

Linear Equations 1

You have been using a prepaid cell phone for a long while and have started to realize it is getting too expensive. To check out various cell phone plans to see which one is cheapest, you search online and are immediately overwhelmed by all of the choices. Is an unlimited talk time plan really the best way to go? Do you really need texting or Web capabilities? You are on a tight budget as a student and want to make the best choice possible so money will be left over each month for other expenses. The mathematics of this chapter provides the background for making the best decision to save the most money. The Chapter Project at the end of the chapter will help you choose the most economical plan to suit your needs.

OUTLINE

- 1.1** Lines
- 1.2** Pairs of Lines
- 1.3** Applications in Business and Economics
- 1.4** Scatter Diagrams; Linear Curve Fitting
 - Chapter Review
 - Chapter Project
 - Mathematical Questions from Professional Exams

A Look Forward

In this chapter we make the connection between algebra and geometry through the rectangular coordinate system. The idea of using a system of rectangular coordinates dates back to ancient times, when such a system was used for surveying and city planning. Apollonius of Perga, in 200 B.C., used a form of rectangular coordinates in his work on conics, although this use does not stand out as clearly as it does in modern treatments. Sporadic use of rectangular coordinates continued until the 1600s. By that time, algebra had developed sufficiently so that René Descartes (1596–1650) and Pierre de Fermat (1601–1665)

could take the crucial step, which was the use of rectangular coordinates to translate geometry problems into algebra problems, and vice versa. This step was important for two reasons. First, it allowed both geometers and algebraists to gain critical new insights into their subjects, which previously had been regarded as separate but now were seen to be connected in many important ways. Second, the insights gained made possible the development of calculus, which greatly enlarged the number of areas in which mathematics could be applied and made possible a much deeper understanding of these areas.

1.1 Lines

PREPARING FOR THIS SECTION Before getting started, review the following:

- Algebra Essentials (Appendix A, Section A.2, pp. A–16 to A–20 and A–24)



NOW WORK THE “ARE YOU PREPARED” PROBLEMS ON PAGE 16

- OBJECTIVES**
- 1 Graph linear equations (p. 3)
 - 2 Graph a vertical line (p. 6)
 - 3 Find the slope of a line and interpret it (p. 7)
 - 4 Graph a line given a point on the line and the slope (p. 10)
 - 5 Use the point–slope form of a line (p. 10)
 - 6 Find the equation of a horizontal line (p. 11)
 - 7 Find the equation of a line given two points (p. 12)
 - 8 Use the slope–intercept form of a line (p. 12)
 - 9 Solve applied problems involving linear equations (p. 14)

Rectangular Coordinates

We locate a point on the real number line by assigning it a single real number, called the *coordinate of the point*. For work in a two-dimensional plane, we locate points by using two numbers.

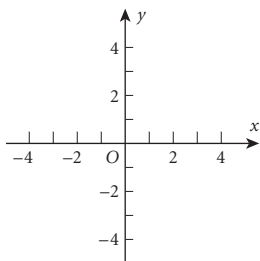
We begin with two real number lines located in the same plane: one horizontal and the other vertical. Call the horizontal line the **x -axis**, the vertical line the **y -axis**, and the point of intersection the **origin O** . Now assign coordinates to every point on these number lines as shown in Figure 1, using a convenient scale. We usually use the same scale on each axis. However, if the graph represents an application problem, scales appropriate to the application are used, which often results in different scales being used on each axis.

The origin O has a value of 0 on both the x -axis and the y -axis. We follow the usual convention that points on the x -axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Points on the y -axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. In Figure 1, the x -axis and y -axis are labeled as x and y , respectively, and we have used an arrow at the end of each axis to denote the positive direction.

The coordinate system described here is called a **rectangular** or **Cartesian*** coordinate system. The plane formed by the x -axis and y -axis is sometimes called the **xy -plane**, and the x -axis and y -axis are referred to as the **coordinate axes**.

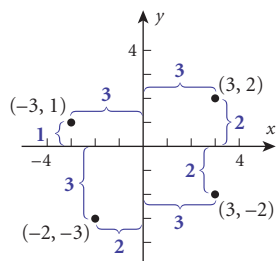
Any point P in the xy -plane can then be located by using an **ordered pair** (x, y) of real numbers. Let x denote the signed distance of P from the y -axis (*signed* in the sense that, if P is to the right of the y -axis, then $x > 0$, and if P is to the left of the y -axis, then $x < 0$), and let y denote the signed distance of P from the x -axis. The ordered pair (x, y) , also called the **coordinates** of P , then gives us enough information to locate the point P in the plane.

FIGURE 1



* Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

FIGURE 2



For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the x -axis to the left of O and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the x -axis has coordinates of the form $(x, 0)$, and any point on the y -axis has coordinates of the form $(0, y)$.

If (x, y) are the coordinates of a point P , then x is called the **x -coordinate** of P , and y is the **y -coordinate** of P . We identify the point P by its coordinates (x, y) by writing $P = (x, y)$, referring to it as “the point (x, y) ,” rather than “the point whose coordinates are (x, y) .”

The coordinate axes divide the xy -plane into four sections, called **quadrants**, as shown in Figure 3. In quadrant I, both the x -coordinate and the y -coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.

FIGURE 3

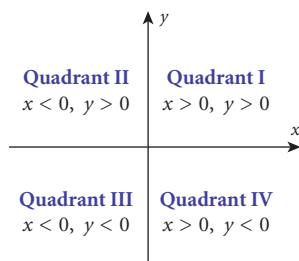
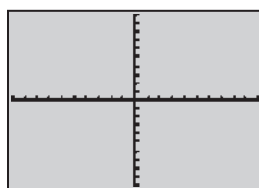
**NOW WORK PROBLEM 9.**

FIGURE 4



COMMENT On a graphing utility, you can set the scale on each axis. Once this has been done, you obtain the **viewing rectangle**. See Figure 4 for a typical viewing rectangle. You should now read Section C.1, The Viewing Rectangle, in Appendix C. ■

1 Graph Linear Equations**Definition**

A **linear equation in two variables** x and y is an equation equivalent to one of the form

$$Ax + By = C \quad (1)$$

where A, B, C are real numbers and A and B are not both zero.

Examples of linear equations are

$$3x - 5y - 6 = 0 \quad \text{This equation can be written as} \\ 3x - 5y = 6 \quad A = 3, B = -5, C = 6$$

$$-3x = 2y - 1 \quad \text{This equation can be written as} \\ -3x - 2y = -1 \quad A = -3, B = -2, C = -1 \\ \text{or as} \\ 3x + 2y = 1 \quad A = 3, B = 2, C = 1$$

$$y = \frac{3}{4}x - 5$$

Here we can write

$$-\frac{3}{4}x + y = -5 \quad A = -\frac{3}{4}, B = 1, C = -5$$

or

$$3x - 4y = 20 \quad A = 3, B = -4, C = 20$$

$$y = -5$$

Here we can write

$$0 \cdot x + y = -5 \quad A = 0, B = 1, C = -5$$

$$x = 4$$

Here we can write

$$x + 0 \cdot y = 4 \quad A = 1, B = 0, C = 4$$

The **graph** of an equation is the set of all points (x, y) whose coordinates satisfy the equation. For example, $(0, 4)$ is a point on the graph of the equation $3x + 4y = 16$, because when we substitute 0 for x and 4 for y in the equation, we get

$$3 \cdot 0 + 4 \cdot 4 = 16 \quad 3x + 4y = 16, x = 0, y = 4$$

which is a true statement.

It can be shown that if A , B , and C are real numbers, with A and B not both zero, then the graph of the equation

$$Ax + By = C$$

is a **line**. This is the reason we call it a **linear equation**.

Conversely, any line is the graph of an equation of the form $Ax + By = C$.

Since any line can be written as an equation in the form $Ax + By = C$, we call this form the **general equation** of a line.

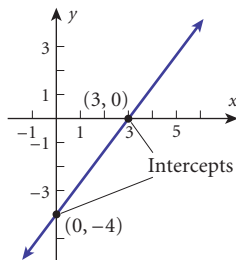
Given a linear equation, we can obtain its graph by plotting two points that satisfy its equation and connecting them with a line. Usually the easiest two points to use are the *intercepts*.

Definition

Intercepts

The points at which the graph of a linear equation crosses the axes are called **intercepts**. The ***x*-intercept** is the point at which the graph crosses the *x*-axis; the ***y*-intercept** is the point at which the graph crosses the *y*-axis.

FIGURE 5



For example, the line shown in Figure 5 has the intercepts $(0, -4)$ and $(3, 0)$.

Steps for Finding the Intercepts of a Linear Equation

To find the intercepts of a linear equation $Ax + By = C$, where $A \neq 0$ or $B \neq 0$, follow these steps:

STEP 1 Let $y = 0$ and solve for x . This determines the *x*-intercept of the line.

STEP 2 Let $x = 0$ and solve for y . This determines the *y*-intercept of the line.

EXAMPLE 1 Finding the Intercepts of a Linear Equation

Find the intercepts of the equation $2x + 3y = 6$. Graph the equation.

SOLUTION

STEP 1 To find the x -intercept, we need to find the number x for which $y = 0$. We let $y = 0$ in the equation and proceed to solve for x :

$$\begin{aligned} 2x + 3y &= 6 \\ 2x + 3(0) &= 6 & y = 0 \\ 2x &= 6 & \text{Simplify.} \\ x &= 3 & \text{Solve for } x. \end{aligned}$$

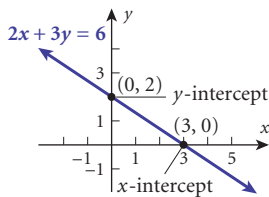
The x -intercept is $(3, 0)$.

STEP 2 To find the y -intercept, we let $x = 0$ in the equation and solve for y :

$$\begin{aligned} 2x + 3y &= 6 \\ 2(0) + 3y &= 6 & x = 0 \\ 3y &= 6 & \text{Simplify.} \\ y &= 2 & \text{Solve for } y. \end{aligned}$$

The y -intercept is $(0, 2)$.

Since the equation is a linear equation, its graph is a line. We use the two intercepts $(3, 0)$ and $(0, 2)$ to graph it. See Figure 6. ■

FIGURE 6**EXAMPLE 2** Graphing a Linear Equation

Graph the equation: $y = 2x + 5$

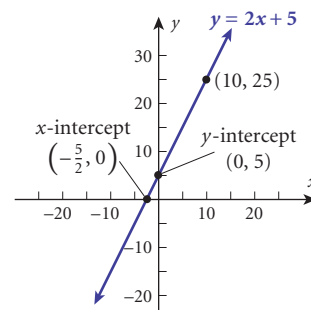
SOLUTION This equation can be written as

$$-2x + y = 5$$

This is a linear equation, so its graph is a line. The intercepts are $(0, 5)$ and $(-\frac{5}{2}, 0)$, which you should verify. To find a third point, arbitrarily let $x = 10$. Then $y = 2x + 5 = 2(10) + 5 = 25$, so $(10, 25)$ is a third point on the graph. See Figure 7.

FIGURE 7

x	y
0	5
$-\frac{5}{2}$	0
10	25



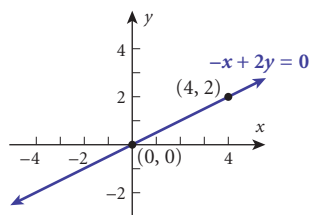
NOW WORK PROBLEM 13.

When a line passes through the origin, it has only one intercept. To graph such lines, we need to locate an additional point on the graph.

EXAMPLE 3 Graphing a Linear Equation

Graph the equation: $-x + 2y = 0$

FIGURE 8



SOLUTION

This is a linear equation, so its graph is a line. The only intercept is $(0, 0)$. To locate another point on the graph, let $x = 4$. (This choice is arbitrary; any choice of x other than 0 could also be used.) Then

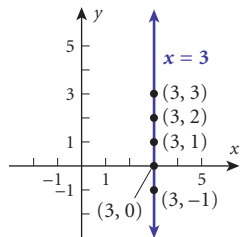
$$\begin{aligned} -4 + 2y &= 0 & -x + 2y = 0, x = 4 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

So, $y = 2$ when $x = 4$ and $(4, 2)$ is a point on the graph. See Figure 8. ■

2 Graph a Vertical Line

EXAMPLE 4 Graphing a Vertical Line

FIGURE 9



Graph the equation: $x = 3$

SOLUTION This is a linear equation ($1 \cdot x + 0 \cdot y = 3$), so its graph is a line. Since $x = 3$, no matter what y -coordinate is used, the corresponding x -coordinate always equals 3. Consequently, the graph of the equation $x = 3$ is a *vertical* line with x -intercept $(3, 0)$ as shown in Figure 9. ■

As suggested by Example 4, we have the following result:

Theorem

Equation of a Vertical Line

A vertical line is given by an equation of the form

$$x = a$$

where $(a, 0)$ is the x -intercept.

EXAMPLE 5 Find

Find an equation for the vertical line containing the point $(-1, 6)$.

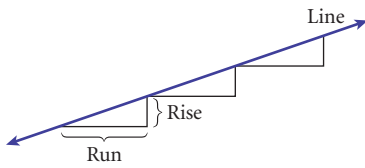
SOLUTION

The x -coordinate of any point on a vertical line is always the same. Since $(-1, 6)$ is a point on the vertical line, its equation is $x = -1$. ■



NOW WORK PROBLEM 17(a).

FIGURE 10



3 Find the Slope of a Line and Interpret It

Consider the staircase illustrated in Figure 10. Each step contains exactly the same horizontal **run** and the same vertical **rise**. The ratio of the rise to the run, called the **slope**, is a numerical measure of the steepness of the staircase. For example, if the run is increased and the rise remains the same, the staircase becomes less steep. If the run is kept the same, but the rise is increased, the staircase becomes more steep. This important characteristic of a line is best defined using rectangular coordinates.

Definition

Slope of a Line

Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope** m of the nonvertical line containing P and Q is defined by the formula

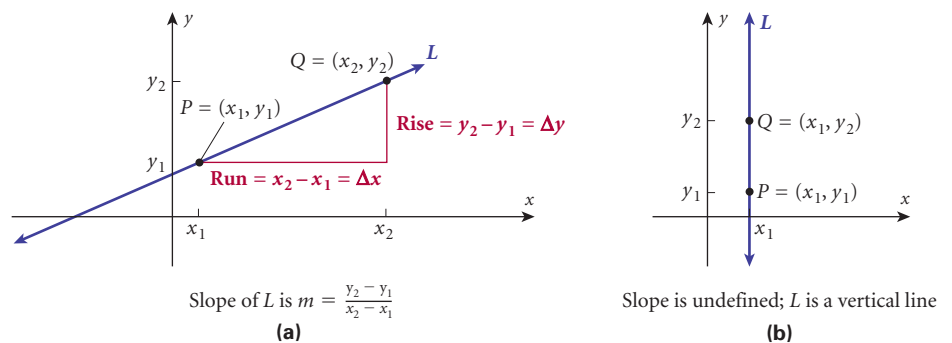
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad x_1 \neq x_2 \quad (2)$$

If $x_1 = x_2$, the slope m is undefined (since $x_1 = x_2$ results in division by 0) and the line is **vertical**.

If $y_1 = y_2$, the slope m is 0 and the line is **horizontal**.

Figure 11(a) provides an illustration of the slope of a nonvertical line; Figure 11(b) illustrates a vertical line.

FIGURE 11



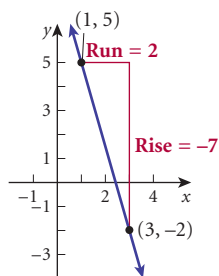
As Figure 11(a) illustrates, the slope m of a nonvertical line may be given as

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Change in } y}{\text{Change in } x}$$

The change in y is usually denoted by Δy , read “delta y ,” and the change in x is denoted by Δx .

The slope m of a nonvertical line measures the amount y changes, Δy , as x changes from x_1 to x_2 , Δx . This is called the **average rate of change of y with respect to x** . Then the slope m is

$$m = \frac{\Delta y}{\Delta x} = \text{Average rate of change of } y \text{ with respect to } x$$

EXAMPLE 6 Finding the Slope of a Line and Interpreting It**FIGURE 12**

The slope m of the line containing the points $(1, 5)$ and $(3, -2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{5 - (-2)}{1 - 3} = \frac{7}{-2} = \frac{-7}{2} = -\frac{7}{2}$$

We interpret the slope to mean that for every 2-unit change in x , then y will change by -7 units. That is, if x increases by 2 units, then y decreases by 7 units. The average rate of change of y with respect to x is $-\frac{7}{2}$. See Figure 12. ■



NOW WORK PROBLEMS 23 AND 27.

