1

Introduction

1.1 The Case for Cost Optimization

A great majority of structural optimization papers deal with the minimization of the weight of the structure (Vanderplaats, 1984; Arora, 1989; Adeli and Kamal, 1993; Adeli, 1994 – to mention a few). While weight of a structure constitutes a significant part of the cost, minimization of the cost is the final objective for optimum use of available resources. For concrete structures the optimization problem has to be formulated as a cost minimization problem because different materials are involved. In contrast, for steel structures the optimization problem can be formulated as a weight minimization problem. Only a small fraction of hundreds of papers published on optimization of steel structures deals with cost optimization; the great majority deal with minimization of the weight of the structure. In reality, a minimum weight design may not be a minimum cost design. Besides material cost there are many other factors that influence the total construction cost of a structure. Up to the late 1990s, little research work had been reported on the optimization of the overall cost of a three-dimensional steel structure subjected to the constraints of the commonly used design specifications such as AISC ASD and LRFD specifications (AISC, 1995, 2001).

Ideally, the optimization problem should be formulated in terms of the life-cycle cost, which includes the costs of materials, fabrication, erection, maintenance, and disassembling the structure at the end of its life cycle. Some methodologies for determining life-cycle costs and decision making are discussed by Wilson *et al.* (1997). However, prior to the authors' research, little

Cost Optimization of Structures: Fuzzy Logic, Genetic Algorithms, and Parallel Computing H. Adeli and K. C. Sarma

work on cost optimization of steel structures was reported in the literature. Optimization of total cost as well as the life-cycle cost are very important from the economic point of view and should be the prime focus of structural optimization in the new millennium.

In the traditional optimization algorithms, constraints are satisfied within a tolerance defined by a crisp number. In actual engineering practice constraint evaluation involves many sources of imprecision and approximation. When an optimization algorithm is forced to satisfy the design constraints exactly it can miss the global optimum solution within the confines of commonly acceptable approximations. By taking into account the fuzziness and imprecision in the constraints (the input of the optimization problem) and employing the fuzzy set theory of Zadeh (1965, 1978), one can further reduce the objective function (the output of the optimization problem) and increase the probability of finding the actual global optimum solution substantially.

For the structural optimization methodology in general, and the cost optimization approach in particular, to be embraced by the structural engineering community, the focus of future research should be on large structures subjected to the actual constraints of a commonly used design code such as the AISC ASD (AISC, 1995) or the AISC LRFD (AISC, 2001) code. The true benefit of optimization is realized for large structures with hundreds of members.

In cost minimization additional difficulties are encountered. They include the definition of the cost function and uncertainties and fuzziness involved in determining the cost parameters. As a result, only a small fraction of the structural optimization papers published deal with the minimization of the cost. In this chapter, a chronological review of papers is presented on the cost optimization of concrete and steel structures published in archival journals.

1.2 Cost Optimization of Concrete Structures

Hundreds of papers have been published on optimization of structures during the past four decades. However, only a small fraction of them deal with cost optimization of structures. The great majority of the structural optimization papers are concerned with minimization of the weight of the structure. For concrete structures the objective function to be minimized should be the cost since they are made of more than one material. A review of articles on cost optimization of concrete structures published in archival journals is presented in this section, where interesting and important results and conclusions are summarized. Most of these papers deal with structural elements such as beams. Few journal papers are found on cost optimization of rather realistic three-dimensional structures. As such, there is a need to perform research on cost optimization of realistic three-dimensional structures, especially large structures with hundreds of members where optimization can result in substantial savings. The results of such research efforts will be of great value to practicing engineers.

Concrete structures include reinforced concrete (RC), prestressed concrete, and fiber-reinforced concrete structures. In concrete structures at least three different cost items should be considered in optimization: costs of concrete, steel, and the formwork. This review is presented in terms of different types of concrete structures. Reliability-based cost optimization is reviewed in the last section.

1.2.1 Concrete Beams and Slabs

Most of the papers published on cost optimization of concrete structures are about beams or girders. The general cost function for reinforced, fiber, or prestressed concrete beams can be expressed in the following form:

$$C_{\mathrm{m}} = C_{\mathrm{cb}} + C_{\mathrm{sb}} + C_{\mathrm{pb}} + C_{\mathrm{fb}} + C_{\mathrm{sbv}} + C_{\mathrm{fib}} \tag{1.1}$$

where C_{m} is the total material cost, C_{cb} is the cost of concrete in the beam, C_{sb} is the cost of reinforcing steel, C_{pb} is cost of prestressing steel, C_{fb} is the cost of the formwork, C_{sbv} is the cost of shear steel, and C_{fib} is the cost of fiber in concrete. For a pretensioned beam equation (1.1) can be written as

$$\begin{aligned} C_{\mathrm{m}} = {} & w_{\mathrm{c}} L_{\mathrm{b}} \left(A_{\mathrm{cb}} - A_{\mathrm{sb}} - A'_{\mathrm{sb}} - A_{\mathrm{pb}}\right) c_{\mathrm{c}} + w_{\mathrm{s}} L_{\mathrm{b}} \left(A_{\mathrm{sb}} + A'_{\mathrm{sb}}\right) c_{\mathrm{s}} \\ & + w_{\mathrm{p}} L_{\mathrm{b}} A_{\mathrm{pb}} c_{\mathrm{p}} + L_{\mathrm{b}} p_{\mathrm{fb}} c_{\mathrm{f}} + C_{\mathrm{sbv}} + C_{\mathrm{fib}} \end{aligned} \tag{1.2}$$

where L_{b} is the length of the beam; w, A, c are unit weights, cross-sectional areas, and unit costs, respectively; subscripts b, c, s, p, and f refer to beam, concrete, steel, prestressing, and formwork, respectively; the prime indicates compression steel; and p_{fb} is the cross-sectional perimeter of the form. Equations (1.1) and (1.2) can be reduced for special cases. For example, in the case of an RC beam with no prestressing and fiber, the quantities A_{pb} and C_{fib} are set to zero in equation (1.2).

Goble and Lapay (1971) minimize the cost of post-tensioned prestressed concrete T-section beams based on the ACI code (ACI, 1963) by using the gradient projection method (Arora, 1989). The cost function includes the first four terms in equation (1.1). They state that the optimum design seems to be unaffected by the changes in the cost coefficients. However, subsequent researchers to be discussed later rebut this conclusion.

Kirsch (1972) presents the minimum cost design of continuous two-span prestressed concrete beams subjected to constraints on the stresses, prestressing force, and the vertical coordinates of the tendon by linearizing the nonlinear optimization problem approximately and solving the reduced linear problem by the linear programming (LP) method. His cost function includes only the first and third terms in equation (1.1). Kirsch (1973) extends this work to prestressed concrete slabs.

Friel (1974) finds closed-form solutions for the optimum ratio of steel to concrete for minimum cost simply supported rectangular RC beams using the ultimate moment constraints of the ACI code (ACI, 1971). The cost function is similar to equation (1.1), but neglecting the costs of prestressing steel (C_{pb}) and fiber (C_{fib}) and adding an additional term for increasing the cost due to the increase in the building height. The author concludes that the costs of the formwork and the increase in the height do not influence the optimum cost significantly.

Brown (1975) presents an iterative method for minimum cost selection of the thickness of simply supported uniformly loaded one-way slabs using only the flexural constraints of the ACI code (ACI, 1971). The cost function includes only the first two terms in equation (1.1). The author reports cost savings of up to 17 %.

Naaman (1976) compares minimum cost designs with minimum weight designs for simply supported prestressed rectangular beams and one-way slabs based on the ACI code (ACI, 1971). The cost function includes the first, third, and fourth terms in equation (1.1) and is optimized by a direct search technique (Siddall, 1972). He concludes that the minimum weight and minimum cost solutions give approximately similar results only when the ratio of cost of concrete per cubic yard to the cost of prestressing steel per pound is more than 60. Otherwise, the minimum cost approach yields a more economical solution, and for ratios much smaller than 60 the cost optimization approach yields substantially more economical solutions. He also points out that for most projects in the US the aforementioned ratio is less than 60.

Chou (1977) uses the Lagrange multiplier method for the minimum cost design of a singly reinforced T-beam using the ACI code (ACI, 1971). The author defines only two design variables: effective depth and area of steel reinforcement. The cost function includes the first two terms in equation (1.1).

In the formulation it is assumed that the neutral axis is located inside the flange of the T-section. The author reports a cost reduction up to 14 % of the cost of the beams with a maximum steel ratio.

Gunaratnam and Sivakumaran (1978) present the minimum cost design of RC slabs satisfying the limit state requirements of the British code (CP110, 1972) for members having uniform, triangular, or parabolic moment distribution using a combination of the Lagrange multiplier and graphical methods. Their cost function includes only the first two terms in equation (1.1). They present curves for optimum design parameters as a function of the thickness of the slab. They point out the significant influence of the serviceability limit state of deflection on the optimum design parameters.

Kirsch (1983) presents a simplified three-level iterative procedure for cost optimization of multi-span continuous RC beams with rectangular cross-sections using a cost function consisting of only the first two terms in equation (1.1). In the first level the amount of the reinforcement is found in each critical section for given concrete dimensions and design moments. In the second level the concrete dimensions of each element are found. In the third level design moments are optimized. The author does not, however, consider the constraints of a concrete design code used in practice.

