1

Introduction

1.1 Array Signal Processing

Array signal processing is one of the major areas of signal processing and has been studied extensively in the past due to its wide applications in various areas ranging from radar, sonar, microphone arrays, radio astronomy, seismology, medical diagnosis and treatment, to communications (Allen and Ghavami, 2005; Brandstein and Ward, 2001; Fourikis, 2000; Haykin, 1985; Hudson, 1981; Johnson and Dudgeon, 1993; Monzingo and Miller, 2004; Van Trees, 2002). It involves multiple sensors (microphones, antennas, etc.) placed at different positions in space to process the received signals arriving from different directions. An example for a simple array system consisting of four sensors with two impinging signals is shown in Figure 1.1 for illustrative purposes, where the direction of arrival (DOA) of the signals is characterized by two parameters: an elevation angle θ and an azimuth angle ϕ.

We normally assume the array sensors have the same characteristics and they are omnidirectional (or isotropic), i.e. their responses to an impinging signal are independent of their DOA angles. According to the relative locations of the sensors, arrays can be divided into three classes (Van Trees, 2002):

- one-dimensional (1-D) arrays or linear arrays;
- two-dimensional (2-D) arrays or planar arrays;
- three-dimensional (3-D) arrays or volumetric arrays.

Each of them can be further divided into two categories:

- regular spacing, including uniform and nonuniform spacings;
- irregular or random spacing.

Our study in this book will be based on arrays with regular spacings.

For the impinging signals, we always assume that they are plane waves, i.e. the array is located in the far field of the sources generating the waves and the received signals have a planar wavefront.

Now consider a plane wave with a frequency f propagating in the direction of the z-axis of the Cartesian coordinate system as shown in Figure 1.2. At the plane defined

Wideband Beamforming Wei Liu and Stephan Weiss

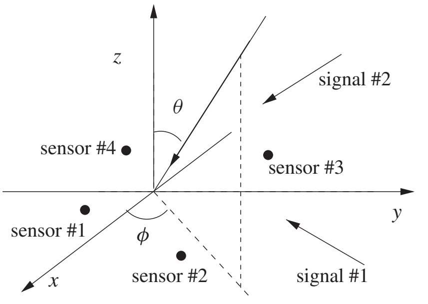

Figure 1.1 An illustrative array example with four sensors and two impinging signals

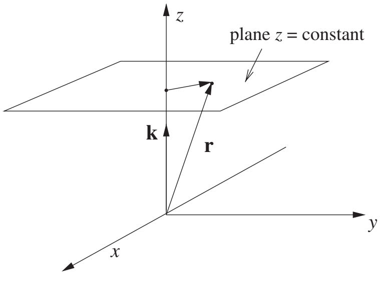

Figure 1.2 A plane wave propagating in the direction of the z-axis of the Cartesian coordinate system

by $z =$ constant, the phase of the signal can be expressed as:

$$\phi(t, z) = 2\pi f t - kz \tag{1.1}$$

where t is time and the parameter k is referred to as the wavenumber and defined as (Crawford, 1968):

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \tag{1.2}$$

where ω is the (temporal) angular frequency, c denotes the speed of propagation in the specific medium and λ is the wavelength. Similar to ω, which means that in a temporal interval t the phase of the signal accumulates to the value ωt, the interpretation of k is that over a distance z, measured in the propagation direction, the phase of the signal accumulates to kz radians. As a result, k can be referred to as the spatial frequency of a signal.

Different from the temporal frequency ω, which is one-dimensional, the spatial frequency k is three-dimensional and its direction is opposite to the propagating direction of the signal. In a Cartesian coordinate system, it can be denoted by a three-element vector:

$$\mathbf{k} = [k_x, k_y, k_z]^T \tag{1.3}$$

with a length of:

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} \tag{1.4}$$

This vector is referred to as the wavenumber vector. In the case shown in Figure 1.2, we have $k_x = k_y = 0$ and $k_z = -k$. Let $\hat{\mathbf{z}} = [0, 0, 1]^T$ denote the unit vector along the z-axis direction, then we have $\mathbf{k} = -k\hat{\mathbf{z}}$.

These two quantities are not independent of each other and as shown in Equation (1.2), they are related by the following equation:

$$k = \frac{2\pi f}{c} \tag{1.5}$$

Any point in a 3-D space can be represented by a vector $\mathbf{r} = [r_x, r_y, r_z]^T$, where r_x, r_y and r_z are the coordinates of this point in the Cartesian coordinate system. With the definition of the wavenumber vector $\mathbf{k}$, the phase function $\phi(t, \mathbf{r})$ of a plane wave can be expressed in a general form:

$$\phi(t, \mathbf{r}) = 2\pi f t + \mathbf{k}^T \mathbf{r} \tag{1.6}$$

For the case in Figure 1.2, we have:

$$\mathbf{k}^T \mathbf{r} = -k(\hat{\mathbf{z}}^T \mathbf{r}) = -k r_z \tag{1.7}$$

Therefore, as long as the points have the same coordinate r_z in the z-axis direction, they have the same phase value at a fixed time instant t.

For the general case, where the signal impinges upon the array from an elevation angle θ and an azimuth angle ϕ, as shown in Figure 1.1, the wavenumber vector $\mathbf{k}$ is given by:

$$\mathbf{k} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} = k \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \tag{1.8}$$

Then the time independent phase term $\mathbf{k}^T \mathbf{r}$ changes to:

$$\mathbf{k}^T \mathbf{r} = k(r_x \sin\theta\cos\phi + r_y \sin\theta\sin\phi + r_z \cos\theta) \tag{1.9}$$

The wavefront of the signal is still represented by the plane perpendicular to its propagation direction.

There are three major research areas for array signal processing:

1. Detecting the presence of an impinging signal and determine the signal numbers.

2. Finding the DOA angles of the impinging signals.

3. Enhancing the signal of interest coming from some known/unknown directions and suppress the interfering signals (if present) at the same time.

The third research area is the task of beamforming, which can be divided into narrowband beamforming and wideband beamforming depending on the bandwidth of the impinging signals, and wideband beamforming will be the focus of this book. In the next sections, we will first introduce the idea of narrowband beamforming and then extend it to the wideband case.

1.2 Narrowband Beamforming

In beamforming, we estimate the signal of interest arriving from some specific directions in the presence of noise and interfering signals with the aid of an array of sensors. These sensors are located at different spatial positions and sample the propagating waves in space. The collected spatial samples are then processed to attenuate/null out the interfering signals and spatially extract the desired signal. As a result, a specific spatial response of the array system is achieved with 'beams' pointing to the desired signals and 'nulls' towards the interfering ones.

