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Overview of Chaos

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This chapter gives an overview of chaos, including the definition of chaos, the development of chaology, and the research of chaos. In particular, the research and development of chaos in the field of electrical engineering – with an emphasis on electric drive systems – are discussed in detail.

1.1 What is Chaos?

The etymology of the word “chaos” is a Greek word “$\chi\alpha'\varsigma$” (Nagashima and Baba, 1999) which means “the nether abyss, or infinite darkness,” and was personified as “the most ancient of the gods.” Namely, the god Chaos was the foundation of all creation. From this god arose Gaea (god of the earth), Tartarus (god of the underworld) and Eros (the god of love). Eros drew Chaos and Gaea together so that they could produce descendants, the first born of whom was Uranus (the god of the sky). This also resulted in the creation of the elder gods known as Titans. The interaction of these gods resulted in the creation of other gods, including such well-known figures as Aphrodite, Hades, Poseidon, and Zeus.

There is a Chinese myth of chaos (Liu, 1998), taken from one of the ancient Chinese classics *Chuang-Tzu*: “The god of the Southern Sea was called *Shu* (Change), the god of the Northern Sea was called *Hu* (Suddenness), and the god of the Central was called *Hun-tun* (Chaos). *Shu* and *Hu* often came together for a meeting in the Central, and *Hun-tun* treated them generously. *Shu* and *Hu* determined to repay his kindness, and said, ‘Mankind has seven holes for seeing, hearing, eating and breathing; but *Hun-tun* has none of them; let us bore the holes for him!’ So, every day they bored one hole in his head. On the seventh day, *Hun-tun* died.” This myth not only indicates the disorder-like or random-like behavior of chaos, but also implies that chaos is the natural state of the world and should not be disrupted by a sudden change.

There are many myths relating to the god of chaos in different cradles of civilization, such as Greece, China, Egypt, and India, but in the modern world chaos is no longer a god. In 1997, its meaning in the *Oxford English Dictionary Online* was updated as “Behavior of a system which is governed by deterministic laws but is so unpredictable as to appear random, owing to its extreme sensitivity to changes in parameters or its dependence on a large number of independent variables; a state characterized by such behavior” (Simpson, 2004).

The general perception on chaos is equivalent to disorder or even random. It should be noted that chaos is not exactly disordered, and its random-like behavior is governed by a rule – mathematically,

Chaos in Electric Drive Systems: Analysis, Control and Application, First Edition. K.T. Chau and Zheng Wang.
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a deterministic model or equation that contains no element of chance. Actually, the disorder-like or random-like behavior of chaos is due to its high sensitivity on initial conditions.

Similar to many terms in science, there is no standard definition of chaos. Nevertheless chaos has some typical features:

- **Nonlinearity:** Chaos cannot occur in a linear system. Nonlinearity is a necessary, but not sufficient condition for the occurrence of chaos. Essentially, all realistic systems exhibit certain degree of nonlinearity.
- **Determinism:** Chaos must follow one or more deterministic equations that do not contain any random factors. The system states of past, present and future are controlled by deterministic, rather than probabilistic, underlying rules. Practically, the boundary between deterministic and probabilistic systems may not be so clear since a seemingly random process might involve deterministic underlying rules yet to be found.
- **Sensitive dependence on initial conditions:** A small change in the initial state of the system can lead to extremely different behavior in its final state. Thus, the long-term prediction of system behavior is impossible, even though it is governed by deterministic underlying rules.
- **Aperiodicity:** Chaotic orbits are aperiodic, but not all aperiodic orbits are chaotic. Almost-periodic and quasi-periodic orbits are aperiodic, but not chaotic.

1.2 Development of Chaology

Chaology means the study of chaos theory and chaotic systems. The development of chaology started in mathematics and physics, and expanded into chemistry, biology, engineering, and social sciences. In particular, there have been growing interests in practical applications based on various aspects of chaotic systems (Ditto and Munakata, 1995).

Jules Henri Poincaré first introduced the idea of chaos in 1890 when he participated in a contest sponsored by King Oscar II of Sweden. Although the King Oscar prize was granted to the first person who could solve the *n*-body problem to prove the stability of the solar system, Poincaré won the prize by being closest to a solution of the problem and discovered that the orbit of a three-body celestial system can exhibit unpredictable behavior. To show how visionary Poincaré was, the description of chaos – sensitive dependence on initial conditions – in his 1903 book *Science and Method* (Poincaré, 1996) is quoted below:

> A very small cause which escapes our notice determines a considerable effect that we cannot fail to see, and then we say that that effect is due to chance. If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. But, even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation **approximately**. If that enabled us to predict the succeeding situation **with the same approximation**, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.

Until Edward Norton Lorenz rediscovered a chaotic deterministic system in 1963, Poincaré's finding did not receive the attention it deserved. Lorenz studied air convection in the atmosphere, for which he built a simple mathematical model. He discovered that the weather did not always change in accordance with prediction, and observed that small changes in the initial conditions of variables in his primitive computer weather model could result in very different weather patterns. This model was described by a simple system of equations (Lorenz, 1963):

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = -\sigma x + \sigma y \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -xz + rx - y \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = xy - bz \end{cases} \tag{1.1}$$

where σ is the Prandtl number and r is the Rayleigh number. The system exhibits the well-known Lorenz attractor as shown in Figure 1.1, when $\sigma = 10$, $b = 8/3$, and $r = 28$.

