



Fundamentals

“Give me six hours to chop down a tree and I will spend the first four sharpening the axe.”

– US President Abraham Lincoln

From the earliest days of recorded history, humans have dreamed of soaring into the sky. It is believed that the first kite was invented in China in circa 1000 BC. This was followed by the invention of the first rockets in China in circa 200 BC. Thirteen centuries later and about 8000 km away in Italy, Renaissance artist Leonardo da Vinci drew several designs for flying machines, inspired by his anatomical study of birds. In 1783, two French men named Jean-François de Rozier and Marquis D’Arlandes made the first free aerial flight in a Montgolfier hot-air balloon. However, it wasn’t until 1903 that the era of manned, powered flight was begun by two American brothers from Dayton, Ohio. Their Wright Flyer was



“Bird’s Eye View” The Wright brothers proved that great things can be accomplished if you have a willingness to work hard, search for the important facts, and dare to have an optimistic hope about the future.

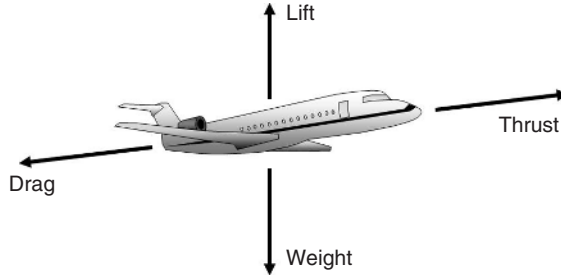


Figure 1.1 Forces acting on an aircraft

the starting point for the design of manned aircraft which today have drawn together the far expanses of this planet. However, what is often overlooked is that they also developed the first operational aircraft engine to power it.

The word **propulsion** means driving forward. Therefore a propulsion system is a machine that produces thrust to drive an object forward. On air and space vehicles, a thrusting force is produced by applying Newton's 3rd Law of Action and Reaction. The action is accomplished by accelerating a working gas, normally by adding heat due to chemical combustion. The reaction to this acceleration (\dot{V}) produces a net thrust force (F_N) that propels an air vehicle forward (Figure 1.1).

$$\sum F = \sum m\dot{V} \quad (1.1)$$

$$\sum F = Thrust - Drag = m\dot{V} \quad (1.2)$$






$$\dot{V} = \frac{F_N - D}{m} \quad (1.3)$$

Thrust (F_N) is the driving force that propels an aircraft, helicopter, missile, or rocket forward. An aerospace propulsion system (engine or motor) is simply a device that converts power into thrust to propel an aerospace vehicle. Table 1.1 shows the most prevalent types of aerospace propulsion systems used today. Each of these engine types is covered in this book.

1.1 Fundamental Equations




Most aerospace propulsion systems operate in a cyclic manner to produce a net work output from a supply of heat. Engines convert heat energy from available sources (such as combustion of chemical fuels) into mechanical work, according to the laws of fluid mechanics and thermodynamics. Therefore, the performance of propulsion systems is governed by the conservation of mass, momentum, and energy. This section presents a review of these fundamental equations and terms.

Table 1.1 Types of aerospace propulsion systems

Types	Applications	Characteristics	
Rocket motors	<ul style="list-style-type: none"> • Space launchers • Missiles 	<ul style="list-style-type: none"> • Can operate outside Earth's atmosphere • Capable of very high thrusts • Can operate at high supersonic speeds • Heavy, non-air-breather (must carry oxidizer propellant) • Most are expendable (one-time use) 	
Piston aerodynamic engines	<ul style="list-style-type: none"> • General aviation aircraft 	<ul style="list-style-type: none"> • Relatively low cost • Relatively simple maintenance • Only capable of low thrusts • Limited to low subsonic speeds • Limited to low altitudes 	
Turbojet engines	<ul style="list-style-type: none"> • Outdated military fighters • Long-range missiles (e.g., cruise and anti-ship missiles) 	<ul style="list-style-type: none"> • Capable of high thrusts • Capable of supersonic speeds (normally an afterburner must be used) • Less fuel efficiency than turbofan engines. 	
Turbofan engines	<ul style="list-style-type: none"> • Commercial aircraft • Business jets • Most modern military combat aircraft 	<ul style="list-style-type: none"> • Widely used today • Capable of medium to high thrusts • Capable of supersonic speeds (normally an afterburner must be used) • Better fuel efficiency than turbojet engines 	
Turboprop engines	<ul style="list-style-type: none"> • Short-range commercial aircraft • Cargo transports • Military troop and cargo transports 	<ul style="list-style-type: none"> • Fuel efficient • Short take-off and landing distances • Low to medium altitude limit • Subsonic speed limitation • Noisy, high vibration 	

(continued)

Table 1.1 (Continued)

Turboshaft engines	<ul style="list-style-type: none"> • Helicopters • Auxiliary power units (APU) 	<ul style="list-style-type: none"> • Optimized to produce shaft power • Short in length • Generally not suitable for fixed-wing aircraft 	
Ramjet engines	<ul style="list-style-type: none"> • Long-range supersonic missiles • Specialized aircraft 	<ul style="list-style-type: none"> • Mechanically very simple • Can operate efficiently at high supersonic Mach numbers (2.5–5.0) • Generally cannot operate at subsonic speeds, so requires a booster rocket 	
Scramjet engines	<ul style="list-style-type: none"> • Experimental hypersonic vehicles 	<ul style="list-style-type: none"> • Many difficult technical challenges, no operational models yet • Can operate at hypersonic Mach numbers (5.0–15.0) • Cannot operate at subsonic or low supersonic speeds, so requires a booster rocket 	

(Images courtesy of NASA, USAF, and National Museum of the USAF.)

1.1.1 Review of Terms

1.1.1.1 Systems

A **system** is an identifiable collection of matter that is under investigation. The mass or expanse outside of a system is called the **surroundings**. Systems can be moveable or fixed. Types of systems are listed below:

- **Isolated System** – A system that is completely uninfluenced by its surroundings. It has a fixed mass, and no heat or work can cross the boundary (or control volume) of the system
- **Closed System** – One in which a fixed mass (control mass) is contained within a boundary at all times. A piston cylinder with closed intake and exhaust manifolds is an example of such a system (Figure 1.2). The piston moves and compresses the air, but the mass of air in the cylinder remains fixed. Energy (e.g., heat and work) can cross the boundary, but with the manifolds closed, no mass can enter or leave the system.

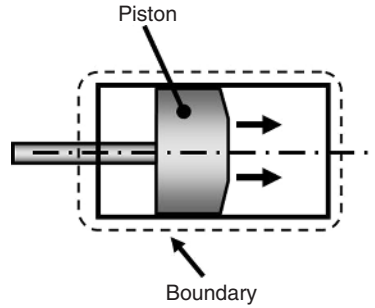
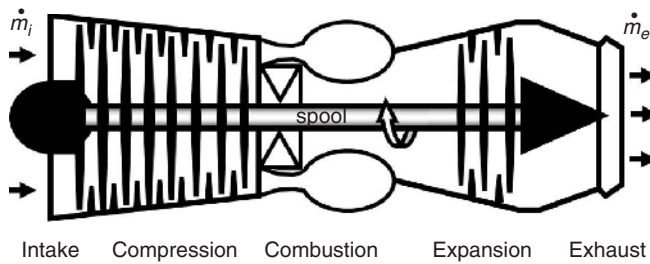


Figure 1.2 Closed system



If $\dot{m}_i = \dot{m}_e$, then it is a **steady flow system**

Figure 1.3 Open system

- **Open Systems** (also called **flow systems**) – A system where matter (mass) and energy are transferred across the boundary of a control volume (Figure 1.3). A **steady flow** open system requires that the mass flows at entry, exit, and any intermediate point within the boundary are the same. A gas turbine engine operating at steady speed in fixed ambient conditions is an example of such a system.

1.1.1.2 Working Fluid

Most aerospace propulsion systems operate in a thermodynamic cycle that involves transferring heat to and from a **working fluid**. Atmospheric air (or air mixed with combustion gases) is the predominant working fluid used in air-breathing propulsion systems. Air-breathing engines (such as gas turbine, ramjet, and scramjet engines) are open systems that draw in ambient air and use it as an oxidizer to burn fuel. However, the properties of atmospheric air change with altitude. Because of this, these engines can operate only over a certain range of altitudes and velocities (Mach numbers) which correspond to differing atmospheric pressures, temperatures, and densities. This range is known as the engine's flight envelope. Figure 1.4 shows approximate flight envelopes of aircraft with various types of propulsion systems.

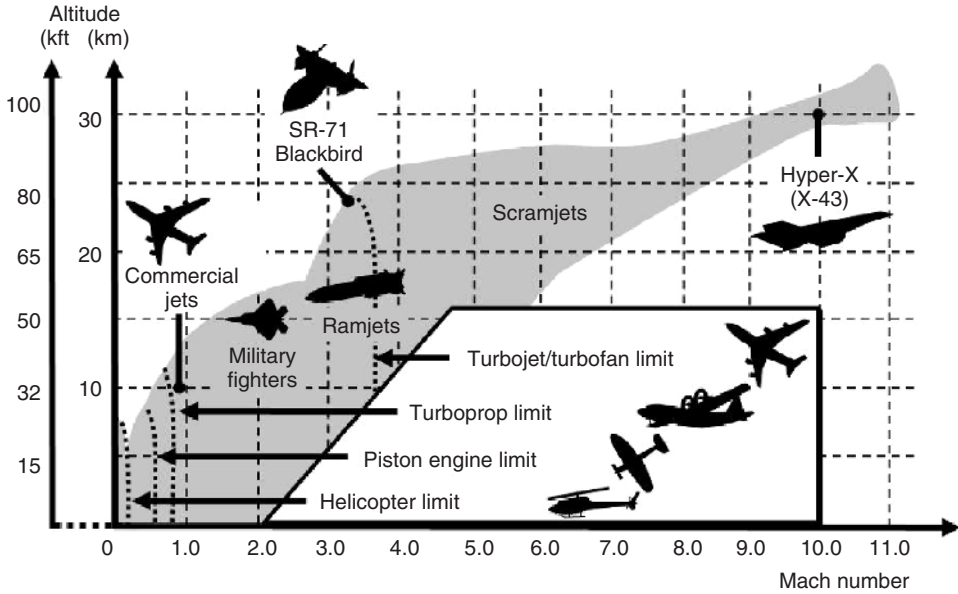


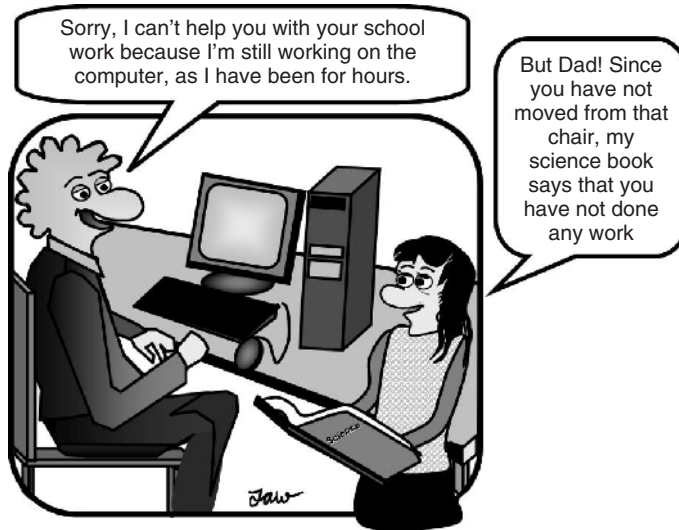
Figure 1.4 Flight envelopes of aircraft with different engine types

Rockets are non-air-breathing systems, so are not limited by altitude, so they can even operate in the vacuum of space. The working fluid of a chemical rocket is the combustion gases produced by combustion of its propellants.

The properties used to define the state of a working fluid are: pressure (P), temperature (T), volume (V), enthalpy (H), density (ρ), internal energy (U), and entropy (S). Since atmospheric air properties vary with altitude, weather, location, time, and other factors; the Standard Atmospheric Tables are typically used by engine designers as a standard reference [1]. Table A.1 in Appendix A gives the properties of the standard atmosphere in SI units. Values such as temperature, pressure, density, and viscosity are given at different altitudes. An altitude of 0 km represents the sea-level condition.

1.1.1.3 Work and Power

Work (W) is generated when a force moves something in the direction it is being applied. It is a scalar quantity that is simply defined as the force applied to a body times the displacement of that body in that direction. If there is no displacement (or movement), there is no work. It has the SI (Standard International) units newton-meter (N·m), which is also called a joule (J). In Imperial units it is written in foot-pound (ft·lbf), Btu (British thermal units), or calories (cal). The generally accepted sign convention for work interaction is that work done by a system (gain) is positive and work done on a system (loss) is negative.



“Hard Work” The definition of mechanical work requires movement to occur. The girl in this cartoon makes a great point, but she obviously did not see her dad typing on the keyboard, moving his mouse or putting paper in the printer.

Power (\dot{W}) is the rate of doing work. It is normally expressed in joules per second (J/s) or Watts (W). In Imperial units it can be expressed as horsepower (hp), Btu per hour (Btu/hr) or in calories per second (cal/s).

1.1.1.4 Energy

Energy is a scalar physical quantity that describes the capacity of a system to produce work. It is expressed in SI units as joules (J). It can exist in many different forms which are often named after a related force. Some examples are kinetic, potential, thermal, mechanical, electrical, chemical, magnetic, and nuclear. The **total energy** (E) of a system is the sum of all the forms of energy applicable to that system. The various forms of energy can be categorized into two groups: macroscopic and microscopic. The macroscopic forms of energy are those that a system has as a whole with respect to an outside frame of reference. This would include the overall system's kinetic and gravitational potential energies. The microscopic forms of energy are those that are related to the molecular structure and activity of a system, independent of an outside frame of reference.

The sum of all microscopic energy is called the **internal energy** (U). This includes the sum of the kinetic and potential energy of the molecules. Molecular kinetic energy consists of atoms or electrons that may be vibrating, translating, rotating, or spinning (depending on the phase of the matter). The portion of the internal energy related to the kinetic energy is called the **sensible energy**. The level of activity (velocity and momentum) of the molecules increases with temperature. As temperature increases, the sensible energy

increases causing the system to have a higher internal energy. Internal energy is also associated with the binding forces between molecules. A sufficient amount of energy can break molecular bonds causing a phase change (such as a liquid into a gas). The internal energy related to this phase change is called the **latent energy**. (A phase change process can occur without changing the chemical composition of the system.) The combination of the sensible and latent internal energy of a system is commonly referred to as its **thermal energy**.

The energy associated with the atomic bonds in a system is called the **chemical energy**. During a combustion process some chemical bonds are formed while others are destroyed, changing the internal energy of the matter. **Nuclear energy** involves changes to the strong bonds within the nucleus of an atom. (An atom remains the same during a chemical reaction, but changes in a nuclear reaction.) **Mechanical energy** is a form of energy that is converted into work by a mechanical device, such as a turbine. In a gas turbine propulsion system, a propeller or compressor transfers mechanical work into a working fluid by raising its pressure.

1.1.1.5 Heat

Energy can be transferred from one system to another as either work or heat. Heat (Q) is defined as the energy transferred between molecules of one system to those of another due to a temperature difference. Like work, heat is expressed in SI units as joules (J). By convention, a positive value of heat indicates a heat gain to a system and a negative value indicates a heat loss.

