# 1

# Nature of Variability



There is no engineering product so simple that only one source of variability affects its dimensions or properties. Take two examples of products which are relatively simple in their physical appearance – high-carbon steel wire and milk bottles.

The tensile strength of steel wire depends on numerous factors: the carbon content of the ingot from which rods were made in the rolling mill; the temperature of the heat treatment furnace through which the rods were passed; the rate of passage through the furnace; the temperature of the quenching bath; the ambient temperature in the heat treatment shop; the number of dies through which the rods were drawn to finished wire size; the rate of drawing; the ambient temperature in the wire mill, etc. Variability in any of these factors is likely to generate variability in tensile strength.

One of the hazards of a milkman's life is the possibility of being stopped in the street by a weights and measures inspector. Milk bottles are filled to a predetermined level on automatic machines. The capacity at that level is determined by the external profile of the bottle and by its wall thickness. The bottles are made on multi-head automatic machines by dropping gobs of molten glass into metal moulds (one at each work station), piercing them hollow, then inflating them with compressed air until they fill the moulds. The external profile can be affected by different settings at each work station, by mould differences, by fluctuations in air pressure, by sagging after release from the moulds, and by malfunctioning of the automatic timing gear which

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controls the various functions. The wall thickness is determined by the setting of the gob feeder and this in its turn is affected by the viscosity of the glass, the forehearth temperature, and the action of the shears which cut off successive gobs from the continuous flow of the feeder. Variability in any of these process factors may contribute to variability in the volumetric capacity of bottles in continuous production.

It must be assumed that most engineering products which are infinitely more complex than steel wire or milk bottles will be equally susceptible to a multitude of factors located in raw materials, components, processes and the environment which are capable of affecting the properties and dimensions of a finished product. It is therefore important for engineers to have an understanding of the way in which random combinations of independent sources can affect the variability of a finished product.

This can be demonstrated with random combinations of the variables $R$, $Y$ and $B$ in Table 1.1. These single-digit numbers in the range 0–9 were generated by throwing twenty-faced icosahedron coloured dice (red, yellow and blue) with the numbers zero to nine engraved twice on each die. The dice were invented in 1950/60 by Mr Yasushi Ishida and patented by Tokyo-Shibaura Electric Company. They were marketed and distributed by the Japanese Standards Association for demonstrating the principles of statistical quality control. In the discussion that follows the data in Table 1.1 will be used to demonstrate some of the phenomena of variability that are encountered in engineering data without resort to the mathematics of probability theory. It is hoped this will help the reader to understand the relevance of statistical methods to be described later.

**Table 1.1** Dice scores

| $R$ | $Y$ | $B$ | $R+Y+B$ | Mean | Range | $R \times Y$ |
|---|---|---|---|---|---|---|
| 0 | 6 | 5 | 11 | | | 0 |
| 0 | 8 | 9 | 17 | | | 0 |
| 4 | 6 | 5 | 15 | 13.8 | 6 | 24 |
| 7 | 0 | 6 | 13 | | | 0 |
| 9 | 4 | 0 | 13 | | | 36 |
| 1 | 9 | 4 | 14 | | | 9 |
| 7 | 0 | 3 | 10 | | | 0 |
| 7 | 3 | 6 | 16 | 12.2 | 9 | 21 |
| 2 | 4 | 1 | 7 | | | 8 |
| 1 | 9 | 4 | 14 | | | 9 |

Continued for one hundred trials

One hundred trials were conducted, but only the first ten are recorded in the table.

Readers who are not convinced that the trials are properly reported are at liberty to conduct their own time-consuming experiments. Also recorded in the table are the sums $R + Y + B$, and the products $R \times Y$, along with the *mean* and the *range* of groups of five. In statistical terms, the mean of a set of data is the sum of the individuals divided by the number of individuals. The range is the difference between the largest and smallest individuals.