Cohn and MacRae (1984a) consider the minimum cost design of simply supported RC and partially or fully pre-tensioned and post-tensioned concrete beams of fixed cross-sectional geometry subjected to serviceability and ultimate limit state constraints including constraints on flexural strength, deflection, ductility, fatigue, cracking, and minimum reinforcement, based on the ACI code (ACI, 1977) or the Canadian building code (CSA, 1977) using the feasible conjugate-direction method (Kirsch, 1993). The beam can be of any cross-sectional shape subjected to distributed and concentrated loads. Their cost function is similar to equation (1.1) with the exception of the term for the cost of fibers. For the examples considered they conclude that for post-tensioned members partial prestressing appears to be more economical than complete prestressing for a prestressing-to-reinforcing steel cost ratio greater than 4. For pretensioned beams, on the other hand, complete prestressing seems to be the best solution. For partially prestressed concrete they also conclude that for a prestressing-to-reinforcing steel cost ratio in the range of 0.5 to 6, the optimal solutions vary little. Cohn and MacRae (1984b) perform parametric studies on 240 simply supported, reinforced, partially, or completely pre- and post-tensioned prestressed concrete beams with different dimensions, depth-to-span ratios, and live load intensities. They conclude that, in general, RC beams are the most cost-effective at high depth-to-span ratios and low live load intensities. On the other hand, completely prestressed beams are the most cost-effective at low depth-to-span ratios and high live load intensities. For intermediate values, partial prestressing is the most cost-effective option.

Saouma and Murad (1984) present the minimum cost design of simply supported, uniformly loaded, partially prestressed, I-shaped beams with unequal flanges subjected to the constraints of the 1977 ACI code (ACI, 1977). The optimization problem is formulated in terms of nine design variables: six geometrical variables plus areas of tensile, compressive, and prestressing steel. The constrained optimization problem is transformed to an unconstrained optimization problem using the interior penalty function method (Kirsch, 1993) and is solved by the quasi-Newton method (Vanderplaats, 1984) of the IMSL library (IMSL, 1980). They found the optimum solutions for several beams with spans ranging from 6 m to 42 m, assuming both cracked and uncracked sections, and reported cost reductions in the range of 5 % to 52 %. They also conclude that allowing cracking to occur does not reduce the cost by any significant measure.

Using integer programming, Jones (1985) formulates the minimum cost design of precast, prestressed concrete simply supported box girders used in a multi-beam highway bridge and subjected to the AASHTO (1977) loading assuming that the cross-sectional geometry and the gridwork of strands are given and fixed. The design variables are the concrete strength, and the number, location, and draping of strands (moving the strands up at the end of the beam). The constraints used are release and service load stresses, ultimate moment capacity, cracking moment capacity, and release camber. The cost function includes only the first and third terms in equation (1.1).

Abendroth and Salmon (1986) present a parametric study on the sensitivity of the optimum cost of partially or fully end-restrained RC T-section beams in terms of various parameters such as allowable deflections, material strength, support conditions, and unit material costs. The constrained minimization problem is converted to an unconstrained one using an internal penalty function and solved by the quasi-Newton–Raphson method (Kirsch, 1993). The design constraints are given in the ACI code (ACI, 1983). The cost function includes the first, second, fourth, and fifth terms in equation (1.1). In addition, they add a penalty cost parameter in order to take into account the various factors associated with increased floor thickness. They assume the unit cost of stirrups (shear reinforcement) to be one and a half times the cost of longitudinal reinforcement. They found that the optimum cost is less with metallic forms than wooden forms even when the latter is used as many as four times. They state that shear reinforcement does not have a significant role in reducing the total cost and therefore may be neglected in the optimization formulation. They report a 5 % savings in the total cost by increasing the strength of concrete from 17.2 MPa (2.5 ksi) to 48.2 MPa (7.0 ksi) and 15 % savings by increasing the yield strength of steel from 275.6 MPa (40 ksi) to 516.8 MPa (75 ksi). They compare an elastic design with a partial limit state design and state that the latter does not produce significantly more economical beams.

Park and Harik (1987) present the minimum cost design of horizontally curved two-way RC slabs with rigid boundaries based on the British code (CP110, 1980) using the sequential LP method. The cost function includes the first two terms in equation (1.1). They consider the constraints on deflections, minimum effective depths, and design moments as the three dominant factors in the optimization process.

MacRae and Cohn (1987) present the optimization of prestressed concrete flat slabs based on the Canadian code for concrete structures (CSA, 1977) and the recommendation of an ACI-ASCE Committee (ACI-ASCE, 1974) using the conjugate-direction method. Notwithstanding the importance of shear in the design of flat slabs, they only consider the flexural reinforcement in the optimization formulation. They pose the problem as one of finding the reinforcing and prestressing steel in the flat slab of a given story for given concrete dimensions. Their cost function contains only the first three terms in equation (1.1). They conducted a parametric study by varying the depth-to-span ratio, live load, cable layout, and limit state and allowable tensile stresses. They conclude that using cables in clusters (groups of cable) and using high-strength steel reduce the total cost.

Prakash *et al.* (1988) present minimum cost designs of singly and doubly reinforced rectangular and T-shape RC beams, using Lagrangian and Simplex methods per limit state conditions of the prevailing Indian code. The cost function includes the first two terms in equation (1.1). They state that a two-way slab is more economical than a T beam floor for spans up to 6 m in a residential type building, whereas for heavier loads or longer spans the reverse is true.

Paul *et al.* (1990) present the minimum cost design of a modular floor system with precast prestressed voided and solid slabs simply supported on steel beams, using the general geometric programming method (Beightler and Phillips, 1976). The design is given in the British codes (BS449, 1969; CP110, 1976). The cost function includes the cost of fabrication of the slabs including the cost of concrete, prestressing steel, and forms, cost of steel beams, and the cost of erection. They conclude that for optimum cost designs the prestressing force required for a solid slab is less than that for a voided slab.

Kanagasundaram and Karihaloo (1990) describe the minimum cost design of simply supported and continuous rectangular, L- and T-section RC beams according to the Australian code (AS3600, 1988) using two different methods: sequential LP and sequential convex programming (Arora, 1989). The constraints used are stability, strength, serviceability, durability, and fire resistance. Their cost function includes the first, second, and fourth terms in equation (1.1). For the examples considered they state that the costs of concrete and the reinforcing steel are about the same but the cost of formwork is more than twice the cost of concrete and steel combined, thus concluding the significant contribution of the formwork cost to the total cost. A sensitivity analysis of the cost optimization with respect to the cost of formwork is performed and found that the minimum cost design is not affected by the variations in the relative cost of formwork.

Kanagasundaram and Karihaloo (1991a) consider the strength of concrete (f_c') as a design variable in addition to cross-sectional dimensions and the steel ratio for the cost optimization of simply supported and multi-span beams with rectangular and T-sections. The cost of concrete is related to the concrete strength through nonlinear regression analysis and using a cubic function. They conclude that higher strength concrete (up to 60 MPa) resulting in shallower sections yields more economical beams. Kanagasundaram and Karihaloo (1991b) present the minimum cost design of RC multi-span beams subjected to earth pressure, liquid pressure, wind, or earthquake loads in addition to dead and live loads. The design constraints and cost function are the same as before. The conclusions are also similar.

Ezeldin (1991) presents the minimum cost design of rectangular, reinforced fiber concrete beams with four variables: width and depth of the beam, steel fiber content and area of bending reinforcing bars. The cost function can be obtained from equation (1.2) by setting the prestressing steel (A_{pb}), compression steel (A'_{sb}), and shear reinforcement (C_{sbv}) to zero. A direct search method is used for optimization. As an extension of this work, Ezeldin and Hsu (1992) formulate the minimum cost design of rectangular, reinforced fiber concrete beams with two additional variables, cross-sectional area and spacing of stirrups, and thus the cost function includes the cost of shear reinforcement. They conclude that the variations of the costs of concrete and form appear to have a more significant influence on the minimum cost than those of steel reinforcement and fibers.

Chakrabarty (1992a) presents the minimum cost design of RC rectangular beams using the geometric programming (Kirsch, 1993) and Newton–Rapson methods. The cost function includes the first, second, and fourth terms in equation (1.1). In the context of the Indian condition where the labor is cheap, the author found that at optimum solution the cost of concrete and steel are about the same but the cost of formwork is about one-fourth of the cost of concrete or steel. The reverse was reported earlier by Kanagasundaram and Karihaloo (1990) for countries like Australia or the USA where the labor cost is high. The author observes that in most cases the optimum design yields ductile beams, which is desirable for withstanding dynamic forces like earthquakes. In an extension of this work Chakrabarty (1992b) concludes that the minimum cost design of a rectangular singly reinforced beam is increased by 36 % when the width-to-depth ratio is increased from 0.25 to 0.67. The author observes that optimized sections are often deeper sections to satisfy displacement constraints and hence become more ductile with less steel reinforcement. Erbatur *et al.* (1992) discuss the minimum cost design of prestressed concrete beams with rectangular and flanged sections. They solve the non-linear optimization problem approximately by using the LP approach.

Cohn and Lounis (1993) present the minimum cost design of partially and fully prestressed concrete continuous beams and one-way slabs. The optimization is based on the limit state design and projected Lagrangian algorithm. They simultaneously satisfy both collapse and serviceability limit state criteria based on the ACI code (ACI, 1989). The material nonlinearity is idealized by an elastoplastic constitutive relationship. A constant prestressing force and prestressing losses are assumed. Their cost function includes the first three terms in equation (1.1). They report that the total cost decreases with the increase in the allowable tensile stress (f_t).