Two comments about computing the slope of a nonvertical line may prove helpful:

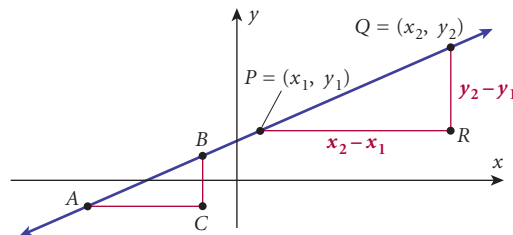
1. Any two distinct points on the line can be used to compute the slope of the line. (See Figure 13 for justification.)
2. The slope of a line may be computed from $P = (x_1, y_1)$ to $Q = (x_2, y_2)$ or from Q to P because

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

FIGURE 13 Triangles ABC and PQR are similar (they have equal angles). So ratios of corresponding sides are proportional. Then

$$\text{Slope using } P \text{ and } Q = \frac{y_2 - y_1}{x_2 - x_1} = \frac{d(B, C)}{d(A, C)} = \text{Slope using } A \text{ and } B$$

where $d(B, C)$ denotes the distance from B to C and $d(A, C)$ denotes the distance from A to C .

**EXAMPLE 7** Finding the Slopes of Various Lines Containing the Same Point P

Compute the slopes of the lines L_1, L_2, L_3 , and L_4 containing the following pairs of points. Graph all four lines on the same set of coordinate axes.

$$L_1: P = (2, 3) \quad Q_1 = (-1, -2)$$

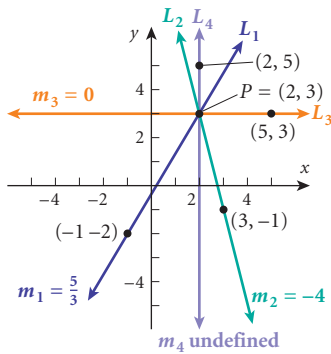
$$L_2: P = (2, 3) \quad Q_2 = (3, -1)$$

$$L_3: P = (2, 3) \quad Q_3 = (5, 3)$$

$$L_4: P = (2, 3) \quad Q_4 = (2, 5)$$

SOLUTION Let m_1, m_2, m_3 , and m_4 denote the slopes of the lines L_1, L_2, L_3 , and L_4 , respectively.

FIGURE 14



Then

$$m_1 = \frac{-2 - 3}{-1 - 2} = \frac{-5}{-3} = \frac{5}{3} \quad \text{A rise of 5 divided by a run of 3}$$

$$m_2 = \frac{-1 - 3}{3 - 2} = \frac{-4}{1} = -4 \quad \text{A rise of } -4 \text{ divided by a run of 1}$$

$$m_3 = \frac{3 - 3}{5 - 2} = \frac{0}{3} = 0 \quad \text{A rise of 0 divided by a run of 3}$$

m_4 is undefined The x-coordinates of P and Q_4 are equal (both = 2).

The graphs of these lines are given in Figure 14. ■

As Figure 14 illustrates,

1. When the slope m of a line is positive, the line slants upward from left to right (L_1).
2. When the slope m is negative, the line slants downward from left to right (L_2).
3. When the slope m is 0, the line is horizontal (L_3).
4. When the slope m is undefined, the line is vertical (L_4).



COMMENT Now read Section C.3, Square Screens, in Appendix C. ■



SEEING THE CONCEPT On the same square screen, graph the following equations:

$$Y_1 = 0 \quad \text{Slope of line is 0.}$$

$$Y_2 = \frac{1}{4}x \quad \text{Slope of line is } \frac{1}{4}.$$

$$Y_3 = \frac{1}{2}x \quad \text{Slope of line is } \frac{1}{2}.$$

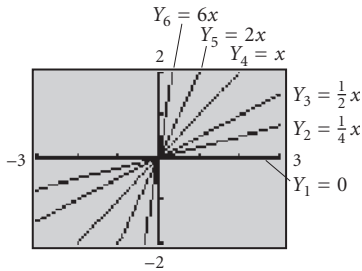
$$Y_4 = x \quad \text{Slope of line is 1.}$$

$$Y_5 = 2x \quad \text{Slope of line is 2.}$$

$$Y_6 = 6x \quad \text{Slope of line is 6.}$$

See Figure 15. ■

FIGURE 15



SEEING THE CONCEPT On the same square screen, graph the following equations:

$$Y_1 = 0 \quad \text{Slope of line is 0.}$$

$$Y_2 = -\frac{1}{4}x \quad \text{Slope of line is } -\frac{1}{4}.$$

$$Y_3 = -\frac{1}{2}x \quad \text{Slope of line is } -\frac{1}{2}.$$

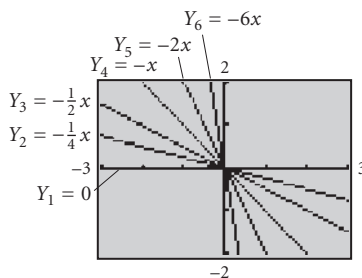
$$Y_4 = -x \quad \text{Slope of line is } -1.$$

$$Y_5 = -2x \quad \text{Slope of line is } -2.$$

$$Y_6 = -6x \quad \text{Slope of line is } -6.$$

See Figure 16. ■

FIGURE 16



Figures 15 and 16 illustrate that the closer the line is to the vertical position, the greater the magnitude of the slope.

The next example illustrates how the slope of a line can be used to graph the line.

4 Graph a Line Given a Point on the Line and the Slope

EXAMPLE 8 Graphing a Line When Its Slope and a Point Are Given

Draw a graph of the line that contains the point $(3, 2)$ and has a slope of

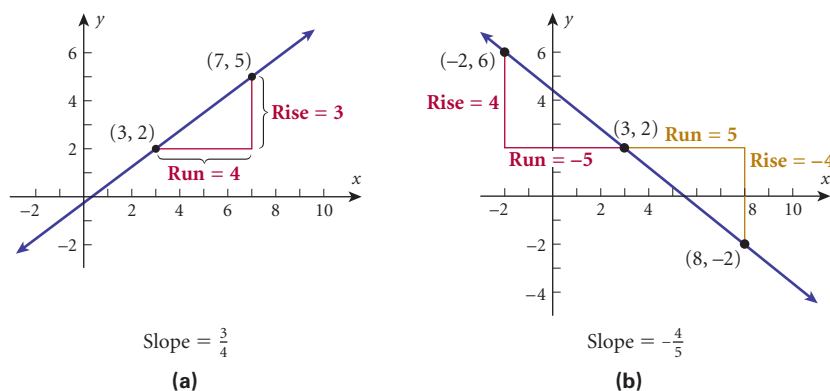
(a) $\frac{3}{4}$ (b) $-\frac{4}{5}$

SOLUTION (a) Slope = $\frac{\text{rise}}{\text{run}}$. The fact that the slope is $\frac{3}{4}$ means that for every horizontal movement (run) of 4 units to the right, there will be a vertical movement (rise) of 3 units. If we start at the given point $(3, 2)$ and move 4 units to the right and 3 units up, we reach the point $(7, 5)$. By drawing the line through this point and the point $(3, 2)$, we have the graph. See Figure 17(a).

(b) The fact that the slope is $-\frac{4}{5} = \frac{-4}{5}$ means that for every horizontal movement of 5 units to the right, there will be a corresponding vertical movement of -4 units (a downward movement). If we start at the given point $(3, 2)$ and move 5 units to the right and then 4 units down, we arrive at the point $(8, -2)$. By drawing the line through these points, we have the graph. See Figure 17(b).

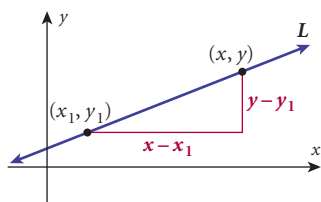
Alternatively, we can set $-\frac{4}{5} = \frac{4}{-5}$ so that for every horizontal movement of -5 units (a movement to the left), there will be a corresponding vertical movement of 4 units (upward). This approach brings us to the point $(-2, 6)$, which is also on the graph shown in Figure 17(b).

FIGURE 17



NOW WORK PROBLEM 35.

FIGURE 18



5 Use the Point–Slope Form of a Line

Let L be a nonvertical line with slope m and containing the point (x_1, y_1) . See Figure 18. Since any two distinct points on L can be used to compute slope, for any other point (x, y) on L , we have

$$m = \frac{y - y_1}{x - x_1} \quad \text{or} \quad y - y_1 = m(x - x_1)$$

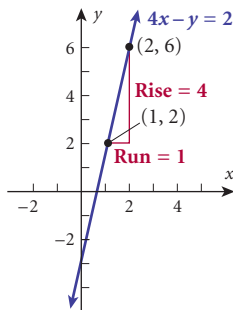
Theorem ▶ Point–Slope Form of an Equation of a Line

An equation of a nonvertical line with slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad (3)$$

EXAMPLE 9 Using the Point–Slope Form of a Line

FIGURE 19



An equation of the line with slope 4 and containing the point $(1, 2)$ can be found by using the point–slope form with $m = 4$, $x_1 = 1$, and $y_1 = 2$:

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-slope form} \\ y - 2 &= 4(x - 1) && m = 4, x_1 = 1, y_1 = 2 \\ y - 2 &= 4x - 4 && \text{Simplify.} \\ 4x - y &= 2 && \text{General equation} \end{aligned}$$

See Figure 19.



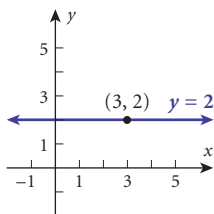
NOW WORK PROBLEMS 17(c) AND 47.

6 Find the Equation of a Horizontal Line

EXAMPLE 10 Finding the Equation of a Horizontal Line

Find an equation of the horizontal line containing the point $(3, 2)$. Graph the line.

FIGURE 20



SOLUTION

The slope of a horizontal line is 0. To get an equation, we use the point–slope form with $m = 0$, $x_1 = 3$, and $y_1 = 2$:

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-slope form} \\ y - 2 &= 0 \cdot (x - 3) && m = 0, x_1 = 3, y_1 = 2 \\ y - 2 &= 0 && \\ y &= 2 && \end{aligned}$$

See Figure 20 for the graph.

As suggested by Example 10, we have the following result:

Theorem ▶ Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b$$

where $(0, b)$ is the y -intercept.



NOW WORK PROBLEM 17(b).

7 Find the Equation of a Line Given Two Points

EXAMPLE 11 Finding an Equation of a Line Given Two Points

Find an equation of the line containing the points $(2, 3)$ and $(-4, 5)$. Graph the line.

SOLUTION Since two points are given, we first compute the slope of the line:

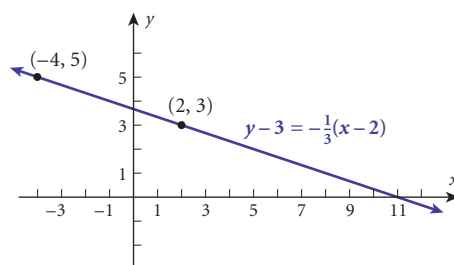
$$m = \frac{5 - 3}{-4 - 2} = \frac{2}{-6} = \frac{1}{-3} = -\frac{1}{3}$$

We use the point $(2, 3)$ and the fact that the slope $m = -\frac{1}{3}$ to get the point–slope form of the equation of the line:

$$y - 3 = -\frac{1}{3}(x - 2)$$

See Figure 21 for the graph.

FIGURE 21



In the solution to Example 11 we could have used the point $(-4, 5)$ instead of the point $(2, 3)$. The equation that results, although it looks different, is equivalent to the equation we obtained in the example. (Try it for yourself.)

The general form of the equation of the line in Example 11 can be obtained by multiplying both sides of the point–slope equation by 3 and collecting terms:

$$y - 3 = -\frac{1}{3}(x - 2) \quad \text{Point–slope equation}$$

$$3(y - 3) = 3\left(-\frac{1}{3}\right)(x - 2) \quad \text{Multiply by 3.}$$

$$3y - 9 = -1(x - 2) \quad \text{Simplify.}$$

$$3y - 9 = -x + 2 \quad \text{Simplify.}$$

$$x + 3y = 11 \quad \text{General equation}$$

This is the general form of the equation of the line.



NOW WORK PROBLEM 51.

8 Use the Slope–Intercept Form of a Line

Another useful equation of a line is obtained when the slope m and y -intercept $(0, b)$ are known. In this case we know both the slope m of the line and a point $(0, b)$ on the line. Then we can use the point–slope form, Equation (3), to obtain the following equation:

$$\begin{aligned}
 y - y_1 &= m(x - x_1) && \text{Point-slope form} \\
 y - b &= m(x - 0) && x_1 = 0, y_1 = b \\
 y &= mx + b && \text{Simplify and solve for } y.
 \end{aligned}$$

Theorem **Slope–Intercept Form of an Equation of a Line**

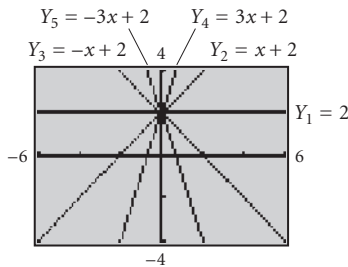
An equation of a line with slope m and y -intercept $(0, b)$ is

$$y = mx + b \tag{4}$$



SEEING THE CONCEPT To see the role that the slope m plays in the equation $y = mx + b$, graph the following lines on the same square screen.

FIGURE 22



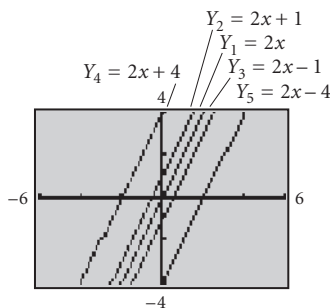
- $Y_1 = 2$
- $Y_2 = x + 2$
- $Y_3 = -x + 2$
- $Y_4 = 3x + 2$
- $Y_5 = -3x + 2$

See Figure 22. What do you conclude about the lines $y = mx + 2$? ■



SEEING THE CONCEPT To see the role of b in the equation $y = mx + b$, graph the following lines on the same square screen.

FIGURE 23



- $Y_1 = 2x$
- $Y_2 = 2x + 1$
- $Y_3 = 2x - 1$
- $Y_4 = 2x + 4$
- $Y_5 = 2x - 4$

See Figure 23. What do you conclude about the lines $y = 2x + b$? ■

When an equation of a line is written in slope–intercept form, it is easy to find the slope m and y -intercept $(0, b)$ of the line. For example, suppose the equation of the line is

$$y = -2x + 3$$

Compare it to $y = mx + b$:

$$\begin{array}{ccc}
 y = & -2x & + & 3 \\
 & \uparrow & & \uparrow \\
 y = & mx & + & b
 \end{array}$$

The slope of this line is -2 and its y -intercept is $(0, 3)$.

EXAMPLE 12 Finding the Slope and y -Intercept of a Line

Find the slope m and y -intercept $(0, b)$ of the line $2x + 4y = 8$. Graph the line.

SOLUTION To obtain the slope and y -intercept, we transform the equation into its slope–intercept form. To do this, we need to solve for y :

$$\begin{aligned} 2x + 4y &= 8 \\ 4y &= -2x + 8 \\ y &= -\frac{1}{2}x + 2 \end{aligned}$$

The coefficient of x , $-\frac{1}{2}$, is the slope, and the y -intercept is $(0, 2)$.

We can graph the line in either of two ways:

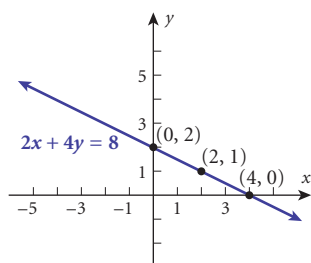
1. Use the fact that the y -intercept is $(0, 2)$ and the slope is $-\frac{1}{2}$. Then, starting at the point $(0, 2)$, go to the right 2 units and then down 1 unit to the point $(2, 1)$. Plot these points and draw the line containing them. See Figure 24.
2. Locate the intercepts. The y -intercept is $(0, 2)$. To obtain the x -intercept, we let $y = 0$ in the equation $2x + 4y = 8$ and solve for x . When $y = 0$, we have

$$\begin{aligned} 2x + 4 \cdot 0 &= 8 & 2x + 4y &= 8; y = 0 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

The intercepts are $(4, 0)$ and $(0, 2)$. Plot these points and draw the line containing them. Look again at Figure 24. ■

NOTE The second method, locating the intercepts, only produces one point when the line passes through the origin. In this case some other point on the line must be found in order to graph the line. Refer back to Example 3. ■

FIGURE 24



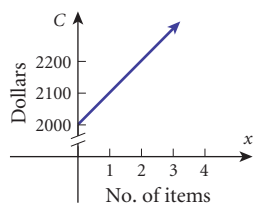
NOW WORK PROBLEM 69.

