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Introduction to Piezoelectric Energy Harvesting

This chapter provides an introduction to vibration-based energy harvesting using piezoelectric transduction. Following a summary of the basic transduction mechanisms that can be used for vibration-to-electricity conversion, the advantages of piezoelectric transduction over the other alternatives (particularly electromagnetic and electrostatic transductions) are discussed. Since the existing review articles mentioned in this chapter present an extensive review of the literature of piezoelectric energy harvesting, only the self-charging structure concept that uses flexible piezoceramics and thin-film batteries is summarized as a motivating example of multifunctional aspects. The focus is then placed on summarizing the literature of mathematical modeling of these devices for various problems of interest, ranging from exploiting mechanical nonlinearities to aeroelastic energy harvesting. Along with historical notes, the mathematical theory of linear piezoelectricity is briefly reviewed in order to derive the constitutive equations for piezoelectric continua based on the first law of thermodynamics, which are later simplified to reduced forms for use throughout this text. An outline of the remaining chapters is also presented.

1.1 Vibration-Based Energy Harvesting Using Piezoelectric Transduction

Vibration-based energy harvesting has received growing attention over the last decade. The research motivation in this field is due to the reduced power requirement of small electronic components, such as the wireless sensor networks used in passive and active monitoring applications. The ultimate goal in this research field is to power such small electronic devices by using the vibrational energy available in their environment. If this can be achieved, the requirement of an external power source as well as the maintenance costs for periodic battery replacement and the chemical waste of conventional batteries can be reduced.

As stated by Williams and Yates [1] in their early work on harvesting vibrational energy for microsystems, the three basic vibration-to-electric energy conversion mechanisms are the

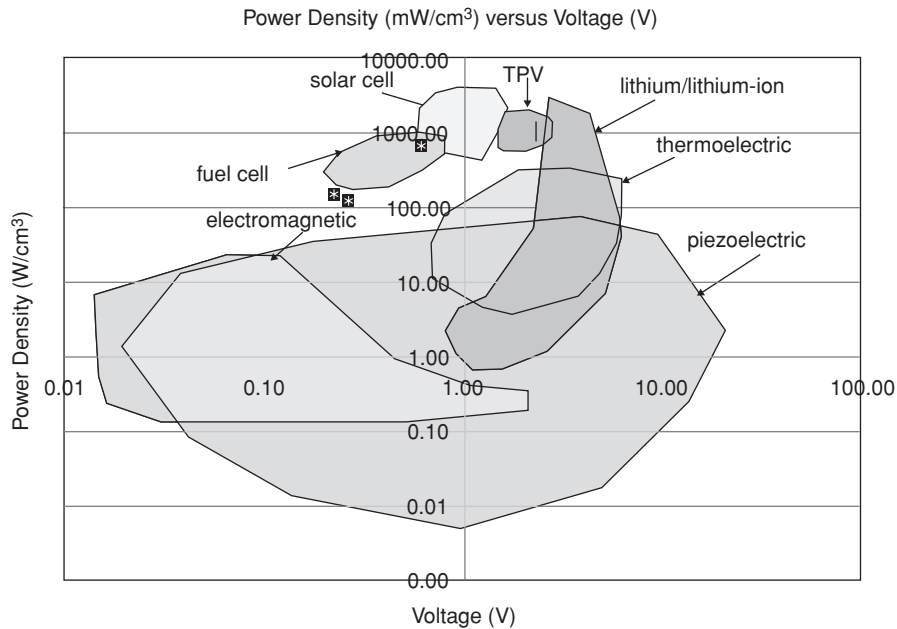


Figure 1.1 Power density versus voltage comparison of common regenerative and lithium/lithium-ion power supply strategies (from Cook-Chennault *et al.* [13], reproduced by permission of IOP © 2008)

electromagnetic [1–3], electrostatic [4, 5] and piezoelectric [6, 7] transductions.¹ Over the last decade, several articles have appeared on the use of these transduction mechanisms for low power generation from ambient vibrations. Two of the review articles covering mostly the experimental research on all transduction mechanisms are given by Beeby *et al.* [12] and Cook-Chennault *et al.* [13]. Comparing the number of publications that have appeared using each of these three transduction alternatives, it can be seen that piezoelectric transduction has received the greatest attention, especially in the last five years. Several review articles [13–16] have appeared in four years (2004–2008) with an emphasis on piezoelectric transduction to generate electricity from vibrations.

The main advantages of piezoelectric materials in energy harvesting (compared to using the other two basic transduction mechanisms) are their large power densities and ease of application. The power density² versus voltage comparison given in Figure 1.1 (due to Cook-Chennault *et al.* [13]) shows that piezoelectric energy harvesting covers the largest area in the graph with power density values comparable to those of thin-film and thick-film lithium-ion

¹ Other techniques of vibration-based energy harvesting include magnetostriction [8,9] and the use of electroactive polymers [10,11].

² The *power density* of an energy harvester is the power output divided by the device volume for a given input. In vibration-based energy harvesting, the input (excitation) is often characterized by the acceleration level and the frequency (usually a single harmonic is considered). Therefore, unless the input energy is provided, the power density is an insufficient parameter to compare different energy harvesters, although this has been frequently done in the literature.