Lounis and Cohn (1993b) present a multi-objective optimization formulation for minimizing the cost and maximizing the initial camber of post-tensioned floor slabs with serviceability and ultimate limit state constraints of the ACI code (ACI, 1989). The cost objective function is chosen as the primary objective and the camber objective function is transformed into a constraint with specified lower and upper bounds. The resulting single optimization problem is then solved by the projected Lagrangian method. The cost function for the slab includes only the first and third terms in equation (1.1).

Khaleel and Itani (1993) present the minimum cost design of simply supported partially prestressed concrete unsymmetrical I-shaped girders per ACI code (ACI, 1983). The objective function is similar to equation (1.1) but with the exception of the last term. The sequential quadratic programming method is used to solve the nonlinear optimization problem assuming both cracked and uncracked sections. They conclude that an increase in the concrete strength does not reduce the optimum cost significantly, and higher strength in prestressing steel reduces the optimum cost to a certain extent. They state that some amount of reinforcing steel facilitates the development of cracking in the concrete, which reduces the cost of materials and improves ductility.

Al-Salloum and Siddiqi (1994) present the minimum cost design of singly reinforced rectangular concrete beams per ACI code (ACI, 1989). The cost function includes only the first, second, and fourth terms in equation (1.1). They obtained a closed-form solution for the steel areas and depth in terms of the cost and strength parameters by taking the derivatives of the augmented Lagrangian function with respect to the area of steel reinforcement, depth of beam, and four Lagrange multipliers for constraints on flexural strength, lower and upper bounds on ductility, and the side constraint.

Adamu *et al.* (1994) outline a continuum-type optimality criteria approach (Rozvany *et al.* 1994) for the minimum cost design of singly reinforced RC beams with rectangular cross sections based on the European code (CEB/FIB, 1990). The cost function includes only the first, second, and fourth terms in equation (1.1). The necessary cost minimality criteria are obtained by applying the calculus of variation to an augmented Lagrangian function. They applied the method to a propped cantilever beam (fixed support at one end and simple support at the other end) with variable depth and width. As an extension of this work, Adamu and Karihaloo (1994a) used the discretized continuum-type optimality criteria (DCOC) method for the minimum cost design of RC beams with varying cross-sections using the depth or the depth and steel reinforcement ratio as design variables. They applied the method to two example problems: a propped cantilever beam and a three-span continuous beam. Adamu and Karihaloo (1994b) discuss the minimum cost design of rectangular RC beams with uniform cross-sections and variable steel ratio in each span. Adamu and Karihaloo (1995a) consider the minimum cost design of nonprismatic RC simply supported T-beams and propped cantilever rectangular beams with segmentation. In each segment of the beam, the cross-section is either constant or varies linearly or quadratically.

Han *et al.* (1995) discuss the minimum cost design of partially prestressed concrete rectangular, and T-shape beams based on the Australian code (AS3600, 1988) using the DCOC method. The cost function includes the first four terms in equation (1.1). They conclude that for a simply supported beam, a T-shape is more economical than a rectangular section. Han *et al.* (1996) use the DCOC method to minimize the cost of continuous, partially prestressed and singly reinforced T-beams with constant cross-sections within each span. A three-span and a four-span continuous beam example is presented.

1.2.2 Concrete Columns

Few papers have been published on the cost optimization of concrete columns. The general cost function for a concrete column can be written in a similar way to equation (1.1) for concrete beams:

$$C_m = C_{cc} + C_{sc} + C_{pc} + C_{fc} + C_{tc} \tag{1.3}$$

where C_{cc}, C_{sc}, C_{pc}, C_{fc}, and C_{tc} are the costs of concrete, reinforcing steel, prestressing steel, formwork and lateral ties in columns, respectively. For pre-tensioned columns, equation (1.3) can be written as

$$C_m = w_c H_c \left(A_{cc} - A_{sc} - A_{pc}\right) c_c + w_s H_c A_{sc} c_s + w_p H_c A_{pc} c_p + H_c p_{fc} c_f + V_{tc} c_s \tag{1.4}$$

where H_c is the height of the column, A_{cc} is the cross-sectional area of the column, A_{sc} is the cross-sectional area of the steel reinforcement, A_{pc} is the cross-sectional area of the prestressing steel, p_{fc} is the cross-sectional perimeter of the form, and V_{tc} is the volume of the lateral ties.

Kanagasundaram and Karihaloo (1990, 1991a) present the minimum cost design of rectangular RC columns subjected to an axial compressive force and single or biaxial bending based on the Australian code (AS3600, 1988) using sequential LP and sequential convex programming methods. Both short and long columns are considered, taking into account their slenderness ratio. The cost function is similar to equation (1.3) but without the prestressing cost. Both objective function and constraints are approximated by Taylor's series expansions. In a subsequent paper, Kanagasundaram and Karihaloo (1991b) include the concrete strength as a design variable in addition to cross-sectional dimensions and the area of the longitudinal reinforcement.

Zielinski *et al.* (1995) present the cost optimization of RC short tied rectangular columns based on the Canadian code (CSA, 1984) using the internal penalty function method. The cost function includes the first, second, and fourth terms in equation (1.3). Kocer and Arora (1996) present the minimum cost design of prestressed concrete transmission poles based on the PCI (1983) and the ACI (1977) codes using (a) a combination of branch and bound, enumeration, and sequential quadratic programming methods and (b) a genetic algorithm (Goldberg, 1989; Adeli and Hung, 1995). Their cost function includes the first, third, and fourth terms in equation (1.3). Their results indicate a genetic algorithm to be more efficient than the other approach used. They report savings in the neighborhood of 25 % compared with conventional designs.

1.2.3 Concrete Frame Structures

The overall or total cost of a concrete structure (C_T) can be expressed in the following form:

$$C_T = C_m + C_f + C_t + C_s + C_{cd} + C_e \tag{1.5}$$

where C_f, C_t, C_s, C_{cd}, and C_e are the costs of fabrication (or placement), transportation, substructure (or foundation), cladding, and erection, respectively. Only a few papers have been published on the cost optimization of reinforced concrete frame structures. All of them deal with two-dimensional frames with two exceptions.

Andam and Knapton (1980) discuss the minimum cost design of portal precast RC frames but without presenting much detail. Krishnamoorthy and Mosi (1981) present the cost optimization of two-dimensional frames with rectangular cross-sections using the sequential unconstrained minimization technique (SUMT) (Fiacco and McCormick, 1968) and Davidon-Fletcher-Powell method (Arora, 1989). They considered nonlinear constitutive relationships but no actual design code. Their cost function includes only the material costs of concrete, steel reinforcement, and formwork. They present examples of single-, double-, and triple-bay and two-, four-, and six-story frames.

Huanchun and Zheng (1985) present a two-level minimum cost design approach for two-dimensional RC frames according to the Chinese building code. In the first level they try to find the most flexible structure satisfying the global constraints, such as the lateral drift using the sequential LP method. In the second level the cost of the frame is minimized by considering the local constraints for each member of the structure and using a discrete search method for cross-sectional widths and depths. Their cost function includes the material costs for beams and columns only. Choi and Kwak (1990) minimize the costs of rectangular beams and columns of RC frames by using a direct search method to select appropriate design sections from some predetermined discrete sections based on the ACI (1977) and Korean codes. Their cost function includes the material costs of concrete, steel, and the formwork.

Spires and Arora (1990) discuss the optimal design of tall tubular RC framed structures with double symmetry in the plan based on the ACI code

(ACI, 1983) using a sequential quadratic programming procedure. However, they reduce the doubly symmetric structure into the equivalent plane frame using the approximate finite element approach proposed by Khan (1974). As such, they optimize the cost of regular symmetric two-dimensional frames. Their cost function includes the material costs of concrete, steel, and framework for beams and columns. They also consider the frequency constraint in order to limit wind and earthquake forces. They present examples of five- and forty-story two-dimensional frames.

Dinno and Mekha (1993) discuss the minimum cost design of one-and two-story RC frames based on the ACI code (ACI, 1983) using an inelastic trilinear moment–rotation relationship for beams and columns, and SUMT. They consider the material costs of concrete, reinforcement, and formwork. They conclude that optimal designs using inelastic analysis results in somewhat more economical designs.

Moharrami and Grierson (1993) present the minimum cost design of RC building frames subjected to vertical and lateral loading based on the ACI code (ACI, 1989) using the optimality criteria approach. The columns have rectangular cross-sections and the beams can be rectangular, L, or T shapes. The design variables are the width, depth, and longitudinal steel reinforcement of the beams and columns. Their cost function includes the material costs of the concrete, reinforcement, and the formwork. Their largest example is a five-story single-bay RC frame. They conclude that the optimality criteria approach converges slowly when stiffness constraints are included in the formulation.

Adamu and Karihaloo (1995b) used the discretized continuum-type optimality criteria (DCOC) method for the minimum cost design of two-dimensional multi-bay and multi-story RC frames based on Australian (AS3600, 1988) and European (CEB/FIB, 1990) limit state design codes. The cost function includes the material costs of concrete, reinforcing steel, and the formwork. The design variables are cross-sectional dimensions and steel ratios. For economical reasons they assume uniform beam and column dimensions in every story but vary the steel ratios in each member. This also reduces the cost of the formwork because the formwork can be re-used more frequently. They present the optimum cost design of a seven-story RC frame with setbacks. In a companion paper, Adamu and Karihaloo (1995c) take into account the biaxial bending of the corner columns approximately, but still considering plane frames.

