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General Introductory Aspects for Thermal Engineering

1.1 Introduction

Thermal energy storage (TES) is one of the key technologies for energy conservation, and therefore, it is of great practical importance. One of its main advantages is that it is best suited for heating and cooling thermal applications. TES is perhaps as old as civilization itself. Since recorded time, people have harvested ice and stored it for later use. Large TES systems have been employed in more recent history for numerous applications, ranging from solar hot water storage to building air conditioning systems. The TES technology has only recently been developed to a point where it can have a significant impact on modern technology.

In general, a coordinated set of actions has to be taken in several sectors of the energy system for the maximum potential benefits of thermal storage to be realized. TES appears to be an important solution to correcting the mismatch between the supply and demand of energy. TES can contribute significantly to meeting society's needs for more efficient, environmentally benign energy use. TES is a key component of many successful thermal systems, and a good TES should allow little thermal losses, leading to energy savings, while permitting the highest reasonable extraction efficiency of the stored thermal energy.

There are mainly two types of TES systems, that is, sensible (e.g., water and rock) and latent (e.g., water/ice and salt hydrates). For each storage medium, there is a wide variety of choices depending on the temperature range and application. TES via latent heat has received a great deal of interest. Perhaps, the most obvious example of latent TES is the conversion of water to ice. Cooling systems incorporating ice storage have a distinct size advantage over equivalent capacity chilled water units because of the ability to store large amount of energy as latent heat. TES deals with the storing of energy, usually by cooling, heating, melting, solidifying, or vaporizing a substance, and the energy becomes available as heat when the process is reversed. The selection of a TES is mainly dependent on the storage period required, that is, diurnal or seasonal, economic viability, operating conditions, and so on. In practice, many research and development activities related to energy have concentrated on efficient energy use and energy savings, leading to energy conservation. In this regard, TES appears to be an attractive thermal application. Furthermore, exergy analysis is an important tool for analyzing TES performance.

We begin this chapter with a summary of fundamental definitions, physical quantities, and their units, dimensions, and interrelations. We consider introductory aspects of thermodynamics, fluid flow, heat transfer, energy, entropy, and exergy.

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1.2 Systems of Units

There are two main systems of units: the *International System of Units* (*Le Systéme International d'Unités*), which is normally referred to as *SI units*, and the *English System of Units*. SI units are used most widely throughout the world, although the English System is traditional in the United States. In this book, SI units are primarily employed. Note that the relevant unit conversions and relationships between the International and English unit systems concerning fundamental properties and quantities are listed in Appendix A.

1.3 Fundamental Properties and Quantities

In this section, we briefly cover several general aspects of thermodynamics to provide adequate preparation for the study of TES systems and applications.

1.3.1 Mass, Time, Length, and Force

Mass is defined as a quantity of matter forming a body of indefinite shape and size. The fundamental unit of mass is the kilogram (kg) in SI units and the pound mass ($\mathrm{lb_m}$) in English units. The basic unit of time for both unit systems is the second.

In thermodynamics, the unit *mole* (mol) is commonly used and defined as a certain amount of a substance as follows:

$$n = \frac{m}{M} \tag{1.1}$$

where n is the number of moles, m is the mass, and M is the molecular weight. If m and M are given in units of gram and gram per mole, we obtain n in moles. For example, one mole of water, having a molecular weight of 18 (compared to 12 for carbon-12), has a mass of 0.018 kg.

The basic unit of length is the meter (m) in SI units and the foot (ft) in the English system.

A force is a kind of action that brings a body to rest or changes its speed or direction of motion (e.g., a push or a pull). The fundamental unit of force is the Newton (N).

The four aspects, for example, mass, time, length, and force, are related by the Newton's second law of motion, which states that the force acting on a body is proportional to the mass and the acceleration in the direction of the force, as given in Equation 1.2:

$$F = ma \tag{1.2}$$

Equation 1.2 shows the force required to accelerate a mass of one kilogram at a rate of one meter per second per second as $1\,\mathrm{N} = 1\,\mathrm{kg\,m/s^2}$.

It is important to note that the value of the earth's gravitational acceleration is $9.80665\,\mathrm{m/s^2}$ in the SI system and $32.174\,\mathrm{ft/s^2}$ in the English system, and it indicates that a body falling freely toward the surface of the earth is subject to the action of gravity alone.

1.3.2 Pressure

When we deal with liquids and gases, pressure becomes one of the most important quantities. Pressure is the force exerted on a surface, per unit area, and is expressed in bar or Pascal (Pa). The related expression is

$$P = \frac{F}{A} \tag{1.3}$$

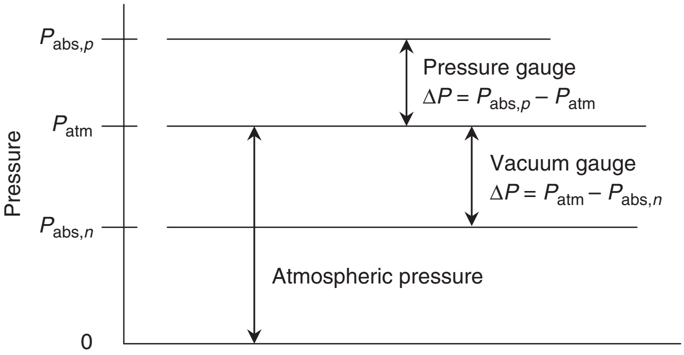

Figure 1.1 Illustration of pressures for measurement

The SI unit for pressure is the force of one Newton acting on a square meter area (or the *Pascal*). The unit for pressure in the English system is pound-force per square foot, lb_f/ft^2.

Here, we introduce basic pressure definitions, and a summary of basic pressure measurement relationships is shown in Figure 1.1.

Atmospheric Pressure The atmosphere that surrounds the earth can be considered a reservoir of low-pressure air. Its weight exerts a pressure which varies with temperature, humidity, and altitude. Atmospheric pressure also varies from time to time at a single location because of the movement of weather patterns. While these changes in barometric pressure are usually less than one-half inch of mercury, they need to be taken into account when precise measurements are required.

Gauge Pressure The *gauge pressure* is any pressure for which the base for measurement is atmospheric pressure expressed as kPa (gauge). Atmospheric pressure serves as a reference level for other types of pressure measurements, for example, gauge pressure. As shown in Figure 1.1, the gauge pressure is either positive or negative depending on its level above or below atmospheric level. At the level of atmospheric pressure, the gauge pressure becomes zero.

Absolute Pressure A different reference level is utilized to obtain a value for absolute pressure. The absolute pressure can be any pressure for which the base for measurement is a complete vacuum, and is expressed in kPa (absolute). Absolute pressure is composed of the sum of the gauge pressure (positive or negative) and the atmospheric pressure as follows:

$$pressure\ (gauge) + atmospheric\ pressure = pressure\ (absolute) \tag{1.4}$$

For example, to obtain the absolute pressure, we simply add the value of atmospheric pressure to gauge pressure. The absolute pressure is the most common one used in thermodynamic calculations, despite the fact that what is read by most pressure gauges and indicators is the pressure difference between the absolute pressure and the atmospheric pressure existing in the gauge.

Vacuum A vacuum is a pressure lower than atmospheric pressure and occurs only in closed systems, except in outer space. It is also called *negative gauge pressure*. In fact, a vacuum is the pressure differential produced by evacuating air from the closed system. A vacuum is usually divided into four levels: (i) low vacuum representing pressures above 1 Torr absolute (a large number of mechanical pumps in industries are used for this purpose; flow is viscous), (ii) medium vacuum varying between 1 and 10^{-3} Torr absolute (most pumps serving this range are mechanical; fluid is in transition between viscous and molecular phases), (iii) high vacuum ranging between 10^{-3} and 10^{-6} Torr absolute (nonmechanical ejector or cryogenic pumps are used; flow is molecular or Newtonian), and (iv) very high vacuum representing absolute pressure below 10^{-6} Torr (primarily for laboratory applications and space simulation).

It is important to note an additional term, the *saturation pressure*, which is the pressure of a liquid or vapor at saturation conditions.

1.3.3 Temperature

Temperature is an indication of the thermal energy stored in a substance. In other words, we can identify hotness and coldness with the concept of temperature. The temperature of a substance may be expressed in either relative or absolute units. The two most common temperature scales are Celsius (°C) and Fahrenheit (°F). The Celsius scale is used with the SI unit system and the Fahrenheit scale with the English system of units. There are two additional scales, the Kelvin scale (K) and the Rankine scale (R), which are absolute temperature scales and are often employed in thermodynamic applications.

The degree Kelvin is a unit of temperature measurement; zero kelvin (0 K) is absolute zero and is equal to −273.15 °C. Increments of temperature in units of K and °C are equal. For instance, when the temperature of a product is decreased to −273.15 °C (or 0 K), known as *absolute zero*, the substance contains no thermal energy and all molecular movement stops.

Temperature can be measured in a large number of ways by devices. In general, the following devices are commonly used:

- **Thermometers.** Thermometers contain a volume of fluid which expands when subjected to heat, thereby raising its temperature. In practice, thermometers work over a certain temperature range. For example, the common thermometer fluid, mercury, becomes solid at −38.8 °C and its properties change dramatically at that condition.
- **Resistance thermometers.** A resistance thermometer (or detector), also known as a wire-wound thermometer, has great accuracy for wide temperature ranges. The wire used has to have known, repeatable, electrical characteristics so that the relationship between the temperature and resistance value can be predicted precisely. The measured value of the resistance of the detector can then be used to determine the value of an unknown temperature. Among metallic conductors, pure metals exhibit the greatest change of resistance with temperature. For applications requiring higher accuracy, especially where the temperature measurement is between −200 and +800 °C, the resistance thermometer comes into its own. The majority of such thermometers are made of platinum. In industries, in addition to platinum, nickel (−60 to +180 °C) and copper (−30 to +220 °C) are frequently used to manufacture resistance thermometers. Resistance thermometers can be provided with two, three, or four wire connections, and for higher accuracy at least three wires are required.
- **Averaging thermometers.** An averaging thermometer is designed to measure the average temperature of bulk stored liquids. The sheath contains a number of elements of different lengths, all starting from the bottom of the sheath, The longest element that is fully immersed is connected to the measuring circuit to allow a true average temperature to be obtained. For this type of thermometer, several parameters are significant, namely, sheath material (stainless steel for the temperature range from −50 to +200 °C or nylon for the temperature range from −50 to +90 °C), sheath length (to suit the application), termination (flying leads or terminal box), element length, element calibration (to copper or platinum curves), and operating temperature

ranges. In many applications, where a multielement thermometer is not required, such as in air ducts, cooling water and gas outlets, a single-element thermometer stretched across the duct or pipe work can provide a true average temperature reading. Despite the working range from 0 to 100 °C, the maximum temperature may reach 200 °C. To maintain high accuracy, these units are normally supplied with three-wire connections. However, up to 10 elements can be mounted in the averaging bulb fittings, and they can be made of platinum, nickel, or copper, and fixed at any required position.

- **Thermocouples.** A thermocouple consists of two electrical conductors of different materials connected together at one end (the so-called *measuring junction*). The two free ends are connected to a measuring instrument, for example, an indicator, a controller, or a signal conditioner, by a reference junction (the so-called *cold junction*). The thermoelectric voltage appearing at the indicator depends on the materials of which the thermocouple wires are made and on the temperature difference between the measuring junction and the reference junction. For accurate measurements, the temperature of the reference junction must be kept constant. Modern instruments usually incorporate a cold junction reference circuit and are supplied ready for operation in a protective sheath, to prevent damage to the thermocouple by any mechanical or chemical means. Table 1.1 lists several types of thermocouples along with their maximum absolute temperature ranges. As can be seen in Table 1.1, a copper–constantan thermocouple has an accuracy of ±1 °C, and is often employed for control systems in refrigeration and food processing applications. The iron–constantan thermocouple with its maximum of 850 °C is used in applications in the plastics industry. The chromel–alumel-type thermocouples, with a maximum of about 1100 °C, are suitable for combustion applications in ovens and furnaces. In addition, it is possible to reach about 1600 or 1700 °C using platinum rhodium–platinum thermocouples, which are particularly useful in steel manufacturing. It is worth noting that one advantage that the thermocouple has over most other temperature sensors is that it has a small thermal capacity, and thus a prompt response to temperature changes. Furthermore, its small thermal capacity rarely affects the temperature of the body under examination.
- **Thermistors.** These devices are made of semiconductors and act as thermal resistors with a high (usually negative) temperature coefficient. In use, thermistors are either self-heated or externally heated. Self-heated units employ the heating effect of the current flowing through them to raise and control their temperature and thus their resistance. This operating mode is useful in such devices as voltage regulators, microwave power meters, gas analyzers, flow meters, and automatic volume and power level controls. Externally heated thermistors are well suited for precision temperature measurement, temperature control, and temperature compensation due to the large changes in resistance versus temperature. These are generally used for applications in the range

Table 1.1 Some of the most common thermocouples

Type	Common names	Temperature range (°C)
T	Copper–constantan (C/C)	−250 to 400
J	Iron–constantan (I/C)	−200 to 850
E	Nickel chromium–constantan or Chromel–constantan	−200 to 850
K	Nickel chromium–nickel aluminum or Chromel–alumel (C/A)	−180 to 1100
–	Nickel 18% molybdenum–nickel	0 to 1300
N	Nicrosil–nisil	0 to 1300
S	Platinum 10% rhodium–platinum	0 to 1500
R	Platinum 13% rhodium–platinum	0 to 1500
B	Platinum 30% rhodium–platinum 6% rhodium	0 to 1600

-100 to $+300\,^\circ$C. Despite early thermistors having tolerances of ±20 or ±10%, modern precision thermistors are of a higher accuracy, for example, $\pm 0.1\,^\circ$C (less than ±1%).

- **Digital display thermometers.** A wide range of digital display thermometers, for example, hand-held battery-powered displays and panel-mounted mains or battery units, are available commercially. Displays can be provided for use with all standard thermocouples or platinum resistance thermometers with several digits and 0.1 °C resolution.

It is important to emphasize that before temperature can be controlled, it must be sensed and measured accurately. For temperature measurement devices, there are several potential sources of error, including not only sensor properties but also contamination effects, lead lengths, immersion, heat transfer, and controller interfacing. In temperature control, there are many sources of error that can be minimized by careful consideration of the type of sensor, its working environment, the sheath or housing, extension leads, and the instrumentation. An awareness of potential errors is vital in many applications dealt with in this book. Selection of temperature measurement devices is a complex task and has been discussed only briefly here. It is important to remember the following: “choose the right tool for the right job.”

1.3.4 Specific Volume and Density

The *specific volume* v is the volume per unit mass of a substance, usually expressed in cubic meters per kilogram (m^3/kg) in the SI system and in cubic feet per pound (ft^3/lb) in the English system. The *density* ρ of a substance is defined as the mass per unit volume, and is therefore the inverse of the specific volume:

$$\rho = \frac{1}{v} \tag{1.5}$$

The units of density are kg/m^3 in the SI system and lb/ft^3 in the English system. Specific volume is also defined as the volume per unit mass, and density as the mass per unit volume, that is,

$$v = \frac{V}{m} \tag{1.6}$$

$$\rho = \frac{m}{V} \tag{1.7}$$

Both specific volume and density are intensive properties and are affected by temperature and pressure.

1.3.5 Mass and Volumetric Flow Rates

Mass flow rate is defined as the mass flowing per unit time (kg/s in the SI system and lb/s in the English system). Volumetric flow rates are given in m^3/s in the SI system and ft^3/s in the English system. The following expressions can be written for the flow rates in terms of mass, specific volume, and density:

$$\dot{m} = \dot{V}\rho = \frac{\dot{V}}{v} \tag{1.8}$$

$$\dot{V} = \dot{m}v = \frac{\dot{m}}{\rho} \tag{1.9}$$

1.4 General Aspects of Thermodynamics

In this section, we briefly introduce some general aspects of thermodynamics that are related to energy storage systems and applications.

1.4.1 Thermodynamic Systems

A thermodynamic system is a device or combination of devices that contains a certain quantity of matter. It is important to carefully define a system under consideration and its boundaries. We can define three important types of systems as follows:

- **Closed system.** This is defined as a system across the boundaries of which no material crosses. It, therefore, contains a fixed quantity of matter. In some books, it is also called a *control mass*.
- **Open system.** This is defined as a system in which material (mass) is allowed to cross its boundaries. The term open system is also called a *control volume*.
- **Isolated system.** This is a closed system that is not affected by the surroundings. No mass, heat, or work crosses its boundary.

1.4.2 Process

A process is a physical or chemical change in the properties of matter or the conversion of energy from one form to another. In some processes, one property remains constant. The prefix “iso” is employed to describe such a process, for example, isothermal (constant temperature), isobaric (constant pressure), and isochoric (constant volume).

1.4.3 Cycle

A cycle is a series of thermodynamic processes in which the end-point conditions or properties of the matter are identical to the initial conditions.

1.4.4 Thermodynamic Property

This is a physical characteristic of a substance, which is used to describe its state. Any two properties usually define the state or condition of a substance, from which all other properties can be derived. Some examples are temperature, pressure, enthalpy, and entropy. Thermodynamic properties are classified as intensive properties (independent of the mass, e.g., pressure, temperature, and density) and extensive properties (dependent on the mass, e.g., mass and total volume). Extensive properties on a per unit mass basis, such as specific volume, become intensive properties. Property diagrams of substances can be presented in graphical form and summarize the main properties listed in property tables, for example, refrigerant tables.