9 Solve Applied Problems Involving Linear Equations

EXAMPLE 13 Daily Cost of Production

A factory that manufactures microwave ovens has daily fixed overhead expenses of \$2000. Each microwave oven produced costs \$100. Find an equation that relates the daily cost C to the number x of microwaves produced each day.

FIGURE 25



SOLUTION The fixed overhead expense of \$2000 represents the fixed cost, the cost incurred no matter how many microwaves are produced. Since each microwave produced costs \$100, the variable cost of producing x microwaves is $100x$. Then the total daily cost C of production is

$$C = 100x + 2000$$

The graph of this equation is given by the line in Figure 25. Notice that the fixed cost \$2000 is represented by the y -intercept, while the \$100 cost of producing each microwave is the slope. Also notice that a different scale is used on each axis. ■

EXAMPLE 14 Predicting the Cost of a Home

In 2008 the cost of an average home in Chicago was \$251,364. In 2009 the cost was \$222,960.

- (a) Assuming that the relationship between time and cost is linear, develop a formula for predicting the cost of an average home in 2010.
 (b) Comment on whether you think this trend will continue.

Source: Sperling's Best Places; www.bestplaces.net

SOLUTION (a) We let x represent the year and y represent the cost. We seek a relationship between x and y . The assumption is that the equation relating x and y is linear. Two points on the graph of the linear equation relating x and y are

$$(2008, 251,364) \text{ and } (2009, 222,960)$$

The slope of this line is

$$\frac{222,960 - 251,364}{2009 - 2008} = -28,404$$

Using this fact and the point $(2008, 251,364)$, the point-slope form of the equation of the line is

$$\begin{aligned} y - 251,364 &= -28,404(x - 2008) & y - y_1 &= m(x - x_1) \\ y &= 251,364 - 28,404(x - 2008) \end{aligned}$$

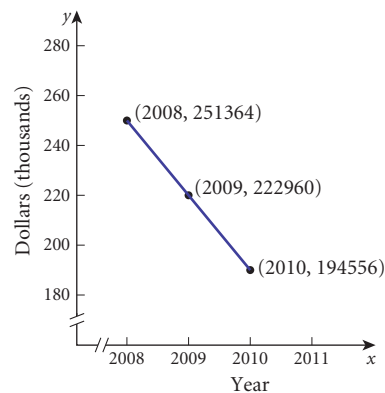
For $x = 2010$ we predict the cost of an average home to be

$$\begin{aligned} y &= 251,364 - 28,404(x - 2008) \\ &= 251,364 - 28,404(2010 - 2008) & x &= 2010 \\ &= 251,364 - 56,808 \\ &= 194,556 \end{aligned}$$

We predict the average cost of a home in Chicago in 2010 to be \$194,556.

- (b) This prediction of future cost is based on the assumption that annual changes in price remain the same. In this example, the assumption is that each year the cost of a house will go down \$28,404 (the slope of the line). If this assumption is not correct, the predicted cost may be incorrect. Also, it is not reasonable to expect home prices to decline for an extended length of time.

FIGURE 26



NOW WORK PROBLEM 113.

SUMMARY The graph of a linear equation, $Ax + By = C$, where A and B are not both zero, is a line. In this form it is referred to as the general equation of a line.

- Given the general equation of a line, information can be found about the line:
 - Place the equation in slope–intercept form $y = mx + b$ to find the slope m and y -intercept $(0, b)$.
 - Let $x = 0$ and solve for y to find the y -intercept.
 - Let $y = 0$ and solve for x to find the x -intercept.
- Given information about a line, an equation of the line can be found. The form of the equation to use depends on the given information. See the table below.

Given	Use	Equation
Point (x_1, y_1) , slope m	Point–slope form	$y - y_1 = m(x - x_1)$
Two points $(x_1, y_1), (x_2, y_2)$	If $x_1 = x_2$, the line is vertical If $x_1 \neq x_2$, find the slope m : $m = \frac{y_2 - y_1}{x_2 - x_1}$ Then use the point–slope form	$x = x_1$ $y - y_1 = m(x - x_1)$
Slope m , y -intercept $(0, b)$	Slope–intercept form	$y = mx + b$

EXERCISE 1.1 Answers Begin on Page AN–1.

'Are You Prepared?' Problems Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

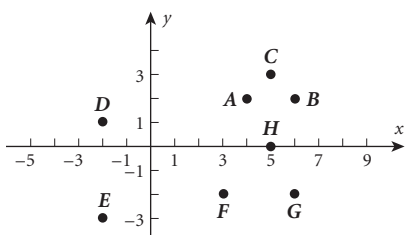
- True or False** On the real number line, the coordinate of the origin is the number 0. (pp. A–16 to A–20)
- Solve the equation: $2x + 6 = 10$ (p. A–24)

Concepts and Vocabulary

- True or False** To find the y -intercept of a linear equation, let $x = 0$ and solve for y .
- The equation of a vertical line with x -intercept at $(-3, 0)$ is _____.
- If the slope of a line is undefined, the line is _____.
- The line $y = -4x + 6$ has slope _____ and y -intercept _____.
- The point–slope form of the equation of a line with slope m containing the point (x_1, y_1) is _____.
- If the graph of a line slants downward from left to right, its slope m is (positive, negative, zero).

Skill Building

- Give the coordinates of each point in the following figure. Assume each coordinate is an integer.
- Plot each point in the xy -plane. Tell in which quadrant or on what coordinate axis each point lies.



- $A = (-3, 2)$
- $B = (6, 0)$
- $C = (-2, -2)$
- $D = (6, 5)$
- $E = (0, -3)$
- $F = (6, -3)$

11. Plot the points $(2, 0)$, $(2, -3)$, $(2, 4)$, $(2, 1)$, and $(2, -1)$. Describe the collection of all points of the form $(2, y)$, where y is a real number.

12. Plot the points $(0, 3)$, $(1, 3)$, $(-2, 3)$, $(5, 3)$, and $(-4, 3)$. Describe the collection of all points of the form $(x, 3)$, where x is a real number.

In Problems 13–16, use the given equation to fill in the missing values in each table. Use these points to graph the equation.

13. $y = 2x + 4$

x	0		2	-2	4	-4
y		0				

14. $y = -3x + 6$

x	0		2	-2	4	-4
y		0				

15. $2x - y = 6$

x	0		2	-2	4	-4
y		0				

16. $x + 2y = 8$

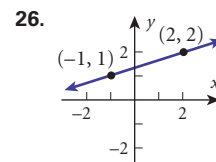
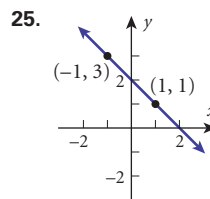
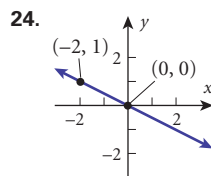
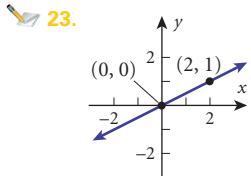
x	0		2	-2	4	-4
y		0				

In Problems 17–22 a point is given.

- (a) Find the equation of the vertical line containing the given point.
 (b) Find the equation of the horizontal line containing the given point.
 (c) Find the general equation of a line with slope 5 containing the given point.

17. $(2, -3)$ 18. $(5, 4)$ 19. $(-4, 1)$ 20. $(-6, -3)$ 21. $(0, 3)$ 22. $(-6, 0)$

In Problems 23–26, find the slope of the line. Give an interpretation of the slope.



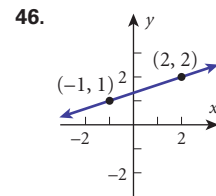
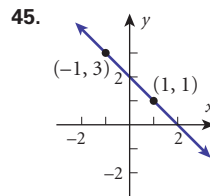
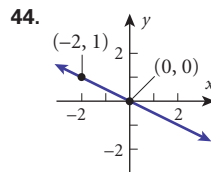
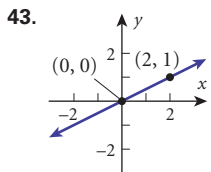
In Problems 27–34, plot each pair of points and find the slope of the line containing them. Interpret the slope and graph the line.

27. $(2, 3); (1, 0)$ 28. $(1, 2); (3, 4)$ 29. $(-2, 3); (2, 1)$ 30. $(-1, 1); (2, 3)$
 31. $(-3, -1); (2, -1)$ 32. $(4, 2); (-5, 2)$ 33. $(-1, 2); (-1, -2)$ 34. $(2, 0); (2, 2)$

In Problems 35–42, graph the line containing the point P and having slope m .

35. $P = (1, 2); m = 2$ 36. $P = (2, 1); m = 3$ 37. $P = (2, 4); m = -\frac{3}{4}$ 38. $P = (1, 3); m = -\frac{2}{3}$
 39. $P = (-1, 3); m = 0$ 40. $P = (2, -4); m = 0$ 41. $P = (0, 3);$ slope undefined 42. $P = (-2, 0);$ slope undefined

In Problems 43–64, find the general equation of each line; that is, write the equation in the form $Ax + By = C$.



47. Slope = 2; containing the point $(-4, 1)$
 48. Slope = 3; containing the point $(-3, 4)$
 49. Slope = $-\frac{2}{3}$; containing the point $(1, -1)$
 50. Slope = $\frac{1}{2}$; containing the point $(3, 1)$
 51. Containing the points $(1, 3)$ and $(-1, 2)$
 52. Containing the points $(-3, 4)$ and $(2, 5)$

18 Chapter 1 Linear Equations

53. Slope = -2 ; y -intercept = $(0, 3)$

55. Slope = 3 ; x -intercept = $(-4, 0)$

57. Slope = $\frac{4}{5}$; containing the point $(0, 0)$

59. x -intercept = $(2, 0)$; y -intercept = $(0, -1)$

61. Slope undefined; containing the point $(1, 4)$

63. Slope = 0 ; containing the point $(1, 4)$

54. Slope = -3 ; y -intercept = $(0, -2)$

56. Slope = -4 ; x -intercept = $(2, 0)$

58. Slope = $\frac{7}{3}$; containing the point $(0, 0)$

60. x -intercept = $(-4, 0)$; y -intercept = $(0, 4)$

62. Slope undefined; containing the point $(2, 1)$

64. Slope = 0 ; containing the point $(2, 1)$


In Problems 65–80, find the slope and y -intercept of each line. Graph the line.

65. $y = 2x + 3$

66. $y = -3x + 4$

67. $\frac{1}{2}y = x - 1$

68. $\frac{1}{3}x + y = 2$

 69. $2x - 3y = 6$

70. $3x + 2y = 6$

71. $x + y = 1$

72. $x - y = 2$

73. $x = -4$

74. $y = -1$

75. $y = 5$


76. $x = 2$

77. $y - x = 0$

78. $x + y = 0$

79. $2y - 3x = 0$

80. $3x + 2y = 0$

 In Problems 81–88, use a graphing utility to graph each linear equation. Be sure to use a viewing rectangle that shows the intercepts. Then locate each intercept rounded to two decimal places.

81. $1.2x + 0.8y = 2$

82. $-1.3x + 2.7y = 8$

83. $21x - 15y = 53$


84. $5x - 3y = 82$

85. $\frac{4}{17}x + \frac{6}{23}y = \frac{2}{3}$

86. $\frac{9}{14}x - \frac{3}{8}y = \frac{2}{7}$

87. $\pi x - \sqrt{3}y = \sqrt{6}$

88. $x + \pi y = \sqrt{15}$

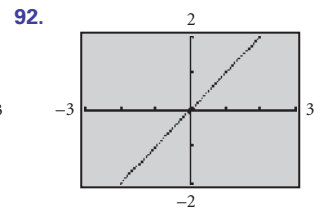
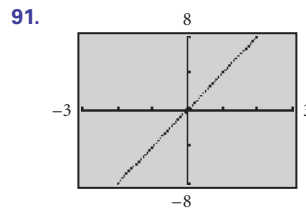
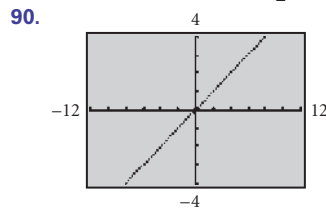
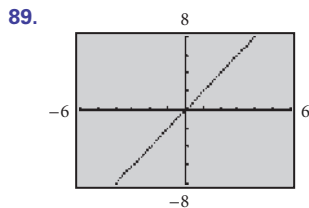
 In Problems 89–92, match each graph with the correct equation:


(a) $y = x$

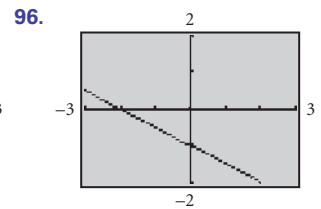
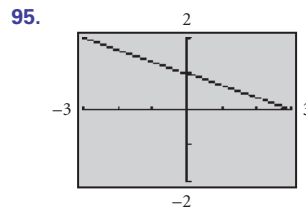
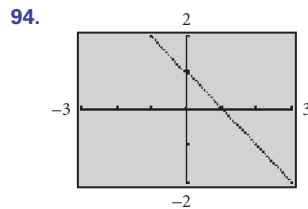
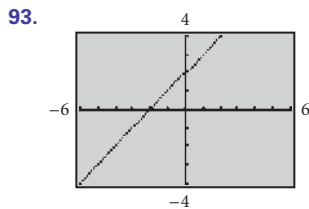
(b) $y = 2x$

(c) $y = \frac{x}{2}$

(d) $y = 4x$



 In Problems 93–96, write an equation of each line. Express your answer using either the general form or the slope-intercept form of the equation of a line, whichever you prefer.



Applications

97. **Cost of Operating a Car** According to the American Automobile Association (AAA), the average annual cost of operating a standard-sized car, including gasoline, oil, tires, and maintenance decreased to \$0.54 per mile in 2009.

- (a) Find a linear equation that relates the average cost C of operating a standard-sized car and the number x of miles it is driven.

(b) What is the annual cost if you drive 15,000 miles?

(c) Graph the linear equation for $0 \leq x \leq 50,000$.

(d) Write a practical sentence expressing how C changes for every unit increase in x .

