1

A Simple Flight Model

1.1 Introduction

1.1.1 General Introduction to Volume 2

Welcome to Volume 2 of *Computational Modelling and Simulation of Aircraft and the Environment*. This volume will present and explain the main theories that enable the dynamics of fixed-wing aircraft to be modelled using mathematical and computational methods. The aim is to establish the heuristic basis for education in aeronautical engineering that provides a 'handbook' of concepts and interpretations, together with a formulary to support practical application. It is appropriate and convenient to commence with a simple flight model that brings together all the essential components without too much detail. This covers aircraft motion, atmosphere, aerodynamics, and propulsion. More detailed expositions are given in Chapters 2–5. These focus on Equations of Motion, Wing Aerodynamics, Longitudinal Flight and Gas Turbines.

The significant omission is lateral-directional aerodynamics, apart from rolling a wing in flight (later in Chapter 1). This is because the formulary tends to be complicated and abstract, with no easily recognisable link to the underlying physics. Also, there is no inherent value in just repeating what other books [e.g. Pamadi] already provide. Also, supersonic flight is not discussed because it is a specialised area of aircraft design. The vast majority of aircraft are not supersonic.

The final chapter offers a brief introduction to several topics that are important in whole-aircraft modelling but that sit outside the usual scope of flight physics. The discussion is brief because these subjects have substantial content and could easily expand to fill another two or three textbooks.

1.1.2 What Chapter 1 Includes

This chapter includes:

- Equations of motions expressed with respect to flight path parameters.
- Summary of the International Standard Atmosphere up to 20 km (roughly 50 000 ft).
- Simple propulsion model that enables thrust calculations at given altitude and Mach number.

Computational Modelling and Simulation of Aircraft and the Environment: Aircraft Dynamics,
First Edition, Volume II. Dominic J. Diston.

- Simple aerodynamic model that is applicable to idealised wing geometry plus trailing-edge flaps.
- A short introduction to spanwise lift distribution for an idealised wing.
- Aerodynamic model for wing/tail combinations (as an approximation to a complete aircraft).
- A set of airspeed definitions.
- One of many possible architectures for a flight model (i.e. a whole-aircraft model).

1.1.3 What Chapter 1 Excludes

This chapter excludes:

- Six degree-of-freedom (6-DOF) equations of motion [go to Chapter 2].
- Generalised wing configurations (e.g. taper, twist) [go to Chapter 3].
- Flight mechanics of wing/tail combinations [go to Chapter 4].
- Fuselage aerodynamic effects [go to Chapter 4].
- Physics-based models of gas turbines [go to Chapter 5].
- Lateral-directional aerodynamics [not covered by this book].
- Supersonic flight [not covered by this book].

1.1.4 Overall Aim

Chapter 1 should provide 'enough of everything' that is needed to create a complete representation of aircraft flight behaviour, from ground up to 20 km and from low-speed up to about 0.85 Mach number. This includes the essential flight physics without too much detail, such that computations can be verified by manual calculation and that parametric trend should be readily discernible. In short, this should provide a compact aircraft model for the purpose of preliminary concept evaluation and simulation.

1.2 Flight Path

The simplest possible flight path model is shown in Figure 1.1. This represents symmetric flight (with wings level) in a vertical plane. Motion parameters are defined at the centre of mass for an instantaneous pull-up (which is turn in the vertical plane). Airspeed V is aligned (or tangential) with the flight path, which is normal to the radius of turn. The tangential acceleration varies the airspeed while the centripetal acceleration varies the flight path angle. The pitch angle θ defines the orientation of the aircraft horizontal datum and the angle of attack (AOA) is defined by:

$$\alpha = \theta - \gamma \tag{1.1}$$

where γ is the climb/dive angle. The rate of change of pitch angle is the pitch rate $\dot{\theta} = q$, such that

$$\dot{\alpha} = q - \dot{\gamma} \tag{1.2}$$

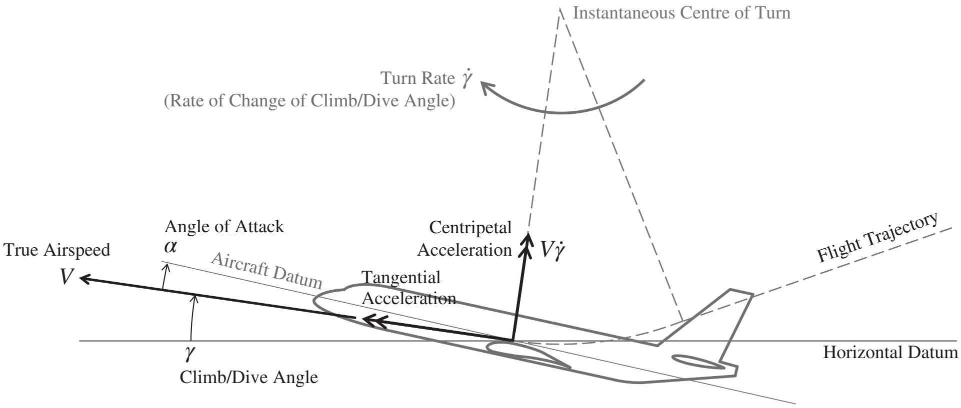


Figure 1.1 Symmetric Flight Trajectory.

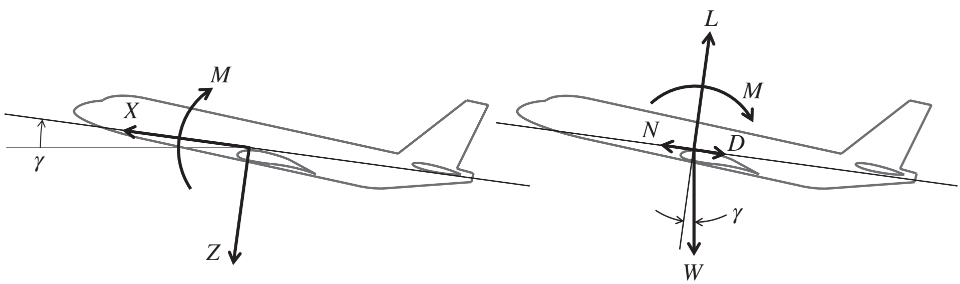


Figure 1.2 Symmetric Force/Moment System.

The force/moment system is shown in Figure 1.2 (referred to the centre of gravity, CG). Thus, aircraft motion is governed by the following equations when aircraft mass is constant:

$$m\dot{V} = X \qquad mV\dot{\gamma} = -Z \qquad J\dot{q} = M \tag{1.3}$$

where m is aircraft mass, J is moment of inertia, X is tangential force, Z is normal force and M is pitching moment. Altenatively, these equations can be written as:

$$\dot{V} = \frac{X}{m} \qquad \dot{\gamma} = -\frac{Z}{mV} \qquad \dot{\alpha} = \frac{M}{J} - \dot{\gamma} \tag{1.4}$$

The forces X and Z are composed as:

$$X = T - D - W\sin\gamma \qquad Z = W\cos\gamma - L \tag{1.5}$$

where L is total lift, D is total drag, W is aircraft weight and T is the total nett thrust from all engines. For convenience, the thrust line is drawn through the centre of mass. Also, for convenience, the thrust is aligned with the velocity vector and not the aircraft datum. This is true if AOA is zero (which it rarely is) and almost true if AOA is small (which it usually is).

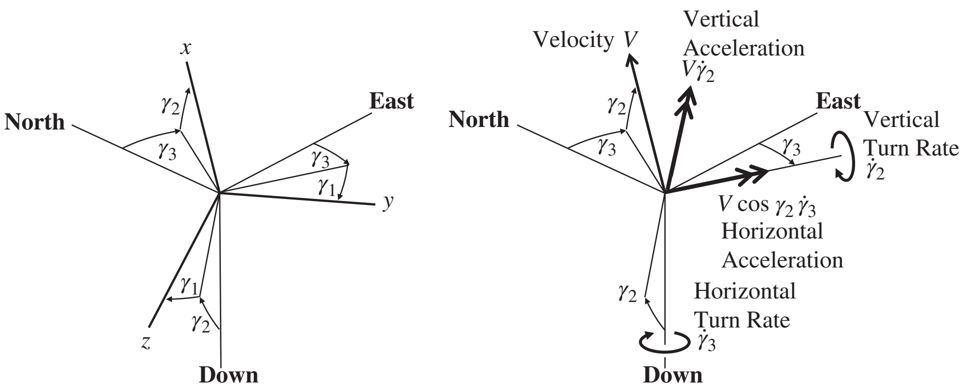


Figure 1.3 Generalised Flight Path Parameters.