This sensitive dependence on initial conditions is the essence of chaos, and is known as the "butterfly effect", which is often ascribed to Lorenz. In a 1963 paper for the New York Academy of Sciences, Lorenz said:

> One meteorologist remarked that if the theory were correct, one flap of a seagull's wings would be enough to alter the course of the weather forever.

By the time of his 1972 speech at the meeting of the American Association for the Advancement of Science in Washington, DC, Lorenz used a more poetic butterfly statement (Hilborn, 1994):

> Predictability: Does the Flap of a Butterfly's Wings in Brazil set off a Tornado in Texas?

Yoshisuke Ueda first encountered chaos in analog simulations of a nonlinear oscillator at the end of 1961 when he was a doctoral student. With his subsequent studies of the Duffing equation, he published pioneering articles in 1970 and 1973 (Ueda, 1992), illustrating the tangled invariant manifold structure in both its transient and steady-state manifestations. Ueda utilized the following Duffing equation to describe a series-resonant circuit containing a saturable inductor under the supply of DC and sinusoidal voltage:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = y \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -ky - x^3 + B\cos t + B_0 \end{cases} \tag{1.2}$$

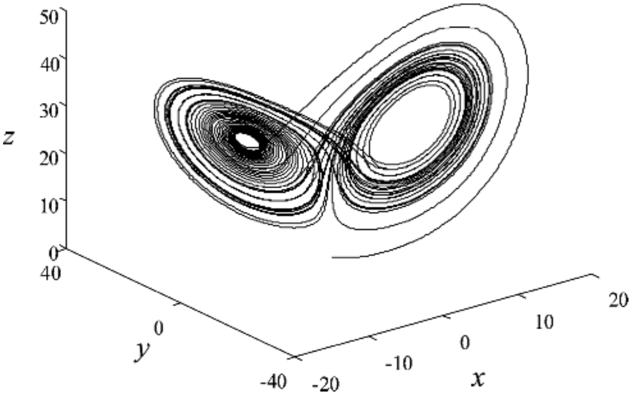

Figure 1.1 The Lorenz attractor

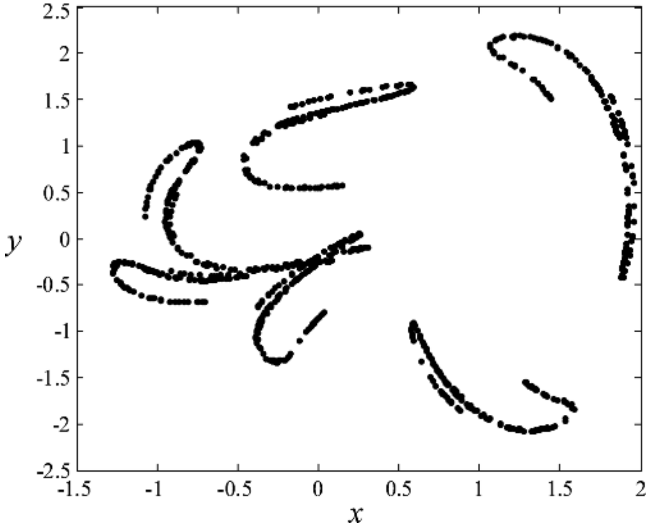

Figure 1.2 The Ueda attractor

where $k = 0.2$, $B = 1.2$, and $B_0 = 0.85$. Figure 1.2 shows the corresponding strange attractor. The fundamental nature of steady-state chaos was described in terms of unstable periodic motions (Abraham and Ueda, 2000).

Robert M. May introduced the idea of chaos to population biology in 1974, which was actually the first, modern, technical use of the term “chaos.” He published a paper “Biological populations with non-overlapping generations: stable point, stable cycles and chaos” in *Science* (May, 1974) in which he outlined that the “evolution of populations (in this case) can be apparently random, even when the underlying equations have nothing random about them.” In his population study, the outside influences (such as climate, food and health) could all factor into the randomness of the population changes. By applying the logistic equation to model the relationship population, May discovered that the population growth exhibited chaotic behavior – the “sensitive dependence on initial conditions.” Namely, if the initial population level was changed, all other successive populations would change; and even where two initial population levels were extremely close, their later population levels could become severely different (May, 1976).

In 1975, Tien-Yien Li and James A. Yorke first coined the mathematical definition of chaos in a ground-breaking paper “Period three implies chaos” in the *American Mathematical Monthly* (Li and Yorke, 1975). This Li–Yorke Theorem states that any continuous one-dimensional system which exhibits a period-three orbit must also display orbits of every other length, as well as completely chaotic orbits. Although A.N. Sharkovskii had published the equivalent of the first part of the Li–Yorke Theorem a decade earlier, the latter part of the Li–Yorke Theorem thoroughly unveiled the nature and characteristics of chaos: the sensitive dependence on initial conditions and the resulting unpredictable nature.

In 1975, Mitchell Jay Feigenbaum discovered that the ratio of the difference between the values at which successive period-doubling bifurcations occur tends to be a constant of around 4.6692, based on his primitive HP-65 computer. He then mathematically proved that the same constant would occur in a wide class of nonlinear mappings when approaching a chaotic level. The logistic map is a well-known example of the mappings that Feigenbaum studied in his famous 1978 paper “Quantitative universality for a class of nonlinear transformations” (Feigenbaum, 1978). Named after Feigenbaum, there are two constants which are depicted in the bifurcation diagram of the logistic map, as shown in Figure 1.3.

