CHAPTER 1

BASIC CONCEPTS OF STRUCTURAL DYNAMICS

COPYRIGHTED MATERIAL

1.1 THE DYNAMIC ENVIRONMENT

Structural engineers are familiar with the analysis of structures for static loads in which a load is applied to the structure and a single solution is obtained for the resulting displacements and member forces. When considering the analysis of structures for dynamic loads, the term *dynamic* simply means "time-varying." Hence, the loading and all aspects of the response vary with time. This results in possible solutions at each instant during the time interval under consideration. From an engineering standpoint, the maximum values of the structural response are usually the ones of particular interest, especially when considering the case of structural design.

Two different approaches, which are characterized as either deterministic or nondeterministic, can be used for evaluating the structural response to dynamic loads. If the time variation of the loading is fully known, the analysis of the structural response is referred to as a *deterministic analysis*. This is the case even if the loading is highly oscillatory or irregular in character. The analysis leads to a time history of the displacements in the structure corresponding to the prescribed time history of the loading. Other response parameters such as member forces and relative member displacements are then determined from the displacement history.

If the time variation of the dynamic load is not completely known but can be defined in a statistical sense, the loading is referred to as a *random dynamic loading*, and the analysis is referred to as *nondeterministic*. The nondeterministic analysis provides information about the displacements in a statistical sense, which results from the statistically defined loading. Hence, the time variation of the displacements is not determined, and other response parameters must be evaluated directly from an independent nondeterministic analysis rather than from the displacement results. Methods for nondeterministic analysis are described in books on random vibration. In this text, we only discuss methods for deterministic analysis.

1.2 TYPES OF DYNAMIC LOADING

Most structural systems will be subjected to some form of dynamic loading during their lifetime. The sources of these loads are many and varied. The ones that have the most effect on structures can be classified as environmental loads that arise from winds, waves, and earthquakes. A second group of dynamic loads occurs as a result of equipment motions that arise in reciprocating and rotating machines, turbines, and conveyor systems. A third group is caused by the passage of vehicles and trucks over a bridge. Blast-induced loads can arise as the result of chemical explosions or breaks in pressure vessels or pressurized transmission lines.

For the dynamic analysis of structures, deterministic loads can be divided into two categories: periodic and nonperiodic. Periodic loads have the same time variation for a large number of successive cycles. The basic periodic loading is termed *simple harmonic* and has a sinusoidal variation. Other forms of periodic loading are often more complex and nonharmonic. However, these can be represented by summing a sufficient number of harmonic components in a Fourier series analysis. Nonperiodic loading varies from very short duration loads (air blasts) to long-duration loads (winds or waves). An air blast caused by some form of chemical explosion generally results in a high-pressure force having a very short duration (milliseconds). Special simplified forms of analysis may be used under certain conditions for this loading, particularly for design. Earthquake loads that develop in structures as a result of ground motions at the base can have a duration that varies from a few seconds to a few minutes. In this case, general dynamic analysis procedures must be applied. Wind loads are a function of the wind velocity and the height, shape, and stiffness of the structure. These characteristics give rise to aerodynamic forces that can be either calculated or obtained from wind tunnel tests. They are usually represented as equivalent static pressures acting on the surface of the structure.

1.3 BASIC PRINCIPLES

The fundamental physical laws that form the basis of structural dynamics were postulated by Sir Isaac Newton in the *Principia* (1687).[1] These laws are also known as *Newton's laws of motion* and can be summarized as follows:

First law: A particle of constant mass remains at rest or moves with a constant velocity in a straight line unless acted upon by a force.

Second law: A particle acted upon by a force moves such that the time rate of change of its linear momentum equals the force.

Third law: If two particles act on each other, the force exerted by the first on the second is equal in magnitude and opposite in direction to the force exerted by the second on the first.

Newton referred to the product of the mass, m, and the velocity, dv/dt, as the quantity of motion that we now identify as the *momentum*. Then Newton's second law of *linear momentum* becomes

$$\frac{d(m\dot{v})}{dt} = f \tag{1.1}$$

where both the momentum, $m(dv/dt)$, and the driving force, f, are functions of time. In most problems of structural dynamics, the mass remains constant, and Equation (1.1) becomes

$$m\left(\frac{d\dot{v}}{dt}\right) = ma = f \tag{1.2}$$

An exception occurs in rocket propulsion in which the vehicle is losing mass as it ascends. In the remainder of this text, time derivatives will be denoted by dots over a variable. In this notation, Equation (1.2) becomes $m\ddot{v} = f$.

[1] I. Newton, *The Principia: Mathematical Principles of Natural Philosophy*, 1687.