1.4.5 Sensible and Latent Heats

It is known that all substances can hold a certain amount of heat; this property is their thermal capacity. When a liquid is heated, its temperature rises to the boiling point. This is the highest temperature that the liquid can reach at the measured pressure. The heat absorbed by the liquid in raising the temperature to the boiling point is called *sensible heat*. The heat required to convert the liquid to vapor at the same temperature and pressure is called *latent heat*. This is the change in enthalpy during a state change (the amount of heat absorbed or rejected at constant temperature at any pressure, or the difference in enthalpies of a pure condensable fluid between its dry saturated state and its saturated liquid state at the same pressure).

1.4.6 Latent Heat of Fusion

Fusion is associated with the melting and freezing of a material. For most pure substances, there is a specific melting/freezing temperature, relatively independent of the pressure. For example, ice begins to melt at 0 °C. The amount of heat required to melt one kilogram of ice at 0 °C to one kilogram of water at 0 °C is called the *latent heat of fusion* of water, and equals 334.92 kJ/kg. The removal of the same amount of heat from one kilogram of water at 0 °C changes it back to ice.

1.4.7 Vapor

A vapor is a gas at or near equilibrium with the liquid phase – a gas under the saturation curve or only slightly beyond the saturated vapor line. *Vapor quality* is theoretically assumed; that is, when vapor leaves the surface of a liquid, it is pure and saturated at the particular temperature and pressure. In actuality, tiny liquid droplets escape with the vapor. When a mixture of liquid and vapor exists, the ratio of the mass of the liquid to the total mass of the liquid and vapor mixture is called the *quality*, and is expressed as a percentage or decimal fraction. *Superheated vapor* is the saturated vapor to which additional heat has been added, raising the temperature above the boiling point. Let us consider a mass m with a quality x. The volume is the sum of the volumes of both the liquid and the vapor, as defined below:

$$V = V_{liq} + V_{vap} \tag{1.10}$$

Equation 1.10 can also be written in terms of specific volumes as

$$mv = m_{liq}v_{liq} + m_{vap}v_{vap} \tag{1.11}$$

Dividing all terms by the total mass yields

$$v = (1-x)v_{liq} + xv_{vap} = v_{liq} + xv_{liq,vap} \tag{1.12}$$

where $v_{liq,vap} = v_{vap} - v_{liq}$.

1.4.8 Thermodynamic Tables

The thermodynamic tables were first published in 1936 as steam tables by Keenan and Keyes, and later in 1969 and 1978, these were revised and republished. The use of thermodynamic tables of many substances ranging from water to refrigerants is very common in process design calculations. In the literature, they are also called either *steam tables* or *vapor tables*. In this book, we will refer to the thermodynamic tables. These tables are normally given as different distinct phases (parts), for example, four different parts for water, such as saturated water, superheated vapor water, compressed liquid water, saturated solid-saturated vapor water; and two distinct parts for R-134a, such as saturated and superheated. Each table is listed according to the values of temperature and pressure, with the remainder containing values of various other thermodynamic parameters such as specific volume, internal energy, enthalpy, and entropy. Normally, when we have values for two independent variables, we may obtain other data from the respective table. In learning how to use these tables, an important point is to specify the state using any two independent parameters. In some design calculations if we do not have the exact values of the parameters, we use interpolation to find the necessary values.

Beyond thermodynamic tables, recently, much attention has been paid to computerized tables for such design calculations. Although computerized tables can eliminate several reading problems for data, they may not provide students with an understanding of the concepts and a good comprehension of the subject. That is why in thermodynamics courses, it is important for the students to know how to obtain thermodynamic data from the appropriate thermodynamic tables. The *Handbook of Thermodynamic Tables* by Raznjevic (1995) is one of the most valuable sources for several solids, liquids, and gaseous substances.

1.4.9 State and Change of State

The state of a system or substance is defined as the condition of the system or substance characterized by certain observable macroscopic values of its properties, such as temperature and pressure. The term *state* is often used interchangeably with the term *phase*, for example, solid phase or gaseous phase of a substance. Each of the properties of a substance in a given state has only one definite value, regardless of how the substance reaches the state. For example, when sufficient heat is added or removed at a certain condition, most substances undergo a state change. The temperature remains constant until the state change is complete. This can be from solid to liquid, liquid to vapor, or vice versa. Figure 1.2 depicts typical examples of ice melting and water boiling.

A clearer presentation of solid, liquid, and vapor phases of water is provided on a temperature–volume (T–v) diagram in Figure 1.3. The constant pressure line ABCD represents the states that water passes through as follows:

- **A–B.** Represents the process where water is heated from the initial temperature to the saturation temperature (liquid) at constant pressure. At point B, the water is a fully saturated liquid with a quality $x = 0$, but no water vapor has formed.
- **B–C.** Represents a constant-temperature vaporization process in which there is only phase change from a saturated liquid to a saturated vapor. As this process proceeds, the vapor quality

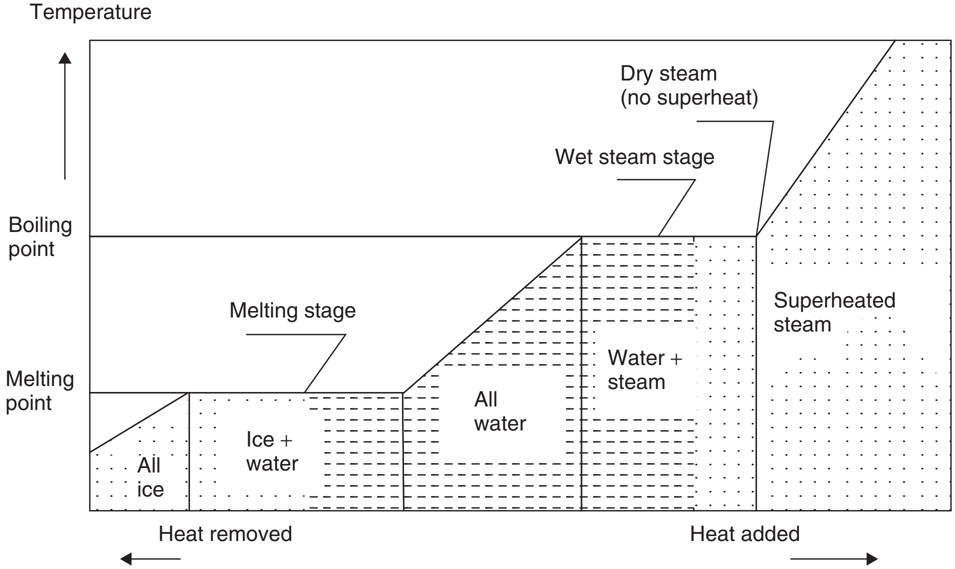

Figure 1.2 The state-change diagram of water

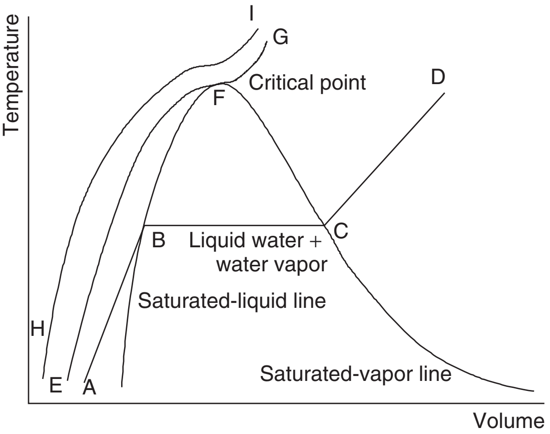

Figure 1.3 Temperature–volume diagram for the phase change of water

varies from 0 to 100%. Within this zone, the water is a mixture of liquid and vapor. At point C, we have a completely saturated vapor and the quality is 100%.

- **C–D.** Represents the constant-pressure process in which the saturated water vapor is superheated with increasing temperature.
- **E–F–G.** Represents a nonconstant-temperature vaporization process. In this constant-pressure heating process, point F is called the *critical point* where the saturated liquid and saturated vapor states are identical. The thermodynamic properties at this point are called *critical thermodynamic properties*, for example, critical temperature, critical pressure, and critical specific volume.
- **H–I.** Represents a constant-pressure heating process in which there is no change from one phase to another (only one is present). However, there is a continuous change in density during this process.

The other process which may occur during melting of water is *sublimation*, in which the ice directly passes from the solid phase to the vapor phase. Another important point is that the solid, liquid, and vapor phases of water may be present together in equilibrium, leading to the *triple point*.

1.4.10 Specific Internal Energy

Internal energy represents a molecular state type of energy. Specific internal energy is a measure per unit mass of the energy of a simple system in equilibrium as a function of $c_v dT$. For many thermodynamic processes in closed systems, the only significant energy changes are internal energy changes, and the significant work done by the system in the absence of friction is the work of pressure–volume expansion, such as in a piston–cylinder mechanism. The specific internal energy of a mixture of liquid and vapor can be written in a form similar to Equation 1.12:

$$u = (1 - x)u_{liq} + xu_{vap} = u_{liq} + xu_{liq,vap} \tag{1.13}$$

where $u_{liq,vap} = u_{vap} - u_{liq}$.

1.4.11 Specific Enthalpy

Enthalpy is another measure of the energy per unit mass of a substance. Specific enthalpy, usually expressed in kJ/kg or Btu/lb, is normally expressed as a function of $c_p dT$. Since enthalpy is a state function, it is necessary to measure it relative to some reference state. The usual practice is to determine the reference values that are called the *standard enthalpy of formation* (or the heat of formation), particularly in combustion thermodynamics. The specific enthalpy of a mixture of liquid and vapor components can be written as Equation 1.12:

$$h = (1 - x)h_{liq} + xh_{vap} = h_{liq} + xh_{liq,vap} \tag{1.14}$$

where $h_{liq,vap} = h_{vap} - h_{liq}$.

1.4.12 Specific Entropy

Entropy is the ratio of the heat added to a substance to the absolute temperature at which it was added, and is a measure of the molecular disorder of a substance at a given state. The specific enthalpy of a mixture of liquid and vapor components can be written as Equation 1.12:

$$s = (1 - x)s_{liq} + xs_{vap} = s_{liq} + xs_{liq,vap} \tag{1.15}$$

where $s_{liq,vap} = s_{vap} - s_{liq}$.

1.4.13 Pure Substance

A pure substance is defined as the one that has a homogeneous and invariable chemical composition. Despite having the same chemical composition throughout, it may be in more than one phase, namely, liquid, a mixture of liquid and vapor (e.g., steam), and a mixture of solid and liquid. Each phase has the same chemical composition. However, a mixture of liquid air and gaseous air cannot be considered a pure substance because the composition of each phase differs from that of the other. A thorough understanding of the pure substance is of significance, particularly for TES applications. Thermodynamic properties of water and steam can be obtained from tables and charts that are present in most thermodynamics books, based on experimental data or real-gas equations of state, or obtained through computer calculations. It is important to note that the properties of low-pressure water are of great significance in TES systems for cooling applications, since water vapor existing in the atmosphere typically exerts a pressure less than 1 psi (6.9 kPa). At such low pressures, it is known that water vapor exhibits ideal-gas behavior.

1.4.14 Ideal Gases

In many practical thermodynamic calculations, gases such as air and hydrogen can often be treated as ideal gases, particularly for temperatures much higher than their critical temperatures and for pressures much lower than their saturation pressures at given temperatures. An ideal gas can be described in terms of three parameters: the volume that it occupies, the pressure that it exerts, and its temperature. In fact, all gases or vapors, including water vapor, at very low pressures exhibit ideal-gas behavior. The practical advantage of treating real gases as ideal is that a simple equation of state with only one constant can be applied in the following form:

$$Pv = RT \tag{1.16}$$

and

$$PV = mRT \tag{1.17}$$

The ideal-gas equation of state was originally established from experimental observations, and is also called a *P–v–T relationship* for gases. It is generally considered as a concept rather than a reality. It requires only a few data values to define a particular gas over a wide range of its possible thermodynamic equilibrium states.

The gas constant R is different for each gas depending on its molecular weight M:

$$R = \frac{\overline{R}}{M} \tag{1.18}$$

where $\overline{R}$= 8.314 kJ/kg K.

Equations 1.17 and 1.18 may be written on a mole-basis as follows:

$$P\overline{v} = RT \tag{1.19}$$

and

$$PV = n\overline{R}T \tag{1.20}$$

The other simplifying feature of ideal-gas behavior is that, if assumed that the constant-pressure and constant-volume specific heats are constant, changes in specific internal energy and specific enthalpy can be calculated simply without referring to thermodynamic tables and graphs from the following expressions:

$$\Delta u = (u_2 - u_1) = c_v(T_2 - T_1) \tag{1.21}$$

$$\Delta h = (h_2 - h_1) = c_p(T_2 - T_1) \tag{1.22}$$

The following is another useful relation for ideal gases obtained from the expression, $h = u + Pv = u + RT$:

$$c_v - c_p = R \tag{1.23}$$

For the entire range of states, the ideal-gas model may be found unsatisfactory. Therefore, the compressibility factor (Z) is introduced to measure the deviation of a real substance from the ideal-gas equation of state. The compressibility factor is defined by the relation:

$$Pv = ZRT \quad \text{or} \quad Z = \frac{Pv}{RT} \tag{1.24}$$

Figure 1.4 shows a generalized compressibility chart for simple substances. In the chart, we have two important parameters: the reduced temperature ($T_r = T/T_c$) and the reduced pressure ($P_r = P/P_c$). To calculate the compressibility factor, the values of T_r and P_r should be calculated using the critical temperature and pressure values of the respective substance, which can easily be obtained from thermodynamics books. As can be seen in Figure 1.4, at all temperatures $Z \to 1$ as $P_r \to 0$. This means that the behavior of the actual gas closely approaches ideal-gas behavior, as the pressure approaches zero. For real gases, Z takes on values between 0 and 1. If $Z = 1$, Equation 1.24 becomes Equation 1.16. In the literature, there are also several equations of state for accurately representing the P–v–T behavior of a gas over the entire superheated vapor region, for example, the Benedict–Webb–Rubin equation, the van der Waals equation, and the Redlich and Kwong equation. However, some of these equations of state are complicated due to the number of empirical constants they contain, and are more conveniently used with computer software to obtain results.

There are some special cases if one of P, v, and T is constant. At a fixed temperature, the volume of a given quantity of ideal gas varies inversely with the pressure exerted on it (in some books, this is called *Boyle's law*), describing compression as

$$P_1V_1 = P_2V_2 \tag{1.25}$$

where the subscripts refer to the initial and final states.

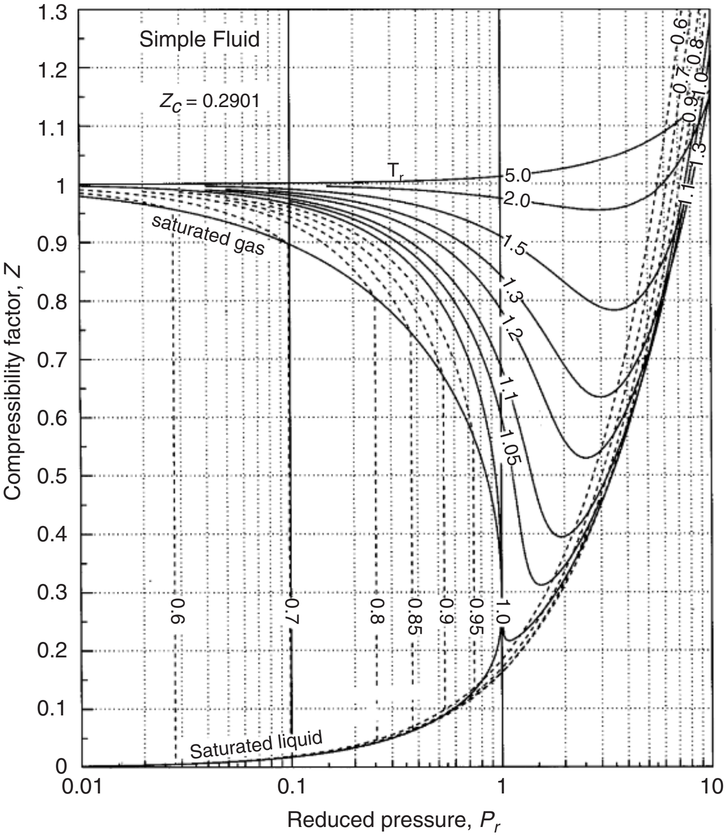

Figure 1.4 Generalized compressibility chart for simple substances (Borgnakke and Sonntag, 2008)

Equation 1.25 is employed by designers in a variety of situations: when selecting an air compressor, for calculating the consumption of compressed air in reciprocating air cylinders, and for determining the length of time required for storing air. Nevertheless, use of Equation 1.25 may not always be practical due to temperature changes. If temperature increases with compression, the volume of a gas varies directly with its absolute temperature in K as:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \tag{1.26}$$

If temperature increases at constant volume, the pressure of a gas varies directly with its absolute temperature in K as:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \tag{1.27}$$

Equations 1.26 and 1.27 are known as *Charles' law*. If both temperature and pressure change at the same time, the combined ideal-gas equation can be written as:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \tag{1.28}$$

For a given mass, the internal energy of an ideal gas can be written as a function of temperature, since c_{v0} is constant, as shown below:

$$dU = mc_{v0}dT \tag{1.29}$$

and the specific internal energy becomes

$$du = c_{v0}dT \tag{1.30}$$

The enthalpy equation for an ideal gas, based on $h = u + Pv$, can be written as

$$dH = mc_{p0}dT \tag{1.31}$$

and the specific enthalpy then becomes

$$dh = c_{p0}dT \tag{1.32}$$

The entropy change of an ideal gas, based on the general entropy equation in terms of $Tds = du + Pdv$ and $T\,ds = dh - v\,dP$ as well as on the ideal-gas equation $Pv = RT$, can be obtained in two ways by substituting Equations 1.29 and 1.30:

$$s_2 - s_1 = c_{v0} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) \tag{1.33}$$

$$s_2 - s_1 = c_{p0} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \tag{1.34}$$

For a reversible adiabatic process, the ideal-gas equation in terms of the initial and final states under $Pv^k =$ constant can be written as:

$$Pv^k = P_1v_1^k = P_2v_2^k \tag{1.35}$$

where k denotes the adiabatic exponent (the *specific heat ratio*) as a function of temperature:

$$k = \frac{c_{p0}}{c_{v0}} \tag{1.36}$$

On the basis of Equation 1.35 and the ideal-gas equation, the following expression can be obtained:

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1} = \left(\frac{v_1}{v_2}\right)^k = \left(\frac{V_1}{V_2}\right)^k \tag{1.37}$$

Consider a closed system containing an ideal gas, undergoing an adiabatic reversible process. The gas has constant specific heats. The work can be derived from the first law of thermodynamics (FLT) as follows:

$$W_{1-2} = \frac{mR(T_2 - T_1)}{1-k} = \frac{(P_2V_2 - P_1V_1)}{1-k} \tag{1.38}$$

Equation 1.38 can also be derived from the general work relation, $W = PdV$.

