that are characterized by multiple, usually conflicting, attributes. The main peculiarity of multiattribute decision-making problems is that there are usually a limited number of predetermined alternatives, which are associated with a level of achieving the attributes. Based on the attributes, the final decision is made.

Finally, we should discuss in detail the concept of *alternatives*. How to generate alternatives is a significant part of the process of multiobjective and multiattribute decision-making model building (Lu *et al.*, 2007). In almost all multiobjective decision-making models, the alternatives can be generated automatically by the models. In the case of multiattribute decision-making models, however, it is necessary to generate alternatives manually. Sometime, the essence of the problem defines the number of alternatives. However, in general, how and when to stop generating alternatives becomes a very important issue. Generating alternatives significantly depends on the availability and cost of information, and also requires reliance on expertise in the problem area. Alternatives can be generated with the use of heuristics as well, and they could come from either individuals or groups.

The issues related to the necessity of setting up and solving multicriteria problems as well as the classification of decision-making situations, which need the application of the multicriteria approach, have been discussed in many works (for instance, Larichev, 1984; Gomes, Gomes, and Almeida, 2002). It is possible to identify two major types of situations, which call for the application of a multicriteria approach:

- Problems whose solution consequences cannot be estimated with a single criterion: these problems are associated with the analysis of models including economic as well as physical indices (when alternatives cannot be reduced to comparable form) and also by the need to consider indices whose cost estimation is hampered (for example, many power engineering problems are considered on the basis of technological, economical, ecological, and social nature criteria).
- Problems that may be solved on the basis of a single criterion (or several criteria). However, if the uncertainty of information does not permit the derivation of unique solutions, it is possible to reduce these problems to multicriteria decision-making by applying additional criteria, including those of a qualitative character (for example, "flexibility of development", "complexity of maintenance", "attractiveness of investments", and so on, whose utilization is based on the knowledge, experience, and intuition of involved experts). This can serve as a convincing means to contract the corresponding decision uncertainty region. It could be regarded as an intuitively appealing approach exercised in the practice of decision-making.

According to the major types of situations outlined above, two classes of models, so-called $\langle \mathbf{X}, \mathbf{M} \rangle$ models and $\langle \mathbf{X}, \boldsymbol{R} \rangle$ models (Ekel, 2001; Ekel, 2002) can be constructed. Both of these classes of models are comprehensively discussed in the book. The $\langle \mathbf{X}, \mathbf{M} \rangle$ models correspond to multiobjective decision-making problems. In the book, their analysis is illustrated by considering the problems of multicriteria allocation of resources or their shortages (with the presentation of an adaptive interactive decision-making system (AIDMS1), which is dedicated to their solution) as well as important classes of power engineering problems (multiobjective power and energy shortage allocation as applied to load management, multiobjective power system operation, multiobjective optimization of network configurations in distribution systems, and energetically effective (bicriteria) voltage control in distribution systems). At the same time, the $\langle \mathbf{X}, \boldsymbol{R} \rangle$ models correspond to the multiattribute decision-making problems and include a vector of fuzzy preference relations (Orlovsky, 1981; Fodor and Roubens, 1994), which play the role of attributes. We will present an interactive system for multicriteria decision-making (MDMS) dedicated to the analysis of the $\langle \mathbf{X}, \boldsymbol{R} \rangle$ models. In the book, the construction and application of the $\langle \mathbf{X}, \boldsymbol{R} \rangle$ models is illustrated by considering the problems of substation planning in power systems, reactive power source choice at a power system bus, energy planning (selection of the most appropriate technology in a renewable energy diffusion plan) as well as managerial activities.

Finally, the $\langle \mathbf{X}, \boldsymbol{R} \rangle$ models are also used in the present book in problems of group decision-making, which are briefly discussed in the next section.

1.4   Group Decision-Making

Group decision-making is defined as a decision situation in which there is more than one individual involved. The group members have their own attitudes and motivations, recognize

the existence of a common problem, and attempt to reach a collective decision (Lu *et al.*, 2007). The necessity of applying procedures of group decision-making is associated with the following considerations.