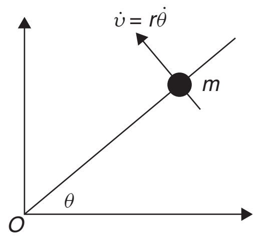

Figure 1.1 Rotation of a mass about a fixed point (F. Naeim, *The Seismic Design Handbook*, 2nd ed. (Dordrecht, Netherlands: Springer, 2001), reproduced with kind permission from Springer Science+Business Media B.V.)

Newton's second law can also be applied to rotational motion, as shown in Figure 1.1. The *angular momentum*, or moment of momentum, about an origin O can be expressed as

$$L = r(m\dot{v}) \tag{1.3}$$

where L = the angular momentum
r = the distance from the origin to the mass, m
$\dot{v}$ = the velocity of the mass

When the mass is moving in a circular arc about the origin, the angular speed is $\dot{\theta}$, and the velocity of the mass is $r\dot{\theta}$. Hence, the angular momentum becomes

$$L = mr^2\dot{\theta} \tag{1.4}$$

The time rate of change of the angular momentum equals the torque:

$$\text{torque} = N = \frac{dL}{dt} = mr^2\ddot{\theta} \tag{1.5}$$

If the quantity mr^2 is defined as the moment of inertia, I_θ, of the mass about the axis of rotation (mass moment of inertia), the torque can be expressed as

$$I_\theta\ddot{\theta} = N \tag{1.6}$$

where $d^2\theta/dt^2$ denotes the angular acceleration of the moving mass; in general, $I_\theta = \int \rho^2 dm$. For a uniform material of mass density μ, the mass moment of inertia can be expressed as

$$I_\theta = \mu \int \rho^2 dV \tag{1.7}$$

The rotational inertia about any reference axis, G, can be obtained from the parallel axis theorem as

$$I_G = I_\theta + mr^2 \tag{1.8}$$

Example 1.1 Consider the circular disk shown in Figure 1.2a. Determine the mass moment of inertia of the disk about its center if it has mass density (mass/unit volume) μ, radius r, and thickness t. Also determine the mass moment of inertia of a rectangular rod rotating about one end, as shown in Figure 1.2b. The mass density of the rod is μ, the dimensions of the cross section are $b \times d$, and the length is r.

$$I_0 = \mu \int \rho^2 dV \quad \text{where} \quad dV = \rho(d\theta)(d\rho)t$$

$$I_0 = \mu t \int_0^{2\pi} \int_0^r \rho^3 d\rho d\theta = \mu t \pi \frac{r^4}{2}$$

The mass of the circular disk is $m = \pi r^2 t \mu$.

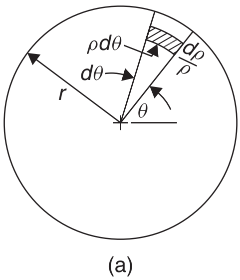

Figure 1.2a Circular disk

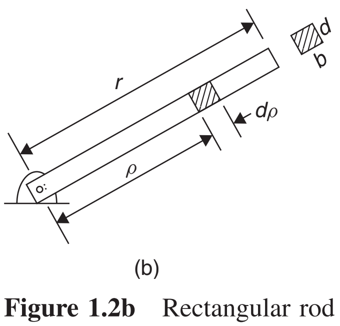

Figure 1.2b Rectangular rod

Hence, the mass moment of inertia of the disk becomes

$$I_0 = \frac{mr^2}{2}$$

$$I_0 = \mu \int \rho^2 dV \quad \text{where, for a rectangular rod:} \quad dV = (bd)d\rho$$

$$I_0 = \mu bd \int_0^r \rho^2 d\rho = \mu bd \frac{r^3}{3}$$

The mass of the rod is $m = bdr\mu$, and the mass moment of inertia of the rod becomes

$$I_0 = \frac{mr^2}{3}$$

The rigid-body mass properties of some common structural geometric shapes are summarized in Figure 1.3.

The difference between *mass* and *weight* is sometimes confusing, particularly to those taking a first course in structural dynamics. The mass, m, is a measure of the quantity of matter, whereas the weight, w, is a measure of the force necessary to impart a specified acceleration to a given mass. The acceleration of gravity, g, is the acceleration that the gravity of the earth would impart to a free-falling body at sea level, which is 32.17 ft/sec^2 or 386.1 in/sec^2. For engineering calculations, the acceleration of gravity is often rounded to 32.2 ft/sec^2, which results in 386.4 in/sec^2 when multiplied by 12 in/ft. Therefore, mass does not equal weight but is related by the expression $w = mg$. To keep this concept straight, it is helpful to carry units along with the mathematical operations.