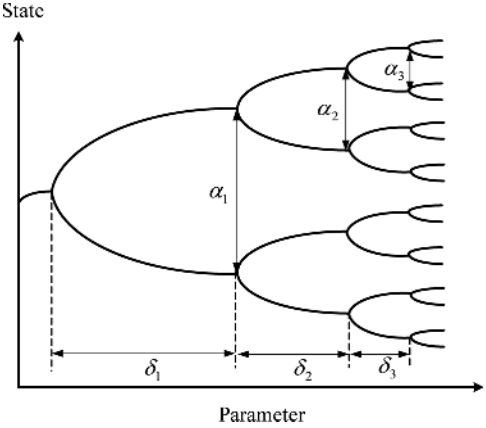

Figure 1.3 The definition of Feigenbaum constants

The first Feigenbaum constant is given by:

$$\delta = \lim_{k \to \infty} \frac{\delta_k}{\delta_{k+1}} = 4.66920160910299067185320382\cdots \tag{1.3}$$

which is the ratio between successive bifurcation intervals.

The second Feigenbaum constant is given by:

$$\alpha = \lim_{k \to \infty} \frac{\alpha_k}{\alpha_{k+1}} = 2.502907875095892822283902873218\cdots \tag{1.4}$$

which is the ratio between successive tine widths.

Benoît B. Mandelbrot almost single-handedly created the fractal geometry that is critical to an understanding of the laws of chaos. He also showed that fractals have fractional dimensions, rather than whole number dimensions. With the aid of computer graphics, he plotted the ground-breaking image of a simple mathematical formula: $z_{n+1} = z_n^2 + c$, which is now called the Mandelbrot set. His work was first fully elaborated in his book *The Fractal Geometry of Nature* in 1982. The Mandelbrot set is an iterative calculation of complex numbers with zero as the starting point. The order behind the chaotic generation of numbers can only be seen by a graphical portrayal of these numbers; otherwise, the generated numbers seem to be random. As shown in Figure 1.4, in which the stable points are represented by black points, the order is strange and beautiful, exhibiting self-similar recursiveness over an infinite scale (Mandelbrot, 1982).

Edward Ott, Celso Grebogi, and James A. Yorke kicked off the control of chaos (Ott, Grebogi, and Yorke, 1990). They developed a method known (after their initials) as the OGY technique, which functions to apply tiny disturbances to chaotic attractors so that the selected unstable orbits can be stabilized – namely, from chaos to order. This OGY technique has been widely accepted since it requires no analytical model and all necessary dynamics are estimated from past observations of the system.

In 1997, Guanrong Chen coined the term "anticontrol" of chaos, and discussed its potential in nontraditional applications (Chen, 1997). Instead of stabilizing a chaotic system, the anticontrol of chaos (namely, from order to chaos) has received great interest. The idea of the anticontrol of chaos was first proposed experimentally to prevent the periodic behavior of the neuronal population bursting of an *in vitro* rat brain (Schiff *et al.*, 1994). It is interesting to note that the new words "chaotify" and "chaotification" were created to represent "to generate chaos" and "the generation of chaos," respectively.

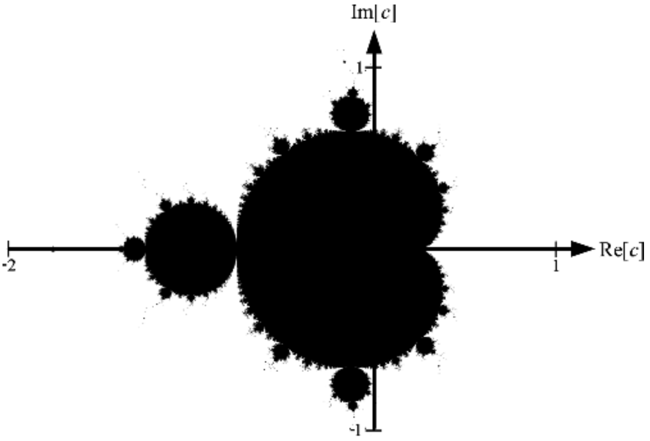

Figure 1.4 An image of the Mandelbrot set

Table 1.1 Milestones in chaology

1890	In a contest sponsored by the King Oscar II of Sweden, Poincaré was awarded the prize for his discovery that the orbit of a three-body celestial system is deterministic but can exhibit unpredictable behavior – the first realization of chaos.
1963	Lorenz developed a simple system of equations to model weather, hence displaying the first chaotic attractor, which he then poetically termed the well-known butterfly effect.
1970	Ueda published a pioneering paper to illustrate the tangled invariant manifold structure of the Duffing equation.
1974	May identified chaotic behavior in population biology, and first introduced the term "chaos" as is technically used today.
1975	Li and Yorke published a ground-breaking paper "Period three implies chaos," and coined the mathematical definition of chaos.
1978	Feigenbaum unveiled the Feigenbaum constant of a wide class of nonlinear mappings prior to the onset of chaos.
1982	Mandelbrot coined the word "fractal" and laid the foundation of fractal geometry to elaborate chaos.
1990	Ott, Grebogi, and Yorke opened the research theme on the control of chaos.
1997	Chen coined the term "anticontrol" of chaos, and began his research in this direction.

Actually, there are formal words "chaoize" and "chaoization" that carry these meanings, although they were rarely used.

