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Introduction

1.1 Motivations and Historical Clues

Structural dynamics deals with the problems of response analysis, reliability evaluation and system control of any given type of structure subjected to dynamic actions.¹ Structures (such as buildings, bridges, aircraft, ships and so on) refer to those bodies or systems composed of various materials in a certain way that are capable of bearing loads and actions. On the other hand, when we say an action applied on structures is dynamic, this not only indicates that the action is time varying, but also that the induced inertial effects cannot be ignored. For example, earthquakes, wind, sea waves, jet noise and turbulence in the boundary layer and the like are typical dynamic actions. The task of dynamic response analysis of structures is to capture the internal forces, deformations or other state quantities of structures when they are subjected to dynamic actions. At the same time, we may need to study whether the structural response meets some specified limit in a sense, which is generally referred to as reliability evaluation. Furthermore, to make a structure subjected to dynamic actions response in a desired way to an extent is what to be done in system control.

Most dynamic actions exhibit appreciable randomness. Actually, investigators frequently find that the results observed under almost identical conditions have obvious deviation, but simultaneously exhibit some statistical rules. In essence, the randomness results from the uncontrollability of causation of the realized phenomenon. For example, consider wind turbulence in the atmospheric boundary layer. It is well known that the observed wind speeds recorded at the same position but during different time intervals are quite different (Figure 1.1). However, if the statistics of a large number of samples are examined, then we find that the probabilistic characteristics of the wind speed are relatively stable (Figure 1.2). In fact, the randomness involved stems from a complicated physical mechanism in the wind flows, say the mechanism of turbulence. The underlying reason is the uncontrollable nature of the motion of air molecules.

In addition, the randomness involved in the physical parameters of structures is also one of the sources that induce randomness in the dynamic responses of structures. For instance, in the

¹The dynamic properties of structures, such as the frequencies and mode shapes, are also research topics of structural dynamics. But, in a general sense, the dynamic properties of structures can be regarded as part of the dynamic analysis of structures.

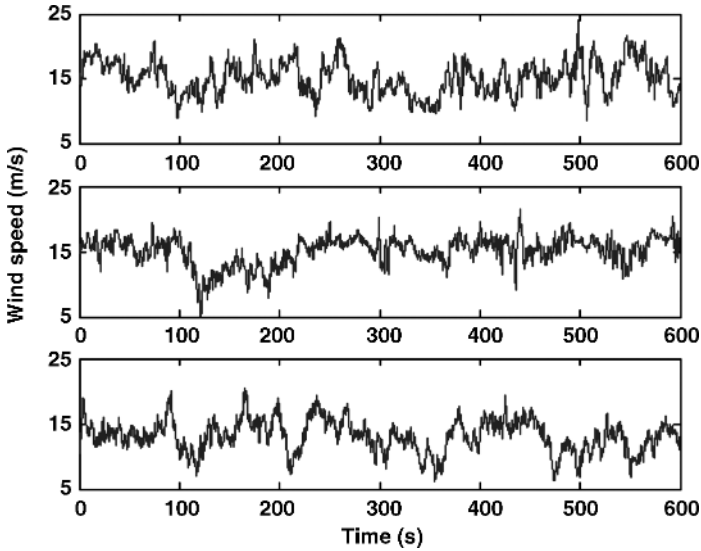


Figure 1.1 Records of wind speed.

dynamic response analysis of building structures, the soil–structure interaction is one of the basic problems where the properties of soil must be considered in the establishment of a reasonable structural analysis model. Evidently, it is impossible to measure the physical properties of soil completely at all points in the groundwork. Thus, a reasonable modeling approach is to regard the physical properties of soil, such as the shear wave speed and the damping ratio, as random variables or random fields. This will lead to the structural analysis involving random parameters, usually known as stochastic structural analysis.

Stochastic dynamic response analysis, reliability evaluation and system control compose the basic research scope of the stochastic dynamics of structures.

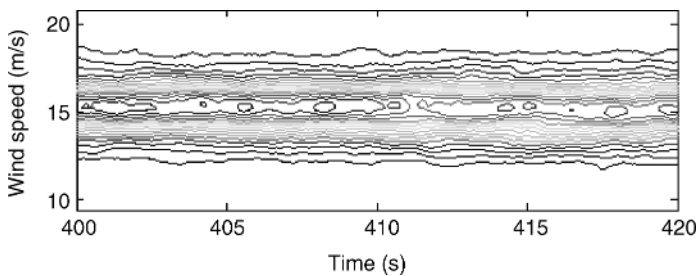


Figure 1.2 Contour of probability density of wind speed.

Although the studies on stochastic dynamical systems can be dated back to the investigations on statistical mechanics by Gibbs and Boltzmann (Gibbs, 1902; Cercignani, 1998), it is generally considered more reasonable to regard the studies on Brownian motion by Einstein (1905) as the origin of the discipline.