Fadaee and Grierson (1996) present the minimum cost design of three-dimensional RC frames with members subjected to biaxial moments and shear forces using the optimality criteria approach based on the ACI code

(ACI, 1995). Beams and columns are assumed to have rectangular sections. The cost function includes the material costs of concrete, steel, and the formwork. The focus of this work is formulation of the appropriate constraints for combinations of the axial load, biaxial bending moment, and biaxial shear. Their example is only a one-bay and one-story space frame. They conclude that the biaxial shear is an important consideration for the design of columns and its inclusion increases the cost of the optimum structure significantly.

Balling and Yao (1997) present a comparative study of optimization of three-dimensional RC frames with rectangular columns, and rectangular, T-, or L-shape beams according to the ACI code (ACI, 1989) using one-, two-, and four-story frames subjected to vertical and lateral loads, and employing the sequential quadratic programming or a gradient-based method. For steel reinforcement they consider two different definitions for design variables. In the first definition, the area of steel in each member is the only design variable used for steel in that member. In the second definition, they consider the number, diameter, and longitudinal distribution of the reinforcing bars, and perform a two-level optimization. They include the costs of materials, fabrication, and placement in the cost function by assuming that (a) the material and fabrication costs of the steel reinforcement are proportional to the weight and (b) the placement cost is proportional to the number of bars, stirrups, and ties. They conclude that the optimum costs based on the two definitions are very close to each other and thus there is no need to include the second more computationally costly definition in the optimization formulation. Based on this conclusion, the authors then discuss a simplified approach for cost optimization of space RC frames.

1.2.4 Bridge Structures

As one of the first papers on cost optimization of structures, Torres *et al.* (1966) present the minimum cost design of prestressed concrete highway bridges subjected to AASHTO loading by using a piecewise LP method. The independent design variables are the number and depth of girders, prestressing force, and tendon eccentricity. They further define dependent design variables as the spacing of girders, tendon cross-sectional area, initial prestress, and the slab thickness and reinforcement. They claim their cost function includes the costs of transportation, erection, and bearings in addition to the material costs of concrete and steel, but do not give any detail. They present results for bridges with spans ranging from

20 ft to 110 ft (6.1 m to 33.5 m) and with widths of 25 ft (7.6 m) and 50 ft (15.2 m).

Yu *et al.* (1986) present the minimum cost design of a prestressed concrete box bridge girder used in a balanced cantilever bridge (consisting of two end cantilever and overhang spans and one middle simple span) based on the British code (CP110, 1976) and using general geometric programming (Beightler and Phillips, 1976). The cost function includes the material costs of concrete, prestressing steel, and the metal formwork. They also include the labor cost of the metal formwork, roughly as 1.5 times the cost of the material for the formwork. The design variables are the prestressing forces, the eccentricities, and the girder depths for all spans. Barr *et al.* (1989) also use the general geometric programming method to minimize the cost of a continuous three-span bridge RC slab with an overall span length of 16.6 m subjected to the constraints of AASHTO (1983) and Ohio Department of Transportation bridge design regulations (ODOT, 1982). The cost function includes the material costs of concrete and steel.

Lounis and Cohn (1993a) present the minimum cost design of short and medium span highway bridges consisting of RC slabs on precast, post-tensioned, prestressed concrete I-girders satisfying the serviceability and ultimate limit state constraints of the Ontario Highway Bridge Design Code (OHBDC, 1983). They use a three-level optimization approach. In the first level they deal with the optimization of the bridge components including dimensions of the girder cross-sections, slab thickness, amounts of reinforcing and prestressing steel, and tendon eccentricities by the projected Lagrangian method (Haftka and Gurdal, 1992). In the second level, they consider the optimization of the longitudinal layout such as the number of spans, restraint type and span length ratios, and transverse layout such as the number of girders and slab overhang length. In the third level, they consider various structural systems such as solid or voided slabs on precast I- or box girders. They use a sieve-search technique (Kirsch, 1993) for the second and third levels of optimization. Their cost function includes the material costs of concrete, reinforcement, and connections at piers. They also include the costs of fabrication, transportation, and erection of girders assuming a constant value per length of the girder. They conclude by optimizing a complete set of bridge system results in a more economical structure than optimizing the individual components of the bridge. Based on their optimization studies they recommend simply supported girders for prestressed concrete bridges of up to 27 m (89 ft) long, two-span continuous girders for span lengths of 28 m (92 ft) to 44 m (144 ft), three-span continuous girders for span lengths of 55 m (180 ft) to 100 m (328 ft), and two- or three-span continuous girders for an intermediate range of 44 m (144 ft) to 55 m (180 ft).

Cohn and Lounis (1994) apply the above three-level cost optimization approach to multi-objective optimization of partially and fully prestressed concrete highway bridges with span lengths of 10 m to 15 m and widths of 8 m to 16 m. Their objective functions include the minimum superstructure cost, minimum weight of prestressing steel, minimum volume of concrete, maximum girder spacing, minimum superstructure depth, maximum span-to-depth ratio, maximum feasible span length, and minimum superstructure camber. For a four-lane 20 m length single-span bridge, they conclude that the voided slab and the precast I-girder systems are more economical than the solid slab and one- and two-cell box girders. Lounis and Cohn (1995a) also conclude that voided slab decks are more economical than box girders for short spans (less that 20 m) and wide decks (greater than 12 m), and single-cell box girders are more economical for medium spans (more than 20 m) and narrow decks (less than 12 m). The single-cell box girder, however, results in the deepest superstructure, which may be a drawback when there is restriction on the depth of the deck. Multi-criteria cost optimization of bridge structures is further discussed by Lounis and Cohn (1995b, 1996). They suggest that the criteria of minimax and minimum Euclidean distance can be used by designers for selection of the *best* solution.

Fereig (1996) presents the minimum cost preliminary design of single-span bridge structures consisting of cast-in-place RC deck and girders based on the AASHTO code (AASHTO, 1992). The author linearizes the problem by approximating the nonlinear constraints by straight lines and solves the resulting linear problem by the Simplex method. The author concludes that 'it is always more economical to space the girder at the maximum practical spacing'.

1.2.5 Water Tanks

Saxena *et al.* (1987) present the minimum cost design of RC water tanks based on the Indian and ACI (1969) codes using the heuristic flexible tolerance method (Himmelblau, 1972). The cost function includes the material costs of concrete, steel, and the formwork. They conclude that a larger percentage in cost savings can be achieved for water tanks with larger capacities.

Using a direct search method and the SUMT, Tan *et al.* (1993) present the minimum cost design of RC cylindrical water tanks based on the British code for water tanks. The cost function includes the material costs of concrete and steel only. The tank wall thickness is idealized with piecewise linear slopes with the maximum thickness at the base.

1.2.6 Folded Plates and Shear Walls

Lakshmy and Bhavikatti (1995) present the minimum cost design of simply supported trough-type folded plate roofs based on the Indian code using a combination of sequential LP and the SUMT. The cost function includes the material costs of concrete and steel only.

Hajek and Frangopol (1991) describe a computer program for the minimum cost design of concrete shear wall systems based on the ACI code (ACI, 1983) using the folded plate theory and the method of feasible directions (Vanderplaats, 1984). The cost function includes the costs of concrete and the formwork (including transport and labor) but excluding the cost of reinforcement.

1.2.7 Concrete Pipes

Thakkar and Sridhar Rao (1974) discuss cost optimization of composite-type prestressed concrete pipes based on the Indian code. They approximate the constraints by linear functions and solve the resulting problem by the LP method. The cost function includes the material costs of concrete and steel only. Heinloo and Kaliszky (1981) present a closed-form approximate solution for the minimum material cost design of thick-walled plastically rigid RC pipes subjected to internal pressure.

1.2.8 Concrete Tensile Members

Naaman (1982) presents the minimum cost design of prestressed concrete tensile members based on the ACI code (ACI, 1977). He approximates the nonlinear optimization problem to a linear one and solves it by the LP method. The cost function includes the material costs of concrete and the prestressing steel. Optimization of a 30.48 m (100 ft) long tie member of an arch structure subjected to an axial tensile force of 444.8 kN (100 kip) is presented.

1.2.9 Cost Optimization Using the Reliability Theory

All the aforementioned references use a deterministic approach to cost optimization. A few researchers have used the reliability theory to include the uncertainties in the computation of the design loads and resistances. In deterministic optimization a structure is optimized only for a given predetermined set of loadings. In reliability-based design the loads and the structural strengths are considered as random variables, and safety is related to some probability of exceeding the structural capacity by the applied loading. In reliability-based optimization an attempt is made to consider different failure modes under different loading scenarios simultaneously. The reliability-based optimization arguably can incorporate the interactions among various failure modes. However, the major bottleneck in the reliability-based optimization is the computation of the probability of failure, which often cannot be done consistently due to insufficient statistical data.