The aforementioned milestones in chaology are summarized in Table 1.1 in which they are focused on theoretical developments and systems, rather than practical developments and systems. As the practicability of chaology covers many disciplines and fields, our discussion will focus on the field of electrical engineering.

1.3 Chaos in Electrical Engineering

In electrical engineering, research on chaos has covered a very wide range of areas, including, but not limited to, electronic circuits, telecommunications, power electronics, power systems, and electric drive systems. Overviews will be given to these selected areas, with emphasis on the chaos in electric drive systems.

1.3.1 Chaos in Electronic Circuits

The electronic circuit was the natural extension from theoretical chaos to practical electrical engineering. Basically, the investigation of chaos in electronic circuits can be grouped as one-dimensional map circuits, higher-dimensional map circuits, continuous-time autonomous circuits, and continuous-time non-autonomous circuits (Van Wyk and Steeb, 1997).

There were some electronic circuits that can be described by one-dimensional maps (Mishina, Kohmoto, and Hashi, 1985). Among them, the switched-capacitor circuit was one of the simplest, which is composed of a voltage source, a linear capacitor, a nonlinear switched-capacitor component and three analog switches (Rodríguez-Vázquez, Huertas, and Chua, 1985). For a specific choice of circuit parameters, its dynamics is equivalent to the well-known logistic map which is actually the simplest chaotic polynomial discrete map.

There were many electronic circuits that can be described by higher-dimensional maps. Among them, the infinite impulse response (IIR) digital filter, which utilized a two's complement adder with overflow, exhibits nonlinear dynamics ranging from limit cycles to chaos (Chua and Lin, 1988; Kocarev and Chua, 1993). Also, the adaptation algorithm for the filter weights was demonstrated to be one of the elements which may cause the filter to behave chaotically (Macchi and Jaidane-Saidane, 1989).

The continuous-time autonomous circuits have no external inputs so that such chaotic circuits are a kind of oscillators. The famous Chua circuit is the representative of this kind of oscillators. It is considered to be the simplest electronic circuit that can exhibit chaos and bifurcation phenomena (Chua, 2007), which have been verified by theoretical analysis (Chua, Komuro, and Matsumoto, 1986), computer simulation (Matsumoto, 1984) and experimental measurement (Zhong and Ayrom, 1985). The Chua circuit was invented in 1983 (Chua, 1992), which is shown in Figure 1.5. It is composed of five circuit elements, namely four passive elements, including an inductor, a resistor and two capacitors, as well as an active nonlinear element termed the Chua diode. This Chua diode is characterized by a piecewise-linear odd-symmetric characteristic, which can be realized by various circuits (Cruz and Chua, 1993; Gandhi *et al.*, 2009). The Chua circuit exhibits a number of distinct routes to chaos and a chaotic attractor termed the double-scroll attractor which is multi-structural. Other well-known autonomous circuits that show chaotic behavior include the Shinriki circuit (Shinriki *et al.*, 1981), the Saito circuit (Saito, 1985), and the Matsumoto circuit (Matsumoto, Chua, and Kobayashi, 1986).

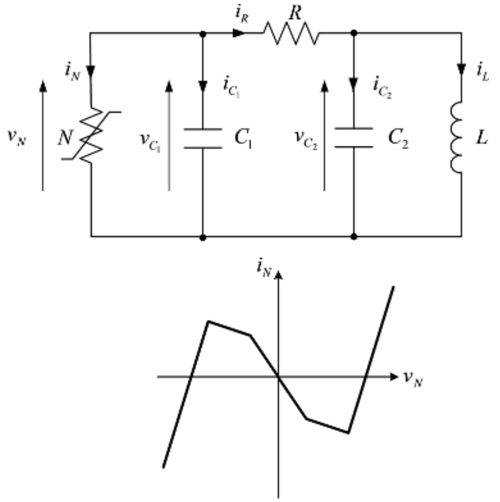

Figure 1.5 The Chua circuit

The continuous-time non-autonomous circuits refer to those circuits that are driven by an external source. The well-known phase-locked loop circuit is the representative of these types of driven circuit. It consists of three basic functional blocks, namely a voltage-controlled oscillator, a phase detector, and a loop filter, where it can perform various tasks such as frequency modulation and demodulation, frequency synthesis, and data synchronization. It has been identified that a second-order phase-locked loop circuit used as a frequency modulation demodulator exhibits chaotic behavior (Endo and Chua, 1988; Endo, Chua, and Narita, 1989). Other well-known non-autonomous circuits that show chaotic behavior include the periodically driven ferro-resonant circuit (Chua *et al.*, 1982), the triggered astable multivibrator (Tang, Mees, and Chua, 1983), the automatic gain control loop circuit (Chang, Twu, and Chang, 1993), and the cellular neural network circuit (Zou and Nossek, 1991).

1.3.2 Chaos in Telecommunications

As opposed to other areas in electrical engineering in which the investigation was started from the identification of chaos, the study of chaos in telecommunications was based on practical applications. In recent years, there has been tremendous interest in the use of chaos in telecommunications, as depicted in Figure 1.6. Chaotic signals possess inherent wideband characteristics, which can readily encode and spread narrowband information, resulting in spread-spectrum signals having the merits of difficulty in uninformed detection, mitigation of multipath fading, and an antijamming capability. Moreover, chaos provides the definite advantage of low-cost implementation. Basically, the chaos-based telecommunications can be grouped into three main types: chaotic masking, chaotic modulation, and chaotic switching.