The *frequency distributions* are as follows;

| $R$, $Y$ and $B$ | *Frequency* | $R + Y + B$ | *Frequency* |
|---|---|---|---|
| 0 | 30 | 0, 1 | 0 |
| 1 | 38 | 2, 3 | 1 |
| 2 | 20 | 4, 5 | 2 |
| 3 | 38 | 6, 7 | 7 |
| 4 | 29 | 8, 9 | 12 |
| 5 | 31 | 10, 11 | 15 |
| 6 | 29 | 12, 13 | 24 |
| 7 | 32 | 14, 15 | 17 |
| 8 | 21 | 16, 17 | 4 |
| 9 | 32 | 18, 19 | 9 |
| | | 20, 21 | 5 |
| | | 22, 23 | 3 |
| | | 24, 25 | 1 |
| | | 26, 27 | 0 |

These can be represented graphically in Figures 1.1 and 1.2.

In a perfect world one might expect Figure 1.1 to display 30 scores in each of the 10 categories 0–9, but the bar chart (or *histogram*, to use a statistical term) shows some degree of irregularity. If bias was suspected it would be necessary to run a much more extensive series of trials to show whether the dice were loaded in favour of scores 1 and 3 at the expense of scores 2 and 8. In the absence of such evidence it can be assumed that the scoring conforms to a rectangular distribution and that the irregularity is no more than is commonly encountered in real life collections of data.

In sharp contrast, the bar chart for the sum of the three colours (Figure 1.2) shows an entirely different pattern of distribution. There is a marked central tendency around a mean score of 13.5 which is easy to explain. All possible combinations of scores on the three dice are equally likely. There are many