The reliability factor in the cost optimization is considered either directly or indirectly. In the direct approach, the reliability factor is included directly in the objective function. Moses (1977) presents the total cost (C_T) as the summation of the initial cost (C_I), which is a function of design variables, and the expected failure cost (C_F) multiplied by a probability of failure (P_F), which is also considered a function of design variables:

$$C_T = C_I + P_F C_F \tag{1.6}$$

subjected to the design constraints:

$$h_i(\mathbf{x}) = 0, \qquad i = 1, 2, \ldots, N_{ch} \tag{1.7}$$

$$g_i(\mathbf{x}) \geq 0, \qquad i = 1, 2, \ldots, N_{cg} \tag{1.8}$$

where N_{ch} and N_{cg} are the total number of equality and inequality constraints, respectively. The second term in equation (1.6) represents the risk of the loading on the structure exceeding its capacity. The expected failure cost includes the cost associated with the failure of the structure, such as replacement cost, damage to properties, casualties, business interruption, litigation costs, etc.

In the indirect method the objective function is only the initial cost. The reliability term is considered indirectly in the form of a constraint or constraints in addition to the design constraints, equations (1.7) and (1.8), such as

$$P_F \leq P_{F\ \text{allowable}} \tag{1.9}$$

Thus, in this approach a deterministic optimization procedure can be converted into a reliability-based optimization procedure by adding one or more additional probability constraints.

Moses (1977) uses both direct and indirect approaches for the minimum cost design of RC beams and highway girders subjected to fatigue loading using SUMT and a direct search procedure. The probability of failure is calculated from a safety index, which is in turn computed from the mean values and the standard deviations of the random strength and load parameters (Frangopol and Moses, 1994). The expected failure costs are chosen in advance somewhat arbitrarily.

Surahman and Rojiani (1983) present a reliability-based optimization of four- and ten-story RC building frames by including the reliability term in the cost function. By varying the probabilities of failures between the range 0.000 001 to 0.01 and assuming different values for the expected failure cost, they arrive at an *optimum* probability of failure. SriVidya and Ranganathan (1995) discuss the reliability-based cost optimization of single-story single-bay RC frames based on different live and wind load conditions and the Indian code. They perform an elastoplastic analysis and include both component- and system-level probabilities of failure in the form of constraints. They simply assume values for probabilities of failure. Lin and Frangopol (1996) present the reliability-based minimum cost design of simply supported RC T-girders for highway bridges based on AASHTO provisions (AASHTO, 1992). Their initial cost is only the material costs of concrete and steel. The optimization approach is the method of feasible directions. They point out that only about 4 % of the structural optimization papers are about concrete and composite structures.

Koskisto and Ellingwood (1997) present the minimum life-cycle cost optimization of prefabricated concrete structures using the reliability theory. They define the total life-cycle cost as

$$C_L = C_D + C_P + C_C + C_Q + C_M + P_F C_F \tag{1.10}$$

where C_D is the planning and design cost, C_P is the production cost, C_C is the construction cost, C_Q is the quality assurance and quality control costs, and C_M is the preventive and corrective maintenance costs. For an example problem of a hollow core slab, they assume the design cost as 2.5 % of the production cost, where the production cost is the sum of materials and the labor costs. They ignore the C_Q value and assume that the labor cost is 43 % of the material costs (C_m) and the construction cost (C_C) is 0.01 times the span length and thickness of the slab. They use the projected Lagrangian method to solve the optimization problem.

1.2.10 Concluding Comments

The great majority of papers on cost optimization of concrete structures includes the material costs of concrete, steel, and formwork. Some researchers ignore the cost of the formwork. However, this cost is significant in industrialized countries and should not be ignored. Other costs such as the cost of labor, fabrication, placement, and transportation are often ignored. Additional research needs to be done on life-cycle cost optimization of structures where the life-cycle cost of the structure over its lifetime is minimized instead of its initial cost of construction only.

The researchers of reliability-based optimization make a valid argument about the inclusion of uncertainties in loads and resistances in the optimization process. However, at present (and in the foreseeable future) the probabilities of failure and the *expected failure* costs cannot be calculated with any measure of certainty due to insufficient statistical data; they have to be chosen somewhat arbitrarily or in some magical way!

1.3 Cost Optimization of Steel Structures

In this section, a chronological review of papers is presented on the cost optimization of steel structures published in archival journals. This review is divided into three subsections: deterministic, reliability-based, and fuzzy logic-based cost optimization of steel structures. In deterministic cost optimization of steel structures, where the great majority of the papers are published, optimization is performed for a predetermined set of loadings based on code-specified constraints. In reliability-based cost optimization, loads and resistances are considered to be random and the optimization is performed for a given safety factor or probability of exceeding the structural capacity. In a fuzzy logic-based optimization an attempt is made to take into account the imprecisions in determining the cost parameters and constraints using the theory of fuzzy sets (Zadeh, 1965).

1.3.1 Deterministic Cost Optimization

For steel structures a general total cost function (C_T) can be defined in the following form:

$$C_T = C_m + C_f + C_t + C_e \tag{1.11}$$

where C_{m} is the material cost of structural members (beams, columns, and bracings), C_{f} is the fabrication cost (including the material costs of connection elements, bolts, and electrode, and the labor cost), C_{t} is the cost of transporting the fabricated pieces to the construction field, and C_{e} is the erection cost (including the material costs of connection elements, bolts, and electrode, and the labor cost).

When only the material cost of structural members is included (the first term in equation (1.11)) the cost function can be presented as proportional to the volume or weight of the structure:

$$C_{\mathrm{m}} = c_{\mathrm{m}} \rho_{\mathrm{s}} V = c_{\mathrm{m}} W \tag{1.12}$$

where ρ_{s} is the unit weight of steel, c_{m} is the cost per unit weight of steel, V is the volume of the structure, and W is the total weight of the structure. In this case the cost optimization problem is simply transformed to the weight optimization problem. This simplification also assumes that various hot-rolled shapes commonly used for beams, columns, and bracings have the same unit price, which may not be the case.

Some authors use equation (1.12) as their objective function and refer to the resulting problem as the 'cost' optimization problem. In this work, those papers are considered as a weight optimization problem and consequently excluded from this review. The review in this section is classified based on the type of steel structures.

1.3.1.1 Beams and Plate Girders

For beams and plate girders a general cost function can be defined in the following form:

$$C_{\mathrm{T}} = C_{\mathrm{mb}} + C_{\mathrm{fb}} + C_{\mathrm{tb}} + C_{\mathrm{eb}} \tag{1.13}$$

where C_{mb}, C_{fb}, C_{tb}, and C_{eb} are the material, fabrication, transportation, and erection costs of the beams or plate girders, respectively. The great majority of published articles include only the first two terms in equation (1.13) in the cost optimization formulation and use the following reduced cost function:

$$C_{\mathrm{T}} = C_{\mathrm{mb}} + C_{\mathrm{fb}} \tag{1.14}$$

An early attempt on cost optimization of steel girders is presented by Razani and Goble (1966). They optimize doubly symmetric I-shaped welded plate girders with a constant web depth based on design requirements similar to AASHO (1961) using the dynamic programming method (Adeli and Ge, 1989; Adeli, 1994). Their cost function includes both the material cost of the girder, including stiffeners and splices, and the fabrication cost. They balance the material cost with the fabrication cost to minimize the total cost by smoothening the variations in the flange thickness along the span and minimizing the use of flange splices, resulting in less welding. They present two example problems: a simple-span girder with overhangs on both ends subjected to a uniformly distributed load and a three-span continuous girder under moving AASHO (1961) loads.

Goble and DeSantis (1966) present the minimum cost design of composite, continuous welded plate girders used in highway bridges with unequal flanges and variable-thickness flange and web plates but with constant depth. The basis of design is AASHO (1961). The design variables are the flange thicknesses and widths, the web thicknesses and depth, and the distances between the web plate splices and the bottom and top flange splices. They find the minimum cost web height and flange width for a given arrangement of splice points. They mention various costs for fabrication and welding including the cost of pre-heating steel, the cost of preparing and aligning edges of the plates before welding, the cost of welding rods, and the cost of weld metal depositing (labor cost). The material cost for a steel plate is considered to be made of three components: a basic cost per pound, an extra cost per inch of thickness, and an extra cost per inch of width, assuming a higher cost for thicker and wider plates. However, the authors present no actual cost function. The optimization technique used is dynamic programming. They present an example of a two-span continuous plate girder with a span length of 60.96 m (200 ft) under moving AASHO (1961) loads. Moses and Goble (1970) point out that using similar cross-sections for many members can reduce the fabrication cost of a framed steel structure. In other words, the minimum cost structure is often somewhat heavier than the minimum weight structure. They describe the use of dynamic programming for minimum cost selection of member sizes without actually presenting any structure made of those members.

Annamalai *et al.* (1972) present the minimum cost design of simply supported welded plate girders subjected to concentrated and uniformly distributed loads based on the AISC specifications (AISC, 1969). They use commercially available plates and a discrete optimization method called ‘backtrack programming’ (Golomb and Baumert, 1965). They provide a table of material and labor costs for different components of the plate girder but present no cost function. For welding and splices they do not present separate material and labor costs; instead they combine both material and labor costs into one cost item. As examples, they present the cost optimization of a 36.6 m (120 ft) simply supported plate girder with and without flange splices.

Anderson and Chong (1986) present the minimum cost design of homogeneous and hybrid stiffened steel plate girders according to the AISC code (AISC, 1978). They consider two factors that raise the cost of a stiffened hybrid girder over the cost of an unstiffened homogeneous girder: (1) the additional labor cost for cutting and welding stiffeners, and (2) the cost of higher strength steel for the flange plates. They present analytical functions for the *optimum* depth of the web plate by making a number of assumptions such as neglecting the tension field effect and the shear–tension interaction.