For a reversible polytropic process, the only difference is the polytropic exponent n which shows the deviation in a log P and log V diagram, leading to the slope. Equations 1.35, 1.37, and 1.38 can be rewritten with the polytropic exponent under $Pv^n =$ constant as:

$$Pv^n = P_1v_1^n = P_2v_2^n \tag{1.39}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{n/n-1} = \left(\frac{v_1}{v_2}\right)^n = \left(\frac{V_1}{V_2}\right)^n \tag{1.40}$$

$$W_{1-2} = \frac{mR(T_2 - T_1)}{1-n} = \frac{(P_2V_2 - P_1V_1)}{1-n} \tag{1.41}$$

To provide a clear understanding of the polytropic exponent, it is important to show the values of n for four types of polytropic processes for ideal gases (Figure 1.5):

- $n = 0$ for isobaric process (P = constant)
- $n = 1$ for isothermal process (T = constant)
- $n = k$ for isentropic process (s = constant)
- $n = \infty$ for isochoric process (v = constant)

As can be seen in Figure 1.5, there are two quadrants where n varies from zero to infinity and where it has a positive value. The slope of any curve is an important consideration when a reciprocating engine or compressor cycle is under consideration.

In thermodynamics, a number of problems involve the mixture of pure substances (e.g., ideal gases). In this regard, it is important to understand related aspects accordingly. Tables 1.2 and 1.3 summarize the relevant expressions and two ideal-gas models: the Dalton model and the Amagat model. In the comparison presented, it is assumed that each gas is unaffected by the presence of other gases, and each is treated as an ideal gas. With respect to entropy, it is noted that an increase in entropy is dependent only upon the number of moles of ideal gases, and is independent of the chemical composition. Of course, when the gases in the mixture are distinguished, the entropy increases.

1.4.15 Energy Transfer

Energy can be viewed as the capacity for doing work. Energy can take a number of forms during transfer such as thermal (heat), mechanical (work), electrical, and chemical. Thermal energy flows only from a higher to a lower temperature level unless external energy is added to reverse the process. The rate of energy transfer per unit time is called *power*.

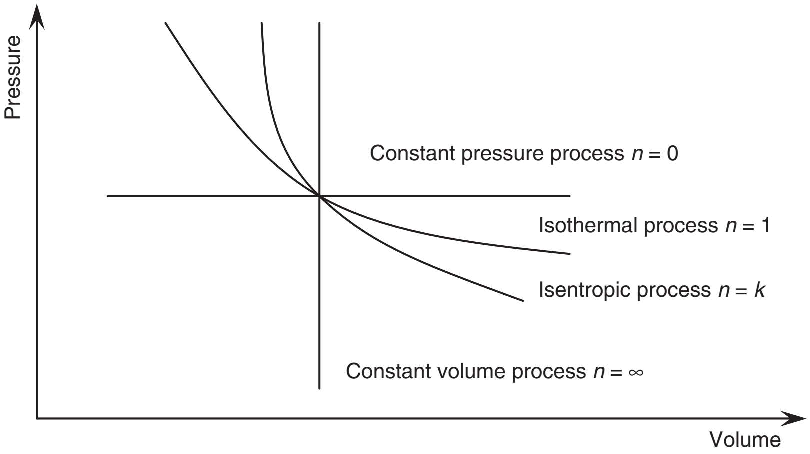

Figure 1.5 Representation of four polytropic processes on a pressure–volume diagram

Table 1.2 Equations for gas and gas mixtures and relevant models

Definition	Dalton model and Amagat model
Total mass of a mixture of N components	$m_{tot} = m_1 + m_2 + \ldots + m_N = \sum m_i$
Total number of moles of a mixture of N components	$n_{tot} = n_1 + n_2 + \ldots + n_N = \sum n_i$
Mass fraction for each component	$c_i = m_i / m_{tot}$
Mole fraction for each component	$y_i = n_i / n_{tot} = P_i / P_{tot} = V_i / V_{tot}$
Molecular weight of the mixture	$M_{mix} = m_{tot}/n_{tot} = \sum n_i M_i / n_{tot} = \sum y_i M_i$
Internal energy of the mixture	$U_{mix} = n_1 \overline{U_1} + n_2 \overline{U_2} + \ldots + n_N \overline{U_N} = \sum n_i \overline{U_i}$
Enthalpy of the mixture	$H_{mix} = n_1 \overline{H_1} + n_2 \overline{H_2} + \ldots + n_N \overline{H_N} = \sum n_i \overline{H_i}$
Entropy of the mixture	$S_{mix} = n_1 \overline{S_1} + n_2 \overline{S_2} + \ldots + n_N \overline{S_N} = \sum n_i \overline{S_i}$
Entropy difference for the mixture	$S_2 - S_1 = -\overline{R}(n_1 \ln y_1 + n_2 \ln y_2 + \ldots + n_N \ln y_N)$

Table 1.3 Comparison of Dalton and Amagat models

Definition	Dalton model	Amagat model
P, V, T for the mixture	T and V are constant $P_{\text{tot}} = P = P_1 + P_2 + \ldots + P_N$	T and P are constant $V_{\text{tot}} = V = V_1 + V_2 + \ldots + V_N$
Ideal-gas equation for the mixture	$PV = n\overline{R}T$	
Ideal-gas equations for the components	$P_1 V = n_1 \overline{R}T$ $P_2 V = n_2 \overline{R}T$ $\vdots$ $P_N V = n_N \overline{R}T$	$PV_1 = n_1 \overline{R}T$ $PV_2 = n_2 \overline{R}T$ $\vdots$ $PV_N = n_N \overline{R}T$

1.4.16 Heat

The definitive experiment that showed that heat was a form of energy convertible into other forms was carried out by a Scottish physicist, James Joule. Heat is the thermal form of energy, and heat transfer takes place when a temperature difference exists within a medium or between different media. Heat always requires a difference in temperature for its transfer. Higher temperature differences provide higher heat transfer rates. The units for heat are joules or kilojoules in the International system and the foot pound-force or British thermal unit (Btu) in the English system. Following a common convention in thermodynamic calculations, heat transfer to a system is considered *positive*, while heat transfer from a system is *negative*. If there is no heat transfer involved in a process, it is called *adiabatic*.

1.4.17 Work

Work is the energy that is transferred by a difference in pressure or force of any kind and is subdivided into shaft work and flow work. Shaft work is the mechanical energy used to drive a mechanism such as a pump, compressor, or turbine. Flow work is the energy transferred into a system by fluid flowing into, or out of, the system. Both forms are usually expressed in kilojoules and on a unit mass basis as kJ/kg. By usual convention, work done by a system is considered positive and work done on a system (work input) is considered negative. The unit for power or rate of work is joule per second, which is a Watt (W).

1.4.18 The First Law of Thermodynamics

Thermodynamics is the science of energy and entropy, and the basis of thermodynamics is experimental observation. In thermodynamics, such observations were formed into four basic laws: the zeroth, first, second, and third laws of thermodynamics. The first and second laws of thermodynamics are the most common tools in practice due to fact that transfers and conversions of energy are governed by these two laws, and in this chapter, we focus on these two laws.

The FLT can be defined as the law of conservation of energy and states that in a closed system energy can be neither created nor destroyed. For a change of state from an initial state 1 to a final state 2 with a constant amount of matter, the first law can be formulated as follows:

$$Q_{1-2} = (E_2 - E_1) + W_{1-2} = (U_2 - U_1) + (\mathrm{KE}_2 - \mathrm{KE}_1) + (\mathrm{PE}_2 - \mathrm{PE}_1) + W_{1-2} \qquad (1.42)$$

where $(U_2 - U_1) = mc_v(T_2 - T_1)$, $(\mathrm{KE}_2 - \mathrm{KE}_1) = m(V_2^2 - V_1^2)/2$, $(\mathrm{PE}_2 - \mathrm{PE}_1) = mg(Z_2 - Z_1)$.

As is clear in Equation 1.42, we broaden the definition of energy to include kinetic and potential energies in addition to internal energy. An important consequence of the first law is that the internal energy change resulting from a process is independent of the thermodynamic path followed by the system, and of the paths followed by the processes, for example, heat transfer and work. In turn, the rate at which the internal energy content of the system changes is dependent only on the rates at which heat is added and work is done (when kinetic and potential energies are neglected).

1.4.19 The Second Law of Thermodynamics

As mentioned earlier, the first law is the energy-conservation principle. The second law of thermodynamics (SLT) is instrumental in determining the inefficiencies of practical thermodynamic systems, and indicates that it is impossible to have 100% efficiency in energy conversion. The classical statements, such as the Kelvin–Plank statement and the Clausius statement, help us formulate the second law:

- **The Kelvin–Plank statement.** It is impossible to construct a device operating in a cycle (e.g., heat engine), that accomplishes only the extraction of heat from some source and its complete conversion to work. This statement describes the impossibility to have a heat engine with a thermal efficiency of 100%.
- **The Clausius statement.** It is impossible to construct a device operating in a cycle (e.g., refrigerator and heat pump), that transfers heat from a low-temperature (cooler) region to a high-temperature (hotter) region.

A simple way to illustrate the implications of both the first and second laws is a desktop game that consists of several pendulums (made of metal balls), one in contact with the other. When you raise the first of the balls, you give energy to the system in the form of potential energy. Releasing this ball allows it to gain kinetic energy at the expense of potential energy. When this ball hits the second ball, a small elastic deformation transforms the kinetic energy again into a form of potential energy. The energy is transferred from one ball to the other. The last ball again gains kinetic energy, which allows it to rise. The cycle continues, with the ball rising every time to a slightly lower level, until it finally stops. The first law concerns why the balls keep moving, while the second law explains why they do not do it forever. In this game, the energy is lost in the form of sound and heat, as the motion declines.

The second law also states that the entropy in the universe always increases. As mentioned before, entropy is a measure of degree of disorder, and every process happening in the universe increases the entropy of the universe to a higher level. The entropy of a state of a system is proportional to (depends on) its probability, which gives us an opportunity to define the second law in a broader manner as "the entropy of a system increases in any heat transfer or conversion of energy within a closed system." That is why all energy transfers or conversions are irreversible. From the entropy perspective, the basis of the second law is the statement that the sum of the entropy of a system changes and that of its surroundings must always be positive. Recently, much effort has been invested in minimizing the entropy generation (irreversibilities) in thermodynamic systems and applications.

Moran and Shapiro (2007) noted that the second law and deductions from it are useful because they provide a means for

- predicting the direction of processes;
- establishing conditions for equilibrium;
- determining the best performance of thermodynamic systems and applications;
- quantitatively evaluating the factors that preclude the attainment of the best theoretical performance level;
- defining a temperature scale, independent of the properties of the substance; and
- developing tools for evaluating some thermodynamic properties, for example, internal energy and enthalpy, using available experimental data.

Consequently, the second law is the linkage between entropy and the usefulness of energy. The second law analysis has found applications in a wide variety of disciplines, for example, chemistry, economics, ecology, environment, and sociology, far removed from engineering thermodynamics applications.

1.4.20 Reversibility and Irreversibility

These two concepts are highly important to thermodynamic processes and systems. *Reversibility* is defined by the statement that only for a reversible process can both a system and its surroundings be returned to their initial states. Such a process is only theoretical. The irreversibility during a process describes the destruction of useful energy or availability. Without new inputs, both the system and its surroundings cannot be returned to their initial states because of the irreversibilities that have occurred, for example, friction, heat transfer or rejection, and electrical and mechanical effects. For instance, an actual system provides an amount of work that is less than the ideal reversible work, so the difference between these two values gives the irreversibility of that system. In real applications, there are always such differences, and therefore real processes and cycles are always irreversible.

1.4.21 Exergy

Exergy is defined as the maximum amount of work (also called *availability*, see Table 1.4) that can be produced by a stream of matter or energy (heat, work, etc.) as it comes to equilibrium with a reference environment. Exergy is a measure of the potential of a flow or system to cause change as a consequence of not being in complete stable equilibrium relative to a reference environment. For exergy analysis, the state of the reference environment, or the reference state, must be specified completely. This is commonly done by specifying the temperature, pressure, and chemical composition of the reference environment. Exergy is not subject to a conservation law. Rather exergy is consumed or destroyed because of irreversibilities in any process. Table 1.5 compares energy and exergy from a thermodynamics point of view.

As pointed out by Dincer and Rosen (1999, 2007), exergy is a measure of the usefulness, quality or potential of a flow or system to cause change, and is therefore a type of measure of the potential of a substance to impact on the environment.

Exergy analysis is a method that uses the conservation of mass and conservation of energy principles together with the second law of thermodynamics for the design and analysis of systems and processes. The exergy method can be suitable for furthering the goal of more efficient

Table 1.4 Relations among essergy, availability, exergy, and free energy

Name	Function	Remarks
Essergy	$E + P_0V - T_0S - \sum_i \mu_{i0}N_i$	Formulated for the special case in 1878 by Gibbs and in general in 1962, and changed from available energy to exergy in 1963, and from exergy to essergy (i.e., essence of energy) in 1968 by Evans.
Availability	$E + P_0V - T_0S - (E_0 + P_0V_0 - T_0S_0)$	Formulated by Keenan in 1941 as a special case of the essergy function.
Exergy	$E + P_0V - T_0S - (E_0 + P_0V_0 - T_0S_0)$	Introduced by Darrieus in 1930 and Keenan in 1932; called the availability in steady flow by him, and exergy by Rant in 1956 as a special case of essergy.
Free Energy	Helmholtz: $E - TS$ Gibbs: $E + PV - TS$	Introduced by von Helmholtz and Gibbs in 1873 as the Legendre transforms of energy to yield useful alternate criteria of equilibrium, as measures of the potential work of systems representing special cases of the essergy function.

Source: Szargut *et al.* (1988).

Table 1.5 Comparison between energy and exergy

Energy	Exergy
• Dependent on the parameters of matter or energy flow only, and independent of the environment parameters.	• Dependent both on the parameters of matter or energy flow and on the environment parameters.
• Has values different from zero (which is equal to mc^2 in accordance with Einstein's equation).	• Equal to zero (in dead state by virtue of being in equilibrium with the environment).
• Guided by the first law of thermodynamics for all processes.	• Guided by the first and second laws of thermodynamics for reversible processes only (in irreversible processes, it is destroyed partly or completely).
• Limited by the second law of thermodynamics for all processes (including reversible ones).	• Not limited for reversible processes owing to the second law of thermodynamics.
• Conserved in all processes.	• Not conserved in all processes.

energy-resource use, for it enables the locations, types, and true magnitudes of wastes and losses to be determined. Therefore, exergy analysis can reveal whether or not, and by how much, it is possible to design more efficient energy systems by reducing the sources of inefficiency in existing systems. In the past, exergy was called *essergy, availability, and available energy*. Table 1.4 lists some relations among essergy, availability, exergy, and free energy.

