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Introduction

1.1 Importance of Heat and Mass Transfer

The subject of heat and mass transfer is of fundamental importance in many branches of engineering. A *mechanical engineer* may be interested to know the mechanisms of heat transfer involved in the operation of equipment, for example, boilers, condensers, air preheaters, economizers etc., in a thermal power plant in order to improve their performance. Nuclear power plants require precise information on heat transfer as safe operation is an important factor in their design. Refrigeration and air-conditioning systems also involve heat-exchanging devices, which need careful design. *Electrical engineers* are keen to avoid material damage in electric motors, generators and transformers due to hot spots, developed by improper heat transfer design. An *electronic engineer* is interested in knowing efficient methods of heat dissipation from chips and semi-conductor devices so that they function within safe operating temperatures. A *computer hardware engineer* is interested to know the cooling requirements of circuit-boards, as the miniaturization of computing devices is advancing at a rapid rate. *Chemical engineers* are interested in heat and mass transfer processes in various chemical reactions. A *metallurgical engineer* would be interested in knowing the rate of heat transfer required for a particular heat treatment process, e.g. the rate of cooling in a casting process has a profound influence on the quality of the final product. *Aeronautical engineers* are interested in knowing the heat transfer rate in rocket nozzles and in heat shields used in re-entry vehicles. An *agricultural engineer* would be interested in the drying of food grains, food processing and preservation. A *civil engineer* would need to be aware of the thermal stresses developed in quick setting concrete, the influence of heat and mass transfer on building and building materials as well as the effect of heat on nuclear containment and buildings etc. An *environmental engineer* is concerned with the effect of heat on dispersion of pollutants in air, transport of pollutants in soils, lakes and seas and their impact on life. A *bioengineer* is often interested in the heat and

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mass transfer processes, such as hypothermia and hyperthermia associated with the human body.

The above-mentioned applications are only a sample of heat and mass transfer applications. The solar system and the associated energy transfer from the sun are the principal factors for existence of life on Earth. It is not untrue to say that it is extremely difficult, often impossible, to avoid some form of heat transfer in any process on Earth.

The study of heat and mass transfer provides economical and efficient solutions for many critical problems encountered in diverse engineering items of equipment. For example, we can consider the development of heat pipes which can transport heat at a much greater rate than that of copper or silver rods of the same dimensions and even at almost isothermal conditions. The development of present-day gas turbine blades, where the gas temperature exceeds the melting point of the blade material, is possible by providing efficient cooling systems. This is another example of the success of heat transfer design methods. The design of computer chips, which encounter heat flux of the order occurring in re-entry vehicles, especially when the surface temperature of the chips is limited to less than 100 °C, is again a success story of heat transfer design.

Although there are many successful heat transfer designs, further developments on heat and mass transfer studies are necessary in order to increase the life span and efficiency of the many devices discussed previously, which can lead to many more new inventions. Also, if we are to protect our environment, it is essential to understand the many heat and mass transfer processes involved and if necessary to take appropriate action.

1.2 Heat Transfer Modes

Heat transfer is that section of engineering science that studies the energy transport between material bodies due to temperature difference (Bejan 1993; Holman 1989; Incropera and Dewitt 1990; Sukhatme 1992). The three modes of heat transfer are:

- (a) conduction
- (b) convection and
- (c) radiation.

The conduction mode of heat transport occurs either because of an exchange of energy from one molecule to another without actual motion of the molecules, or is due to the motion of free electrons if they are present. Therefore, this form of heat transport depends heavily on the properties of the medium and takes place in solids, liquids and gases if a difference in temperature exists.

Molecules present in liquids and gases have freedom of motion and by moving from a hot to a cold region, they carry energy with them. The transfer of heat from one region to another due to such macroscopic motion in a liquid or gas, added to the energy transfer by conduction within the fluid, is called heat transfer by convection. Convection may be either free, forced or mixed. When fluid motion occurs due to a density variation caused by temperature differences, the situation is said to be a free or natural convection. When the fluid motion is caused by an external force, such as pumping or blowing, the state is defined as being forced convection.

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A mixed convection state is one in which both natural and forced convection are present. Convection heat transfer also occurs in boiling and condensation processes.

All bodies emit thermal radiation at all temperatures. This is the only mode which does not require a material medium for heat transfer to occur. The nature of thermal radiation is such that a propagation of energy, carried by *electromagnetic waves*, is emitted from the surface of the body. When these electromagnetic waves strike other body surfaces, a part is reflected, a part transmitted and the remaining part is absorbed.

All modes of heat transfer are generally present in varying degrees in a real physical problem. The important aspects in solving heat transfer problems are to identify the significant modes and to decide whether the heat transferred by other modes can be neglected.

1.3 The Laws of Heat Transfer

It is important to quantify the amount of energy being transferred per unit time and for that we require the use of rate equations. For heat conduction, the rate equation is known as *Fourier's law* (Fourier 1955) which is expressed for one dimension, as

$$q_x = -k \frac{dT}{dx}, \quad (1.1)$$

where q_x is the heat flux in the x direction (W/m^2); k is the thermal conductivity (W/mK , a property of the material, see Table 1.1) and dT/dx the temperature gradient (K/m).