Currently, the flight path is constrained to lie within a single vertical plane, tracing a straight line course across the surface of the Earth. Horizontal turns would be useful! So, a reference system is defined for the Earth, with its axes aligned with North, East, and Down, shown in Figure 1.3. Flight path angles are defined as γ_3 (setting the course direction), γ_2 (setting the climb/dive angle), and γ_1 (setting a rotation about the velocity vector). The resulting 'flight path axes' are shown as *xyz*. The vertical turn rate is now written as $\dot{\gamma}_2$ [cf. Figure 1.1] and a horizontal turn rate is introduced as $\dot{\gamma}_3$. In fact, $\dot{\gamma}$ can be redefined as the variation in flight path angle measured in the plane of symmetry (which is inclined at an angle γ_1 with respect to the vertical):

$$\dot{\gamma} = \dot{\gamma}_2 \cos\gamma_1 + \dot{\gamma}_3 \cos\gamma_2 \sin\gamma_1 \tag{1.6}$$

The force/moment system is modified and extended, as shown in Figure 1.4. The lift vector is inclined at an angle γ_1 with respect to the vertical. This generates the horizontal acceleration, thereby providing a bank-to-turn capability. Rotation about the velocity vector is produced by a rolling moment K about the velocity vector, such that the roll rate p is equal to $\dot{\gamma}_1$.

The generalised equations of motion are given by:

$$\dot{V} = \frac{N-D}{m} - g\sin\gamma_2 \qquad \dot{\gamma}_2 = \frac{L}{mV}\cos\gamma_1 - \frac{g}{V}\cos\gamma_2 \qquad \dot{\gamma}_3 = \frac{L}{mV}\frac{\sin\gamma_1}{\cos\gamma_2} \tag{1.7}$$

$$\dot{p} = \frac{K}{J_1} \qquad \dot{\gamma}_1 = p \qquad \dot{q} = \frac{M}{J_2} \qquad \dot{\alpha} = q - \dot{\gamma} \tag{1.8}$$

where m is the aircraft mass, g is the gravitational acceleration, J_1 is the roll moment of inertia, J_2 is the pitch moment of inertia, and the other symbols have their previously defined meanings.

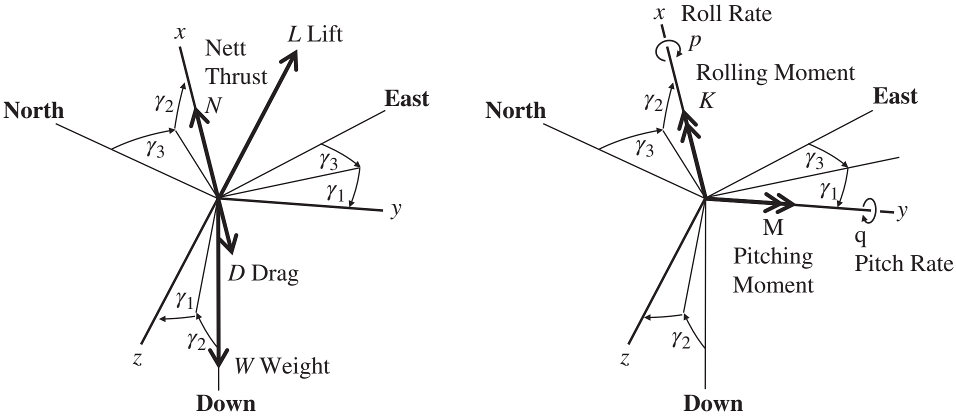


Figure 1.4 Generalised Force/Moment System.

The flight path angles are $(\gamma_1, \gamma_2, \gamma_3)$. In addition, aircraft position (expressed as e = East, n = North, and h = Altitude) is given by simple trigonometry:

$$\dot{e} = V\cos\gamma_2\sin\gamma_3 \qquad \dot{n} = V\cos\gamma_2\cos\gamma_3 \qquad \dot{h} = -V\sin\gamma_2 \tag{1.9}$$

1.3 Flight Environment <20 km

Atmosphere models were explained and developed in Volume 1. The International Standard Atmosphere is underpinned by parametric values given in Table 1.1. Temperature T [measured in Kelvin] decreases with altitude up to 11 km (at the top of the troposphere) and remains constant up to 20 km (at the top of the lower stratosphere), such that:

$$T = \max(T_0 + L_{01}H, T_1) \qquad \text{for} \quad H \leq 20\,000\ \text{m} \tag{1.10}$$

where H is the geopotential altitude. This altitude scale is used because the variation of pressure P can be expressed with a constant value for gravitational acceleration g_0 (at sea level):

$$dP = -\rho g_0 dH \tag{1.11}$$

where ρ is air density. The relationship with altitude h (from Section 1.2) is as follows:

$$H = \frac{r_0\, h}{r_0 + h} \qquad h = \frac{r_0\, H}{r_0 - H} \tag{1.12}$$

and where r_0 is the mean radius of the Earth.

The relationship between pressure P and temperature T is given by the Ideal Gas Law:

$$P = \rho RT \tag{1.13}$$

Accordingly, Equation 1.11 gives:

$$\int \frac{dP}{P} = -\frac{g_0}{R}\int \frac{1}{T}dH$$

Table 1.1 Selected Parameters for the International Standard Atmosphere.

Parameter	Symbol	Value	Units
Mean Earth radius	r_0	6356766	m
Gravitational acceleration at sea level	g_0	9.80665	$\mathrm{m\,s^{-2}}$
Gas constant (dry air)	R	287.05287	$(\mathrm{J\,kg^{-1}})\ \mathrm{K^{-1}}$
Adiabatic gas constant (dry air)	γ	1.4	
Pressure at sea level	P_0	101325	Pa
Pressure at Tropopause	P_1	22632	Pa
Temperature at sea level	T_0	288.15	K
Temperature at Tropopause	T_1	216.65	K
Temperature gradient across Troposphere	L_{01}	-0.0065	$\mathrm{K\,m^{-1}}$
Density at sea level	$\rho_0 = P_0/RT_0$	1.225	$\mathrm{kg\,m^{-3}}$
Speed of sound at sea level	$a_0 = \sqrt{\gamma R T_0}$	340.294	$\mathrm{m\,s^{-1}}$

Recalling Equation 1.10, the pressure variation across the troposphere is determined as:

$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{-\frac{g_0}{RL_{01}}} \tag{1.14a}$$

and the pressure variation across the lower stratosphere (at constant temperature) is:

$$\frac{P}{P_1} = \exp\left[-\frac{g_0}{RT_1}(H - 11\,000)\right] \tag{1.14b}$$

Air density ρ is then determined from the Ideal Gas Law.