Heat is transferred as it crosses the boundaries of one system into another. Once the heat transfers into a system it becomes part of the internal energy of that system. Heat transfer (\dot{Q}) occurs by three mechanisms: conduction, convection, and radiation. Conductive heat transfer occurs when an energetic substance (high thermal energy) is in contact with a less energetic substance. Conduction can take place in solids, liquids, or gases. In solids it occurs due to the combinations of vibrations of the molecules in their lattice and the energy transport of free electrons. In liquids and gases, conduction takes place due to collisions and diffusion of the molecules randomly in motion. Convective heat transfer occurs due to the combined effects of molecular motion (conduction) and the bulk motion of fluids (liquids or gases). An understanding of heat convected between a bounding surface and a moving fluid, when both are at different temperatures, is particularly important in the analysis of propulsion systems (e.g., cooling). Radiation heat transfer is the energy emitted by matter in the form of electromagnetic waves (or photons). Unlike conduction or convection, radiation does not require the presence of an intervening medium, so it can occur across the vacuum of space.

The **thermal efficiency** (η_{th}) of a heat engine is the ratio of its work output to the total heat added into the system. If the total work is equal to the change of heat in the system, then this is expressed as Equation 1.4.

Thermal efficiency:

$$\eta_{th} = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \quad (1.4)$$

1.1.1.6 Cycles

A system undergoes a **process** when its state changes from one equilibrium condition to another. If a system undergoes a number of processes so that its final state is the same as its initial state (in all respects), then the system has undergone a **cycle**.

Figure 1.5 shows a pressure–volume plot ($P - V$ diagram) of an expansion process, involving non-repetitive translational motion of a piston cylinder. This is an example of a non-cyclic process from a high pressure/low volume state to a low pressure/high volume state.

Figure 1.6 shows an ideal gas turbine cycle. The numbered labels refer to different processes in the cycle. The last process ($4 \rightarrow 1$) is not actually physically possible inside a gas turbine engine. The exhaust gases diffuse outside in the ambient air surroundings. However, since this is an open system, the engine operates in a cycle because ambient air continuously flows into the engine through its intake (which effectively restarts the cycle at point 1).

1.1.1.7 Isentropic Processes

Ideal processes are **reversible**. This means that the original state of the system can be restored, leaving no residual change in either the system or its surroundings. Reversible

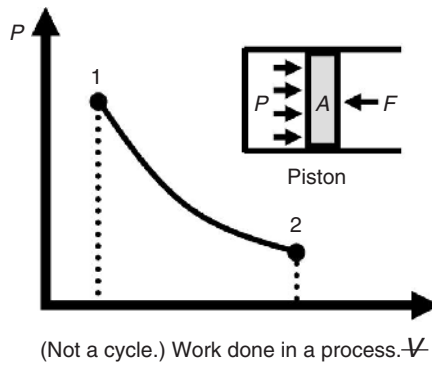


Figure 1.5 Non-cyclic process

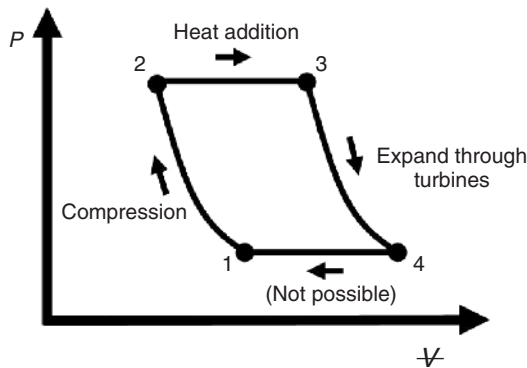


Figure 1.6 Cyclic process

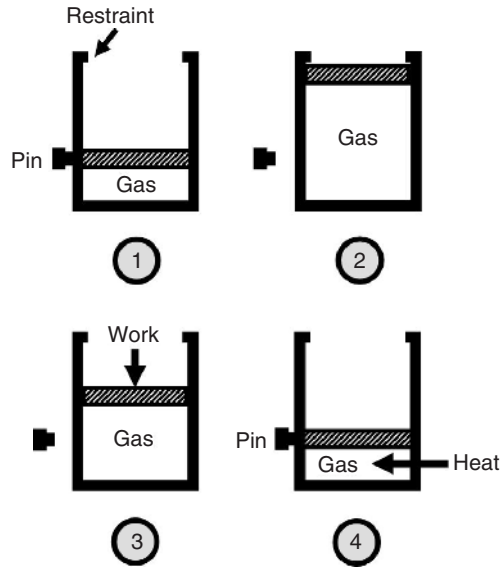


Figure 1.7 Irreversible process

processes do not actually occur in nature. Consider the example illustrated in Figure 1.7. (1) A gas is held under pressure by a weightless piston that is pinned in place. (2) When the pin is released, the pressurized gas pushes the piston upwards until a restraint stops it. Since the piston is weightless, there is no work done by the gas on the piston to move it. (3) In order to reverse this system to its original state a force (or work) must be applied to push the piston down and compress the gas. (4) Heat must also be transferred to the gas in order to restore its original internal energy. The addition of heat and work to restore the system will change the surroundings. Since this process is not reversible, it is called an **irreversible process**.

As the example shows, reversible processes do not occur in real systems because of the presence of friction and finite temperature and pressure differences create irreversible conditions. Reversible systems are simply idealizations of an actual system. A work-producing device (e.g., turbine) will deliver the maximum amount of work if a reversible process is used. Similarly a work-consuming device (e.g., compressor) will consume a minimum amount of work when reversible processes are used. Therefore, reversible processes can be considered as theoretical limits for corresponding real, irreversible processes. The more an actual process approximates a reversible process, the more efficient the design. Therefore reversible processes are often used to represent an ideal model of a system that can be compared with the actual irreversible system.

A process in which no heat transfer occurs is called an **adiabatic process**. Normally an adiabatic process can occur in only one of two ways: either the system is so well insulated that only a small (negligible) amount of heat can pass between the system and its surroundings, or both the system and its surroundings are at the same temperature. An adiabatic, reversible process is called **isentropic**.

The Second Law of Thermodynamics states that it is impossible to construct a system that will operate in a cycle, extract heat from a reservoir and do an equivalent amount of work on the surroundings. Thus the Second Law implies that if a system is to undergo a cycle and produce work, it must operate between two heat reservoirs of different temperatures. Therefore, an airplane driven by an engine extracting heat from the ambient air cannot exist, because it would have to reject heat to a lower temperature than its surroundings in order to produce work. The fact that no system can truly be irreversible (and therefore isentropic) is shown by the Clausius inequality, which states:

$$\oint \frac{\delta Q}{T} \leq 0 \quad (1.5)$$

If no irreversibilities occur within the system then the cycle can be reversed. This is called an internally reversible system, which is defined as:

$$\oint \left(\frac{\delta Q}{T} \right)_{rev} = 0 \quad (1.6)$$

In 1867, German physicist Rudolf Clausius defined the thermodynamic property, shown in Equation 1.5, as entropy (S) (Equation 1.6.) Entropy is expressed in SI units as J/K (or J/°C). In Imperial units it can have units such as cal/°R (or cal/°F). Entropy per unit mass or **specific entropy** is represented by the symbol s .

$$dS = \left(\frac{\delta Q}{T} \right)_{rev} \quad (1.7)$$

The Clausius inequality shows that entropy change (ΔS) of an isolated system during an irreversible process is always a positive quantity. In other words, some entropy is always generated in an isolated, irreversible process. Entropy is constant in an isolated, reversible process. Therefore, an isentropic process can be defined as an adiabatic process in which the entropy remains constant. This is known as the increase of entropy principle and is summarized as:

$$\Delta S > 0 \text{ Irreversible processes} \quad (1.8)$$

$$\Delta S = 0 \text{ Reversible (isentropic) processes} \quad (1.9)$$

$$\Delta S < 0 \text{ Impossible} \quad (1.10)$$

Entropy is a useful property in analyzing propulsion systems, but unlike other properties (such as temperature, pressure, or velocity) which have directly observable physical effects, entropy is a difficult concept for many people. It can be indirectly observed by measuring the losses incurred in other properties as a system undergoes a process. Entropy can be thought of as a measure of disorder, chaos, or randomness. Since no real, isolated system can undergo an isentropic process, the entropy or disorder of the universe is always increasing.

Many systems used in engines such as compressors, turbines, nozzles and diffusers can be ideally approximated by considering them to operate as an adiabatic process. Since the performance of these systems is optimized when irreversibilities (such as friction) are minimized, an isentropic (or ideal) model is often used to compare with the actual

irreversible process. This comparison is done by defining **isentropic efficiencies** that compare the actual performance to the idealized performance.



“Irreversible Mess” The Second Law of Thermodynamics says that processes will naturally become more disordered. By intentionally adding heat or work to a system we can bring more order to a particular attribute of the system. However, even though one attribute of the system may become more ordered, there will always be a net entropy gain (or increase of disorder) due to other irreversibilities (such as friction or heat loss).

1.1.2 Equation of State for a Perfect Gas

The properties of gases (especially air) are very important in analyzing propulsion systems. An equation that relates the pressure, temperature, and density of a gas (or any substance) is called an **equation of state**. In 1662, Englishman Robert Boyle discovered that the pressure of a gas is inversely proportional to its volume, at constant temperature. Boyle’s Law is shown in Equation 1.11.

$$P \propto \frac{1}{V} \quad (1.11)$$

In 1787, Frenchman Jacques Charles discovered that the volume of a gas is directly proportional to its temperature, at constant pressure. In 1802, Joseph Louis Gay-Lussac published this finding (citing Charles’s unpublished work). This is now known as Charles’s Law, shown in Equation 1.12.

$$T \propto V \quad (1.12)$$

Combining the results of Boyle’s and Charles’s Laws, an equation for a **perfect** (or **ideal**) **gas** is defined in Equations 1.13, 1.14, and 1.15.

**Equation of state for a perfect gas:
(Ideal Gas Law)**

$$\begin{array}{l}
 P\upsilon = RT \\
 P = \rho RT \\
 P\mathcal{V} = mRT
 \end{array}
 \left\{
 \begin{array}{l}
 P \equiv \text{Pressure of gas (N/m}^2\text{)} \\
 \upsilon \equiv \frac{\mathcal{V}}{m} \equiv \text{Specific volume of gas (m}^3\text{/kg)} \\
 T \equiv \text{Temperature of gas (K)} \\
 \rho \equiv \frac{1}{\upsilon} \equiv \text{Density of gas (kg/m}^3\text{)} \\
 \mathcal{V} \equiv \text{Volume of gas (m}^3\text{)} \\
 m \equiv \text{Mass of gas (kg)}
 \end{array}
 \right.
 \begin{array}{l}
 (1.13) \\
 (1.14) \\
 (1.15)
 \end{array}$$

The constant of proportionality (R) is called the **gas constant** and is different for every gas. It can be determined from a **universal gas constant** (R°) shown in Equation 1.16. The ideal gas equation of state is particularly useful for analysis of propulsion systems, because air is closely approximated to a perfect gas. For pure air, R is approximately equal to 287 J/(kg·K).

$$R = \frac{R^\circ}{M_w} \left\{ \begin{array}{l} R^\circ \equiv \text{Universal gas constant [= 8.3145 kJ/(kmole}\cdot\text{K)]} \\ M_w \equiv \text{Molecular weight of gas (kg/kmole)} \end{array} \right. \quad (1.16)$$

1.1.3 Law of the Conservation of Mass

The Law of the Conservation of Mass (also known as the continuity equation) applied to a fluid crossing the boundary of a fixed control volume is shown schematically in Figure 1.8.

The conservation of mass integral equation is shown in Equation 1.17.

$$\frac{\partial}{\partial t} \iiint_{\text{control volume}} \rho \, d\mathcal{V} + \iint_{\text{control surface}} \rho \vec{V} \, dA = 0 \quad (1.17)$$

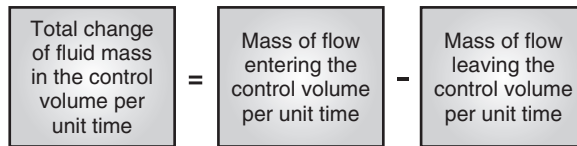


Figure 1.8 Law of the conservation of mass

If there is a **steady flow** through the control volume then the time rate of change of mass in the control volume is zero or:

$$\frac{\partial}{\partial t} \iiint_{\text{control volume}} \rho dV = 0 \quad (1.18)$$

So the **steady flow continuity equation** becomes:

$$\iint_{\text{control surface}} \rho \vec{V} dA = 0 \quad (1.19)$$

If the velocity (\vec{V}) does not vary in either magnitude or direction across a cross-sectional area (A) that is normal to the flow direction, then:

$$\dot{m} = \rho VA = \text{constant} \quad (1.20)$$

Example 1.1

Liquid oxygen (LOX) and liquid hydrogen (LH2) are steadily injected into a rocket thrust chamber at 8 kg/s and 1 kg/s respectively and ignited as shown in Figure 1.9. The combustion products are expelled from the rocket through a nozzle with a diameter of 25 cm.

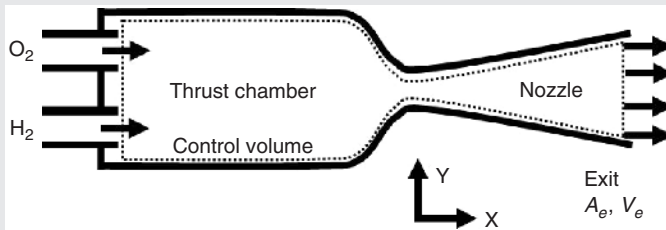


Figure 1.9

If the density of the combustion gases is 0.175 kg/m^3 . Determine the exit velocity of the combustion gases.

Solution

The control volume of the rocket thrust chamber and nozzle is shown by the dotted line in Figure 1.9. Since the propellants are flowing at a steady rate, the conservation of mass equations are reduced to:

$$\frac{\partial}{\partial t} \iiint_{\text{control volume}} \rho dV + \iint_{\text{control surface}} (\rho \vec{V} dA) = 0 \quad \text{0, steady flow}$$

$$\begin{aligned}
 \underbrace{\rho_{comb} A_e V_e}_{\dot{m}_{exit}} &= \underbrace{\dot{m}_{O_2} + \dot{m}_{H_2}}_{\dot{m}_{in}} \\
 V_e &= \frac{\dot{m}_{O_2} + \dot{m}_{H_2}}{A_e \rho_{comb\ prod}} = \frac{\dot{m}_{O_2} + \dot{m}_{H_2}}{\frac{\pi d^2}{4} \left(\rho_{comb\ prod} \right)} \\
 &= \frac{8 \frac{\text{kg}}{\text{s}} + 1 \frac{\text{kg}}{\text{s}}}{\frac{\pi}{4} (0.25 \text{ m})^2 \left(0.175 \frac{\text{kg}}{\text{m}^3} \right)} = 1047.7 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

1.1.4 Law of the Conservation of Linear Momentum

The conservation of linear momentum equation applied to a fixed control volume is shown schematically in Figure 1.10. The conservation of linear momentum integral equation is shown in Equation 1.21:

$$\sum F = \frac{\partial}{\partial t} \iiint_{\text{control volume}} \rho \vec{V} dV + \iint_{\text{control surface}} \vec{V} (\rho \vec{V} dA) \quad (1.21)$$

In this equation, $\sum F$ is the summation of all the external forces acting on the control volume, which may include gravity, pressure forces, or viscous forces. If steady flow conditions exist, then the time rate of change of momentum in the control volume is zero or:

$$\frac{\partial}{\partial t} \iiint_{\text{control volume}} \rho \vec{V} dV = 0 \quad (1.22)$$

So the **steady flow momentum equation** becomes:

$$\sum F = \iint_{\text{control surface}} \vec{V} (\rho \vec{V} dA) \quad (1.23)$$

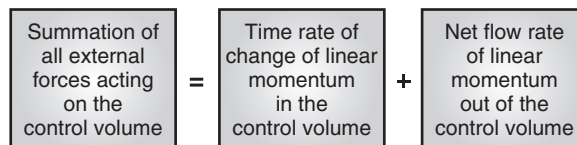


Figure 1.10 Conservation of linear momentum

Example 1.2

A rocket motor burning on a test stand steadily exhausts 20 kg/s of combustion gases at an exit velocity of 750 m/s, as shown in Figure 1.11. The static pressure of the exhaust gases exiting the nozzle is 120 kPa. Assume an ambient air pressure of 101.3 kPa. Determine the force (or thrust) the rocket produces.

