

FUNDAMENTAL PRINCIPLES

Convective heat transfer, or simply, *convection*, is the study of heat transport processes effected by the flow of fluids. The very word *convection* has its roots in the Latin verbs *convecto-are* and *convěho-věhěre* [1],* which mean to bring together or to carry into one place. Convective heat transfer has grown to the status of a contemporary science because of our need to understand and predict how a fluid flow acts as a “carrier” or “conveyor belt” for energy and matter.

Convective heat transfer is clearly a field at the interface between two older fields: heat transfer and fluid mechanics. To study the *interdisciplinary* is valuable, but it must come after one possesses the *disciplines*, not the other way around. For this reason, the study of any convective heat transfer problem must rest on a solid understanding of basic heat transfer and fluid mechanics principles. The objective in this chapter is to review these principles in order to establish a common language for the more specific issues addressed in later chapters.

Before reviewing the foundation of convective heat transfer methodology, it is worth reexamining the historic relationship between fluid mechanics and heat transfer. Especially during the past 100 years, heat transfer and fluid mechanics have enjoyed a symbiotic relationship in their development, a relationship where one field was stimulated by the curiosity and advance in the other field. Examples of this symbiosis abound in the history of boundary layer theory and natural convection. The field of convection grew out of this symbiosis, and if we are to learn anything from history, important advances in convection will continue to result from this symbiosis. Thus, the student and the future researcher would be well advised to devote equal attention to fluid mechanics and heat transfer literature.

*Numbers in brackets indicate references at the end of each chapter.

1.1 MASS CONSERVATION

The first principle to review is undoubtedly the oldest: It is the conservation of mass in a closed system or the “continuity” of mass through a flow (open) system. From engineering thermodynamics, we recall the mass conservation statement for a control volume [2]:

$$\frac{\partial M_{cv}}{\partial t} = \sum_{\text{inlet ports}} \dot{m} - \sum_{\text{outlet ports}} \dot{m} \quad (1.1)$$

where M_{cv} is the mass that is trapped instantaneously inside the control volume (cv), while the \dot{m} 's are the mass flow rates associated with flow into and out of the control volume. In convective heat transfer, we are usually interested in the velocity and temperature distribution in a flow region near a solid wall; hence, the control volume to consider is the infinitesimally small $\Delta x \Delta y$ box drawn around a fixed location (x, y) in a flow field. In Fig. 1.1, as in most of the problems analyzed in this book, the flow field is two-dimensional (i.e., the same in any plane parallel to the plane of Fig. 1.1). In a three-dimensional flow field, the control volume would be the parallelepiped $\Delta x \Delta y \Delta z$. Taking u and v as the local velocity components at point (x, y) , the mass conservation equation (1.1) requires that

$$\begin{aligned} \frac{\partial}{\partial t}(\rho \Delta x \Delta y) &= \rho u \Delta y + \rho v \Delta x - \left[\rho u + \frac{\partial(\rho u)}{\partial x} \Delta x \right] \Delta y \\ &\quad - \left[\rho v + \frac{\partial(\rho v)}{\partial y} \Delta y \right] \Delta x \end{aligned} \quad (1.2)$$

or, dividing through by the constant size of the control volume ($\Delta x \Delta y$),

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (1.3)$$

In a three-dimensional flow, an analogous argument yields

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (1.4)$$

where w is the velocity component in the z direction. The local mass conservation statement (1.4) can also be written as

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} + \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0 \quad (1.5)$$