In chaotic masking (Kocarev *et al.*, 1992; Cuomo and Oppenheim, 1993), the information signal is added to a chaotic signal at the transmitter. At the receiver, the original chaotic signal is reconstructed using chaos synchronization to allow the information signal to be extracted by subtracting the reconstructed chaotic signal from the incoming signal. Consequently, various techniques were proposed such as the feedback-based chaotic masking (Milanović and Zaghloul, 1996) and observer-based chaotic masking (Boutayeb, Darouach, and Rafaralahy, 2002; Liu and Tang, 2008).

In chaotic modulation (Kocarev and Parlitz, 1993; Itoh and Murakami, 1995; Tao and Chua, 1996), the communication is based on a modulated transmitter parameter – that is, the information signal is fed into a chaotic system to modulate its parameter, hence varying its dynamics. At the receiver, the change in dynamics of the chaotic signal is tracked in order to retrieve the information signal (Song and Yu, 2000; Feng and Tse, 2001).

In chaotic switching (Lau and Tse, 2003), the basic principle is to map bits or symbols to the basis functions of chaotic signals emanating from one or more chaotic attractors. One of the earliest chaotic-switching techniques is based on chaos shift keying (CSK) which maps different symbols to different basis functions of chaotic signals by varying a bifurcation parameter. If synchronized copies of the basis functions are available at the receiver, coherent detection can easily be achieved by evaluating the synchronization error (Parlitz *et al.*, 1992; Dedieu, Kennedy, and Hasler, 1993) or on the basis of

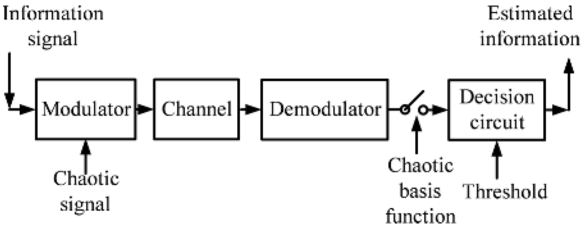

Figure 1.6 A chaos-based communication system

correlation (Kolumbán, Kennedy, and Chua, 1998). On the other hand, if there are no synchronized copies of the basis functions at the receiver, detection needs to be done by noncoherent means (Kolumbán and Kennedy, 2000). Another widely accepted chaotic-switching technique is based on differential chaos shift keying (DCSK) (Kolumbán *et al.*, 1996) which contains a basis function in half of its symbol period serving as the reference signal, thus avoiding synchronization between the transmitter and the receiver. Consequently, many other chaotic-switching techniques are derived from the CSK and DCSK, such as chaotic on-off-keying (Kolumbán, Kennedy, and Kis, 1997), correlation delay shift keying, symmetric CSK (Sushchik, Tsimring, and Volkovskii, 2000), quadrature CSK (Galias and Maggio, 2001), and frequency-modulated DCSK (Kolumbán, Jako, and Kennedy, 1999; Min *et al.*, 2010).

1.3.3 Chaos in Power Electronics

Power electronic systems are essentially piecewise-switched circuits. Their operation is characterized by the cyclic switching of circuit topologies, and as this results in a wide variety of nonlinear dynamics, they naturally prefer the use of nonlinear methods for design and analysis.

Starting from the late 1980s, chaos has been identified to be a real phenomenon in power electronics (Hamill and Jefferies, 1988; Wood, 1989). Furher discussions on instability and chaos in power electronic circuits and systems were reported in 1990 (Krein and Bass, 1990; Deane and Hamill, 1990), and since then much interest has been aroused in the investigation of the chaotic behavior of power electronic systems, especially DC–DC converters (Di Bernardo and Tse, 2002).

The milestone of investigations of chaos in power electronics was the paper by Hamill, Deane, and Jefferies (1992) which analyzed the occurrence of chaos in a simple buck converter, as shown in Figure 1.7. By using iterated nonlinear mappings, the occurrence of chaos was successfully modeled and simulated. This derivation was further extended to study the chaos in a current-mode controlled boost converter (Deane, 1992). The investigation was then extended to chaos in a DC–DC converter operating in discontinuous conduction mode (Tse, 1994) and chaos in a fourth-order Ćuk (Tse, Fung, and Kwan, 1996).

Further works on chaos in the buck converter were reported. For example, the bifurcation behavior under different variations of circuit parameters, including storage inductance, output capacitance, and load resistance, was specifically studied (Chakrabarty, Poddar, and Banerjee, 1996), while a detailed analytical description of converter dynamics – and, hence, its chaotic attractor – were discussed (Fossas and

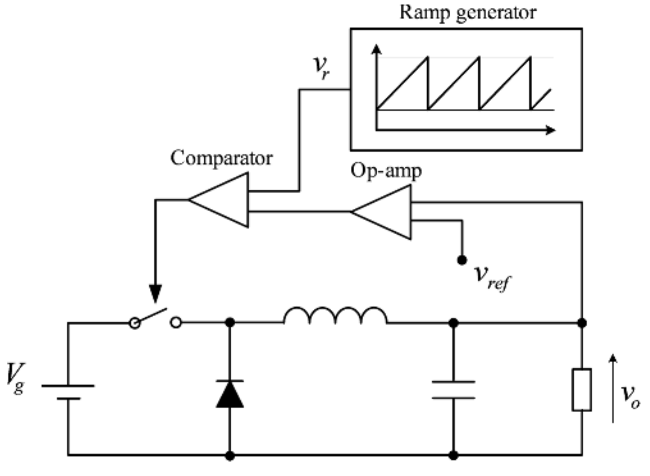

Figure 1.7 Chaos in a simple buck converter

Olivar, 1996). Moreover, some investigations on the sudden jump transition from periodic solution to chaos were conducted (Di Bernardo *et al.*, 1998; Yuan *et al.*, 1998), showing that it is due to a special class of bifurcation (the border-collision bifurcation) that is unique to switching systems.