There are many situations, for instance at the high managerial levels of organizations, when the decision problems involve wide domains of knowledge which are beyond a single individual (this is particularly true when the decision environment becomes more complex and multifaceted). As a consequence, it is usually necessary to allocate more than one professional to the decision process. This is particularly valid in environments with a diverse workforce, where decisions require multiple perspectives and different areas of expertise of the individuals represented in the group. The following are among the advantages of group over individual decision-making (Tan, Teo, and Wei, 1995):

- Group decision-making allows more intellectual resources to be gathered to support the decision. The resources available to the group include the individual competencies, intuition, and knowledge.
- With the participation of multiple experts, it becomes possible to distribute among them the labor related to acquiring and processing the vast amount of information pertaining to the decision.
- If the group members exhibit divergent interests, the final decision tends to be more representative of the needs of the organization.

It is possible to indicate some important characteristics of group decision-making as follows (Lu *et al.*, 2007):

- the group performs a decision-making task;
- group decision-making may cover the whole process of transfer from generating ideas for solving a problem to implementing solutions;
- group members may be located at the same place or at different places;
- group members may work at the same or different times;
- group members may work for the same or different departments or organizations;
- the group can be at any managerial level;
- there may be conflicting opinions in the group decision process among group members;
- the decision might have to be accomplished in a short time;
- group members might not have complete information for the decision;
- some required data, information, or knowledge for a decision may be located in many sources and some may be external to the organization.

Quite often, the group members may be at different locations and may be working at different times. Thus, they need to communicate, collaborate on, and access a diverse set of information sources, which can be met by the development of the Internet and its derivates (intranets and extranets). The questions of constructing and utilizing Web-based group decision support systems are discussed, for instance, in Lu *et al.* (2007).

With regard to the common goals and interests of the experts in group decision-making, it is worthy distinguishing two different environments, namely *cooperative* and *noncooperative* work. In cooperative decision-making, all the experts are supposed to work together, in order to achieve a decision for which they will share the responsibility. In noncooperative

decision-making, the experts play the role of antagonists or disputants over some common interest for which they must negotiate (Lu *et al.*, 2007). It should be made clear that this book addresses problems of group decision-making in the cooperative environment.

As in cooperative work the experts share responsibility for the decision (and, as indicated above, they also may participate in the implementation of the selected solution), it is important to guarantee that each member is satisfied with the selected solution. Clearly the commitment of the group to the implementation of the outcomes depends upon the level of consensus achieved by the group. Under this perspective, a group decision constructed by means of domination and enforced concessions should be considered inferior to an individual decision, because it will probably face more difficulties in its implementation. Therefore, achieving a genuine consensus on the solution is an important task for the group. However, it should be indicated that achieving perfect concordance among the experts is extremely rare. Although, ideally, the condition for terminating the decision-making process under group settings should be the achievement of a unanimous solution, in reality, because that unanimous solution hardly ever exists, it is sufficient to meet the alternative that is the most satisfactory for the group as a whole. Otherwise, the decision will probably take longer than is admissible or affordable.

Among the reasons for the occurrence of discordance among the group, we can identify the following:

- Although group members are supposed to share the primary goal, which obviously is to meet the solution which most benefits the organization, their secondary goals may be just partially shared. For instance, when each expert is representative of a different department, it is natural that they would have specific interests associated with the priorities and needs of their respective departments.
- Each expert usually has a distinct perception of the problem and intuition which may be difficult to formalize and communicate to the other members.
- In general, no single expert knows the entire domain of the decision problem. Each expert usually has access to different profiles of information. In particular, certain members of the group may have privileged access to secure information.