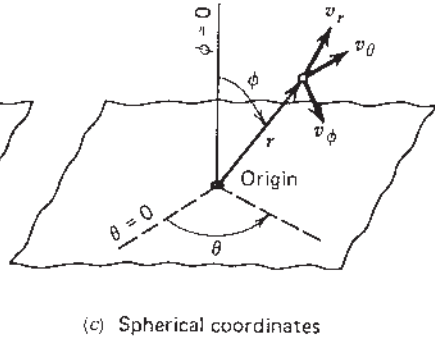
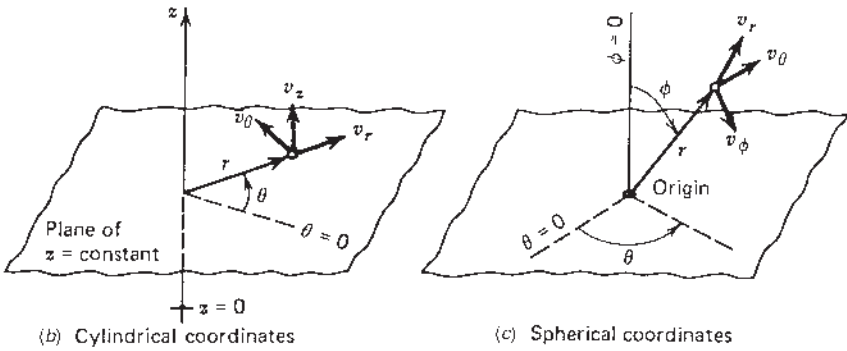
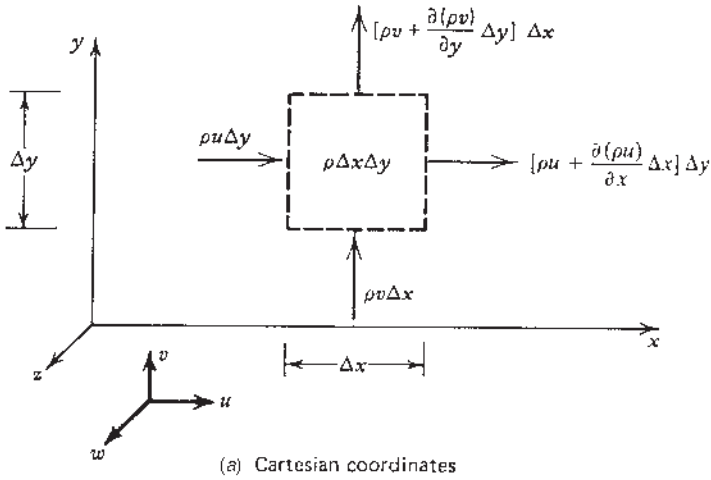


Figure 1.1 Mass conservation and systems of coordinates.

or

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0 \tag{1.6}$$

In expression (1.6), \mathbf{v} is the velocity vector (u, v, w), and D/Dt represents the “material derivative” operator,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \tag{1.7}$$

Of particular interest to *classroom* and *fundamental* treatment of the convection problem is the wide class of flows in which temporal and spatial variations

in density are negligible relative to the local variations in velocity. For this class, the mass conservation statement reads

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1.8)$$

The equivalent forms of eq. (1.8) in cylindrical and spherical coordinates are (Fig. 1.1)

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \quad (1.9)$$

and

$$\frac{1}{r} \frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{\sin \phi} \frac{\partial}{\partial \phi}(v_\phi \sin \phi) + \frac{1}{\sin \phi} \frac{\partial v_\theta}{\partial \theta} = 0 \quad (1.10)$$

It is tempting to think that eqs. (1.8)–(1.10) are valid only for incompressible fluids; in fact, their derivation shows that they apply to *flows* (not fluids) where the density and velocity gradients are such that the $D\rho/Dt$ terms are negligible relative to the $\rho \nabla \cdot \mathbf{v}$ terms in eq. (1.6). Most of the gas flows encountered in heat exchangers, heated enclosures, and porous media obey the simplified version of the mass conservation principle [eqs. (1.8)–(1.10)].

1.2 FORCE BALANCES (MOMENTUM EQUATIONS)

From the dynamics of thrust or propulsion systems, we recall that the instantaneous force balance on a control volume requires that (see Ref. 3, p. 15)

$$\frac{\partial}{\partial t}(Mv_n)_{cv} = \sum F_n + \sum_{\text{inlet ports}} \dot{m}v_n - \sum_{\text{outlet ports}} \dot{m}v_n \quad (1.11)$$

where n is the direction chosen for analysis and v_n and F_n are the projections of fluid velocity and forces in the n direction. Equation (1.11) is recognized in the literature as the *momentum principle* or *momentum theorem*. In essence, eq. (1.11) is the control volume formulation of Newton's second law of motion, where in addition to terms accounting for *forces* and *mass* \times *acceleration*, we now have the *impact* due to the flow of momentum into the control volume, plus the *reaction* associated with the flow of momentum out of the control volume. In the two-dimensional flow situation of Fig. 1.2, we can write two force balances of type (1.11), one for the x direction and the other for the y direction.