Lorenz (1988) discusses the minimum cost design of composite beams based on the AISC Load and Resistance Factor Design (LRFD) code (AISC, 1986). He suggests that the true advantage of the LRFD code can be realized in a minimum cost design. The author is concerned with the trade-off between steel weight and the number of studs needed, without considering the cost of concrete, and presents an equivalent 'cost-rated beam weight' to take into account the costs of beam and studs for conditions limited to uniformly distributed loading, ASTM A36 steel, concrete strength of 20.7 MPa (3 ksi), and a particular size of studs.

Farkas (1991) presents closed-form solutions for optimum cost values of the cross-sectional variables for simply supported welded box girders subjected to a uniformly distributed load and simplified noncode constraints on bending stress, local flange and web buckling, shear fatigue for longitudinal fillet welds, and deflection. The cost function is similar to equation (1.14) where the cost of fabrication (C_{fb}) is expressed as a function of different labor times required for (a) preparation, assembly, and tacking, (b) welding, (c) electrode changing, weld slagging, and chipping, and (d) post-treatment of welds (toe burr grinding). The author presents empirical equations for various items using empirical data. In order to come up with the closed-form solutions, the author makes a number of simplifying assumptions, e.g. assuming the relation $b = 2h/3$ between the flange width (b) and web depth (h). The primary conclusion of this work is that the fabrication details and costs play an important part in the optimum cost design of welded steel structures. Further, for an example box girder with a span of 10 m subjected to a distributed load of 60 kN/m, the author reports that the minimum cost design is about 11 % more economical than the minimum weight design.

Bhatti (1996) presents the minimum cost design of simply supported partially or fully composite I-shaped steel beams with concrete slabs subjected to a uniformly distributed load, and strength, deflection, and vibration constraints of the AISC LRFD specifications (AISC, 1994) using the Lagrange multiplier approach. The resulting equations are solved by using a symbolic algebra program such as Mathematica (Wolfram, 1988). The cost function is similar to equation (1.14). However, the fabrication cost includes the cost of field-installed studs only. The cost optimization is formulated in terms of the relative cost of field-fabricating a stud to the cost per pound of the rolled steel (a ratio varying in the range of 6 to 12). Graphical solutions of several examples with span lengths of 7.6 m and 12.2 m are presented.

1.3.1.2 Trusses

Lipson and Russell (1971) discuss the minimum cost design of a roof structural system consisting of welded parallel-chord trusses, purlins, deck, and wall cladding above the bottom chord based on the Canadian code (CSA, 1965). The top and bottom chord members are T-section, and web members are double angles. The design variables are member sizes, spacing of trusses, depth-to-span ratio of trusses, number of panels of trusses, and the spacing of purlins. Their cost function includes the cost of materials for trusses, decking, purlins, and the wall cladding, and the cost of fabrication including the costs of preparation of chord and web members, splicing, welding, and labor. The labor cost is expressed in terms of the number of members rather than the weight of the truss. The cost of wall cladding is expressed as a step function of truss depth and spacing. The costs of decking and purlins are expressed as step functions of the spacing of trusses and purlins. The optimization approach is a modified Simplex method for a nonlinearly constrained optimization problem dubbed the ‘Complex’ method (Box, 1965). Lipson and Gwin (1977) discuss the minimum cost design of steel space trusses subjected to the AISC constraints (AISC, 1970). Their cost function includes the first two terms in equation (1.11). The fabrication cost includes the cost of galvanization. The optimization approach is the same ‘Complex’ method. They present a 25-member space truss example made of steel angles.

Thomas and Brown (1977) discuss the cost optimization of a truss roof system consisting of a number of identical one-way two-dimensional trusses, open-web joists, and standard 22 gage decking materials subjected to the AISC specification (AISC, 1970). Their cost function includes the first, second, and fourth terms in equation (1.11) for the aforementioned components. The material and erection costs of the roof decking are assumed to be proportional to the roof area. The fabrication and erection costs of both the open-web joists and primary trusses are assumed to be proportional to their weights. The optimization method is the sequential unconstrained minimization technique (SUMT) (Arora, 1989) and the Davidon–Fletcher–Powell method (Fletcher and Powell, 1963). The largest example truss presented has 37 members and covers a span of 32.6 m (1283.5 in).

Imai (1983) presents a mini-max dual approach for minimum weight and cost optimization of trusses made of steel and aluminum members subjected to explicit displacement and stress constraints. The theoretical idea is to combine the lightweight but more expensive aluminum with heavier but less expensive steel in an economical way. The cost function is the sum of the scaled material costs and weights of the components of a structure. The author acknowledges the difficulty in dealing with the discontinuous nature of the material properties and approximates the problem using the first-order Taylor series expansion for a displacement response. A 72-bar space truss example is presented.

1.3.1.3 Plane Frames

Ridha and Wright (1967) discuss the minimum cost design of two-dimensional steel frames using the mechanism method of the simple plastic analysis and the plastic design requirements of the AISC (1963) code assuming adequate bracing against buckling in the weak axis direction. The cost function includes the first two terms in equation (1.11) but only the cost of welded connections is included in the fabrication cost. The authors assume that the connection cost is a linear function of the shear force and the bending moment resisted by the connection. Further, the connection cost includes another component as a function of the size of the connected members intended to represent the costs of detail drawing, making templates, shear angles, and reaming. They report that compared with the minimum weight design the minimum material and connection cost design results in a heavier frame but a lower total cost, indicating the relative importance of the connection cost. A two-bay and two-story frame and a single-bay and three-story frame are presented as examples. They report savings in the range of 7 % to 26 % for the minimum cost design versus the minimum weight design.

Anderson and Islam (1979) attempt to present approximate closed-form solutions for the minimum cost design of multi-story rectangular rigid frames with limiting values on the lateral deflections. They oversimplify the problem by a number of assumptions, including neglecting the effect of vertical loads on lateral displacements and assuming inflection points at the midpoints of beams and columns. As such the frame becomes statically determinate and only one tier is considered for optimization.

Crawford and Jenkins (1980) present the minimum cost design of seven different types of steel single-span gable frame roof structures based on the British code (BSI, 1977) using a combination of the Complex method mentioned earlier and the pattern search of Hooke and Jeeves (1961). The single-span gable structures consist of two steel columns and a roof made of hot-rolled steel sections with or without haunches, plate girders, Warren truss, or trussed beam. Only the roof structure is optimized, excluding the columns, purlins, and sheeting. The authors studied the relative cost advantages of the various roof structures for a span range of 10 m to 50 m in the construction environment of the United Kingdom and provide relative cost curves and recommendations for practicing engineers. They also present curves for optimum length-to-span ratios and the number of panels versus the span length fitting through data. This paper demonstrates how cost optimization algorithms can directly help practicing engineers.

Majid *et al.* (1980) present the minimum cost topological design of rigid frames subjected primarily to lateral deflection constraints. The nonlinear optimization problem is approximated linearly by the Taylor series expansion and solved by the Simplex method. The cost function is the summation of the material cost and a constant value representing roughly the construction cost. They present examples of two-, three-, and five-story and multi-bay frames. Topological optimization is carried by simply removing some of the columns. Nakamura and Takenaka (1983) also discuss an analytical method for the minimum cost design of rectangular multi-story multi-span frames without considering any actual code constraints. Their cost function includes the first two terms in equation (1.11), but the fabrication cost includes the cost of connections only. Such highly limited analytical solutions have academic values only.

Douty (1980) describes the minimum cost design of three different types of bolted and welded connections used in steel frames based on the AISC specifications (AISC, 1970): shear angle-framed connection, and flange and end plate moment-resisting connections. The cost function is presented in terms of the connection variables, such as the diameter of the bolts, flange plate width and thickness, shear plate length and thickness, and the leg size of the fillet weld for connecting the shear plate to the column flange and for the flange plate moment connection. A weighting penalty is included in the size of the welds and bolt diameter, assuming that the cost is increased for larger size bolts and welds. The nonlinear programming problem is approximately linearized using the Taylor series expansion and then solved by a linear programming approach. Cheng and Juang (1989) present the minimum cost design of multi-story rigid frames subjected to static wind and earthquake forces according to the Uniform Building Code (UBC, 1984). They include the PΔ effect in the formulation and solve the problem using the optimality criteria approach (Adeli, 1994). They present empirical functions for costs of members, painting, and welded connections. Their examples include a two-bay, fifteen-story rigid frame.

Thurston and Sun (1993) present the multi-criteria optimization of two-dimensional steel frames without using any actual design code constraints. They attempt to minimize both cost and lateral drift using a combination of the Pareto optimization approach (Koski, 1994) and the multi-attribute utility theory. The cost function is presented as a function of the length of the steel members and volume of the concrete used in a rectangular floor deck. An example of a one-bay, three-story frame is presented.

Xu and Grierson (1993) present the minimum cost design of steel frames with semi-rigid connections based on the AISC code (AISC, 1978) using the augmented Lagrangian method. The cost function includes the material cost of the members (proportional to their weights) and the cost of each connection, assumed to be proportional to its rotational stiffness. Examples of one-bay and two-story, and three-bay and ten-story steel frames are given. Based on the limited examples and the aforementioned assumption about the cost of connections, they report that for low-rise frames with insignificant lateral displacements, semi-rigid connections 'may sometimes' result in a lighter design compared with the more common rigid connection design. However, the total cost of the semi-rigid frame may be more than that of the corresponding rigid frame because the authors did not include the actual fabrication cost of semi-rigid connections (including the labor cost). When lateral loads dominate the design, such as in the case of tall frames, the authors state that the fully rigid design will probably yield a lighter design because it provides a greater lateral stiffness. Examples of optimal cost designs of semi-rigid, low-rise industrial frames are also given in Xu *et al.* (1995).