Table 1.1 Typical values of thermal conductivity of some materials in W/mK at 20°C .

Material	Thermal conductivity, k
<i>Metals:</i>	
Pure silver	410
Pure copper	385
Pure aluminium	200
Pure iron	73
<i>Alloys:</i>	
Stainless steel (18% Cr, 8% Ni)	16
Aluminium alloy (4.5% Cr)	168
<i>Non metals:</i>	
Plastics	0.6
Wood	0.2
<i>Liquid:</i>	
Water	0.6
<i>Gasses:</i>	
Dry air	0.025 (at atmospheric pressure)

Table 1.2 Typical values of heat transfer coefficient in $\text{W/m}^2\text{K}$

Gases (stagnant)	15
Gases (flowing)	15–250
Liquids (stagnant)	100
Liquids (flowing)	100–2000
Boiling liquids	2000–35 000
Condensing vapors	2000–25 000

For convective heat transfer, the rate equation is given by *Newton's law* of cooling (Whewell 1866) as

$$q = h(T_w - T_a), \quad (1.2)$$

where q is the convective heat flux; (W/m^2); ($T_w - T_a$) the temperature difference between the wall and the fluid and h is the convection heat transfer coefficient ($\text{W/m}^2\text{K}$) (or film coefficient, see Table 1.2).

The convection heat transfer coefficient frequently appears as a boundary condition in the solution of heat conduction through solids, where h is often known (Table 1.2).

The maximum flux that can be emitted by radiation from a black surface is given by the *Stefan-Boltzmann Law* (Boltzmann 1884; Stefan 1879), that is,

$$q = \sigma T_w^4, \quad (1.3)$$

where q is the radiative heat flux (W/m^2); σ is the Stefan-Boltzmann constant (5.669×10^{-8}), in $\text{W/m}^2\text{K}^4$ and T_w is the surface temperature (K).

The heat flux emitted by a real surface is less than that of a black surface and is given by

$$q = \epsilon \sigma T_w^4, \quad (1.4)$$

where ϵ is the radiative property of the surface and is referred to as the emissivity. The net radiant energy exchange between any two surfaces 1 and 2 is given by

$$Q = F_\epsilon F_G \sigma A_1 (T_1^4 - T_2^4), \quad (1.5)$$

where F_ϵ is a factor which takes into account the nature of the two radiating surfaces; F_G a factor which takes into account the geometric orientation of the two radiating surfaces and A_1 is the area of surface 1.

When a heat transfer surface, at temperature T_1 , is completely enclosed by a much larger surface at temperature T_2 , the net radiant exchange can be calculated by

$$Q = qA_1 = \epsilon_1 \sigma A_1 (T_1^4 - T_2^4). \quad (1.6)$$

With respect to the laws of thermodynamics, only the first law (Clausius 1850) is of interest in heat transfer problems. The increase of energy in a system is equal to the difference

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between the energy transfer by heat to the system and the energy transfer by work done on the surroundings by the system, that is,

$$dE = dQ - dW, \quad (1.7)$$

where Q is the total heat entering the system and W is the work done by the system on the surroundings. Since we are interested in the rate of energy transfer in heat transfer processes, we can restate the first law of thermodynamics as:

“The rate of increase of the energy of the system is equal to the difference between the rate at which energy enters the system and the rate at which the system does work on the surroundings,” that is,

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}, \quad (1.8)$$

where t is the time.

1.4 Mathematical Formulation of Some Heat Transfer Problems

In analyzing a thermal system, the engineer should be able to identify the relevant heat transfer processes and only then can the system behavior be quantified properly. In this section, some typical heat transfer problems are formulated by identifying the appropriate heat transfer mechanisms.

1.4.1 Heat Transfer from a Plate Exposed to Solar Heat Flux

Consider a plate of size $L \times B \times d$ exposed to the solar flux of intensity q_s as shown in Figure 1.1. In many solar applications, such as a solar water heater, solar cooker etc., the temperature of the plate is a function of time. The plate loses heat by convection and radiation to the ambient air, which is at temperature T_a . Some heat flows through the plate and is convected

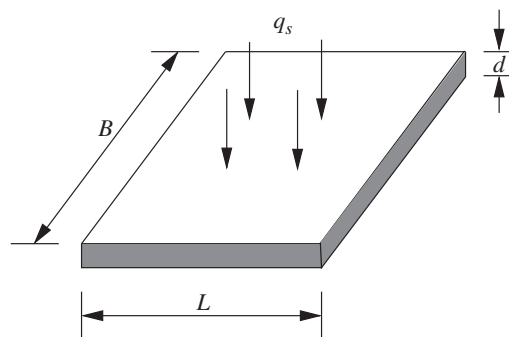


Figure 1.1 Heat transfer from a plate subjected to solar heat flux.

to the atmosphere from the bottom side. We shall apply the law of conservation of energy to derive an appropriate equation, the solution of which gives the temperature of the plate with respect to time.