Consider now the special form taken by eq. (1.11) when applied to the finitesize control volume $\Delta x \Delta y$ drawn around point (x, y) in Fig. 1.2. Consider first the balance of forces in the x direction. In Fig. 1.2a, showing the $\Delta x \Delta y$ control volume, we see the sense of the impact and reaction forces associated with the flow of momentum through the control volume. In Fig. 1.2b, we see the more classical forces represented by the normal stress (σ_x), tangential stress

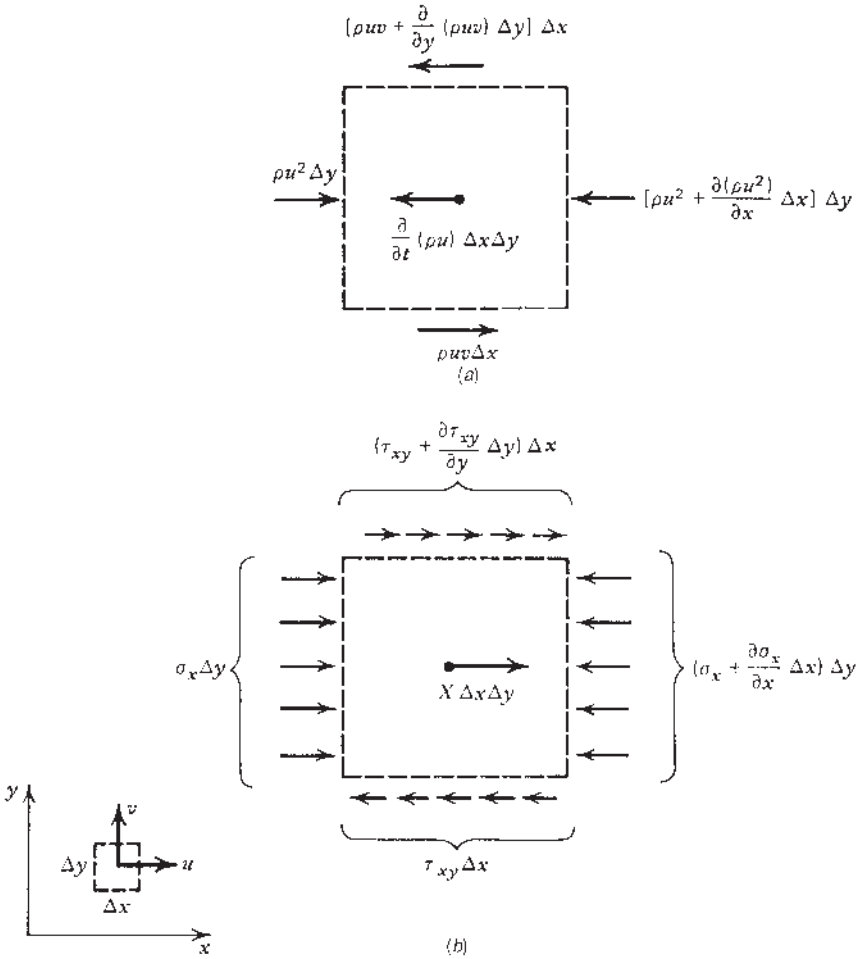


Figure 1.2 Force balance in the x direction on a control volume in two-dimensional flow.