Source: AAA.com

- 98. Cost of Renting a Truck** In January 2010, the cost of renting a truck in Naples, FL, was \$224 per week plus a charge of \$0.52 per mile driven.
- Find a linear equation that relates the cost C for a weekly rental in which the truck is driven x miles.
 - What is the rental cost if you drive 500 miles in a week?
 - Graph the linear equation for $0 \leq x \leq 2000$.
 - Write a practical sentence expressing how C changes for every unit increase in x .
 - Write a practical sentence about the y -intercept.
- Source:** Budget, Tamiami Trail, Naples
- 99. Electricity Rates in Illinois** Commonwealth Edison Company supplies electricity to residential customers for a monthly customer charge of \$8.23 plus 10.438 cents per kilowatt-hour for up to 400 kilowatt-hours.
- Find a linear equation that relates the monthly charge C , in dollars, to the number x of kilowatt-hours used in a month, $0 \leq x \leq 400$.
 - Graph this equation.
 - What is the monthly charge for using 100 kilowatt-hours?
 - What is the monthly charge for using 300 kilowatt-hours?
 - Write a practical sentence expressing how C changes for every unit change in x .
- Source:** Commonwealth Edison Company, January 2010.
- 100. Electricity Rates in Florida** Florida Power & Light Company supplies electricity to residential customers for a monthly customer charge of \$5.69 plus 8.735 cents per kilowatt-hour for up to 1000 kilowatt-hours.
- Find a linear equation that relates the monthly charge C , in dollars, to the number x of kilowatt-hours used in a month, $0 \leq x \leq 1000$.
 - Graph this equation.
 - What is the monthly charge for using 200 kilowatt-hours?
 - What is the monthly charge for using 500 kilowatt-hours?
 - Write a practical sentence expressing how C changes for every unit change in x .
- Source:** Florida Power & Light Company, January 2010.
- 101. Temperature Conversion** The relationship between Celsius ($^{\circ}\text{C}$) and Fahrenheit ($^{\circ}\text{F}$) degrees for measuring temperature is linear.
- Find a linear equation relating $^{\circ}\text{C}$ and $^{\circ}\text{F}$ if 0°C corresponds to 32°F and 100°C corresponds to 212°F .
 - Use the equation to find the Celsius measure of 68°F .
- 102. Temperature Conversion** The Kelvin (K) scale for measuring temperature is obtained by adding 273 to the Celsius temperature.
- Find a linear equation relating K and $^{\circ}\text{C}$.
 - Find a linear equation relating K and $^{\circ}\text{F}$ (see Problem 101).
- 103. Water Preservation** At Harlan County Dam in Nebraska, the U.S. Bureau of Reclamation reports that the storage content of the reservoir increased from 315,000 acre-feet (102.7 billion gallons of water) on December 21, 2009 to 319,300 acre-feet (104.1 billion gallons of water) on January 20, 2010. Suppose that the rate of increase of water remains constant.
- Find a linear equation that relates the amount A of water, in billions of gallons, to the time t , in days. Use $t = 0$ for December 21, $t = 1$ for December 22, and so on.
 - How much water was in the reservoir on December 31 ($t = 10$)?
 - Interpret the slope.
 - How much water is predicted to be in the reservoir on January 31, 2010 ($t = 41$)?
 - The capacity of the dam is 814,111 acre-feet (265.5 billion gallons of water). If the outflow of water is not controlled, when will the reservoir overflow and cause flooding?
 - Comment on your answer to part (e).
- Source:** U.S. Bureau of Reclamation.
- 104. Product Promotion** A cereal company finds that the number of people who will buy one of its products the first month it is introduced is linearly related to the amount of money it spends on advertising. If it spends \$4000 on advertising, 100,000 boxes of cereal will be sold, and if it spends \$6000, then 300,000 boxes will be sold.
- Find a linear equation describing the relation between the amount A spent on advertising and the number N of boxes sold.
 - How much advertising is needed to sell 200,000 boxes of cereal?
 - Write a practical sentence expressing how A changes for each unit increase in N .
- 105. Predicting Sales** Suppose the sales of a company are given by
- $$S = \$5000x + \$80,000$$
- where x is measured in years and $x = 0$ corresponds to the year 2006.
- Find S when $x = 0$.
 - Find S when $x = 3$.
 - Find the predicted sales in 2012, assuming this trend continues.
 - Find the predicted sales in 2015, assuming this trend continues.
- 106. Disease Propagation** Research indicates that in a controlled environment, the number of diseased mice will increase linearly each day after one of the mice in the cage is infected with a particular type of disease-causing germ. There were 8 diseased mice 4 days after the first exposure and 14 diseased mice after 6 days. Write a linear equation that will give the number of diseased mice after any given number of days. If there were 40 mice in the cage, how long will it take until they are all infected?
- 107. Wages of a Car Salesperson** In 2008, median earnings, including commissions, for car salespersons was \$21.43 per hour or \$857.31 per week. Dan receives \$400 per week for selling new and used cars. In addition, he receives 5% of the profit on any sales he generates.
- Find a linear equation that relates Dan's weekly salary S when he has sales that generate a profit of x dollars.
 - If Dan has sales that generate a profit of \$4000, what are his weekly earnings?

- (c) To equal the median earnings of a car salesperson, Dan would have to have sales that generate a profit of how many dollars?

Source: Bureau of Labor Statistics.

108. Oil Depletion The Alaskan oil fields, in operation since 1977, had an estimated reserve of 4.9 billion barrels in 1999. In 2009, the fields had an estimated reserve of 3.5 billion barrels. Assume the rate of depletion is constant.

- (a) Find a linear equation that relates the amount A , in millions of barrels, of oil left in the fields at any time t , where t is the year.
 (b) If the trend continues, when will the fields dry out?
 (c) Write a practical sentence expressing how A changes for every unit change in t .
 (d) The Jack Field, discovered in the Gulf of Mexico off the coast of Louisiana in 2006, is estimated to contain up to 15 billion barrels of oil. At the same rate of depletion, how long will this field last?

Source: Energy Information Administration

109. SAT Scores The average score on the mathematics portion of the SAT has been increasing over the past 10 years. In 1999 the average SAT mathematics score was 475, while in 2009 the average SAT mathematics score was 496. Assume the rate of increase is constant.

- (a) Find a linear equation that relates the average SAT mathematics score S at any time t , where t is the year.
 (b) If the trend continues, what will the average SAT mathematics score be in 2011?

Source: The College Board

110. Financial Statement In November 2005, SBC Communications acquired AT&T through a merger and formally adopted the AT&T name. After the merger, AT&T's net income increased from \$1.45 billion at the end of the first financial quarter in 2006 to \$1.81 billion at the end of the second financial quarter in 2006. Assume the rate of increase is constant.

- (a) Find a linear equation that relates the net income I , in billions of dollars, and the time t . Use $t = 1$ for the first quarter of 2006, $t = 2$ for the second quarter of 2006, and so on.
 (b) What is the projected net income for the fourth quarter?
 (c) Write a practical sentence expressing how I changes for every unit change in t .

111. Percent of Population with Bachelor's Degrees In 1998 the percent of people over 25 years old who had a bachelor's degree or higher was 24.4%. By 2008, the percent of people over 25 with a bachelor's degree or higher was 29.4%. Assume the rate of increase is constant.

- (a) Find a linear equation that relates the percent P of people over 25 with a bachelor's degree or higher at any time t , where t is the year.
 (b) If the trend continues, estimate the percentage of people over 25 who will have a bachelor's degree or higher by 2011.


- (c) Write a practical sentence expressing how P changes for every unit change in t .

Source: *Digest of Education Statistics*, 2008, National Center for Education Statistics.

112. College Degrees In 2000, 1,237,875 bachelor's degrees were conferred by colleges and universities in the United States. In 2007, 1,524,092 bachelor's degrees were awarded. Suppose we assume the relationship between time and degrees conferred is linear.

- (a) Find a linear equation that relates the number N of bachelor's degrees awarded in the year t .
 (b) If the trend continues, estimate the number of bachelor's degrees that are to be awarded in 2011.
 (c) Write a practical sentence expressing how N changes for every unit change in t .

Source: *Digest of Education Statistics*, 2008. National Center for Education Statistics.

 **113. Predicting the Cost of a Home** In 2008, the average cost of a home in Memphis was \$94,823. In 2009, the average cost was \$88,280.

- (a) Assuming that the relationship between time and cost is linear, develop a formula for predicting the average cost of a home.
 (b) What will be the average cost of a home in 2011?

Source: National Association of Realtors

114. Weight–Height Relation in the U.S. Army Assume the recommended weight w of females aged 17–20 years in the U.S. Army is linearly related to their height h . If an Army female who is 67 inches tall should weigh 139 pounds and an Army female who is 70 inches tall should weigh 151 pounds, find a linear equation that expresses recommended weight in terms of height.

115. Predicting Sales The total sales and other operating income for Chevron Corporation were \$214.091 billion in 2007 and \$264.908 billion in 2008.

- (a) Assuming that the dollar amount of annual increase in total sales and other operating income will remain constant, find a linear equation expressing total sales and other operating income S for Chevron Corporation in terms of the time t in years.
 (b) Use this equation to predict total sales and other operating income for Chevron Corporation in the year 2011.

Source: Chevron Corporation

116. Predicting Revenue The net revenue for Dell, Inc. for the fiscal year ending in 2009 was \$61.1 billion and its net revenue for the fiscal year ending in 2008 was \$61.13 billion.

- (a) Assuming that the dollar amount of annual increase in net revenue will remain constant, find a linear equation expressing net revenue R for Dell, Inc. in terms of the time t in years. Let $t = 0$ correspond to the fiscal year ending in 2008.
 (b) Use the equation found in part (a) to predict the net revenue for Dell, Inc. for the fiscal year ending in 2011.

Source: Dell, Inc.

117. Cost of Gasoline The average price of a gallon of regular gasoline in the United States on January 18, 2010 was \$2.739, and one year earlier, the average price of a gallon of regular gasoline was \$1.847. Suppose that the Smith family drives a 2010 GMC Yukon 1500 4WD, which has a fuel usage of 17 miles per gallon (city and highway combined).

- Find a linear equation that gives this family's annual fuel cost C as a function of the number x of miles driven in one year, using the price of a gallon of regular gasoline on January 18, 2010.
- Find a similar equation using the price of a gallon of regular gasoline one year earlier.
- Assuming the Smith family drives 15,000 miles annually, determine their annual fuel cost using the price of gasoline on January 18, 2010.
- Determine the Smith family's annual fuel cost using the price of gasoline one year earlier.
- What is the difference between these annual fuel costs?

Source: Energy Information Administration, United States Department of Energy and www.fueleconomy.gov

118. Cost of Gasoline Repeat parts (a)–(e) of Problem 117 for the Jones family who drive a 2010 Ford Fusion FWD that has a fuel usage of 39 miles per gallon (city and highway combined).

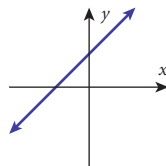
- 119. Credit and Debit Card Growth** At the end of the first quarter of 2009, there were $N = 1147.5$ million credit and signature debit cards in force on the four major U.S. credit card networks and at the end of the first quarter of 2006, there were 954.7 million credit and signature debit cards in force. Let $t = 0$ correspond to the end of the first quarter of 2006.
- Find an equation of the line containing the points $(0, 954.7)$ and $(3, 1147.5)$.
 - Give an interpretation of the slope of this line as an average rate of change.
 - Use the equation of the line found in part (a) to estimate the number of credit and signature debit cards that will be in force on the four major U.S. credit card networks at the end of the first quarter of 2011.
 - Use the equation of the line found in part (a) to estimate the year when the number of credit and signature debit cards that will be in force on the four major U.S. credit card networks will first exceed 1.5 billion cards.

Source: CardData

Discussion and Writing

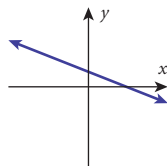
120. Which of the following equations might have the graph shown? (More than one answer is possible.)

- $2x + 3y = 6$
- $-2x + 3y = 6$
- $3x - 4y = -12$
- $x - y = 1$
- $x - y = -1$
- $y = 3x - 5$
- $y = 2x + 3$
- $y = -3x + 3$



121. Which of the following equations might have the graph shown? (More than one answer is possible.)

- $2x + 3y = 6$
- $2x - 3y = 6$
- $3x + 4y = 12$
- $x - y = 1$
- $x - y = -1$
- $y = -2x + 1$
- $y = -\frac{1}{2}x + 10$
- $y = x + 4$



122. Which form of the equation of a line do you prefer to use? Justify your position with an example that shows that your choice is better than another. Have reasons.

123. Can every line be written in slope–intercept form? Explain.

124. Does every line have two distinct intercepts? Explain. Are there lines that have no intercepts? Explain.

125. What can you say about two lines that have equal slopes and equal y -intercepts?

126. What can you say about two lines with the same x -intercept and the same y -intercept? Assume that the x -intercept is not $(0, 0)$.

127. If two lines have the same slope, but different x -intercepts, can they have the same y -intercept?

128. If two lines have the same y -intercept, but different slopes, can they have the same x -intercept? What is the only way that this can happen?

129. Can a line have two distinct x -intercepts? Can a line have infinitely many x -intercepts?

130. Can a line have no x -intercept? Can a line have neither an x -intercept nor a y -intercept?

131. The accepted symbol used to denote the slope of a line is the letter m . Investigate the origin of this symbolism. Begin by consulting a French dictionary and looking up the French word *monter*. Write a brief essay on your findings.

'Are You Prepared?' Answers

- True
- $\{2\}$

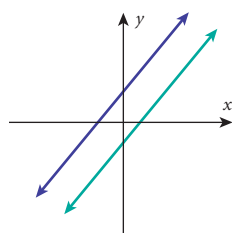
1.2 Pairs of Lines

- OBJECTIVES**
- 1 Show that two lines are coincident (p. 22)
 - 2 Show that two lines are parallel (p. 23)
 - 3 Show that two lines intersect (p. 25)
 - 4 Find the point of intersection of two intersecting lines (p. 25)
 - 5 Solve applied problems involving pairs of lines (p. 26)

Suppose we look at two lines in the same plane. See Figure 27. Then exactly one of the following must hold:

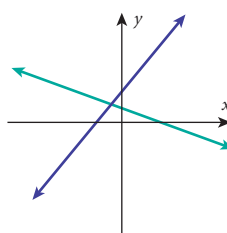
1. They have no points in common, in which case the lines are **parallel**.
2. They have one point in common, in which case the lines **intersect**.
3. They have two points in common, in which case the lines are **coincident** or **identical**, and all the points on one of the lines are the same as the points on the other line.

FIGURE 27



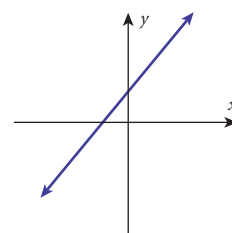
Parallel Lines: No points in common

(a)



Intersecting Lines: One point in common

(b)



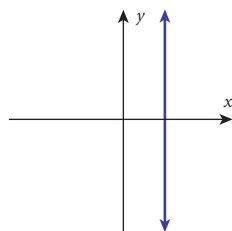
Coincident Lines: All points on each line are the same

(c)

1 Show That Two Lines Are Coincident

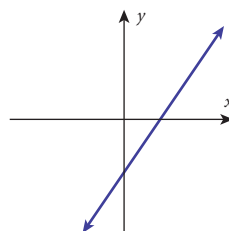
Figure 28 illustrates vertical coincident lines and nonvertical coincident lines.

FIGURE 28



Coincident lines that are vertical

(a)



Coincident lines that are nonvertical

(b)

We are led to the following result.

Theorem ▶ Coincident Lines

Coincident lines that are vertical have undefined slope and the same x -intercept. Coincident lines that are nonvertical have the same slope and the same intercepts.

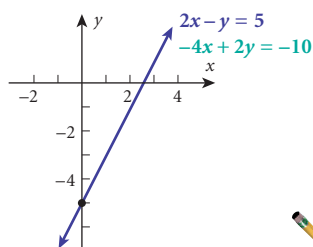
To show that two nonvertical lines are coincident only requires that you show they have the same slope and the same y -intercept. Do you see why?

EXAMPLE 1 Showing That Two Lines Are Coincident

Show that the lines given by the following equations are coincident.

$$L: 2x - y = 5 \quad M: -4x + 2y = -10$$

FIGURE 29



SOLUTION

Write each equation in slope–intercept form:

$$\begin{array}{ll} L: 2x - y = 5 & M: -4x + 2y = -10 \\ -y = -2x + 5 & 2y = 4x - 10 \\ y = 2x - 5 & y = 2x - 5 \end{array}$$

The lines L and M have the same slope 2 and the same y -intercept $(0, -5)$ so they are coincident. See Figure 29. ■

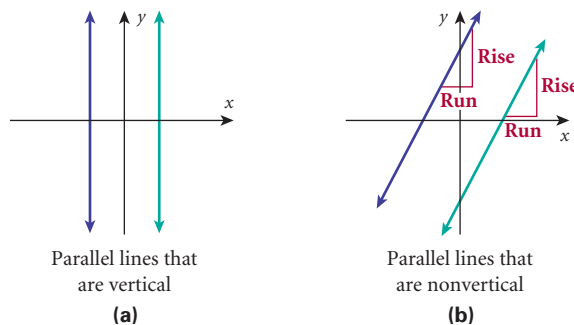


NOW WORK PROBLEM 7.

2 Show That Two Lines Are Parallel

Look at the parallel lines in Figure 30.

FIGURE 30



We see in Figure 30(a) that the two vertical parallel lines have different x -intercepts. For the two nonvertical parallel lines in Figure 30(b), equal runs result in equal rises. As a result, nonvertical parallel lines have the same slope. Since they also have no points in common, they will have different x - and y -intercepts.

Theorem ▶ Parallel Lines

Parallel lines that are vertical have undefined slope and different x -intercepts. Parallel lines that are nonvertical have the same slope and different intercepts.

To show that two nonvertical lines are parallel only requires that you show they have the same slope and different y -intercepts. Do you see why?

EXAMPLE 2 Showing That Two Lines Are Parallel

Show that the lines given by the following equations below are parallel.