Apart from the aforementioned analysis of chaos, the control of chaos in DC–DC converters was also investigated. By using parameter perturbation or by changing the switching instant, the chaos that occurred in the current-mode controlled buck converter was successfully stabilized (Poddar, Chakrabarty, and Banerjee, 1998). Also, a resonant parametric perturbation was applied to control chaos in the buck converter (Zhou *et al.*, 2003) and the buck-boost converter (Kavitha and Uma, 2008). Moreover, a recurrence plot analysis was employed to identify the set of unstable periodic orbits and, hence, use a resonant parameter perturbation to control chaos in the buck converter (Ivan and Serbanescu, 2009). On the other hand, the well-known time-delay feedback control was successfully applied to stabilize chaos in the buck converter (Batlle, Fossas, and Olivar, 1999). Also, by applying the control input to the system intermittently, the partial time-delay feedback control was proposed to save the control energy consumption (Bouzahir *et al.*, 2008).

In recent years, some applications of chaotic power electronic systems have been identified. For instance, chaos was positively employed to spread the noise spectrum and hence improve the electromagnetic interference in DC–DC converters (Deane and Hamill, 1996; Tse *et al.*, 2003). Another possible application is the use of chaotic transitions between different controlled states of the current-mode controlled boost converter, hence achieving very quick targeting (Aston, Deane, and Hamill, 1997).

1.3.4 Chaos in Power Systems

The investigation of chaos in power systems probably began in 1990, and spread over different areas, including power system stability and control, power flow optimization, unit commitment scheduling, load forecasting, and fault analysis.

Firstly, by decreasing the frequency of excitation in a single-machine quasi-infinite busbar system, it was shown that oscillatory solutions might lose their stability through period-doubling bifurcations, leading to chaos and unbounded motions (Hamdan, Nayfeh, and Nayfeh, 1990). Chaotic behavior was then observed in a simple power system over a range of reactive power loading conditions. As shown in Figure 1.8, this simple power system was composed of two generator buses and a load bus at which the load is modeled by an induction motor with a constant $P - Q$ load in parallel. By taking Q as the bifurcation parameter, four different kinds of bifurcation were identified, namely the subcritical Hopf

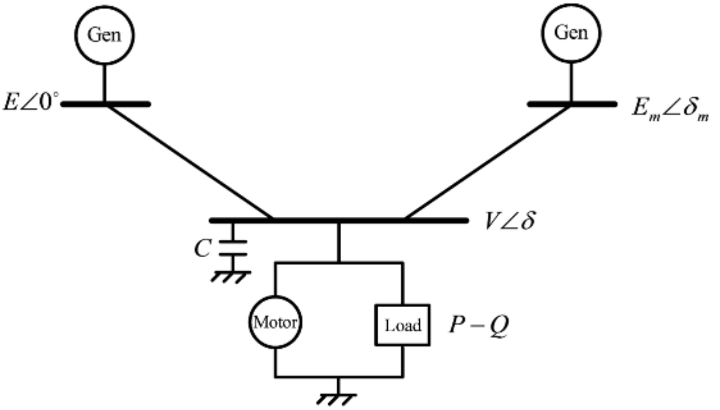

Figure 1.8 Chaos in a simple power system

bifurcation, period-doubling bifurcation, supercritical Hopf bifurcation, and saddle-node bifurcation. Hence, the period-doubling routes to chaos were confirmed by calculating the relevant Lyapunov exponents and broad-band spectrum (Chiang *et al.*, 1993). Similar period-doubling routes to chaos in a power system were also observed and discussed (Lee and Ajjarapu, 1993). Rather than varying the excitation or loading conditions, the Hopf bifurcation, the period-doubling bifurcation, and the chaos caused by the line resistance were investigated (Niu and Qiu, 2002). On the other hand, the banded chaos, namely the strange attractor with a band-like structure, was also reported in an actual power system with ferro-resonance (Ben-Tal, Kirk, and Wake, 2001).

Facing the occurrence of chaos in power systems, measures on how to avoid or control such chaotic behavior were brought up, aiming to reduce the risk of catastrophic failure (Wildberger, 1994). Then, the use of flexible AC transmission system (FACTS) devices was proposed to damp out the Hopf bifurcation and chaos in power systems (Srivastava and Srivastava, 1998). Moreover, an improved OGY method was also proposed to stabilize chaos in a power system (Okuno, Takeshita, and Kanari, 2002).

Although chaos has been identified to be undesirable for system stability, it can be positively utilized to benefit the power system. Since traditional optimization techniques generally suffer from the problem that optimal power flow is easily trapped by a local minimum solution, a chaos optimization algorithm was proposed to provide more robust convergence (Liu, Wang, and Hou, 2003). Similarly, a chaos search algorithm was proposed to provide the most economical solution for unit commitment scheduling (Liao and Tsao, 2004).