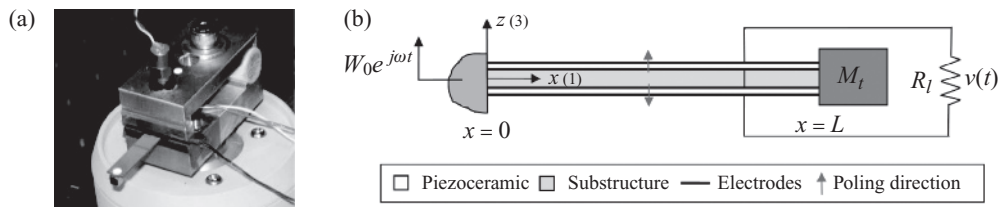


Figure 1.2 (a) A cantilevered piezoelectric energy harvester tested under base excitation and (b) its schematic representation

batteries and thermoelectric generators. As can be seen in Figure 1.1, voltage outputs in electromagnetic energy harvesting are typically very low and often multistage post-processing is required in order to reach a voltage level that can charge a storage component. In piezoelectric energy harvesting, however, usable voltage outputs can be obtained directly from the piezoelectric material itself. When it comes to electrostatic energy harvesting, an input voltage or charge needs to be applied so that the relative vibratory motion of the capacitor elements creates an alternating electrical output [4,5]. On the other hand, the voltage output in piezoelectric energy harvesting emerges from the constitutive behavior of the material, which eliminates the requirement of an external voltage input. As another advantage, unlike electromagnetic devices, piezoelectric devices can be fabricated both in macro-scale and micro-scale due to the well-established thick-film and thin-film fabrication techniques [7,17]. Poor properties of planar magnets and the limited number of turns that can be achieved using planar coils are some of the main practical limitations in enabling micro-scale electromagnetic energy harvesters [13].

Most piezoelectric energy harvesters are in the form of cantilevered beams with one or two piezoceramic layers (i.e., a unimorph or a bimorph).³ The harvester beam is located on a vibrating host structure and the dynamic strain induced in the piezoceramic layer(s) results in an alternating voltage output across their electrodes. An example of a cantilever tested under base excitation is shown in Figure 1.2 along with its schematic. An alternating voltage output is obtained due to the harmonic base motion applied to the structure. In the mechanics research on piezoelectric energy harvesting as well as in the experimental research conducted to estimate the device performance for AC power generation, it is common practice to consider a resistive load in the electrical domain [6,7,20–27] to represent an electronic load as depicted in Figure 1.2b.

From an electrical engineering point of view, the alternating voltage output should be converted to a stable rectified voltage through a rectifier bridge and a smoothing capacitor (which constitute an AC–DC converter) for charging a small battery or a capacitor by using the harvested energy. Often a second stage (DC–DC converter) is employed to regulate the voltage output of the rectifier so that the power transfer to the storage device can be maximized (Figure 1.3). These electrical circuit and power electronics aspects [28–34] are addressed in

³ Typical exceptions are the stack [18] and the cymbal [19] transducers used under direct force excitation in limited cases or patches excited by surface strain fluctuations (Chapter 7). Polyvinylidene fluoride (PVDF) membranes can also be used for piezoelectric power generation; however, they exhibit very low electromechanical coupling compared to piezoceramics and they are not discussed here.

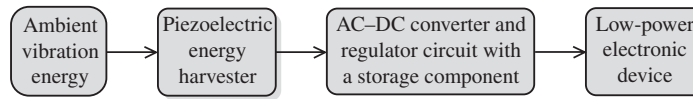


Figure 1.3 Schematic representation of the concept of a piezoelectric energy harvesting system

the last chapter of this book with a brief review of the relevant literature. The majority of this text takes a perspective from the mechanical engineering standpoint with a focus on detailed electromechanical modeling of piezoelectric energy harvesters (with simplified circuits) for various forms of excitation and engineering applications of practical importance.

1.2 An Example of a Piezoelectric Energy Harvesting System

Although examples from the literature of piezoelectric energy harvesting will not be reviewed here (the reader is referred to the existing review articles [13–16]), it is worth discussing a specific application of multidisciplinary aspects. The concept of *self-charging structures* refers to structures composed of elastic substructures (usually metallic or carbon fiber), flexible piezoceramics embedded in kapton layers, and flexible thin-film battery layers [35–38]. Proof-of-concept prototypes of self-charging structures with aluminum and carbon-fiber substructures are shown in Figure 1.4a. The goal of this concept is to use these structures in load-bearing applications to improve *multifunctionality* [39, 40] for low-power applications (such as powering a wireless sensor in the vicinity of the load-bearing structure using the harvested and stored energy). That is, if the existing load-bearing structure can be modified using flexible piezoceramics and thin-film batteries, electrical energy can be generated from the dynamic loads and stored inside the structure itself. As long as the load-bearing capacity of the modified structure is within an acceptable margin, this concept provides a multifunctional structure that can find applications for remote structural systems with battery-powered wireless electronic components. Dynamic excitation of a self-charging structure to charge its battery layers is shown in Figure 1.4b and its schematic view is depicted in Figure 1.4c. Energy-efficient charging of the battery layers (Figure 1.4d) by using the electrical output of the piezoceramic layers requires sophisticated considerations in the electrical domain. A regulator circuit that operates in the discontinuous conduction mode [34] for resistive impedance matching is shown in Figure 1.3e. In order to claim multifunctionality, it is required to test and characterize the load-bearing capacity of the multilayer structure. Three-point bending tests (Figure 1.4f) verify that the most critical layers are the brittle piezoceramics, although the kapton layers prevent disintegration of the structure after the fracture [41]. A primary motivation for self-charging structures is to use them for powering small electronic components in unmanned aerial vehicle (UAV) applications (Figure 1.4g).