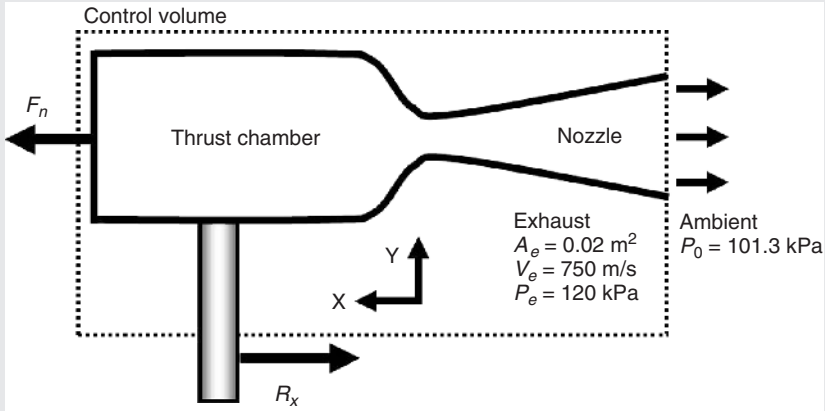


Figure 1.11

Solution

The external reaction force (R_x) which holds the rocket in place on the test stand is equal in magnitude but opposite in direction to the thrust force produced by the rocket.

$$F_N = -R_x$$

The control volume encompassing the rocket and test stand is shown by the dotted line in Figure 1.11. Since the exhaust gas is flowing at a steady rate, the conservation of momentum equations reduce to:

$$\begin{aligned}
 F_N - (P_e - P_0)A_e &= \frac{\partial}{\partial t} \iiint_{\text{control volume}} \rho \vec{V} dV + \iint_{\text{control surface}} \vec{V} (\rho \vec{V} \cdot d\Lambda) \\
 F_N &= \dot{m} V + A_e (P_e - P_0) \\
 &= \frac{\left(20 \frac{\text{kg}}{\text{s}}\right) \left(750 \frac{\text{m}}{\text{s}}\right)}{1,000 \frac{\text{N}}{\text{kN}}} + (0.02 \text{ m}^2) (120 \text{ kPa} - 101.3 \text{ kPa}) \\
 &= 15.4 \text{ kN}
 \end{aligned}$$

This example shows how the thrust equation for a rocket is derived from the Law of the Conservation of Momentum.

1.1.5 Law of the Conservation of Energy

The Law of the Conservation of Energy is also known as the First Law of Thermodynamics. For a system consisting of a fixed mass of particles, the Law of the Conservation of Energy of the system is shown schematically in Figure 1.12.

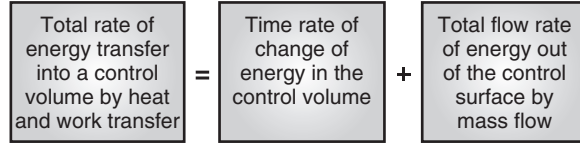


Figure 1.12 Conservation of energy

The total heat (Q) and work (W) are defined in the energy equation as forms of energy crossing the control volume boundary of a system. Total work can include mechanical work, electrical work, or magnetic work. The total energy (E) includes all energy forms of the system at a given state. The forms of energy include internal energy (U), which is the random motion of molecules in the system; the potential energy (E_{PE}); kinetic energy (E_{KE}), due to the position and motion of the entire system; and storable forms of energy such as chemical energy or electrical (capacitance) energy. Each of these energy forms must be applied to a control volume encompassing the system to derive an energy integral equation. The total energy per unit mass is symbolized by e and the total internal energy per unit mass (or **specific internal energy**) is symbolized by u . If the system has only internal, potential, and kinetic energies, the integral equation is expressed as Equation 1.24.

$$\frac{d}{dt} (Q - W) = \frac{\partial}{\partial t} \iiint_{\text{control volume}} e \rho dV + \iint_{\text{control surface}} \left(u + \frac{\vec{v}^2}{2} + gz \right) (\rho \vec{v} dA) \quad (1.24)$$

If the system consists of mass flows that cross the control volume boundaries, the total work (W) can be written in two parts. The first part is the flow work. This is the work required to overcome the external pressure forces and drive the mass across the boundaries. The second part lumps together all other work crossing the boundaries such as mechanical (or shaft) work, viscous shear work, electrical work, and so on. The equation for the flow work is shown in Equation 1.25.

$$W_{\text{flow}} = P \Delta V = \frac{P}{\rho} \Delta m \quad (1.25)$$

Therefore the flow work done per unit mass is:

$$w_{\text{flow}} = \frac{P}{\rho} \quad (1.26)$$

Under these conditions, the energy equation is normally written by combining the specific internal energy (u) and the flow work per unit mass (w_{flow}) into a single property, called

the **specific enthalpy** (h) defined in Equation 1.27.

$$h = u + \frac{P}{\rho} \quad (1.27)$$

The energy equation in this form is shown in Equation 1.28.

$$\frac{d}{dt} (Q - W') = \frac{\partial}{\partial t} \iiint_{\text{control volume}} \epsilon \rho dV + \iint_{\text{control surface}} \left(h + \frac{\vec{V}^2}{2} + gz \right) (\rho \vec{V} dA) \quad (1.28)$$

In this equation W' is the total work excluding the flow work (w_{flow}).

Another important parameter used in relation to the energy equation is the specific heat. The specific heat of a solid or liquid is defined as the amount of heat required to raise a unit mass through a 1°C temperature rise. When the volume is held constant, this known as the **constant volume specific heat** (C_v). Since the specific internal energy (u) is a function of only the temperature of a perfect gas, this can be expressed as shown in Equation 1.29.

$$C_v = \left[\frac{\partial u}{\partial T} \right]_v \quad (1.29)$$

When the pressure is held constant, this is known as the **constant pressure specific heat** (C_p). However, from the definition of enthalpy (h) in Equation 1.27, it can be seen that for a perfect gas (such as air):

$$dh = du + d \left(\frac{P}{\rho} \right) \quad (1.30)$$

Since this is a perfect gas, Equation 1.14 can be substituted into Equation 1.30 to give:

$$dh = du + RdT \quad (1.31)$$

Therefore the enthalpy of a perfect gas is also only a function of temperature, so the constant pressure specific heat can be expressed as Equation 1.32.

$$C_p = \left[\frac{\partial h}{\partial T} \right]_p \quad (1.32)$$

Rearranging Equation 1.31 gives:

$$R = \frac{dh}{dT} - \frac{du}{dT} = C_p - C_v \quad (1.33)$$

Also the **ratio of specific heats** (γ) is defined as:

$$\gamma = \frac{C_p}{C_v} \quad (1.34)$$

Therefore:

$$\frac{C_v}{R} = \frac{C_v}{C_p - C_v} = \frac{1}{\frac{C_p}{C_v} - 1} = \frac{1}{\gamma - 1} \quad (1.35)$$

and likewise:

$$\frac{C_p}{R} = \frac{\gamma}{\gamma - 1} \quad (1.36)$$

Table A.2 in Appendix A lists properties of gases at a reference temperature of 25°C (298 K).

A **calorically perfect gas** is a perfect gas with constant specific heats. If a calorically perfect gas is involved in a thermodynamic process between two equilibrium states, then the specific internal energy or enthalpy difference can be determined by integrating Equations 1.29 and 1.32.

$$\Delta u = u_2 - u_1 = \int_1^2 C_v dT = C_v (T_2 - T_1) \quad (1.37)$$

$$\Delta h = h_2 - h_1 = \int_1^2 C_p dT = C_p (T_2 - T_1) \quad (1.38)$$

Another useful thermodynamic equation for a pure substance is:

$$T ds = dh - \frac{dP}{\rho} \quad (1.39)$$

Therefore the entropy change of a perfect gas is:

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - R \ln \left(\frac{P_2}{P_1} \right) \quad (1.40)$$

For an isentropic process $\Delta s = 0$, therefore:

$$\int_1^2 C_p \frac{dT}{T} = R \ln \left(\frac{P_2}{P_1} \right) \quad (1.41)$$

$$\frac{C_p}{R} \ln \left(\frac{T_2}{T_1} \right) = \ln \left(\frac{P_2}{P_1} \right) \quad (1.42)$$

Example 1.3

Air enters an adiabatic compressor of a turbojet engine at 70 kg/s and at a temperature (T_1) of 30°C, as shown in Figure 1.13. It flows steadily through the compressor with no change in velocity and exits at a temperature (T_2) of 350°C. Assume that the constant pressure specific heat (C_p) of air is 1.005 kJ/(kg·K). Determine the minimum power that must be generated by a turbine in order to drive the compressor at these conditions.

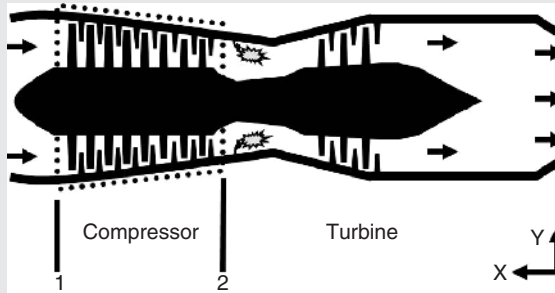


Figure 1.13

Solution

The control volume around the compressor section is shown by the dotted line in Figure 1.13. The energy equation for steady flow is:

$$\frac{\partial}{\partial t} \iiint_{\text{control volume}} e \rho dV + \iint_{\text{control surface}} \left(h + \frac{\vec{v}^2}{2} + gz \right) (\rho \vec{V} \cdot dA) = \frac{d}{dt} (Q - W')$$

0, steady flow No velocity change No height change 0, adiabatic

Therefore the power required by the compressor to pressurize the air (and thereby also increase its temperature) is equal to the minimum power required by the turbine to drive it.

$$T_1 = 30^\circ\text{C} + 273 = 303 \text{ K}$$

$$T_2 = 350^\circ\text{C} + 273 = 623 \text{ K}$$

$$\begin{aligned} \dot{W}_{\text{turbine}} &= -\dot{W}_{\text{compressor}} = \dot{m}_{\text{air}} (h_2 - h_1) = \dot{m}_{\text{air}} C_p (T_2 - T_1) \\ &= \left(70 \frac{\text{kg}}{\text{s}} \right) \left(1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right) (623 \text{ K} - 303 \text{ K}) \\ &= 22\,512 \frac{\text{kJ}}{\text{s}} \text{ or kW} \end{aligned}$$

1.2 Isentropic Equations

1.2.1 Isentropic Relationship between Temperature and Pressure

Isentropic equations are often used in the analysis of perfect gas turbine engines and rocket motors. For a simple, stationary compressible system the **state postulate** specifies that the state of a system is completely specified by two independent, intensive (independent of mass) properties, such as pressure (P) and temperature (T). (For a complete description of the state postulate, see [2].) This means that the properties of a unit mass of a gas (a single-phase system) can be determined from knowledge of just two independent, intensive properties. However, if the gas is in motion, three intensive properties are required, such as velocity (V), pressure (P), and temperature (T). The tables in Appendix B list how several property ratios vary ideally (isentropically) with Mach number. These ratios are determined from isentropic equations, derived by first applying the conservation of energy equation to a closed stationary system (fixed mass) containing a compressible working fluid (such as air). An internally reversible process involving this closed system is expressed in Equation 1.43.

$$dQ - dW = dU \quad (1.43)$$

Since:

$$dQ = Tds \quad (1.44)$$

$$dW = Pd\psi \quad (1.45)$$

$$dU = C_v dT \quad (1.46)$$

Substituting Equations 1.44, 1.45, and 1.46 into Equation 1.43 and recognizing that for an isentropic process $T ds = 0$, gives the following expression:

$$Tds = C_v dT + Pd\psi = 0 \quad (1.47)$$

Therefore:

$$dT = -\frac{Pd\psi}{C_v} \quad (1.48)$$

Rearranging Equation 1.13 (for a perfect gas):

$$T = \frac{P\psi}{R} \quad (1.49)$$

Differentiating Equation 1.49 with respect to temperature gives:

$$dT = \frac{1}{R}(Pd\psi + \psi dP) \quad (1.50)$$

Substituting Equation 1.50 into Equation 1.48 gives:

$$-\frac{Pd\psi}{C_v} = \frac{1}{R}(Pd\psi + \psi dP) \quad (1.51)$$

$$\frac{C_v}{R}(Pd\psi + \psi dP) + Pd\psi = 0 \quad (1.52)$$

Since Equation 1.35 states:

$$\frac{C_v}{R} = \frac{1}{\gamma - 1}$$

Therefore:

$$\frac{1}{\gamma - 1}(Pd\psi + \psi dP) + Pd\psi = 0 \quad (1.53)$$

$$Pd\psi + \psi dP + (\gamma - 1)Pd\psi = 0 \quad (1.54)$$

$$Pd\psi + \psi dP + \gamma Pd\psi - Pd\psi = 0 \quad (1.55)$$

$$\frac{dP}{P} + \gamma \frac{d\psi}{\psi} = 0 \quad (1.56)$$

$$\ln(P) + \gamma \ln(\psi) = 0 \quad (1.57)$$

These equations can be simplified by using the following logarithmic identities:

$$\log(x^n) = n \log(x) \quad (1.58)$$

$$\log(xy) = \log(x) + \log(y) \quad (1.59)$$

By applying these identities, Equation 1.57 can be rewritten as:

$$\ln(P) + \ln(\psi^\gamma) = \ln(P\psi^\gamma) = 0 \quad (1.60)$$

Therefore:

$$P\psi^\gamma = \text{constant} = C \quad (1.61)$$

Since by the Ideal Gas Law (Equation 1.13):

$$\psi = \frac{RT}{P} \quad (1.62)$$

$$P\psi^\gamma = P \left(\frac{RT}{P} \right)^\gamma = \frac{T^\gamma R^\gamma}{P^{(\gamma-1)}} = \text{constant} = C \quad (1.63)$$

$$\frac{T_1^\gamma R^\gamma}{P_1^{(\gamma-1)}} = \frac{T_2^\gamma R^\gamma}{P_2^{(\gamma-1)}} \quad (1.64)$$

$$\frac{T_1}{T_2} = \left[\frac{P_1}{P_2} \right]^{\frac{(\gamma-1)}{\gamma}} \quad (1.65)$$

The isentropic relationship between temperature and pressure in a gas turbine engine is commonly illustrated by a T - S diagram of the engine cycle (Figure 1.14). The T - S diagram is generally preferred to the P - ψ diagram for illustrating propulsion analysis of gas turbine, ramjet, or scramjet engines. The ideal (or isentropic) processes representing the compressor and turbine are shown by the vertical arrows and labeled with a prime symbol ('). The actual (or non-isentropic) processes are represented by dashed arrows.