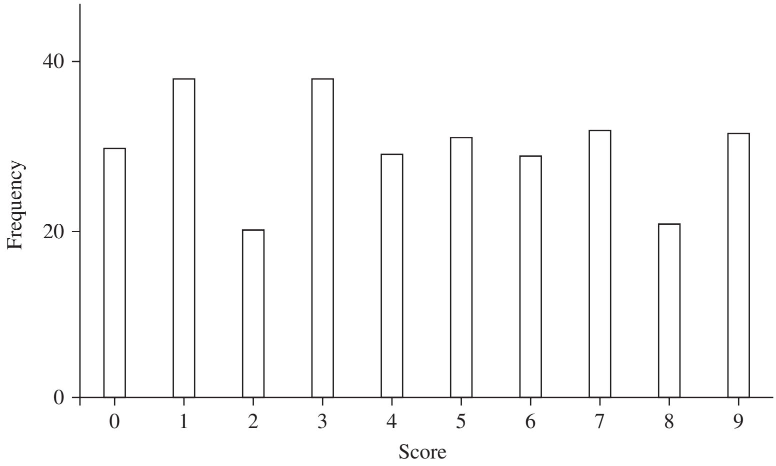


**Figure 1.1** Individual dice scores

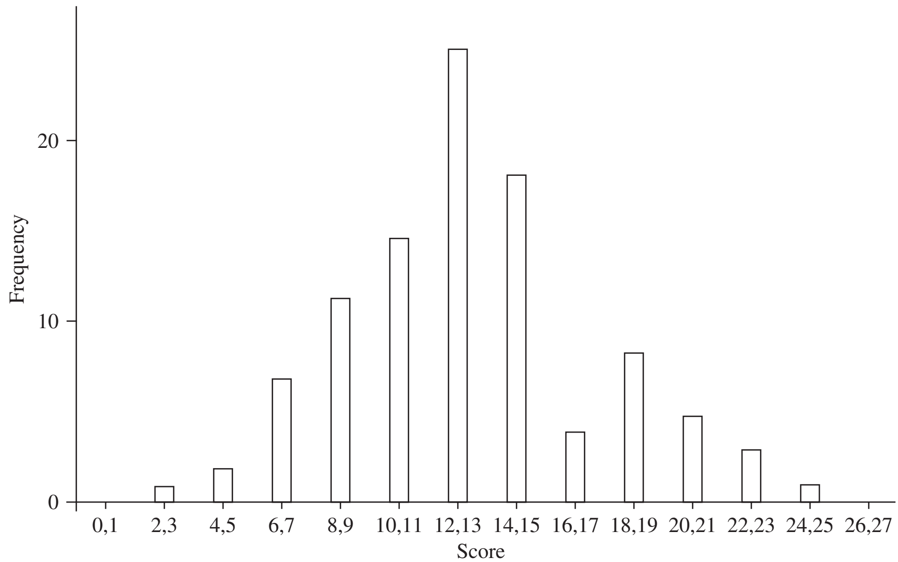


**Figure 1.2** $R + Y + B$ dice scores

different combinations, yielding totals of 10, 11, 12, 13, 14 or 15, but very few which can yield extreme values of 0, 1, 2, 3 or 24, 25, 26, 27. In fact there is only one combination $0+0+0$ which could yield 0 and only one other combination $9+9+9$ which could yield 27, and neither occurred in this relatively small set of trials.

Symmetrical bell-shaped distributions exhibiting a central tendency are commonplace in engineering data. It is not unreasonable to argue these are indicative of random combination of independent factors contributing to the variability of the data and to suggest that analytical statistical methods might be used to identify and control them.

However, it must not be assumed that other patterns of distribution will not occur in engineering data. The distribution of products of red and yellow scores, $R \times Y$ is highly *skewed* (i.e. asymmetric) as shown in Figure 1.3.

Skewed distributions do occur in engineering when the effect of a contributory factor is one-sided. For example, in a thermionic valve electrons are emitted from the heated cathode and are attracted by a positive voltage on the anode. They have to pass through the grid (a helix of fine wire) to which a

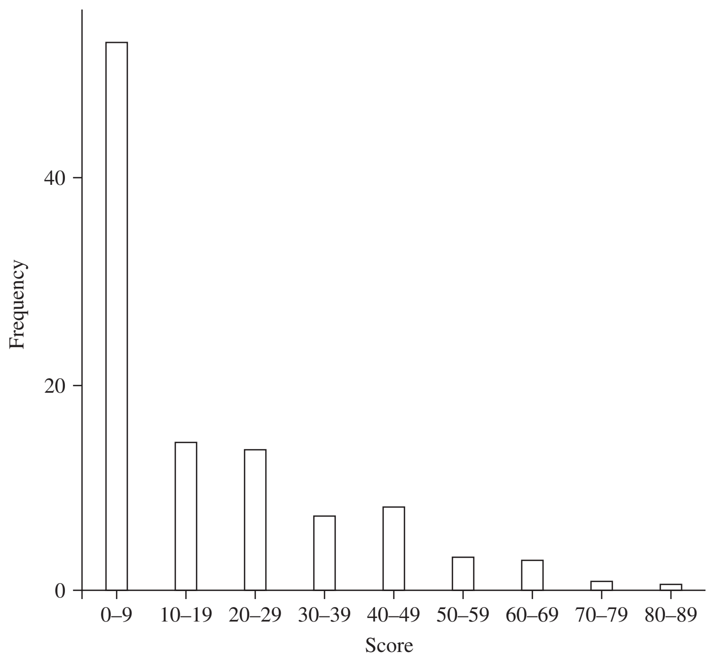


**Figure 1.3** $R \times Y$ dice scores

negative voltage is applied to control the current. Any lack of uniformity in the grid helix can only increase, not reduce, the anode current. Again, in a cylindrical mechanical product zero ovality is the ultimate degree of perfection. Any finite degree of ovality is positive if it is regarded as the excess of the major diameter over the minor without regard to orientation. In such circumstances skewed data distributions are inevitable.

Fortunately statistical methods are available which are not confined exclusively to data that conforms to a symmetrical distribution. When skewed distributions are encountered in engineering data they can often be handled more easily by making a logarithmic transformation of the data.