More recently, flexible solar panels have also been combined with piezoceramics, thin-film batteries, and metallic layers as shown in Figure 1.5 [42]. The purpose is to utilize both sunlight and structural vibrations for charging the battery layers. Multifunctionality is again due to the load-bearing capability of the entire assembly with flexible layers. The electrical energy that can be harvested through the flexible solar cells depends mainly on the amount of the solar irradiance level, while the piezoelectric power output strongly depends on the electromechanical frequency response of the structure due to the *resonance* phenomenon.

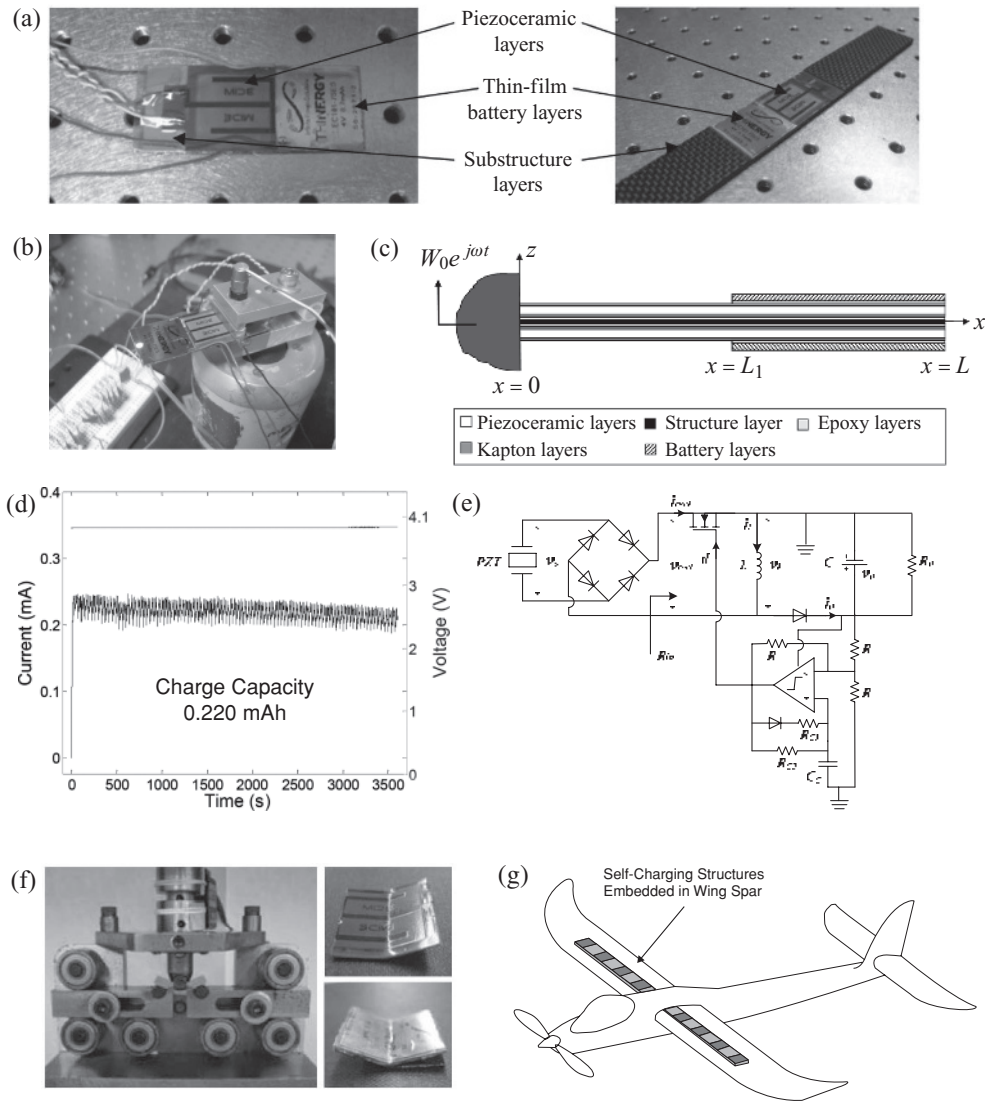


Figure 1.4 (a) Self-charging structure prototypes; (b) a cantilevered self-charging structure under base excitation; (c) schematic representation of the self-charging structure; (d) time history of charging a thin-film battery; (e) resistive impedance matching circuit; (f) experimental characterization of the mechanical strength; (g) an application for unmanned aerial vehicles

For a given vibration input (prescribed acceleration or dynamic strain at a certain frequency and amplitude or with certain statistical characteristics), the piezoelectric power output of the self-charging structure shown in Figure 1.5 is a complicated function of the geometric and material properties of its layers. Although the electrical response due to structural vibrations