Heat entering the top surface of the plate:

$$q_s A_T. \quad (1.9)$$

Heat loss from the plate to the surroundings:

Top surface:

$$hA_T(T - T_a) + \epsilon\sigma A_T(T^4 - T_a^4), \quad (1.10)$$

Side surface:

$$hA_S(T - T_a) + \epsilon\sigma A_S(T^4 - T_a^4), \quad (1.11)$$

Bottom surface:

$$hA_B(T - T_a) + \epsilon\sigma A_B(T^4 - T_a^4), \quad (1.12)$$

where the subscripts T , S and B refer respectively to the top, side and bottom surface areas. The topic of radiation exchange between a gas and a solid surface is not simple. Readers are referred to appropriate texts for details (Holman 1989; Siegel and Howell 1992). Under steady-state conditions, the heat received by the plate is lost to the surroundings, thus

$$q_s A_T = hA_T(T - T_a) + \epsilon\sigma A_T(T^4 - T_a^4) + hA_S(T - T_a) + \epsilon\sigma A_S(T^4 - T_a^4) + hA_B(T - T_a) + \epsilon\sigma A_B(T^4 - T_a^4). \quad (1.13)$$

This is a nonlinear algebraic equation because of the presence of the T^4 term. The solution of this equation results in the steady-state temperature of the plate. If we want to calculate the temperature of the plate as a function of time, t , then we have to consider the rate of rise in the internal energy of the plate. Substituting $E = \text{volume} \times \rho \times c_p \times T$ into the LHS of the Equation (1.8) gives

$$(\text{volume}) \times \rho c_p \frac{dT}{dt} = (LBd)\rho c_p \frac{dT}{dt}, \quad (1.14)$$

where ρ is the density and c_p is the specific heat of the plate. Thus, at any instant of time, the difference between the heat received and lost (work done on the surroundings) by the plate will be equal to the rate of change in internal energy heat stored (Equation (1.8)). Thus,

$$(LBd)\rho c_p \frac{dT}{dt} = q_s A_T - [hA_T(T - T_a) + \epsilon\sigma A_T(T^4 - T_a^4) + \epsilon\sigma A_S(T^4 - T_a^4) + hA_B(T - T_a) + \epsilon\sigma A_B(T^4 - T_a^4)]. \quad (1.15)$$

This is a first-order nonlinear differential equation, which requires an initial condition, viz.,

$$\text{at } t = 0, T = T_a. \quad (1.16)$$

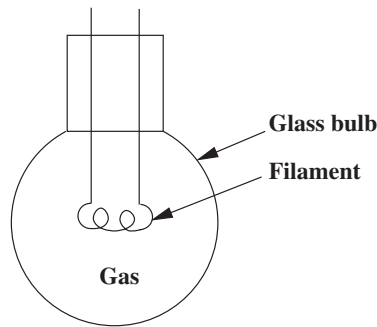


Figure 1.2 Energy balance in an incandescent light source.

The solution is determined iteratively because of the nonlinearity of the problem. Equation (1.15) can be simplified by substituting relations for the surface areas. It should be noted, however, that this is a general equation, which can be used for similar systems.

It is important to note that the spatial variation of temperature within the plate is neglected here. However, this variation can be included via Fourier's law of heat conduction, Equation (1.1). Such a variation is necessary if the plate is not thin enough to reach equilibrium instantly (Section 1.5).

1.4.2 Incandescent Lamp

Figure 1.2 shows an idealized incandescent lamp. The filament is heated to a temperature T_f by an electric current. Heat is convected to the surrounding gas and is radiated to the wall, which also receives heat from the gas by convection. The wall in turn convects and radiates heat to the ambient at T_a . A formulation of equations, based on energy balance, is necessary in order to determine the temperature of the gas and the wall with respect to time.

1.4.2.1 Gas

Rise in internal energy of the gas:

$$\rho_g c_{pg} \frac{dT_g}{dt}. \quad (1.17)$$

Convection from the filament to the gas:

$$h_f A_f (T_f - T_g). \quad (1.18)$$

Convection from the gas to the wall:

$$h_g A_g (T_g - T_w). \quad (1.19)$$

Radiation from the filament to the gas:

$$\epsilon_f A_f \sigma (T_f^4 - T_g^4). \quad (1.20)$$

Now, the energy balance for the gas gives

$$\rho_g c_{pg} \frac{dT_g}{dt} = h_f A_f (T_f - T_g) - h_g A_g (T_g - T_w) + \epsilon_f A_f \sigma (T_f^4 - T_g^4). \quad (1.21)$$

1.4.2.2 Wall

Rise in internal energy of the wall:

$$\rho_w c_{pw} \frac{dT_w}{dt}. \quad (1.22)$$

Radiation from the filament to the wall:

$$\epsilon_f \sigma A_f (T_f^4 - T_w^4). \quad (1.23)$$

Convection from the wall to ambient:

$$h_w A_w (T_w - T_a). \quad (1.24)$$

Radiation from the wall to ambient:

$$\epsilon_w \sigma A_w (T_w^4 - T_a^4). \quad (1.25)$$

Energy balance for the wall gives

$$\rho_w c_{pw} \frac{dT_w}{dt} = h_g A_g (T_g - T_w) + \epsilon_f \sigma A_f (T_f^4 - T_w^4) - h_w A_w (T_w - T_a) - \epsilon_w \sigma A_w (T_w^4 - T_a^4), \quad (1.26)$$

where ρ_g is the density of the gas in the bulb; c_{pg} the specific heat of the gas; ρ_w the density of the wall of the bulb; c_{pw} the specific heat of the wall; h_f the heat transfer coefficient between filament and gas; h_g the heat transfer coefficient between gas and wall; h_w the heat transfer coefficient between wall and ambient and ϵ the emissivity. The subscripts f , w , g and a respectively indicate the filament, wall, gas and ambient.