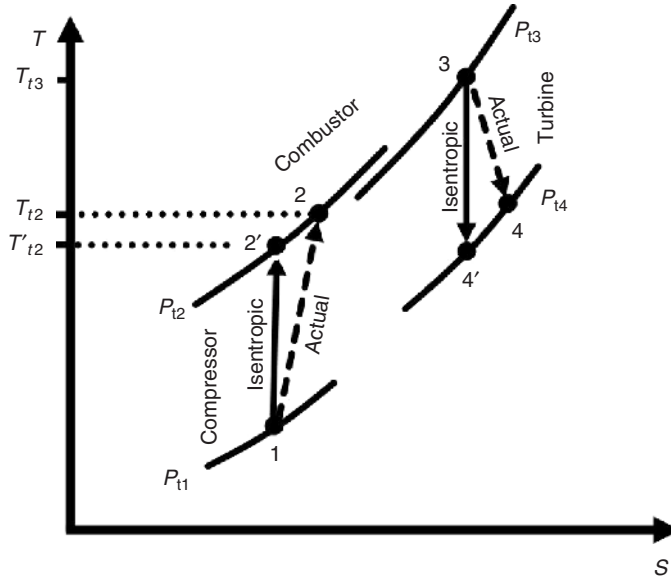


Figure 1.14 T - S Diagram for a gas turbine engine

1.2.2 Isentropic Relationships with Specific Volume

It is also useful to develop isentropic expressions that relate the specific volume (υ) to pressure (P) and temperature (T). Starting with the definition of enthalpy (h), shown in Equation 1.30:

$$dh = du + d\left(\frac{P}{\rho}\right) = du + d(P\upsilon) \quad (1.66)$$

$$dh = du + P d\upsilon + \upsilon dP \quad (1.67)$$

Rearranging Equation 1.48:

$$C_v dT = -P d\upsilon \quad (1.68)$$

Substituting Equation 1.67 into Equation 1.68 gives:

$$0 = (dh - P d\upsilon - \upsilon dP) + P d\upsilon \quad (1.69)$$

$$0 = dh - \upsilon dP \quad (1.70)$$

$$dh = \upsilon dP \quad (1.71)$$

Since also (Equation 1.32):

$$dh = C_p dT \quad (1.72)$$

then:

$$\upsilon dP = C_p dT \quad (1.73)$$

Dividing Equation 1.68 by Equation 1.73 results in the following equation:

$$\frac{-P d\vartheta}{\vartheta dP} = \frac{C_v}{C_p} \quad (1.74)$$

$$\frac{dP}{P} = \frac{-C_p}{C_v} \frac{d\vartheta}{\vartheta} = -\gamma \frac{d\vartheta}{\vartheta} \quad (1.75)$$

Integrating this equation between points 1 and 2 gives:

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\gamma \int_{\vartheta_1}^{\vartheta_2} \frac{d\vartheta}{\vartheta} \quad (1.76)$$

$$\ln\left(\frac{P_2}{P_1}\right) = -\gamma \ln\left(\frac{\vartheta_2}{\vartheta_1}\right) \quad (1.77)$$

$$\ln\left(\frac{P_2}{P_1}\right) = \ln\left(\frac{\vartheta_2}{\vartheta_1}\right)^{-\gamma} \quad (1.78)$$

$$\frac{P_2}{P_1} = \left(\frac{\vartheta_2}{\vartheta_1}\right)^{-\gamma} = \left(\frac{\vartheta_1}{\vartheta_2}\right)^{+\gamma} \quad (1.79)$$

Recall from Equation 1.13 for a perfect gas:

$$P = \frac{RT}{\vartheta}$$

This can be substituted in to give a relationship with temperature.

$$\frac{P_2}{P_1} = \frac{\frac{RT_2}{\vartheta_2}}{\frac{RT_1}{\vartheta_1}} = \left(\frac{\vartheta_1}{\vartheta_2}\right)^\gamma \quad (1.80)$$

$$\frac{T_2}{T_1} \left(\frac{\vartheta_1}{\vartheta_2}\right) = \left(\frac{\vartheta_1}{\vartheta_2}\right)^\gamma \quad (1.81)$$

$$\frac{T_2}{T_1} = \left(\frac{\vartheta_1}{\vartheta_2}\right)^{-1} \left(\frac{\vartheta_1}{\vartheta_2}\right)^\gamma \quad (1.82)$$

$$\frac{T_2}{T_1} = \left(\frac{\vartheta_1}{\vartheta_2}\right)^{\gamma-1} \quad (1.83)$$

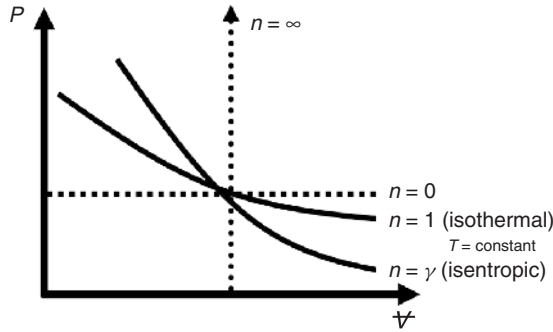


Figure 1.15 Polytropic expression definition

1.3 Polytropic Processes

In the previous section, relations were derived using isentropic processes. These ideal relationships are often used to compare actual processes by use of isentropic efficiencies (Figure 1.14). Isentropic efficiencies will be defined and described in greater detail in subsequent chapters of this book. These efficiencies will be used as a measure of the irreversibility of turbomachinery processes. However, there are other propulsion processes that approach the isentropic law. Polytropic processes follow laws that form the relation:

$$P\psi^n = \text{constant} = C \quad (1.84)$$

Where n lies between 0 and γ . Equation 1.84 is used to create P - ψ diagram plots for three different profiles shown in Figure 1.15.

The different curves shown in this plot (created by changing the value of n) illustrate alternative methods of defining efficiencies, called **polytropic efficiencies**. Figure 1.15 shows that the isothermal (or constant temperature) process can be considered as a special case of the polytropic process when $n = 1$. Isentropic processes can be considered a special polytropic case, when $n = \gamma$. For simplicity, this book deals only with isentropic efficiencies. These are defined as the ratio of the ideal (isentropic) work to the actual work for given pressure ratios. Other references make more use of polytropic efficiencies [3]. Polytropic efficiencies are defined as the ratio of the ideal work to the actual work for a differential pressure change.

1.4 Total (or Stagnation) Properties

When a fluid in motion is isentropically brought to rest a temperature and pressure rise occurs. The properties of a fluid at this stagnation point are called stagnation properties (Figure 1.16). In the absence of hydrostatic pressures (i.e., the elevation effects of fluid weight on pressure), stagnation properties are equivalent to total properties. **Total properties** (e.g., P_t , T_t , h_t) are useful as a reference state for compressible flow in propulsion systems.

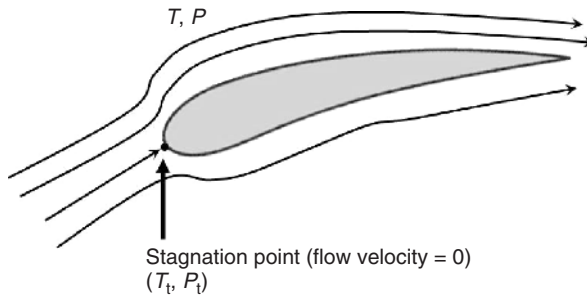


Figure 1.16 Definition of total (or stagnation) properties

A definition for the total temperature can be derived by applying the energy equation to a unit mass flow in a duct where:

- there is no change in potential energy
- flow is adiabatic ($Q = 0$)
- no work is done ($W = 0$).

Applying the Law of the Conservation of Energy gives:

$$u_1 + P_1 v_1 + \frac{V_1^2}{2} = u_2 + P_2 v_2 + \frac{V_2^2}{2} \quad (1.85)$$

$$C_v T_1 + RT_1 + \frac{V_1^2}{2} = C_v T_2 + RT_2 + \frac{V_2^2}{2} \quad (1.86)$$

Substituting Equation 1.33 into this equation gives:

$$(C_p - R)T_1 + RT_1 + \frac{V_1^2}{2} = (C_p - R)T_2 + RT_2 + \frac{V_2^2}{2} \quad (1.87)$$

$$T_1 + \frac{V_1^2}{2C_p} = T_2 + \frac{V_2^2}{2C_p} \quad (1.88)$$

The sum of the terms on the two sides of this equation are identical. Therefore these summations can be defined as a new term, as shown in Equations 1.89 and 1.90.

$$T_{t1} = T_{t2} \quad (1.89)$$

The term T_t in Equation 1.89 is the total (or stagnation) temperature (Figure 1.16). Therefore the **total temperature** is defined as:

Total (or stagnation) temperature:

Total temperature = Static temperature + Dynamic temperature

$$T_t = T + \frac{V^2}{2C_p} \quad (1.90)$$

In this equation, the **dynamic temperature** is the temperature equivalent of the kinetic energy ($V^2/2C_p$) of the flow. The total to static pressure ratio (P_t/P) can be related to the total to static temperature ratio (T_t/T) by inspection of Equation 1.65:

$$\frac{P_t}{P} = \left(\frac{T_t}{T}\right)^{\frac{\gamma}{\gamma-1}} \quad (1.91)$$

The **total enthalpy per unit mass** (h_t) for a perfect gas with constant specific heats is therefore defined as:

$$h_t - h = C_p (T_t - T) \quad (1.92)$$

Substituting Equation 1.90 into Equation 1.92:

$$h_t = h + \frac{V^2}{2} \quad (1.93)$$

It is often inconvenient to use the velocity of the gas in aerospace propulsion analysis. The **Mach number** (M) is a non-dimensional parameter that is often used in place of the velocity to describe the state of a flowing gas. The Mach number is defined as the ratio of the velocity (V) over the **velocity of sound** (a).

Mach number:

$$M = \frac{V}{a} \quad (1.94)$$

The velocity of sound (a) for a perfect gas is derived by encompassing a sound expansion wave within a control volume and applying the Law of the Conservation of Mass and the Law of the Conservation of Momentum to it. Combining these two equations results in an equation for the velocity of sound (for a weak compression wave), shown in Equation 1.95 (the derivation is given in [4]).

$$a^2 = \left(\frac{\partial P}{\partial \rho}\right)\bigg|_s = \frac{dP}{d\rho} \quad (1.95)$$

The velocity of sound for a perfect gas is simplified from this expression by applying Equation 1.79, which states:

$$\frac{P_2}{P_1} = \left(\frac{\vartheta_1}{\vartheta_2}\right)^\gamma = \left(\frac{\rho_2}{\rho_1}\right)^\gamma \quad (1.96)$$

or:

$$\frac{P_1}{\rho_1^\gamma} = \frac{P_2}{\rho_2^\gamma} = \frac{P}{\rho^\gamma} = \text{constant} = C \quad (1.97)$$

Solving this equation for P and substituting this back into Equation 1.95 yields:

$$a^2 = \frac{dP}{d\rho} = C \frac{d(\rho^\gamma)}{d\rho} = \frac{\gamma C \rho^\gamma}{\rho} \quad (1.98)$$

Substituting the value of P (found in Equation 1.97) into Equation 1.98 results in:

$$a^2 = \frac{\gamma P}{\rho} \quad (1.99)$$

Simplifying this equation and substituting for (P/ρ) by using the equation of state for a perfect gas (Equation 1.14), results in:

Velocity of sound (perfect gas):

$$a = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma RT} \quad (1.100)$$

The total to static ratios of temperature and pressure can be written in terms of Mach number. Combining Equations 1.99 and 1.100 together gives:

$$V^2 = M^2 a^2 = M^2 (\gamma RT) \quad (1.101)$$

Substituting this equation into Equation 1.91 yields:

$$\frac{T_t}{T} = 1 + \frac{M^2 \gamma R}{2C_p} \quad (1.102)$$

Recall from Equations 1.33 and 1.34:

$$R = C_p - C_v \quad \text{and} \quad \gamma = \frac{C_p}{C_v}$$

Combining these equations together gives:

$$\frac{R}{C_v} = \frac{C_p}{C_v} - 1 = \gamma - 1 \quad (1.103)$$

$$R = C_v (\gamma - 1) \quad (1.104)$$

Substituting Equation 1.104 into Equation 1.102 gives:

$$\frac{T_t}{T} = 1 + \frac{M^2 \gamma C_v (\gamma - 1)}{2C_v \gamma} \quad (1.105)$$

Canceling out terms in this equation gives an equation relating total to static temperature ratio and Mach number.

$$\frac{T_t}{T} = 1 + \frac{M^2 (\gamma - 1)}{2} \quad (1.106)$$

Substituting Equation 1.91 into Equation 1.106 gives an equation that relates the total to static pressure ratio and the Mach number.

$$\frac{P_t}{P} = \left[1 + \frac{M^2(\gamma - 1)}{2} \right]^{\frac{\gamma}{\gamma - 1}} \quad (1.107)$$

Like Mach number (M), there are several other non-dimensional parameters that are useful in engine analysis. One commonly used parameter is called the **X-function**. It is derived by applying the Law of the Conservation of Mass to describe a steadily, flowing gas in a parallel duct (Equation 1.108).

$$\dot{m} = \rho AV \quad (1.108)$$

Rearranging the equation of state for a perfect gas (Equation 1.14):

$$\rho = \frac{m}{V} = \frac{P}{RT} \quad (1.109)$$

Rearranging Equation 1.101:

$$V = M\sqrt{\gamma RT} \quad (1.110)$$

Substituting Equation 1.109 and 1.110 into Equation 1.108 gives:

$$\dot{m} = \frac{PAM\sqrt{\gamma}}{\sqrt{RT}} \quad (1.111)$$

Rearranging Equation 1.106 gives:

$$T = \frac{T_t}{\left(1 + \frac{\gamma + 1}{2} M^2 \right)} \quad (1.112)$$

Rearranging Equation 1.107 gives:

$$P = \frac{P_t}{\left(1 + \frac{\gamma + 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}}} \quad (1.113)$$

Substituting Equations 1.112 and 1.113 into Equation 1.111 gives:

$$\dot{m} = \frac{P_t AM\sqrt{\gamma}}{\sqrt{RT_t}} \left[1 + \frac{(\gamma - 1) M^2}{2} \right]^{\frac{-(\gamma + 1)}{2(\gamma - 1)}} \quad (1.114)$$

A portion of Equation 1.114 is defined as the **non-dimensional mass flow rate** or simply as the **X-function** (X) (shown in Equation 1.115). The X -function is a useful term to aid

analysis of fluid flows because it is a unique function of Mach number in a calorically perfect gas.

X-function (non-dimensional mass flow rate):

$$X = M\sqrt{\gamma} \left[1 + \frac{\gamma - 1}{2} M^2 \right]^{\frac{-(\gamma+1)}{2(\gamma-1)}} \quad (1.115)$$

Substituting the definition of the X -function (Equation 1.115) into Equation 1.114 gives:

$$\dot{m} = \frac{P_t A X}{\sqrt{RT_t}} \quad (1.116)$$

Therefore the X -function can also be expressed by rearranging Equation 1.116, which gives:

X-function (non-dimensional mass flow rate):

$$X = \frac{\dot{m}\sqrt{RT_t}}{A P_t} \quad (1.117)$$

The X -function is often used in the analysis of propulsion systems. Likewise a ***Y-function*** and ***Z-function*** can also be used. These are defined as:

$$X = YZ \quad (1.118)$$

The Y - and Z -functions are shown in Equations 1.119 and 1.120.