Simoes (1996) presents the minimum cost design of semi-rigid steel frames subjected to stress and displacement constraints but without using an actual design code. The nonlinear programming problem is approximated by the Taylor series expansion and solved by the segmented linear programming approach. The cost of the members is assumed to be proportional to the weight. The cost of connections is taken as a quadratic function of the connection fixity factor in the range of 0 (for simple pinned connections) to 1 (for moment-resisting connections) and empirical ad hoc values are used for the coefficients of the cost function. The pinned and moment connections are assumed to add 20 % and 60 %, respectively, to the cost of each member, and the additional cost of semi-rigid connections is assumed to fall within this range. The largest example presented is a two-bay, three-story semi-rigid frame. For the small low-rise frames presented and for the assumed cost functions the authors assert that both the weight and cost of a semi-rigid frame are less than those of the corresponding moment-resisting frame.

1.3.1.4 Industrial Buildings

Bradley *et al.* (1974) discuss the minimum cost design of one-story industrial framed structures using the simple plastic theory and geometric programming technique (Beightler and Phillips, 1976; Abuyounes and Adeli, 1986) without using any actual design code. Their focus is computation of the cost terms. Lee and Knapton (1974) also describe their investigation of the minimum cost design of industrial building structures made of steel portal frames based on the British code (BSI, 1969) using the simple plastic theory but without presenting an explicit cost function. The design variables are the number of bays, frame spacing, eaves height, roof pitch, purlin spacing, and building length and width. The simplified optimization problem is solved approximately by a revised Simplex method.

Russell and Choudhary (1980) present the minimum cost design of one-story industrial buildings made of roof trusses in the transverse direction, braced frames in the longitudinal direction, and the footings under the columns based on the Canadian code (CSA, 1975). The problem is first decomposed into three optimization subproblems. Then, three interface variables are defined as the number of panels in the transverse trusses, the number of bays in the longitudinal direction, and the depth-to-span ratio of trusses, and the overall cost optimization problem is solved by using the aforementioned Complex method (Box, 1965). The cost function includes the costs of materials, labor, equipment, overheads, and profit. Profit and overheads are included as a fraction of the other three direct costs. The costs of labor and equipment are presented as functions of man-hours needed in various operations. Empirical equations are presented for times required for various operations as functions of design parameters such as the number of braced frames, the number of panels in the transverse trusses, and the truss weight, based on the curve fitting of the previous data in the Canadian construction environment. The authors present optimization of an industrial building covering a rectangular 30.5 m × 159 m(100 ft × 520 ft) area and the clear height to the underside of the trusses of 7.6 m (25 ft).

Jendo and Paczkowski (1993) describe the single- and multi-objective minimum cost design of one-story industrial buildings made of roof space double-layer trusses consisting of tubular sections subjected to explicit constraints on displacements, stresses, and buckling, using the metric and utility function methods (Jendo, 1990). Similar to Russell and Choudhary (1980), the problem is decomposed into several subproblems for optimization of roof covering (purlins and corrugated sheet), space trusses, columns, and walls (corrugated sheets). They attempt to synthesize the various optimization subproblems by two global or interface variables: the height of the trusses and the 'mesh density', defined as the ratio of the span length to the distance between the truss nodes. The multi-criteria are the minimization of the weight of the truss structure and the wall and column elements, the maximum vertical displacement, and the labor cost expressed empirically.

1.3.1.5 Guyed Towers

Bell and Brown (1976) discuss a heuristic approach for the minimum cost design of cable-supported steel guyed towers with a height in the range 30 m to 150 m used for supporting heavy microwave antennas subjected to wind loading and the AISC specifications (AISC, 1970) by assuming independence of design variables in various subspaces of the design. Optimum cable areas and tower mast sections are found independently using the Powell search method (Powell, 1964) and the branch and bound algorithm. They consider only the material costs and for simplicity transform the minimum cost design problem to a quasi-minimum weight design problem by assuming a fixed ratio for the relative costs of the cable and tower steel. The design variables are cable cross-sectional area, initial cable tension, mast cross-sectional area, and anchor and tie locations. The cross-section of the mast is either square with four angles or triangular with three angles. They do not seem to include the weight of the cross bracings in the formulation. However, the nonlinear behavior of the cables is included in the formulation.

1.3.1.6 Steel Transmission Poles

Kocer and Arora (1997) formulate the cost optimization of self-supporting steel transmission poles made of two overlapping tapering dodecagonal tubes with constant thickness. In addition to the dead load of the pole, the National

Electric Safety Code's light loading, ASCE ice and wind loading, and broken conductor loading are considered. The constraints in the design are given in the ASCE guidelines (ASCE, 1990). The design variables are the outside diameter at the top of the pole, tapering of the pole, and thicknesses of the two overlapping pieces. The cost function is similar to equation (1.14) with the fabrication cost formulated as the welding cost with three different cost items. They are the costs of total labor and overheads, total electrode used, and the power and equipments needed for welding. They calculate the total labor and overhead costs for welding from the length of the weld, hourly labor and overhead charge, welding done by one worker per hour, and a so-called operating factor. The power and equipment cost is assumed to be 20 % of the total electrode costs. The authors include the secondary moment effects due to lateral displacements in the formulation and solve the problem using three different approaches: (a) genetic algorithm (Adeli and Cheng, 1993, 1994a; Adeli and Hung, 1995), (b) simulated annealing (Aarts and Korst, 1989), and (c) the enumeration method. By including the additional labor costs, the optimum values of the diameter at the top of the upper tube and the thicknesses of both tubes are increased, and the optimum value of the tapering slope is decreased. They report that the genetic algorithm is the best of the three methods used in terms of computational efficiency and finding the global optimum solution.

1.3.1.7 Cellular Plates

Farkas and Jarmai (1994) present the minimum cost design of laterally loaded welded cellular steel plates using three different approaches: the backtracking method, the hill-climbing method, and feasible sequential quadratic programming (Farkas and Jarmai, 1997). The cellular plate is created by sandwiching and welding a grid of cold-formed channels or I-beams between two parallel plates. The design constraints are defined explicitly for bending stresses and local buckling of rib webs due to bending and shear without using any actual design code. The cost function includes the material and fabrication costs. The latter is calculated by multiplying the total time required for fabrication by a fabrication cost factor. The total time required for welding is the sum of the times required for (a) preparation, assembly, and tacking, (b) welding, and (c) electrode changing, weld deslagging, and chipping. Empirical equations based on local fabrication conditions are used for various conditions. They conclude that the hill-climbing approach is quick but sensitive to initial solutions and the feasible sequential quadratic programming is 'robust' even when the starting point is infeasible.

1.3.1.8 Bridge Structures

Memari *et al.* (1991) present the minimum cost design of a continuous, multispan concrete reinforced concrete–steel girder highway bridge subjected to the AASHTO code (AASHTO, 1983) using the method of feasible directions. The cost function is expressed as the material costs of the superstructure including the costs of the steel girders, longitudinal and transverse stiffeners, studs, and the reinforced concrete slabs. Their unit costs of materials are intended to include other costs such as fabrication, transportation, and erection indirectly. An example of a three-span and two-lane bridge structure is presented. They conclude that the dimensions of the flange and web plates have the greatest impact on the minimum cost solution.

1.3.2 Cost Optimization Using the Reliability Theory

Papers published on the reliability-based cost optimization of structures all take an academic, theoretical, and idealistic approach to the problem. The examples presented in these publications are usually small, academic two-dimensional structures. None of them uses an actual widely used design code such as the AISC specifications (AISC, 1995). Use of the probabilistic concepts in structural design was presented by Benjamin (1968). One of the first papers published on the reliability-based structural cost optimization is Mau and Sexsmith (1972). They minimize the expected cost of simple statically determinate two-dimensional steel trusses as defined by equation (1.6). They make a number of simplifying assumptions such as limiting each member to only one type of failure and ignoring partial failure and serviceability criterion. The initial cost of the structure, the first term in equation (1.6), is taken as the material cost only, which is expressed as a function of the weight of the structure. The cost of failure is assumed known and taken as proportional to the initial material cost of the structure. They point out that the criterion of minimum expected cost is equivalent to minimization of weight with an allowable probability of failure.

Ravindra and Lind (1973) describe the use of the probability theory in design code optimization with an attempt to balance the safety and cost. They apply the concept by finding a set of optimal load factors for single-story single-bay steel frames subjected to dead, snow, and wind loads using the hill climbing approach (Rosenbrock, 1960). Moses (1977) introduces the general concepts of the reliability theory into structural optimization in the context of the two approaches presented in Section 1.2.9. Rao (1980) presents the minimum cost design of a cable-stayed cantilevered steel box beam with a probabilistic objective function and constraints using the indirect approach. The external loadings and the ultimate stresses are considered as random variables. The author transforms the stochastic formulation to an equivalent deterministic nonlinear programming problem by assuming that the random variables follow a normal distribution with small standard deviations, expanding the objective function about the mean values of the random variables by Taylor's series expansion, and approximating the series by the first two terms. The equivalent deterministic nonlinear programming problem is solved by SUMT (Arora, 1989) with an interior penalty function. A probability of failure of 0.0001 is assumed.