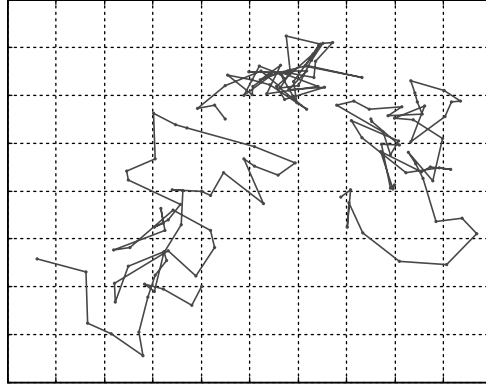


Figure 1.3 Typical trajectories of Brownian motions.

In 1905, Einstein studied the problem of the irregular motion of particles suspended in fluids, which was first observed by the Scottish botanist Robert Brown in 1827 (Figure 1.3). Einstein believed that Brownian motion of the particles was induced by the highly frequent random impacts of the fluid molecules. Based on this physical interpretation, Einstein made the following assumptions:

- (a) the motion of different Brownian particles is mutually independent;
- (b) the motion of Brownian particles is isotropic and no external actions except the collision of fluid molecules are applied;
- (c) the collision of fluid molecules is instantaneous, such that the time of collision can be ignored (rigid collision).

Based on the above assumptions, the probability density of the particle group at two different instants of time can be derived by examining the phenomenological evolution process of the particle group; that is:

$$f(x, t + \tau) = \int_{-\infty}^{+\infty} f(x + r, t) \Phi(r) dr \quad (1.1)$$

where $f(x, t + \tau)$ is the probability density of the position of the particles at time $t + \tau$, $f(x + r, t)$ is the probability density by transition of the particles with distance r during the time interval τ , and $\Phi(r)$ is the probability density of displacement of the particles.

Using the rigid collision assumption, expanding the functions by using the Taylor series and retaining the first-order term with respect to $f(x, t + \tau)$ and the second-order term with respect to $f(x + r, t)$ will yield

$$\frac{\partial f(x, t)}{\partial t} = D \frac{\partial^2 f(x, t)}{\partial x^2} \quad (1.2)$$

where

$$D = \frac{1}{\tau} \int_{-\infty}^{+\infty} \frac{1}{2} r^2 \Phi(r) dr \quad (1.3)$$

Clearly, Equation (1.2) is a diffusion equation, where D is the diffusion coefficient.

In 1914 and 1917, Fokker and Planck respectively introduced the drift term for a similar physical problem, leading to the so-called Fokker–Planck equation (Fokker, 1914; Planck, 1917; Gardiner, 1983), of which the rigorous mathematical basis was later established by Kolmogorov (1931).²

We note that, initially, the studies on Brownian motion were based on physical concepts; however, a statistical phenomenological interpretation was subsequently introduced in the deductions. In this book, we call this historical clue the *Einstein–Fokker–Planck tradition* or *phenomenological tradition*. In this tradition, a large number of studies on the probability density evolution of stochastic dynamical systems have been done (Kozin, 1961; Lin, 1967; Roberts and Spanos, 1990; Zhu, 1992, 2003; Lin and Cai, 1995). However, for the multi-degree-of-freedom (MDOF) systems or multidimensional problems, advancement is still quite limited (Schuëller, 1997, 2001).

Soon after Einstein’s work, Langevin (1908) came up with a completely different research approach. In his investigation, the physical interpretation of Brownian motion is the same as that of Einstein, but Langevin contributed to two basic aspects. He:

- (a) introduced the assumption of random forces;
- (b) employed Newton’s equation of motion to govern the motion of the Brownian particles.

Based on this, he established the stochastic dynamics equation, which was later called the Langevin equation:

$$m\ddot{x} = -\gamma\dot{x} + \xi(t) \quad (1.4)$$

where m is the mass of the Brownian particles, \ddot{x} and \dot{x} are the acceleration and velocity of motion respectively, γ is the viscous damping coefficient and $\xi(t)$ is the force induced by the collision of the fluid molecules, which is randomly fluctuating.

Using the ensemble average, Langevin obtained a diffusion coefficient identical to that given by Einstein.

In contrast to the diffusion equation derived by Einstein, the Langevin equation is more direct and more physically intuitive. However, the physical features of the random forces are not completely clear in Langevin’s work.

In 1923, Wiener proposed a stochastic process model for Brownian motion (Wiener, 1923). Around 20 years later, Itô introduced the Itô integral and gave the more generic Langevin equation based on the Wiener process (Itô, 1942, 1944; Itô and McKean, 1965):

$$dx(t) = a[x(t), t] dt + b[x(t), t] dW(t) \quad (1.5)$$

where $a(\cdot)$ and $b(\cdot)$ are known deterministic functions and $W(t)$ is a Wiener process.