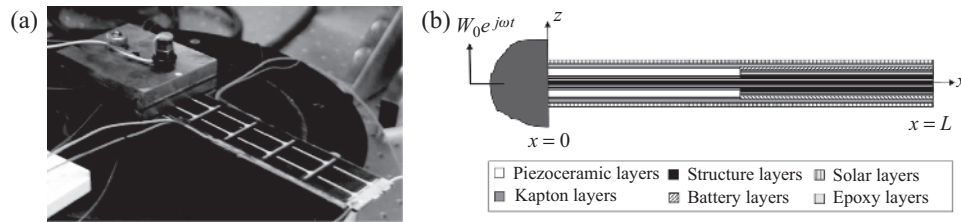


Figure 1.5 (a) A self-charging structure prototype with flexible metallic, piezoceramic, thin-film battery, solar, kapton, and epoxy layers, and (b) its schematic representation

is obtained from the piezoceramics layers, it is the dynamics of the entire composite structure that determine the resonance frequencies, that is, the effective frequencies of maximum power generation. Therefore, it is important to model the structure with substantial accuracy from the mechanical standpoint so that the resulting electromechanical model can provide reliable estimates of the acceleration level required to reach a desired voltage output as well as the matched resistance (for the circuit design) and the maximum acceleration that can be sustained by the piezoceramic layers (for the structural design), among other design parameters at the systems level. Moreover, often the vibratory motion of the energy harvester can be more sophisticated than simple harmonic oscillations at a single frequency; for example, it might involve nonlinear dynamical aspects or coupling with the surround airflow. The aim of this book is also to provide an introduction to such advanced topics in vibration-based energy harvesting.

1.3 Mathematical Modeling of Piezoelectric Energy Harvesters

As in the particular case of the self-charging structure concept summarized in Figures 1.4 and 1.5, research in the area of piezoelectric energy harvesting is strongly connected to various disciplines of engineering. Consequently, this promising way of powering small electronic components and remote sensors has attracted researchers from different disciplines of engineering, including mechanical, aerospace, electrical, and civil, as well as researchers from the field of materials science [13–16], and various modeling approaches have appeared as summarized in the following. A comprehensive mathematical model should be as simple as possible yet sophisticated enough to capture the important phenomena needed to represent and predict the dynamics of the physical system as required by the application of interest.

In the early mathematical modeling treatments, researchers [20, 21] employed lumped-parameter solutions with a single mechanical degree of freedom to predict the coupled system dynamics of piezoelectric energy harvesters (such as the one shown in Figure 1.2). Lumped-parameter modeling is a convenient modeling approach since the electrical domain already consists of lumped parameters: a capacitor due to the internal (or inherent) capacitance of the piezoceramic and a resistor due to an external load resistance. Hence, the only requirement is to obtain the lumped parameters representing the mechanical domain so that the mechanical equilibrium and the electrical loop equations can be coupled through the piezoelectric constitutive equations [43] and a transformer relation can be established. This was the main procedure followed by Roundy and Wright [20] and duToit *et al.* [21] in their lumped-parameter model derivations. Although lumped-parameter modeling gives initial insight into the problem by allowing simple expressions, it is an approximation limited to a single vibration mode and it lacks some important aspects of the coupled physical system,

such as the dynamic mode shapes and accurate strain distribution as well as their effects on the electrical response.

Since cantilevered energy harvesters are basically excited under base motion, the well-known lumped-parameter harmonic base excitation relation taken from elementary vibration texts [44, 45] has been used widely in the energy harvesting literature for both modeling [21] and studying the maximum power generation and parameter optimization [46, 47]. In both lumped-parameter models [20, 21] (derived for the transverse vibrations and longitudinal vibrations, respectively), the contribution of the distributed mass (spring mass in the lumped-parameter sense) to the forcing amplitude in the base excitation problem is neglected. The inertial contribution of the distributed mass to the excitation amplitude can be important, especially if the harvester does not have a large proof mass [48].

As an improved modeling approach, the Rayleigh–Ritz type [44] of discrete formulation originally derived by Hagood *et al.* [49] for piezoelectric actuation (using the generalized Hamilton’s principle for electromechanical systems given by Crandall *et al.* [50]) was employed by Sodano *et al.* [22] and duToit *et al.* [21, 26] for modeling cantilevered piezoelectric energy harvesters (based on the Euler–Bernoulli beam theory). The Rayleigh–Ritz model gives a spatially discretized model of the distributed-parameter system and is a more accurate approximation compared to lumped-parameter modeling with a single mechanical degree of freedom. The Rayleigh–Ritz model gives an approximate representation of the distributed-parameter system (Figure 1.2) as a discretized system by reducing its mechanical degrees of freedom from infinity to a finite dimension and usually it is computationally more expensive than the analytical solution (if available).

In order to obtain analytical expressions, several others [23–25, 27] used the vibration mode shapes obtained from the Euler–Bernoulli beam theory along with the piezoelectric constitutive equation that gives the electric displacement to relate the electrical outputs to the vibration mode shape. The important deficiencies in these mathematical modeling attempts and others were summarized in the literature [51] and they include the lack of consideration of the resonance phenomenon, ignorance of modal expansion, misrepresentation of the forcing due to base excitation, oversimplified modeling of piezoelectric coupling in the beam equation as viscous damping, and use of the static sensing/actuation equations in a fundamentally dynamic problem.