(τ_{xy}), and the x -direction body force per unit volume (X). Projecting all these forces on the x axis, we obtain

$$\begin{aligned}
 & -\frac{\partial}{\partial t}(\rho u \Delta x \Delta y) + \rho u^2 \Delta y - \left[\rho u^2 + \frac{\partial}{\partial x}(\rho u^2) \Delta x \right] \Delta y \\
 & + \rho uv \Delta x - \left[\rho uv + \frac{\partial}{\partial y}(\rho uv) \Delta y \right] \Delta x \\
 & + \sigma_x \Delta y - \left(\sigma_x + \frac{\partial \sigma_x}{\partial x} \Delta x \right) \Delta y - \tau_{xy} \Delta x \\
 & + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} \Delta y \right) \Delta x + X \Delta x \Delta y = 0
 \end{aligned} \tag{1.12}$$

or, dividing by $\Delta x \Delta y$ in the limit $(\Delta x, \Delta y) \rightarrow 0$,

$$\rho \frac{Du}{Dt} + u \left[\frac{D\rho}{Dt} + \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] = -\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X \quad (1.13)$$

According to the mass conservation equation (1.6), the quantity in brackets is equal to zero; hence,

$$\rho \frac{Du}{Dt} = -\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X \quad (1.14)$$

Next, we relate the stresses σ_x and τ_{xy} to the local flow field by recalling the constitutive relations

$$\sigma_x = P - 2\mu \frac{\partial u}{\partial x} + \frac{2}{3}\mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \quad (1.15)$$

$$\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (1.16)$$

These relations are of empirical origin: They summarize the experimental observation that a fluid packet offers no resistance to a change of shape but resists the time rate of a change of shape. Equations (1.15) and (1.16) serve as definition for the measurable coefficient of *viscosity* μ . Combining eqs. (1.14)–(1.16) yields the *Navier—Stokes equation*,

$$\begin{aligned} \rho \frac{Du}{Dt} = & -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left[2\mu \frac{\partial u}{\partial x} - \frac{2\mu}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \\ & + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + X \end{aligned} \quad (1.17)$$

Of particular interest is the case when the *flow* may be treated as *incompressible* and the viscosity μ may be regarded as constant. Then the *x momentum equation* reduces to

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + X \quad (1.18)$$

A similar equation can be derived from the force balance in the *y* direction. For a three-dimensional flow in the (x, y, z) , (u, v, w) Cartesian system, the three momentum equations for $(\rho, \mu) \cong \text{constant}$ flows are

$$\begin{aligned} \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \\ = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + X \end{aligned} \quad (1.19a)$$

$$\begin{aligned} \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) \\ = -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + Y \end{aligned} \quad (1.19b)$$

$$\begin{aligned} \rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) \\ = -\frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + Z \end{aligned} \quad (1.19c)$$

Alternative forms of eqs. (1.19) are:

Vectorial notation:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla P + \mu \nabla^2 \mathbf{v} + \mathbf{F} \quad (1.20)$$

where \mathbf{F} is the body force vector per unit volume (X, Y, Z).

Cylindrical coordinates (Fig. 1.1b):

$$\begin{aligned} \rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) \\ = -\frac{\partial P}{\partial r} + \mu \left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right) + F_r \end{aligned} \quad (1.21a)$$

$$\begin{aligned} \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) \\ = -\frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right) + F_\theta \end{aligned} \quad (1.21b)$$

$$\begin{aligned} \rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) \\ = -\frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + F_z \end{aligned} \quad (1.21c)$$

where (v_r, v_θ, v_z) and (F_r, F_θ, F_z) are the velocity and body force vectors.

Spherical coordinates (Fig. 1.1c):

$$\begin{aligned} \rho \left(\frac{Dv_r}{Dt} - \frac{v_\phi^2 + v_\theta^2}{r} \right) \\ = -\frac{\partial P}{\partial r} + \mu \left(\nabla^2 v_r - \frac{2v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\phi}{\partial \phi} - \frac{2v_\phi \cot \phi}{r^2} - \frac{2}{r^2 \sin \phi} \frac{\partial v_\theta}{\partial \theta} \right) + F_r \end{aligned} \quad (1.22a)$$