It is noted that the local speed of sound is defined by $\sqrt{\gamma RT}$ (as discussed in Chapter 5) and, thus, airspeed V can be expressed by its Mach number M_N:

$$M_N = \frac{\mathrm{V}}{\sqrt{\gamma RT}} \tag{1.15}$$

1.4 Simple Propulsion Model

1.4.1 Reference Parameters

Aeronautical engineering makes use of equivalent sea-level parameters in order to enable comparisons to be made at different altitudes. It is common to relate pressure, temperature, and density to their respective sea-level values. The resulting quantities are called relative pressure δ, relative temperature θ and relative density σ:

$$\delta = \frac{P}{P_0} \qquad \theta = \frac{T}{T_0} \qquad \sigma = \frac{\rho}{\rho_0} \tag{1.16}$$

where the subscript 0 denotes Sea Level. It is noted that the Ideal Gas Law (cf. Equation 1.13) can now be rewritten as:

$$\delta = \sigma\theta \tag{1.17}$$

Referred Airspeed V_0 gives the same Mach number at sea level as for the airspeed V at a given altitude, such that:

$$\frac{V}{V_0} = \frac{M_N\sqrt{\gamma RT}}{M_N\sqrt{\gamma RT_0}} = \sqrt{\frac{T}{T_0}} = \sqrt{\theta} \tag{1.18}$$

Referred Massflow Q_0 is based on referred airspeed for a given cross-sectional flow area A:

$$\frac{Q}{Q_0} = \frac{\rho AV}{\rho_0 AV_0} = \sigma\sqrt{\theta} = \frac{\delta}{\sqrt{\theta}} \tag{1.19}$$

Referred Thrust X_0 is based on relative pressure applied over a surface area A:

$$\frac{X}{X_0} = \frac{AP}{AP_0} = \delta \tag{1.20}$$

Referred Fuel Flow F_0 (which is energy flow) is based on the momentum force that is calculated as the product of thrust X and airspeed V:

$$\frac{F}{F_0} = \frac{XV}{X_0V_0} = \delta\sqrt{\theta} \tag{1.21}$$

1.4.2 Simple Jet Engine Performance

A simple engine model is presented here that was created so many years ago that the original source document has been lost and the team that produced it has been forgotten. It was used in the 1970s for research into autoland systems and it serves a useful purpose here because of its simplicity.

Engine parameters are Gross Thrust X, Intake Massflow Q, Fuel Massflow F and Engine Speed Ω. These are referred to sea level as follows:

$$X_0 = \frac{X}{\delta} \qquad Q_0 = Q\frac{\sqrt{\theta}}{\delta} \qquad F_0 = \frac{F}{\delta\sqrt{\theta}} \qquad \Omega_0 = \frac{\Omega}{\sqrt{\theta}} \tag{1.22}$$

where δ is relative pressure and θ is relative temperature. Note that engine speed (which is an angular quantity) is treated in the same manner as airspeed (which is a linear quantity). Maximum values of these parameters are X_{max}, Q_{max}, F_{max} and Ω_{max}. The corresponding nondimensional parameters are:

$$x = \frac{X_0}{X_{max}} \qquad q = \frac{Q_0}{Q_{max}} \qquad f = \frac{F_0}{F_{max}} \qquad \omega = \frac{\Omega_0}{\Omega_{max}} \tag{1.23}$$

Engine performance is defined empirically as follows:

$$q = 1.10\omega - 0.10 \tag{1.24}$$

$$x = 1.95\omega^2 - 1.20\omega + 0.25 \tag{1.25}$$

$$f = max((0.60\omega - 0.06), (2.40\omega - 1.40)) \tag{1.26}$$

Equation 1.25 can only be solved for $x > 0.06539$ which corresponds with $\omega > 0.3077$. Nett thrust is the gross thrust minus the intake momentum drag:

$$N = X - QV \tag{1.27}$$

Using referred parameters:

$$\frac{N}{\delta} = X_0 - Q_0 V_0 \tag{1.28}$$

Applying nondimensional parameters:

$$\frac{N}{\delta} = xX_{max} - qQ_{max}a_0M_N \tag{1.29}$$

where M_N is Mach number and a_0 is the speed of sound at sea level. Maximum nett thrust is given when $x = 1$ and $q = 1$:

$$\frac{N_{max}}{\delta} = X_{max} - Q_{max}a_0M_N \tag{1.30}$$

Thus, using Equations 1.22 and 1.23, the nett thrust at any flight condition is given by:

$$\frac{N}{\delta} = \left(1.95\omega^2 - 1.20\omega + 0.25\right)X_{max} - (1.10\omega - 0.10)Q_{max}a_0M_N \tag{1.31}$$

As an example, assume the following values associated with maximum thrust at sea level:

$X_{max} = 55\,000\ \mathrm{N}$ $\quad Q_{max} = 95\ \mathrm{kg \cdot s^{-1}}$ $\quad F_{max} = 0.9\ \mathrm{kg \cdot s^{-1}}$ $\quad \Omega_{max} = 9\,000\ \mathrm{rpm}$

This corresponds with a specific fuel consumption of 58.9 $(\mathrm{kg\,h^{-1}})\ \mathrm{kN^{-1}}$. Nett thrust is given by:

$$\frac{1}{\delta}\left(\frac{N}{55\,000}\right) = 1.95\omega^2 - (1.20 + 0.64658M)\omega + (0.25 + 0.05878M)$$

From this formula, the variation of referred net thrust against referred engine speed and Mach number is calculated and shown in Figure 1.5. The accuracy of this model diminishes significantly as altitude increases but it is adequate for its original purpose, investigating automatic control for low-altitude, low-speed flight.

1.4.3 'Better' Jet Engine Performance

Better thrust estimates are available but, for illustrative purposes, only one will be considered here. The variation with Mach number is given by Bartel and Young (2007), following the method of Torenbeek (1982):

$$\frac{N}{N_{TO}} = 1 - k_1M + k_2M^2 \tag{1.32}$$

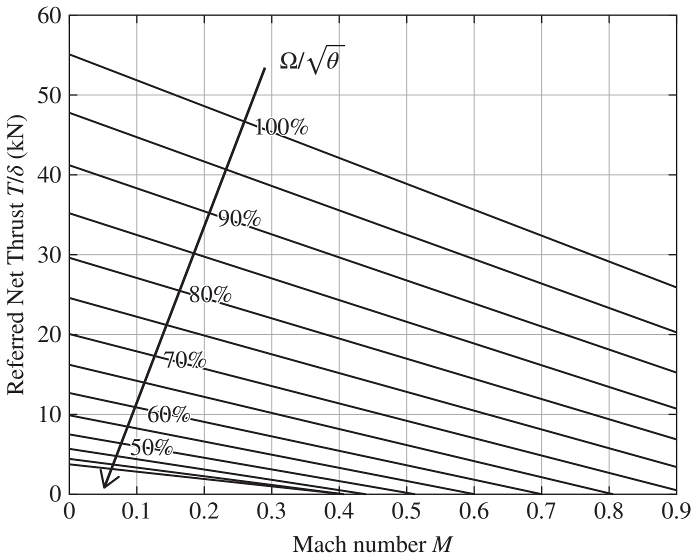


Figure 1.5 Thrust Performance for Simple Jet Engine Model.

where N is nett thrust, N_{TO} is nett thrust at take-off and M is Mach number. The factors k_1 and k_2 were determined from studying a number of two-shaft engines:

$$k_1 = \frac{0.377(1+\lambda)}{\sqrt{(1+0.82\lambda)G}} \qquad k_2 = 0.19\lambda + 0.23 \tag{1.33}$$

where λ is the bypass ratio and G is the so-called gas generator function:

$$G = 0.06\sqrt{\lambda} + 0.64$$

For example, a Rolls-Royce Spey has $\lambda = 0.64$ and so $k_1 = 0.559$ and $k_2 = 0.678$.

Equation 1.32 is extended to give the variation of thrust with Mach number and altitude:

$$\frac{N}{N_{TO}} = A_0 - A_1 k_1 M + A_2 k_2 M^2 \tag{1.34}$$

where the new coefficients are stated as:

$$A_0 = -0.4327\delta^2 + 1.3855\delta + 0.0472$$
$$A_1 = 0.9106\delta^3 - 1.7736\delta^2 + 1.8697\delta$$
$$A_2 = 0.1377\delta^3 - 0.4374\delta^2 + 1.3003\delta$$

in which δ is the relative pressure that was defined in Equation 1.14.

The variation of thrust-specific fuel consumption (TSFC) is given by Howe (2000):

$$c = c_2\left(1 - 0.15\lambda^{0.65}\right)\left(1 + 0.28\left(1 + 0.063\lambda^2\right)M\right)\sigma^{0.08} \tag{1.35}$$

where c is TSFC, λ is bypass ratio, M is Mach number and σ is relative density (ρ/ρ_0). The factor c_2 is chosen as a constant that matches the performance of a specific engine but, in the absence of real data, it is claimed that preliminary analysis can use $c_2 = 24\,(\mathrm{mg\,s^{-1}})\,\mathrm{N^{-1}}$ for a low-bypass engine and $c_2 = 20$ (mg s^{-1}) N^{-1} for a high-bypass engine. In more practical units, $c_2 = 86.4$ (kg h^{-1}) kN^{-1} for a low-bypass engine and $c_2 = 72$ (kg h^{-1}) kN^{-1} for a high-bypass engine.