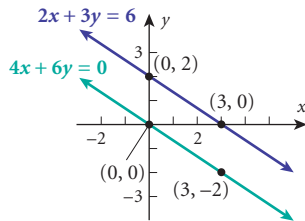
$$L: 2x + 3y = 6 \quad M: 4x + 6y = 0$$

SOLUTION To see if these lines have equal slopes, write each equation in slope–intercept form:

$$\begin{aligned} L: 2x + 3y &= 6 & M: 4x + 6y &= 0 \\ 3y &= -2x + 6 & 6y &= -4x \\ y &= -\frac{2}{3}x + 2 & y &= -\frac{2}{3}x \\ \text{Slope} &= -\frac{2}{3} & \text{Slope} &= -\frac{2}{3} \\ y\text{-intercept} &= (0, 2) & y\text{-intercept} &= (0, 0) \end{aligned}$$

Since each has slope $-\frac{2}{3}$ but different y -intercepts, the lines are parallel. See Figure 31. ■

FIGURE 31



NOW WORK PROBLEM 3.

EXAMPLE 3 Finding the Equation of a Line Parallel to a Given Line

Given the line $x - 4y = 8$, find an equation for the line that contains the point $(2, 1)$ and is parallel to the given line.

SOLUTION First find the slope of the line $x - 4y = 8$ by writing it in slope–intercept form $y = mx + b$:

$$\begin{aligned} x - 4y &= 8 \\ -4y &= -x + 8 && \text{Proceed to solve for } y. \\ y &= \frac{1}{4}x - 2 && y = mx + b; m = \frac{1}{4}; b = -2 \end{aligned}$$

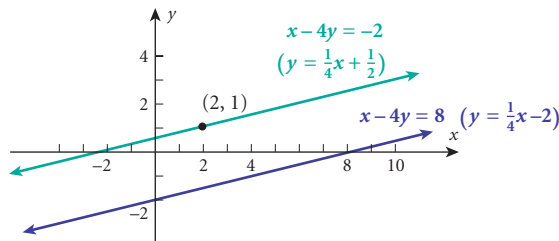
The slope of the line is $\frac{1}{4}$.

We seek a line parallel to the given line that contains the point $(2, 1)$. The slope of this line must be $\frac{1}{4}$. (Do you know why?) Using the point–slope form of the equation of a line, we have

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= \frac{1}{4}(x - 2) && m = \frac{1}{4}, x_1 = 2, y_1 = 1 \\ y - 1 &= \frac{1}{4}x - \frac{1}{2} \\ y &= \frac{1}{4}x + \frac{1}{2} && \text{Slope–intercept form} \\ x - 4y &= -2 && \text{General form} \end{aligned}$$

Figure 32 illustrates the solution.

FIGURE 32



NOW WORK PROBLEM 29.

3 Show That Two Lines Intersect

If two lines have exactly one point in common, the common point is called the **point of intersection**. The slopes of intersecting lines are unequal. Do you know why?

Theorem **Intersecting Lines**

Intersecting lines have different slopes.

EXAMPLE 4 Showing That Two Lines Intersect

Show that the lines given by the following equations below intersect.

$$L: 2x - y = 5 \quad M: x + y = 4$$

SOLUTION Write each equation in slope–intercept form.

$$\begin{aligned} L: 2x - y &= 5 & M: x + y &= 4 \\ -y &= -2x + 5 & y &= -x + 4 \\ y &= 2x - 5 & & \end{aligned}$$

The slope of L is 2 and the slope of M is -1 , so the lines intersect. ■



NOW WORK PROBLEM 5.

4 Find the Point of Intersection of Two Intersecting Lines

EXAMPLE 5 Find

Find the point of intersection of the two lines

$$L: 2x - y = 5 \quad M: x + y = 4$$

SOLUTION The slope–intercept form of each line, as found in Example 4, is

$$L: y = 2x - 5 \quad M: y = -x + 4$$

If (x_0, y_0) denotes the point of intersection, then (x_0, y_0) is a point on both L and M . As a result, we must have

$$y_0 = 2x_0 - 5 \quad \text{and} \quad y_0 = -x_0 + 4$$

Set these equal and solve for x_0 .

$$\begin{aligned} 2x_0 - 5 &= -x_0 + 4 \\ 3x_0 &= 9 \\ x_0 &= 3 \end{aligned}$$

Substituting $x_0 = 3$ in $y_0 = 2x_0 - 5$ (or in $y_0 = -x_0 + 4$), we find

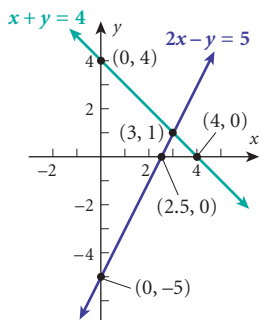
$$y_0 = 2x_0 - 5 = 2(3) - 5 = 1$$

The point of intersection of L and M is $(3, 1)$. See Figure 33.

✓ **CHECK:** To verify that the point $(3, 1)$ is on both L and M , we check to see if $x = 3$, $y = 1$ satisfies each equation.

$$L: 2x - y = 2(3) - 1 = 6 - 1 = 5 \quad M: x + y = 3 + 1 = 4 \quad \blacksquare$$

FIGURE 33



NOW WORK PROBLEM 15.

5 Solve Applied Problems Involving Pairs of Lines

EXAMPLE 6 **Mixing Peanuts**

A store that specializes in selling nuts sells cashews for \$5 per pound and peanuts for \$2 per pound. At the end of the month the manager finds that the peanuts are not selling well. In order to sell 30 pounds of peanuts more quickly, the manager decides to mix the 30 pounds of peanuts with some cashews and sell the mixture of peanuts and cashews for \$3 a pound. How many pounds of cashews should be mixed with the peanuts so that the revenue remains the same as it would be selling the nuts separately?

SOLUTION There are two unknowns: the number of pounds of cashews (call this x) and the number of pounds of the mixture (call this y). Since we know that the number of pounds of cashews plus 30 pounds of peanuts equals the number of pounds of the mixture, we can write

$$x + 30 = y \quad \text{or} \quad y = x + 30$$

Also, in order to keep revenue the same, we must have

$$\begin{pmatrix} \text{Price} \\ \text{per} \\ \text{pound} \\ \$5 \end{pmatrix} \cdot \begin{pmatrix} \text{Pounds} \\ \text{of} \\ \text{cashews} \\ x \end{pmatrix} + \begin{pmatrix} \text{Price} \\ \text{per} \\ \text{pound} \\ \$2 \end{pmatrix} \cdot \begin{pmatrix} \text{Pounds} \\ \text{of} \\ \text{peanuts} \\ 30 \end{pmatrix} = \begin{pmatrix} \text{Price} \\ \text{per} \\ \text{pound} \\ \$3 \end{pmatrix} \cdot \begin{pmatrix} \text{Pounds} \\ \text{of} \\ \text{mixture} \\ y \end{pmatrix}$$

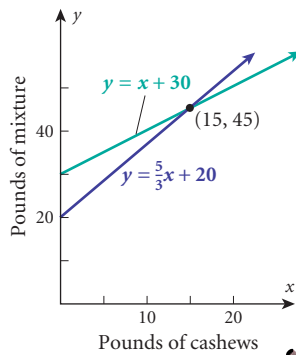
That is,

$$\begin{aligned} 5x + 2(30) &= 3y \\ \frac{5}{3}x + 20 &= y \quad \text{Divide both sides by 3.} \end{aligned}$$

We now have two equations

$$y = \frac{5}{3}x + 20 \quad \text{and} \quad y = x + 30$$

FIGURE 34



Since the number of pounds of the mixture, y , is the same in each case, we have

$$\frac{5}{3}x + 20 = x + 30$$

$$\frac{2}{3}x = 10$$

$$x = 15$$

The manager should mix 15 pounds of cashews with 30 pounds of peanuts. See Figure 34. Notice that the point of intersection $(15, 45)$ represents the pounds of cashews (15) in the mixture (45 pounds). ■



NOW WORK PROBLEM 37.

EXAMPLE 7 Financial Planning

Kathleen has \$80,000 to invest and wants to earn \$2400 from it. She can invest in a safe, government-insured Certificate of Deposit, but it only pays 1% per year (\$800 per year.) To obtain a higher return, she agrees to invest some of her money in noninsured corporate bonds paying 6% per year. How much should be placed in each investment to achieve her goals?

SOLUTION To earn \$2400 on an \$80,000 investment requires a rate of interest of 3%. The question is how should Kathleen split her money between the two investments in order to realize her goal of earning a 3% rate of return on her money? We need two dollar amounts: the amount to invest in corporate bonds (call this x) and the amount to invest in the Certificate of Deposit (call this y). Since we know that the amount she invests in corporate bonds plus the amount she invests in the Certificate of Deposit equals \$80,000, we can write

$$x + y = \$80,000$$

Solving this for y , we get

$$y = \$80,000 - x$$

which is the amount that will be invested in the Certificate of Deposit. See Table 1.

TABLE 1

	Amount \$	Rate	Time yr	Interest \$
Bonds	x	6% = 0.06	1	$0.06x$
Certificate	$80,000 - x$	1% = 0.01	1	$0.01(80,000 - x)$
Total	80,000	3% = 0.03	1	$0.03(80,000) = 2400$

Since the total interest from the investments is equal to $0.03(\$80,000) = \2400 , the equation relating the interest earned on the accounts is given as

Interest earned on the bonds + Interest earned on the certificate = Total interest earned
 $0.06x + 0.01(80,000 - x) = 2400$

(Note that the units are consistent: The unit is dollars on both sides of the equation.)
Now solve the equation for x , the amount invested in corporate bonds.

$$0.06x + 800 - 0.01x = 2400$$

$$0.05x + 800 = 2400$$

$$0.05x = 1600$$

$$x = 32,000$$

Kathleen should invest \$32,000 in corporate bonds and $\$80,000 - \$32,000 = \$48,000$ in the Certificate of Deposit. ■



NOW WORK PROBLEM 45.

SUMMARY

Pair of Lines

Conclusion: The lines are

Both vertical

(a) Coincident, if they have the same x -intercept

(b) Parallel, if they have different x -intercepts

One vertical, one nonvertical

Intersecting

Neither vertical

Write the equation of each line in slope–intercept form:

$$y = m_1x + b_1, \quad y = m_2x + b_2$$

(a) Coincident, if $m_1 = m_2, b_1 = b_2$

(b) Parallel, if $m_1 = m_2, b_1 \neq b_2$

(c) Intersecting, if $m_1 \neq m_2$

EXERCISE 1.2 Answers Begin on Page AN–3.

Concepts and Vocabulary

1. If two lines have no points in common, they are _____.

2. If two lines have different slopes, they will _____.

Skill Building

In Problems 3–14, determine whether the given pairs of lines are parallel, coincident, or intersecting.

3. L: $x + y = 10$

M: $3x + 3y = 6$

4. L: $x - y = 5$

M: $-2x + 2y = 8$

5. L: $2x + y = 4$

M: $2x - y = 8$

6. L: $2x + y = 8$

M: $2x - y = -4$

7. L: $-x + y = 2$

M: $2x - 2y = -4$

8. L: $x + y = -4$

M: $3x + 3y = -12$

9. L: $2x - 3y = -8$

M: $6x - 9y = -2$

10. L: $4x - 2y = -7$

M: $-2x + y = -2$

11. L: $3x - 4y = 1$

M: $x - 2y = -4$

12. L: $4x + 3y = 2$

M: $2x - y = -1$

13. L: $x = 3$

M: $y = -2$

14. L: $x = 4$

M: $x = -2$

In Problems 15–26, the given pairs of lines intersect. Find the point of intersection. Graph each pair of lines.

15. L: $x + y = 5$

M: $3x - y = 7$

16. L: $2x + y = 7$

M: $x - y = -4$

17. L: $x - y = 2$

M: $2x + y = 7$

18. L: $2x - y = -1$

M: $x + y = 4$

19. L: $4x + 2y = 4$

M: $4x - 2y = 4$

20. L: $4x - 2y = 8$

M: $6x + 3y = 0$

21. L: $3x - 4y = 2$

M: $x + 2y = 4$

22. L: $4x + 3y = 2$

M: $2x - y = 1$

23. L: $3x - 2y = -5$

M: $3x + y = -2$

24. L: $4x + y = 6$

M: $4x - 2y = 0$

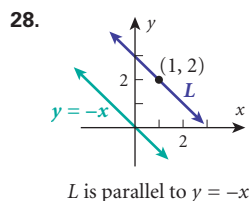
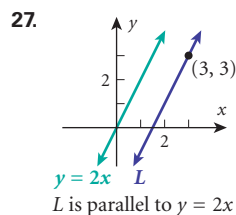
25. L: $x = 4$

M: $y = -2$

26. L: $x = 0$

M: $y = 0$

In Problems 27 and 28, find an equation for the line L . Express the answer using the general form or the slope–intercept form, whichever you prefer.



In Problems 29–34, find an equation for the line with the given properties. Express your answer using the general form or the slope–intercept form, whichever you prefer.

29. Parallel to the line $y = 4x$; containing the point $(-1, 2)$
30. Parallel to the line $y = -3x$; containing the point $(-1, 2)$
31. Parallel to the line $2x - y = -2$; containing the point $(0, 0)$
32. Parallel to the line $x - 2y = -5$; containing the point $(0, 0)$
33. Parallel to the line $x = 3$; containing the point $(4, 2)$
34. Parallel to the line $y = 3$; containing the point $(4, 2)$
35. Find the equation of the line containing the point $(-2, -5)$ and parallel to the line containing the points $(-2, 9)$ and $(3, -10)$.
36. Find the equation of the line containing the point $(-2, -5)$ and parallel to the line containing the points $(-4, 5)$ and $(2, -1)$.

Applications

37. **Mixing Candy** Sweet Delight Candies sells boxes of candy consisting of creams and caramels. Each box sells for \$8.00 and holds 50 pieces of candy (all pieces are the same size). If the caramels cost \$0.10 to produce and the creams cost \$0.20 to produce, how many caramels and creams should be in each box for no profit or loss? Would you increase or decrease the number of caramels in order to obtain a profit?
38. **Mixing Nuts** The manager of Nutt's Nuts regularly sells cashews for \$6.50 per pound, pecans for \$7.50 per pound, and peanuts for \$2.00 per pound. How many pounds of cashews and pecans should be mixed with 40 pounds of peanuts to obtain a mixture of 100 pounds that will sell for \$4.89 a pound so that the revenue is unchanged?
39. **Financial Planning** Mr. Nicholson has just retired and needs \$10,000 per year in supplementary income. He has \$150,000 to invest and can invest in AA bonds at 10% annual interest or in Savings and Loan Certificates at 5% interest per year. How much money should be invested in each so that he realizes exactly \$10,000 in extra income per year?
40. **Financial Planning** Mr. Nicholson finds after 2 years that because of inflation he now needs \$12,000 per year in supplementary income. How should he transfer his funds to achieve this amount? (Use the data from Problem 39.)
41. **Mixing Coffee** California Coffee Roasters sells Kona coffee for \$22.95 per pound and Colombian coffee for \$6.75 per pound. Suppose they offer a blend of those two coffees for a price of \$10.80 per pound. What amounts of Kona and Colombian coffees should be blended to obtain the desired mixture? *Hint:* Assume that the total weight of the blend is 100 pounds.
Source: California Coffee Roasters.
42. **Livestock Feed** Both cornmeal and soybean meal are popular feed for livestock. The protein content of cornmeal is 22% and the protein content of soybean meal is 44%. A farmer wants a 300 pound mixture that is 30% protein. How much of each feed should he use to obtain the desired mixture? (Round answer to the nearest integer.)
43. **Mixing Acid** One solution is 15% acid and another is 5% acid. How many cubic centimeters of each should be mixed to obtain 100 cubic centimeters of a solution that is 8% acid?
44. **Financial Planning** A bank loaned \$10,000, some at an annual rate of 8% and some at an annual rate of 12%. If the income from these loans was \$1000, how much was loaned at 8%? How much at 12%?
45. **Investing in Gold** Suppose an investor purchased x ounces of gold in January 2006 at the prevailing price of \$549.86 per ounce and then sold this amount of gold in January 2010 for \$1112.30 per ounce.
(a) Find an equation to express the realized gain, y , in terms of x .
(b) Find the point of intersection of the graph of this equation with the graph of $y = 10,000$ to determine the number of ounces of gold (rounded to the nearest tenth) that would result in a gain of \$10,000.
Source: The Financial Forecast Center
46. **Comparing Gasoline Costs** On January 21, 2010, the average price of a gallon of regular gasoline was \$2.775. The Environmental Protection Agency's fuel economy estimate for a 2010 Honda Civic Hybrid is 42 miles per gallon (city and highway combined), while the EPA's fuel economy estimate for a 2010 Ford Fusion Hybrid is 39 mpg (city and highway combined).