The data in Table 1.1 can be used to demonstrate relationships between *samples* and *populations*. This is a matter of considerable importance to engineers who often have to draw valid conclusions from quite small samples of data. For example, in the early stages of development of a new product it is necessary to check measurements of a few prototypes to determine whether the population will be on target and whether the (unavoidable) spread of variability will lie comfortably within specification tolerance limits. In this instance the prototype data can be treated as a sample from a population that does not yet exist, yet a prediction has to be made.

This situation is simulated in the third and fourth columns of Table 1.1 by taking the mean value and range of $R+Y+B$ scores in successive groups of five trials. This resulted in the following 20 mean values, not one of which coincided with the mean of the original set of $R+Y+B$ scores (12.9). The nearest was 13.2, but the extreme examples were 10.2 and 15.2. Clearly, there were many instances in which the sample mean would not have given a good estimate of the population mean.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 13.8 | 12.2 | 13.4 | 11.2 | 12.2 | 11.2 | 12.0 | 14.0 | 10.8 | 15.0 |
| 13.8 | 14.4 | 15.2 | 12.2 | 14.0 | 12.2 | 13.2 | 14.2 | 13.2 | 10.2 |

The range of $R+Y+B$ scores over each group of five trials gave the following results.

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 9 | 13 | 16 | 10 | 15 | 4 | 9 | 8 | 12 | 15 | 19 | 12 | 5 | 13 | 7 | 9 | 17 | 13 | 7 |

If the range is taken as a crude measure of overall variability (as many development engineers have been known to do in the past when writing specification tolerances) it is clear that not even the highest value (19) recorded in this set of trials would embrace the span of the distribution shown in Figure 1.2. Most of the others would fall very far short of this requirement.

The relatively small sets of data used by engineers at the development stage of a new product can be regarded as samples from a population which will exist when full-scale production starts. The discrepancies in mean value and variability which can exist between a sample, and the population from which it is drawn, identify a serious hazard along the road from design, through development to production of manufactured products. It is to be hoped that the straightforward demonstration of the risks given above will alert engineers to the dangers and persuade them to listen more carefully to the advice of statisticians, or (better still) develop some statistical skill on their own account. So, if range is not to be regarded as a satisfactory measure of overall variability what else can we do? Consider the following small set of data:

| | | | | |
|---|---|---|---|---|
| 16 | 18 | 16 | 10 | 14 |

The location of the data on a scale of measurement can be identified by calculating the mean value.

$$(16 + 18 + 16 + 10 + 14)/5 = 74/5 = 14.8$$

The *deviates* of the individuals from the mean are

$$\begin{aligned} 16.0 - 14.8 &= \phantom{-}1.2 \\ 18.0 - 14.8 &= \phantom{-}3.2 \\ 16.0 - 14.8 &= \phantom{-}1.2 \\ 10.0 - 14.8 &= -4.8 \\ 14.0 - 14.8 &= -0.8 \end{aligned}$$

The sum of these deviates, taking account of positive and negative signs, will be zero. Suppose we square them before adding them together?

$$\begin{aligned} 1.2^2 + 3.2^2 + 1.2^2 + (-4.8)^2 + (-0.8)^2 &= 1.44 + 10.24 + 1.44 + 23.04 + 0.64 \\ &= 36.80 \end{aligned}$$

This *sum of squares* is a powerful overall measure of variability which gives equal weight to all of the individuals, not just the extreme values. It does, however, respond to the size of the data. If data from the same source had 10 values the sum of squares would be (roughly) twice as large.

This can be overcome by dividing the sum of squares by the number of individuals to give a *mean square*:

$$\frac{36.80}{5} = 7.36$$

In some situations the divisor should be one less than the number of individuals, but more of that later in Section 2.2!

Summing squares to measure variability is the foundation on which statistical analysis is built. In modern usage 'statistics' implies much more than simply recording events. In the hands of a competent engineer statistical analysis is a powerful tool which should not be neglected. Now read on!