In general, these factors can be diminished by promoting discussions among the experts, in an attempt to pool all relevant information pertaining to the decision. Indeed, by pooling the undistributed information, it is possible to increase the chances of achieving better decisions than each member could obtain without help. However, the existence of abundant intellectual resources is not sufficient to guarantee high-quality decisions, as the group may fail to wisely consider, evaluate, and integrate the profiles of information and perspectives held by the other members of the group (Bonner, Baumannb, and Dalal, 2002; van Ginkel and van Knippenberg, 2009). The current literature identifies some factors that can adversely affect the decision process, leading to low-quality decisions. We can distinguish the following:

- The pressure for early consensus that is due to the need to obtain a solution rapidly.
- The pressure of concordant majorities on the other experts, which is reflected by the group's tendency to prematurely converge on a single solution, once a majority supports a position (even if such solution is not good).
- The problem of critical pooling of nondistributed (centralized) information, which can be described as follows: the information supporting the best alternative is not shared among

all experts, whereas all experts have information supporting the inferior alternatives. Under these circumstances, the group may prematurely achieve a consensus on a bad solution that is apparently good, as the information shared among most experts has more chance of being recalled than the information that is available to just a few experts. One way of reducing this specific problem is to stimulate each member to focus on information related to their respective areas of expertise during the discussion (Stasser and Vaughan, 2000).

In this context, it is important to stress the importance of the moderator (or facilitator) in the discussion among the experts. As indicated in Wong and Aiken (2003), the participation of a moderator, which may be human or automated, in the decision process, always results in better outcomes. The moderator is supposed to act as an arbiter responsible for controlling the information flow across the group. In this way, the moderator does not participate directly in the decision, but is supposed to enhance the ability of the group to make decisions (Griffith, Fuller, and Northcraft, 1998).

Among the tasks of a moderator we can identify the following: (1) define the rules for the group decision process and the tasks of each member, select the appropriate group technology, support the group in formulating the problems, and define the outcomes to be achieved; (2) develop the schedule to be accomplished, identify controversial opinions across the group, identify conflicting topics that should be focused on in the discussion, and verify if the current level of concordance among experts is acceptable (Ngwenyama, Bryson, and Mobolurin, 1996).

It is important to indicate that, in real-world applications, sometimes it is impossible to promote the consensus and thereby the exchange of information among the experts, due to logistic, timing, or monetary constraints. In this case, the invited professionals may give their opinions individually and then the group decision is dictatorially constructed with the use of an aggregation rule, despite the existence of substantial discordances among the experts. We can distinguish the following most common approaches for dealing with this situation:

- the use of a majority rule, according to which the group decision is constructed in concordance with the opinion of the majority in the group (Lu *et al.*, 2007);
- the use of a rule determined by a member of the group with authority to make the ultimate decision for the group (Lu *et al.*, 2007);
- the search for a collective opinion that minimizes the major discordance in the group, in such a way that no expert is extremely dissatisfied with the group outcomes (Parreiras *et al.*, 2010).

1.5   Fuzzy Sets and their Role in Decision-Making Processes

As elaborated in Section 1.1, various types of uncertainty are commonly met in a wide range of decision-making problems, which are inherently encountered in the project, planning, operation, and control of complex systems. Taking these types of uncertainty into account when constructing mathematical models serves as a vehicle for increasing the adequacy of the models and, as a result, the credibility and factual efficiency of decisions based on their analysis. Considering this, it is necessary to note that the starting point in the formation of mathematical models is the requirement of a strict correspondence of these models to the level of uncertainty of information used for their construction. Observing just this correspondence, we can talk

about the adequacy of the presentation of the object, system, or process and the possibility of obtaining a real effect as a result of solving the corresponding problems of an optimization character. Any simplification of reality or its idealization, undertaken with the purpose of using rigorous mathematical models, distorts the nature of many problems and diminishes the practical value of results obtained on the basis of analyzing these models. Following this line of thought, researchers (for instance, Belyaev and Krumm, 1983; Rommelfanger, 2004), for a number of reasons, have doubts about the validity or, at least, the expediency of taking into account the uncertainty factor within the framework of traditional approaches (first of all, approaches based on probability theory, for instance, Dantzig, 1955; Grassman, 1981; Wagner, 1982). In particular, Belyaev and Krumm (1983) indicate that, similar to the solution of problems on the basis of deterministic methods, when we assume exact knowledge of the information, which usually does not correspond to reality, the application of probabilistic methods also supposes exact knowledge of the distribution laws and their parameters, which does not always correspond to the real possibilities of obtaining the entire spectrum of the probabilistic description.

In general, the approaches highlighted above do not ensure an adequate or sufficiently rational consideration of the uncertainty factor along with an entire spectrum of its manifestations.