Figure 1.3 shows a simple beamforming structure based on a linear array, where M sensors sample the wave field spatially and the output $y(t)$ at time t is given by an instantaneous linear combination of these spatial samples $x_m(t)$, $m = 0, 1, \ldots, M-1$, as:

$$y(t) = \sum_{m=0}^{M-1} x_m(t) w_m^* \tag{1.10}$$

where * denotes the complex conjugate.

The beamformer associated with this structure is only useful for sinusoidal or narrowband signals, where the term 'narrowband' means that the bandwidth of the impinging signal should be narrow enough to make sure that the signals received by the opposite ends of the array are still correlated with each other (Compton, 1988b), and hence it is termed a narrowband beamformer.

We now analyse the array's response to an impinging complex plane wave $e^{j\omega t}$ with an angular frequency ω and a DOA angle θ, where $\theta \in [-\pi/2\ \pi/2]$ is measured with respect to the broadside of the linear array, as shown in Figure 1.3. For convenience, we assume the phase of the signal is zero at the first sensor. Then the signal received by the first sensor is $x_0(t) = e^{j\omega t}$ and by the mth sensor is $x_m(t) = e^{j\omega(t-\tau_m)}$, $m = 1, 2, \ldots, M-1$, where τ_m is the propagation delay for the signal from sensor 0 to sensor m and is a function of θ. Then the beamformer output is:

$$y(t) = e^{j\omega t} \sum_{m=0}^{M-1} e^{-\omega\tau_m} w_m^* \tag{1.11}$$

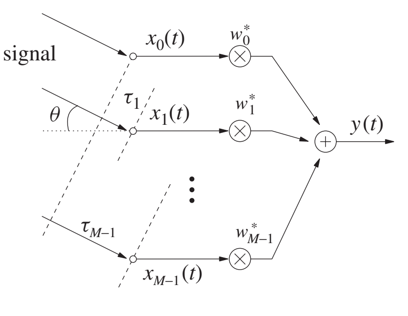

Figure 1.3 A general structure for narrowband beamforming

with $\tau_0 = 0$. The response of this beamformer is given by:

$$P(\omega, \theta) = \sum_{m=0}^{M-1} \mathrm{e}^{-j\omega\tau_m} w_m^* = \mathbf{w}^H \mathbf{d}(\omega, \theta) \tag{1.12}$$

where the weight vector $\mathbf{w}$ holds the M *complex conjugate* coefficients of the sensors, given by:

$$\mathbf{w} = [w_0 \; w_1 \; \ldots \; w_{M-1}]^T \tag{1.13}$$

and the vector $\mathbf{d}(\omega, \theta)$ is given by:

$$\mathbf{d}(\omega, \theta) = \left[1 \;\; \mathrm{e}^{-j\omega\tau_1} \;\; \ldots \;\; \mathrm{e}^{-j\omega\tau_{M-1}}\right]^T \tag{1.14}$$

We refer to $\mathbf{d}(\theta, \omega)$ as the array response vector, which is also known as the steering vector or direction vector (Van Veen and Buckley, 1988). We will use the term 'steering vector' to avoid confusion with the response vector used in linearly constrained minimum variance beamforming introduced later in Section 2.2 of Chapter 2.

In our notation, we generally use lowercase bold letters for vector valued quantities, while uppercase bold letters denote matrices. The operators $\{\cdot\}^T$ and $\{\cdot\}^H$ represent transpose and Hermitian transpose operations, respectively.

Based on the steering vector, we briefly discuss the spatial aliasing problem encountered in array processing. In analogue to digital conversion, we sample the continuous-time signal temporally and convert it into a discrete-time sequence. In this temporal sampling process, aliasing is referred to as the phenomenon that signals with different frequencies have the same discrete sample series, which occurs when the signal is sampled at a rate lower than the Nyquist sampling rate, i.e. twice the highest frequency of the signal (Oppenheim and Schafer, 1975). With temporal aliasing, we will not be able to recover the original continuous-time signal from their samples. In array processing, the sensors sample the impinging signals spatially and if the signals from different spatial locations are not sampled by the array sensors densely enough, i.e. the inter-element spacing of the array is too large, then sources at different locations will have the same array steering vector and we cannot uniquely determine their locations based on the received array signals. Similar to the temporal sampling case, now we have a spatial aliasing problem, due to the ambiguity in the directions of arrival of source signals.

For signals having the same angular frequency ω and the corresponding wavelength λ, but different DOAs θ_1 and θ_2 satisfying the condition $(\theta_1, \theta_2) \in [-\pi/2 \; \pi/2]$, aliasing implies that we have $\mathbf{d}(\theta_1, \omega) = \mathbf{d}(\theta_2, \omega)$, namely:

$$\mathrm{e}^{-j\omega\tau_m(\theta_1)} = \mathrm{e}^{-j\omega\tau_m(\theta_2)} \tag{1.15}$$

For a uniformly spaced linear array with an inter-element spacing d, we have $\tau_m = m\tau_1 = m(d \sin\theta)/c$ and $\omega\tau_m = m(2\pi d \sin\theta)/\lambda$. Then Equation (1.15) changes to:

$$\mathrm{e}^{-jm(2\pi d \sin\theta_1)/\lambda} = \mathrm{e}^{-jm(2\pi d \sin\theta_2)/\lambda} \tag{1.16}$$

In order to avoid aliasing, the condition $|2\pi(\sin\theta)d/\lambda|_{\theta=\theta_1,\theta_2} < \pi$ has to be satisfied. Then we have $|d/\lambda \sin\theta| < 1/2$. Since $|\sin\theta| \leq 1$, this requires that the array distance d should be less than $\lambda/2$.

In the following, we will always set $d = \lambda/2$, unless otherwise specified, then $\omega\tau_m = m\pi \sin\theta$ and the response of the uniformly spaced narrowband beamformer is given by:

$$P(\omega, \theta) = \sum_{m=0}^{M-1} \mathrm{e}^{-jm\pi \sin\theta} w_m^* \tag{1.17}$$

Note for an FIR (finite impulse response) filter with the same set of coefficients (Oppenheim and Schafer, 1975), its frequency response is given by:

$$P(\Omega) = \sum_{m=0}^{M-1} \mathrm{e}^{-jm\Omega} w_m^* \tag{1.18}$$

with $\Omega \in [-\pi\ \pi]$ being the normalized frequency. For the response of the beamformer given by Equation (1.17), when θ changes from $-\pi/2(-90^\circ)$ to $\pi/2(90^\circ), \pi \sin\theta$ changes from $-\pi$ to π accordingly, which is in the same range as Ω in Equation (1.18). With this correspondence, the design of uniformly spaced linear arrays can be achieved by the existing FIR filter design approaches directly.