Equations (1.21) and (1.26) are first-order nonlinear differential equations. The initial conditions required are

At $t = 0$,

$$T_g = T_a \quad \text{and} \quad T_w = T_a. \quad (1.27)$$

The simultaneous solution of Equations (1.21) and (1.26), along with the above initial condition, results in the temperatures of the gas and the wall as functions of time.

1.4.3 Systems with a Relative Motion and Internal Heat Generation

The extrusion of plastics, drawing of wires and artificial fiber (optical fiber), suspended electrical conductors of various shapes, continuous casting etc. can be treated alike.

In order to derive an energy balance for such a system, we consider a small differential control volume of length, Δx , as shown in Figure 1.3. In this problem, the heat lost to the environment by radiation is assumed to be negligibly small. The energy is conducted, convected

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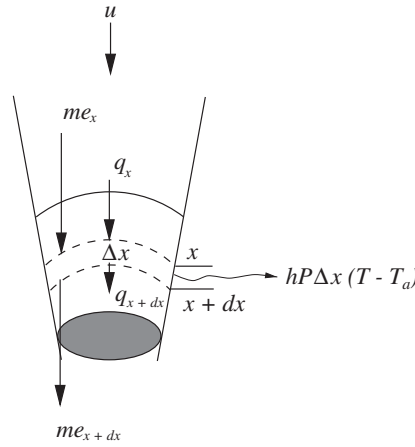


Figure 1.3 Conservation of energy in a moving body.

and transported with the material in motion. With reference to Figure 1.3, we can write the following equations of conservation of energy, that is,

$$Q_x + me_x + GA\Delta x = Q_{x+dx} + me_{x+dx} + hP\Delta x(T - T_a), \quad (1.28)$$

where $Q = Aq$ is the total heat; m is the mass flow ρAu and is assumed to be constant; e_x is the specific energy; ρ the density of the material; A the cross-sectional area; P the perimeter of the control volume; G is the heat generated per unit volume and u is the velocity at which the material is moving. Using the Taylor series of expansion we obtain

$$m(e_x - e_{x+dx}) = -m \frac{de_x}{dx} \Delta x = -mc_p \frac{dT}{dx} \Delta x. \quad (1.29)$$

Note that $de_x = c_p dT$ at constant pressure. Similarly, using Fourier's law (Equation (1.1)),

$$Q_x - Q_{x+dx} = -\frac{dQ_x}{dx} \Delta x = \frac{d}{dx} \left[kA \frac{dT}{dx} \right] \Delta x. \quad (1.30)$$

On substituting Equations (1.29) and (1.30) into Equation (1.28), we obtain the following conservation equation,

$$\frac{d}{dx} \left[kA \frac{dT}{dx} \right] - hP(T - T_a) - \rho c_p Au \frac{dT}{dx} + GA = 0. \quad (1.31)$$

In the above equation, the first term is derived from the heat diffusion (conduction) within the material, the second term is due to convection from the material surface to ambient, the third term represents the heat transport due to the motion of the material, and finally the last term is added to account for heat generation within the body.

1.5 Heat Conduction Equation

The determination of temperature distribution in a medium (solid, liquid, gas or combination of phases) is the main objective of a conduction analysis, that is, to know the temperature in the medium as a function of space at steady state and as a function of time during the transient state. Once this temperature distribution is known, the heat flux at any point within the medium, or on its surface, may be computed from Fourier's law, Equation (1.1). A knowledge of the temperature distribution within a solid can be used to determine the structural integrity via a determination of the thermal stresses and distortion. The optimization of the thickness of an insulating material and the compatibility of any special coatings or adhesives used on the material can be studied by knowing the temperature distribution.

We shall now derive the conduction equation in Cartesian coordinates by applying the energy conservation law to a differential control volume as shown in Figure 1.4. The solution of the resulting differential equation, with prescribed boundary conditions, gives the temperature distribution in the medium.

The Taylor series expansion gives:

$$\begin{aligned} Q_{x+\Delta x} &= Q_x + \frac{\partial Q_x}{\partial x} \Delta x \\ Q_{y+\Delta y} &= Q_y + \frac{\partial Q_y}{\partial y} \Delta y \\ Q_{z+\Delta z} &= Q_z + \frac{\partial Q_z}{\partial z} \Delta z. \end{aligned} \quad (1.32)$$