Y-function (non-dimensional specific internal thrust reciprocal):

$$Y = \frac{M\sqrt{\gamma} \sqrt{1 + \left(\frac{\gamma - 1}{2}\right) M^2}}{1 + \gamma M^2} \quad (1.119)$$

Z-function (non-dimensional internal thrust):

$$Z = \frac{(1 + \gamma M^2)}{\left[1 + \frac{\gamma - 1}{2} M^2 \right]^{\frac{\gamma}{\gamma - 1}}} \quad (1.120)$$

1.5 Isentropic Principles in Engine Components

1.5.1 Ducts

Steady, isentropic flow through a frictionless duct is one of the simplest fluid dynamics systems to define and analyze. No work can be extracted from a duct. If the flow is isentropic (and therefore adiabatic) there is no heat transfer (in or out), and the energy equation becomes:

$$\dot{Q} - \dot{W} = 0 \quad (1.121)$$

Since there is no heat transfer, the total temperature is constant ($\Delta T_t = 0$). For steady flow ($\Delta \dot{m} = 0$) in a frictionless duct the total pressure is also constant ($\Delta P_t = 0$). If a frictional duct is considered (non-isentropic), there is a loss in total pressure because of the change in entropy (Δs) caused by frictional losses. This can be seen in Equation 1.122.

$$\Delta s = \frac{\Delta S}{m} = -R \ln \left(\frac{P_{t2}}{P_{t1}} \right) \quad (1.122)$$

Since the change in entropy (Δs) cannot fall in value (Equation 1.8), then there must be a loss in total pressure ($P_{t2} < P_{t1}$) in a frictional duct. But in this simple case, flow through the duct is non-frictional and isentropic ($\Delta s = 0$). Therefore:

$$AX = \frac{\dot{m} \sqrt{RT_t}}{P_t} = \text{constant} = C \quad (1.123)$$

The only forces acting on the gas in this duct are pressure forces (the duct exerts no axial force). So applying Newton's Second Law gives:

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1) \quad (1.124)$$

or

$$\dot{m}V_1 + AP_1 = \dot{m}V_2 + AP_2 \quad (1.125)$$

Therefore at any plane in the duct:

$$\dot{m}V + AP = \text{constant} \quad (1.126)$$

This constant, known as the internal stream thrust (F_{int}), is expressed as:

$$F_{int} = \dot{m}V + AP \quad (1.127)$$

Rearranging the equation for mass flow derived from the Law of the Conservation of Mass (Equation 1.20) gives:

$$\dot{m} = \frac{PAV}{RT} \quad (1.128)$$

Substituting Equation 1.128 into Equation 1.127 results in:

$$F_{int} = \frac{PAV^2}{RT} + AP = PA \left(\frac{V^2}{RT} + 1 \right) \quad (1.129)$$

Substituting the Mach number definition (Equation 1.94) for velocity into Equation 1.128 gives:

$$F_{\text{int}} = PA (\gamma M^2 + 1) \quad (1.130)$$

Finally substituting Equation 1.107 into this equation gives:

$$F_{\text{int}} = \frac{P_t A (\gamma M^2 + 1)}{\left(1 + M^2 \frac{(\gamma - 1)}{2}\right)^{\frac{\gamma}{\gamma - 1}}} \quad (1.131)$$

Substituting the Z -function into this equation gives:

Steady frictionless flow in a parallel duct:

$$F_{\text{int}} = P_t AZ \quad (1.132)$$

This equation reveals the reason that the Z -function is known as the non-dimensional internal thrust, since:

$$Z = \frac{F_{\text{int}}}{P_t A} \quad (1.133)$$

The Law of the Conservation of Mass or continuity equation (Equation 1.19) for one-dimensional steady flow through a varying area duct can be written as:

$$(\rho + d\rho)(A + dA)(V + dV) - \rho AV = 0 \quad (1.134)$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad (1.135)$$

Applying the steady flow, linear momentum equation (Equation 1.23) for this same control volume gives the equation:

$$pA + \left(P + \frac{dP}{2}\right) dA - (P + dP)(A + dA) = (\rho AV) dV \quad (1.136)$$

$$dP + \rho V dV = 0 \quad (1.137)$$

Multiplying Equation 1.135 by (ρV^2) and rearranging gives:

$$\rho V dV = -d\rho V^2 - \frac{\rho V^2 dA}{A} \quad (1.138)$$

Substituting Equation 1.138 into Equation 1.137 results in:

$$dP + \rho V^2 \left[-\frac{d\rho}{\rho} - \frac{dA}{A} \right] = 0 \quad (1.139)$$

This equation is simplified by solving the general definition of the velocity of sound (Equation 1.95) for $d\rho$ and then substituted into Equation 1.137, resulting in:

$$dP + \rho V^2 \left(-\frac{dP}{\rho a^2} - \frac{dA}{A} \right) = 0 \quad (1.140)$$

Further simplification of Equation 1.140 gives a relationship for isentropic flow in a varying duct or channel.

$$dP (1 - M^2) = \rho V^2 \frac{dA}{A} \quad (1.141)$$

Equation 1.141 is significant because it shows the effect that Mach number has on flow inside a varying area duct or channel (Figure 1.17). For subsonic flow ($M < 1$), it can be

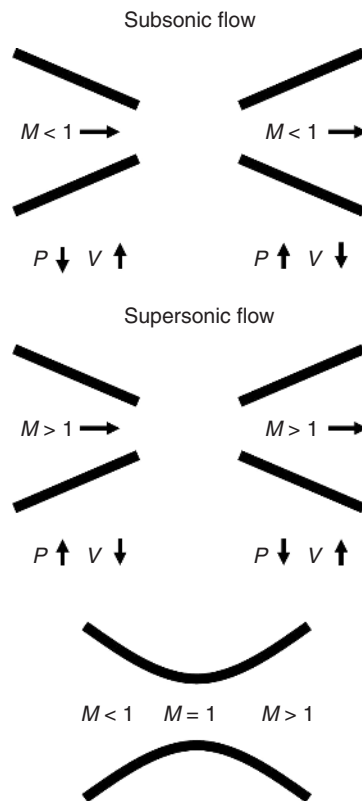


Figure 1.17 Flow through a varying area duct (John, J. E. A., *Gas Dynamics*, 2nd Edition, © 1984. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ [4].)

seen from this equation that a decrease in area (dA) (called a converging duct), results in a decrease in pressure (dP) and an increase in velocity (dV). Conversely an increase in area (diverging duct) increases the pressure and decreases the velocity.

For supersonic flow ($M > 1$), the opposite trends occur. These results show that subsonic flow cannot be accelerated to supersonic flow in a converging duct or nozzle. A convergent–divergent (condi) duct or nozzle must be used to achieve this (see Section 1.5.4).

1.5.2 Turbomachinery

Turbomachinery (such as compressors or turbines) are designed to transfer work (not heat). Compressors are used to increase the pressure of a flow while turbines are used to extract work (or energy) from the flow. In a gas turbine engine, energy extracted from turbines is used to power the compressors. The power required to drive the compressor is determined by the energy equation (derived in Example 1.3):

$$\dot{W}_c = \dot{m}C_p (T_{t2} - T_{t1}) \quad (1.142)$$

Ideally, the temperature ratio in compressors and turbines is the minimum associated with the pressure changes in the devices. So ideal turbomachinery can be considered isentropic and Equation 1.65 can be applied (but for total pressure and temperature ratios), as shown in Equation 1.143.

$$\frac{T_{t1}}{T_{t2}} = \left[\frac{P_{t2}}{P_{t1}} \right]^{\frac{\gamma-1}{\gamma}} \quad (1.143)$$

Actual turbomachinery has irreversibilities due to friction on all of the wetted surfaces. However, it is difficult to account for these irreversibilities. Instead ideal (isentropic) properties can be used as a basis for the engine's performance analysis. Isentropic efficiencies (η) are then used to determine actual properties from the ideal. (This will be discussed in much greater detail in subsequent chapters.)

1.5.3 Combustion Chambers (Combustors)

Combustion chambers (or thrust chambers in rockets) generate heat by greatly increasing the temperature of the flow. For gas turbine engines and rockets, this is generally done by burning fuel or propellant. Combustion chambers are designed to be steady flow devices, so they are essentially ducts with the capacity for heat addition. No work can be extracted. The heat produced by the combustor is therefore:

$$\dot{Q} = \dot{m}C_p (T_{t2} - T_{t1}) \quad (1.144)$$

1.5.4 Nozzles

Two types of nozzles are primarily used in aerospace propulsion systems (shown in Figure 1.18). A **convergent nozzle** is shaped to have a continuously decreasing cross-sectional area in the flow direction. A **convergent–divergent** (or **condi**) **nozzle** is a convergent nozzle followed by a divergent nozzle (continuous increase in cross-sectional

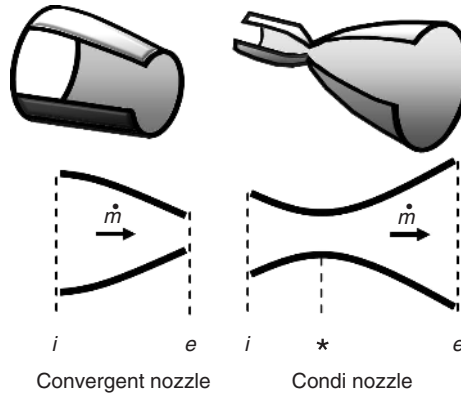


Figure 1.18 Types of nozzles

area in the flow direction). The Laval nozzle was originally invented by a Swedish engineer named Karl Gustaf de Laval for use in steam turbines and is therefore also known as the **de Laval nozzle**.

(The symbol ‘*’ is generally used as a label to indicate the location of the **throat**, which is the point between the convergent and divergent portions of a Laval nozzle. A Laval nozzle is optimally designed so that $M^* = 1.0$ at the throat.)

1.5.4.1 Convergent Nozzles

Isentropic flow through a convergent nozzle can be best understood by examining a nozzle with a constant chamber pressure (P_c) and applying decreasing back pressures (points A \rightarrow D) on it (Figure 1.19). If the nozzle is exhausting gases into the atmosphere, this back pressure is equal to the ambient pressure (P_0). Therefore a continuous decrease in the ambient back pressure is equivalent to climbing in altitude (Table A.1, Appendix A).

Point A illustrates a limiting case where the back pressure equals the chamber pressure ($P_0 = P_c$), so there is no mass flow through the nozzle. As the ambient back pressure is lowered to point B and beyond ($P_0 < P_c$), the static pressure through the nozzle decreases and the mass flow through the nozzle increases. The Mach number at the nozzle exit plane also increases. Under these conditions, the static pressure of the flow exiting the nozzle is equal to the ambient back pressure ($P_e = P_0$). This is called **fully expanded flow**, and is an optimal condition for propulsive convergent nozzles, because it maximizes the momentum thrust component and therefore the net thrust.

This trend continues until point C is reached, where the fluid exiting the nozzle is equal to the velocity of sound ($M_e = 1.0$). As illustrated in Figure 1.17, flow through a convergent nozzle cannot be accelerated from subsonic velocities to supersonic velocities, therefore as the back pressure continues to decrease past point C (to point D and beyond) no additional mass can flow through the nozzle. (For a physical description of this phenomenon, see [4].) This is called **choked flow**. For choked flow, the exit static pressure is not equal to the back pressure ($P_e \neq P_0$). Under these conditions the sonic gases will dissipate through a shock system. If the pressure differences are large enough, these shocks will form outside the nozzle.

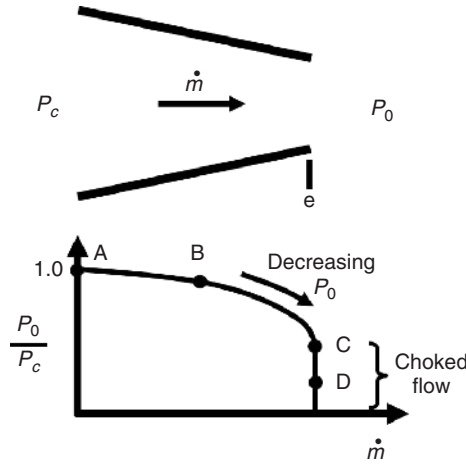


Figure 1.19 Choked flow in a convergent nozzle

Choked (or underexpanded) flow occurs when the pressure ratio (P_e/P_0) is greater than or equal to a critical value (PR_{crit}). This places a maximum limit on the gas mass flow passing through the nozzle. As previously stated, optimum (or maximum) thrust occurs when the exhaust gases fully expand to the ambient pressure ($P_e = P_0$). Full expansion maximizes the momentum thrust, which maximizes the net thrust (F_N). Choked flow results in a loss of momentum thrust, but creates a smaller pressure thrust component since ($P_e > P_0$). This lost momentum thrust may only be recovered by adding a divergent surface (e.g., a condi nozzle).



“Choking!” Choked flow means that the mass flow rate has reached a maximum value.

For isentropic flow, Equation 1.107 can be used to derive a **critical pressure ratio** (PR_{crit}) that is necessary to just choke the nozzle, so it is the maximum pressure ratio (P_t/P) that can be achieved in the nozzle. This will occur when $M_e = 1.0$ (for a convergent nozzle) and is defined in Equation 1.145.

Critical pressure ratio:

$$PR_{crit} = \frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma}{\gamma - 1}} \quad (1.145)$$

The critical pressure ratio is a function of γ only (e.g., for $\gamma = 1.333$ then $PR_{crit} = 1.852$ and if $\gamma = 1.4$ then $PR_{crit} = 1.893$). Equation 1.145 provides a test to see if a convergent nozzle is choked or not. If the ideal pressure ratio (achieved by full expansion of the flow through the nozzle) exceeds the critical pressure ratio than this ideal ratio cannot be achieved because flow through the nozzle is choked.

Nozzle choke test:

$$\text{Choked if: } \frac{P_{te}}{P_0} \geq PR_{crit} \quad (1.146)$$

If the nozzle is choked, then the exhaust pressure ratio (P_{te}/P_e) is equal to the maximum or critical pressure ratio (PR_{crit}). Therefore the static pressure of the exhaust gases is:

$$P_e = P_{te} \left(\frac{P_e}{P_{te}}\right) = \frac{P_{te}}{PR_{crit}} \quad (1.147)$$

If the nozzle is not choked, flow is subsonic throughout the nozzle ($M_e < 1$). Flow through the nozzle can adjust to changes in ambient back pressure (altitude). Ambient pressure changes will propagate upstream from the nozzle exhaust plane at the speed of sound. So for all unchoked flows in a convergent nozzle, the exit pressure will be equal to the ambient back pressure ($P_e = P_0$). The modes of operation of a convergent nozzle are summarized in Table 1.2.