From the point of view of energy and exergy efficiency, it is important to note that if a fossil fuel-based energy source is used for a low-temperature thermal application like space heating or cooling, there would be a great difference between the corresponding energy and exergy efficiencies, perhaps by as much as 50–70% for the energy efficiency and 5% for the exergy efficiency (Dincer, 1998). One may ask why, and to address that question, we provide the following:

- High quality (e.g., high temperature) energy sources such as fossil fuels are often used for relatively low-quality (e.g., low-temperature) processes like water and space heating or cooling.

- Exergy efficiency permits a better matching of energy sources and uses, leading to high-quality energy being reserved for performing high-quality tasks and not used for low-quality end uses.

1.5 General Aspects of Fluid Flow

For a good understanding of the operation of thermal energy storage systems and their components, as well as the behavior of fluid flows, an extensive background on fluid mechanics is essential. In addition to learning the principles of fluid flow, the student and/or engineer should develop an understanding of the properties of fluids and be able to solve practical thermodynamic problems.

In practice, engineers are regularly faced with a large variety of fluid-flow problems:

- subcooled liquids, like water and brine;
- mixtures of boiling liquids and the ensuing vapor;
- mixtures of refrigerants and absorbents;
- mixtures of air and water vapor as humid air; and
- low- and high-pressure gases.

To deal effectively with fluid-flow systems, it is necessary to identify flow categories, defined in predominantly mathematical terms, that allow the appropriate analysis to be undertaken by identifying suitable and acceptable simplifications. Examples of the categories to be introduced include variation of the flow parameters with time (steady or unsteady) or variations along the flow path (uniform or nonuniform). Similarly, compressibility effects may be important in high-speed gas flows, but may be ignored in many liquid flow situations.

1.5.1 Classification of Fluid Flows

Various criteria allow fluid flows to be classified into the following categories:

- uniform or nonuniform,
- one-, two-, or three-dimensional,
- steady- or unsteady-state,
- laminar or turbulent, and
- compressible or incompressible.

Also, liquids flowing in open channels may be classified according to their regions, for example, subcritical, critical, or supercritical, and gas flows may be categorized as subsonic, transonic, supersonic, or hypersonic.

Uniform Flow and Nonuniform Flow

If the velocity and cross-sectional area are constant in the direction of flow, the flow is uniform. Otherwise, the flow is nonuniform.

One-, Two-, and Three-Dimensional Flow

The flow of real fluids occurs in three dimensions. However, in the analysis the conditions are often simplified to either one- or two-dimensional, depending on the flow problem under consideration. If all fluid and flow parameters (velocity, pressure, elevation, temperature, density, viscosity, etc.) are considered to be uniform throughout any cross-section and vary only along the direction of

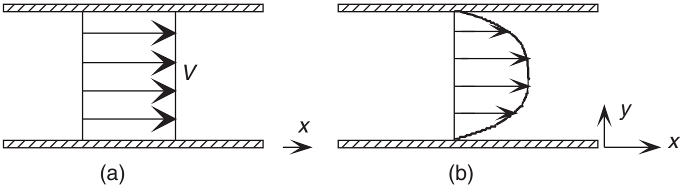

Figure 1.6 Velocity profiles for flows: (a) one-dimensional flow, (b) two-dimensional flow

flow (Figure 1.6a), the flow is one-dimensional. Two-dimensional flow occurs when the fluid and flow parameters have spatial gradients in two directions, that is, x and y axes (Figure 1.6b). In three-dimensional flow, the fluid and flow parameters vary in three directions, that is, x, y, and z axes, and the gradients of the parameters occur in all three directions.

Steady Flow

Steady flow is defined as a flow in which the flow conditions do not change with time. However, we may have a steady flow in which the velocity, pressure, and cross-section of the flow vary from point to point but do not change with time. This requires us to distinguish by dividing such a flow into *steady*, *uniform flow* and *steady*, *nonuniform flow*. In a steady, uniform flow, all conditions (e.g., velocity, pressure, and cross-sectional area) are uniform and do not vary with time or position. For example, uniform flow of water in a duct of constant cross-section is considered a steady, uniform flow. If the conditions (e.g., velocity and cross-sectional area) change from point to point (e.g., from cross-section to cross-section) but not with time, we have a steady, nonuniform flow. For example, a liquid flows at a constant rate through a tapering pipe running completely full.

Unsteady Flow

If the conditions vary with time, the flow becomes unsteady. If at a given time the velocity at every point in the flow field is the same, but the velocity changes with time, we have an *unsteady*, *uniform flow*. An example is an accelerating flow of a fluid through a pipe of uniform bore running full. In an unsteady, uniform flow, the conditions in cross-sectional area and velocity vary with time from point to point, for example, a wave traveling along a channel.

Laminar Flow and Turbulent Flow

This classification is one of the most important in fluid flow and depends primarily upon the arbitrary disturbances, irregularities, or fluctuations in the flow field, based on the internal characteristics of the flow. In this regard, there are two significant parameters such as velocity and viscosity. If the flow occurs at a relatively low velocity and/or with a highly viscous fluid, resulting in a fluid flow in an orderly manner without fluctuations, the flow is referred to as laminar. As the flow velocity increases and/or the viscosity of fluid decreases, the fluctuations take place gradually, referring to a *transition state* which is dependent on the fluid viscosity, the flow velocity, and geometric details. The Reynolds number Re is introduced to represent the characteristics of the flow conditions relative to the transition state. As the flow conditions deviate more from the transition state, a more chaotic flow field, that is, turbulent flow, occurs. Increasing Reynolds number increases the chaotic nature of the turbulence. Turbulent flow is, therefore, defined as a characteristic representative of the irregularities in the flow field.

The differences between laminar flow and turbulent flow can be distinguished by the Reynolds number, which is expressed as

$$\mathrm{Re} = \frac{VD}{\upsilon} = \frac{\rho VD}{\mu} \tag{1.43}$$

The Reynolds number indicates the ratio of inertial force to viscous force. At high Reynolds numbers the inertia forces dominate, resulting in turbulent flow, while at low Reynolds numbers the viscous forces become dominant, making the flow laminar. In a circular duct, the flow is laminar when Re is less than 2100 and turbulent when Re is greater than 4000. In a duct with a rough surface, the flow is turbulent at Re values as low as 2700.

Compressible Flow and Incompressible Flow

All actual fluids are normally compressible, leading to a change in their density with pressure. However, in many cases it is assumed during analysis that changes in density are negligibly small. This refers to incompressible flow.

1.5.2 Viscosity

Viscosity is one of the most significant fluid properties, and is defined as a measure of the fluid's resistance to deformation. In gases, the viscosity increases with increasing temperature, resulting in a greater molecular activity and momentum transfer. The viscosity of an ideal gas is a function of molecular dimensions and absolute temperature only, based on the kinetic theory of gases. However, in fluids, molecular cohesion between molecules considerably affects the viscosity, and the viscosity decreases with increasing temperature because of the fact that the cohesive forces are reduced by increasing the temperature of the fluid (causing a decrease in shear stress). This phenomenon results in an increase in the rate of molecular interchange, leading to a net result of a reduction in viscosity. The coefficient of viscosity of an ideal fluid is zero, meaning that an ideal fluid is inviscid, so that no shear stresses occur in the fluid, despite the fact that shear deformations are finite. Nevertheless, all real fluids are viscous.

As a fluid moves past a solid boundary or wall, the velocity of the fluid particles at the wall must equal the velocity of the wall; the relative velocity between the fluid and the wall at the surface of the wall is zero, which is called the *no-slip* condition, and results in a varying magnitude of the flow velocity (e.g., a velocity gradient), as one moves away from the wall (see Figure 1.7).

There are two types of viscosities, namely, *dynamic viscosity*, which is the ratio of a shear stress to a fluid strain (velocity gradient), and *kinematic viscosity*, which is defined as the ratio of dynamic viscosity to density.

The dynamic viscosity, based on a two-dimensional boundary layer flow and the velocity gradient du/dy occurring in the direction normal to the flow, as shown in Figure 1.7, leading to the shear

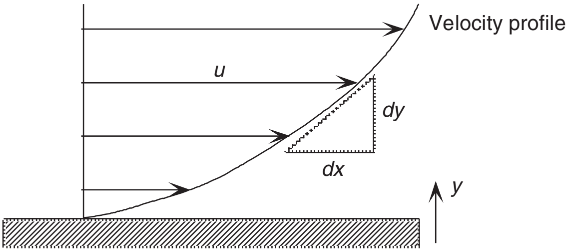

Figure 1.7 Schematic of velocity profile moving away from a wall (i.e., as y increases)

stress within a fluid being proportional to the spatial rate of change of fluid strain normal to the flow, is expressed as

$$\mu = \frac{\tau}{(du/dy)} \tag{1.44}$$

where the units of μ are Ns/m^2 or kg/ms in the SI system and lb$_f$s/ft^2 in the English system.

The kinematic viscosity then becomes

$$\upsilon = \frac{\mu}{\rho} \tag{1.45}$$

where the units of ν are m^2/s in the SI system and ft^2/s in the English system.

From the viscosity perspective, the types of fluids may be classified into the two groups that follow below.

Newtonian Fluids

These fluids have a dynamic viscosity dependent upon temperature and pressure and independent of the magnitude of the velocity gradient. For such fluids, Equation 1.44 is applicable. Some examples are water and air.

Non-Newtonian Fluids

Fluids that cannot be represented by Equation 1.44 are called *non-Newtonian fluids*. These fluids are very common in practice and have a more complex viscous behavior due to the deviation from Newtonian behavior. There are several approximate expressions to represent their viscous behavior. Some examples of such fluids are slurries, polymer solutions, oil paints, toothpaste, and sludges.

1.5.3 Equations of Flow

The basic equations of fluid flow may be derived from important fundamental principles, namely, conservation of mass, conservation of momentum (i.e., Newton's second law of motion), and conservation of energy. Although general statements of these laws can be written (applicable to all substances, e.g., solids and fluids), in fluid flow these principles can be formulated as a function of flow parameters, namely, pressure, temperature, and density. The equations of motion may be classified into two general types: the equations of motion for inviscid fluids (i.e., frictionless fluids) and the equations of motion for viscous fluids. In this regard, we deal with the Bernoulli equations and Navier–Stokes equations.

Continuity Equation

This is based on the *conservation of mass* principle. The requirement that mass be conserved at every point in a flowing fluid imposes certain restrictions on the velocity *u* and density ρ. Therefore, the rate of mass change is zero, so that for a steady flow, the mass of fluid in the control volume remains constant, and therefore the mass of fluid entering per unit time is equal to the mass of fluid exiting per unit time. We now apply this idea to a steady flow in a stream tube (Figure 1.8). The continuity equation for the flow of a compressible fluid through a stream tube is

$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2 = \text{constant} \tag{1.46}$$

where $\rho_1 \delta A_1 u_1$ is the mass entering per unit time (at section 1) and $\rho_2 \delta A_2 u_2$ is the mass exiting per unit time (at section 2).

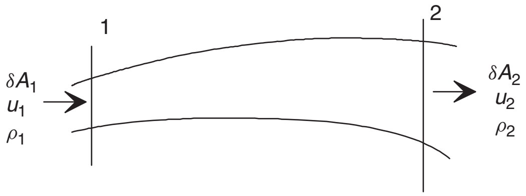

Figure 1.8 Fluid flow in a stream tube

In practice, for the flow of a real fluid through a pipe or a conduit, the mean velocity is used since the velocity varies from wall to wall. Then, Equation 1.46 can be rewritten as

$$\rho_1 A_1 \overline{u}_1 = \rho_2 A_2 \overline{u}_2 = \dot{m} \tag{1.47}$$

where $\overline{u_1}$ and $\overline{u_2}$ are the mean velocities at sections 1 and 2.

For fluids that are considered as incompressible, Equation 1.47 is simplified to the following, since $\rho_1 = \rho_2$:

$$A_1 \overline{u}_1 = A_2 \overline{u}_2 = \dot{V} \tag{1.48}$$

The various forms of the continuity equation for steady-state and unsteady-state cases are summarized below:

- The steady-state continuity equation for an incompressible fluid in a stream tube:

$$V \cdot A = \dot{V} \tag{1.49}$$

- The unsteady-state continuity equation for an incompressible fluid in a stream tube:

$$\left(\frac{dm}{dt}\right)_{sys} = \frac{d}{dt}\int_{cv} \rho dV + \int_{cs} \rho \overline{V} d\overline{A} \tag{1.50}$$

- The steady-state continuity equation for an incompressible fluid in cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1.51}$$

- The unsteady-state continuity equation for an incompressible fluid in cartesian coordinates:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = \frac{\partial \rho}{\partial t} \tag{1.52}$$

- The steady-state continuity equation for an incompressible fluid in cylindrical coordinates:

$$\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0 \tag{1.53}$$

- The steady-state continuity equation for a compressible fluid in a stream tube:

$$\rho V \cdot A = \dot{m} \tag{1.54}$$

- The steady-state continuity equation for a compressible fluid in cartesian coordinates:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{1.55}$$

- The steady-state continuity equation for a compressible fluid in cylindrical coordinates:

$$\frac{\partial(\rho v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} + \frac{\rho v_r}{r} = 0 \tag{1.56}$$

- The unsteady-state continuity equation for a compressible fluid in a stream tube:

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho V \cdot A)}{\partial s} = 0 \tag{1.57}$$

- The unsteady-state continuity equation for a compressible fluid in cartesian coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{1.58}$$

- The unsteady-state continuity equation for a compressible fluid in cylindrical coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} + \frac{\rho v_r}{r} = 0 \tag{1.59}$$

Momentum Equation

The analysis of fluid-flow phenomena is fundamentally dependent on the application of Newton's second law of motion, which is more general than the momentum principle, stating that when the net external force acting on a system is zero, the linear momentum of the system in the direction of the force is conserved in both magnitude and direction (the so-called *conservation of linear momentum*). In fact, the momentum principle is concerned only with external forces, and provides useful results in many situations without requiring much information on the internal processes within the fluid. The momentum principle finds applications in various types of flows (e.g., steady or unsteady, compressible or incompressible).

The motion of a particle must be described relative to an inertial coordinate frame. The one-dimensional momentum equation at constant velocity can be written as follows:

$$\Sigma F = \frac{d}{dt}(mV) \tag{1.60}$$

where ΣF stands for the sum of the external forces acting on the fluid, and mV stands for the kinetic momentum in that direction. Equation 1.60 states that the time rate of change of the linear momentum of the system in the direction of V equals the resultant of all forces acting on the system in the direction of V. The linear momentum equation is a vector equation and is therefore dependent on a set of coordinate directions.

The rate of change of momentum of a control mass can be related to the rate of change of momentum of a control volume via the continuity equation. Then, Equation 1.60 becomes

$$\Sigma F_t = \Sigma F_{cv} + \Sigma F_{cs} = \frac{d}{dt}\int_{cv} V\rho dV - \int_{cs} V(\rho \overline{V} d\overline{A}) \tag{1.61}$$

Here, the sum of forces acting on the control volume in any direction is equal to the rate of change of momentum of the control volume in that direction plus the net rate of momentum flux from the control volume through its control surface in the same direction.

For a steady flow, if the velocity across the control surface is constant, the momentum equation in scalar form becomes

$$\Sigma F_x = (\dot{m} V_x)_e - (\dot{m} V_x)_i \tag{1.62}$$

If the mass flow rate $\dot{m}$ is constant, Equation 1.62 can be written as

$$\Sigma F_x = \dot{m}(V_{x_e} - V_{x_i}) \tag{1.63}$$

Similar expressions can be written for the y and z directions.

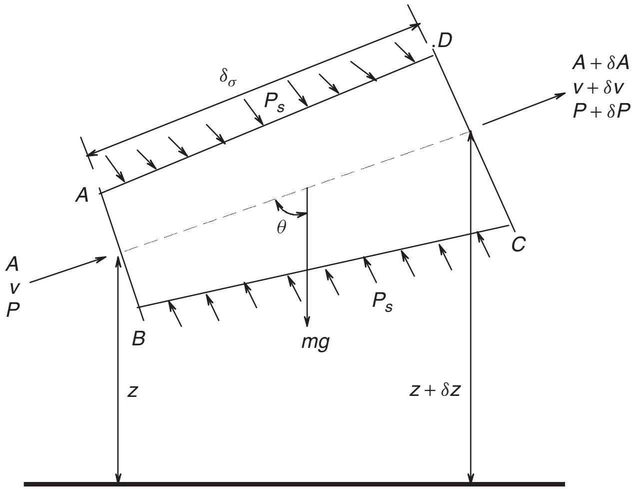

Figure 1.9 Relationship between velocity, pressure, elevation, and density for a stream tube

Euler's Equation

Euler's equation is a mathematical statement of Newton's second law of motion, and finds application in an inviscid fluid continuum. This equation states that the product of mass and acceleration of a fluid particle can be equated vectorially with the external forces acting on the particle. Consider a stream tube, as shown in Figure 1.9, with a cross-sectional area small enough for the velocity to be considered constant along the tube.