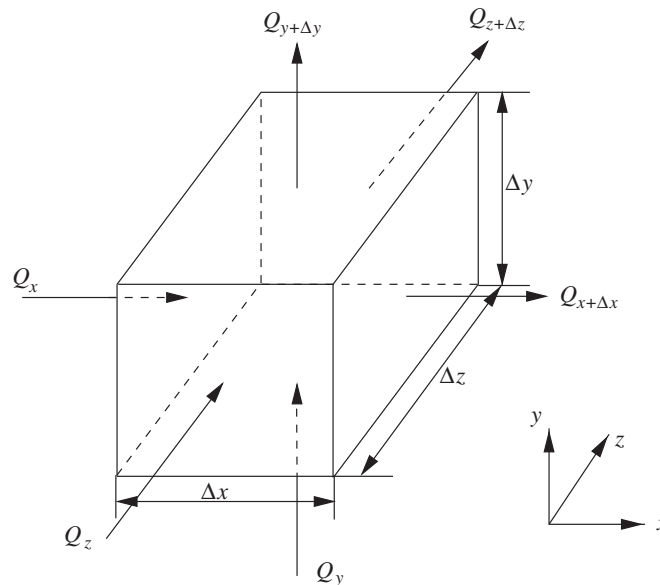


Figure 1.4 A differential control volume for heat conduction analysis.

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Note that second and higher order terms are neglected in the above equation. The heat generated in the control volume is $G\Delta x\Delta y\Delta z$ and the rate of change in energy storage is given as

$$\rho c_p(\Delta x\Delta y\Delta z)\frac{\partial T}{\partial t}. \quad (1.33)$$

Now, with reference to Figure 1.4, we can write the energy balance as

"energy inlet + energy generated = energy stored + energy exit"

that is:

$$(Q_x + Q_y + Q_z) + G(\Delta x\Delta y\Delta z) = \rho(\Delta x\Delta y\Delta z)\frac{\partial T}{\partial t} + Q_{x+dx} + Q_{y+dy} + Q_{z+dz}. \quad (1.34)$$

Substituting Equation (1.32) into the previous equation and rearranging results in;

$$-\frac{\partial Q_x}{\partial x}\Delta x - \frac{\partial Q_y}{\partial y}\Delta y - \frac{\partial Q_z}{\partial z}\Delta z + G(\Delta x\Delta y\Delta z) = \rho c_p(\Delta x\Delta y\Delta z)\frac{\partial T}{\partial t}. \quad (1.35)$$

The total heat transfer Q in each direction can be expressed as (area perpendicular to heat flux direction \times heat flux):

$$\begin{aligned} Q_x &= (\Delta y\Delta z)q_x = -k_x(\Delta y\Delta z)\frac{\partial T}{\partial x} \\ Q_y &= (\Delta x\Delta z)q_y = -k_y(\Delta x\Delta z)\frac{\partial T}{\partial y} \\ Q_z &= (\Delta x\Delta y)q_z = -k_z(\Delta x\Delta y)\frac{\partial T}{\partial z}. \end{aligned} \quad (1.36)$$

Substituting Equation (1.36) into Equation (1.35) and dividing by the volume, $\Delta x\Delta y\Delta z$, we get

$$\frac{\partial}{\partial x}\left[k_x\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z\frac{\partial T}{\partial z}\right] + G = \rho c_p\frac{\partial T}{\partial t}. \quad (1.37)$$

Equation (1.37) is the transient heat conduction equation for a stationary system expressed in Cartesian coordinates. The thermal conductivity, k , in the above equation is a vector. In its most general form, the thermal conductivity can be expressed as a tensor, that is,

$$\mathbf{k} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}. \quad (1.38)$$

The preceding Equations (1.37) and (1.38) are valid for solving heat conduction problems in anisotropic materials with directional variation in thermal conductivities. In many situations, however, thermal conductivity can be taken as a nondirectional property, that is, the material

is isotropic in nature. In such materials, the heat conduction equation is written as (constant thermal conductivity):

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{G}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad (1.39)$$

where $\alpha = k/\rho c_p$ is the *thermal diffusivity*, which is an important parameter in transient heat conduction analyses. If the analysis is restricted only to steady-state heat conduction without heat generation, the equation is reduced to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0. \quad (1.40)$$

For a one-dimensional case, the steady-state heat conduction equation is further reduced to

$$\frac{d}{dx} \left(k \frac{dT}{dx} \right) = 0. \quad (1.41)$$

The heat conduction equation for a cylindrical coordinate system is given by

$$\frac{1}{r} \frac{\partial}{\partial r} \left[k_r r \frac{\partial T}{\partial r} \right] + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left[k_\phi \frac{\partial T}{\partial \phi} \right] + \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] + G = \rho c_p \frac{\partial T}{\partial t}. \quad (1.42)$$

In cylindrical coordinates, the heat fluxes can be expressed as

$$\begin{aligned} q_r &= -k_r \frac{\partial T}{\partial r} \\ q_\phi &= -\frac{k_\phi}{r} \frac{\partial T}{\partial \phi} \\ q_z &= -k_z \frac{\partial T}{\partial z}, \end{aligned} \quad (1.43)$$

where r , ϕ and z are the cylindrical coordinate directions. The heat conduction equation for a spherical coordinate system is given by

$$\begin{aligned} \frac{1}{r^2} \frac{\partial}{\partial r} \left[k_r r^2 \frac{\partial T}{\partial r} \right] + \left(\frac{1}{r^2 \sin^2 \theta} \right) \frac{\partial}{\partial \phi} \left[k_\phi \frac{\partial T}{\partial \phi} \right] + \\ \left(\frac{1}{r^2 \sin \theta} \right) \frac{\partial}{\partial \theta} \left[k_\theta \sin \theta \frac{\partial T}{\partial \theta} \right] + G = \rho c_p \frac{\partial T}{\partial t}. \end{aligned} \quad (1.44)$$