The analytical solutions based on distributed-parameter electromechanical modeling were given by Erturk and Inman [52, 53] along with experimental validations. Convergence of the Rayleigh–Ritz solution to the analytical solution was shown by Elvin and Elvin [54], who combined the lumped parameters obtained from the Rayleigh–Ritz formulation with circuit simulation software to investigate more sophisticated circuits in the time domain. Finite-element simulations given by Rupp *et al.* [55], De Marqui *et al.* [56], Elvin and Elvin [57], and Yang and Tang [58] were also shown to agree with the analytical solutions [52, 53] and they were introduced for various purposes ranging from topology optimization [55] and added mass optimization [56] to analysis of nonlinear circuits [57, 58].

Later, researchers focused on exploiting mechanical nonlinearities in vibration-based energy harvesting and the linear electromechanical models were modified accordingly. Erturk *et al.* [59] used the large-amplitude periodic attractor of a bistable *piezomagnetoelastic* device⁴ and observed a substantially improved broadband energy harvesting performance that

⁴This device is based on the magnetoelastic structure introduced by Moon and Holmes [60, 61] to study chaotic vibrations in structural mechanics.

had a power output an order of magnitude larger than the conventional cantilever configuration. The mechanical component of the nonlinear electromechanical equations [59] obeys the form of the *bistable* Duffing oscillator [62–65].⁵ Contemporaneously, Stanton *et al.* [66] also combined the piezoelastic cantilever configuration with magnets to create softening and hardening stiffness effects in the form of the *monostable* Duffing oscillator.⁶ In addition, they [70] studied an alternative bistable structural configuration focusing on chaotic vibrations as well as other dynamic responses emerging from the bifurcations of the system with respect to excitation frequency and base acceleration amplitude. Parametric excitation [63] of a nonlinear piezoelectric energy harvester was theoretically and experimentally investigated by Daqaq *et al.* [71].

The effect of inherent piezoelectric nonlinearity (formerly pointed out by Crawley and Anderson [72] and Crawley and Lazarus [73] for large electric fields in structural actuation) was implemented by Triplett and Quinn [74] for energy harvesting. Stanton and Mann [75] presented a Galerkin solution by taking into account the geometric and piezoelastic nonlinearities for weak electric fields [76] (since the electric field levels in energy harvesting are not as high as in actuation). For relatively stiff cantilevers, inherent piezoelastic nonlinearities can be pronounced even when the oscillations are geometrically linear.⁷ Therefore the geometrically and piezoelectrically nonlinear modeling framework [75] was later simplified and used for quantitatively identifying the piezoelastic nonlinearities in the absence of geometric nonlinearity for a bimorph cantilever [77].

Another modeling problem of interest is the *stochastic* excitation of vibration-based energy harvesters since waste vibrational energy often appears in nondeterministic [78, 79] forms. Scruggs [80] investigated the optimal control of a linear energy harvester network for increased power flow to a storage system under stochastic excitation. An analysis of random vibrations of a linear piezoelectric energy harvester was carried out by Adhikari *et al.* [81] based on lumped-parameter modeling with a single mechanical degree of freedom. Daqaq [82] considered hardening stiffness in the lumped-parameter formulation (for electromagnetic energy harvesting) and showed that the monostable Duffing oscillator does not provide any enhancement over the typical linear oscillators under white Gaussian and colored excitations.⁸ Cottone *et al.* [84] and Gammaitoni *et al.* [85] reported that nonlinear oscillators can outperform the linear ones under noise excitation both for bistable [84] and monostable [85] configurations. Utilization of nonlinear *stochastic resonance* [86–88] for vibration-based

⁵The terms *bistable* and *monostable* are used in this text for the *static* equilibrium of the respective configuration (i.e., statically bistable and statically monostable). In this context, for instance, the dynamic response of a monostable Duffing oscillator can have multiple (coexisting) stable solutions but it is statically monostable.

⁶Implementations of the monostable Duffing oscillator with hardening stiffness were formerly presented by Burrow *et al.* [67] and Mann and Sims [68] for electromagnetic energy harvesting. Use of the hardening stiffness in the monostable form of the Duffing oscillator was also discussed by Ramlan *et al.* [69] along with snap-through in a bistable mass–spring–damper mechanism.

⁷Recently, it has been observed that the nonlinearities in piezoelastic behavior and dissipation can become effective if the base excitation level exceeds a few hundreds of milli-*g* acceleration [75, 77]. In the absence of other nonlinearities, the manifestation of piezoelastic nonlinearities is a softening stiffness effect; that is, the peak of the amplitude moves toward the left on the frequency axis with increasing base acceleration.

⁸His theoretical observation agrees with the experimental results contemporaneously reported by Barton *et al.* [83] for electromagnetic energy harvesting with stiffness nonlinearity of Duffing-type (where the majority of the high-energy branch obtained under periodic forcing is not reached under random forcing).

energy harvesting was formerly discussed by McInnes *et al.* [89] as well, without focusing on a specific transduction mechanism. More recently, Litak *et al.* [90] employed the lumped-parameter nonlinear equations given by Erturk *et al.* [59] and presented numerical simulations of stochastic resonance in the piezomagnetoelastic energy harvester under noise excitation.