The following is a simple form of Euler's equation for a steady flow along a stream tube, representing the relationship in differential form between pressure p, velocity v, density ρ, and elevation z, respectively:

$$\frac{1}{\rho}\frac{dp}{ds}+v\frac{dv}{ds}+g\frac{dz}{ds}=0 \tag{1.64}$$

For an incompressible fluid (ρ is constant), the integration of the above equation gives the following expression along the streamline (with respect to s) for an inviscid fluid:

$$\frac{p}{\rho}+\frac{v^2}{2}+gz=\text{constant} \tag{1.65}$$

For a compressible fluid, the integration of Equation 1.64 can only be completed to provide the following:

$$\int\frac{dp}{\rho g}+\frac{v^2}{2g}+z=H \tag{1.66}$$

Note that the relationship between ρ and p needs to be known for the given case, and that for gases the relationship can be in the form $p\rho^n=\text{constant}$, varying from adiabatic to isothermal conditions, while for a liquid, $\rho(dp/d\rho)=K$, which is an adiabatic modulus.

Bernoulli's Equation

This equation can be written for both incompressible and compressible flows. Under certain flow conditions, Bernoulli's equation for incompressible flow is often referred to as a *mechanical-energy* *equation* because of the fact that there is similarity to the steady-flow energy equation obtained from the FLT for an inviscid fluid with no external heat transfer and no external work. It is necessary to point out that for inviscid fluids viscous forces and surface tension forces are not taken into consideration, leading to negligible viscous effects. The Bernoulli equation is commonly used in a variety of practical applications, particularly in flows in which the losses are negligibly small, for example, in hydraulic systems. The following is the general Bernoulli equation per unit mass for inviscid fluids between any two points:

$$\frac{u_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{u_2^2}{2g} + \frac{p_2}{\rho g} + z_2 = H \tag{1.67}$$

Here, each term has a dimension of a length or head scale. In this regard, $u^2/2g$ (kinetic energy per unit mass) is referred to as the *velocity head*, $p/\rho g$ (pressure energy per unit mass) as the *pressure head*, z (potential energy per unit mass) as the *potential head* (constant total head), and H (total energy per unit mass) as the *total head* in meters. Subscripts 1 and 2 denote where the variables are evaluated on the streamline.

The terms in Equation 1.67 represent energy per unit mass and have the unit of length. Bernoulli's equation can be obtained by dividing each term in Equation 1.65 by g. These terms, both individually and collectively, indicate the quantities that may be directly converted to produce mechanical energy.

In summary, if we compare Equation 1.67 with the general energy equation, we see that the Bernoulli equation contains even more restrictions than might first be realized, due to the following main assumptions:

- steady flow (common assumption applicable to many flows);
- incompressible flow (acceptable if the Mach number is less than 0.3);
- frictionless flow along a single streamline (highly restrictive);
- no external shaft work or heat transfer occurs between 1 and 2.

Navier–Stokes Equations

The Navier–Stokes equations are the differential expressions of Newton's second law of motion, and are known as constitutive equations for viscous fluids. These equations were named after C.L.M.H. Navier and Sir G.G. Stokes, who are credited with their derivation.

For viscous fluids, two force aspects, namely, a body force and a pressure force on their surface, are taken into consideration. The solution of these equations is dependent upon what flow information is known. The solutions evolving now for such problems have become extremely useful. Recently numerical software packages have been developed in the field of fluid flow for many engineering applications.

Exact solutions to the nonlinear Navier–Stokes equations are limited to a few cases, particularly for steady, uniform flows (either two-dimensional or with radial symmetry) or for flows with simple geometries. However, approximate solutions may be undertaken for other one-dimensional simple flow cases which require only the momentum and continuity equations in the flow direction for the solution of the flow field. Here, we present a few cases: uniform flow between parallel plates, uniform free surface flow down a plate, and uniform flow in a circular tube.

Uniform Flow between Parallel Plates Consider a two-dimensional uniform, steady flow between parallel plates, which extend infinitely in the z direction, as shown in Figure 1.10. We also consider the plates to be oriented at an angle θ to the horizontal plane, which results in a body force per unit mass as the gravitational term $g \sin\theta$. Therefore, the pressure gradient is specified as $(\partial p/\partial y) = \text{constant} = P^*$. After making necessary simplifications and integrating, we find the total

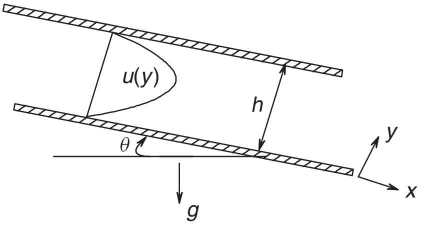

Figure 1.10 Uniform flow between two stationary parallel plates

flow rate per unit width and the velocity profile as follows:

$$q_f = \frac{h^3}{12\mu}(-P^* + \rho g \sin\theta) \tag{1.68}$$

$$u = \frac{1}{2\mu}(-P^* + \rho g \sin\theta)(hy - y^2) \tag{1.69}$$

If the plates are horizontally located (i.e., $\sin\theta = 0$), the above equations reduce to

$$q_f = -\frac{P^* h^3}{12\mu} \tag{1.70}$$

$$u = -\frac{P^*}{2\mu}(hy - y^2) \tag{1.71}$$

Uniform Free Surface Flow Down a Plate This case is, by nature, similar to the previous case, except that the upper plate has been removed, as shown in Figure 1.11. The boundary condition at the lower boundary is the same no-slip boundary condition as before, so that the velocity is zero. However, the boundary condition at the free surface can no longer be specified as no-slip. The total flow rate per unit width becomes

$$q_f = \frac{\rho g d^3 \sin\theta}{3\mu} \tag{1.72}$$

while the flow velocity is

$$u = \frac{\rho g \sin\theta}{\mu}(yd - y^2/2) \tag{1.73}$$

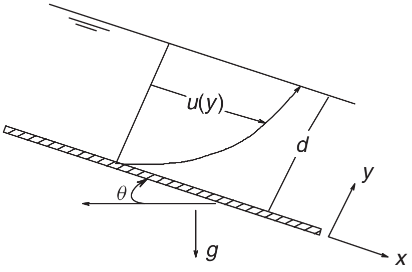

Figure 1.11 Uniform flow down a plate

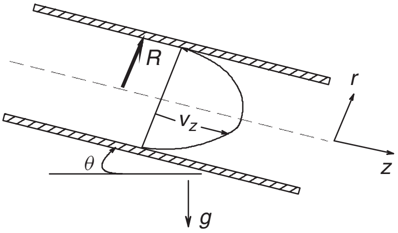

Figure 1.12 Uniform flow in a pipe

Furthermore, the average velocity can be obtained by dividing the discharge by the flow area or depth as follows:

$$V = \frac{\rho g d^2 \sin\theta}{3\mu} \tag{1.74}$$

Uniform Flow in a Circular Tube This case concerns uniform fluid flow in a pipe of radius R, as shown in Figure 1.12, which is the most common example in practical applications associated with pipe flows. Despite having the flow field as three-dimensional, the assumption of radial symmetry makes the problem two-dimensional. Therefore, the parabolic velocity distribution with the maximum velocity at the center of the pipe can be found as follows:

$$v_z = \frac{(R^2 - r^2)}{4\mu}(-P^* + \rho g \sin\theta) \tag{1.75}$$

which is known as the Hagen-Poiseuille equation. The total volumetric flow rate can be calculated if the pressure gradient along with other flow conditions is specified and vice versa, as follows:

$$Q = \frac{\pi R^4}{8\mu}(-P^* + \rho g \sin\theta) \tag{1.76}$$

If the pipe is horizontally located (i.e., $\sin\theta = 0$), the above equations result in

$$v_z = \frac{(R^2 - r^2)}{4\mu} \tag{1.77}$$

$$Q = \frac{\pi R^4}{8\mu} \tag{1.78}$$

1.5.4 Boundary Layer

If there is an equivalence between fluid and surface velocities at the interface between a fluid and a surface, it is called a "no-slip" condition, which is entirely associated with viscous effects, as mentioned earlier. In practice, any real fluid flow shows a region of retarded flow near a boundary in which the velocity relative to the boundary varies from zero at the boundary to a value that may be estimated by the potential-flow solution some distance away. This region of retarded flow is known as the *boundary layer*, which was first introduced by Prandtl in 1904. His hypothesis of a boundary layer was arrived at by experimental observations of the flow past solid surfaces.

Of course, the boundary layer can be taken as that region of the fluid that is close to the surface immersed in the flowing fluid, and the boundary layer development takes place in both internal and external flows. In internal flows, it occurs until the entire fluid is encompassed, such as in pipe flow and open-channel flow. Boundary layer development is important for external flows, which

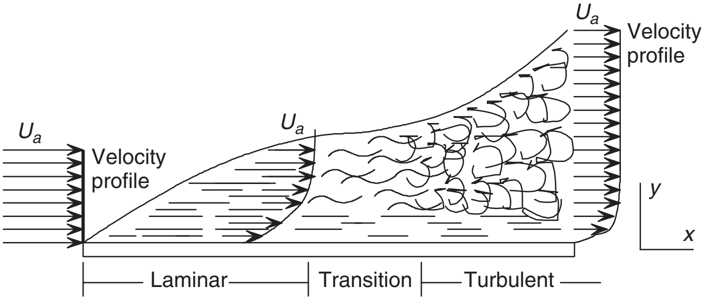

Figure 1.13 Development of boundary layer in a viscous flow along a plate

exhibit a continued growth due to the absence of a confining boundary, such as a flow along a flat plate. It is, therefore, important to assume that the velocity at some distance from the boundary is unaffected by the presence of the boundary, referring to the free-stream velocity u_s.

We now consider a uniform flow of incompressible fluid at a free-stream velocity approaching the plate, as shown in Figure 1.13. Since the plate is stationary with respect to the earth, when the fluid is in contact with the plate surface, it has zero velocity (i.e., the "no-slip" condition). Later, the boundary layer thickens in the direction of flow, and a velocity gradient at a distance δ over an increasingly greater distance normal to the plate takes place between the fluid in the free stream and the plate surface. The rate of change of velocity determines the velocity gradient at the surface as well as the shear stresses. The shear stress for the laminar boundary layer becomes

$$\tau = \mu \left(\frac{du}{dy} \right)_{y=0} \tag{1.79}$$

which varies with distance along the plate by the change in velocity. Further along the plate, the shear force is gradually increased, as the laminar boundary layer thickens, because of the increasing plate surface area affected, and the fluid becomes retarded, so that a turbulent boundary layer occurs as instabilities set in. Thus, the shear stress for the turbulent flow can be approximated as

$$\tau = (\mu + \varepsilon) \left(\frac{du}{dy} \right)_{y=0} \tag{1.80}$$

Experimental studies indicate that there are two boundary layer flow regimes; a *laminar flow regime* and a *turbulent flow regime*, which can be characterized by the Reynolds number, as pointed out earlier. The transition from a laminar to a turbulent boundary layer is dependent mainly upon the following:

- $\mathrm{Re} = u_s x_c / \nu$;
- the roughness of the plate; and
- the turbulence level in the free stream.

There are various boundary-layer parameters to be considered, such as boundary-layer thickness, the local wall shear stress (or local friction or drag coefficient), and the average wall shear stress (or average friction or drag coefficient). The boundary-layer thickness may be expressed in several ways. The simplest approach is that the velocity u within the boundary layer approaches the free-stream velocity u_s. From experimental measurements, it was observed that the boundary-layer thickness δ can be defined as the distance from the boundary to the point at which $u = 0.99 u_s$.

Table 1.6 gives the values of the boundary-layer thicknesses for laminar flow along a flat plate as a function of dimensionless coordinates $\eta = y(u_s/\nu x)$. These values are of practical interest in momentum analysis of fluid flow.

The momentum equations for velocity profiles can be summarized with respect to the momentum thickness, the average skin-drag coefficient, and the displacement thickness (see Table 1.7 for a flat plate). As can be pointed out from the table, the laminar boundary-layer thickness increases with $x^{1/2}$ from the leading edge and inversely with $u_s^{1/2}$, the local and average skin-drag coefficients change inversely with $x^{1/2}$ and $u_s^{1/2}$, and the total drag force, $F = C_f \rho u_s^2 x/2$ per unit width, changes as the 1.5 power of u_s and the square root of the length x. Normally, fluid flow along a flat plate is laminar for Reynolds number values up to about 300,000–500,000, depending on the plate roughness and the level of turbulence in the free stream, as mentioned earlier.

Table 1.8 presents additional momentum equations for a boundary turbulent layer along a flat plate, including additional pipe flow velocity profiles, and summarizes the following for a turbulent boundary layer on a flat plate:

- The boundary-layer thickness increases as the 4/5 power of the distance from the leading edge, as compared with $x^{1/2}$ for a laminar boundary layer.
- The local and average skin-friction coefficients vary inversely as the fifth root of both x and u_s, as compared with the square root for a laminar boundary layer.
- The total drag varies as $u_s^{9/5}$, and $x^{4/5}$, as compared with values of corresponding parameters for a laminar boundary layer.

Table 1.6 Values of laminar boundary-layer thicknesses for laminar flow over a flat plate

η	0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4	6.0
δ	0.000	0.200	0.394	0.575	0.729	0.846	0.924	0.967	0.988	0.996	0.999

Source: Olson and Wright (1991).

Table 1.7 Momentum equations for laminar boundary layer

Velocity profile	δ/x	C_f	δ^*/x
$u/u_s = y/\delta$	$3.46/\mathrm{Re}_x^{1/2}$	$1.156/\mathrm{Re}_x^{1/2}$	$1.73/\mathrm{Re}_x^{1/2}$
$u/u_s = 2(y/\delta) - (y/\delta)^2$	$5.48/\mathrm{Re}_x^{1/2}$	$1.462/\mathrm{Re}_x^{1/2}$	$1.83/\mathrm{Re}_x^{1/2}$
$u/u_s = 1.5(y/\delta) - 0.5(y/\delta)^3$	$4.64/\mathrm{Re}_x^{1/2}$	$1.292/\mathrm{Re}_x^{1/2}$	$1.74/\mathrm{Re}_x^{1/2}$
$u/u_s = \sin \pi y/2\delta$	$4.80/\mathrm{Re}_x^{1/2}$	$1.310/\mathrm{Re}_x^{1/2}$	$1.74/\mathrm{Re}_x^{1/2}$
Blasius exact solution	$4.91/\mathrm{Re}_x^{1/2}$	$1.328/\mathrm{Re}_x^{1/2}$	$1.73/\mathrm{Re}_x^{1/2}$

Source: Olson and Wright (1991).

Table 1.8 Momentum equations for a turbulent boundary layer for flat plate flow and for pipe flow

Re_D	F	u/u_s	V/u_s	C_f	Re_x
$<10^5$	$0.316/\mathrm{Re}_D^{1/4}$	$(y/R)^{1/7}$	49/60	$0.074/\mathrm{Re}_x^{1/5}$	$5 \times 10^5 - 10^7$
$10^4 - 10^6$	$0.180/\mathrm{Re}_D^{1/5}$	$(y/R)^{1/8}$	128/153	$0.045/\mathrm{Re}_x^{1/6}$	$1.8 \times 10^5 - 4.5 \times 10^7$
$10^5 - 10^7$	$0.117/\mathrm{Re}_D^{1/6}$	$(y/R)^{1/10}$	200/231	$0.0305/\mathrm{Re}_x^{1/7}$	$2.9 \times 10^6 - 5 \times 10^8$

Source: Olson and Wright (1991).

Initially, as the boundary layer develops, it will be laminar in form. The boundary layer will become turbulent, based on the ratio of inertial and viscous forces acting on the fluid, referring to the value of the Reynolds number. For example, in pipe flow, for the values of Re < 2300 the flow is laminar. If the Reynolds number increases, the flow becomes turbulent. Compared to flow along a flat plate, the major difference in pipe flow is that there is a limit to the growth of the boundary-layer thickness because of the pipe radius.

Many empirical pipe flow equations have been developed, particularly for water. The velocity V and volumetric flow rate $\dot{V}$equations of Hazen–Williams are the most widely used, and are as follows:

$$V = 0.850 C R_h^{0.63} S^{0.54} \tag{1.81}$$

$$\dot{V} = 0.850 C R_h^{0.63} S^{0.54} A \tag{1.82}$$

where R_h is the hydraulic radius of the pipe, P is wetted perimeter (A/P, for example, $R_h = D/4$ for a round pipe), S is the slope of the total head line, h_f/L, A is the pipe cross-sectional area, and C is the roughness coefficient. The coefficient C takes different values for the pipes, for example, $C = 140$ for very badly corroded iron or steel pipes.

1.6 General Aspects of Heat Transfer

Thermal processes involving the transfer of heat from one point to another are often encountered in industries. The heating and cooling of gases, liquids, and solids, the evaporation of water, and the removal of heat liberated by chemical reaction are common examples of processes that involve heat transfer. Engineers, scientists, technologists, researchers, and others need to understand the physical phenomena and practical aspects of heat transfer, and have a good knowledge of the basic laws, governing equations, and related boundary conditions.

In order to transfer heat, there must be a driving force, which is the temperature difference between the locations where heat is taken and where the heat originates. For example, consider that a long slab of food product is subjected to heating on the left side; the heat flows from the left side to the right side, which is colder. Heat tends to flow from a point of high temperature to a point of low temperature, owing to the temperature difference driving force.

Many of the generalized relationships used in heat transfer calculations have been determined by means of dimensional analysis and empirical considerations. It has been found that certain standard dimensionless groups repeatedly appear in the final equations. It is necessary for people working in heat transfer to recognize the more important of these groups. Some of the most commonly used dimensionless groups that appear frequently in the heat transfer literature are given in Table 1.9.