As a simple example, if we want to form a flat beam response pointing to the directions $\theta \in [-\pi/6\ \pi/6]([-30^\circ\ 30^\circ])$, while suppressing signals from directions $\theta \in [-\pi/2\ -\pi/4]$ and $[\pi/4\ \pi/2]$, then it is equivalent to designing an FIR filter with a passband of $\Omega \in [-0.5\pi\ 0.5\pi]$ and a stopband of $\Omega \in [-\pi\ \ -0.71\pi]$ and $[0.71\pi\ \pi]$ ($\sin\pi/6 = 0.5$ and $\sin\pi/4 = 0.71$). We can use the MATLAB© function *remez* to design such a filter (Mat, 2001), and then use the result directly as the coefficients of the desired beamformer. One of the design results is given by ($M = 10$):

$$\begin{aligned}\mathbf{w}^H = [&0.0422\ \ 0.0402\ \ -0.1212\ \ 0.0640\ \ 0.5132\\ &0.5132\ \ 0.0640\ \ -0.1212\ \ 0.0402\ \ 0.0422]\end{aligned} \tag{1.19}$$

Substituting this result into Equation (1.17), we can draw the resultant amplitude response $|P(\theta, \omega)|$ of the beamformer with respect to the DOA angle θ. $|P(\theta, \omega)|$ is called the beam pattern of the beamformer to describe the sensitivity of the beamformer with respect to signals arriving from different directions and with different frequencies. Figure 1.4 shows the beam pattern (BP) in dB, which is defined as follows:

$$\mathrm{BP} = 20\log_{10}\frac{|P(\theta, \omega)|}{\max|P(\theta, \omega)|} \tag{1.20}$$

For the general case of $d = \alpha\lambda/2$, $\alpha \le 1$, the response of the beamformer given by Equation (1.17), will change to:

$$P(\omega, \theta) = \sum_{m=0}^{M-1} \mathrm{e}^{-jm\alpha\pi \sin\theta} w_m^* \tag{1.21}$$

Its design can be obtained in a similar way as above and the only difference is that the FIR filter can have an arbitrary response over the regions $\Omega \in [-\pi\ \ -\alpha\pi]$ and $[\alpha\pi\ \pi]$ without affecting that of the narrowband beamformer.

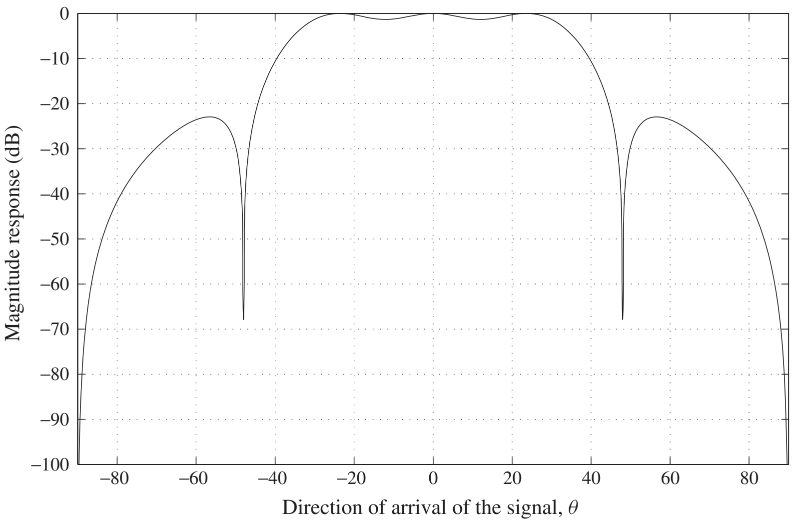

Figure 1.4 The beam pattern of the resultant narrowband beamformer with $M = 10$ sensors

1.3 Wideband Beamforming

The beamforming structure introduced in the last section works effectively only for narrowband signals. When the signal bandwidth increases, its performance will degrade significantly. This can be explained as follows.

Suppose there are in total M impinging signals $s_m(t)$, $m = 0, 1, \ldots, M-1$, from directions of θ_m, $m = 0, 1, \ldots, M-1$, respectively. The first one $s_0(t)$ is the signal of interest and the others are interferences. Then the array's steering vector $\mathbf{d}_m$ for these signals is given by:

$$\mathbf{d}_m(\omega, \theta) = \begin{bmatrix} 1 & e^{-j\omega\tau_1(\theta_m)} & \ldots & e^{-j\omega\tau_{M-1}(\theta_m)} \end{bmatrix}^T \tag{1.22}$$

Ideally, for beamforming, we aim to form a fixed response to the signal of interest and zero response to the interfering signals. Note for simplicity, we do not consider the effect of noise here. This requirement can be expressed as the following matrix equation:

$$\begin{pmatrix} 1 & e^{-j\omega\tau_1(\theta_0)} & \ldots & e^{-j\omega\tau_{M-1}(\theta_0)} \\ 1 & e^{-j\omega\tau_1(\theta_1)} & \ldots & e^{-j\omega\tau_{M-1}(\theta_1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\omega\tau_1(\theta_{M-1})} & \ldots & e^{-j\omega\tau_{M-1}(\theta_{M-1})} \end{pmatrix} \begin{pmatrix} w_0^* \\ w_1^* \\ \vdots \\ w_{M-1}^* \end{pmatrix} = \begin{pmatrix} \text{constant} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{1.23}$$

Obviously, as long as the matrix on the left has full rank, we can always find a set of array weights to cancel the $M - 1$ interfering signals and the exact value of the weights for complete cancellation of the interfering signals is dependent on the signal frequency (certainly also on their directions of arrival).

For wideband signals, since each of them consists of infinite number of different frequency components, the value of the weights should be different for different frequencies and we can write the weight vector in the following form:

$$\mathbf{w}(\omega) = [w_0(\omega)\ w_1(\omega)\ \ldots,\ w_{M-1}(\omega)]^T \tag{1.24}$$

This is why the narrowband beamforming structure with a single constant coefficient for each received sensor signal will not work effectively in a wideband environment.

The frequency dependent weights can be achieved by sensor delay-lines (SDLs), which were proposed only recently and will be studied in Chapter 7. Traditionally, an easy way to form such a set of frequency dependent weights is to use a series of tapped delay-lines (TDLs), or FIR/IIR filters in its discrete form (Compton, 1988a; Frost, 1972; Mayhan et al., 1981; Monzingo and Miller, 2004; Rodgers and Compton, 1979; Van Veen and Buckley, 1988; Vook and Compton, 1992).