Some of the interesting applications of piezoelectric energy harvesting involve fluid–structure interactions. For the piezoaeroelastic problem of energy harvesting from the air-flow excitation of a cantilevered plate with embedded piezoceramics, De Marqui *et al.* [91, 92] presented finite-element models based on the vortex-lattice method [91] and the doublet-lattice method [92] of aeroelasticity [93–96]. Time-domain simulations [91] were given for a cantilevered plate with embedded piezoceramics for various airflow speeds below the linear flutter speed and at the flutter boundary. Frequency-domain simulations [92] considering resistive and resistive–inductive circuits were also presented focusing on the linear response at the flutter boundary. Bryant and Garcia [97, 98] studied the aeroelastic energy harvesting problem for a typical section by using the finite state theory of Peters *et al.* [99]. Erturk *et al.* [100] presented an experimentally validated lumped-parameter model for a wing section (airfoil) with piezoceramics attached onto plunge stiffness members using the unsteady aerodynamic model proposed by Theodorsen [101]. Piezoelectric power generation at the flutter boundary and its effect on the linear flutter speed have also been discussed [100]. In addition to their possible integration with aerospace structures, aeroelastic energy harvesters are considered as a scalable alternative to the conventional windmill design [102]. As an alternative to airfoil-based and wing-based configurations, St. Clair *et al.* [103] presented a design that uses a piezoelectric beam embedded within a cavity under airflow and described the resulting limit-cycle oscillations through the governing equations of the Van der Pol oscillator [63–65, 104]. Vortex-induced oscillations of piezoelectric cantilevers located behind bluff bodies were investigated by Robbins *et al.* [105], Pobering *et al.* [106] and Akaydin *et al.* [107, 108] through experiments and numerical simulations. Other than airflow excitation, Elvin and Elvin [109] theoretically investigated the flutter response of a cantilevered pipe with piezoceramics for power generation from liquid flow and its effect on the flutter instability.

The foregoing summary includes some of the major papers on the mathematical modeling of piezoelectric energy harvesters for various applications where the presence of waste vibrational energy might have some value. Before reviewing the outline of the material covered in this text, it is useful to summarize the theory of linear piezoelectricity with brief historical notes.

1.4 Summary of the Theory of Linear Piezoelectricity

Piezoelectricity is a form of coupling between the mechanical and the electrical behaviors of ceramics and crystals belonging to certain classes. These materials exhibit the *piezoelectric effect*, which is historically divided into two phenomena as the *direct* and the *converse* piezoelectric effects. In simplest terms, when a piezoelectric material is mechanically strained, electric polarization that is proportional to the applied strain is produced. This is called the direct piezoelectric effect and it was discovered by the Curie brothers in 1880. When the same material is subjected to an electric polarization, it becomes strained and the amount of strain is proportional to the polarizing field. This is called the converse piezoelectric effect (sometimes called the “inverse” piezoelectric effect) and it was deduced mathematically from

the fundamental principles of thermodynamics by Lippmann⁹ in 1881 [110] and then confirmed experimentally by the Curie brothers in the same year. It is important to note that these two effects usually coexist in a piezoelectric material. Therefore in an application where the direct piezoelectric effect is of particular interest (which is the case in vibration-based energy harvesting), ignoring the presence of the converse piezoelectric effect would be thermodynamically inconsistent.¹⁰

Several natural crystals were observed to exhibit the piezoelectric effect in the first half of the last century, for example, Rochelle salt, quartz, and so on [111]. However, in order to use them in engineering applications, the *electromechanical coupling* between the mechanical and the electrical behaviors of the material has to be sufficiently strong. Piezoelectric ceramics were developed in the second half of the last century and they exhibit much larger coupling compared to natural crystals. The most popular of engineering piezoceramics, PZT (lead zirconate titanate), was developed at the Tokyo Institute of Technology in the 1950s and various versions of it (particularly PZT-5A and PZT-5H) are today the most commonly used engineering piezoceramics. As far as energy harvesting research is concerned, PZT-5A and PZT-5H are the most widely implemented piezoceramics according to the literature [13].

Decades after the fundamental contributions to the field of piezoelectricity (by Voigt [111], Cady [112], Heising [113], Mason [114], Mindlin and Tiersten [115–117], among others cited in their work), today's commonly used IEEE Standard on Piezoelectricity [43] was published.¹¹ It is useful to review the derivation of the linear constitutive equations of a piezoelectric material [43], which will later be simplified to reduced forms (Appendix A) for use in various chapters of this text.

The first law of thermodynamics (the principle of conservation of energy) for a linear piezoelectric continuum leads to [117]

$$\dot{U} = T_{ij}\dot{S}_{ij} + E_i\dot{D}_i \quad (1.1)$$

where U is the stored energy density of the piezoelectric continuum, T_{ij} is the stress tensor, S_{ij} is the strain tensor, E_i is the electric field tensor, D_i is the electric displacement tensor and the overdot represents differentiation with respect to time.¹²

The electric enthalpy density H is given by

$$H = U - E_i D_i \quad (1.2)$$

Substituting Equation (1.1) into the time derivative of Equation (1.2) gives

$$\dot{H} = T_{ij}\dot{S}_{ij} - D_i\dot{E}_i \quad (1.3)$$

⁹ Lippmann's relevant article [110] on the application of thermodynamic principles to reversible processes involving electrical quantities includes electrostriction and pyroelectricity as well (which also have their converse effects).