- (a) Using the price of gasoline on January 21, 2010, find an equation to express the annual gasoline cost, y , in terms of the total annual miles driven, x , in a 2010 Honda Civic Hybrid.
- (b) Find a similar equation for a 2010 Ford Fusion Hybrid.
- (c) Graph each of these equations and find the point of intersection of these graphs with the graph of the line $x = 15,000$.
- (d) Give an interpretation of the distance between these two points of intersection.

Source: US Energy Information Association and www.fueleconomy.gov

47. HD Radio On February 1, 2008, there were 1615 High Definition radio stations broadcasting a digital signal alongside their analog signal. On February 1, 2009, there were 1877 such stations. Assume the rate of increase is constant.

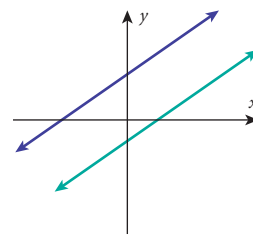
- (a) Find an equation to express the number N of High Definition radio stations in terms of x , the number of days after February 1, 2008. Use 365 days in a year.
- (b) Find the point of intersection of the graph of this equation with the graph of $y = 2000$ to predict the date on which the 2000th HD radio station began broadcasting.

Source: *This Week in Consumer Electronics*, March 9, 2009

Discussion and Writing

48. The figure below shows the graph of two parallel lines. Which of the following pairs of equations might have such a graph?

- (a) $x - 2y = 3$
 $x + 2y = 7$
- (b) $x + y = 2$
 $x + y = -1$
- (c) $x - y = -2$
 $x - y = 1$
- (d) $x - y = -2$
 $2x - 2y = -4$
- (e) $x + 2y = 2$
 $x + 2y = -1$



1.3 Applications in Business and Economics

- OBJECTIVES**
- 1 Solve problems involving the break-even point (p. 30)
 - 2 Solve problems involving supply and demand equations (p. 33)

1 Solve Problems Involving the Break-Even Point

In many businesses the cost C of production and the number x of items produced can be expressed as a linear equation $C = mx + b$, where m represents the unit **variable cost** of producing each item and b is the **fixed cost**. Similarly, sometimes the revenue R obtained from sales and the number x of items produced can also be expressed as a linear equation of the form $R = px$, where p is the price charged for each unit sold. When the cost C of production exceeds the revenue R from the sales, the business is operating at a loss; when the revenue R exceeds the cost C , there is a profit; and when the revenue R and the cost C are equal, there is no profit or loss. The point at which $R = C$, that is, the point of intersection of the two lines, is usually referred to as the **break-even point**.

EXAMPLE 1 Find

Sweet Delight Candies, Inc., has daily fixed costs from salaries, rent, and other operations of \$300. Each pound of candy produced costs \$1 and is sold for \$2.

- (a) Find the cost C of production for x pounds of candy.
- (b) Find the revenue R from selling x pounds of candy.
- (c) What is the break-even point? That is, how many pounds of candy must be produced and sold daily to guarantee no loss and no profit?
- (d) Graph C and R and label the break-even point.

SOLUTION

- (a) The cost C of production is the fixed cost of \$300 plus the variable cost of producing x pounds of candy at \$1 per pound. That is,

$$C = \$1 \cdot x + \$300 = x + 300$$

- (b) The revenue R realized from the sale of x pounds of candy at the price of \$2 per pound is

$$R = \$2 \cdot x = 2x$$

- (c) The break-even point is the point where $R = C$.

$$R = C$$

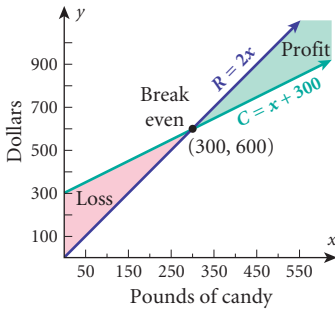
$$2x = x + 300 \quad R = 2x, C = x + 300$$

$$x = 300 \quad \text{Solve for } x.$$

That is, 300 pounds of candy must be produced and sold daily to break even. The break-even point is $(300, 600)$.

- (d) Figure 35 shows the graphs of C and R and the break-even point. Notice that for $x > 300$, the revenue R always exceeds the cost C so that a profit results. Similarly, for $x < 300$, the cost exceeds the revenue, resulting in a loss. ■

FIGURE 35



NOW WORK PROBLEMS 3 AND 11.

EXAMPLE 2 Analyzing Break-Even Points

After negotiations with employees of Sweet Delight Candies and an increase in the price of sugar, the daily cost C of production for x pounds of candy changed to

$$C = \$1.05x + \$330$$

- (a) If each pound of candy is sold for \$2.00, how many pounds must be produced and sold daily to make a profit?
- (b) If the selling price is increased to \$2.25 per pound, what is the break-even point?
- (c) If it is known that 325 pounds of candy can be produced and sold daily, what price should be charged per pound to guarantee no loss?

SOLUTION

- (a) If each pound is sold for \$2.00, the revenue R from sales is

$$R = \$2x$$

where x represents the number of pounds produced and sold daily. Set $R = C$ to find the break-even point.

$$R = C$$

$$2x = 1.05x + 330 \quad R = 2x, C = 1.05x + 330$$

$$0.95x = 330 \quad \text{Subtract } 1.05x \text{ from each side.}$$

$$x = \frac{330}{0.95} = 347.37 \quad \text{Solve for } x.$$

If 347 pounds or less of candy are produced and sold daily, a loss is incurred; if 348 pounds or more are sold daily, a profit results.

- (b) If the selling price is increased to \$2.25 per pound, the revenue
- R
- from sales is

$$R = \$2.25x$$

The break-even point is the point where $R = C$.

$$\begin{aligned} R &= C \\ 2.25x &= 1.05x + 330 && R = 2.25x, C = 1.05x + 330 \\ 1.2x &= 330 \\ x &= \frac{330}{1.2} = 275 \end{aligned}$$

With the new selling price, the break-even point occurs when 275 pounds of candy are produced and sold daily.

- (c) If we know that 325 pounds of candy will be produced and sold daily, the price per pound
- p
- needed to guarantee no loss (that is, to guarantee at worst a break-even point) is the solution of the equation

$$\begin{aligned} R &= C \\ xp &= 1.05x + 330 && R = xp, C = 1.05x + 330 \\ 325p &= (1.05)(325) + 330 && x = 325 \\ 325p &= 671.25 && \text{Simplify.} \\ p &= \$2.07 && \text{Solve for } p. \end{aligned}$$

The company should charge \$2.07 per pound to guarantee no loss, provided at least 325 pounds will be produced and sold daily. ■

EXAMPLE 3 Analyzing Break-Even Points

A producer sells items for \$0.30 each.

- (a) Determine the revenue R from selling x items.
 (b) If the cost for production is

$$C_1 = \$0.15x + \$105$$

where x is the number of items sold, find the break-even point.

- (c) If the cost can be changed to

$$C_2 = \$0.12x + \$110$$

would it be advantageous?

- (d) Graph
- R
- ,
- C_1
- , and
- C_2
- together.

SOLUTION (a) The revenue R from selling x items is

$$R = \$0.30x$$

- (b) If the cost for production is
- $C_1 = \$0.15x + \105
- , then the break-even point is the point where
- $R = C_1$
- .

$$\begin{aligned} R &= C_1 \\ 0.3x &= 0.15x + 105 && R = 0.3x, C_1 = 0.15x + 105 \\ 0.15x &= 105 && \text{Simplify.} \\ x &= 700 && \text{Solve for } x. \end{aligned}$$

The break-even point occurs when 700 items are sold.

- (c) If the revenue received remains at $R = \$0.3x$, but the cost for production changes to $C_2 = \$0.12x + \110 , then the break-even point is the point where $R = C_2$.

$$\begin{aligned}
 R &= C_2 \\
 0.3x &= 0.12x + 110 && R = 0.3x, C_2 = 0.12x + 110 \\
 0.18x &= 110 && \text{Simplify.} \\
 x &= 611.11 && \text{Solve for } x.
 \end{aligned}$$

The break-even point for the cost in (a) was 700 items. Since the cost in (b) will require fewer items to be sold in order to break even, management should probably change over to the new cost.

- (d) Figure 36 shows the graphs of R , C_1 , and C_2 .

FIGURE 36

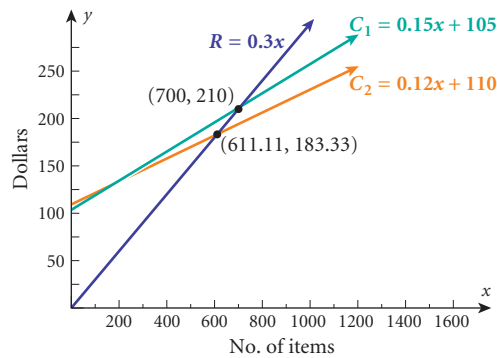
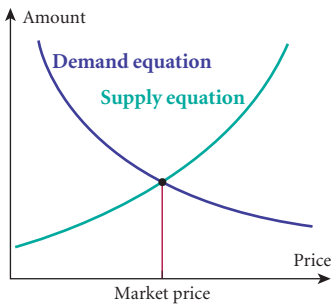


FIGURE 37



2 Solve Problems Involving Supply and Demand Equations

The **supply equation** in economics is used to specify the amount of a particular commodity that sellers are willing to offer in the market at various prices. The **demand equation** specifies the amount of a particular commodity that buyers are willing to purchase at various prices.

An increase in price p usually causes an increase in the supply S and a decrease in demand D . On the other hand, a decrease in price brings about a decrease in supply and an increase in demand. The **market price** is defined as the price at which supply and demand are equal (the point of intersection).

Figure 37 illustrates a typical supply/demand situation.

EXAMPLE 4 Supply and Demand

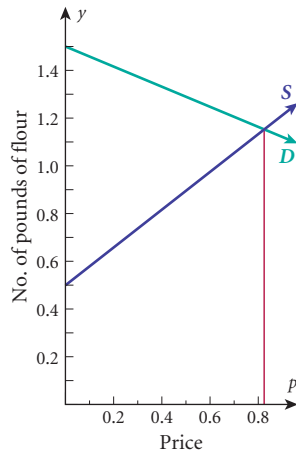
The supply and demand for flour have been estimated as being given by the equations

$$S = 0.8p + 0.5 \quad D = -0.4p + 1.5$$

where p is measured in dollars and S and D are measured in pound units of flour. Find the market price and graph the supply and demand equations.

FIGURE 38

SOLUTION The market price p is the solution of the equation



$$\begin{aligned}
 S &= D \\
 0.8p + 0.5 &= -0.4p + 1.5 \\
 1.2p &= 1 \\
 p &= 0.83
 \end{aligned}$$

At a price of \$0.83 per pound, supply and demand for flour are equal. The graphs are shown in Figure 38. ■



NOW WORK PROBLEMS 9 AND 15.

EXERCISE 1.3 Answers Begin on Page AN-4.

Concepts and Vocabulary

- 1. **True or False** The break even point is the point of intersection of the revenue graph and the profit graph.
- 2. **True or False** An increase in price usually causes an increase in supply and a decrease in demand.

Skill Building

In Problems 3–6, find the break-even point for the cost C of production and the revenue R . Graph each result. Indicate the break-even point and where a profit results and where a loss results.

- 3. $C = \$10x + \600 $R = \$30x$
- 4. $C = \$5x + \200 $R = \$8x$
- 5. $C = \$0.20x + \50 $R = \$0.30x$
- 6. $C = \$1800x + \3000 $R = \$2500x$

In Problems 7–10, find the market price for each pair of supply and demand equations.

- 7. $S = p + 1$ $D = 3 - p$
- 8. $S = 2p + 3$ $D = 6 - p$
- 9. $S = 20p + 500$ $D = 1000 - 30p$
- 10. $S = 40p + 300$ $D = 1000 - 30p$

Applications

- 11. **Break-Even Point** A manufacturer produces game-day pennants at a cost of \$0.75 per item and sells them for \$1 per item. The daily operational overhead is \$300. What is the break-even point? Graph your result.
- 12. **Break-Even Point** If the manufacturer in Problem 11 is able to reduce the cost per item to \$0.65, but with a resultant increase to \$350 in operational overhead, is it advantageous to do so? Give reasons. Graph your result.
- 13. **Market Price of Sugar** The supply and demand equations for sugar have been estimated to be given by the equations

$$S = 0.7p + 0.4 \quad D = -0.5p + 1.6$$
 where p is the price in dollars per pound and S and D are in millions of pounds.
 - (a) Find the market price.
 - (b) What quantity of supply is demanded at this market price?
 - (c) Graph both the supply and demand equations.
 - (d) Interpret the point of intersection of the two lines.
- 14. **Supply and Demand** The market price for a certain product is \$5.00 per unit and occurs when 14,000 units are produced. At a price of \$1, no units are manufactured and, at a price of \$19.00, no units will be purchased. Find the supply and demand equations, assuming they are linear.
- 15. **Supply and Demand** For a certain commodity the supply equation is given by

$$S = 2p + 5$$
 At a price of \$1, there is a demand for 19 units of the commodity. If the demand equation is linear and the market price is \$3, find the demand equation.
- 16. **Supply and Demand** For a certain commodity the demand equation is given by

$$D = -3p + 20$$
 At a price of \$1, four units of the commodity are supplied. If the supply equation is linear and the market price is \$4, find the supply equation.

17. **DVD Club** A DVD club offers four DVDs for \$0.49 each plus shipping and handling and additional DVDs for \$17.95 each plus shipping and handling. Shipping and handling is \$2.31 per DVD. A discount retailer offers the same DVDs for \$14.95 each plus 7% sales tax. What is the maximum number of DVDs that can be ordered from the club while keeping it a better deal than the discount retailer?
18. **Broadcasting Profits** The following table gives the operating expenses and revenues for radio and television broadcasting for the years 2004 and 2007.

		2004	2007
Radio	Revenue (in billions)	\$13.817	\$13.624
	Cost (in billions)	\$9.920	\$9.988
	Profit		

		2004	2007
Television	Revenue (in billions)	\$35.599	\$37.008
	Cost (in billions)	\$28.312	\$29.803
	Profit		

- (a) Fill in the profit rows in each table.
- (b) Find equations that relate the profit P_r for radio broadcasting and the profit P_t for television broadcasting in terms of the year t . Assume the relationships are linear.
- (c) Using the equations from part (a), determine when the two media had the same profit.

Source: U.S. Census Bureau, 2010 *Statistical Abstract*

1.4 Scatter Diagrams; Linear Curve Fitting

- OBJECTIVES**
- 1 Draw and interpret scatter diagrams (p. 35)
 - 2 Distinguish between linear and nonlinear relations (p. 36)
 - 3 Use a graphing utility to find the line of best fit (p. 38)

1 Draw and Interpret Scatter Diagrams

A **relation** is a correspondence between two sets. If x and y are two elements in these sets and if a relation exists between x and y , then we say that x **corresponds to** y or that y **depends on** x and write $x \rightarrow y$. We may also write $x \rightarrow y$ as the ordered pair (x, y) . Here, y is referred to as the **dependent** variable and x is called the **independent** variable.

Often we are interested in specifying the type of relation (such as an equation) that might exist between two variables. The first step in finding this relation is to plot the ordered pairs using rectangular coordinates. The resulting graph is called a **scatter diagram**.


EXAMPLE 1 Drawing a Scatter Diagram

The data listed in Table 2 represent the apparent temperature versus the relative humidity in a room whose actual temperature is 72° Fahrenheit.

TABLE 2

Relative Humidity (%), x	Apparent Temperature, y	(x, y)	Relative Humidity (%), x	Apparent Temperature, y	(x, y)
0	64	(0, 64)	60	72	(60, 72)
10	65	(10, 65)	70	73	(70, 73)
20	67	(20, 67)	80	74	(80, 74)
30	68	(30, 68)	90	75	(90, 75)
40	70	(40, 70)	100	76	(100, 76)
50	71	(50, 71)			

(a) Draw a scatter diagram by hand.