The heat fluxes in a spherical coordinate system can be expressed as

$$\begin{aligned} q_r &= -k_r \frac{\partial T}{\partial r} \\ q_\phi &= -\frac{k_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \\ q_\theta &= -\frac{k_\theta}{r} \frac{\partial T}{\partial \theta}, \end{aligned} \quad (1.45)$$

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where r , ϕ and θ are the spherical coordinate directions. It should be noted that for both cylindrical and spherical coordinate systems (Equations (1.42) and (1.44)) can be derived in a similar fashion as for Cartesian coordinates by considering the appropriate differential control volumes.

1.6 Mass Transfer

When a concentration gradient exists in a fluid mixture, mass transfer takes place from a higher concentration to a lower concentration location. Such mass transport often takes place at the molecular level in the form of mass diffusion. The mass transport at the macroscopic level is referred to as mass convection. Thus, the modes of mass transfer are very similar to the first two modes of heat transfer, that is, conduction (diffusion) and convection. Mass diffusion is often described using Fick's law of mass transport (Fick 1855). This states that the mass flux of a constituent per unit area is proportional to the concentration gradient, that is,

$$J_A = \frac{\dot{m}_A}{A} = -D_{AB} \frac{dC_A}{dx}, \quad (1.46)$$

where \dot{m}_A is the mass flux per unit time, D_{AB} is the diffusion coefficient and C_A is the mass concentration of the component A . As seen, this expression is very similar to Fourier's law of heat conduction (Equation (1.1)). The convective mass flux per unit area may be defined as

$$\frac{\dot{m}_A}{A} = h_A(C_A - C_{A\infty}), \quad (1.47)$$

where h_A is the mass transfer coefficient and $C_A - C_{A\infty}$ is the concentration difference through which mass transfer occurs. Equation (1.47) is analogous to the Newton's law of cooling for heat transfer (Equation (1.2)). Further details on mass transfer are given in Chapter 10.

1.7 Boundary and Initial Conditions

The heat conduction equations discussed in Section 1.5 will be complete for any problem only if the appropriate boundary and initial conditions are stated. With the necessary boundary and initial conditions, a solution to the heat conduction equation is possible. The boundary conditions for the conduction equation can be of two types or a combination of these: the *Dirichlet* condition, in which the temperature on the boundaries is known and/or the *Neumann* condition, in which the heat flux is imposed, that is (see Figure 1.5):

Dirichlet condition:

$$T = T_o \quad \text{on} \quad \Gamma_T. \quad (1.48)$$

Neumann condition:

$$q = -k \frac{\partial T}{\partial n} = \bar{q} \quad \text{on} \quad \Gamma_{qf}. \quad (1.49)$$

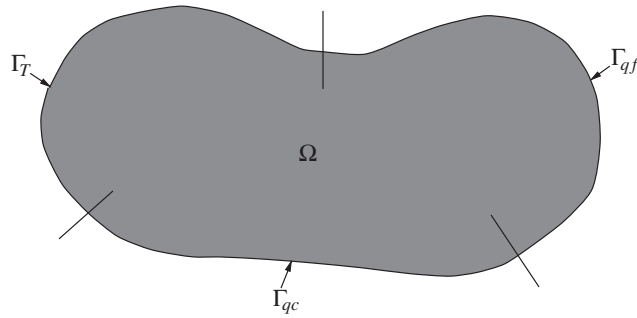


Figure 1.5 Boundary conditions.

In the above equations (Equations (1.48) and (1.49)), T_o is the prescribed temperature; Γ the boundary surface; n is the outward direction normal to the surface and \bar{q} is the constant flux given. The insulated, or adiabatic, condition can be obtained by substituting $\bar{q} = 0$. The convective heat transfer boundary condition also falls into the *Neumann* category and can be expressed as

$$-k \frac{\partial T}{\partial n} = h(T_w - T_a) \quad \text{on} \quad \Gamma_{qc}. \quad (1.50)$$

It should be observed that the heat conduction equation has second-order terms and hence faces two types of boundary conditions. Since the time appears as a first-order term, at least one initial value (i.e., at some instant of time all temperatures must be known) is to be specified for the entire body, that is,

$$T = T_0 \quad \text{all over the domain} \quad \Omega \quad \text{at} \quad t = t_0, \quad (1.51)$$

where t_0 is a reference time.

The constant or variable temperature conditions are generally easy to implement as temperature is a scalar. However, the implementation of surface fluxes is not as straightforward. Equation (1.49) can be rewritten with direction cosines of the outward normals as

$$-\left(k_x \frac{\partial T}{\partial x} \bar{l} + k_y \frac{\partial T}{\partial y} \bar{m} + k_z \frac{\partial T}{\partial z} \bar{n}\right) = \bar{q} \quad \text{on} \quad \Gamma_{qf}. \quad (1.52)$$

Similarly, Equation (1.50) can be rewritten as

$$-\left(k_x \frac{\partial T}{\partial x} \bar{l} + k_y \frac{\partial T}{\partial y} \bar{m} + k_z \frac{\partial T}{\partial z} \bar{n}\right) = h(T - T_a) \quad \text{on} \quad \Gamma_{qc}, \quad (1.53)$$

where \bar{l} , \bar{m} and \bar{n} are the direction cosines of the appropriate outward surface normals.