Table 1.9 Some of the most important heat transfer dimensionless parameters

Name	Symbol	Definition	Application
Biot number	Bi	hY/k	Steady- and unsteady-state conduction
Fourier number	Fo	at/Y^2	Unsteady-state conduction
Graetz number	Gz	$GY^2 c_p/k$	Laminar convection
Grashof number	Gr	$G\beta \Delta T Y^3/\nu^2$	Natural convection
Rayleigh number	Ra	Gr × Pr	Natural convection
Nusselt number	Nu	hY/k_f	Natural or forced convection, boiling, or condensation
Peclet number	Pe	$UY/a =$ Re × Pr	Forced convection (for small Pr)
Prandtl number	Pr	$c_p\mu/k = \nu/a$	Natural or forced convection, boiling, or condensation
Reynolds number	Re	UY/ν	Forced convection
Stanton number	St	$h/\rho U c_p =$ Nu/Re Pr	Forced convection

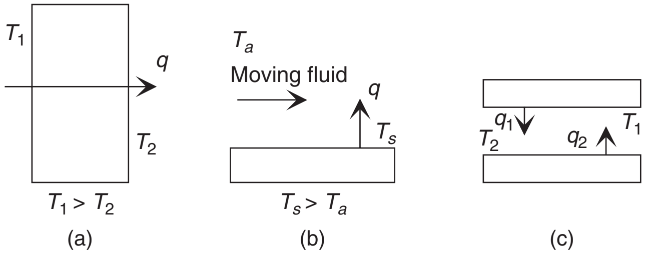

Figure 1.14 Representations of heat transfer modes: (a) conduction through a solid, (b) convection from a surface to a moving fluid, (c) radiation between two surfaces

In the utilization of these groups, care must be taken to use equivalent units so that all the dimensions cancel out. Any system of units may be used in a dimensionless group as long as all units cancel in the final result.

Basically, heat is transferred in three ways: conduction, convection, and radiation (the so-called modes of heat transfer). In many cases, heat transfer takes place by all three of these methods simultaneously. Figure 1.14 shows the different types of heat transfer processes as modes. When a temperature gradient exists in a stationary medium, which may be a solid or a fluid, the heat transfer occurring across the medium is by conduction, the heat transfer occurring between a surface and a moving fluid at different temperatures is by convection, and the heat transfer occurring between two surfaces at different temperatures, in the absence of an intervening medium (or presence of a non-obscuring medium), is by radiation, where all surfaces of finite temperature emit energy in the form of electromagnetic waves.

1.6.1 Conduction Heat Transfer

Conduction is a mode of transfer of heat from one part of a material to another part of the same material, or from one material to another in physical contact with it, without appreciable displacement of the molecules forming the substance. For example, the heat transfer in a solid object subject to cooling in a medium is by conduction. In solid objects, the conduction of heat is partly due to the impact of adjacent molecules vibrating about their mean positions and partly due to internal radiation. When the solid object is a metal, there are also large numbers of mobile electrons that can easily move through the matter, passing from one atom to another, and they contribute to the redistribution of energy in the metal object. The contribution of the mobile electrons predominates in metals, which explains the relation that is observed between the thermal and electrical conductivities of such materials.

Fourier's Law of Heat Conduction

Fourier's law states that the instantaneous rate of heat flow through a homogeneous solid object is directly proportional to the cross-sectional area A (i.e., the area at right angles to the direction of heat flow) and to the temperature difference driving force across the object with respect to the length of the path of the heat flow, dT/dx. This is an empirical law based on observation.

Figure 1.15 presents an illustration of Fourier's law of heat conduction. Here, a thin slab object of thickness dx and surface area F has one face at a temperature T and the other at a lower temperature $(T - dT)$. Heat flows from the high-temperature side to the low-temperature side,

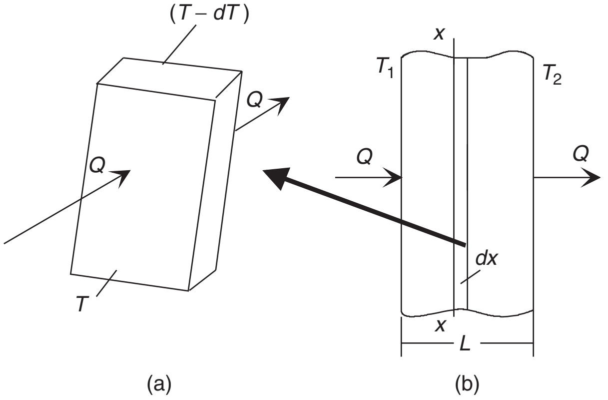

Figure 1.15 Conduction in a slab (a) and in a thin slice of the slab (b)

with a temperature change dT in the direction of the heat flow. Therefore, under Fourier's law we obtain the heat transfer as

$$Q = -kA\frac{dT}{dx} \tag{1.83}$$

Here, we have a term *thermal conductivity*, k, of the object, which can be defined as the heat flow per unit area per unit time when the temperature decreases by one degree over a unit distance. The SI units of thermal conductivity are usually W/m °C or W/m K.

Integrating Equation 1.83 from T_1 to T_2 for dT and 0 to L for dx yields

$$Q = -k\frac{A}{L}(T_2 - T_1) = k\frac{A}{L}(T_1 - T_2) \tag{1.84}$$

Equation 1.84 can be solved when the variation of thermal conductivity with temperature is known. For most solids, thermal conductivity values are approximately constant over a broad range of temperatures, and can be taken as constants.

1.6.2 Convection Heat Transfer

Convection is the heat transfer mode that occurs within a fluid by mixing one portion of the fluid with another. Convection heat transfer may be classified according to the nature of the flow. When the flow is caused by some mechanical or external means such as a fan, a pump, or atmospheric wind, it is called *forced convection*. On the other hand, for *natural* (*free*) *convection* the flow is induced by buoyancy forces in the fluid that arise from density variations caused by temperature variations in the fluid. For example, when a hot object is exposed to the atmosphere, natural convection occurs, whereas in a cold place with a fan-driven air flow, forced-convection heat transfer takes place between air flow and the object subject to this flow. The transfer of heat through solid objects is by conduction alone, whereas the heat transfer from a solid surface to a liquid or gas takes place partly by conduction and partly by convection. Whenever there is an appreciable movement of the gas or liquid, the heat transfer by conduction in the gas or liquid becomes negligibly small compared with the heat transfer by convection. However, there is always a thin boundary layer of fluid on a surface, and through this thin film the heat is transferred by conduction. The convection heat transfer occurring within a fluid is due to the combined effects of conduction and bulk fluid motion. In general, the heat that is transferred is the *sensible* or internal thermal heat of the fluid. However, there are convection processes for which there is also *latent* heat exchange, which is generally associated with a phase change between the liquid and vapor states of the fluid.

Newton's Law of Cooling

Newton's law of cooling states that the heat transfer from a solid surface to a fluid is proportional to the difference between the surface and fluid temperatures, and the surface area. This is a particular type of convection heat transfer, and is expressed as

$$Q = hA(T_s - T_f) \tag{1.85}$$

where h is referred to as the *convection heat transfer coefficient* (the *heat transfer coefficient*, the *film coefficient*, or the *film conductance*). It encompasses all effects that influence the convection mode and depends on conditions in the boundary layer, which is affected by factors such as surface geometry, the nature of the fluid motion, and thermal and physical properties (Figure 1.16).

In Equation 1.85, a radiation term is not included. The calculation of radiation heat transfer is discussed later. In many heat transfer problems, the radiation effect on the total heat transfer is negligible compared with the heat transferred by conduction and convection from a surface to a fluid. When the surface temperature is high, or when the surface loses little heat by natural convection, then the heat transfer due to radiation is often of a similar magnitude to that lost by convection.

To better understand Newton's law of cooling, consider the heat transfer from a high-temperature fluid A to a low-temperature fluid B through a wall of thickness x (Figure 1.16). In fluid A, the temperature decreases rapidly from T_A to T_{s1} in the region of the wall, and similarly in fluid B from T_{s2} to T_B. In most cases, the fluid temperature is approximately constant throughout its bulk, apart from a thin film (Δ_A or Δ_B) of fluid near each solid surface. The heat transfer per unit surface area from fluid A to the wall and that from the wall to fluid B can be expressed as

$$q = h_A(T_A - T_{s1}) \tag{1.86}$$

$$q = h_B(T_{s2} - T_B) \tag{1.87}$$

Also, the heat transfer in thin films is by conduction only, as given below:

$$q = \frac{k_A}{\Delta_A}(T_A - T_{s1}) \tag{1.88}$$

$$q = \frac{h_B}{\Delta_B}(T_{s2} - T_B) \tag{1.89}$$

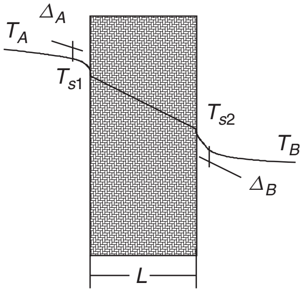

Figure 1.16 A wall subject to convection heat transfer on both sides

Equating Equations 1.86–1.89, the convection heat transfer coefficients can be found to be $h_A = k_A/\Delta_A$, and $h_B = k_B/\Delta_B$. Thus, the heat transfer in the wall per unit surface area becomes

$$q = \frac{k}{L}(T_{s1} - T_{s2}) \tag{1.90}$$

For the case of steady-state heat transfer, Equation 1.86 is equal to Equation 1.87, and hence to Equation 1.90:

$$q = h_A(T_A - T_{s1}) = h_B(T_{s2} - T_B) = \frac{k}{L}(T_{s1} - T_{s2}) \tag{1.91}$$

which yields

$$q = \frac{(T_A - T_B)}{(1/h_A + L/k + 1/h_B)} \tag{1.92}$$

An analogy can be made with Equation 1.85, allowing Equation 1.92 to become

$$Q = HA(T_A - T_B) \tag{1.93}$$

where $1/H = (1/h_A + L/k + 1/h_B)$. H is the overall heat transfer coefficient and includes various heat transfer coefficients.

1.6.3 Radiation Heat Transfer

An object emits radiant energy in all directions unless its temperature is absolute zero. If this energy strikes a receiver, part of it may be absorbed, part may be transmitted, and part may be reflected. Heat transfer from a hot to a cold object in this manner is known as *radiation heat transfer*. The higher the temperature, the greater is the amount of energy radiated. If, therefore, two objects at different temperatures are placed so that the radiation from each object is intercepted by the other, then the body at the lower temperature will receive more energy than it radiates, and thereby its internal energy will increase; in conjunction with this, the internal energy of the object at the higher temperature will decrease. Radiation heat transfer frequently occurs between solid surfaces, although radiation from gases also takes place. Certain gases emit and absorb radiation at certain wavelengths only, whereas most solids radiate over a wide range of wavelengths. The radiative properties of many gases and solids may be found in heat transfer books.

Radiation striking an object can be absorbed by the object, reflected from the object, or transmitted through the object. The fractions of the radiation absorbed, reflected, and transmitted are called the *absorptivity* a, the *reflectivity* r, and the *transmissivity* t, respectively. By definition, $a + r + t = 1$. For many solids and liquids in practical applications, the transmitted radiation is negligible, and hence $a + r = 1$. A body that absorbs all radiation striking it is called a *blackbody*. For a blackbody, $a = 1$ and $r = 0$.

The Stefan–Boltzman Law

This law was found experimentally by Stefan, and proved theoretically by Boltzmann. It states that the emissive power of a blackbody is directly proportional to the fourth power of its absolute temperature. The Stefan–Boltzmann law enables calculation of the amount of radiation emitted in all directions and over all wavelengths simply from the knowledge of the temperature of the blackbody. This law is expressible as follows:

$$E_b = \sigma T_s^4 \tag{1.94}$$

where σ denotes the Stefan–Boltzmann constant, which has a value of $5.669 \times 10^{-8}\ \mathrm{W/m^2\,K^4}$, and T_s denotes the absolute temperature of the surface.

The energy emitted by a non-blackbody becomes

$$E_{\mathrm{nb}} = \varepsilon\sigma T_s^4 \tag{1.95}$$

Then, the heat transferred from an object's surface to its surroundings per unit area is

$$q = \varepsilon\sigma(T_s^4 - T_a^4) \tag{1.96}$$

Note that if the emissivity of the object at T_s is much different from the emissivity of the object at T_a, then this gray object approximation may not be sufficiently accurate. In this case, it is a good approximation to take the absorptivity of object 1 when receiving radiation from a source at T_a as being equal to the emissivity of object 1 when emitting radiation at T_a. This results in

$$q = \varepsilon_{Ts}\sigma T_s^4 - \varepsilon_{Ta}\sigma T_a^4 \tag{1.97}$$

There are numerous applications for which it is convenient to express the net radiation heat transfer (radiation heat exchange) in the following form:

$$Q = h_r A(T_s - T_a) \tag{1.98}$$

After combining Equations 1.97 and 1.98, the radiation heat transfer coefficient can be found as follows:

$$h_r = \varepsilon\sigma(T_s + T_a)(T_s^2 + T_a^2) \tag{1.99}$$

Here, the radiation heat transfer coefficient is seen to strongly depend on temperature, whereas the temperature dependence of the convection heat transfer coefficient is generally weak.

The surface within the surroundings may also simultaneously transfer heat by convection to the surroundings. The total rate of heat transfer from the surface is the sum of the convection and radiation modes:

$$Q_t = Q_c + Q_r = h_c A(T_s - T_a) + \varepsilon\sigma A(T_s^4 - T_a^4) \tag{1.100}$$

1.6.4 Thermal Resistance

There is a similarity between heat flow and electricity flow. While electrical resistance is associated with the conduction of electricity, thermal resistance is associated with the conduction of heat. The temperature difference providing heat conduction plays a role analogous to that of the potential difference or voltage in the conduction of electricity. Below we give the *thermal resistance for heat conduction*, based on Equation 1.84, and similarly the *electrical resistance for electrical conduction* according to Ohm's law:

$$R_{t,cd} \equiv \frac{(T_1 - T_2)}{Q_{cd}} = \frac{L}{kA} \tag{1.101}$$

$$R_e \equiv \frac{(E_1 - E_2)}{I} = \frac{L}{\sigma A} \tag{1.102}$$

It is also possible to write the *thermal resistance for convection*, based on Equation 1.85, as follows:

$$R_{t,c} \equiv \frac{(T_s - T_f)}{Q_c} = \frac{1}{hA} \tag{1.103}$$

In a series of connected objects through which heat is transferred, the total thermal resistance can be written in terms of the overall heat transfer coefficient. The heat transfer expression for a composite wall is discussed next.

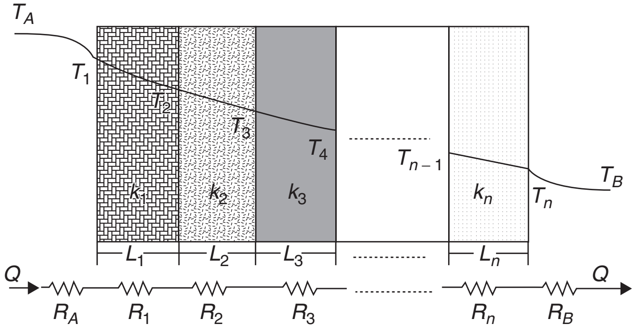

Figure 1.17 A composite wall with many layers in series

1.6.5 The Composite Wall

In practice, there are many cases in the form of a composite wall, for example, the wall of a cold storeroom. Consider that we have a general form of the composite wall as shown in Figure 1.17. Such a system includes any number of series and parallel thermal resistances because of the existence of layers of different materials. The heat transfer rate is related to the temperature difference and resistance associated with each element as follows:

$$Q = \frac{(T_A - T_1)}{(1/h_1 A)} = \frac{(T_1 - T_2)}{(L_1/k_1 A)} = \cdots = \frac{(T_n - T_B)}{(1/h_n A)} \tag{1.104}$$

Therefore, the one-dimensional heat transfer rate for this system can be written as

$$Q = \frac{(T_A - T_B)}{\Sigma R_t} = \frac{\Delta T}{\Sigma R_t} \tag{1.105}$$

where $\Sigma R_t = R_{t,t} = 1/HA$. Therefore, the overall heat transfer coefficient becomes

$$H = \frac{1}{R_{t,t} A} = \frac{1}{(1/h_1 + L_1/k_1 + \cdots + 1/h_n)} \tag{1.106}$$

1.6.6 The Cylinder

A practical common object is a hollow cylinder, and a commonly encountered problem is the case of heat transfer through a pipe or cylinder. Consider that we have a cylinder of internal radius r_1 and external radius r_2, whose inner and outer surfaces are in contact with fluids at different temperatures (Figure 1.18). In a steady-state form with no heat generation, the governing heat conduction equation is written as

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0 \tag{1.107}$$

Based on Fourier's law, the rate at which heat is transferred by conduction across the cylindrical surface in the solid is expressed as

$$Q = -kA\frac{dT}{dr} = -k(2\pi r L)\frac{dT}{dr} \tag{1.108}$$

where $A = 2\pi rL$ is the area normal to the direction of heat transfer.