1.4.4 Simple Jet Engine Dynamics

The engine speed for a given fuel flow is defined by the inverse of Equation 1.26. The solution can be interpreted as a speed demand:

$$\omega_{demand} = min\left(\frac{f + 0.06}{0.6}, \frac{f + 1.40}{2.40}\right) \tag{1.36}$$

For preliminary calculations, It is appropriate to model engine dynamics as a first-order response to a speed demand, with a time constant (τ) and an acceleration limit (A):

$$\frac{d\omega}{dt} = \frac{1}{\tau}(\omega_{demand} - \omega)\Big]_{-A}^{A} \tag{1.37}$$

As an initial guess, choose a time constant of 0.5 second for small perturbations together with a rate limit that is compatible with an acceleration from ‘flight idle’ (defined as $\omega = 0.325$) up to maximum speed ($\omega = 1.00$) nominally in 5 seconds.

1.5 Simple Aerodynamic Model

1.5.1 Idealised Aircraft

An idealised aircraft configuration provides the essential constituents for building a model of aircraft flight physics. So, this section will assume a conventional wing/tail combination, such as shown in Figure 1.6. This enables a compact set of aerodynamic calculations to be defined, without becoming buried under an avalanche of empirical detail.

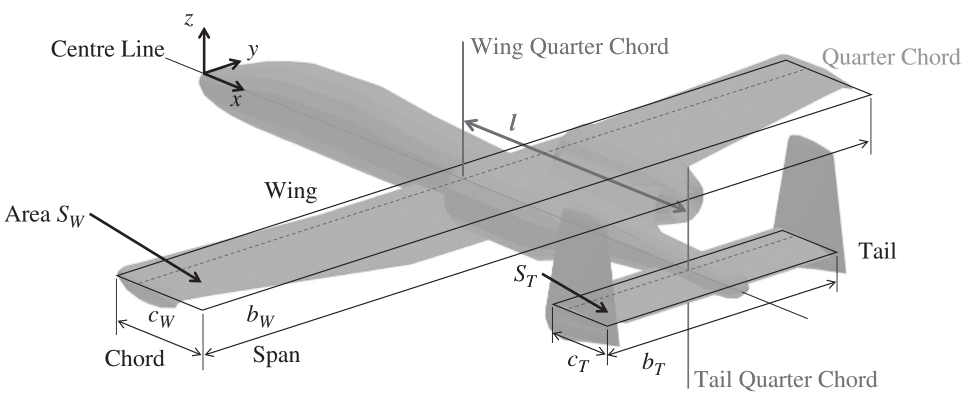


Figure 1.6 Idealised Aircraft.

Geometric axes for airframe definition are defined by the ***xyz*** triad with its point of origin at the intersection of the aircraft centre line, the horizontal datum and the traverse datum (which is located at the nose in this example). For simplicity, the lifting surfaces have a rectangular planform with span b, chord length c, area $S = bc$ and aspect ratio $A = b/c$. Subscripts W and T denote wing and tail, respectively. Reference points are defined at the quarter-chord positions for the wing and tail, at which aerodynamic forces and moments are calculated. The distance between these points is l. The product of area and distance is the *tail volume* ($V_T = lS_T$), which determines the effectiveness of the tail as a producer of pitching moment. For convenience, the reference points are assumed to lie on the intersection of the aircraft centre line and the horizontal datum, together with the aircraft CG. This simplifies the analysis of flight stability.

1.5.2 Idealised Wing

Figure 1.3 summarises the aerodynamics of a simple wing (generating lift L, drag D, and pitching moment M) for true airspeed V and AOA α. In 'normal' flight, lift variation is linear with respect to AOA and also AOA is reasonably small (probably less than 10° in most conditions). The pitching moment is nominally constant when calculated about the quarter-chord, for reasons that are discussed in Chapter 3. If the wing cross-section is symmetrical (as it is in Figure 1.7), then the pitching moment is zero and, in addition, the wing generates no lift at zero AOA.

Lift and drag are defined as follows:

$$L = QS\,C_L \qquad\qquad D = QS\,C_D \tag{1.38}$$

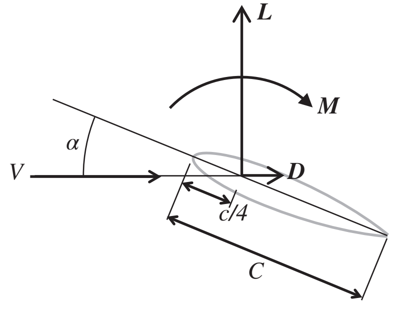
where the dynamic pressure is $Q = \rho V^2/2$ and S is the wing area. The quantities C_L and C_D are aerodynamic coefficients for lift and drag, respectively. For this simplified (symmetric) wing, the lift coefficient is defined as:

$$C_L = a\alpha \tag{1.39}$$

where a is the lift-curve gradient and α is the AOA. The product $a\alpha$ is called the *induced lift* (i.e. lift induced by AOA).

Conceptually, a two-dimensional (2D) wing is the cross-section of a wing with infinite span. The local flow around any section will show an identical pattern of streamlines,

Figure 1.7 Wing Force/Moment System.



with no variation across the span. The 2D lift-curve gradient at low speed is crudely approximated as:

$$a_{2D} = 2\pi + 4.9\left(\frac{t}{c}\right) \tag{1.40}$$

where M is the flight Mach number and t/c is the thickness/chord ratio (which is often assumed to be 0.1 in the absence of real data). The 'ideal' lift-curve gradient for a thin wing is 2π.

A real wing has finite span and the flow over upper and lower surfaces has to mix in order to equalise the pressure at the wing tips. This is achieved by flow under the wing being drawn around the tips by the lower pressure above the wing. For the outer wing, the flow direction turns outwards over the lower surface and inwards over the upper surface. The 2D model is no longer usable because the flow has a three-dimensional (3D) pattern. The 3D lift-curve gradient for a rectangular wing [which was introduced as a in Equation 1.39] can be approximated using DATCOM[1]:

$$a = \frac{2\pi A}{2 + \sqrt{4 + (A/\kappa)^2\left(1 - M_N^2\right)}} \tag{1.41}$$

where M_N is the flight Mach number, A is the aspect ratio, and $\kappa = a_{2D}/2\pi$. The variation of this quantity is shown in Figure 1.8 for a wing with thickness/chord ratio of 0.1. It is worth noting that:

$$a \longrightarrow \frac{a_{2D}}{\sqrt{1 - M_N^2}} \qquad \text{as} \qquad A \longrightarrow \infty$$

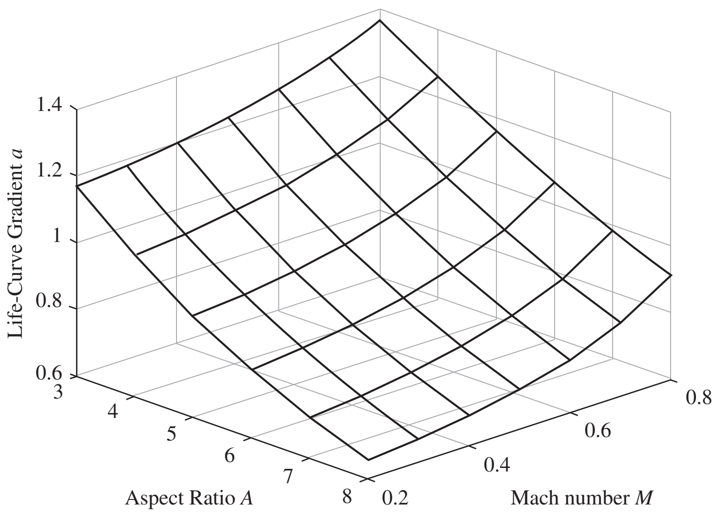


Figure 1.8 Variation of Lift-Curve Gradient.