Also from Equation 1.106, the critical temperature ratio can be found for choked flow as:

$$TR_{crit} = \frac{\gamma + 1}{2} \quad (1.148)$$

The critical temperature ratio is a function of γ only (e.g., for $\gamma = 1.333$ then $TR_{crit} = 1.167$ and if $\gamma = 1.4$ then $TR_{crit} = 1.2$). The exit static temperature for a choked nozzle can be determined from the critical temperature ratio:

$$T_e = \frac{T_t}{TR_{crit}} \quad (1.149)$$

Table 1.2 Modes of operation – convergent nozzle

	Underexpanded (choked)	Just choked	Fully expanded (not choked)
P_{te}/P_0	$>PR_{crit}$	PR_{crit}	$<PR_{crit}$
P_{te}/P_e	PR_{crit}	PR_{crit}	$<PR_{crit}$
M_e	1	1	<1
P_e	$>P_0$	P_0	P_0
$A_e(P_e - P_0)$	>0	0	0

1.5.4.2 Convergent-Divergent (Condi) Nozzles

As was done for convergent nozzles in Figure 1.19, isentropic flow through a condi nozzle can be understood by examining a nozzle with a constant chamber pressure (P_c) and applying decreasing ambient back pressures (P_0) (points A → D) on it as shown in Figure 1.20. Again Point A illustrates a limiting case where the ambient back pressure equals the chamber pressure ($P_0 = P_c$), so there is no mass flow through the nozzle. Similar to Figure 1.19, as the ambient back pressure is lowered to point B and beyond ($P_0 < P_c$), the static pressure through the nozzle decreases and the mass flow through the nozzle increases.

In this range of ambient back pressures, the flow is fully expanded so the static pressure of the flow exiting the nozzle is equal to the ambient back pressure ($P_e = P_0$). Flow in both the convergent and divergent portions of the nozzle is subsonic. This trend continues until the ambient back pressure at point C is reached. At this point, the fluid at the throat flows at the velocity of sound ($M^* = 1.0$). Since the flow through the convergent portion of the nozzle cannot be accelerated from subsonic velocities to supersonic velocities (Figure 1.17); the condi nozzle becomes choked at all pressure ratios below point C (point D).

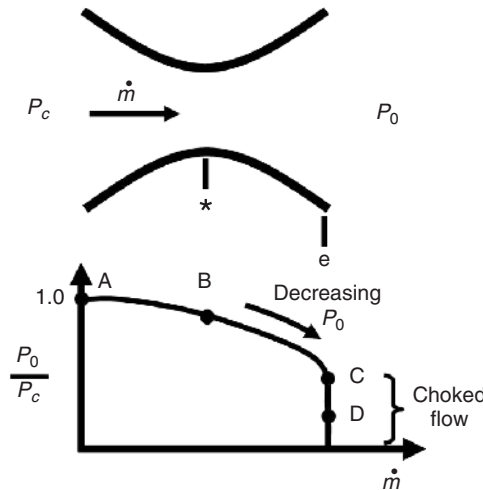


Figure 1.20 Choked flow in a condi nozzle

When $M^* = 1.0$ at the throat, there are two possible isentropic solutions for a given area ratio (A/A^*). The flow can either decelerate to a subsonic exit Mach number ($M_e < 1$) or accelerate to a supersonic exit Mach number ($M_e > 1$). Point D (and lower) represents the ambient back pressure condition where the flow accelerates to a supersonic Mach number in the diverging section of the nozzle. Therefore for ambient back pressures lower than the point D, the pressure will decrease in both the convergent and divergent sections of the condi nozzle resulting in supersonic exhaust flow. This is the objective of condi nozzle designs, because a supersonic exhaust gas velocity greatly increases the thrust of a propulsion system. For back pressures in between points C and D, an isentropic solution is not possible because shock waves are formed and this is an irreversible process. In this case, shock equations would have to be used to determine the flow properties.

As just stated, condi nozzles are designed to be choked at the throat ($M^* = 1.0$) so exhaust gases can be accelerated to a supersonic exit velocity in the diverging section. The same choke test derived for convergent nozzles (Equation 1.146) can also be applied to the throat section of condi nozzles (assuming that the diverging section does become a subsonic diffuser). Optimal thrust occurs when the nozzle is sized so that the exhaust gases are fully (or perfectly) expanded ($P_e = P_0$). Imperfect nozzle expansion is caused by not having an ideal nozzle expansion ratio (ε) for a particular operating altitude. The flow is underexpanded if $P_e > P_0$ and overexpanded if $P_e < P_0$. Underexpansion is caused by a less than optimal nozzle expansion ratio, resulting in a loss in momentum thrust. Overexpansion is caused by having a greater than optimal nozzle expansion ratio, which may result in flow separation, which forms shocks inside the nozzle. Nozzle performance losses due to overexpanded flow are generally much larger than losses due to underexpanded flow [5].

Full expansion of an exhaust jet in a fixed-geometry condi nozzle can only be achieved when it is operating at its design pressure ratio. Consequently, fixed-geometry condi nozzles are typically only used in missiles that spend the majority of their flight at a predictable constant supersonic cruising velocity. Most other aerospace propulsion systems equipped with condi nozzles are designed with variable geometry (VG). This allows the area ratio to be variably optimized over a range of flight conditions, improving the condi nozzle's effectiveness at generating thrust. A variable area nozzle is critically important at aircraft speeds below the design speed, where severe losses can occur in the divergent section (of the condi nozzle) due to overexpansion. Additionally most supersonic aircraft require an afterburner to achieve supersonic velocities. Afterburning engines are designed so that the operating conditions upstream are unchanged, which necessitates the use of a variable geometry nozzle.

Variable geometry nozzles have physical limitations that stop them from being able to fully expand the exhaust gases at all the possible flight conditions required of an aircraft. When perfect expansion is not achieved the exhaust flow will adjust outside the nozzle by forming expansion or compression waves. Figure 1.21 shows how this occurs at different conditions.

In an overexpanded condition, since the flow inside the nozzle is less than the ambient pressure ($P_e < P_0$), oblique shocks (compression waves) form at the nozzle exit to raise the pressure to the ambient value. The flow at the exit plane is assumed to be uniform and parallel, so by symmetry there should be no flow across the centerline. In other words, there is no velocity component normal to the centerline. The pressure of the exhaust

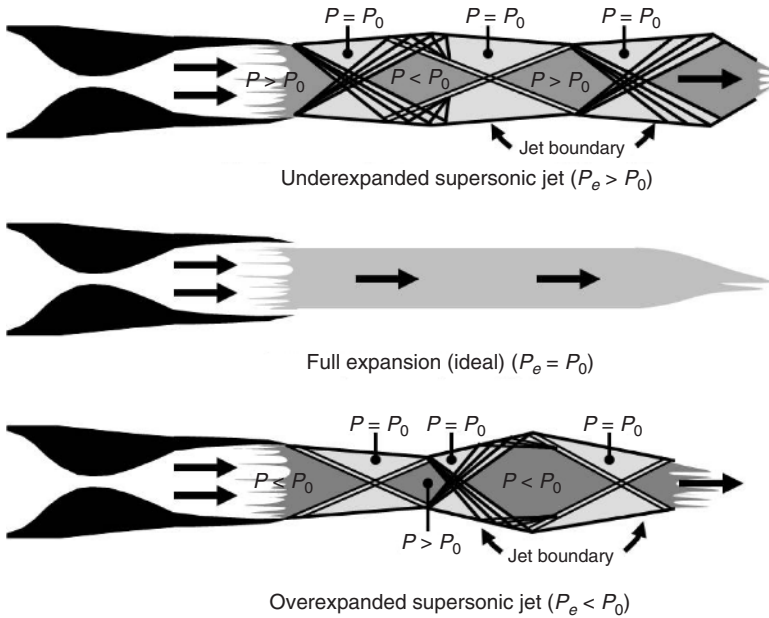


Figure 1.21 Exhaust flow out a conical nozzle (John, J. E. A., *Gas Dynamics*, 2nd Edition, © 1984. Reprinted by permission of Pearson Education, Inc., Upper Saddle River NJ [4].)

gases is raised to the ambient pressure as the flow goes through the first set of shocks. However, the flow in this region is turned away from the centerline. Since there can be no normal velocity component at the centerline, the flow must turn back toward the horizontal. Therefore the intersection of the shock waves at the centerline reflects another set of shock waves. As the flow passes through this second set of shock waves the pressure of the exhaust gases rises above the ambient pressure, which causes a set of expansion waves to reflect from the ambient air. The expansion waves cause the pressure of the flow to once again be equal to the ambient pressure, but the flow is turned away from the centerline. The intersection of the expansion waves at the centerline requires another set of expansion waves to turn the flow back towards the horizontal. These expansion waves then reflect from the ambient air as shock waves. This cycle continues to repeat itself until the exhaust gases completely mix and dissipate into the ambient air at the jet boundaries.

In an underexpanded condition the flow behaves oppositely to the overexpanded case. Since the underexpanded flow inside the nozzle was unable to decrease to the ambient pressure ($P_e > P_0$), expansion fans form at the nozzle exit plane to reduce the pressure to the ambient value. The flow at the exit plane is assumed to be uniform and parallel, so by symmetry there should be no flow across the centerline. In other words, there is no velocity component normal to the centerline. The pressure of the exhaust gases is reduced to the ambient pressure as the flow goes through the first expansion wave. However, the flow in this region is turned away from the centerline. Since there can be no normal velocity component at the centerline, the flow must turn back toward the horizontal there. Therefore the intersection of the expansion waves at the centerline reflects another set of expansion waves. As the flow passes through this second set of expansion waves the



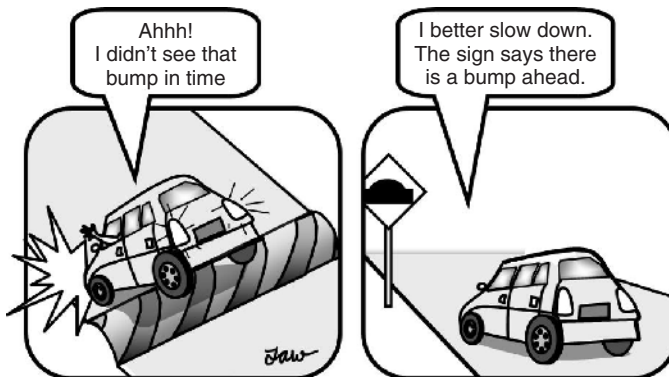
Figure 1.22 SR-71B with shock diamonds in the exhaust (Courtesy of NASA.)

pressure of the exhaust gases is reduced below the ambient pressure, which causes the expansion waves to reflect from the ambient air as a set of oblique shocks (or compression waves). The shocks cause the pressure of the flow to once again be equal to the ambient pressure, but the flow is turned away from the centerline. The intersection of the shocks requires another set of oblique shocks to turn the flow back towards the horizontal. These shocks then reflect from the ambient air as expansion waves. This cycle also continues to repeat itself until the exhaust gases dissipate into the ambient air [4].

In both overexpanded and underexpanded conditions, the flow pattern behind the nozzle appears as a series of diamonds. This is often visible, as shown in Figure 1.22.

1.6 Shock Waves

A body traveling at a subsonic speed ($M < 1$) through a compressible fluid (such as air) creates a disturbance that is propagated throughout the fluid by a wave traveling at the local velocity of sound (relative to the body). This creates gradual changes in the fluid properties (such as density, pressure, and temperature) along smooth, continuous streamlines as it approaches the body. However, if the body is traveling at a supersonic speed ($M > 1$) then the fluid is unable to gradually change ahead of the body. Therefore the supersonic body induces a sudden change in fluid properties due to a **shock wave**.



“Shocking Encounter” The first panel shows a situation similar to supersonic flow over a body. Like the driver, the flow has no time to prepare for it. The second case is like subsonic flow over a body. Like the driver, the flow ahead of the body gradually changes to prepare for it.

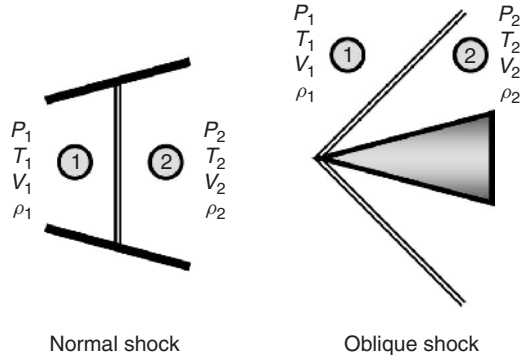


Figure 1.23 Types of shock waves

Consideration of shock waves is important in the design of intakes, nozzles, and ducts of aerospace propulsion systems capable of supersonic velocities. There are two types of shock waves (shown in Figure 1.23). The simplest type of shock, the **normal shock** (or plane shock), occurs normal to the flow direction. Therefore changes to the flow properties across a normal shock occur only in the flow direction. An **oblique shock** occurs at an inclined angle to the flow direction. The net change in fluid properties across a shock wave can be determined by encompassing the shock within a control volume. Since there are no temperature gradients inside the control volume the shock process is adiabatic. Therefore, there is no change in the total temperature (T_t) across the shock (Equation 1.150). However, there is a change in Mach number (velocity), static temperature (T), total and static pressure (P_t and P), and entropy (S) across the shock. So although flow through a shock wave is adiabatic, it is not isentropic because it is irreversible.

$$T_{t1} = T_{t2} \sim \text{Across a shock} \quad (1.150)$$

1.6.1 Normal Shocks

Suppose a normal shock occurs in a one-dimensional, steady flow. The shock is assumed to be thin, so that there is a negligible change in area across the shock. Applying the conservation of mass and linear momentum across the normal shock yields the following equations, respectively:

$$\rho_1 V_1 = \rho_2 V_2 \quad (1.151)$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 \quad (1.152)$$

If the fluid medium is air (or some other gas that can be approximated as a perfect or ideal gas), then Equations 1.13, 1.94, and 1.100 can substituted into Equations 1.151 and 1.152 to transform them into the following equations:

$$\frac{P_1}{RT_1} M_1 \sqrt{\gamma RT_1} = \frac{P_2}{RT_2} M_2 \sqrt{\gamma RT_2} \quad (1.153)$$

$$P_1 (\gamma M_1^2 + 1) = P_2 (\gamma M_2^2 + 1) \quad (1.154)$$

There is no change in total temperature across a shock (Equation 1.150). Therefore Equation 1.106 can be substituted into Equation 1.150 to obtain:

$$T_1 \left(1 + \frac{\gamma - 1}{2}\right) M_1^2 = T_2 \left(1 + \frac{\gamma - 1}{2}\right) M_2^2 \quad (1.155)$$

Rearranging Equation 1.155 gives:

$$\frac{P_1}{P_2} = \frac{M_2}{M_1} \sqrt{\frac{T_1}{T_2}} \quad (1.156)$$

Substituting Equation 1.154 and 1.155 into Equation 1.156 gives:

$$\frac{1 + \gamma M_2^2}{1 + \gamma M_1^2} = \frac{M_2}{M_1} \frac{\sqrt{1 + \frac{\gamma - 1}{2} M_2^2}}{\sqrt{1 + \frac{\gamma - 1}{2} M_1^2}} \quad (1.157)$$

Rearranging this equation:

$$\frac{M_1}{1 + \gamma M_1^2} \sqrt{1 + \frac{\gamma - 1}{2} M_1^2} = \frac{M_2}{1 + \gamma M_2^2} \sqrt{1 + \frac{\gamma - 1}{2} M_2^2} \quad (1.158)$$

Squaring both sides of Equation 1.158 gives:

$$\frac{M_1^2 \left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{(1 + \gamma M_1^2)^2} = \frac{M_2^2 \left(1 + \frac{\gamma - 1}{2} M_2^2\right)}{(1 + \gamma M_2^2)^2} \quad (1.159)$$

Rearranging Equation 1.159 into a quadratic equation gives:

$$M_2^4 \left(\frac{\gamma - 1}{2} - \frac{\gamma^2 M_1^2 \left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{(1 + \gamma M_1^2)^2} \right) + M_2^2 \left(1 - \frac{2\gamma M_1^2 \left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{(1 + \gamma M_1^2)^2} \right) - \frac{M_1^2 \left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{(1 + \gamma M_1^2)^2} = 0 \quad (1.160)$$

Solving this quadratic equation for M_2 gives:

$$M_2 = \sqrt{\frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}} \quad (1.161)$$

Equation 1.161 shows that for supersonic flow ($M_1 > 1$) then $M_2 < 1$. This shows that across a normal shock the flow abruptly transitions from supersonic flow to subsonic flow. [Note: This equation also shows that if flow is subsonic ($M_1 < 1$) and a shock occurs then $M_2 > 1$. However, this second case is impossible because it violates the Second Law of Thermodynamics by requiring an entropy decrease. Therefore for a shock to occur, the approaching flow (M_1) must be supersonic.]