The concepts of the *work* done by a force, and of the *potential and kinetic energies*, are important in many problems of dynamics. Multiply both sides of Equation (1.2) by *dv/dt* and integrate with respect to time:

$$\int_{t_1}^{t_2} f(t)\dot{v}dt = \int_{t_1}^{t_2} m\ddot{v}\dot{v}dt \tag{1.9}$$

Because $\dot{v}dt = dv$ and $\ddot{v}dt = d\dot{v}$, Equation (1.9) can be written as

$$\int_{v_1}^{v_2} f(t)dv = \frac{1}{2}m\left(\dot{v}_2^2 - \dot{v}_1^2\right) \tag{1.10}$$

The integral on the left side of Equation (1.10) is the area under the force-displacement curve and represents the work done by the force $f(t)$. The two

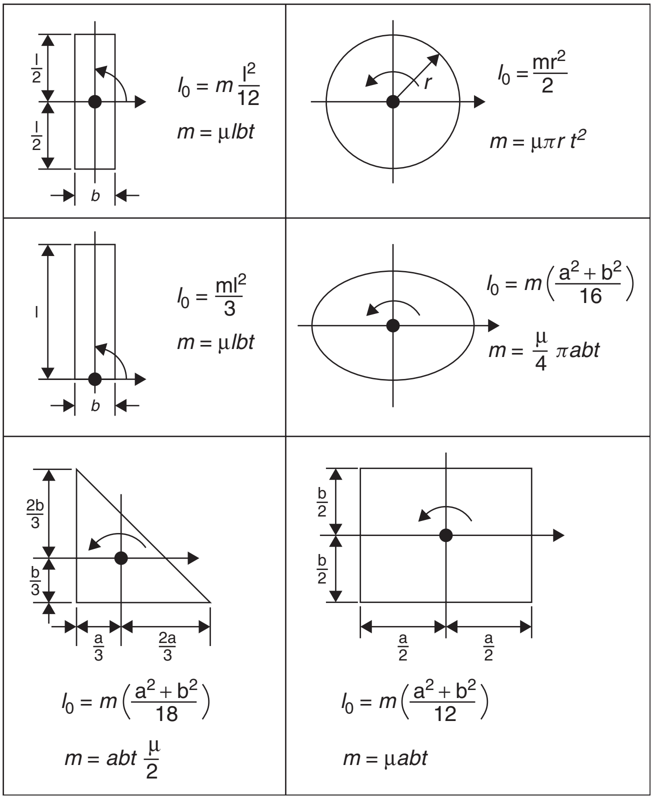

Figure 1.3 Transitional mass and mass moment of inertia (F. Naeim, *The Seismic Design Handbook*, 2nd ed. (Dordrecht, Netherlands: Springer, 2001), reproduced with kind permission from Springer Science+Business Media B.V.)

terms on the right represent the final and initial kinetic energies of the mass. Hence, the work done is equal to the change in kinetic energy.

Consider a force that is acting during the time interval (t_1, t_2). The integral $I = \int f(t)dt$ is defined as the impulse of the force during the time interval. According to Newton's second law of motion, $f = m\ddot{v}$. If both sides are integrated with respect to t,

$$I = \int_{t_1}^{t_2} f(t)dt = m\,(\dot{v}_2 - \dot{v}_1) \tag{1.11}$$

Hence, the impulse, I, is equal to the change in the momentum. This relation will be useful in analyzing the result of applying a large force for a brief interval of time as will be demonstrated in a later section.

Newton's laws of motion lead, in special circumstances, to the following three important properties of motion (conservation laws):

1. If the sum of the forces acting on a mass is zero, the linear momentum is constant in time.
2. If the sum of the external torques acting on a particle is zero, the angular momentum is constant in time.
3. In a conservative force field, the sum of the kinetic and potential energies remains constant during the motion.

It should be noted that nonconservative forces include frictional forces and forces that depend on velocity and time.

The term *degrees of freedom* in a dynamic system refers to the least number of displacement coordinates needed to define the motion of the system. If the physical system is represented as a continuum, an infinite number of coordinates would be needed to define the position of all the mass of the system. The system would thus have infinitely many degrees of freedom. In most structural systems, however, simplifying assumptions can be applied to reduce the degrees of freedom and still obtain an accurate determination of the displacement.

A *constraint* is a restriction on the possible deformed shape of a system, and a *virtual displacement* is an infinitesimal, imaginary change in configuration that is consistent with the constraints.

In 1717, Johann Bernoulli posed his *principle of virtual work*, which is basically a definition of equilibrium that applies to dynamic as well as static systems. The principle of virtual work states that if, for any arbitrary virtual displacement that is compatible with the system constraints, the virtual work under a set of forces is zero, then the system is in equilibrium. This principle can be restated in terms of virtual displacements—a form that is more applicable to structural systems. It states that if a system that is in equilibrium under a set of forces is subjected to a virtual displacement that is compatible with the system constraints, then the total work done by the forces is zero. The vanishing of the virtual work done is equivalent to a statement of equilibrium.