¹⁰ Ignoring the effect of piezoelectric power generation on the dynamics of the energy harvester is a typical oversimplification repeated in the existing literature, as summarized by Erturk and Inman [51].

¹¹ This [43] refers to ANSI/IEEE Std 176-1987. An earlier version of it (ANSI/IEEE Std 176-1978) was published in 1978 (which followed various standards dating back to 1950s).

¹² The electric field and the electric displacement tensors are Cartesian tensors [118] of order 1 (i.e., they are vectors). Although, in principle, the stress and the strain tensors are of order 2, these are symmetric tensors and their subscripts can be relabeled based on Voigt's notation [111] so that they become vectors of six components (Appendix A).

which implies $H = H(S_{ij}, E_i)$ so that the components of the stress and the electric displacement tensors can be derived from the electric enthalpy density as [117]

$$T_{ij} = \frac{\partial H}{\partial S_{ij}}, \quad D_i = -\frac{\partial H}{\partial E_i} \quad (1.4)$$

The form of the electric enthalpy density in the linearized theory of piezoelectricity is

$$H = \frac{1}{2} c_{ijkl}^E S_{ij} S_{kl} - e_{kij} E_k S_{ij} - \frac{1}{2} \varepsilon_{ij}^S E_i E_j \quad (1.5)$$

where c_{ijkl}^E , e_{kij} , and ε_{ij}^S are the elastic, piezoelectric, and permittivity constants, respectively, while the superscripts E and S denote that the respective constants are evaluated at constant electric field and constant strain, respectively.

Using Equations (1.4) and (1.5) along with the relation $\partial S_{ij} / \partial S_{ji} = \delta_{ij}$ (where δ_{ij} is the Kronecker delta defined as being equal to unity for $i = j$ and equal to zero for $i \neq j$), one obtains [43,117]

$$T_{ij} = c_{ijkl}^E S_{kl} - e_{kij} E_k \quad (1.6)$$

$$D_i = e_{ikl} S_{kl} + \varepsilon_{ik}^S E_k \quad (1.7)$$

which is the form of the linear constitutive equations for the unbounded piezoelectric continuum. The following three pairs constitute alternative forms of piezoelectric constitutive equations and they are used for approximations under certain limiting circumstances [43] (usually for bounded piezoelectric media):

$$S_{ij} = s_{ijkl}^E T_{kl} + d_{kij} E_k \quad (1.8)$$

$$D_i = d_{ikl} T_{kl} + \varepsilon_{ik}^T E_k \quad (1.9)$$

and

$$S_{ij} = s_{ijkl}^D T_{kl} + g_{kij} D_k \quad (1.10)$$

$$E_i = -g_{ikl} T_{kl} + \beta_{ik}^T D_k \quad (1.11)$$

and

$$T_{ij} = c_{ijkl}^D S_{kl} - h_{kij} D_k \quad (1.12)$$

$$E_i = -h_{ikl} S_{kl} + \beta_{ik}^S D_k \quad (1.13)$$

where d_{kij} , g_{kij} , and h_{kij} are alternative forms of the piezoelectric constants, s_{ijkl}^E and s_{ijkl}^D are the elastic compliance constants, and β_{ik}^T and β_{ik}^S are the impermittivity constants. Here, the superscripts D and T denote that the respective constants are evaluated at constant electric displacement and constant stress, respectively. The transformations between the elastic, piezoelectric, and dielectric constants under different electrical and mechanical conditions can be found in the IEEE standard [43].

The electromechanical models derived in this book use the form of the constitutive equations given by Equations (1.8) and (1.9) for deriving reduced elastic, piezoelectric, and dielectric constants depending on the structural theory used and the locations of the electrodes (Appendix A). The surviving stress components and the electric displacement components are then used in model derivations as the dependent field variables in the form of Equations (1.6) and (1.7) while the strain and the electric field components are expressed in terms of the displacement field and the voltage output, respectively.

1.5 Outline of the Book

Vibrational energy is available in various environments as an alternative form of waste energy. The modeling treatment required to predict the coupled dynamics of a piezoelectric energy harvester changes dramatically depending on the application and the form of the excitation. For instance, although it is common practice to consider harmonic excitation to characterize the resonance behavior of a piezoelectric energy harvester, scavenging energy from vibrations of an aircraft wing requires a more involved analysis due to the coupling of the piezoelectric structure with the surrounding airstream. The motivation of this book is therefore to cover the basic mathematical modeling treatments for different problems and applications of piezoelectric energy harvesting. The problems discussed here range from nonlinear energy harvesting to energy harvesting from aeroelastic vibrations and the applications of interest include civil, mechanical, and aerospace engineering structures.

Chapter 2 reviews the formal treatment of the base excitation problem since piezoelectric energy harvesters are often designed as cantilevers operated under harmonic base excitation. After showing the inaccuracy of the frequently quoted lumped-parameter base excitation equation for a single mechanical degree of freedom, correction factors are derived to improve the lumped-parameter equations for both the transverse and the longitudinal vibration cases. Experimental validations are given and an amplitude correction factor is introduced to the lumped-parameter piezoelectric energy harvester equations. The distributed-parameter derivations given in this chapter constitute the mechanical background for the modeling of cantilevers excited under base motion in vibration-based energy harvesting.