1 Source: US Airforce Stability and Control Data Compendium (DATCOM).

The drag coefficient is written as the sum of profile drag, induced drag, and wave drag coefficients:

$$C_D = C_{D0} + C_{Di} + C_{Dw} \tag{1.42}$$

The main drag components are summarised as follows:

Profile drag is caused by air forcing its way around the outer geometry of the aircraft. In the absence of real data, a starting assumption might be $C_{D0} = 0.02$.

- Induced drag is caused by deflecting the airflow in order to generate lift. It is estimated as:

$$C_{Di} = \frac{C_L^2}{\pi A e} \tag{1.43}$$

where C_L is the lift coefficient, A is the wing aspect ratio, and e is the wing efficiency. Initial calculations might assume $\pi e \approx 2.5$ or thereabouts.

- Wave drag is caused by the shock waves as the Mach number M_N exceeds the critical Mach number M_{crit} (which is the maximum speed at which the flow field across the entire wing is subsonic). Lock's approximation is as follows:

$$C_{Dw} = 20\,(M_N - M_{crit})^4 \tag{1.44}$$

A rough estimate for M_{crit} is given by:

$$M_{crit} = k_T - \left(\frac{t}{c}\right) - 0.1C_L - 0.1078 \tag{1.45}$$

where t/c is the thickness/chord ratio, C_L is the lift coefficient and the factor k_T is assigned a value of 0.87 (in the absence of specific data). This is discussed in Chapter 3.

1.5.3 Wing/Tail Combination

Figure 1.9 shows an aircraft in straight-and-level flight for a given velocity V and AOA α. Lift vectors are applied on the wing and tail. The aircraft CG is shown as a circle with alternate black and white quadrants. Engine thrust will contribute a vertical force and a pitching

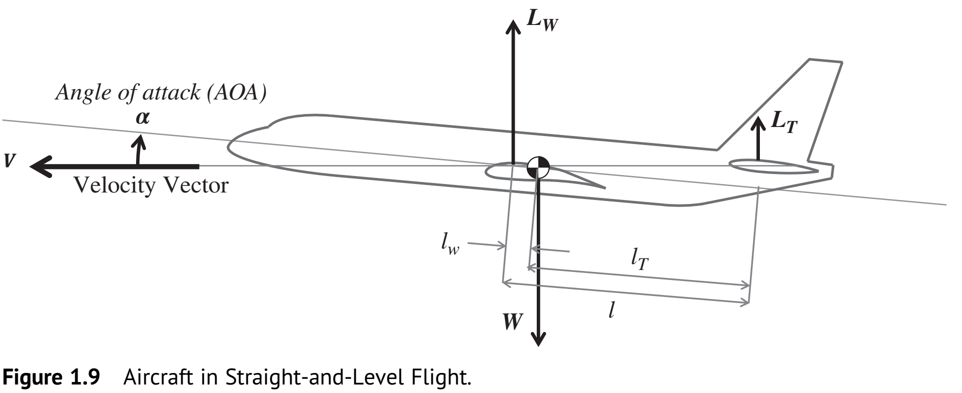


Figure 1.9 Aircraft in Straight-and-Level Flight.

moment to the aircraft but, for simplicity, it is convenient to neglect it (as textbooks usually do).

This aerodynamic force/moment system is evaluated with respect to the CG as follows:

$$L = L_W + L_T \qquad M = l_W L_W - l_T L_T \tag{1.46}$$

where subscripts W and T denote wing and tail, respectively. This can be written in coefficient form:

$$L = Q(S_W C_{LW} + S_T C_{LT}) \qquad M = Q(S_W l_W C_{LW} - S_T l_T C_{LT}) \tag{1.47}$$

where $Q = \rho V^2/2$ is the dynamic pressure, S_W is the wing planform area, and S_T is the tail planform area. The distances l_W and l_T define the respective positions of the wing and tail quarter-chord positions relative to the CG.

The lift coefficients are C_{LW} for the wing and C_{LT} for the tail, where:

$$C_{LW} = a_W \alpha_W \qquad C_{LT} = a_T \alpha_T \tag{1.48}$$

where a_W and a_T are the 3D lift-curve gradients for the wing and tail, respectively, and α_W and α_T are the corresponding angles of attack. In this simple aerodynamic model, the wing plane is aligned with the aircraft datum, such that:

$$\alpha_W = \alpha \tag{1.49}$$

As shown in Figure 1.10, airflow is deflected downwards behind the trailing edge (known as 'downwash') and, accordingly, the tail AOA is approximated as:

$$\alpha_T = \alpha - \varepsilon + \delta_T \tag{1.50}$$

where α is the aircraft AOA, ε is the downwash angle at the tail, and δ_T is the tail rotation angle (assuming that this aircraft has an all-moving tail). There is a time delay $t' = l_T/V$ for the air to flow from the wing to the tail, such that:

$$\alpha_T(t) = \alpha(t) - \varepsilon(t - t') + \delta_T(t)$$

Using a power series expansion for $\varepsilon(t - \tau)$, this becomes:

$$\alpha_T(t) = (1 - \varepsilon_\alpha)\alpha(t) + t'\varepsilon_\alpha\dot{\alpha}(t) + \delta_T(t) \tag{1.51}$$

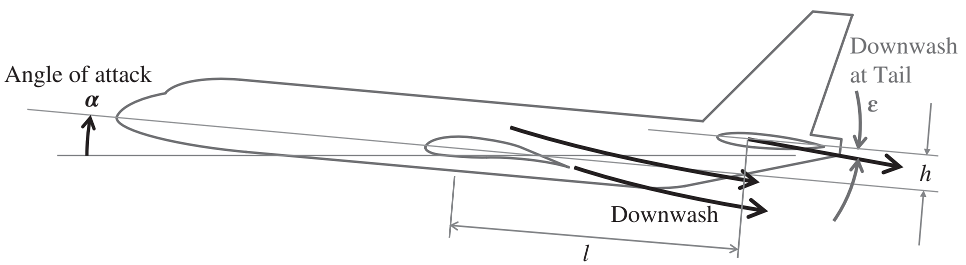


Figure 1.10 Effect of Downwash on Horizontal Tail.

The variation of downwash with AOA can be written as $\varepsilon = \varepsilon_\alpha \alpha$, where ε_α is the downwash gradient given by DATCOM:

$$\varepsilon_\alpha = \frac{d\varepsilon}{d\alpha} = 4.44\,(K_A K_T)^{1.19} \tag{1.52}$$

where

$$K_A = \frac{1}{A_W} - \frac{1}{1 + A_W^{1.7}} \qquad K_T = \frac{1 - |h/b|}{\sqrt[3]{2l/b}}$$

The aspect ratio is A_W, the longitudinal separation of wing and tail is l and the vertical separation (height) is h. Note that l and h are measured relative to the wing chord plane, which is aligned with the aircraft datum in this simplified explanation. Note that Equation 1.51 is applicable to a wing with a rectangular planform. The general case is presented in Chapter 4.

The tail lift coefficient is now expressed as:

$$C_{LT} = a'_T \alpha + a_T(t' \varepsilon_\alpha \dot{\alpha} + \delta_T) \tag{1.53}$$

where the modified lift-curve gradient is $a'_T = a_T(1 - \varepsilon_\alpha)$.