The form of Equation 1.5 is nowadays called the Itô stochastic differential equation. Clearly, this equation is in essence a physical equation. It is generally believed that the Itô equation

² Interestingly, Kolmogorov did not at first know about the work of Fokker and Planck and developed his equation independently.

provides a sample trajectory description for stochastic dynamical systems. In this book, we refer to this historical clue as the *Langevin–Itô tradition* or *physical tradition*. In this approach, the mean-square calculus theory was established, based on which correlation analysis and spectral analysis in classical random vibration analysis were well developed (Crandall, 1958, 2006; Lin, 1967; Zhu, 1992; Øksendal, 2005).

There were intrinsic and countless ties between the phenomenological tradition and the physical tradition in stochastic dynamics. As a matter of fact, upon the assumption that the system inputs are white-noise processes, it is easy to obtain the Fokker–Planck–Kolmogorov (FPK) equation via the Itô equation. This demonstrates that in the physics approach we can discover the intrinsic arcanum of the evolution of stochastic systems. Unfortunately, white noise is physically unrealizable. In other words, although mathematically it plays a fundamental role in a sense, the various singular or even ridiculous features of white noise (say, continuous but nondifferentiable everywhere) are rare in the real world.

The white-noise process is, of course, an idealized model for various real physical processes. Noticing this, we naturally hope to return to the real physical processes themselves. For a specific physical dynamical process, the problem is usually easily resolvable. Thus, once further introducing the intrinsic ties between the sample trajectories and the probabilistic description, we will be led to an approach of studying stochastic systems based on physics. In this approach, we not only can establish the generalized probability density evolution equation (Li and Chen, 2003, 2006c, 2008), but also find that the nowadays available major research results, such as traditional random vibration theory and stochastic finite element methods, can be appropriately brought into the new theoretical frame (Li, 2006). In fact, correlation analysis and the spectral analysis in classical random vibration theory can be regarded as the results of combining the formal solution of physical equations and the evolution of moment characteristics of the response processes. Perturbation theory and orthogonal expansion theory in the analysis of structures with random parameters can also be reasonably interpreted in this sense. The classical FPK equation, as mentioned before, can be viewed as the result of the idealization of physical processes. In addition, the thoughts of physical stochastic system can also be used in modeling of general stochastic process, such as seismic ground motion, wind turbulence and the like (Li and Ai, 2006; Li and Zhang, 2007).

On the basis of the above thoughts on physical stochastic systems, we prefer to entitle this book *Stochastic Dynamics of Structures: a Physical Approach*.

1.2 Contents of the Book

This book deals with the basic problems of the stochastic dynamics of structures in the theoretical frame of physical stochastic systems.

In Chapter 2 the prerequisite fundamentals of probability theory are outlined, including the basic concepts of random variables, stochastic processes, random fields and the orthogonal expansion of random functions.

Chapter 3 deals with stochastic process models for typical dynamic excitations of structures, including the phenomenological and physical modeling of seismic ground motions, fluctuating wind speed and sea waves. Simultaneously, we introduce the standard orthogonal expansion of stochastic processes, which can be applied to random vibration analysis of structures.

The approaches for analysis of structures with random parameters mainly include the random simulation method, the perturbation method and the orthogonal expansion method. These approaches are discussed in detail in Chapter 4.

Chapter 5 deals with the response analysis of deterministic structures subjected to stochastic dynamic excitations, including correlation analysis, spectral analysis, the statistical linearization method and the FPK equation approach. In particular, in this chapter we introduce the pseudo-excitation method for linear systems. We believe these contents are valuable to in-depth understanding of classical random vibration theory.

Probability density evolution analysis of stochastic responses of dynamical systems is an important topic of the book. We will deal with this topic in Chapters 6 and 7. In Chapter 6 we trace in some detail the historical origin of probability density evolution analysis of stochastic dynamical systems. Using the principle of preservation of probability as a unified basis, we derive the Liouville equation, the FPK equation, the Dostupov–Pugachev equation and the generalized probability density evolution equation proposed by the authors. In Chapter 7 we study the numerical methods for probability density evolution analysis in detail, including the finite difference method, the strategy of selecting representative points via tangent spheres, lattices and the number theoretical method. For all these methods, we discuss the problems of numerical convergence and stability where possible.

The aim of structural dynamical analysis is to realize reliability-based design and performance control of structures. We discuss the problem of dynamic reliability and global reliability of structures in Chapter 8. Based on the random event description of the evolution of probability density, the absorbing boundary condition for the first-passage problem is introduced. The theory on evaluation of the extreme value distribution is elaborated through introducing a virtual stochastic process related to the extreme value of the response process. Furthermore, the principle of equivalent extreme value and its application to the global reliability evaluation of structures is discussed. It is worth pointing out that the principle of equivalent extreme value is of significance and applicable to static reliability evaluation of generic systems.

We come to the problem of the dynamic control of structures in Chapter 9. On the basis of classical dynamic control, the concept of stochastic optimal control is introduced and the approach for design of the control systems based on probability density evolution analysis is proposed. For realization of ‘real’ stochastic optimal control of dynamical systems, the proposed approach is undoubtedly promising.