Chapter 3 introduces electromechanically coupled analytical solutions of symmetric bi-morph piezoelectric energy harvester configurations under base excitation for the series and parallel connections of the piezoceramics layers. The formulation is given based on the thin-beam theory since piezoelectric energy harvesters are typically thin structures. The multi-mode solutions (valid for arbitrary excitation frequencies) and the single-mode solutions (approximately valid at or near the resonance frequencies) are obtained and an extensive theoretical case study is provided. The analytical solutions obtained in this chapter are used for several purposes throughout the book.

Chapter 4 provides detailed experimental validations for the analytical expressions derived in Chapter 3. Three case studies are given with a focus on a brass-reinforced PZT-5H cantilever in the absence and presence of a tip mass and a brass-reinforced PZT-5A cantilever. The effect of the rotary inertia of a tip mass is discussed and the single-mode and multi-mode electromechanical frequency response functions (FRFs) are compared to the experimental measurements. Validations of the electromechanical FRFs are given along with the electrical performance diagrams. In addition to validation of the analytical solutions, this chapter

provides a comprehensive discussion of the experimental testing and characterization of a piezoelectric energy harvester.

Chapter 5 presents dimensionless forms of the analytical expressions and detailed mathematical analyses of these equations along with experimental validations. The asymptotic forms of the voltage and displacement FRFs are obtained for the short-circuit and open-circuit conditions, which are then employed to derive closed-form expressions for identifying various system parameters. Analysis of the voltage asymptotes leads to a simple experimental technique for identifying the optimum electrical load using an open-circuit voltage measurement and a single resistor.

Chapter 6 presents approximate analytical solutions using an electromechanical version of the assumed-modes method for relatively complicated structural configurations which do not allow analytical solutions. The Euler–Bernoulli, Rayleigh, and Timoshenko types of solutions are presented. The derivations given in this chapter can be used for energy harvesters with varying cross-section and material properties, asymmetric laminates as well as moderately thick energy harvesters. The experimental cases given in Chapter 4 are revisited for validation of the approximate solutions using the electromechanical assumed-modes method. Convergence of the assumed-modes solution to the analytical solution with increasing number of modes is also shown. Another experimental case study is given for the modeling of a two-segment bimorph energy harvester.

Chapter 7 introduces distributed-parameter electromechanical models for piezoelectric energy harvesting under various forms of dynamic loading. The problems discussed in this chapter are periodic excitation, random excitation in the form of ideal white noise, moving load excitation of slender bridges, excitation due to strain fluctuations on large structures, and general transient base excitation. Detailed derivations are given for representing the input force to be used in the electromechanical problem and for predicting the electrical output under these various excitation forms. Two case studies are presented for the periodic excitation of a bimorph cantilever located on a four-bar mechanism and for strain fluctuations on a civil engineering structure.

Chapter 8 focuses on modeling and exploiting mechanical nonlinearities in piezoelectric energy harvesting. Enhancement of the bandwidth using the monostable form of the Duffing equation with softening and harvesting stiffness effects is discussed and a perturbation-based electromechanical formulation is given. The perturbation solution is verified against the time-domain numerical simulations. After the monostable case, the outstanding broadband power generation performance of a bistable piezomagnetoelastic energy harvester is investigated theoretically and experimentally. Chaotic vibration of the bistable piezoelectric energy harvester configuration is also discussed. Experimental performance results of a bistable carbon-fiber-epoxy plate with piezoceramics are summarized for nonlinear broadband piezoelectric energy harvesting.

Chapter 9 investigates the problem of piezoelectric energy harvesting from aeroelastic vibrations of structures with piezoceramic layers under airflow excitation. The linear piezoaeroelastic models discussed here include a two-degree-of-freedom formulation (for the typical section problem with plunge and pitch degrees of freedom) and assumed-modes formulation (for the distributed-parameter problem of a cantilever with bending and torsion modes). Finite-element piezoaeroelastic models using the vortex-lattice and the doublet-lattice methods are also reviewed in this chapter. Experimental validations are presented for the piezoaeroelastic typical section model and theoretical simulations are given for the finite

element solutions. The effect of piezoelectric power generation on the linear flutter speed is also discussed.

Chapter 10 considers the effects of material constants and mechanical damping on piezoelectric energy harvesting. Various soft piezoelectric ceramics and single crystals are compared for power generation by using the experimentally validated model introduced in Chapter 3. Performance comparisons for resonance excitation are presented and the effect of damping is investigated along with other important material parameters in the beam-type plane-stress formulation. Soft and hard ceramics as well as soft and hard crystals are also compared for resonant and off-resonant energy harvesting.

Chapter 11 provides a brief review of the literature of electrical circuit papers used in piezoelectric energy harvesting. Lumped-parameter modeling of a piezoelectric energy harvester with a standard AC–DC converter is summarized and simulation results are reviewed. Two-stage energy harvesting circuits combining AC–DC and DC–DC converters are also discussed. Finally, the synchronous switch harvesting on inductor technique and its modeling are reviewed for performance enhancement in weakly coupled piezoelectric energy harvesters.

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