$$\begin{aligned} & \rho \left(\frac{Dv_\phi}{Dt} + \frac{v_r v_\phi}{r} - \frac{v_\theta^2 \cot \phi}{r} \right) \\ &= -\frac{1}{r} \frac{\partial P}{\partial \phi} + \mu \left(\nabla^2 v_\phi + \frac{2}{r^2} \frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r^2 \sin^2 \phi} - \frac{2 \cos \phi}{r^2 \sin^2 \phi} \frac{\partial v_\theta}{\partial \theta} \right) + F_\phi \end{aligned} \quad (1.22b)$$

$$\begin{aligned} & \rho \left(\frac{Dv_\theta}{Dt} + \frac{v_\theta v_r}{r} + \frac{v_\phi v_\theta \cot \phi}{r} \right) \\ &= -\frac{1}{r \sin \phi} \frac{\partial P}{\partial \theta} + \mu \left(\nabla^2 v_\theta - \frac{v_\theta}{r^2 \sin^2 \phi} + \frac{2}{r^2 \sin \phi} \frac{\partial v_r}{\partial \theta} \right. \\ & \quad \left. + \frac{2 \cos \phi}{r^2 \sin^2 \phi} \frac{\partial v_\phi}{\partial \theta} \right) + F_\theta \end{aligned} \quad (1.22c)$$

where (v_r, v_ϕ, v_θ) and (F_r, F_ϕ, F_θ) are the velocity and body force vectors, and

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r} + \frac{v_\phi}{r} \frac{\partial}{\partial \phi} + \frac{v_\theta}{r \sin \phi} \frac{\partial}{\partial \theta} \quad (1.23)$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \phi} \frac{\partial}{\partial \phi} \left(\sin \phi \frac{\partial}{\partial \phi} \right) + \frac{1}{r^2 \sin^2 \phi} \frac{\partial^2}{\partial \theta^2} \quad (1.24)$$

are the material derivative and Laplacian operators in spherical coordinates.

1.3 FIRST LAW OF THERMODYNAMICS

The preceding two principles—mass conservation and force balances—are in many cases sufficient for solving the *flow part* of the convective heat transfer problem. Note at this juncture the availability of four equations (mass conservation plus three force balances) for determining four unknowns (three velocity components plus pressure). The exception to this statement is the subject of Chapter 4, where the natural flow is driven by the heat administered to the flowing fluid. In all cases, however, the *heat transfer part* of the convection problem requires a solution for the temperature distribution through the flow, especially in the close vicinity of the solid walls bathed by the heat-carrying fluid stream (Chapter 2). The additional equation for accomplishing this ultimate objective is the first law of thermodynamics or the *energy equation*.

For the control volume of finite size $\Delta x \Delta y$ in Fig. 1.3, the first law of thermodynamics requires that (see Ref. 2, p. 22)

$$\begin{aligned}
 & \left(\text{rate of energy accumulation in the control volume} \right)_1 \\
 &= \left(\text{net transfer of energy by fluid flow} \right)_2 + \left(\text{net heat transfer by conduction} \right)_3 \\
 &+ \left(\text{rate of internal heat generation (e.g., electrical power dissipation)} \right)_4 - \left(\text{net work transfer from the control volume to its environment} \right)_5 \quad (1.25)
 \end{aligned}$$