From Equation 1.106, the following equation can be derived:

$$\frac{T_2}{T_1} = \left(\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \right) \quad (1.162)$$

Substituting Equation 1.161 into this equation results in:

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{\gamma - 1}{2} M_1^2\right) \left(\frac{2\gamma}{\gamma - 1} M_1^2 - 1\right)}{M_1^2 \left(\frac{2\gamma}{\gamma - 1} + \frac{\gamma - 1}{2}\right)} \quad (1.163)$$

Values of other property ratios across a normal shock, such as: P_2/P_1 , P_{t2}/P_{t1} , and ρ_2/ρ_1 shown in Equations 1.164, 1.165, and 1.166 are found in a similar manner. For derivations of these equations, see [4].

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1} \quad (1.164)$$

$$\frac{P_{t2}}{P_{t1}} = \left[\frac{\frac{\gamma + 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_1^2} \right]^{\frac{\gamma}{\gamma - 1}} \left[\frac{1}{\frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}} \right]^{\frac{1}{\gamma - 1}} \quad (1.165)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_1^2}{(\gamma - 1) M_1^2 + 2} \quad (1.166)$$

These property ratios are tabulated in Tables C.1 and C.2 (Normal Shock Tables) in Appendix C [6].

Example 1.4

A normal shock forms on the intake of an aircraft flying at Mach 1.6 at 10 km (Figure 1.24). Assume $\gamma = 1.4$. Determine the Mach number (M_2), total pressure (P_{t2}), static pressure (P_2), total temperature (T_{t2}), and static temperature (T_2) of air after the shock.

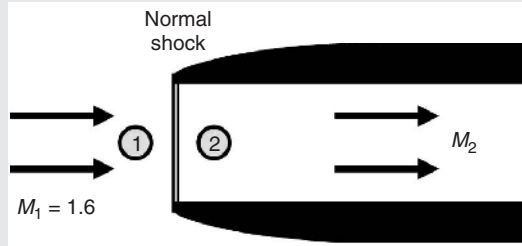


Figure 1.24 Pitot intake

Solution

According to the Standard Atmospheric Table, Appendix A, Table A.1 for 10 km altitude:

$$P_1 = 26.5 \text{ kPa and } T_1 = 223.3 \text{ K}$$

According to Table C.1, Appendix C ($\gamma = 1.4$) for $M_1 = 1.6$:

$$M_2 = 0.6684; \quad \frac{P_{t2}}{P_{t1}} = 0.8952; \quad \frac{P_2}{P_1} = 2.820; \quad \text{and} \quad \frac{T_2}{T_1} = 1.388$$

Assuming isentropic flow outside the intake:

$$\begin{aligned} P_{t1} &= P_1 \left[1 + \frac{M_1^2(\gamma - 1)}{2} \right]^{\frac{\gamma}{\gamma - 1}} \\ &= (26.5 \times 10^3 \text{ Pa}) \left[1 + \frac{1.6^2(1.4 - 1)}{2} \right]^{\frac{1.4}{1.4 - 1}} = 112.6 \text{ kPa} \\ T_{t1} &= T_1 \left[1 + \frac{M_1^2(\gamma - 1)}{2} \right] \\ &= (223.3 \text{ K}) \left[1 + \frac{1.6^2(1.4 - 1)}{2} \right] = 337.6 \text{ K} \end{aligned}$$

Therefore:

$$P_{t2} = P_{t1} \frac{P_{t2}}{P_{t1}} = (112.6 \text{ kPa})(0.8952) = 100.8 \text{ kPa}$$

$$P_2 = P_1 \frac{P_2}{P_1} = (26.5 \text{ kPa})(2.82) = 74.7 \text{ kPa}$$

$$T_2 = T_1 \frac{T_2}{T_1} = (223.3 \text{ K})(1.388) = 309.9 \text{ K}$$

Finally, since there is no change in total temperature across a shock:

$$T_{t2} = T_{t1} = 337.6 \text{ K}$$

1.6.2 Oblique Shocks

The methodology of analyzing flow properties across oblique shocks is very similar to that shown in section 1.6.1 for studying normal shocks. Even though an oblique shock is inclined at an angle to the flow direction it still creates an abrupt change in fluid properties and is adiabatic. Therefore like a normal shock, there is no change in the total temperature (T_t) across an oblique shock. Also similar to a normal shock, the equations of mass, linear momentum, and energy can be used to derive equations relating fluid properties across the shock. The difference is that an additional variable must be introduced to account for the oblique shock's inclination to the flow direction. The two-dimensional, steady flow continuity equation (Equation 1.20) across an oblique shock yields the following relation:

$$\rho_1 A_1 V_{1n} = \rho_2 A_2 V_{2n} \quad (1.167)$$

In Equation 1.167, V_{1n} and V_{2n} are the normal velocity components as shown in Figure 1.25. Since V_{1n} and V_{2n} are the normal components, this is essentially the same relation that was derived for a normal shock (Equation 1.151).

The two-dimensional, steady flow equations for linear momentum (Equation 1.23) can be separately written in component form (since momentum is a vector). This means that momentum can be written for both the tangential and normal directions with respect to the shock wave. Since there is no change in pressure in the tangential direction, the tangential momentum equation is:

$$V_{1t}(\rho_1 A_1 V_{1n}) = V_{2t}(\rho_2 A_2 V_{2n}) \quad (1.168)$$

Equation 1.167 shows that the mass flow in the normal direction does not change across the shock, therefore applying this to Equation 1.168 allows terms to be cancelled and the following relation is obtained:

$$V_{1t} = V_{2t} \quad (1.169)$$

This equation shows that across an oblique shock, there is no change in the tangential velocity. The normal momentum equation is:

$$\rho_1 A_1 V_{1n}^2 + P_1 A_1 = \rho_2 A_2 V_{2n}^2 + P_2 A_2 \quad (1.170)$$

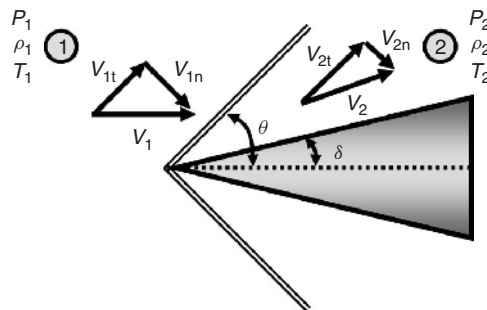


Figure 1.25 Velocity components across an oblique shock

Since the shock is very thin $A_1 = A_2$ and therefore:

$$P_1 - P_2 = \rho_2 V_{2n}^2 - \rho_1 V_{1n}^2 \quad (1.171)$$

The two-dimensional, steady flow energy equation for adiabatic steady flow (Equation 1.28) is:

$$\left(h_1 + \frac{V_1^2}{2} \right) - \left(h_2 + \frac{V_2^2}{2} \right) = \left(h_1 + \frac{V_{1t}^2 + V_{1n}^2}{2} \right) - \left(h_2 + \frac{V_{2t}^2 + V_{2n}^2}{2} \right) = 0 \quad (1.172)$$

Since $V_{1t} = V_{2t}$ (Equation 1.169), this simplifies to:

$$h_1 - h_2 = \frac{V_{2n}^2 - V_{1n}^2}{2} \quad (1.173)$$

Since Equations 1.172 and 1.173 do not contain tangential velocity components, these equations are essentially the same as the equations derived for a normal shock. This means that the components normal to an oblique shock act just like a normal shock, while the components tangential to an oblique shock do not change. Therefore, the fluid property ratios across an oblique shock can be determined by calculating the components normal to the oblique shock and using the normal shock tables found in Tables C.1 or C.2 (Appendix C).

However, oblique shock tables (Figures C.1 and C.2, Appendix C) have also been made for the usual case, when the wave angle (δ) is unknown [6]. In order to use these tables, it is more convenient to write the components of Mach number as:

$$M_{1n} = M_1 \sin \theta \quad (1.174)$$

$$M_{1t} = M_1 \cos \theta \quad (1.175)$$

$$M_{2n} = M_2 \sin(\theta - \delta) \quad (1.176)$$

$$M_{2t} = M_2 \cos(\theta - \delta) \quad (1.177)$$

Figures C.1 and C.2 (Appendix C) both show that there are two possible solutions or none at all. The three oblique shock types associated with these conditions are known as: strong shocks, weak shocks, or detached shocks.

A **strong shock** has a large value of θ and a large pressure ratio across the shock. It generally occurs when the downstream pressure (or back pressure) of a supersonic flow is extremely high. A strong shock can be expected to occur on the spike of supersonic inlet if no flow is allowed to pass through the inlet. (However, this system is unstable and will normally degenerate into the weaker solution.) A strong shock will always slow the supersonic flow velocity to a subsonic speed. The limiting case of a strong shock is a normal shock.

A **weak shock** is one that has a relatively small value of θ , a smaller pressure ratio across the shock, and a small back pressure. A weak solution occurs more frequently

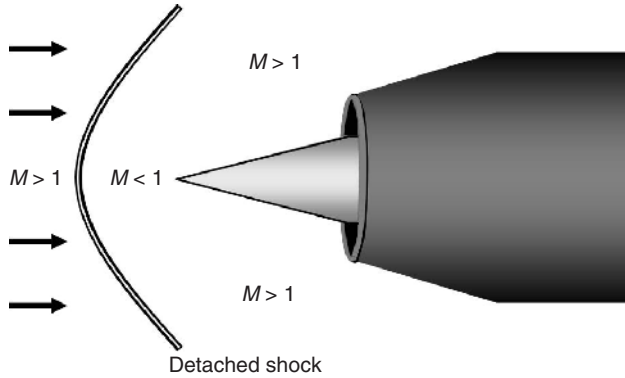


Figure 1.26 Detached oblique shock

on aerospace system designs than a strong shock. Normally, weak shocks will occur on wings, open inlets, and planar surfaces. A weak shock will always slow the flow velocity to a lower but still supersonic speed. The limiting case of a weak shock is isentropic flow ($\delta = 0$).

A third possibility is that there is no solution at all. This can occur if there is a great enough wedge angle (δ). In this case the shock detaches from the body and may occur in front of it. An example of a **detached bow shock** is shown in Figure 1.26.

Example 1.5

Compare the loss in total pressure ratio incurred by a two-dimensional, two-shock spike diffuser and a three-shock diffuser operating at Mach 2.0, as shown in Figure 1.27. Assume that each oblique shock turns the flow through an angle (δ) of 10° .

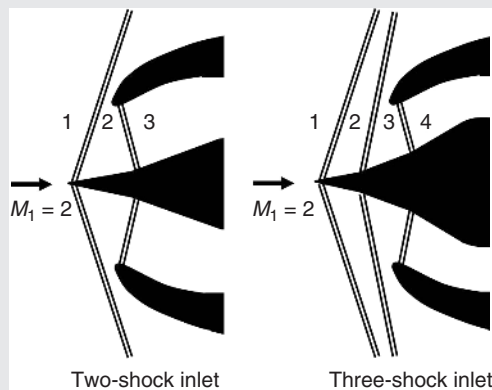


Figure 1.27

Solution**(a) Two-shock inlet calculations:**

From oblique flow charts (Figures C.1a and b, Appendix C) for $M_1 = 2.0$, $\gamma = 1.4$, and $\delta = 10^\circ$, the weak shock solution is:

$$\theta = 39.4^\circ$$

$$M_2 = 1.64$$

Therefore:

$$\begin{aligned} M_{1n} &= M_1 \sin \theta \\ &= (2.0) \sin(39.4^\circ) = 1.27 \end{aligned}$$

The normal shock tables (Table C.1, Appendix C) can now be used for $M_{1n} = 1.27$:

$$\left(\frac{P_{t2}}{P_{t1}} \right) = 0.9842$$

For the normal shock $M_2 = 1.64$, then again from the normal shock tables:

$$\left(\frac{P_{t3}}{P_{t2}} \right) = 0.8799$$

So the total pressure recovery across the one-shock inlet is:

$$\left(\frac{P_{t3}}{P_{t1}} \right)_{2 \text{ shock inlet}} = \frac{P_{t3}}{P_{t2}} \frac{P_{t2}}{P_{t1}} = (0.8779)(0.9842) = 0.864$$

(b) Three-shock inlet calculations:

This is done similarly to the one-shock inlet. From the oblique shock tables (Figures C.1a and b, Appendix C) again for $M_1 = 2.0$, $\gamma = 1.4$, and $\delta = 10^\circ$:

$$\theta = 39.4^\circ$$

$$M_2 = 1.64$$

Therefore, once again:

$$M_{1n} = M_1 \sin \theta = 1.27$$

Again using the normal shock tables for $M_{1n} = 1.27$:

$$\left(\frac{P_{t2}}{P_{t1}} \right) = 0.9842$$

For the second oblique shock for $M_2 = 1.64$, $\gamma = 1.4$, and $\delta = 10^\circ$ (Figures C.1a and b, Appendix C), $\theta = 49.4^\circ$ and $M_3 = 1.28$.

$$M_{2n} = 1.64 \sin(49.5^\circ) = 1.25$$

Again using the normal shock tables for $M_{2n} = 1.25$:

$$\left(\frac{P_{t3}}{P_{t2}}\right) = 0.9871$$

For the normal shock, using the normal shock tables for $M_3 = 1.28$:

$$\left(\frac{P_{t4}}{P_{t3}}\right) = 0.9827$$

Therefore:

$$\begin{aligned} \left(\frac{P_{t4}}{P_{t1}}\right)_{3 \text{ shock inlet}} &= \frac{P_{t4}}{P_{t3}} \frac{P_{t3}}{P_{t2}} \frac{P_{t2}}{P_{t1}} \\ &= (0.9827)(0.9871)(0.9842) = 0.9547 \end{aligned}$$

Thus there is about a *10% improvement* in total pressure ratio gained by using the three-shock inlet over a two-shock inlet at $M_1 = 2.0$. If we were to repeat this calculation for $M_1 = 4.0$, there would be a *62% improvement*. Thus the improvement increases with higher speeds.

Example 1.5 illustrates a supersonic intake design feature for gas turbine engines and ramjets. At supersonic speeds, a shock wave or series of shock waves will form at the intake slowing the airflow to a subsonic speed inside the engine. As already shown, normal shocks decrease the airflow from supersonic to subsonic speeds, while weak oblique shocks only decrease the velocity to lower supersonic speeds. Therefore for these inlet designs, a normal shock will generally occur after an oblique shock (or a series of oblique shocks), so that flow entering the engine will be subsonic. However, there is a loss of pressure across each shock. To increase the performance of the engine this pressure loss must be minimized. A much larger pressure loss occurs across normal shocks than across oblique shocks. As the supersonic speed of the flow increases, the pressure loss across a normal shock increases exponentially, creating a stronger normal shock. This pressure loss can be reduced by slowing the flow velocity ahead of the normal shock with a weak oblique shock (or a series of weak oblique shocks). Weak oblique shocks can be induced ahead of the normal shock by appropriately designing the inlet geometry.