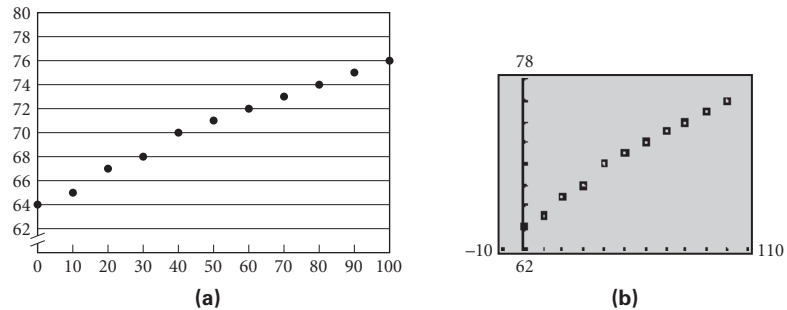
 (b) Use a graphing utility to draw a scatter diagram.*

(c) Describe what happens to the apparent temperature as the relative humidity increases.

SOLUTION (a) To draw a scatter diagram by hand, we plot the ordered pairs listed in Table 2, with the relative humidity as the x -coordinate and the apparent temperature as the y -coordinate. See Figure 39(a). Notice that the points in a scatter diagram are not connected.

 (b) Figure 39(b) shows the scatter diagram using a graphing utility.

FIGURE 39



(c) We see from the scatter diagrams that, as the relative humidity increases, the apparent temperature increases. ■

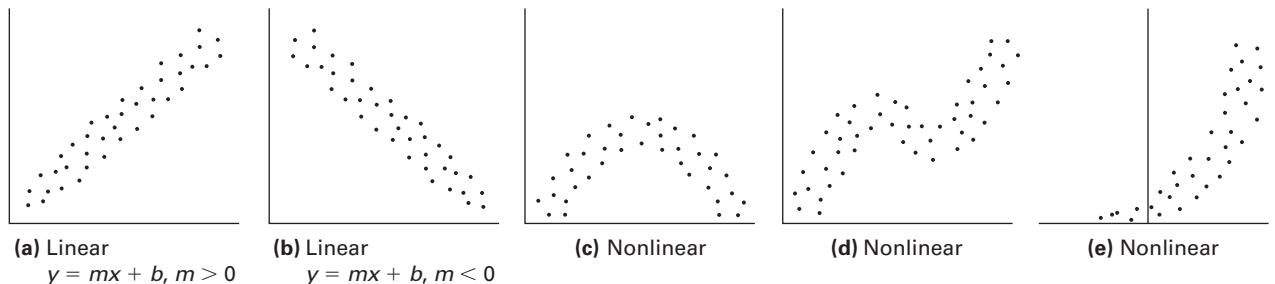


NOW WORK PROBLEMS 9(a) and 9(d).

2 Distinguish Between Linear and Nonlinear Relations

Scatter diagrams are used to help us see the type of relation that may exist between two variables. In this text, we concentrate on distinguishing between linear and nonlinear relations. See Figure 40.

FIGURE 40

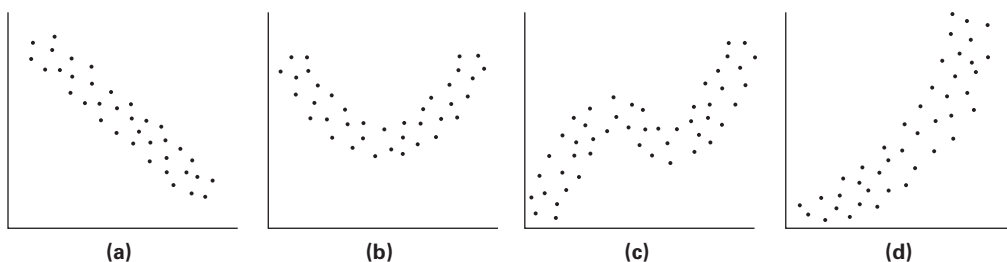


* Consult your owner's manual for the proper keystrokes.

EXAMPLE 2 Distinguishing Between Linear and Nonlinear Relations

Determine whether the relation between the two variables in Figure 41 is linear or nonlinear.

FIGURE 41



SOLUTION (a) Linear (b) Nonlinear (c) Nonlinear (d) Nonlinear ■

**NOW WORK PROBLEM 3.**

In this book we only study data whose scatter diagrams imply that a linear relation exists between the two variables.

Suppose that the scatter diagram of a set of data appears to be linearly related as in Figure 40(a) or (b). We might wish to find an equation of a line that relates the two variables. One way to obtain an equation for such data is to draw a line through two points on the scatter diagram and find the equation of the line.

EXAMPLE 3 Find an Equation for Linearly Related Data

Using the data in Table 2 from Example 1, select two points from the data and find an equation of the line containing the points.

(a) Graph the line on the scatter diagram obtained in Example 1(a).



(b) Graph the line on the scatter diagram obtained in Example 1(b).

SOLUTION Select two points, say (10, 65) and (70, 73). (You should select your own two points and complete the solution.) The slope of the line joining the points (10, 65) and (70, 73) is

$$m = \frac{73 - 65}{70 - 10} = \frac{8}{60} = \frac{2}{15}$$

The equation of the line with slope $\frac{2}{15}$ and passing through (10, 65) is found using the point-slope form with $m = \frac{2}{15}$, $x_1 = 10$, and $y_1 = 65$.

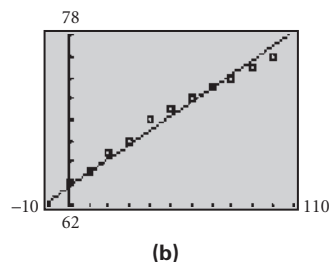
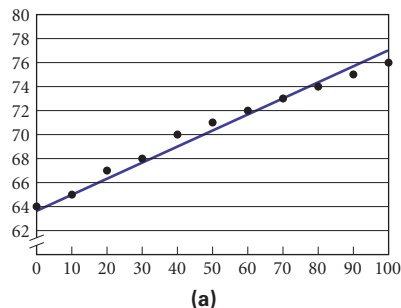
$$y - y_1 = m(x - x_1)$$

$$y - 65 = \frac{2}{15}(x - 10)$$

$$y = \frac{2}{15}x + \frac{191}{3}$$

- (a) Figure 42(a) shows the scatter diagram with the graph of the line drawn by hand.
-  (b) Figure 42(b) shows the scatter diagram with the graph of the line using a graphing utility.

FIGURE 42



NOW WORK PROBLEMS 9(b) and (c).



3 Use a Graphing Utility to Find the Line of Best Fit

The line obtained in Example 3 depends on the selection of points, which will vary from person to person. So the line that we found might be different from the line that you found. Although the line that we found in Example 3 appears to “fit” the data well, there may be a line that “fits better.” Do you think your line fits the data better? Is there a line of *best fit*? As it turns out, there is a method for finding the line that best fits linearly related data (called the *line of best fit*).*

EXAMPLE 4 Finding the Line of Best Fit

Using the data in Table 2 from Example 1,

- (a) Find the line of best fit using a graphing utility.
- (b) Graph the line of best fit on the scatter diagram obtained in Example 1(b).
- (c) Interpret the slope of the line of best fit.
- (d) Use the line of best fit to predict the apparent temperature of a room whose actual temperature is 72°F and relative humidity is 45%.

SOLUTION (a) Graphing utilities contain built-in programs that find the line of best fit for a collection of points in a scatter diagram. (Look in your owner’s manual under Linear Regression or Line of Best Fit for details on how to execute the program.) Upon executing the LINear REGression program on a TI-84 Plus, we obtain the results shown in Figure 43. The output the utility provides shows us the equation $y = ax + b$, where a is the slope of the line and $(0, b)$ is the y -intercept. The line of best fit that relates relative humidity to apparent temperature may be expressed as the line $y = 0.121x + 64.409$.

* We show how this line of best fit is found algebraically in Chapter 2. Here we use a graphing utility to find the line of best fit.

FIGURE 43

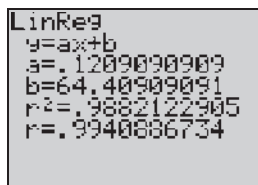
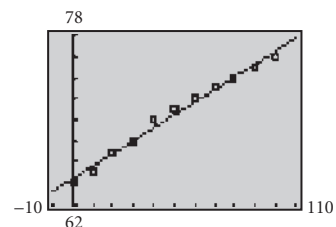


FIGURE 44



- (b) Figure 44 shows the graph of the line of best fit, along with the scatter diagram.
- (c) The slope of the line of best fit is 0.121, which means that, for every 1% increase in the relative humidity, apparent room temperature increases 0.121°F.
- (d) Letting $x = 45$ in the equation of the line of best fit, we obtain $y = 0.121(45) + 64.409 \approx 70^\circ\text{F}$, which is the apparent temperature in the room. ■



NOW WORK PROBLEMS 9(d), (e), AND (f).

Does the line of best fit appear to be a good fit? In other words, does the line appear to accurately describe the relation between temperature and relative humidity?

And just how “good” is this line of best fit? The answers are given by what is called the *correlation coefficient*. Look again at Figure 43. The last line of output is $r = 0.994$. This number, called the **correlation coefficient**, r , $-1 \leq r \leq 1$, is a measure of the strength of the *linear relation* that exists between two variables. The closer that $|r|$ is to 1, the more perfect the linear relationship is. If r is close to 0, there is little or no *linear* relationship between the variables. A negative value of r , $r < 0$, indicates that as x increases y decreases; a positive value of r , $r > 0$, indicates that as x increases y does also. The data given in Example 1, having a correlation coefficient of 0.994, are indicative of a strong linear relationship with positive slope.

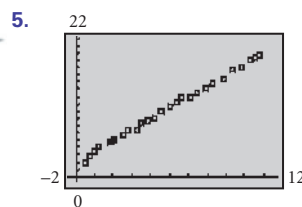
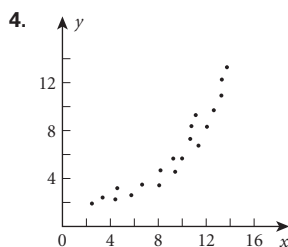
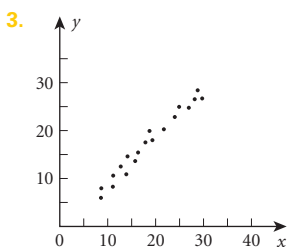
EXERCISE 1.4 Answers Begin on Page AN-4.

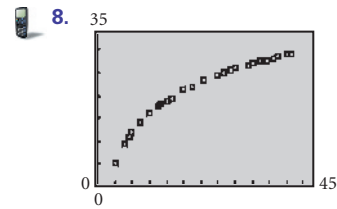
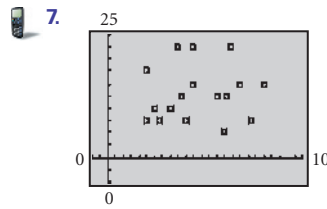
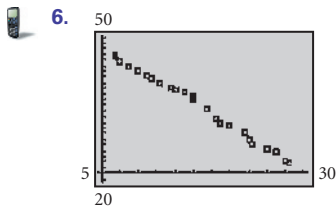
Concepts and Vocabulary

- 1. **True or False** A scatter diagram is used to identify the type of relation that might exist between the independent variable and the dependent variable.
- 2. The mathematical measure of the goodness of fit of a line is called the _____.

Skill Building

In Problems 3–8, examine the scatter diagram and determine whether the type of relation that may exist is linear or nonlinear.





In Problems 9–16,

- Draw a scatter diagram by hand.
- Select two points from the scatter diagram and find the equation of the line containing the points selected.*
- Graph the line found in part (b) on the scatter diagram.
- Use a graphing utility to draw a scatter diagram.
- Use a graphing utility to find the line of best fit.
- Use a graphing utility to graph the line of best fit on the scatter diagram.

9.

x	3	4	5	6	7	8	9
y	4	6	7	10	12	14	16

10.

x	3	5	7	9	11	13
y	0	2	3	6	9	11

11.

x	-2	-1	0	1	2
y	-4	0	1	4	5

12.

x	-2	-1	0	1	2
y	7	6	3	2	0

13.

x	20	30	40	50	60
y	100	95	91	83	70

14.

x	5	10	15	20	25
y	2	4	7	9	11

15.

x	-20	-17	-15	-14	-10
y	100	120	118	130	140

16.

x	-30	-27	-25	-20	-14
y	10	12	13	13	18

Applications

17. Consumption and Disposable Income An economist wishes to estimate a line that relates personal consumption expenditures C and disposable income I . Both C and I are in thousands of dollars. She interviews eight heads of households for families of size 3 and obtains the data below.

I (in thousands)	20	20	18	27	36	37	45	50
C (in thousands)	16	18	13	21	27	26	36	39

Let I represent the independent variable and C the dependent variable.

- Draw a scatter diagram by hand.
- Find a line that fits the data.*
- Interpret the slope. The slope of this line is called the **marginal propensity to consume**.
- Predict the consumption of a family whose disposable income is \$42,000.

(e) Use a graphing utility to find the line of best fit to the data.

18. Marginal Propensity to Save The same economist as in Problem 17 wants to estimate a line that relates savings S and disposable income I . Let $S = I - C$ be the dependent variable and I the independent variable.

- Draw a scatter diagram by hand.
- Find a line that fits the data.
- Interpret the slope. The slope of this line is called the **marginal propensity to save**.
- Predict the savings of a family whose income is \$42,000.
- Use a graphing utility to find the line of best fit.

19. Apparent Room Temperature The data on page 41 represent the apparent temperature versus the relative humidity in a room whose actual temperature is 65° F. Let h represent the independent variable and T the dependent variable.

- Use a graphing utility to draw a scatter diagram of the data.
- Use a graphing utility to find the line of best fit to the data.
- Graph the line of best fit on the scatter diagram drawn in part (a).
- Interpret the slope of the line of best fit.
- Determine the apparent temperature of a room whose actual temperature is 65°F if the relative humidity is 75%.

* Answers will vary. We will use the first and last data points in the answer section.

Relative Humidity, h (%)	Apparent Temperature, T (°F)
0	59
10	60
20	61
30	61
40	62
50	63
60	64
70	65
80	65
90	66
100	67

Source: National Oceanic and Atmospheric Administration

- 20. Apparent Room Temperature** The data in the table represent the apparent temperature versus the relative humidity in a room whose actual temperature is 75°F. Let h represent the independent variable and let T be the dependent variable.

Relative Humidity, h (%)	Apparent Temperature, T (°F)
0	68
10	69
20	71
30	72
40	74
50	75
60	76
70	76
80	77
90	78
100	79

Source: National Oceanic and Atmospheric Administration

- Use a graphing utility to draw a scatter diagram of the data.
- Use a graphing utility to find the line of best fit to the data.
- Graph the line of best fit on the scatter diagram drawn in part (a).

- Interpret the slope of the line of best fit.
 - Determine the apparent temperature of a room whose actual temperature is 75°F if the relative humidity is 75%.
- 21.** The annual residential energy use in the United States from petroleum for the years 1997–2009 is given in the following table.

Year	Annual Residential Energy Use in the United States from Petroleum (in quadrillion btu)
1997	10.71
1998	10.28
1999	10.69
2000	11.24
2001	10.98
2002	11.25
2003	11.61
2004	11.43
2005	11.56
2006	10.8
2007	11.38
2008	11.48
2009	11.22

Source: U.S. Department of Energy, Energy Information Administration

Let E be the dependent variable representing annual energy use from petroleum and let t be the independent variable representing time. Let $t = 0$ correspond to the year 1995.

- Find the line of best fit for this data and then use this line to predict annual energy use from petroleum for the years 2015, 2030, and 2035.
- Compare your results with the following forecasts given by the U.S. Department of Energy. Can you explain any differences?