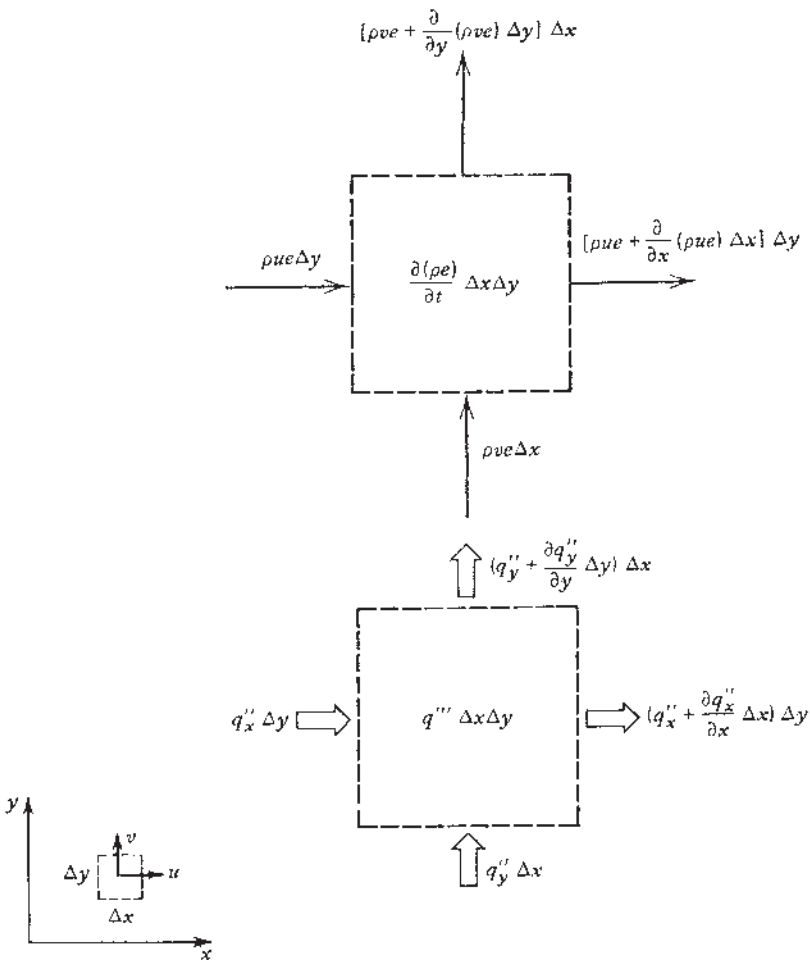


Figure 1.3 First law of thermodynamics applied to a control volume in two-dimensional flow (for work transfer, see Fig. 1.2).

According to the energy flows sketched in Fig. 1.3, the groups of terms above are

$$\begin{aligned}
 \{\cdot\}_1 &= \Delta x \Delta y \frac{\partial}{\partial t} (\rho e) \\
 \{\cdot\}_2 &= -(\Delta x \Delta y) \left[\frac{\partial}{\partial x} (\rho u e) + \frac{\partial}{\partial y} (\rho v e) \right] \\
 \{\cdot\}_3 &= -(\Delta x \Delta y) \left(\frac{\partial q''_x}{\partial x} + \frac{\partial q''_y}{\partial y} \right) \\
 \{\cdot\}_4 &= (\Delta x \Delta y) q''' \\
 \{\cdot\}_5 &= (\Delta x \Delta y) \left(\sigma_x \frac{\partial u}{\partial x} - \tau_{xy} \frac{\partial u}{\partial y} + \sigma_y \frac{\partial v}{\partial y} - \tau_{yx} \frac{\partial v}{\partial x} \right) \\
 &\quad + (\Delta x \Delta y) \left(u \frac{\partial \sigma_x}{\partial x} - u \frac{\partial \tau_{xy}}{\partial y} + v \frac{\partial \sigma_y}{\partial y} - v \frac{\partial \tau_{yx}}{\partial x} \right)_*
 \end{aligned} \tag{1.25'}$$

where e , q''_x , q''_y , and q''' are the specific internal energy, heat flux in the x direction, heat flux in the y direction, and dissipation rate or rate of internal heat generation.

The origin of the dissipation rate term $\{\cdot\}_5$ lies in the work transfer effected by the normal and tangential stresses sketched in Fig. 1.2*b*. For example, the work done per unit time by the normal stresses σ_x on the left side of the $\Delta x \Delta y$ element is negative and equal to the force acting on the boundary ($\sigma_x \Delta y$) times the boundary displacement per unit time (u), which yields $-\sigma_x u \Delta y$. Similarly, the work transfer rate associated with normal stresses acting on the right side of the element is positive and equal to $[\sigma_x + (\partial \sigma_x / \partial x) \Delta x][u + (\partial u / \partial x) \Delta x] \Delta y$. The net work transfer rate due to these two contributions is $[\sigma_x (\partial u / \partial x) + u (\partial \sigma_x / \partial x)] (\Delta x \Delta y)$, as shown in the $\{\cdot\}_5$ term of eq. (1.25').