1.6.3 Conical Shocks

Supersonic flow about a three-dimensional circular cone is more complex than a simple two-dimensional wedge, because after a conical shock the streamlines curve to satisfy the conservation of mass. Therefore a conical shock will be inclined at a lesser angle to the flow direction than a simple two-dimensional oblique shock. This means that a two-dimensional wedge will create a greater flow disturbance than a three-dimensional cone. This is because flow cannot pass around the side of a two-dimensional wedge, since it extends to infinity in the third dimension. Therefore separate flow relations are necessary to analyze a conical shock (see Example 5.1). These relations are illustrated in Figures C.3, C.4, and C.5 in Appendix C.

1.7 Summary

This chapter introduced some fundamental definitions and terminology that are used in the analysis of aerospace propulsion systems. The equations of mass, linear momentum, and energy were presented and applied to basic engine components. Equations for isentropic flow were derived and applied to idealized engine model components. The effect of Mach number on isentropic flow was also derived. Two different types of nozzles were introduced: convergent and convergent–divergent (con-di) nozzles. A limiting factor on the utility of nozzles is choked flow. Choked flow means that no additional mass can flow through the nozzle (maximum mass flow). If a convergent nozzle is choked, the exit Mach number of the exhaust gases is equal to 1.0 (sonic). If a con-di nozzle is choked, the Mach number of gases at the throat is equal to 1.0. Lastly the formation of shock waves in compressible, supersonic flow was introduced. A normal shock represents an abrupt change in fluid properties in the direction of the flow. Although the shock is adiabatic, internal viscous dissipation and heat transfer effects make this an irreversible process. Therefore according to the Second Law of Thermodynamics, entropy will rise across a shock. This means that the flow ahead of a shock must be supersonic. Solutions to flow properties across a normal shock were derived. Shocks that are inclined at an angle to the flow direction are called oblique shocks. It was demonstrated that an oblique shock can be treated as a normal shock in respect to the velocity component perpendicular to the shock wave. Using this approach, the properties of an oblique shock can be analyzed using the equations derived for a normal shock.

References

- [1] *US Standard Atmosphere* 1976; NOAA, NASA, USAF, Washington DC, October 1976.
- [2] Cengel, Y. A. and Boles, M. A., *Thermodynamics, An Engineering Approach*, 5th edition, McGraw-Hill, New York, NY, USA, 2006.
- [3] Farokhi, S., *Aircraft Propulsion*, John Wiley and Sons Inc., Hoboken, NJ, USA, 2008.
- [4] John, J. E. A., *Gas Dynamics*, 2nd edition, Allyn and Bacon Inc., Newton, MA, USA, 1984.
- [5] Gamble, E., Terrell, D., and DeFrancesco, R., *Nozzle Selection and Design Criteria*, 40th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, AIAA-2004-3923, Fort Lauderdale, FLA, USA, 2004.
- [6] NASA Ames Research Staff, *Equation, Tables and Charts for Compressible Flow*, NACA Report 1135, 1953.

Problems

- 1.1 Define the term propulsion and relate it to Newton's Third Law of Motion.
- 1.2 Give a brief definition of a system. Explain the differences between open, closed, and isolated systems.
- 1.3 Briefly describe a working fluid. List some properties used to describe a working fluid.
- 1.4 Describe how the pressure, temperature, density, and speed of sound of atmospheric air vary with altitude.
- 1.5 Describe the difference between a cyclic process and a non-cyclic process.

- 1.6 Explain the difference between a reversible process and an irreversible process.
- 1.7 Briefly describe what characterizes an isentropic process.
- 1.8 Define work. Explain how it is different from power.
- 1.9 Define internal energy. Describe sensible, latent, and thermal energy.
- 1.10 Briefly describe what is meant by the term heat. Describe the three mechanisms by which heat can be transferred.
- 1.11 Define the thermal efficiency of a heat engine.
- 1.12 Define a perfect gas. Describe how different properties of a perfect gas, such as density, volume, mass, temperature, and pressure, are related to one another.
- 1.13 Describe the continuity equation. In steady flow systems, describe what the continuity equation shows about the variation of mass flow through the system.
- 1.14 Describe the conservation of linear momentum. If steady flow conditions exist, describe how the momentum changes with time in a constant control volume.
- 1.15 Describe the process of a convergent and condensing nozzle becoming choked.
- 1.16 Explain why shock waves form when a supersonic, compressible fluid flows over a body such as a wing.
- 1.17 Describe the difference between normal shocks and oblique shocks. State which shock induces the greater loss of pressure.
- 1.18 Briefly describe what happens to the total temperature of a fluid across a shock.
- 1.19 A cylinder contains 1 kg of fluid at a pressure of 100 kPa (point 1 on Figure P1.19). The fluid drives a piston by undergoing a reversible expansion defined by the equation $PV^2 = 4$, until its initial volume (V_1) of 0.2 m^3 doubles to a volume of (V_2) 0.4 m^3 . The fluid is then cooled reversibly at a constant pressure of 25 kPa until the piston reaches its initial position (point 3). Heat is then added reversibly with the piston fixed until the pressure once again reaches 100 kPa (point 1). Calculate the net work done by the fluid.

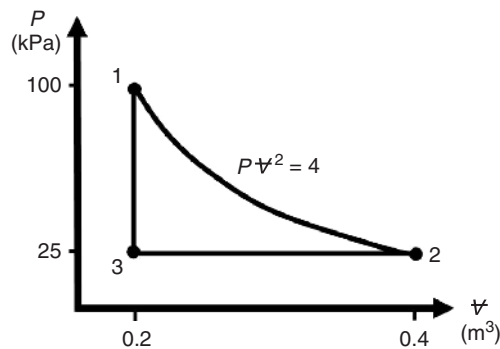


Figure P1.19

- 1.20 A clown uses a 0.25 m^3 tank containing helium (He) to fill balloons. She normally fills the tank so that the pressure of the helium is 200 kPa at 25°C . However, in

order to meet the expected demands of excited children at a large party who all want balloons, she needs to pump an additional 1 kg of helium into the tank. She is a little worried because she knows that the tank will rupture at a pressure of 500 kPa. Helium can be assumed to be a perfect gas and has a molecular weight (M_w) of 4.003 kg/kmole. Calculate the new pressure after allowing the gas to return to an ambient temperature of 25°C. Will the tank rupture and ruin the party?

- 1.21** Oxygen (O_2) at 10 MPa is stored in a pressurized, spherical tank at a temperature of 25°C. The tank has a radius of 25 cm. The maximum allowable pressure of the tank is rated at being 12 MPa. Assuming that oxygen is a perfect gas, calculate the mass stored in the tank. To what temperature can the oxygen be allowed to rise before the pressure limit on the tank is reached?
- 1.22** Calculate the specific internal energy (u) of 1 kg of air that occupies a volume of 1 m³ at 87 kPa. If the temperature is then increased to 500 K as the air is compressed to 200 kPa, calculate the change in internal energy and the new volume occupied by the air. Assume $C_v = 0.718$ kJ/(kg·K), $C_p = 1.008$ kJ/(kg·K), and $R = 287$ J/(kg·K).
- 1.23** Jet fuel steadily flows through a 2 cm diameter pipe at 2 m/s. Assuming that the jet fuel is incompressible, determine the velocity (V_2) of the fuel after the pipe enlarges to a 3 cm diameter (as shown in Figure P1.23).

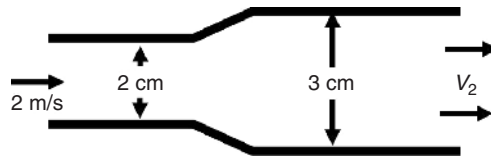


Figure P1.23

- 1.24** A rocket is fired on a static test stand which holds it in place. Exhaust gases are expelled out of the rocket's nozzle at a velocity (V_{jet}) of 1500 m/s (as shown in Figure P1.24). The exhaust nozzle has a 50 cm diameter circular cross-section. The exhaust gas has a density of 0.5 kg/m³. Determine the approximate force (or thrust) generated by the rocket, assuming the exhaust pressure equals the ambient pressure (full expansion condition).



Figure P1.24

- 1.25** Air flows through a compressor at a steady mass flow rate of 0.5 kg/s. The air enters the compressor at a velocity of 15 m/s, pressure of 101.3 kPa, and specific volume of 0.8 m³/kg. The air exits at a velocity of 10 m/s, pressure of 800 kPa, and specific volume of 0.15 m³/kg. The internal energy of the air is increased by

80 kJ/kg. Calculate the power required to drive the compressor, assuming that there is no heat loss due to a cooling system.

- 1.26** Air flows through a turbine at a steady mass flow rate of 15 kg/s. The air enters the turbine at a velocity of 50 m/s and specific enthalpy (h) of 1200 kJ/kg and exits the turbine at a velocity of 140 m/s and specific enthalpy of 300 kJ/kg. Assuming that no heat is rejected from the turbine, calculate the total power generated by the turbine.
- 1.27** An airplane flies at a velocity of 300 m/s at an altitude of 5 km. Assuming $C_p = 1.007$ kJ/(kg·K) for air, determine the total temperature of the air relative to the airplane.
- 1.28** A large commercial jet aircraft is flying at Mach 0.8 at an altitude of 10 km. Air entering the intake of the aircraft's engine is slowed down to a velocity of 100 m/s. Assume that $\gamma = 1.4$ and $C_p = 1.007$ kJ/(kg·K) for air. If this is an isentropic process, determine the static temperature and pressure of the air in the intake.
- 1.29** Exhaust gases exiting the nozzle of a turbojet engine (shown in Figure P1.29) have: a total temperature of 1000 K, total pressure of 350 kPa, and Mach number of 1.0. Assume isentropic flow and $\gamma = 1.33$ and $R = 287$ J/(kg·K). Calculate the static pressure, static temperature, and exhaust jet velocity of the gases.

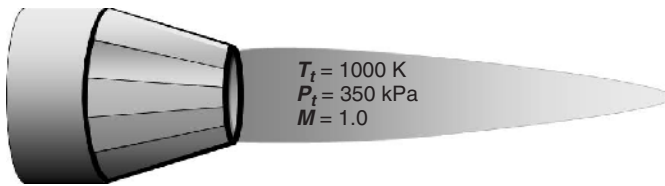


Figure P1.29

- 1.30** A compressor has a total pressure ratio of 12:1. Air with a steady mass flow rate of 50 kg/s enters the compressor at 500 K. If $\gamma = 1.4$, $C_p = 1.007$ kJ/(kg·K), and the air flows isentropically, calculate the total power required to drive this compressor.
- 1.31** Air with a static temperature of 223 K entering a gas turbine engine intake at $V_1 = 300$ m/s accelerates to a new velocity (V_2) and decreases in pressure (P_2) at the exit plane of the intake. The pressure recovery (P_2/P_1) through the intake is 0.833. Assume that the flow through the intake is isentropic.
- Calculate the static temperature (T_2) of air exiting the intake.
 - Find the difference in total temperature (ΔT_t) across the intake.
 - Determine the velocity (V_2) of the air exiting the intake.
- 1.32** A turbojet engine operates at the conditions shown in the T - S diagram (Figure P1.32). Air entering the compressor is at an ambient static temperature (T_1) of 200 K. Assume isentropic diffusion through the inlet and that $\gamma = 1.4$ for the air before it enters the combustion chamber.
- Determine the Mach number (M_1) of the air in the inlet.
 - Determine the total pressure ratio (P_{t2}/P_{t1}) across the compressor.

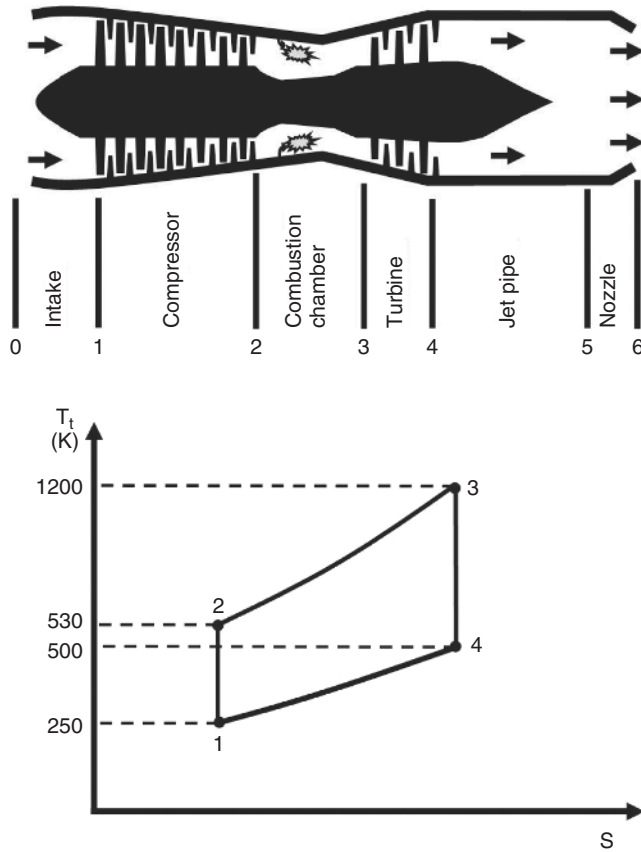


Figure P1.32

- 1.33** Exhaust gases entering a convergent nozzle have a total pressure (P_t) of 200 kPa and total temperature (T_t) of 800 K. The gases exit the nozzle into ambient air at a static pressure (P_0) of 101.3 kPa.
- Assuming that $\gamma = 1.33$ and $R = 287 \text{ J}/(\text{kg}\cdot\text{K})$, determine the critical pressure ratio (PR_{crit}) and evaluate whether the nozzle is choked or not.
 - Calculate the exit static pressure (P_e) and exit velocity (V_e).
- 1.34** A supersonic aircraft flying in air ($\gamma = 1.4$) at Mach 1.8 has an intake type which induces a single normal shock. Calculate the percentage pressure loss and Mach number of the flow entering the intake diffuser after the shock.
- 1.35** Supersonic air ($\gamma = 1.4$) at Mach 2.8 flows over a wedge that is inclined at an angle of 30° . If the ambient pressure is 101.3 kPa and temperature is 25°C . Calculate the Mach number, static pressure, and static temperature after the oblique shock.
- 1.36** A ramjet intake is designed with two ramps (Figure P1.36) so that two oblique shocks and one normal shock occur when it travels at its cruising velocity of Mach 2.8. Calculate the total pressure recovery (P_{t4}/P_{t1}) and the Mach number (M_4) after this shock system. [Assume $\gamma = 1.4$ for air.]

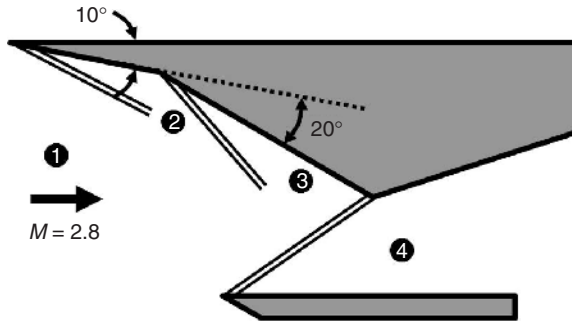


Figure P1.36

- 1.37** An attached conical shock wave forms on the nose cone of a rocket traveling at Mach 8.0 at an altitude of 30 km. The cone's semi-vertex angle (θ) is 15° . Calculate the Mach number (M_c) and static pressure (P_c) of the airflow on the surface of the cone after the conical shock.