In many industrial applications, for example, wire drawing, crystal growth, continuous casting, etc., the material will have a motion in space and this motion may be restricted to one

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direction, as in the example (Section 1.4.3) cited previously. The general energy equation for heat conduction, taking into account the spatial motion of the body, is given by

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) + G = \rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right), \quad (1.54)$$

where u , v and w are the components of the velocity in the three directions, x , y and z respectively.

The governing equations for convection heat transfer are very similar to the above and will be discussed in Chapter 7.

1.8 Solution Methodology

Although a number of analytical solutions for conduction heat transfer problems are available (Carslaw and Jaeger 1959; Ozisik 1968), in many practical situations, the geometry and the boundary conditions are so complex that an analytical solution is not possible. Even if one could develop analytical relations for such complicated cases, these will invariably involve complex series solutions and would thus be practically difficult to implement. In such situations, conduction heat transfer problems do need a numerical solution. Some commonly employed numerical methods are the Finite Difference (Ozisik and Czisik 1994), Finite Volume (Patankar 1980), Finite Element and Boundary Element (Ibanez and Power 2002) techniques. This text will address issues related to the Finite Element Method (FEM) only (Comini *et al.* 1994; Huang and Usmani 1994; Lewis *et al.* 1996, 2004; Reddy and Gartling 2000).

In contrast to an analytical solution, which allows for the temperature determination at any point in the medium, a numerical solution enables the determination of temperature only at discrete points. The first step in any numerical analysis must therefore be to select these points. This is done by dividing the region of interest into a number of smaller regions. These regions are bounded by points. These reference points are termed nodal points and their assembly results in a grid or mesh. It is important to note that each node represents a certain region surrounding it and its temperature is a measure of the temperature distribution in that region. The numerical accuracy of these calculations depends strongly on the number of designated nodal points, which control the number of elements generated. The accuracy approaches an exact value as the mesh size (region size) approaches zero.

Further details on the numerical methods, mesh generation, accuracy and error are discussed in later chapters.

1.9 Summary

In this chapter, the subject of heat transfer was introduced and various modes of heat transport were discussed. The fundamentals of energy conservation principles and the application of such principles to some selected problems were also presented. Finally, the general heat conduction equations in multi-dimensions were derived and the appropriate boundary and

initial conditions were given. Although this chapter has been brief, we trust that it has given the reader the essential fundamental concepts involved in heat transfer in general and some detailed understanding of conduction heat transfer in particular.

1.10 Exercises

Exercise 1.10.1 *Derive the energy balance equation for a rectangular fin of variable cross-section as shown in Figure 1.6. The fin is stationary and is attached to a hot heat source. (Hint: This is similar to the problem given in Section 1.4.3. but without relative motion.)*

Exercise 1.10.2 *The inner body temperature of a healthy person remains constant at 37°C while the temperature and humidity of the environment change. Explain, via heat transfer mechanisms between the human body and the environment, how the human body keeps itself cooler in summer and warmer in winter.*

Exercise 1.10.3 *Discuss the modes of heat transfer that determine the equilibrium temperature of a space shuttle when it is in orbit. What happens when it reenters the Earth's atmosphere?*

Exercise 1.10.4 *A closed plastic container used to serve coffee in a seminar room is made of two layers with an air gap placed between them. List all heat transfer processes associated with the cooling of the coffee in the inner plastic vessel. What steps do you consider for a better container design so as to reduce the heat loss to the ambient?*

Exercise 1.10.5 *A square chip of size 8 mm is mounted on a substrate with the top surface being exposed to a coolant flow at 20°C . All other surfaces of the chip are insulated. The chip temperature must not exceed 80°C in order for the chip to function properly. Determine the maximum allowable power, which can be applied to the chip if the coolant is air with a heat*

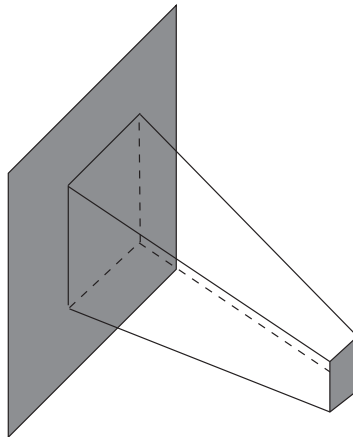


Figure 1.6 Rectangular fin.

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transfer coefficient of $250 \text{ W/m}^2\text{K}$. If the coolant is a dielectric liquid with a heat transfer coefficient of $2500 \text{ W/m}^2\text{K}$, how much additional power can be dissipated compared to air cooling?

Exercise 1.10.6 Consider a person standing in a room, which is at a temperature of 21°C . Determine the total heat rate from this person if the exposed surface area of the person is 1.6 m^2 and the average outer surface temperature of the person is 30°C . The convection coefficient from the surface of the person is $5 \text{ W/m}^2\text{ }^\circ\text{C}$. What is the effect of radiation if the emissivity of the surface of the person is 0.90?