Three more work transfer rates can be calculated in the same manner by examining the effect of the remaining three stresses, τ_{xy} in the x direction and σ_y and τ_{yx} in the y direction. In the $\{\cdot\}_5$ expression above, the eight terms have been separated into two groups. It can be shown that the group denoted as $(\cdot)_*$ reduces to $-\rho(D/Dt)(u^2 + v^2)/2$, which represents the change in kinetic energy of the fluid packet; in the present treatment, this change is considered negligible relative to the internal energy change $\partial(\rho e)/\partial t$ appearing in $\{\cdot\}_1$.

Assembling expressions (1.25') into the energy conservation statement that preceded them, and using constitutive relations (1.15) and (1.16), we obtain

$$\rho \frac{De}{Dt} + e \left(\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} \right) = -\nabla \cdot \mathbf{q}'' + q''' - P \nabla \cdot \mathbf{v} + \mu \Phi \tag{1.26}$$

where \mathbf{q}'' is the heat flux vector (q''_x, q''_y) and Φ is the *viscous dissipation function*, shown later in eq. (1.45a). The quantity between parentheses on the left-hand side of eq. (1.26) is equal to zero [cf. eq. (1.6)]. In the special case where the flow

can be modeled as incompressible and two-dimensional, the viscous dissipation function reduces to

$$\Phi = 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \quad (1.27)$$

To express eq. (1.26) in terms of enthalpy, we use the thermodynamics definition $h = e + (1/\rho)P$; hence,

$$\frac{Dh}{Dt} = \frac{De}{Dt} + \frac{1}{\rho} \frac{DP}{Dt} - \frac{P}{\rho^2} \frac{D\rho}{Dt} \quad (1.28)$$

In addition, we can express the directional heat fluxes q''_x and q''_y in terms of the local temperature gradients, by invoking the *Fourier law of heat conduction*,

$$\mathbf{q}'' = -k \nabla T \quad (1.29)$$

Then, combining eqs. (1.26), (1.28), and (1.29) we obtain

$$\rho \frac{Dh}{Dt} = \nabla \cdot (k \nabla T) + q''' + \frac{DP}{Dt} + \mu \Phi - \frac{P}{\rho} \left(\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} \right) \quad (1.30)$$

Finally, the mass conservation equation (1.6) shows that the last terms in parentheses in eq. (1.30) add up to zero, and the first law of thermodynamics reduces to

$$\rho \frac{Dh}{Dt} = \nabla \cdot (k \nabla T) + q''' + \frac{DP}{Dt} + \mu \Phi \quad (1.31)$$

In order to express the energy equation (1.31) in terms of temperature, it is tempting to replace the specific enthalpy on the left-hand side by the product of specific heat \times temperature. This move is correct only in cases where the fluid behaves like an ideal gas (see the ideal gas model, Table 1.1). In general, the change in specific enthalpy for a single-phase substance is expressed by the canonical relation for enthalpy [2],

$$dh = T ds + \frac{1}{\rho} dP \quad (1.32)$$

where T is the absolute temperature and ds the specific entropy change,

$$ds = \left(\frac{\partial s}{\partial T} \right)_P dT + \left(\frac{\partial s}{\partial P} \right)_T dP \quad (1.33)$$

From the last of Maxwell's relations (see Ref. 2, p. 172), we have

$$\left(\frac{\partial s}{\partial P} \right)_T = - \left[\frac{\partial (1/\rho)}{\partial T} \right]_P = \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial T} \right)_P = - \frac{\beta}{\rho} \quad (1.34)$$