Frangopol (1985) gives two reasons why the reliability-based structural optimization has not been popular as compared with the deterministic structural optimization. First, the lack of a universally acceptable method for incorporating the uncertainties in the structural optimization formulation results in nonuniform reliability levels in similar structural design situations. Second, the diverging opinions on many basic issues include the very definition of reliability-based optimization. The author then advocates a multi-criteria optimization approach with collapse and unserviceability as the failure criteria. The method is applied to a single-story rigid steel frame with random strengths and random vertical and horizontal concentrated loads, assuming 0.000 01 and 0.01 for probabilities of collapse and unserviceability, respectively.

Soltani and Corotis (1988) present single- and multi-objective formulations with initial and failure costs as objectives functions. Design variables are the mean plastic moment capacities of structural members using the simple plastic theory for steel structures. They define the cost of failure as the replacement cost and the cost of compensation for possible damage caused by failure and note that the evaluation of this cost is extremely difficult, especially if human lives are endangered. The multi-objective optimization problem is solved by the so-called 'constraint method', where one of the objectives is treated as a constraint with lower and upper bound limits (Cohon, 1978). The approach is applied to a one-story single-bay steel rigid frame with initial cost treated as an additional constraint.

Kim and Wen (1990) present the reliability-based cost optimization of structures under multiple stochastic (time-varying) loads. The combined effects of the loads, treated as random processes, are included using the load coincidence method (Pearce and Wen, 1984). The optimization problem is solved using SUMT (Arora, 1989) with an interior penalty function. Examples of one-story, single-bay and two-story, two-bay steel frames are presented. Enevoldsen and Sorensen (1994) present the reliability-based cost optimization of structures with component and system reliability constraints, and apply the concepts to a very simple example, a simply supported tubular steel column. Chang *et al.* (1994) discuss reliability-based cost optimization of steel structures subjected to seismic loading of the Uniform Building Code (UBC, 1988) and Newmark's nondeterministic seismic response spectra (Paz, 1991) and apply it to a ten-story, single-bay steel frame assuming rigid floors. The optimization problem is solved by SUMT (Arora, 1989). They conclude that nonstructural costs as well as future failure costs can affect structural cost only at high failure probability levels. Tao *et al.* (1995) use the Markov decision process and structural reliability theory to model the minimum expected lifetime cost of a structure and apply the concepts to a composite five-girder highway bridge.

1.3.3 Fuzzy Optimization

Fuzzy optimization is based on the theory of fuzzy sets developed by Zadeh (1965). Brown and Yao (1983) introduce the application of the fuzzy set theory in structural engineering and state:

> It has been argued that probability theory and statistics are useful in civil engineering but their use is limited in the sense that most civil engineering decisions are made with a shortage of numerical evidence and depend on informed opinions. The fuzzy set theory is intended to deal with the informed opinion, but in no way disperses with countable evidence.

Reliability-based optimization is based on the long-established theory of probability while fuzzy optimization is based on the more recent theory of possibility (Zadeh, 1978) based on the theory of fuzzy sets (Zadeh, 1965). Probability is based on the premise that events or variables are random in nature with a statistical basis, but possibility is based on a fuzzy domain with mostly nonstatistical variables. In the fuzzy optimization, numerical values of the membership functions are used, as opposed to the probabilities in the reliability-based optimization. In structural design, two major sources of fuzziness, imprecision, or uncertainties can be identified, one in the evaluation of the structural behavior and resistance, the other in determining the loadings acting on the structure. In the cost optimization of structures, a third source of fuzziness and imprecision comes into play. That is in the formulation and evaluation of the cost function.

A fuzzy set Y for any set Z is characterized by a membership function $\mu_Y(z)$ which grades each point in Z with a value in the interval [0,1]. This membership function is the grade of membership of z in Y. The nearer the value of $\mu_Y(z)$ to unity the higher is the grade of membership of z in Y. Thus, a fuzzy set Y is defined as

$$Y = \{z, \mu_Y(z)\}|z \in Z \tag{1.15}$$

The fuzzy set theory can be used to model judgments on ambiguous, imprecise, or fuzzy situations. The membership functions of a fuzzy set are used to develop a fuzzy transition from total acceptance to total rejection of certain decision processes.

A number of papers have been published on fuzzy optimization of structures (Wang and Wang, 1985a, 1985b; Rao 1987a, 1987b; Yeh and Hsu, 1990; Rao *et al.*, 1992a, 1992b, 1992c; Yu and Xu, 1994; Shih and Lai, 1994). Most of these papers, however, are on weight optimization. Only a few deal with the cost optimization of structures presenting academic examples. In these papers, the cost function for the fuzzy cost optimization of structures is expressed as the summation of the initial cost (C_{I}) and the expected cost of maintenance and failure (C_{E}):

$$C_{\mathrm{T}} = C_{\mathrm{I}} + C_{\mathrm{E}} \tag{1.16}$$

This equation is somewhat similar to equation (1.6) for reliability-based optimization.

Wang and Wang (1985a) present a simplified fuzzy optimization procedure, dubbed the α-level cut method, by considering the fuzziness in the constraints and using the nonfuzzy cost function defined by equation (1.16). The membership functions for the constraints are restricted to preselected lower limit values of α. As such, the amount of fuzziness in the constraints is limited to preselected ranges. The advantage of this approach is that the problem is readily transformed to ordinary nonfuzzy optimization with expanded lower and upper bound limits, which are functions of α. The disadvantage of this approach is that the α values are selected somewhat arbitrarily. The authors apply the concepts to two small academic examples, a one-bay, two-story shear frame and a three-bar truss. Wang and Wang (1985b) discuss a two-step approach for fuzzy optimum design of aseismic structures considering both construction cost and earthquake-caused loss expectation during the service life of the structure. They introduce the concept of a fuzzy response spectrum and apply the approach to a simple one-bay, two-story shear frame. Yeh and Hsu (1990) also discuss a similar procedure for the cost optimization of structures with fuzzy allowable strength and fuzzy loads using the simple plastic theory and theory of possibility (Zadeh, 1978). They assume exponential functions for the fuzzy allowable strength and fuzzy loads, and apply the concepts to a simple three-bar truss and a one-story, one-bay frame.

1.3.4 Concluding Comments

Only a small fraction of structural steel optimization articles attempt to include any cost other than the weight of the structure. Most of the cost optimization papers are applied to small or academic examples. With the exception of a few that present moderate-size problems, all are really small-scale optimization problems.

A number of articles have been published on the reliability-based cost optimization of steel structures. Practically all present simple academic examples. Nearly three decades ago Moses (1977) acknowledged the weak database for determining statistical parameters needed in a meaningful reliability-based cost optimization. The same problem of an inadequate database exists even today and will exist in the foreseeable future. Another problem is the existence of a large number of possible failure modes, especially for large structures, which makes the evaluation of system reliability in a consistent practical way an impossible task. While the reliability theories can make a real contribution in advancing the development of more realistic design codes, their use in practical cost optimization of realistic structures appears limited at the present time.

So far only a few articles have been published on the cost optimization of steel structures using the fuzzy set theory, which deal with small academic examples. The authors of these papers appear to have been influenced by the reliability-based optimization in formulating the problem. The approach is primarily the α-level cut method, which includes the fuzziness in the constraints only. However, there are significant sources of fuzziness in the cost function as well, and the fuzzy set approach provides an effective way of modeling them.

It is interesting to note that there was a relatively good amount of research activity in the cost optimization of structures in the 1960s and 1970s. This activity dwindled in the 1980s and picked up again in the 1990s to some extent. The cost optimization problem is somewhat ill-defined in a mathematical sense, and in general its solution is less amenable to established algorithmic procedures and is computationally more intensive. With widespread availability of increasingly powerful personal computers and workstations and the development of recent computational paradigms such as the theory of fuzzy sets, structural optimization researchers need to pay closer attention to the cost optimization problem.

Research on cost optimization can encourage the use of the optimization approach in the structural steel design practice for at least two reasons. First, it provides a more realistic way of modeling structural steel design. Second, the consensus of the existing literature is that cost optimization can result in additional savings in the order of 7 % to 26 % compared to the weight optimization problem. These savings can be very significant for large structures.

For the structural optimization methodology in general, and the cost optimization approach in particular, to be embraced by the structural engineering community, the focus of research should be on large structures subjected to the actual constraints of a commonly used design code such as the AISC ASD (AISC, 1995) or the AISC LRFD (AISC, 2001) codes. The true benefit of optimization is realized for large structures with hundreds of members.

An optimization algorithm that works for a small problem, or a large problem but with simplified constraints, may not work for a large structure subjected to the actual highly nonlinear, implicit, and discontinuous constraints of an actual design code such as the AISC LRFD code (AISC, 2001). This significant issue is hardly discussed in the structural optimization literature. It should be known that nonlinear optimization algorithms are highly sensitive to the nature of the constraints and the size of the problem. An algorithm that works for explicit simplified constraints can produce unstable results for complicated implicit and discontinuous constraints. Recently, however, new promising algorithms have been created for solution of large-scale and complicated optimization problems that produce stable results consistently, such as the recently patented neural dynamics model of Adeli and Park (Adeli and Park, 1996; Park and Adeli, 1997a, 1997b; Adeli and Park, 1998) and the evolutionary computing and genetic algorithm that will be presented in subsequent chapters.