Exercise 1.10.7 A thin metal plate has one large insulated bottom surface and the top large surface exposed to solar radiation at a rate of 600 W/m^2 . The surrounding air temperature is 20°C . Determine the equilibrium surface temperature of the plate if the convection heat transfer coefficient from the plate surface is $20 \text{ W/m}^2\text{K}$ and the emissivity of the top surface of the plate is 0.8.

Exercise 1.10.8 A long, thin copper wire of radius r and length L has an electrical resistance of ρ per unit length. The wire is initially kept at a room temperature of T_a and subjected to an electric current flow of I . The heat generation due to the current flow is simultaneously lost to the ambient by convection. Set up an equation to determine the temperature of the wire as a function of time. Mention the assumptions made in the derivation of the equation.

Exercise 1.10.9 In a continuous casting machine, the billet moves at a rate of $u \text{ m/s}$. The hot billet is exposed to an ambient temperature of T_a . Set up an equation to find the temperature of the billet as a function of time in terms of pertinent parameters. Assume that radiation also plays a role in the dissipation of heat to ambient.

Exercise 1.10.10 In a double pipe heat exchanger, hot fluid (mass flow $M \text{ kg/s}$ and specific heat $C \text{ kJ/kg }^\circ\text{C}$) flows inside a pipe and cold fluid (mass flow $m \text{ kg/s}$ and specific heat $c \text{ kJ/kg }^\circ\text{C}$) flows outside in the annular space. The hot fluid enters the heat exchanger at T_{h1} and leaves at T_{h2} whereas the cold fluid enters at T_{c1} and leaves at T_{c2} . Set up the differential equation to determine the temperature variation (along the heat exchanger) for hot and cold fluids.

References

- Bejan A (1993) *Heat Transfer*: John Wiley & Sons, Inc., New York.
- Boltzmann L (1884) Ableitung des Stefan'schen Gesetzes, betreffend die Abhngigkeit der Wrme-strahlung von der Temperatur aus der electromagnetischen Lichttheorie. *Annalen der Physik und Chemie*, Bd. 22, S. 291–294.
- Carlsaw HS and Jaeger JC (1959) *Conduction of Heat in Solids*, 2nd Edition. Oxford University Press, Fairlawn, N.J.
- Clausius R (1850) Ueber die bewegende Kraft der Wrme und die Gesetze, welche sich daraus für die Wrmelehre selbst ableiten lassen, *Annalen der Physik und Chemie* (Poggendorff, Leipzig), 155, 368–394.
- Comini G, del Giudice S and Nonino C (1994) *Finite Element Analysis in Heat Transfer Basic*. Formulation and Linear Problems Series in Computational and Physical Processes in Mechanics and Thermal Sciences. Taylor & Francis, Bristol, PA.

- Fick A (1855) On liquid diffusion. *Philosophical Magazine*, 10, 30.
- Fourier J (1955) *The Analytical Theory of Heat*. Dover Publications, Inc., New York.
- Holman JP (1989) *Heat Transfer*. McGraw Hill, New York.
- Huang H-C and Usmani AS (1994) *Finite Element Analysis for Heat Transfer*. Springer-Verlag, London.
- Ibanez MT and Power H (2002) *Advanced Boundary Elements for Heat Transfer (Topics in Engineering)*. WIT Press, Southampton, UK.
- Incropera FP and Dewitt DP (1990) *Fundamentals of Heat and Mass Transfer*. John Wiley & Sons, Inc., New York.
- Lewis RW, Morgan K, Thomas HR and Seetharamu KN (1996) *Finite Element Methods in Heat Transfer Analysis*. John Wiley & Sons, Inc., New York.
- Lewis RW, Nithiarasu P and Seetharamu KN (2004) *Fundamentals of the Finite Element Method for Heat and Fluid Flow*. John Wiley & Sons, Inc., New York.
- Ozisik MN (1968) *Boundary Value Problems of Heat Conduction*. International Text Book Company, Scranton, Pa.
- Ozisik MN and Czisik MN (1994) *Finite Difference Methods in Heat Transfer*. CRC Press, London.
- Patankar SV (1980) *Numerical Heat Transfer and Fluid Flow*, Hemisphere, Arlington, VA.
- Reddy JN and Gartling GK (2000), *The Finite Element Method in Heat Transfer and Fluid Dynamics*, 2nd Edition. CRC Press, London.
- Siegel R and Howell JR (1992) *Thermal Radiation Heat Transfer*, 3rd Edition. Hemisphere, Arlington, VA.
- Stefan J (1879) Über die Beziehung zwischen der Wärmestrahlung und der Temperatur. *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der kaiserlichen Akademie der Wissenschaften*. Bd. 79 (Wien 1879), S. 391–428.
- Sukhatme SP (1992) *A Text Book on Heat Transfer*, 3rd Edition. Orient Longman Publishers, Hyderabad, India.
- Whewell W (1866). Correction of Newton's law of cooling. In *History of the Inductive Sciences*, pp. 149–150. Appleton, New York.