Giving up the traditional approaches to the construction of mathematical models and, the application of the fuzzy set theory (Dubois and Prade, 1980; Zimmermann, 1996; Pedrycz and Gomide, 1998), established by Zadeh (1965), may play and plays a significant positive role in overcoming the difficulties that are present. The utilization of this theory opens an interesting avenue of giving up "excessive" precision, which is inherent in the traditional modeling approaches, while preserving reasonable rigor. The principle of incompatibility coined by Zadeh (1973) offers an interesting view of the tradeoffs between precision and relevance of the models: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics". Furthermore, operating in a fuzzy parameter space allows one not only to be oriented toward the contextual or intuitive aspect of the qualitative analysis as a fully substantial aspect, but, by means of fuzzy set theory, to use this aspect as a sufficiently reliable source for obtaining quantitative information. Finally, fuzzy sets allow one to reflect in an adequate way on the essence of the decision-making process. In particular, since the "human factor" has a noticeable effect and occupies a very visible position in making decisions in many real-world problems, we can capitalize on the way in which fuzzy sets help quantify the linguistic facet of available data and preferences (Dubois and Prade, 1980; Zimmermann, 1996; Pedrycz and Gomide, 1998).

We also have to bear in mind that the quest for attaining the maximum effectiveness in decision-making in the presence of uncertainty requires, first of all, that a significant effort be directed toward finding ways to remove or, at least, partially overcome the uncertainty factor (Popov and Ekel, 1987). In particular, this can be attained by aggregating information that arrives from different sources, being both formal and informal in nature. This aggregation allows one (Ekel and Popov, 1985) to supplement the characteristics of the uncertain initial information by justified assumptions about the differentiated confidence (reliability) of its various values which could be reflected by choosing appropriate membership functions (Dubois and Prade, 1980; Zimmermann, 1996; Pedrycz and Gomide, 1998).

However, taking the above into account, it is necessary to indicate that the issues related to the relationships between probability theory and fuzzy set theory, as well as an interpretation of membership functions, have been the subject of intensive discussions of methodological and philosophical character over the years. Thus, it should be emphasized that the decision-making approaches based on fuzzy set theory do not compete with probabilistic methods, but these two approaches are orthogonal in nature. Furthermore, we can witness some hybrid approaches in which fuzzy sets and probability are used in a synergistic way. Likewise, recent years have seen intensive investigations which have applied fuzzy set theory in combination with other approaches to deal with diverse facets of uncertainty in problems of an optimization character. These developments offer the advantages of both a fundamental nature (as we exercise the possibility of obtaining more effective, less "cautious" solutions as well as the ability to consider simultaneously different manifestations of the uncertainty factor) and a computational character.

Finally, it is possible to distinguish two principal ways of solving problems under conditions of uncertainty. In applying the first way, one obtains (at least, theoretically) an exact solution for fixed values of the uncertain parameters, and then estimates its stability for variations of such parameters (for example, by performing multivariant computations). The second way presupposes the tracking of the effect of the uncertainty at all stages along the path toward the final decision. This approach can be implemented on the basis of fuzzy set theory. It is much more complicated than the first one, but is also much more fruitful and highly promising.

As mentioned above, in many real-world problems we have to take into account the criteria, constraints, indices, and so on, of a qualitative character. Thus, it should be emphasized that this type of information was taken into account in the past. However, it was used only after obtaining solutions on the basis of the use of formal models, with the disruption of these solutions (to consider information of a qualitative character) and without any sufficient justification. As such approaches reduce the essential value of the obtained solutions, it remains necessary to develop ways of introducing this type of information directly into the decision-making processes. Fuzzy sets can be considered here as a sound way of proceeding along this path.

Returning to the considerations of Section 1.1, it is necessary to highlight that one of the most important criteria for classifying decision-making problems (Larichev, 1987) is the existence or lack of an objective model for the problem. Taking this into consideration, it should be noted that it is not uncommon to encounter situations, as mentioned in Section 1.1, where it is next to impossible to speak about the existence of objective functions in decision-making problems. The corresponding models reflect the "world outlook" of a DM. In these cases, an obvious question is how to choose actions which correspond, in the best way, to the preferences of the individual (Keeney and Raifa, 1976). Considering that the manner of human thinking, including the perception of preferences, is vague and subjective, fuzzy set theory can play an important role in individual and group preference modeling (Fedrizzi and Kacprzyk, 1990; Fodor and Roubens, 1994).