1.5.4 Lift Distribution

Introductory texts do not usually consider the lift distribution over a wing because they deal with consolidated forces and moments. However, this provides a conceptually simple (albeit approximate) method for calculating rolling moments and, in Section 1.5.5, calculating the aerodynamics of flight controls. The general case is presented in Chapter 3 and a cut-down version is given here for a simple flat wing with a rectangular planform. So, the local lift coefficients are given by:

$$C_l(\eta) = C_L\,\partial(\eta) \tag{1.54}$$

where $\eta = y/(b/2)$ is the a nondimensional length parameter, C_L is the 3D lift coefficient, and $\partial(\eta)$ is the lift distribution function:

$$\partial(\eta) = \lambda + (1 - \lambda)\varepsilon(\eta) \tag{1.55}$$

where $e(\eta)$ is an elliptical distribution:

$$e(\eta) = \frac{4}{\pi}\sqrt{1 - \eta^2} \tag{1.56}$$

and the coefficient λ is a function of $f = (2\pi A/a_{2D})/14$, as shown in Figure 1.11:

$$\lambda \approx -0.1535f^3 - 0.3121f^2 + 1.4647f \tag{1.57}$$

With reference to Equation 1.39, the local lift coefficient is determined as:

$$C_l(\eta) = \partial(\eta) a\alpha \tag{1.58}$$

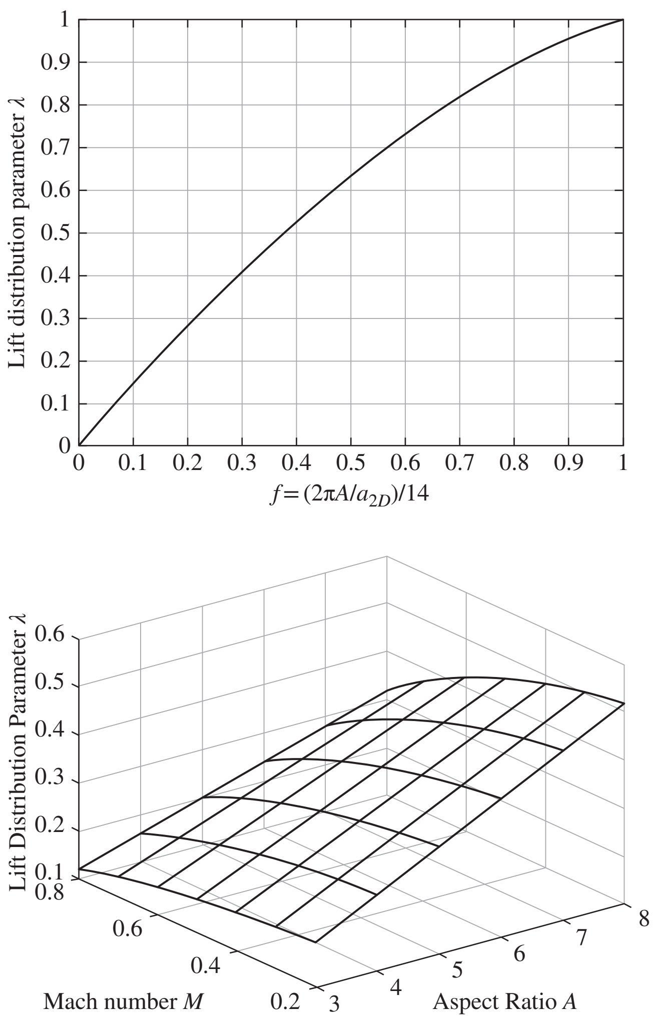


Figure 1.11 Variation of the Lift Distribution Coefficient.

This is valid for symmetric flight. In other condition, it is determined as:

$$C_l(\eta) = \partial(\eta)a\alpha_l(\eta) \tag{1.59}$$

where $\alpha_l(\eta)$ is the local AOA.

As explained in Chapter 3 and as indicated by Figure 1.12, the local AOA can be calculated as:

$$\alpha_l(\eta) = \alpha_s + \eta p' \tag{1.60}$$

where α_s is the symmetric AOA at the wing, $p' = bp/2V$ is the nondimensional roll rate, and η is the nondimensional length scale η. The local lift coefficient becomes:

$$C_l(\eta) = \partial(\eta)a(\alpha_s + \eta p') \tag{1.61}$$

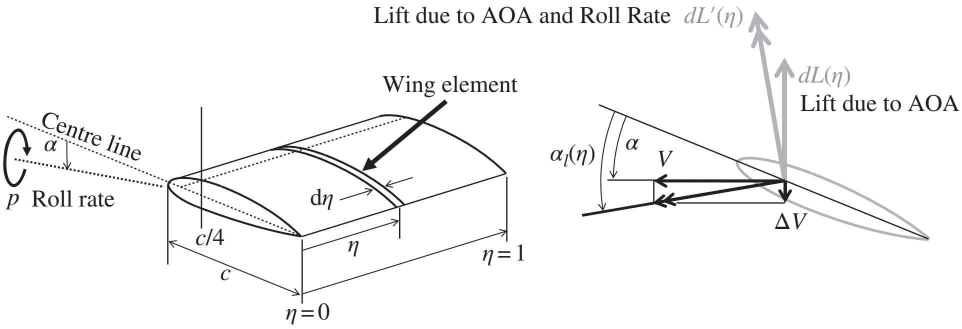


Figure 1.12 Incremental Lift on a Wing Element.

The total lift and total rolling moment on a wing is obtained as follows:

$$L = QS\int_0^1 C_l(\eta)d\eta \qquad K = -QS\frac{b}{2}\int_0^1 C_l(\eta)\eta\, d\eta \tag{1.62}$$

Lift is calculated for a symmetric distribution and rolling moment is calculated for an antisymmetric distribution (noting that a minus sign is needed because the right wing induces a left-handed moment about the velocity vector). Thus, the associated coefficients can be written as:

$$C_L = a(\alpha I_0(1) + p'I_1(1)) \qquad C_K = -a(\alpha I_1(1) + p'I_2(1)) \tag{1.63}$$

where

$$I_N(\eta_1) = \int_0^{\eta_1} \partial(\eta)\eta^N\, d\eta \tag{1.64}$$

Equivalent force vectors can be drawn on the starboard wing span, showing the lines of action for the total lift force and the total rolling moment. These are positioned at nondimensional distances η_L and η_K from the centre line, as defined by:

$$\eta_L = \frac{I_1(1)}{I_0(1)} \qquad \eta_K = \frac{I_2(1)}{I_1(1)} \tag{1.65}$$

Finally, recalling Equation 1.56:

$$\partial(\eta) = \lambda + (1-\lambda)e(\eta)$$

Therefore, the integral Δ_N is evaluated from 0 to $\eta_1 \leq 1$ as follows:

$$I_N(\eta_1) = \lambda C_N(\eta_1) + (1-\lambda)E_N(\eta_1) \tag{1.66}$$

where

$$C_N(\eta_1) = \int_0^{\eta_1} \eta^N\, d\eta \qquad E_N(\eta_1) = \int_0^{\eta_1} e(\eta)\eta^N\, d\eta \tag{1.67}$$

The distribution functions and their integrals are shown in Figure 1.13 for $N = 0, 1, 2$. For reference, $C_0(1) = 1$, $C_1(1) = 1/2$, $C_0(1) = 1/3$, $E_0(1) = 1$, $E_1(1) = 4/3\pi$ and $E_0(1) = 1/3$.

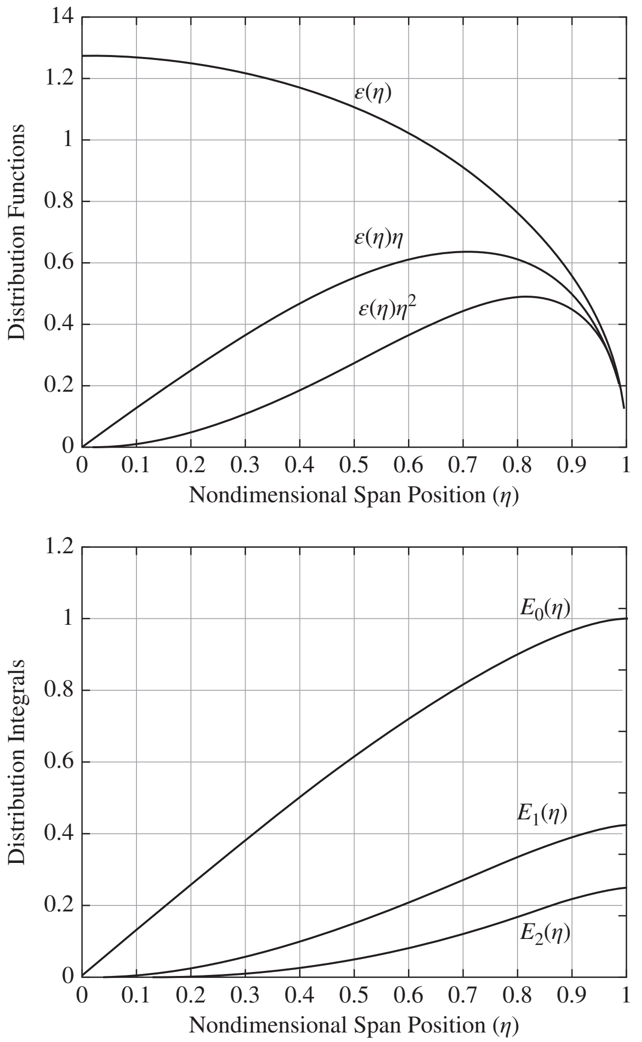


Figure 1.13 Distribution Functions and their Integrals.