Both TDLs and FIR/IIR filters perform a temporal filtering process to form a frequency dependent response for each of the received wideband sensor signals to compensate the phase difference for different frequency components. Such a structure is shown in Figure 1.5. The beamformer obeying this architecture samples the propagating wave field in both space and time. The output of such a wideband beamformer can be expressed as:

$$y(t) = \sum_{m=0}^{M-1} \sum_{i=0}^{J-1} x_m(t - iT_s) \times w_{m,i}^* \tag{1.25}$$

where $J - 1$ is the number of delay elements associated with each of the M sensor channels in Figure 1.5 and T_s is the delay between adjacent taps of the TDLs.

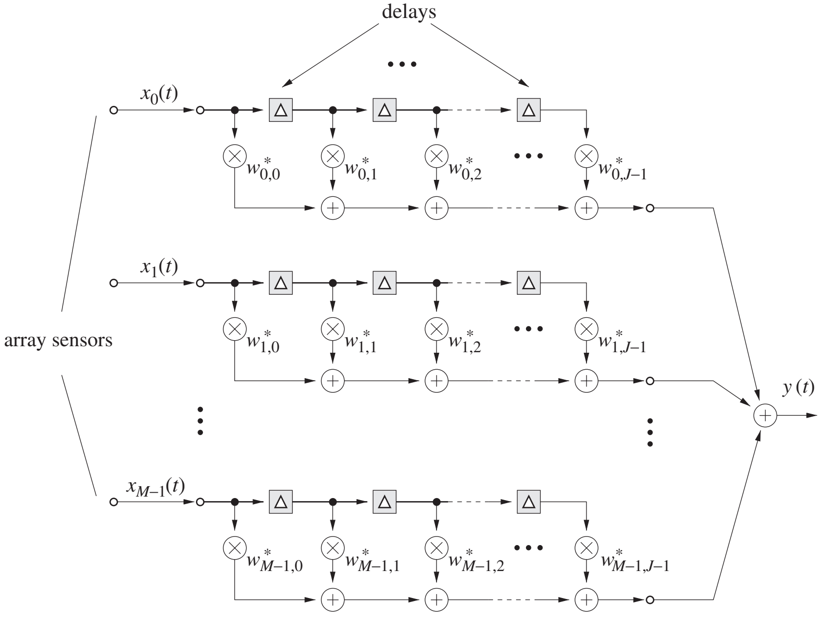

Figure 1.5 A general structure for wideband beamforming

In vector form, Equation (1.25) can be rewritten as:

$$y(t) = \mathbf{w}^H \mathbf{x}(t) \tag{1.26}$$

The weight vector $\mathbf{w}$ holds all MJ sensor coefficients with:

$$\mathbf{w} = \begin{bmatrix} \mathbf{w}_0 \\ \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_{J-1} \end{bmatrix} \tag{1.27}$$

where each vector $\mathbf{w}_i$, $i = 0, 1, \cdots, J-1$, contains the M *complex conjugate* coefficients found at the ith tap position of the M TDLs, and is expressed as:

$$\mathbf{w}_i = [w_{0,i} \; w_{1,i} \; \cdots \; w_{M-1,i}]^T \tag{1.28}$$

Similarly, the input data are also accumulated in a vector form $\mathbf{x}$ as follows:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_0(t) \\ \mathbf{x}_1(t - T_s) \\ \vdots \\ \mathbf{x}_{J-1}(t - (J-1)T_s] \end{bmatrix} \tag{1.29}$$

where $\mathbf{x}_i(t - iT_s)$, $i = 0, 1, \ldots, J-1$, holds the ith data slice corresponding to the ith coefficient vector $\mathbf{w}_i$:

$$\mathbf{x}(t - iT_s) = \left[x_0(t - iT_s) \;\; x_1(t - iT_s) \;\; \cdots \;\; x_{M-1}(t - iT_s)\right]^T \tag{1.30}$$

Note that this notation incorporates the narrowband beamformer with the special case of $J = 1$.

Now, for an impinging complex plane wave signal $\mathrm{e}^{j\omega t}$, assume $x_0(t) = \mathrm{e}^{j\omega t}$. Then we have:

$$x_m(t - iT_s) = \mathrm{e}^{j\omega(t-(\tau_m+iT_s))} \tag{1.31}$$

with $m = 0, 1, \ldots, M-1$, $i = 0, \ldots, J-1$. The array output is given by:

$$\begin{aligned} y(t) &= \mathrm{e}^{j\omega t} \sum_{m=0}^{M-1} \sum_{i=0}^{J-1} \mathrm{e}^{-j\omega(\tau_m+iT_s)} \cdot w_{m,i}^* \\ &= \mathrm{e}^{j\omega t} \times P(\theta, \omega) \end{aligned} \tag{1.32}$$

where $P(\theta, \omega)$ is the beamformer's angle and frequency dependent response. It can be expressed in vector form as:

$$P(\theta, \omega) = \mathbf{w}^H \mathbf{d}(\theta, \omega) \tag{1.33}$$

where $\mathbf{d}(\theta, \omega)$ is the steering vector for this new wideband beamformer and its elements correspond to the complex exponentials $e^{-j\omega(\tau_m + iT_s)}$:

$$\mathbf{d}(\theta, \omega) = [e^{-j\omega\tau_0} \ \ldots \ e^{-j\omega\tau_{M-1}} \ e^{-j\omega(\tau_0+T_s)} \ \ldots \ e^{-j\omega(\tau_{M-1}+T_s)} \\ \ldots \ e^{-j\omega(\tau_0+(J-1)T_s)} \ \ldots \ e^{-j\omega(\tau_{M-1}+(J-1)T_s)}]^T \tag{1.34}$$

For $J = 1$, it is reduced to the steering vector introduced for the narrowband beamformer in Equation (1.14).

For an equally spaced linear array with an inter-element spacing d, we have $\tau_m = m\tau_1$ and $\omega\tau_m = m(2\pi d \sin\theta)/\lambda$ for $m = 0, 1, \ldots, M-1$. To avoid aliasing, $d < \lambda_{\min}/2$, where $\lambda_{\min}$ is the wavelength of the signal component with the highest frequency $\omega_{\max}$. Assume the operating frequency of the array is $\omega \in [\omega_{min} \ \omega_{max}]$ and $d = \alpha\lambda_{\min}/2$ with $\alpha \leq 1$. In its discrete form, T_s is the temporal sampling period of the system and should be no more than half the period $T_{\min}$ of the signal component with the highest frequency according to the Nyquist sampling theorem (Oppenheim and Schafer, 1975), i.e. $T_s \leq T_{\min}/2$.