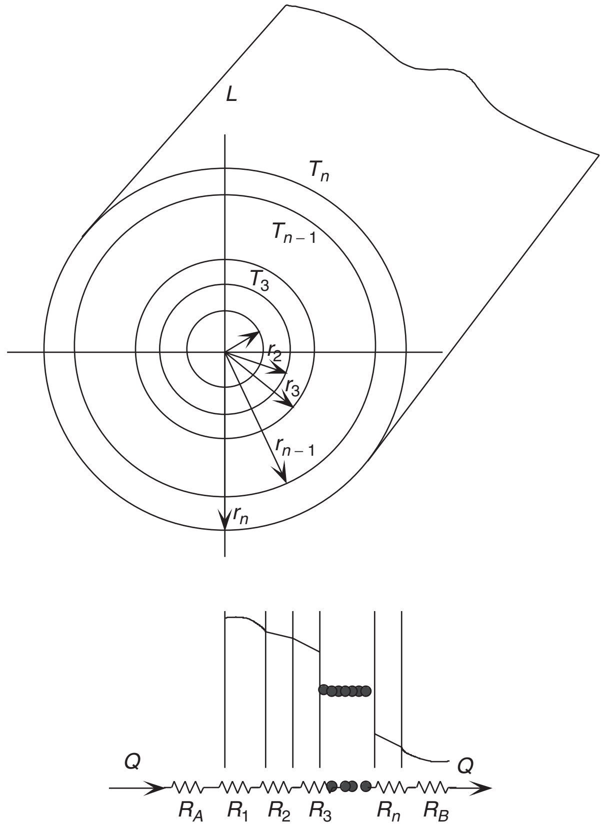

Figure 1.18 A hollow cylinder

To determine the temperature distribution in the cylinder, it is necessary to solve Equation 1.107 under appropriate boundary conditions, by assuming that k is constant. By integrating Equation 1.107 twice, the following heat transfer equation is obtained:

$$Q = \frac{k(2\pi L)(T_1 - T_2)}{\ln(r_1/r_2)} = \frac{(T_1 - T_2)}{R_t} \tag{1.109}$$

If we now consider a composite hollow cylinder, the heat transfer equation is found to be as follows, where interfacial contact resistances are neglected:

$$Q = \frac{(T_1 - T_n)}{R_{t,t}} = HA(T_1 - T_n) \tag{1.110}$$

where $R_{t,t} = (1/2\pi r_1 L h_1) + (\ln(r_2/r_1)/2\pi k_1 L) + (\ln(r_3/r_2)/2\pi k_2 L) + \cdots (1/2\pi r_n L h_n)$.

1.6.7 *The Sphere*

The case of heat transfer through a sphere is not as common as the cylinder problem. Consider a hollow sphere of internal radius r_1 and external radius r_2 (Figure 1.19). Also, consider the inside and outside temperatures to be T_1 and T_2, respectively, and constant thermal conductivity with

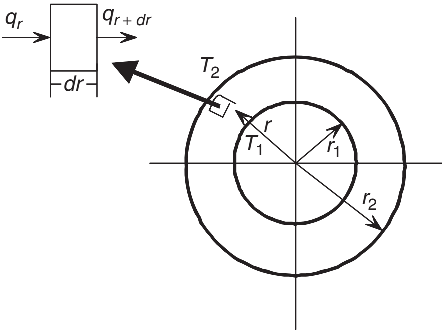

Figure 1.19 Heat conduction in a hollow sphere

no heat generation. We can express the heat conduction across the sphere wall in the form of Fourier's law:

$$Q = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{dT}{dr} \tag{1.111}$$

where $A = 4\pi r^2$ is the area normal to the direction of heat transfer.

After integrating Equation 1.111, we obtain the following expression:

$$Q = \frac{k(4\pi)(T_1 - T_2)}{(1/r_2 - 1/r_1)} = \frac{k(4\pi r_1 r_2)(T_1 - T_2)}{(r_2 - r_1)} = \frac{(T_1 - T_2)}{R_t} \tag{1.112}$$

If we now consider a composite hollow sphere, the heat transfer equation is determined to be as follows, neglecting interfacial contact resistances:

$$Q = \frac{(T_1 - T_n)}{R_{t,t}} = HA(T_1 - T_n) \tag{1.113}$$

where $R_{t,t} = (1/4\pi r_1^2 h_1) + (r_2 - r_1)/(4\pi r_1 r_2 k_1) + (r_3 - r_2)/(4\pi r_2 r_3 k_2) + \cdots (1/4\pi r_2^2 h_2)$.

1.6.8 Conduction with Heat Generation

The Plane Wall

Consider a plane wall, as shown in Figure 1.20a, in which there is uniform heat generation per unit volume. The heat conduction equation becomes

$$\frac{d^2T}{dx^2} + \frac{q_h}{k} = 0 \tag{1.114}$$

By integrating Equation 1.114 with the prescribed boundary conditions, $T(-L) = T_1$ and $T(L) = T_2$. The temperature distribution can be obtained as

$$T(x) = \left(\frac{q_h L^2}{2k}\right)\left(1 - \frac{x^2}{L^2}\right) + \left(\frac{T_2 - T_1}{2}\right)\left(\frac{x}{L}\right) + \left(\frac{T_2 + T_1}{2}\right) \tag{1.115}$$

The heat flux at any point in the wall can be found, depending on x, by using Equation 1.115 with Fourier's law.

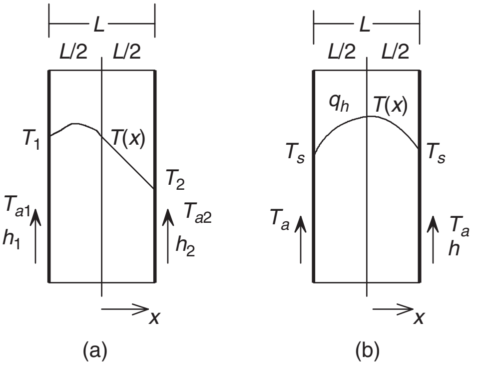

Figure 1.20 Heat conduction in a slab with uniform heat generation: (a) asymmetrical boundary conditions, (b) symmetrical boundary conditions

If $T_1 = T_2 \equiv T_s$, the temperature distribution is symmetrical about the midplane (Figure 1.20b). Then,

$$T(x) = \left(\frac{q_h L^2}{2k}\right)\left(1 - \frac{x^2}{L^2}\right) + T_s \tag{1.116}$$

At the plane of symmetry $dT/dx = 0$, and the maximum temperature at the midplane is

$$T(0) \equiv T_m = \left(\frac{q_h L^2}{2k}\right) + T_s \tag{1.117}$$

After combining Equations 1.116 and 1.117, we find the dimensionless temperature as follows:

$$\frac{(T(x) - T_m)}{(T_s - T_m)} = \left(\frac{x}{L}\right)^2 \tag{1.118}$$

The Cylinder

Consider a long cylinder (Figure 1.18) with uniform heat generation. The heat conduction equation can be rewritten as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{q_h}{k} = 0 \tag{1.119}$$

By integrating Equation 1.119, with the boundary conditions, $dT/dr = 0$, for the centerline ($r = 0$) and $T(r_1) = T_s$, the temperature distribution can be obtained as

$$T(r) = \left(\frac{q_h r_1^2}{4k}\right)\left(1 - \frac{r_2^2}{r_1^2}\right) + T_s \tag{1.120}$$

After combining terms, the dimensionless temperature equation results:

$$\frac{(T(r) - T_m)}{(T_s - T_m)} = 1 - \left(\frac{r_2}{r_1}\right)^2 \tag{1.121}$$

The approach mentioned previously can also be used for obtaining the temperature distributions in solid spheres and spherical shells for a wide range of boundary conditions.

1.6.9 Natural Convection

Heat transfer by natural (or free) convection involving motion in a fluid is due to differences in density and the action of gravity, which causes a natural circulation flow and leads to heat transfer. For many problems involving fluid flow across a surface, the superimposed effect of natural convection is negligibly small. The heat transfer coefficients for natural convection are generally much lower than that for forced convection. When there is no forced velocity of the fluid, heat is transferred entirely by natural convection (when there is negligible radiation). For some practical cases, it is necessary to consider the radiative effect on the total heat loss or gain. Radiation heat transfer may be of the same order of magnitude as natural convection in some circumstances even at room temperatures. Hence, wall temperatures in a room can affect the comfort of occupants.

It is pointed out that in many systems involving multimode heat transfer effects, natural convection provides the largest resistance to heat transfer, and therefore plays an important role in the design or performance of the system. Moreover, when it is desirable to minimize the heat transfer rates or to minimize operating costs, natural convection is often preferred to forced convection.

Natural convection is of significance in a wide variety of heating, cooling, and air-conditioning equipments. Natural convection heat transfer is influenced mainly by the gravitational force from thermal expansion, viscous drag, and thermal diffusion. For this reason, the gravitational acceleration, the coefficient of performance, the kinematic viscosity, and the thermal diffusivity directly affect natural convection. As shown in Table 1.8, these parameters depend on the fluid properties, the temperature difference between the surface and the fluid, and the characteristic length of the surface, which are involved in the Nusselt, Grashof, and Prandtl equations.

The natural convection boundary layers are not restricted to laminar flow. In many cases, there is a transition from laminar to turbulent flow. This is schematically shown in Figure 1.21 for a heated vertical plate.

Transition in a natural convection boundary layer is dependent on the relative magnitude of the buoyancy and viscous forces in the fluid. It is customary to correlate its occurrence in terms of the Rayleigh number. For example, for vertical plates the critical Rayleigh number is $\mathrm{Ra} \approx 10^9$. As in forced convection, transition to turbulence has a strong effect on the heat transfer. Numerous natural convection heat transfer correlations for several plates, pipes, wires, cylinder, and so on, along with a list of heat transfer coefficients, which were compiled from the literature, are given in Table 1.10. To calculate the natural convection heat transfer coefficient, one evaluates the Rayleigh number to determine whether the boundary layer is laminar or turbulent, and then applies the appropriate equation from this table.

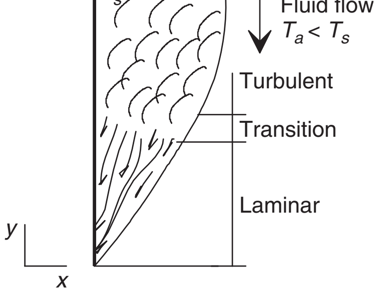

Figure 1.21 Natural convection on a vertical plate

Table 1.10 Natural convection heat transfer equations and correlations

Equation or correlation
• *General equations*
$\mathrm{Nu} = hY/k_f = c\mathrm{Ra}^n$ and $\mathrm{Ra} = \mathrm{Gr\ Pr} = g\beta(T_s - T_a)Y^3/\nu a$
where n is 1/4 for laminar flow and 1/3 for turbulent flow. Y denotes the height for vertical plates or pipes, diameter for horizontal pipes, and radius for spheres. $T_{fm} \equiv (T_s + T_a)/2$.
• *Correlations for vertical plates (or inclined plates, inclined up to 60°)*
$\mathrm{Nu} = [0.825 + 0.387\mathrm{Ra}^{1/6}/(1 + (0.492/\mathrm{Pr})^{9/16})^{4/9}]^2$ for an entire range of Ra
$\mathrm{Nu} = 0.68 + 0.67\mathrm{Ra}^{1/4}/(1 + (0.492/\mathrm{Pr})^{9/16})^{4/9}$ for $0 < \mathrm{Ra} < 10^9$
• *Correlations for horizontal plates* ($Y \equiv A_s/P$)
For upper surface of heated plate or lower surface of cooled plate:
$\mathrm{Nu} = 0.54\mathrm{Ra}^{1/4}$ for $10^4 \leq \mathrm{Ra} \leq 10^7$
$\mathrm{Nu} = 0.15\mathrm{Ra}^{1/3}$ for $10^7 \leq \mathrm{Ra} \leq 10^{11}$
For lower surface of heated plate or upper surface of cooled plate:
$\mathrm{Nu} = 0.27\mathrm{Ra}^{1/4}$ for $10^5 \leq \mathrm{Ra} \leq 10^{10}$
• *Correlations for horizontal cylinders*
$\mathrm{Nu} = hD/k = c\mathrm{Ra}^n$
where
$c = 0.675$ and $n = 0.058$ for $10^{-10} < \mathrm{Ra} < 10^{-2}$
$c = 1.020$ and $n = 0.148$ for $10^{-2} < \mathrm{Ra} < 10^2$
$c = 0.850$ and $n = 0.188$ for $10^2 < \mathrm{Ra} < 10^4$
$c = 0.480$ and $n = 0.250$ for $10^4 < \mathrm{Ra} < 10^7$
$c = 0.125$ and $n = 0.333$ for $10^7 < \mathrm{Ra} < 10^{12}$
$\mathrm{Nu} = [0.60 + 0.387\mathrm{Ra}^{1/6}/(1 + (0.559/\mathrm{Pr})^{9/16})^{8/27}]^2$ for an entire range of Ra
• *Correlations for spheres*
$\mathrm{Nu} = 2 + 0.589\mathrm{Ra}^{1/4}/(1 + (0.469/\mathrm{Pr})^{9/16})^{4/9}$ for $\mathrm{Pr} \geq 0.7$ and $\mathrm{Ra} \leq 10^{11}$
• *Heat transfer correlations*
$\mathrm{Gr\ Pr} = 1.6 \times 10^6 Y^3(\Delta T)$
with Y in m; ΔT in °C.
$h = 0.29(\Delta T/Y)^{1/4}$ for vertical small plates in laminar range
$h = 0.19(\Delta T)^{1/3}$ for vertical large plates in turbulent range
$h = 0.27(\Delta T/Y)^{1/4}$ for horizontal small plates in laminar range (facing upward when heated or downward when cooled)
$h = 0.22(\Delta T)^{1/3}$ for vertical large plates in turbulent range (facing downward when heated or upward when cooled)
$h = 0.27(\Delta T/Y)^{1/4}$ for small cylinders in laminar range
$h = 0.18(\Delta T)^{1/3}$ for large cylinders in turbulent range

Source: Dincer, 1997.

1.6.10 Forced Convection

The study of forced convection is concerned with the heat transfer occurring between a forced moving fluid and a solid surface. To apply Newton's law of cooling as given in Equation 1.85, it is necessary to determine the heat transfer coefficient. For this purpose, the Nusselt–Reynolds

Table 1.11 Forced-convection heat transfer equations and correlations

Equation or correlation		
• *Correlations for flat plate in external flow*		
$\mathrm{Nu} = 0.332\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}$	for $\mathrm{Pr} \geq 0.6$	for laminar; local; T_{fm}
$\mathrm{Nu} = 0.664\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}$	for $\mathrm{Pr} \geq 0.6$	for laminar; average; T_{fm}
$\mathrm{Nu} = 0.565\mathrm{Re}^{1/2}\mathrm{Pr}^{1/2}$	for $\mathrm{Pr} \leq 0.05$	for laminar; local; T_{fm}
$\mathrm{Nu} = 0.0296\mathrm{Re}^{4/5}\mathrm{Pr}^{1/3}$	for $0.6 \leq \mathrm{Pr} \leq 60$	for turbulent; local; T_{fm}, $\mathrm{Re} \leq 10^8$
$\mathrm{Nu} = (0.037\mathrm{Re}^{4/5} - 871)\mathrm{Pr}^{1/3}$	for $0.6 < \mathrm{Pr} < 60$	for mixed flow; average; T_{fm}, $\mathrm{Re} \leq 10^8$
• *Correlations for circular cylinders in cross-flow*		
$\mathrm{Nu} = c\mathrm{Re}^n\mathrm{Pr}^{1/3}$	for $\mathrm{Pr} \geq 0.7$	for average; T_{fm}; $0.4 < \mathrm{Re} < 4 \times 10^6$
where		
$c = 0.989$ and $n = 0.330$ for $0.4 < \mathrm{Re} < 4$		
$c = 0.911$ and $n = 0.385$ for $4 < \mathrm{Re} < 40$		
$c = 0.683$ and $n = 0.466$ for $40 < \mathrm{Re} < 4{,}000$		
$c = 0.193$ and $n = 0.618$ for $4{,}000 < \mathrm{Re} < 40{,}000$		
$c = 0.027$ and $n = 0.805$ for $40{,}000 < \mathrm{Re} < 400{,}000$		
$\mathrm{Nu} = c\mathrm{Re}^n\mathrm{Pr}^s(\mathrm{Pr}_a/\mathrm{Pr}_s)^{1/4}$	for $0.7 < \mathrm{Pr} < 500$	for average; T_a; $1 < \mathrm{Re} < 10^6$
where		
$c = 0.750$ and $n = 0.4$ for $1 < \mathrm{Re} < 40$		
$c = 0.510$ and $n = 0.5$ for $40 < \mathrm{Re} < 1{,}000$		
$c = 0.260$ and $n = 0.6$ for $10^3 < \mathrm{Re} < 2 \times 10^5$		
$c = 0.076$ and $n = 0.7$ for $2 \times 10^5 < \mathrm{Re} < 10^6$		
$s = 0.37$ for $\mathrm{Pr} \leq 10$		
$\mathrm{s} = 0.36$ for $\mathrm{Pr} > 10$		
$\mathrm{Nu} = 0.3 + [(0.62\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3})/(1 + (0.4/\mathrm{Pr})^{2/3})^{1/4}][1 + (\mathrm{Re}/28{,}200)^{5/8}]^{4/5}$ for $\mathrm{RePr} > 0.2$ for average; T_{fm}		
• *Correlations for spheres in cross-flow*		
$\mathrm{Nu}/\mathrm{Pr}^{1/3} = 0.37\mathrm{Re}^{0.6}/\mathrm{Pr}^{1/3}$		for average; T_{fm}; $17 < \mathrm{Re} < 70{,}000$
$\mathrm{Nu} = 2 + (0.4\mathrm{Re}^{1/2} + 0.06\mathrm{Re}^{2/3})\mathrm{Pr}^{0.4}(\mu_a/\mu_s)^{1/4}$		for $0.71 < \mathrm{Pr} < 380$
for average; T_a; $3.5 < \mathrm{Re} < 7.6 \times 10^4$; $1 < (\mu_a/\mu_s) < 3.2$		
• *Correlation for falling drop*		
$\mathrm{Nu} = 2 + 0.6\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}[25(x/D)^{-0.7}]$		for average; T_a

Source: Dincer, 1997.

correlations may be used. The definitions of the Nusselt and Reynolds numbers have been given in Table 1.9. Forced air and water coolers, forced air and water evaporators and condensers, and heat exchangers are examples of equipments commonly involved in forced convection heat transfer.