Furthermore, chaos time series analysis was employed to capture the characteristics of the complicated load behavior in power systems, hence performing short-term load forecasting (Mori and Urano, 1996). Similarly, fractal geometry was also employed to analyze the chaotic properties of high impedance faults during which phase currents and voltages exhibit a certain degree of chaotic behavior (Mamishev, Russell, and Benner, 1996).

1.3.5 Chaos in Electric Drive Systems

The investigation of chaos in electric drive systems can be categorized as three themes, namely the analysis of chaotic phenomena, the control of chaotic behaviors, and the application of chaotic characteristics. The milestones of chaos in electric drive systems are summarized in Table 1.2.

Chaos in electric drive systems was firstly identified in induction drive systems in 1989. That is, the bifurcation of induction motor drives was studied (Kuroe and Hayashi, 1989), which was actually an extension of the instability analysis of pulse-width-modulation (PWM) inverter systems. The bifurcation and chaos resulting from the tolerance-band PWM inverter-fed induction drive system were then investigated (Nagy, 1994; Sütő, Nagy, and Masada, 2000). It was also identified that saddle-node bifurcation, or even Hopf bifurcation, might occur in induction drive systems under indirect field-oriented control (Bazanella and Reginatto, 2000) and, consequently, the control of chaos in induction drive systems was investigated. An attempt was made to use a neural network stabilizing chaos during speed control of induction drive systems (Asakura *et al.*, 2000). On the other hand, an attempt was made to use periodic speed command to stimulate the chaotic motion of induction drive systems (Gao and Chau, 2003a).

Starting from 1997, the investigation of chaos has been accelerated. In 1997, the chaotic behavior in a simple DC drive system was unveiled (Chau *et al.*, 1997a); the dynamic bifurcation in DC drive systems was studied (Chau, Chen, and Chan, 1997b); and the subharmonics and chaos in DC drive systems were analytically modeled (Chau *et al.*, 1997c). Hence, the analysis and experimentation of their chaotic behaviors, including the voltage-mode controlled operation (Chen, Chau, and Chan, 1999) and the current-mode controlled operation (Chen, Chau, and Chan, 2000a), could be conducted. In 2000, the research was extended to the stabilization of chaos in DC drive systems by using a time-delay feedback control (Chen *et al.*, 2000b).

Table 1.2 Milestones of chaos in electric drive systems

1989	Kuroe and Hayashi identified bifurcation of a PWM inverter-fed induction drive system.
1994	Nagy identified bifurcation and chaos resulting from the tolerance-band PWM inverter-fed induction drive system.
1994	Hemati identified strange attractors in a PM brushless DC drive system.
1997	Chau *et al.* identified chaotic behavior and dynamic bifurcation in a simple DC drive system.
1998	Ito and Narikiyo applied chaotic motion for vertical spindle surface grinding.
1999	Chau *et al.* identified subharmonics and chaos in a SR drive system.
2000	Chen *et al.* utilized a time-delay feedback control to stabilize chaos in a DC drive system.
2000	Asakura *et al.* utilized a neural network to stabilize chaos in an induction drive system.
2001	Bellini *et al.* applied a chaotic map to generate chaotic PWM for an induction drive system.
2002	Gao and Chau utilized a time-delay feedback control to stimulate chaotic motion in a PM synchronous drive system.
2003	Gao and Chau utilized a periodic speed command to stimulate the chaotic motion of an induction drive system.
2004	Gao and Chau identified Hopf bifurcation and chaos in a synchronous reluctance drive system.
2004	Gao and Chau identified spontaneous chaotic behavior in a DSPM drive system.
2004	Chau *et al.* applied electrical chaoization to a DC drive system to generate chaotic motion for mixing.
2005	Ye and Chau utilized PM design parameters to stimulate chaotic motion in a PM synchronous drive system.
2005	Huang *et al.* utilized a control strategy to stimulate chaotic motion in a SR drive system.
2005	Chau and Wang applied electrical chaoization to a DC drive system to generate chaotic motion for compaction.
2005	Gao *et al.* applied electrical chaoization to a single-phase induction drive system to generate chaotic motion for cooling.
2006	Ren and Liu utilized nonlinear feedback control to stabilize chaos in a PM synchronous drive system.
2006	Chau and Wang utilized PM design parameters to stimulate chaotic motion in a DSPM drive system.
2006	Wang and Chau applied an extended time-delay auto-synchronization to stabilize chaos in a wiper system.
2006	Ye *et al.* applied electrical chaoization to a single-phase induction drive system for washing machines.

In 1999, the subharmonics and chaos in switched reluctance (SR) drive systems were first identified (Chau *et al.*, 1999) and the corresponding modeling was then developed (Chen *et al.*, 2000c; Chen *et al.*, 2002). Hence, the analysis of their chaotic behaviors under voltage PWM regulation (Chen, Chau, and Jiang, 2001) and current hysteresis regulation (Chau and Chen, 2002) was conducted. Furthermore, the experimentation of chaos in a practical SR drive system was first presented in the literature (Chau and Chen, 2003). In 2005, the research was extended to the stimulation of chaos in SR drive systems by using a control strategy combining piecewise proportional feedback and time-delay feedback (Huang, Chen, and Chau, 2005).