With the normalized frequency $\Omega = \omega T_s$, $\omega(m\tau_1 + iT_s)$ changes to $m\mu\Omega\sin\theta + i\Omega$ with $\mu = d/(cT_s)$, then the steering vector $\mathbf{d}(\theta, \omega)$ changes to:

$$\mathbf{d}(\theta, \omega) = [1 \ \ldots \ e^{-j(M-1)\mu\Omega\sin\theta} \ e^{-j\Omega} \ \ldots \ e^{-j\Omega(\mu\sin\theta(M-1)+1)} \\ \ldots \ e^{-j(J-1)\Omega} \ \ldots \ e^{-j\Omega(\mu\sin\theta(M-1)+J-1)}]^T \tag{1.35}$$

and we have:

$$\begin{aligned} P(\theta, \omega) &= \sum_{m=0}^{M-1}\sum_{i=0}^{J-1} e^{-j\Omega(m\mu\sin\theta + i)} \times w_{m,i}^* \\ &= \sum_{m=0}^{M-1} e^{-jm\mu\Omega\sin\theta} \sum_{i=0}^{J-1} e^{-ji\Omega} \times w_{m,i}^* \\ &= \sum_{m=0}^{M-1} e^{-jm\mu\Omega\sin\theta} \times W_m(e^{j\Omega}) \end{aligned} \tag{1.36}$$

where $W_m(e^{j\Omega}) = \sum_{i=0}^{J-1} e^{-ji\Omega} \times w_{m,i}^*$ is the Fourier transform of the TDL coefficients attached to the mth sensor. For the case where $\alpha = 1$ and $T_s = T_{\min}/2$, we have $\mu = 1$.

Now given the coefficients of the wideband beamformer, we can draw its 3-D beam pattern $|P(\theta, \omega)|$ with respect to frequency and DOA angle, according to Equation (1.36). To calculate the beam pattern for N_θ number of discrete DOA values and N_Ω number of discrete temporal frequencies, an $N_\theta \times N_\Omega$ matrix is obtained holding the response samples on the defined DOA/frequency grid.

As an example, consider an array with $M = 5$ sensors and a TDL length $J = 3$. Suppose the weight vector is given as:

$$\mathbf{W} = [0 \ 0 \ 0 \ 0 \ 0 \ 0.2 \ 0.2 \ 0.2 \ 0.2 \ 0.2 \ 0 \ 0 \ 0 \ 0 \ 0]^T \tag{1.37}$$

The beam pattern of such an array is shown in Figure 1.6 for $N_\Omega = 50$ and $N_\theta = 60$, where the gain is displayed in dB as defined in Equation (1.20).

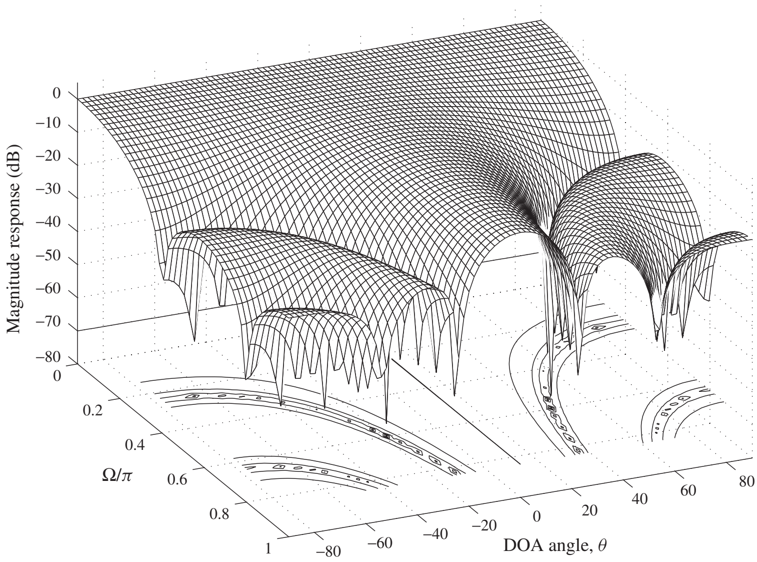

Figure 1.6 A 3-D wideband beam pattern example based on an equally spaced linear array with $M = 5,\ J = 3$ and $\mu = 1$

In the above example, the values of the weight coefficients are fixed and the resultant beamformer will maintain a fixed response independent of the signal/interference scenarios. In statistically optimum beamforming, the weight coefficients need to be updated based on the statistics of the array data. When the data statistics are unknown or time varying, adaptive optimization is required (Haykin, 1996), where according to different signal environments and application requirements, different beamforming techniques may be employed. Both kinds of beamformers will be studied later in this book.

1.4 Wideband Beam Steering

For a narrowband beamformer, we can steer its main beam to a desired direction by adding appropriate steering delays or phase shifts (Johnson and Dudgeon, 1993; Van Trees, 2002). The relationship between the steered response and the original one is simple for a half wavelength spaced linear array: the former one is a circularly shifted version of the latter one, i.e. the sidelobe shifted out from one side is simply shifted back from the other side.

Intuitively, we may think that adding steering delays for wideband beamformers has the same effect as in the narrowband case. However, this is not true and in general there is not a one-to-one correspondence between the original beam response and the steered one (Liu and Weiss, 2008c, 2009a).

In this section we will give a detailed analysis about this relationship. We will see that after adding steering delays to the originally received wideband array signals, the main beam will be shifted to the desired direction; however for the sidelobe region, for one side, it is shifted out of the visible area and for the other side, it is not a simple shifted-back of those shifted out, but exhibits a very complicated pattern.

1.4.1 Beam Steering for Narrowband Arrays

For a uniformly spaced narrowband linear array, its response can be expressed as:

$$P(\sin\theta) = \sum_{m=0}^{M-1} w_m^* \mathrm{e}^{-jm\mu\Omega\sin\theta} \tag{1.38}$$

which is a special case ($J = 1$) of Equation (1.36).

Suppose the set of coefficients w_m^*, $m = 0, 1, \ldots, M-1$, forms a main beam pointing to the broadside of the array ($\theta = 0$). In order to steer the beam to the direction θ_0, we can add a delay of $(M-1)(d\sin\theta_0)/c$ to the first received array signal, a delay of $(M-2)(d\sin\theta_0)/c$ to the second received array signal, and so on. Then the new response with a main beam pointing to θ_0 is given by:

$$P(\sin\theta - \sin\theta_0) = \mathrm{e}^{-j(M-1)\mu\Omega\sin\theta_0} \sum_{m=0}^{M-1} w_m^* \mathrm{e}^{-jm\mu\Omega(\sin\theta - \sin\theta_0)} \tag{1.39}$$

where the term $\mathrm{e}^{-j(M-1)\mu\Omega\sin\theta_0}$ represents a constant delay for all signals and will be ignored in the following equations and discussions.