The various kinds of forced convection, such as flow in a tube, flow across a tube, and flow across a flat plate, may be solved mathematically when certain assumptions are made with regard to the boundary conditions. It is extremely difficult to obtain exact solutions to such problems, especially in the event of turbulent flow, but approximate solutions can sometimes be obtained using appropriate assumptions.

The essential first step in the solution of a convection heat transfer problem is to determine whether the boundary layer is *laminar* or *turbulent*. These conditions affect the convection heat transfer coefficient and hence the convection heat transfer rates.

The conditions of laminar and turbulent flows on a flat plate are shown in Figure 1.13. In the laminar boundary layer, fluid motion is highly ordered and it is possible to identify streamlines along which particles move. Fluid motion in the turbulent boundary layer, on the other hand, is highly irregular, and is characterized by velocity fluctuations that begin to develop in the transition region (after this, the boundary layer becomes completely turbulent). These fluctuations enhance the transfer of momentum, heat, and species, and hence increase surface friction as well as convection transfer rates. In the laminar sublayer, which is nearly linear, transport is dominated by diffusion and the velocity profile. There is an adjoining buffer layer in which diffusion and turbulent mixing are comparable. In the turbulent region, transport is dominated by turbulent mixing.

The *critical Reynolds number* is the value of Re for which transition begins, and for external flow it is known to vary from 10^5 to 3×10^6, depending on the surface roughness, the turbulence level of the free stream, and the nature of the pressure variation along the surface. A representative value of Re is generally assumed for boundary layer calculations:

$$\mathrm{Re}_c = \frac{\rho U_a X_c}{\mu} = \frac{U_a X_c}{\upsilon} = 5 \times 10^5 \tag{1.122}$$

For smooth circular tubes, when the Reynolds number is less than 2100, the flow is laminar, and when it is greater than 10,000, the flow is turbulent. The range between these values represents the transition region.

We give a list of various forced-convection heat transfer correlations (the Nusselt–Reynolds correlations), direct convection heat transfer coefficient equations, along with the relevant parameters and remarks, which are compiled from literature in Table 1.11. In many of these equations, the *film temperature* is used, and is defined as $T_{fm} = (T_s + T_a)/2$.

1.7 Concluding Remarks

In this chapter, a summary is presented of general introductory aspects of thermodynamics, fluid flow, and heat transfer, and related fundamental definitions and physical quantities, to provide a sufficient thermal sciences background for understanding thermal energy storage systems and applications, and their operations. The background provided here is also useful in the energy, exergy, and other analyses presented subsequently.

Nomenclature

a	acceleration, m/s^2; thermal diffusivity, m^2/s; absorptivity
A	cross-sectional area, m^2; surface area, m^2
Bi	Biot number
c	mass fraction; constant in Tables 1.10 and 1.11
c_p	specific heat at constant pressure, kJ/kg K
c_v	specific heat at constant volume, kJ/kg K
C_f	average skin-friction coefficient
d	diameter, m; depth normal to flow, m
D	diameter, m
E	energy, J or kJ; electric potential, V; constant
$\dot{E}$	energy rate, W or kW
F	force; drag force, N
Fo	Fourier number
g	acceleration due to gravity ($= 9.81\,\mathrm{m/s^2}$)
G	mass flow velocity, kg/s m^2
Gz	Graetz number
Gr	Grashof number
h	specific enthalpy, kJ/kg; heat transfer coefficient, W/m^2 °C; head, m
H	enthalpy, kJ; overall heat transfer coefficient, W/m^2 °C; head, m

I	electric current, A
k	thermal conductivity, W/m °C
K	adiabatic modulus
KE	kinetic energy, J or kJ
L	thickness, m
m	mass, kg; constant
$\dot{m}$	mass flow rate, kg/s
M	molecular weight, kg/kmol
n	mole number, kmol; constant exponent in Tables 1.9 and 1.10
Nu	Nusselt number
P	perimeter, m; pressure, Pa or kPa
P^*	constant-pressure gradient, Pa or kPa
Pe	Peclet number
PE	potential energy, J or kJ
Pr	Prandtl number
q	heat rate per unit area, W/m^2; flow rate per unit width or depth
q_h	heat generation rate per unit volume, W/m^3
Q	heat transfer, J or kJ
$\dot{Q}$	heat transfer rate, W or kW
r	reflectivity; radial coordinate; radial distance, m
R	gas constant, kJ/kg K; radius, m
$\overline{\mathrm{R}}$	universal gas constant, kJ/kg K
R_t	thermal resistance, °C/W
Ra	Rayleigh number
Re	Reynolds number
s	specific entropy, kJ/kg; streamline direction; distance, m; constant exponent in Tables 1.10 and 1.11
S	entropy, kJ/K
St	Stanton number
t	time, s; transmissivity
T	temperature, °C or K
T_s	absolute temperature of object surface, K
u	specific internal energy, kJ/kg; velocity in x direction, m/s; variable velocity, m/s
U	internal energy, kJ; flow velocity, m/s
x	quality, kg/kg; cartesian coordinate; variable
X	length for plate, m
v	specific volume, m^3/kg; velocity in y direction, m/s
$\overline{\mathrm{v}}$	molal specific volume, kmol/kg
V	volume, m^3; velocity, m/s
V_x	velocity in x direction, m/s
V_r	velocity in radial direction, m/s
V_y	velocity in y direction, m/s
V_z	velocity in z direction, m/s
V_θ	tangential velocity, m/s
$\dot{\mathrm{V}}$	volumetric flow rate, m^3/s
w	velocity in z direction, m/s
y	mole fraction; cartesian coordinate, variable; coordinate normal to flow
Y	characteristic dimension (length), m
z	cartesian coordinate, variable
Z	compressibility factor (Equation 1.24); elevation, m

Greek Letters

ϕ	temperature difference, °C or K
θ	angle
β	volumetric coefficient of thermal expansion, 1/K
δ	increment; difference
μ	dynamic viscosity, kg/ms; root of the characteristic equation
ρ	density, kg/m^3
ν	kinematic viscosity, m^2/s
Δ	thickness of the stagnant film of fluid on the surface, m
ΔT	temperature difference, K; overall temperature difference, °C or K
σ	Stefan–Boltzmann constant, W/m^2 K^4; electrical conductivity, 1/ohm
ε	surface emissivity, eddy viscosity
τ	shear stress, N/m^2
Σ	summation
π	number (= 3.14159)

Subscripts and Superscripts

a	air; medium; surroundings
av	average
A	fluid A
b	black
B	fluid B
c	convection, critical
cd	conduction
cs	control surface
cv	control volume
D	diameter
e	electrical; end; exit
f	fluid; final; flow; force; friction
fm	film condition
h	heat generation
H	high temperature
hs	heat storage
i	component; input
ie	internal energy
liq	liquid
l	liquid
L	low temperature
m	midplane for plane wall; centerline for cylinder
mix	mixture
n	nth value
nb	nonblack
p	previous
r	radiation
s	surface; near surface; saturation; free stream; in direction parallel to streamline
t	total; thermal
tot	total
x	x direction

v	vapor
vap	vapor
y	y direction
z	z direction
0	surroundings; ambient; environment; reference
1	first value; 1st state; initial
1, 2, 3	points

References

Borgnakke, C. and Sonntag, R.E. (2008). *Fundamentals of Thermodynamics*, 7th edition, Wiley, New York.

Dincer, I. (1997). *Heat Transfer in Food Cooling Applications*, Taylor & Francis, Washington, DC.

Dincer, I. (1998). Thermodynamics, exergy and environmental impact, *Proceedings of the ISTP-11, The Eleventh International Symposium on Transport Phenomena*, pp. 121–125, 29 November-3 December, Hsinchu, Taiwan.

Dincer, I. and Rosen, M.A. (1999). Energy, environment and sustainable development, *Applied Energy* 64, 427–440.

Dincer, I. and Rosen, M.A. (2007). *Energy: Energy, Environment and Sustainable Development*, Elsevier, New York.

Moran, M.J. and Shapiro, H.N. (2007). *Fundamentals of Engineering Thermodynamics*, 6th edition, Wiley, New York.

Olson, R.M. and Wright, S.J. (1991). *Essentials of Engineering Fluid Mechanics*, Harper & Row, New York.

Raznjevic, K. (1995). *Handbook of Thermodynamic Tables*, 2nd edition, Begell House, New York.

Szargut, J., Morris, D.R. and Steward, F.R. (1988). *Exergy Analysis of Thermal, Chemical, and Metallurgical Processes*, Hemisphere, New York.

Study Questions/Problems

Introduction, Thermodynamic Properties

1.1 Why are SI units most widely used throughout the world?

1.2 What is the difference between mass and weight?

1.3 What is specific heat? Define two commonly used specific heats. Is specific heat a function of temperature?

1.4 Explain the operating principle of thermocouples. List some typical applications for different types of thermocouples. What is the main advantage of thermocouples over other temperature sensors?

1.5 Consider the flow of a refrigerant vapor through a compressor, which is operating at steady-state conditions. Do mass flow rate and volume flow rate of the refrigerant across the compressor remain constant?

1.6 Consider a refrigeration system consisting of a compressor, an evaporator, a condenser, and an expansion valve. Is it best to evaluate each component as a closed system or as a control volume, and as a steady-flow system or unsteady-flow system? Explain.

1.7 What is the difference between an adiabatic system and an isolated system?

1.8 Define intensive and extensive properties. Identify the following properties as intensive or extensive: mass, volume, density, specific volume, energy, specific enthalpy, total entropy, temperature, pressure.

1.9 Define the terms system, process, and cycle.

1.10 What is the difference between gauge pressure, absolute pressure, and vacuum? Define atmospheric pressure.

1.11 What is the difference between mass flow rate and volumetric flow rate? How are these related?

1.12 Define the critical point and explain the difference between critical point and triple point.

1.13 Define sensible and latent heats, and latent heat of fusion. What are their units?

1.14 What is the weight of a 10-kg substance in N, kN, kgf, and lb_f?

1.15 The vacuum pressure of a tank is given to be 40 kPa. If the atmospheric pressure is 95 kPa, what is the gauge pressure and absolute pressure in kPa, kN/m^2, lbf/in^2, psi, and mm Hg.

1.16 Express the temperature −40 °C in units of Fahrenheit (°F), Kelvin (K), and Rankine (R).

1.17 The temperature of air changes by 10 °C during a process. Express this temperature change in Kelvin (K), Fahrenheit (°F), and Rankine (R) units.

1.18 The specific heat of water at 25 °C is given to be 4.18 kJ/kg °C. Express this value in kJ/kg K, J/g °C, kcal/kg °C, and Btu/lbm °F.

1.19 A 0.2-kg mass of R134a at 700 kPa pressure and at 4 °C is heated until 50% of mass is vaporized. Determine the temperature at which the refrigerant is vaporized, and the sensible heat and the latent heat are transferred to the refrigerant.

1.20 A 0.5-lbm mass of R134a at 100 psia pressure and 40 °F is heated until 50% of mass is vaporized. Determine the temperature at which the refrigerant is vaporized, and the sensible heat and the latent heat are transferred to the refrigerant.

1.21 A 2-kg mass of ice initially at −18 °C is heated until 75% of the mass is melted. Determine the sensible heat and the latent heat transferred to the water. The specific heat of ice at 0 °C is 2.11 kJ/kg °C, and the latent heat of fusion of water at 0 °C is 334.9 kJ/kg.

1.22 A 2-kg mass of ice initially at −18 °C is heated until it becomes liquid water at 20 °C. Determine the sensible heat and the latent heat transferred to the water. The specific heat of ice at 0 °C is 2.11 kJ/kg °C, and the latent heat of fusion of water at 0 °C is 334.9 kJ/kg.

1.23 Refrigerant 134a enters the evaporator of a refrigeration system at −24 °C with a quality of 25% at a rate of 0.22 kg/s. If the refrigerant leaves the evaporator as a saturated vapor, determine the rate of heat transfer to the refrigerant. If the refrigerant is heated by water in the evaporator, which experiences a temperature rise of 16 °C, determine the mass flow rate of water.

Ideal Gases and the First Law of Thermodynamics

1.24 What is the compressibility factor?

1.25 When can we invoke the ideal gas assumption for real gases?

1.26 Define isothermal, isobaric, and isochoric processes.

1.27 What is an isentropic process? Is a constant-entropy process necessarily reversible and adiabatic?

1.28 What is the difference between heat and work?

1.29 An elastic tank contains 0.8 kmol of air at 23 °C and 600 kPa. Determine the volume of the tank. If the volume is doubled at the same pressure, what is the temperature at the new state?

1.30 A 50-L piston–cylinder device contains oxygen at 52 °C and 170 kPa. If the oxygen is heated until its temperature reaches 77 °C, what is the amount of heat transfer during the process?

1.31 A 50-L rigid tank contains oxygen at 52 °C and 170 kPa. If the oxygen is heated until its temperature reaches 77 °C, what is the amount of heat transfer during the process?

1.32 A 50-L rigid tank contains oxygen at 52 °C and 170 kPa. If the oxygen is heated until the temperature reaches 77 °C, what is the entropy change during the process?

1.33 A rigid tank contains 2.5 kg of oxygen at 52 °C and 170 kPa. If the oxygen is heated in an isentropic process until it reaches 77 °C, what is the pressure at the final state? What is the work interaction during this process?

1.34 A piston–cylinder device contains 2.5 kg oxygen at 52 °C and 170 kPa. If the oxygen is heated until it reaches 77 °C, what is the work done and the amount of heat transfer during the process?

Exergy

1.35 What is the Kelvin–Planck statement of the second law of thermodynamics?

1.36 What is the Clausius statement of the second law of thermodynamics?

1.37 Define the terms energy, exergy, entropy, and enthalpy.

1.38 What is the second-law efficiency? How does it differ from the first-law efficiency?

1.39 What is the relationship between entropy generation and irreversibility?

1.40 What are the two common causes of irreversibility?

1.41 During an irreversible process, do the parameters mass, energy, entropy, and exergy decrease or increase or remain conserved?

1.42 How does an exergy analysis help the goal of more efficient energy-resource use? What are the advantages of using exergy analysis?

General Aspects of Fluid Flow

1.43 What is the physical meaning of the Reynolds number? What makes the flow laminar and what makes it turbulent?

1.44 What is viscosity? How does viscosity change with temperature for gases and for liquids?

General Aspects of Heat Transfer

1.45 What is the difference between heat conduction and heat convection?

1.46 Define the terms forced convection and natural convection, and explain the difference between them.

1.47 Define the term heat generation. Give some examples.

1.48 What are the modes of heat transfer? Explain the physical mechanism of each mode.

1.49 How much energy does it take to convert 10.0 kg of ice at 0 °C to water at 25 °C?

1.50 A 20-cm thick wall of a house made of brick ($k = 0.72$ W/m °C) is subjected to inside air at 22 °C with a convection heat transfer coefficient of 15 W/m^2 °C. The temperature of the inner surface of the wall is 18 °C and the outside air temperature is −1 °C. Determine the temperature of the outer surface of the wall and the heat transfer coefficient at the outer surface.

1.51 A satellite is subjected to solar energy at a rate of 300 W/m^2. The absorptivity of the satellite surface is 0.75 and its emissivity is 0.60. Determine the equilibrium temperature of the satellite.

1.52 An 80-cm-diameter spherical tank made of steel contains liquefied natural gas (LNG) at −160 °C. The tank is insulated with a 4-cm thickness of insulation ($k = 0.015$ W/m °C). The tank is subjected to ambient air at 18 °C with a convection heat transfer coefficient of 20 W/m^2 °C. How long will it take for the temperature of the LNG to decrease to −150 °C. Neglect the thermal resistance of the steel tank. The density and the specific heat of LNG are 425 kg/m^3 and 3.475 kJ/kg °C, respectively.