Without taking power electronic switching into consideration, it was identified that the permanent magnet (PM) brushless DC drive system could be transformed into a Lorenz system, which is well known to exhibit a Hopf bifurcation and chaotic behavior (Hemati, 1994). Also, chaotic behaviors have been identified in PM brushless AC drive systems or the so-called PM synchronous drive systems (Li *et al.*, 2002; Gao and Chau, 2003b). Consequently, the nonlinear feedback control method was developed to control the chaos in a PM synchronous drive system (Ren and Liu, 2006). Furthermore, both set-point and tracking output regulation of PM synchronous drive systems can be achieved by using a simple linear output feedback controller, provided that the operating point for the quadrature axis current is adequately chosen (Loría, 2009). On the other hand, the PM synchronous drive system was chaoized to produce chaotic motion by using a time-delay feedback control (Gao and Chau, 2002) or stator flux regulation (Wang, Chau, and Jian, 2008a). Instead of using control-oriented chaoization, the design-oriented chaoization was applied to a PM synchronous drive system which can spontaneously generate chaotic motion (Ye and Chau, 2005b).

By removing the PM materials and increasing the saliency of the PM synchronous motor, the resulting synchronous reluctance motor gives the definite advantages of high robustness and low cost. Similar to the PM brushless DC drive system, the synchronous reluctance drive system exhibits Hopf bifurcation and chaos (Gao and Chau, 2004a). On the other hand, by incorporating the concept of an SR motor into the PM brushless DC motor, the resulting doubly-salient PM (DSPM) motor offers the advantages of high robustness and immunity from PM thermal problems. It has been shown that the chaos in DSPM drive systems can occur spontaneously, depending on the initial design parameters of the PMs used (Gao and Chau, 2004b). Hence, based on the design of the PMs, the DSPM drive system can be purposely chaoized to produce chaotic motion (Chau and Wang, 2006).

The application of chaos in electric drive systems has focused on the practical use of the control of chaos, including the stabilization of chaos and the stimulation of chaos. For instance, chaotic vibration in an automotive wiper system not only decreases the wiping efficiency but also causes harmful distraction to the drivers (Suzuki and Yasuda, 1998). Thus, the corresponding chaos was directly stabilized by applying an extended time-delay auto-synchronization control to its DC drive (Wang and Chau, 2006; Wang and Chau, 2009a). This approach can be realized experimentally because the armature current of the DC motor can be easily measured by a Hall sensor and the perturbations on the feed-in motor voltage can be readily produced by a power converter.

Recently, there has been increasing attention to the emission of electromagnetic radiation from electric drive systems, which directly affects the electromagnetic interference (EMI) and electromagnetic compatibility, and indirectly creates acoustic noise and mechanical vibration. The EMI of an induction drive system was significantly suppressed by applying a chaotic map, namely the 4-way Bernoulli shift, to generate the so-called chaotic PWM (Bellini *et al.*, 2001). Then, a Chua circuit was employed to generate the desired chaotic sequence for chaotic PWM (Cui *et al.*, 2006). Consequently, a chaotically frequency-modulated signal was proposed to modulate the switching frequency of the PWM, which not only suppresses the peaky EMI, but also avoids the occurrence of low-order noises and mechanical resonance (Wang, Chau, and Liu, 2007). It was then extended to the space vector PWM which offers the additional advantages of less harmonic distortion, less switching loss, and better utilization of the DC supply voltage (Wang and Chau, 2007). Moreover, it was further extended to propose the chaotically amplitude-modulated signal to modulate the switching frequency of the space vector PWM for a closed-loop vector-controlled induction drive system (Wang, Chau, and Cheng, 2008b).

In recent years, chaotic mixing has been proposed to improve the energy efficiency and degree of homogeneity by using mechanical means (Alvarez-Hernández *et al.*, 2002) that are essentially based on the design of impeller vanes to produce chaotic motion. In order to offer the advantages of high flexibility and high controllability, the electrical chaoization was proposed to generate the desired chaotic motion for industrial mixing (Chau *et al.*, 2004; Ye and Chau, 2007). It applied a time-delay feedback control to the DC drive system which serves as the agitator. Similarly, destabilization control was also applied to the DC drive system which electrically generates chaotic motion for mixing (Ye and Chau, 2005a). Furthermore, the design-oriented chaoization was also applied to a PM synchronous drive system that can generate the desired chaotic motion for mixing (Ye and Chau, 2005b).

The application of chaos to compaction was initiated in a mechanical vibrator for compacting soft soil (Long, 2001). In order to offer high flexibility and high controllability, the electrical chaoization was proposed to generate the desired chaotic motion for compaction (Chau and Wang, 2005; Wang and Chau, 2008). Essentially, the PM DC drive system, which directly couples with an eccentric mass to translate the rotational motion to up–down motion, was chaoized by using a proportional time-delay feedback control. The use of a chaotic reference control was then introduced to further improve the ability to control the desired chaotic compaction (Wang and Chau, 2009b). On the other hand, based on the design-oriented chaoization, the DSPM drive system was chaoized to spontaneously produce chaotic motion for compaction (Chau and Wang, 2006).

Although there were many other applications of chaotic motion, most of them relied on a mechanical means to produce chaotic motion, while the electric drive was used as a driving force only. Nevertheless, some of them adopted electrical chaoization for the direct stimulation of chaotic motion, such as the chaoization of a single-phase induction drive system for washing machines (Ye, Chau, and Niu, 2006) and cooling fans (Gao, Chau, and Ye, 2005) as well as a DC drive system for vertical spindle surface grinders (Ito and Narikiyo, 1998).

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