To avoid spatial aliasing, $d = \lambda/2$, where λ is the signal wavelength. We also assume the sampling frequency is twice that of the signal frequency. Then, we have $\mu = 1$ and $\Omega = \pi$. As a result, Equation (1.39) changes to:

$$P(\sin\theta - \sin\theta_0) = \sum_{m=0}^{M-1} w_m^* \mathrm{e}^{-jm\pi(\sin\theta - \sin\theta_0)} \tag{1.40}$$

Since the function $\mathrm{e}^{-jm\pi x}$ is periodic with a period of 2, compared to Equation (1.38), the response given by Equation (1.40) is simply a circularly shifted version of the response in Equation (1.38) for one period $\sin\theta \in [-1\ 1]$. As an example, suppose we have a broadside main beam response $P(\sin\theta)$ with a maximum response at $\sin\theta = 0$ for $\sin\theta \in [-1\ 1]$, as shown in Figure 1.7. Then after shifting it by $\sin\theta_0 < 0$, the new response will be given by Figure 1.8.

Now we consider the effect as a function of θ. For the remaining part of Section 1.4, without loss of generality, we always assume $\theta_0 < 0$. For $-1 < \sin\theta \le (1 + \sin\theta_0)$, we have:

$$-1 < -1 - \sin\theta_0 < \sin\theta - \sin\theta_0 \le 1 \tag{1.41}$$

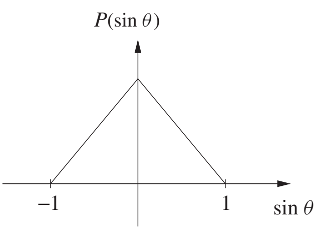

Figure 1.7 A broadside main beam example for an equally spaced narrowband linear array

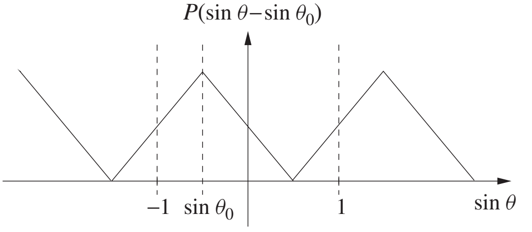

Figure 1.8 The broadside main beam is shifted to the direction θ_0 for the example in Figure 1.7

Then, the response at θ after steering for $-1 < \sin\theta \leq (1 + \sin\theta_0)$ will be the same as the response of the original broadside one at:

$$\hat{\theta} = \arcsin(\sin\theta - \sin\theta_0) \tag{1.42}$$

For $(1 + \sin\theta_0) < \sin\theta \leq 1$, we have:

$$2 \geq (1 - \sin\theta_0) \geq (\sin\theta - \sin\theta_0) > 1 \tag{1.43}$$

Then:

$$0 \geq (\sin\theta - \sin\theta_0 - 2) > -1 \tag{1.44}$$

Therefore, for this range of θ, the steered response at θ will be the same as the response of the original one at:

$$\breve{\theta} = \arcsin(\sin\theta - \sin\theta_0 - 2) \tag{1.45}$$

1.4.2 Beam Steering for Wideband Arrays

1.4.2.1 Wideband Arrays with TDLs or FIR/IIR Filters

As discussed in Section 1.3, for wideband beamforming, we will need the structure shown in Figure 1.5. Recall that its beam response has been given in Equation (1.36) as follows:

$$P(\Omega, \sin\theta) = \sum_{m=0}^{M-1}\sum_{i=0}^{J-1} w_{m,i}^* \times \mathrm{e}^{-jm\mu\Omega\sin\theta} \times \mathrm{e}^{-ji\Omega} \tag{1.46}$$

Suppose the set of coefficients $w_{m,i}^*$, $m = 0, 1, \ldots, M-1$, $i = 0, 1, \ldots, J-1$ forms a broadside main beam ($\theta = 0$), with an example shown in Figure 1.10. In order to steer the beam to the direction θ_0, we add delays in the same way as in the narrowband case and the new response is given by:

$$P(\Omega, \sin\theta - \sin\theta_0) = \sum_{m=0}^{M-1}\sum_{i=0}^{J-1} w_{m,i}^* \times \mathrm{e}^{-jm\mu\Omega(\sin\theta - \sin\theta_0)} \times \mathrm{e}^{-ji\Omega} \tag{1.47}$$

To avoid aliasing, $d = \lambda_{\min}/2$ and $T_s = \pi/\omega_{\max}$. Then we have $\mu = 1$ and:

$$P(\Omega, \sin\theta - \sin\theta_0) = \sum_{m=0}^{M-1}\sum_{i=0}^{J-1} w_{m,i}^* \times e^{-jm\Omega(\sin\theta - \sin\theta_0)} \times e^{-ji\Omega} \tag{1.48}$$

Note the required steering delays can be implemented by some analogue devices and digital interpolation methods (Pridham and Mucci, 1978, 1979; Schafer and Rabiner, 1973), or FIR/IIR filters with fractional delays (Lu and Morris, 1999). A special case is a delay over the whole normalized frequency range $[0\ \pi]$, which can be realized by a series of truncated sinc functions.

As an example, we steer the main beam in Figure 1.9 in this way to the direction $\theta_0 = -30^\circ$ and the result is shown in Fig. 1.10. Although the main beam is indeed steered to the desired direction, there are some problems. The first one is the distorted response at around the frequency $\Omega = \pi$, which is due to the fact that the delay cannot be approximated well by the sinc function at $\Omega = \pi$. More importantly, the relationship between Figure 1.9 and Figure 1.10 is clearly not a simple shift between each other because we can see the irregularity in the steered response at the sidelobe region between about 40° and 90° and we cannot find it in the original broadside main beam in Figure 1.9. This difference indicates that beam steering with delays for wideband beamformers has quite a different effect. In the next section, we will give a detailed analysis about this relationship.