Year	Forecasted Annual Residential Energy Use in the United States from Petroleum (in quadrillion btu)
2015	11.07
2030	11.93
2035	12.12

CHAPTER 1 REVIEW OBJECTIVES

Section	Examples	You should be able to	Review Exercises
1.1	1, 2, 3	1 Graph linear equations (p. 3)	1–8
	4, 5	2 Graph a vertical line (p. 6)	12, 13
	6, 7	3 Find the slope of a line and interpret it (p. 7)	5–8(a), 46(b–e, g) 47(b–e), 48(c, d)
	8	4 Graph a line given a point on the line and the slope (p. 10)	5–8(a), 5–8(c), 9–11
	9	5 Use the point–slope form of a line (p. 10)	9–11, 14
	10	6 Find the equation of a horizontal line (p. 11)	11, 14
	11	7 Find the equation of a line given two points (p. 12)	5–8(b), 15–18
	12	8 Use the slope–intercept form of a line (p. 12)	21–26
	13, 14	9 Solve applied problems involving linear equations (p. 14)	45, 46
1.2	1	1 Show that two lines are coincident (p. 22)	28, 31
	2, 3	2 Show that two lines are parallel (p. 23)	19, 20, 27, 32
	4	3 Show that two lines intersect (p. 25)	29, 30
	5	4 Find the point of intersection of two intersecting lines (p. 25)	33–38
	6, 7	5 Solve applied problems involving pairs of lines (p. 26)	39, 40, 42
1.3	1, 2, 3	1 Solve problems involving the break even point (p. 30)	41
	4	2 Solve problems involving supply and demand equations (p. 33)	45
1.4	1	1 Draw and interpret scatter diagrams (p. 35)	43, 44, 46(a), 47(a), 48(a), 49(a)
	2	2 Distinguish between linear and nonlinear relations (p. 36)	43, 44, 48(b), 49(b)
	4	3 Use a graphing utility to find the line of best fit (p. 38)	46(f), 47(f), 48(e), 49

THINGS TO KNOW

Linear Equation, General Form (pp. 3 and 4)

$$Ax + By = C \quad A, B \text{ not both zero}$$

Vertical Line (p. 6)

$$x = a \quad (a, 0) \text{ is the } x\text{-intercept}$$

Slope of a Line (p. 7)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ if } x_1 \neq x_2; \text{ undefined if } x_1 = x_2$$

Point–Slope Form of the Equation of a Line (p. 11)

$$y - y_1 = m(x - x_1)$$

m is the slope of the line; (x_1, y_1) is a point on the line

Horizontal Line (p. 11)

$$y = b \quad (0, b) \text{ is the } y\text{-intercept}$$

Slope–Intercept Form of the Equation of a Line (p. 13)

$$y = mx + b$$

m is the slope of the line; $(0, b)$ is the y -intercept

Pair of Lines (pp. 22–25)

Conclusion: The lines are

Both vertical

(a) Coincident, if they have the same x -intercept

(b) Parallel, if they have different x -intercepts

One vertical, one nonvertical

Intersecting

Neither vertical

Write the equation of each line in slope–intercept form:

$$y = m_1x + b_1, y = m_2x + b_2$$

(a) Coincident, if $m_1 = m_2, b_1 = b_2$

(b) Parallel, if $m_1 = m_2, b_1 \neq b_2$

(c) Intersecting, if $m_1 \neq m_2$

REVIEW EXERCISES Answers to odd-numbered problems begin on page AN-6.

Blue problem numbers indicate the author's suggestions for use in a practice test.

In Problems 1–4, graph each equation.

1. $y = -2x + 3$

2. $y = 6x - 2$

3. $2y = 3x + 6$

4. $3y = 2x + 6$

In Problems 5–8, (a) find and interpret the slope of the line containing each pair of points; (b) find an equation for the line containing each pair of points. Write the equation using the general form or the slope–intercept form, whichever you prefer. (c) Graph each line.

5. $P = (1, 2)$ $Q = (-3, 4)$

6. $P = (-1, 3)$ $Q = (1, 1)$

7. $P = (-1, 5)$ $Q = (-2, 3)$

8. $P = (-2, 3)$ $Q = (0, 0)$

In Problems 9–20, find an equation of the line having the given characteristics. Write the equation using the general form or the slope–intercept form, whichever you prefer. Graph each line.

9. Slope = -3 ; containing the point $(2, -1)$

10. Slope = 4 ; containing the point $(-1, -3)$

11. Slope = 0 ; containing the point $(-3, 4)$

12. Slope undefined; containing the point $(-3, 4)$

13. Vertical; containing the point $(8, 5)$

14. Horizontal; containing the point $(5, 8)$

15. x -intercept = $(2, 0)$; containing the point $(4, -5)$

16. y -intercept = $(0, -2)$; containing the point $(5, -3)$

17. x -intercept = $(-3, 0)$; y -intercept = $(0, -4)$

18. Containing the points $(3, -4)$ and $(2, 1)$

19. Parallel to the line $2x + 3y = -4$; containing the point $(-5, 3)$

20. Parallel to the line $x + y = 2$; containing the point $(1, -3)$

In Problems 21–26, find the slope and y -intercept of each line. Graph each line.

21. $9x + 2y = 18$

22. $4x + 5y = 20$

23. $4x + 2y = 9$

24. $3x + 2y = 8$

25. $\frac{1}{2}x + \frac{1}{3}y = \frac{1}{6}$

26. $\frac{1}{4}x - \frac{1}{3}y = \frac{5}{12}$

In Problems 27–32, determine whether the two lines are parallel, coincident, or intersecting.

27. $3x - 4y = -12$

28. $2x + 3y = -5$

29. $x - y = -2$

$6x - 8y = -9$

$4x + 6y = -10$

$3x - 4y = -12$

30. $2x + 3y = 5$

31. $4x + 6y = -12$

32. $-3x + y = 0$

$x + y = 2$

$2x + 3y = -6$

$6x - 2y = -5$

In Problems 33–38, the given pair of lines intersect. Find the point of intersection. Graph the lines.

33. L: $x - y = 4$

34. L: $x + y = 4$

35. L: $x - y = -2$

M: $x + 2y = 7$

M: $x - 2y = 1$

M: $x + 2y = 7$

36. L: $2x + 4y = 4$

37. L: $2x - 4y = -8$

38. L: $3x + 4y = 2$

M: $2x - 4y = 8$

M: $3x + 6y = 0$

M: $x - 2y = 1$

39. Financial Planning Karen has just retired and finds she needs an additional \$10,000 per year to live on. Fortunately, she has a nest egg of \$90,000, which she can invest in somewhat risky B-rated bonds at 12% interest per year or in a well-known bank at 5% per year. How much money should she invest in each so that she realizes exactly \$10,000 in interest income each year?

40. Mixing Acid One solution is 20% HCl acid and another is 12% HCl acid. How many cubic centimeters of each solution should be mixed to obtain 100 cubic centimeters of a solution that is 15% HCl acid?

41. Attendance at a Dance A church group is planning a dance in the school auditorium to raise money for its school. The band they will hire charges \$500; the advertising costs are estimated at \$100; and food will be supplied at the rate of \$5 per person. The church group would like to clear at least \$900 after expenses.

- Determine how many people need to attend the dance for the group to break even if tickets are sold at \$10 each.
- Determine how many people need to attend in order to achieve the desired profit if tickets are sold for \$10 each.
- Answer the above two questions if the tickets are sold for \$12 each.

- 42. Mixing Coffee** A coffee manufacturer wants to market a new blend of coffee that will cost \$6.00 per pound by mixing \$5.00 per pound coffee and \$7.50 per pound coffee. What amounts of the \$5.00 per pound coffee and \$7.50 per pound coffee should be blended to obtain the desired mixture?

[Hint: Assume the total weight of the desired blend is 100 pounds.]

In Problems 43 and 44, draw a scatter diagram for each set of data. Then determine whether the relation that may exist is linear or nonlinear.

43.

x	0	1	2	3	4	5	6
y	90	45	21	12	5	3	2

44.

x	3	5	7	9	11	13
y	74	70	67	58	55	51

- 45. Supply and Demand** The supply and demand equations for corn are estimated to be

$$S = 0.8p + 0.2 \quad D = -0.4p + 1.8$$

where p is the price in dollars and S and D are in millions of bushels.

- Find the market price.
 - What quantity is supplied at this price?
 - Graph both S and D .
 - Interpret the point of intersection.
- 46. Concentration of Carbon Monoxide in the Air** The following data represent the average concentration of carbon monoxide in parts per million (ppm) in the air for 1992–2008.

Year	Concentration of Carbon Monoxide (ppm)
1992	5.47
1994	5.34
1996	4.44
1998	4.03
2000	3.51
2002	2.97
2004	2.57
2006	2.22
2008	1.88

Source: U.S. Environmental Protection Agency

- Treating the year as the x -coordinate and the average level of carbon monoxide as the y -coordinate, draw a scatter diagram of the data.
- What is the slope of the line joining the points (1992, 5.47) and (1998, 4.03)?
- Interpret this slope.
- What is the slope of the line joining the points (2000, 3.51) and (2008, 2.57)?
- Interpret this slope.
- Use a graphing utility to find the slope of the line of best fit for these data.
- Interpret this slope.
- How do you explain the differences among the three slopes obtained?
- What is the trend in the data? In other words, as time passes, what is happening to the average level of carbon monoxide in the air? Why do you think this is happening?

- 47. Housing Costs** The following data represent the mean price of houses sold in the United States for 1998–2008.

Year	Price (Dollars)
1998	181,900
1999	195,600
2000	207,000
2001	213,200
2002	228,700
2003	246,300
2004	274,500
2005	297,000
2006	305,900
2007	313,600
2008	292,600


- Treating the year as the x -coordinate and the price of the houses as the y -coordinate, draw a scatter diagram of the data.
- What is the slope of the line joining the points (1998, 181900) and (2002, 228700)?
- Interpret this slope.
- What is the slope of the line joining the points (2002, 228700) and (2008, 292600)?
- Interpret this slope.
- Use a graphing utility to find the slope of the line of best fit for these data.

- (g) Interpret this slope.
- (h) How do you explain the differences among the three slopes obtained?
- (i) What is the trend in the data? In other words, what is happening to the average price of a home in the United States? Why do you think this is happening?

Source: U.S. Census

48. **Value of a Portfolio** The following data represent the value of the Vanguard 500 Index Fund for 2003–2007.

Year	Value per Share
2003	102.67
2004	111.64
2005	114.92
2006	130.59
2007	135.15

- (a) Treating the year as the x -coordinate and the value of the Vanguard 500 Index Fund as the y -coordinate, draw a scatter diagram of the data.
- (b) Do the data appear to be linearly related?
- (c) What is the slope of the line connecting (2003, 102.67) and (2007, 135.15)?
- (d) Interpret the slope.
-  (e) Use a graphing utility to find the line of best fit for these data.

- (f) Assuming the line of best fit truly represents the trend in the data, predict the value of a share of Vanguard 500 Index Fund in the year 2008.

Source: The Vanguard Group, Inc.

49. **Value of a Portfolio** Investment ads typically contain statements such as “past performance is no guarantee of future performance.” Vanguard’s Web site shows the value of a share was \$83.09 in 2008 and \$102.67 in 2009.
- (a) Add these data to the chart in Problem 48 and draw a revised scatter diagram representing the years 2003–2009.
- (b) Do the data appear to be linearly related?
- (c) What would you say about the prediction you made in Problem 48(f)?
- (d) Statements like the one given in this problem are usually meant to warn investors that funds may not always do well. How does the statement apply in this problem?
50. Make up four problems that you might be asked to do given the two points $(-3, 4)$ and $(6, 1)$. Each problem should involve a different concept. Be sure that your directions are clearly stated.
51. Describe each of the following graphs in the xy -plane. Give justification.
- (a) $x = 0$
- (b) $y = 0$
- (c) $x + y = 0$

Chapter 1 Project



CHOOSING A CELLPHONE PLAN*

You have an important decision to make. You want to spend the least amount possible on your cell phone each month but still be able to talk to friends and family whenever you want. The plans found online have either unlimited minutes a month for a set price or they have a certain number of free minutes per month for a specific price and then a cost per minute if you exceed the allowable minutes. You decide to save money by not having texting or Web capabilities and will only compare plans for talk time. As it turns out, the equations involved are all linear, so the cell phone plans can be compared using techniques learned in this chapter.

You decide to compare two companies to start. Metro PCS offers unlimited minutes per month for a flat rate of \$60. This means that you can talk as much as you want and the bill will never change. Verizon offers a plan that costs \$39.99 per month for 450 free minutes and \$0.45 per each additional minute. Of course, if you talk less than 450 minutes a month, Verizon’s plan is the better deal. However, if you talk more than 450 minutes a month, at some point Metro PCS becomes the better deal.

To analyze the situation, let x denote the number of minutes you talk on the cell phone per month.

* All rates quoted have been taken from quotes in February 2010. There are other fees and taxes that are later added to each quote; these fees and taxes are ignored in our analysis.

- Suppose M is the cost per month for the Metro PCS plan. Find a linear equation involving M and x to represent the amount you will have to pay.
- Now let V be the cost per month for the Verizon plan. Find a linear equation involving V and x where $x \leq 450$. Also find a linear equation involving V and x where $x > 450$.
- Graph the three linear equations found in part (1) and part (2) on the same set of coordinate axes. Be careful about the restrictions on x for the equations found in part (2).
- Find the number of minutes at which Metro PCS becomes the cheaper plan each month by finding the point of intersection of two graphs. Label this point on your graph. Round to the nearest minute.
- Explain how you can use the solution in part (4) to see which plan is more economical.
- You are still not convinced that you have the best plan. You look up prices for T-Mobile plans and see that it will charge you \$49.99 a month for up to 650 minutes and \$0.50 for each additional minute. Let T be the cost per month for the T-Mobile plan. Find a linear equation that represents the cost if you talk less than 650 minutes a month and find a linear equation that represents the cost if you talk more than 650 minutes a month.
- Graph the two equations from part (6) on the same graph you found in part (3).
- It is now clear that if you talk less than 450 minutes a month, Verizon is the best plan to save money each month. Who has the cheapest plan if you talk 480 minutes a month?
- For what range of minutes is Metro PCS or T-Mobile the cheapest?
- If you only want to spend \$55 a month and want to use T-Mobile, how many minutes are you limited to per month over the allowable 650 minutes?
- If Metro PCS charges \$10 a month extra to cover the cost of the phone for one year and Verizon's phone is free, how many minutes can you talk a month for the year you are paying the extra charge with Metro PCS where Verizon is the better deal?
- Suppose a new job requires you to be on the phone between 2000 and 2400 minutes a month. You will need to do a lot of traveling and, unfortunately, the Metro PCS service does not have the nationwide coverage you will need. Discuss the suitability of the T-Mobile and Verizon cell phone plans for your new job.

Mathematical Questions from Professional Exams*

- CPA Exam** The Oliver Company plans to market a new product. Based on its market studies, Oliver estimates that it can sell 5500 units in 1992. The selling price will be \$2 per unit. Variable costs are estimated to be 40% of the selling price. Fixed costs are estimated to be \$6000. What is the break-even point?
 - 3750 units
 - 5000 units
 - 5500 units
 - 7500 units
- CPA Exam** The Breiden Company sells rodaks for \$6 per unit. Variable costs are \$2 per unit. Fixed costs are \$37,500. How many rodaks must be sold to realize a profit before income taxes of 15% of sales?
 - 9375 units
 - 9740 units
 - 11,029 units
 - 12,097 units
- CPA Exam** Given the following notations, what is the break-even sales level in units?

$$SP = \text{Selling price per unit}$$

$$FC = \text{Total fixed cost}$$

$$VC = \text{Variable cost per unit}$$
 - $\frac{SP}{FC \div VC}$
 - $\frac{FC}{VC \div SP}$
 - $\frac{VC}{SP - FC}$
 - $\frac{FC}{SP - VC}$

4. **CPA Exam** At a break-even point of 400 units sold, the variable costs were \$400 and the fixed costs were \$200. What will the 401st unit sold contribute to profit before income taxes?
- (a) \$0
 - (b) \$0.50
 - (c) \$1.00
 - (d) \$1.50
5. **CPA Exam** A graph is set up with “depreciation expense” on the vertical axis and “time” on the horizontal axis. Assuming linear relationships, how would the graphs for straight-line and sum-of-the-year’s-digits depreciation, respectively, be drawn?
- (a) Vertically and sloping down to the right
 - (b) Vertically and sloping up to the right
 - (c) Horizontally and sloping down to the right
 - (d) Horizontally and sloping up to the right
6. **CPA Exam** The relationship as shown above is
- (a) Parabolic
 - (b) Curvilinear
 - (c) Linear
 - (d) Probabilistic
 - (e) None of the above
7. **CPA Exam** Y in the above equation is an estimate of
- (a) Total variable costs
 - (b) Total factory overhead
 - (c) Total fixed costs
 - (d) Total direct labor hours
 - (e) None of the above
8. **CPA Exam** The \$2 in the equation is an estimate of
- (a) Total fixed costs
 - (b) Variable costs per direct labor hour
 - (c) Total variable costs
 - (d) Fixed costs per direct labor hour
 - (e) None of the above

The following statement applies to Questions 6–8:

In analyzing the relationship of total factory overhead with changes in direct labor hours, the following relationship was found to exist:

$$Y = \$1000 + \$2X.$$