1.5.5 Adding Flight Controls

Much has been written in textbooks on control surfaces and the body of empirical aerodynamics is considerable, covering incremental forces and moments together with correction factors for every applicable facet of real flow physics. Interested readers are referred to Raymer (1999) and Pamadi (2015), for example. This chapter will opt for a much simpler approach.

Flight controls modify the aerodynamics of a lifting surface. The discussion here is limited to plain flaps on the trailing edge, as drawn in Figure 1.14. These can be full-span or part-span, as seen in Figure 1.15. Symmetric deployment generates lift and antisymmetric deployment generates a rolling moment. Relevant parameters in the calculation of flap

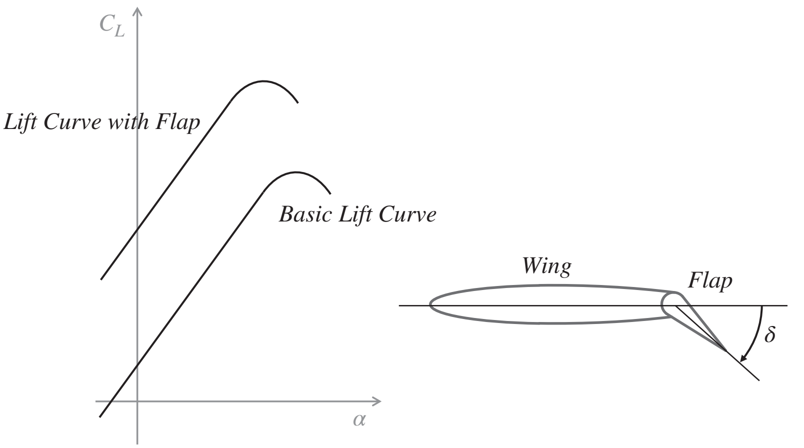


Figure 1.14 Changes in Lift Coefficient due to Flap Deflection.

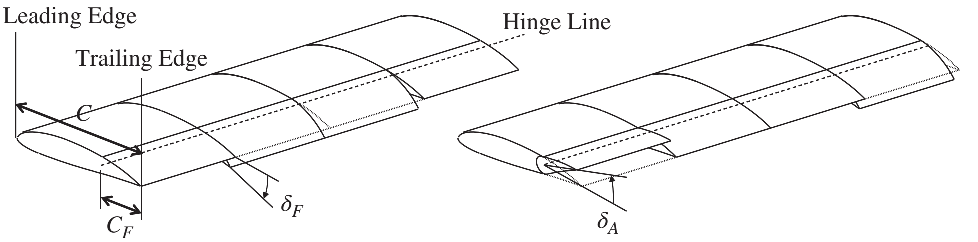


Figure 1.15 Trailing-Edge Control Surfaces.

effects are the chord length c (measured from leading to trailing edge) and flap chord length c_F (measured from hinge line to trailing edge). The flap-chord ratio is c_F/c.

The overall drag increment due to a control deflection δ is estimated in DATCOM as:

$$\Delta C_D = 1.7\left(\frac{c_F}{c}\right)^{1.38}\left(\frac{S_F}{S}\right)\sin^2\delta \tag{1.68}$$

where c_F/c is the flap-wing chord ratio and S_F/S is flap-wing area ratio. The value of S_F is the area calculated across the flap span and S is the area calculated across the wing span. The symbol δ would be replaced by δ_A for an aileron deflection, δ_F for a conventional wing flap deflection, or δ_E for an elevator deflection on the tail.

The aerodynamic effect of a control deflection can be construed as a change in the local AOA:

$$\Delta\alpha_l(\eta) = \alpha_\delta\delta \tag{1.69}$$

where α_δ is the flap effectiveness (shown in Figure 1.16), as calculated using the formula:

$$\alpha_\delta = 1 - \frac{\varphi - \sin\varphi}{\pi} \qquad \text{where} \qquad \cos\varphi = 2\frac{c_F}{c} - 1 \tag{1.70}$$

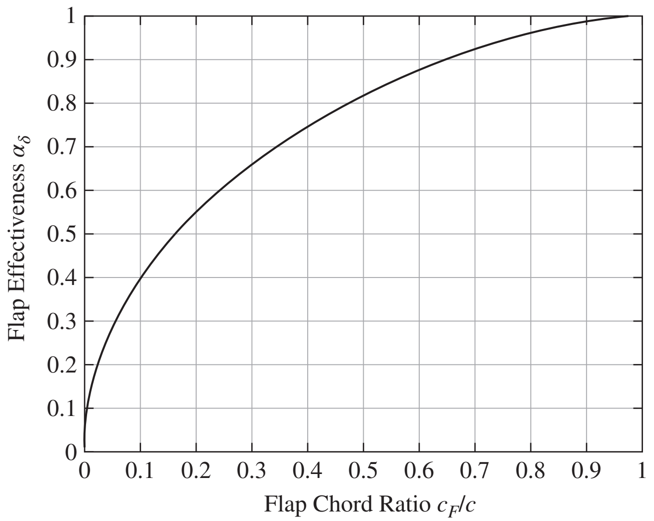


Figure 1.16 Flap Effectiveness.

Recalling Equation 1.58, the incremental change in the local lift coefficient is calculated as:

$$\Delta C_l(\eta) = \partial(\eta) a \alpha_\delta \delta_F \tag{1.71}$$

Recalling Equation 1.65, the resultant changes in lift force and rolling moment over the wing span $\eta_I \leq \eta \leq \eta_O$ (between the inboard and outboard edges of the control surface) are calculated as:

$$\Delta C_L = (I_0(\eta_O) - I_0(\eta_I))\alpha_\delta \delta \qquad \Delta C_K = -(I_1(\eta_O) - I_1(\eta_I))\alpha_\delta \delta \tag{1.72}$$

where the integrals I_0 and I_1 are defined in Equation 1.64.

The control deflection also induces a local pitching moment ΔC_m as follows:

$$\Delta C_m(\eta) = -K_m \Delta C_l(\eta) \tag{1.73}$$

where

$$K_m = 0.3077\left(0.805 - \frac{c_F}{c}\right)$$

This integrates to give the total change in pitching moment as follows:

$$C_M = -K_m \alpha_\delta \delta I_0 \tag{1.74}$$

1.6 Airspeed Definitions

Two definitions of airspeed are in common usage, derived from the true airspeed V. These are summarised below as the final component of the mathematical definition of this simple flight model.

Equivalent airspeed (V_{EAS}) at sea level corresponds with the dynamic pressure at a given altitude.

$$V_{EAS} = V\sqrt{\frac{\rho}{\rho_0}}$$

where the subscript 0 denotes Sea Level. Using relative density, as defined in Equation 1.16:

$$V_{EAS} = V\sqrt{\sigma} \tag{1.75}$$

Calibrated airspeed (V_{CAS}) at sea level corresponds with the impact pressure at a given altitude.

$$V_{CAS} = \sqrt{\frac{2\gamma}{\gamma - 1}RT_0}\left(\frac{Q_c}{P_0} + 1\right)^{\frac{\gamma-1}{2\gamma}} \tag{1.76}$$

where Q_c is the so-called impact pressure:

$$Q_c = P' - P \tag{1.77}$$

This is the difference between the stagnation pressure P' and the static pressure P. These two quantities are interrelated with the adiabatic gas constant γ and the Mach number M_N defined in Equation 1.15:

$$\frac{P'}{P} = \left(1 + \frac{\gamma - 1}{2}M_N^2\right)^{\frac{\gamma}{\gamma-1}} \tag{1.78}$$

where M_N is the flight Mach number. Note that stagnation temperature T' is related to static temperature T as follows:

$$\frac{T'}{T} = 1 + \frac{\gamma - 1}{2}M_N^2 \tag{1.79}$$

These thermodynamic relationships are developed in Chapter 5 (with slightly different notation).