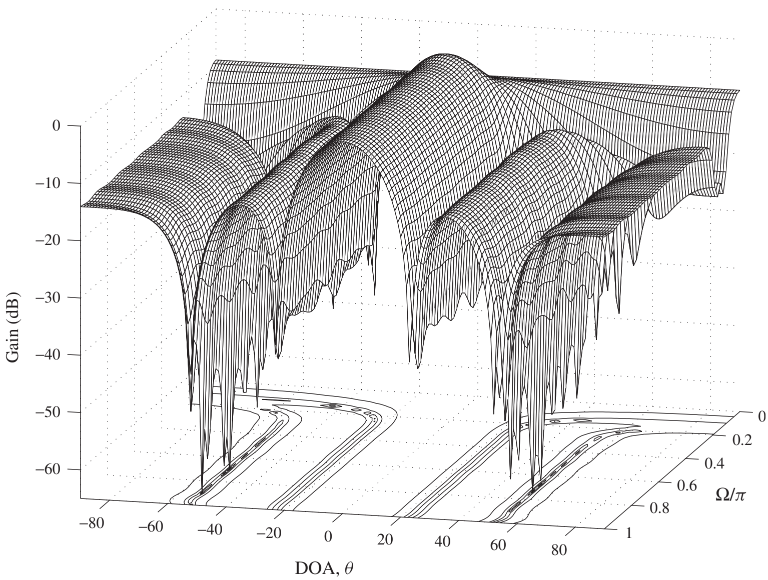

Figure 1.9 A broadside main beam for a linear wideband array with $M = 21$ sensors and $J = 25$ coefficients for each of the attached FIR filters ($\mu = 1$)

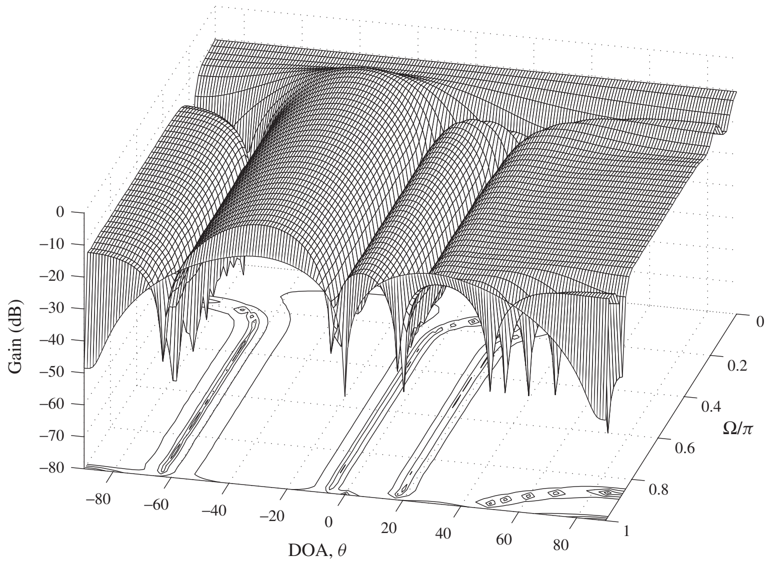

Figure 1.10 The result after the broadside main beam in Figure 1.9 is steered to an off-broadside direction (−30°)

Since $\theta_0 < 0$, for $-1 < \sin\,\theta \leq (1 + \sin\,\theta_0)$, we have:

$$-1 < -1 - \sin\,\theta_0 < \sin\,\theta - \sin\,\theta_0 \leq 1 \tag{1.49}$$

Then the steered response at θ for $-1 < \sin\,\theta \leq (1 + \sin\,\theta_0)$ will be the same as the response of the original broadside main beam design at:

$$\hat{\theta} = \arcsin(\sin\,\theta - \sin\,\theta_0) \tag{1.50}$$

However, for $(1 + \sin\,\theta_0) < \sin\,\theta \leq 1$, we have $(\sin\,\theta - \sin\,\theta_0) > 1$. Then the shift relationship cannot be expressed as Equation (1.50) any more and we need to further consider the following two cases bearing in mind the periodicity of the function $e^{-jm\Omega}$:

1. For $\Omega \leq \pi/(\sin\,\theta - \sin\,\theta_0)$, we have $\Omega(\sin\,\theta - \sin\,\theta_0) \leq \pi$, since $(\sin\,\theta - \sin\,\theta_0) > 1$, it seems that we cannot find any correspondence between the steered response and the original one for this case.

2. For $\Omega > \pi/(\sin\,\theta - \sin\,\theta_0)$, we have:

   $$\Omega(\sin\,\theta - \sin\,\theta_0) > \pi \tag{1.51}$$

   Then we have

   $$e^{-jm\Omega(\sin\,\theta - \sin\,\theta_0)} = e^{-jm\Omega(\sin\,\theta - \sin\,\theta_0 - 2\pi/\Omega)} \tag{1.52}$$

If $\sin\theta - \sin\theta_0 - 2\pi/\Omega < -1$, namely:

$$\Omega < \frac{2\pi}{1 + \sin\theta - \sin\theta_0} \tag{1.53}$$

then we come to the same conclusion as in the first case. Otherwise, we have:

$$\Omega \geq \frac{2\pi}{1 + \sin\theta - \sin\theta_0} \tag{1.54}$$

Then we can assume:

$$\sin\tilde{\theta} = \sin\theta - \sin\theta_0 - 2\pi/\Omega \tag{1.55}$$

Then the steered response for this case will be the same as the response of the original one at frequency Ω and DOA angle $\tilde{\theta} = \arcsin(\sin\theta - \sin\theta_0 - 2\pi/\Omega)$.

Note since $(\sin\theta - \sin\theta_0) > 1$, we have:

$$\frac{2\pi}{1 + \sin\theta - \sin\theta_0} > \frac{\pi}{\sin\theta - \sin\theta_0} \tag{1.56}$$

Then the above two cases for $(1 + \sin\theta_0) < \sin\theta \leq 1$ can be simplified as:

1. For $\Omega < 2\pi/(1 + \sin\theta - \sin\theta_0)$, there is no correspondence between the two beam responses.

2. For $\Omega \geq 2\pi/(1 + \sin\theta - \sin\theta_0)$, the steered response will be the same as the response of the original one at frequency Ω and DOA angle $\tilde{\theta}$.