The application of fuzzy sets to preference modeling and analysis of the corresponding decision-making problems provides a flexible environment which permits us to deal with the inherent fuzziness of perception and, in this manner, to incorporate more human consistency into preference models. Besides, a stimulus for utilizing fuzzy set theory stems, as indicated above, from one of its most important facets that concerns the linguistic aspect commonly applied to different decision-making problems and different preference structures (Herrera and Viedma, 2000; Xu, 2005). In particular, it is possible to distinguish among several directions

in decision-making by applying the linguistic aspect of fuzzy set theory, such as multicriteria decision-making (Buckley, 1995; Rasmy *et al.*, 2002), group decision-making (Yager, 1993; Herrera, Herrera-Viedma, and Verdegay, 1995), diverse consensus schemes (Herrera, Herrera-Viedma, and Verdegay, 1995; Bordogna, Fedrizzi, and Passi, 1997), decision-making on the basis of information granularity (Borisov *et al.*, 1989; Herrera, Herrera-Viedma, and Martínez, 2000), and so on. In principle, all these directions are associated with analyzing the $\langle \mathbf{X}, \mathbf{R} \rangle$ models mentioned above. Taking into account the rationality of analyzing $\langle \mathbf{X}, \mathbf{M} \rangle$ models on the basis of fuzzy sets as well, it is possible to assert that their utilization in the statement and solution of decision-making problems provides answers to the fundamental questions "What should we do?" ($\langle \mathbf{X}, \mathbf{R} \rangle$ models) and "How should we do?" ($\langle \mathbf{X}, \mathbf{M} \rangle$ models) arising in the project, planning, operation, and control of complex systems of diverse nature.

Finally, we should be aware that the development and application of diverse types of uncertainty expressed in the language of fuzzy sets not only serves as a vehicle for improving the adequacy of the constructed models and, consequently, enhancing the credibility and factual efficiency of decisions based on their analysis, but also becomes highly beneficial to the formation of convincing and effective human-oriented (in contrast to machine-oriented) interfaces between a DM and a computer. This aspect becomes crucial given the important and general trend of computerized "intellectualization" of decision-making pursuits.

Although the themes related to fuzzy decision-making have been widely and deeply studied, this area brings about a number of open questions associated not only with methods of decision-making in a fuzzy environment, but also with their combination with other branches of the mathematics of uncertainty (Wang, 2007). From the practical point of view, only some theoretical results have been translated to concrete algorithms and their implementation. In this context, one of the essential objectives of this book is to fill certain theoretical and practical gaps when considering the uncertainty and multicriteria factors in system projects, planning, operation, and control.

1.6 Conclusions

We have discussed the fundamental questions of the appearance and essence of decision-making problems arising in the project, planning, operation, and control of complex systems of diverse nature. The relevance and omnipresence of the uncertainty factor and its influence on the character of the analyzed decision-making models have been considered. The structured, unstructured, and semi-structured problems of decision-making have been classified with a distinct focus on unstructured problems. The main functions of decision support frameworks have been briefly discussed. The fundamental differences between optimization and decision-making problems have also been considered. The models of multicriteria decision-making have been characterized and classified with the split into two main categories of so-called $\langle \mathbf{X}, \mathbf{M} \rangle$ and $\langle \mathbf{X}, \mathbf{R} \rangle$ models, which are the subject of comprehensive considerations in this book. The essence, main concepts, and characteristics of group decision-making have been discussed. Finally, the role of fuzzy set theory in decision-making processes has been discussed, including consideration of its advantages. First of all we stressed the fundamental benefit stemming from the use of fuzzy sets that is the possibility of obtaining more effective, less "cautious" solutions to the decision-making problems, as well as the abilities of incorporating different facets and manifestations of the uncertainty factor.

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