In his book *Traite de Dynamique* (1743)[2], the French mathematician Jean le Rond d'Alembert proposed a principle that would reduce a problem in dynamics to an equivalent one in statics. He developed the idea

[2]J. d'Alembert, *Traite de Dynamique*, 1743, available at http://www.archive.org/details/traitedynamiqu00dalgoog.

that mass develops an inertia force that is proportional to its acceleration and opposing it:

$$f_i = -m\ddot{v} \tag{1.12}$$

d'Alembert's principle also states that the applied forces together with the forces of inertia form a system in equilibrium.

1.4 DYNAMIC EQUILIBRIUM

The basic equation of static equilibrium used in the displacement method of analysis has the form

$$p = kv \tag{1.13}$$

where p = the applied force
k = the stiffness resistance
v = the resulting displacement

If the statically applied force is now replaced by a dynamic or time-varying force, $p(t)$, the equation of static equilibrium becomes one of dynamic equilibrium and has the form

$$p(t) = m\ddot{v}(t) + c\dot{v}(t) + kv(t) \tag{1.14}$$

where the dot represents differentiation with respect to time.

A direct comparison of these two equations indicates that two significant changes that distinguish the static problem from the dynamic problem were made to Equation (1.13) in order to obtain Equation (1.14). First, the applied load and the resulting response are now functions of time; hence, Equation (1.14) must be satisfied at each instant of time during the time interval under consideration. For this reason, it is usually referred to as an *equation of motion*. Second, the time dependence of the displacements gives rise to two additional forces that resist the applied force and have been added to the right side.

The first term is based on Newton's second law of motion and incorporates d'Alembert's concept of an inertia force that opposes the motion. The second term accounts for dissipative or damping forces that are inferred from the observed fact that oscillations in a structure tend to diminish with time once the time-dependent applied force is removed. These forces are generally represented by viscous damping forces that are proportional to the velocity with the constant of proportionality, c, referred to as the *damping coefficient*:

$$f_d = c\dot{v} \tag{1.15}$$

It must also be recognized that all structures are subjected to gravity loads such as self-weight (dead load) and occupancy load (live load) in addition to any dynamic loading. In an elastic system, the principle of superposition can be applied, so that the responses to static and dynamic loadings can be considered separately and then combined to obtain the total structural response. However, if the structural behavior becomes nonlinear, the response becomes dependent on the load path, and the gravity loads must be considered concurrently with the dynamic loading.

Under the action of severe dynamic loading, the structure will most likely experience nonlinear behavior, which can be caused by material nonlinearity and/or geometric nonlinearity. Material nonlinearity occurs when stresses at certain critical regions in the structure exceed the elastic limit of the material. The equation of dynamic equilibrium for this case has the general form

$$p(t) = m\ddot{v}(t) + c\dot{v}(t) + k(t)v(t) \tag{1.16}$$

where the stiffness or resistance, k, is a function of the yield condition in the structure, which, in turn, is a function of time. Geometric nonlinearity is caused by the gravity loads acting on the deformed position of the structure. If the lateral displacements are small, this effect, which is often referred to as the P-Δ *effect*, can be neglected. However, if the lateral displacements become large, this effect must be considered by augmenting the stiffness matrix, k, with the geometric stiffness matrix, k_g, which includes the effect of axial loads.

In order to define the inertia forces completely, it would be necessary to consider the acceleration of every mass particle in the structure and the corresponding displacement. Such a solution would be prohibitively complicated and time-consuming. The analysis procedure can be greatly simplified if the mass of the structure can be concentrated (lumped) at a finite number of discrete points and the dynamic response of the structure can be represented in terms of this limited number of displacement components (degrees of freedom). The number of degrees of freedom required to obtain an adequate solution will depend on the complexity of the structural system. For some structures, a single degree of freedom may be sufficient, whereas, for others, several hundred degrees of freedom may be required.

PROBLEMS

Problem 1.1

Determine the mass moment of inertia of the rectangular and triangular plates when they rotate about the hinges, as shown in Figure 1.4. Assume both plates have a constant thickness. Express your result in terms of the total system mass.

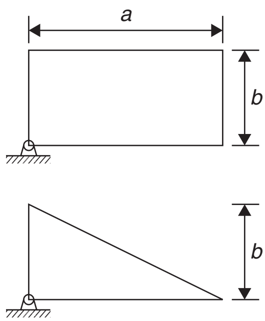

Figure 1.4