1.4.2.2 Wideband Arrays with a Narrowband Structure

Sometimes we also use the narrowband beamforming structure for wideband signals and then Equation (1.46) changes back to the one given in Equation (1.38). In this case, the steered response for $\mu = 1$ is given by:

$$P(\sin\theta - \sin\theta_0) = \sum_{m=0}^{M-1} w_m^* \mathrm{e}^{-jm\Omega(\sin\theta - \sin\theta_0)} \tag{1.57}$$

Now for $-1 < \sin\theta \leq (1 + \sin\theta_0)(\theta_0 < 0)$, the relationship between the steered response and the original one is the same as the one given in Equation (1.50). For $(1 + \sin\theta_0) < \sin\theta \leq 1$, we have $(\sin\theta - \sin\theta_0) > 1$ and we again need to consider two different cases:

1. For $\Omega \leq \pi/(\sin\theta - \sin\theta_0)$, we have:

$$\mathrm{e}^{-jm\Omega(\sin\theta - \sin\theta_0)} = \mathrm{e}^{-jm\pi[\Omega(\sin\theta - \sin\theta_0)]/\pi} \tag{1.58}$$

and:

$$\frac{\Omega(\sin\theta - \sin\theta_0)}{\pi} \leq 1 \tag{1.59}$$

Then the steered beam response for this case will be the same as the response of the original broadside one at frequency $\Omega = \pi$ and DOA angle:

$$\bar{\theta} = \arcsin(\frac{\Omega(\sin\theta - \sin\theta_0)}{\pi}) \tag{1.60}$$

2. For $\Omega > \pi/(\sin\theta - \sin\theta_0)$, we have:

$$e^{-jm\Omega(\sin\theta - \sin\theta_0)} = e^{-jm\pi(\Omega(\sin\theta-\sin\theta_0)-2\pi)/\pi} \tag{1.61}$$

and:

$$\frac{\Omega(\sin\theta - \sin\theta_0) - 2\pi}{\pi} > -1 \tag{1.62}$$

Then the steered beam response for this case will be the same as the response of the original one at frequency $\Omega = \pi$ and a DOA angle of:

$$\check{\theta} = \arcsin\left(\frac{\Omega(\sin\theta - \sin\theta_0) - 2\pi}{\pi}\right) \tag{1.63}$$

1.4.3 A Unified Interpretation

In summary, the relationship between the steered beam response and the original one, given in Sections 1.4.2 and 1.4.2, respectively, is complicated and not as straightforward as in the narrowband one. However, there is another way to understand the relationship between the steered beam response and the original one.

Since $\theta_0 < 0$, we have $|\sin\theta - \sin\theta_0| \le (1 - \sin\theta_0)$ and $1 - \sin\theta_0 = \hat{\mu} > 1$, then Equation (1.48) can be rewritten as:

$$P(\Omega, \sin\theta) = \sum_{m=0}^{M-1}\sum_{i=0}^{J-1} w_{m,i}^* e^{-jm\hat{\mu}\Omega(\sin\theta - \sin\theta_0)/\hat{\mu}} e^{-ji\Omega} \tag{1.64}$$

Since $|(\sin\theta - \sin\theta_0)/\hat{\mu}| \le 1$, we can assume:

$$\sin\ddot{\theta} = \frac{\sin\theta - \sin\theta_0}{\hat{\mu}} \tag{1.65}$$

Then Equation (1.64) changes to:

$$P(\Omega, \theta) = \sum_{m=0}^{M-1}\sum_{i=0}^{J-1} w_{m,i}^* e^{-jm\hat{\mu}\Omega\sin\ddot{\theta}} e^{-ji\Omega} \tag{1.66}$$

When $d_x = \lambda_{\min}/2$, we have $\mu = 1$ in Equation (1.46). Then when $\mu = \hat{\mu}$, it is equivalent to $d_x = \hat{\mu}\lambda_{\min}/2$, i.e. the inter-element spacing is increased by $\hat{\mu} > 1$. As a result, the steered beam response at $\theta \in [-\pi/2\ \pi/2]$ will be the same as the response of the original broadside design at $\ddot{\theta} \in [\arcsin((-1 - \sin\theta_0)/\hat{\mu})\ \pi/2]$ with the inter-element spacing increased by $\hat{\mu}$ and subject to a nonlinear mapping between θ and $\ddot{\theta}$ given in Equation (1.65).

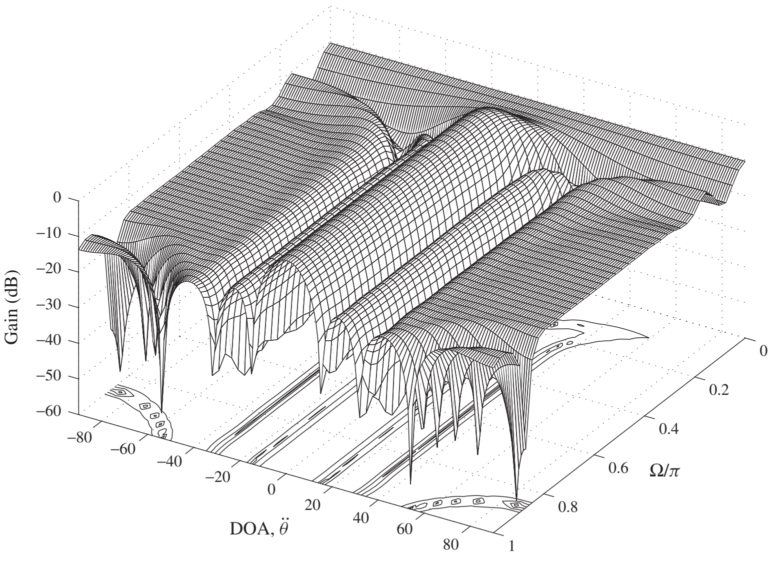

Figure 1.11 The resultant beam pattern with the inter-element spacing increased by 1.5, given the same set of coefficients for obtaining Figure 1.9

Now for the example of $\theta_0 = -30^\circ$, we have $\hat{\mu} = 1.5$ and $\arcsin((-1 - \sin\,\theta_0)/\hat{\mu}) \approx -20^\circ$. We can draw the response of Equation (1.66) given the same set of coefficients for the example of Figure 1.9. The result is shown in Figure 1.11. Compared to the beam pattern of Figure 1.10, we can see a clear match between Figure 1.10 and that of Figure 1.11 for $\ddot{\theta} \in [-20^\circ\ 90^\circ]$, taking into consideration the nonlinear mapping effect of the sinusoidal function.

1.5 Summary

In this chapter, we have given a brief introduction to array signal processing and in particular narrowband beamforming, including how to calculate its beam pattern and obtain a desired beamformer using existing FIR filter design techniques. We then extend the narrowband beamforming structure to the wideband case by considering the need of forming a series of frequency dependent weight coefficients, which can be realized by tapped delay-lines or FIR/IIR filters. Another possibility is to employ sensor delay-lines for wideband beamforming, which will be the topic of Chapter 7.

A detailed analysis is provided at the end for the beam steering process in both narrowband and wideband beamforming. It is shown that unlike the narrowband case, where the steered beam response is a circularly shifted version of the original one given a half wavelength spacing, a more complicated relationship exists for wideband beamformers and a unified interpretation is provided for an easy understanding by considering a wideband beamformer with an increased inter-element spacing.