1.7 Flight Model Architecture

The structure/interface definition of any computational model is a matter of profound importance in the transcription of the model into software (however that is constituted). The principal objective is to partition the calculations into functionally concise blocks and, in so doing, to establish a dataflow map that is reasonably sparse. This is intended to render a model representation that is clear and readable (i.e. not complicated and not cluttered). One possible solution is shown in Figure 1.17, with blocks that correspond with the preceding Sections 1.2–1.5. The visible parameters are those that are transferred from one block to another: all others are contained within the relevant functional block as part of the required calculations.

In this particular model, the Motion and Propulsion blocks contain dynamics, with called state variables governed by differential equations [which are called state equations]. The universal principle is that the outputs from a functional block are fully determined by the current states and inputs. This is depicted in Figure 1.18.

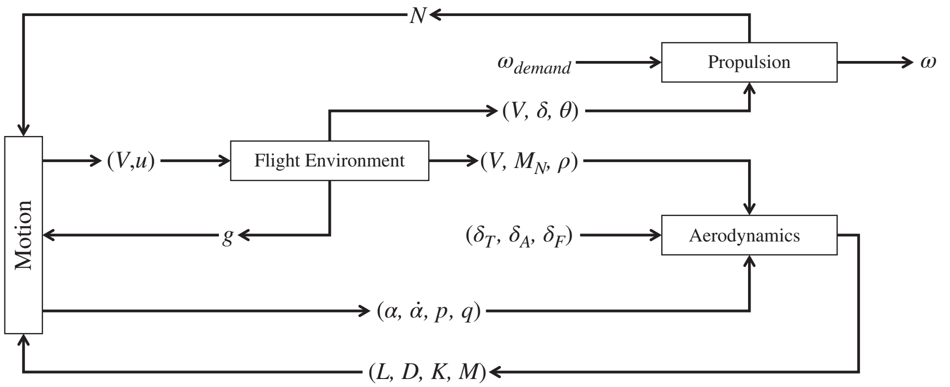


Figure 1.17 Dataflow for the Simple Flight Model.

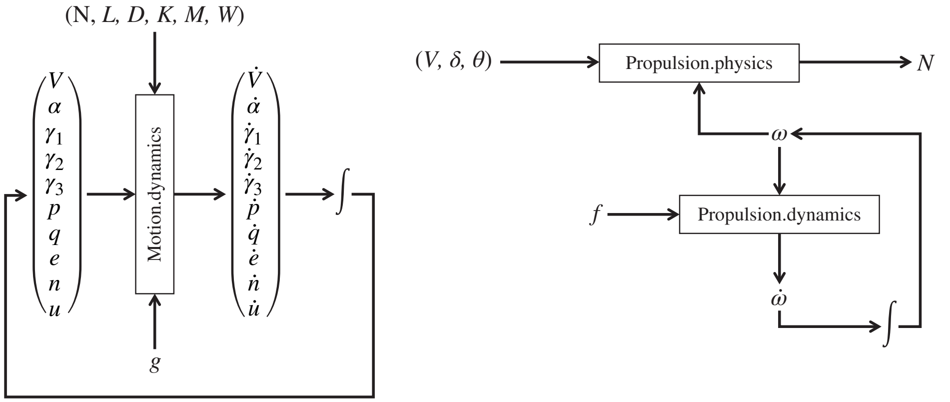


Figure 1.18 Functional Physics and Dynamics for the Simple Flight Model.

The Motion block receives inputs from Aerodynamics, Propulsion, and Flight Environment. These inputs are combined with the state vector, which generically is written as $\boldsymbol{x}$, in order to generate the rate of change $\dot{\boldsymbol{x}}$. In this case, Motion is comprised only of a 'dynamics' subblock and its outputs are the state variables (α, p, q) and the state derivative $\dot{\alpha}$.

The Propulsion block contains a performance calculation, which outputs nett thrust N, plus a state equation for engine speed. This partitions 'physics' and 'dynamics' in a way that many modelling textbooks and toolsets draw the distinction between 'output equations' and 'state equations', respectively. Note that the inputs (V, δ, θ) are taken from the Flight Environment and the control input f is received from outside the model.

There are many ways in which a model like this can be organised and the intent here is NOT to present a methodology, only to indicate the sort of considerations that form part of the underlying thought process. A functional block diagram is an appropriate representation for the Simple Flight Model because it shows clearly where calculations are performed and where parameters are transferred. A summary of 'visible' data is given in Table 1.2. In a full development project, this would be broken down into block-to-block transfers and

Table 1.2 Interface Parameters in the Simple Flight Model.

Symbol	Definition	Units
α	Angle of attack (AOA) [cf. Figure 1.1]	radian
δ	Relative pressure	Not applicable
θ	Relative temperature	Not applicable
ρ	Air density	kg m^{-3}
g	Gravitational acceleration	m s^{-2}
m	Aircraft mass	kg
p	Roll rate (about velocity vector) [cf. Figure 1.4]	radian s^{-1}
q	Pitch rate [cf. Figure 1.4]	radian s^{-1}
u	Altitude measured up from earth surface	m
D	Aerodynamic drag	N
J_1	Aircraft moment of inertia in roll	kg.m^2
J_2	Aircraft moment of inertia in pitch	kg.m^2
L	Aerodynamic lift	N
K	Aerodynamic rolling moment	N.m
M	Aerodynamic pitching moment	N.m
M_N	Mach number	Not applicable
N	Nett thrust	N
V	True airspeed [cf. Figure 1.3]	m s^{-1}
W	Aircraft weight	N

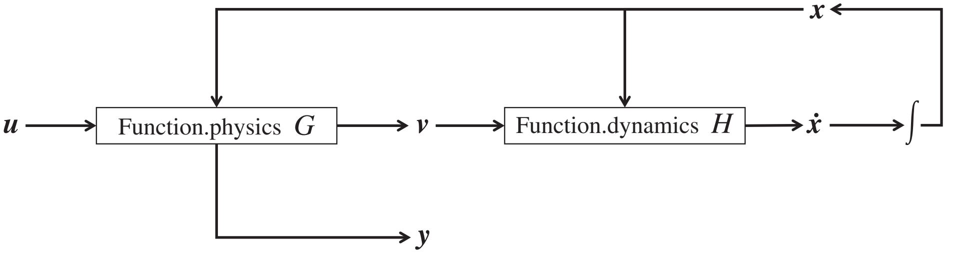


Figure 1.19 Physics and Dynamics of a Generic Functional Block.

input-to-output transformations within each block. The end-result would be a comprehensive statement of content for the model, as a formally declared product model.

A final comment in this Chapter concerns generic computational structure. Figure 1.19 shows one way of defining the contents of a functional block, with the partition of physics and dynamics that has been discussed already. Inputs are supplied as a vector $\boldsymbol{u}$, outputs are delivered as a vector $\boldsymbol{y}$, states are stored in a vector $\boldsymbol{x}$, and state derivatives are updated to a vector $\dot{\boldsymbol{x}}$. The interface between physics and dynamics is via a vector $\boldsymbol{v}$. The constitutive relationships for this scheme are as follows:

$$(\boldsymbol{v}, \boldsymbol{y}) = G(\boldsymbol{u}, \boldsymbol{x}) \tag{1.80}$$

$$\dot{\boldsymbol{x}} = H(\boldsymbol{v}, \boldsymbol{x}) \tag{1.81}$$

where G and H are non-linear functions. Small perturbations about an equilibrium condition would be described in the general form:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{v} \qquad \boldsymbol{y} = C\boldsymbol{x} + D\boldsymbol{u} \qquad \boldsymbol{v} = E\boldsymbol{x} + F\boldsymbol{u} \tag{1.82}$$

This is rationalised as follows:

$$\dot{\boldsymbol{x}} = A'\boldsymbol{x} + B'\boldsymbol{u} \qquad \boldsymbol{y} = C\boldsymbol{x} + D\boldsymbol{u} \tag{1.83}$$

where $A' = A + BE$ and $B' = BF$, which is the form that is most familiar to dynamicists and control engineers.