Table 1.1 Summary of thermodynamic relations^a and models

	Internal Energy $du = T ds - P dv$	Enthalpy $dh = T ds + v dP$	Entropy $ds = \frac{1}{T} du + \frac{P}{T} dv$
Pure substance	$du = c_v dT + \left[T \left(\frac{\partial P}{\partial T} \right)_v - P \right] dv$	$dh = c_p dT + \left[-T \left(\frac{\partial v}{\partial T} \right)_p + v \right] dP$	$ds = \frac{c_p}{T} dT - \left(\frac{\partial v}{\partial T} \right)_p dP$
Ideal gas	$du = c_v dT$	$dh = c_p dT$	$ds = c_p \frac{dT}{T} - R \frac{dP}{P}$ $= c_v \frac{dT}{T} + R \frac{dv}{v}$ $= c_v \frac{dP}{P} + c_p \frac{dv}{v}$
Incompressible liquid	$du = c dT$	$dh = c dT + v dP$	$ds = c \frac{dT}{T}$

Source: Ref. 2.

^aAccording to the classical thermodynamics notation, v is the specific volume, $v = 1/\rho$, and u is the internal energy (e in the text).

where β is the coefficient of thermal expansion,

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \tag{1.35}$$

Table 1.1 also shows that

$$\left(\frac{\partial s}{\partial T} \right)_p = \frac{c_p}{T} \tag{1.36}$$

Together, eqs. (1.32)–(1.36) state that

$$dh = c_p dT + \frac{1}{\rho} (1 - \beta T) dP \tag{1.37}$$

in other words, the left-hand side of the energy equation (1.31) is

$$\rho \frac{Dh}{Dt} = \rho c_p \frac{DT}{Dt} + (1 - \beta T) \frac{DP}{Dt} \tag{1.38}$$

The ‘‘temperature’’ formulation of the first law of thermodynamics is therefore

$$\rho c_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + q''' + \beta T \frac{DP}{Dt} + \mu \Phi \tag{1.39}$$

with the following special forms:

Ideal gas ($\beta = 1/T$):

$$\rho c_P \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + q''' + \frac{DP}{Dt} + \mu \Phi \quad (1.40)$$

Incompressible liquid ($\beta = 0$):

$$\rho c \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + q''' + \mu \Phi \quad (1.41)$$

Most of the convection problems addressed in this book obey an even simpler model: namely, constant fluid conductivity k , zero internal heat generation q''' , negligible viscous dissipation $\mu \Phi$, and negligible compressibility effect $\beta T DP/Dt$. The energy equation for this model is

$$\rho c_P \frac{DT}{Dt} = k \nabla^2 T \quad (1.42)$$

or, in terms of specific coordinate systems (Fig. 1.1):

Cartesian (x, y, z):

$$\rho c_P \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (1.43a)$$

Cylindrical (r, θ, z):

$$\begin{aligned} \rho c_P \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) \\ = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] \end{aligned} \quad (1.43b)$$

Spherical (r, ϕ, θ):

$$\begin{aligned} \rho c_P \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\phi}{r} \frac{\partial T}{\partial \phi} + \frac{v_\theta}{r \sin \phi} \frac{\partial T}{\partial \theta} \right) \\ = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \phi} \frac{\partial}{\partial \phi} \left(\sin \phi \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin^2 \phi} \frac{\partial^2 T}{\partial \theta^2} \right] \end{aligned} \quad (1.43c)$$

If the fluid can be modeled as an incompressible liquid, then, as in eq. (1.41), the specific heat at constant pressure c_P is replaced by the lone specific heat of the incompressible liquid, c (Table 1.1).

When dealing with extremely viscous flows of the type encountered in lubrication problems or the piping of crude oil, the model above is improved by taking into account the internal heating due to viscous dissipation,

$$\rho c_P \frac{DT}{Dt} = k \nabla^2 T + \mu \Phi \quad (1.44)$$

In three dimensions, the viscous dissipation function is expressed as follows:

Cartesian (x, y, z):

$$\begin{aligned} \Phi = & 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \\ & + \left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 \right] \\ & - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 \end{aligned} \quad (1.45a)$$

Cylindrical (r, θ, z):

$$\begin{aligned} \Phi = & 2 \left[\left(\frac{\partial v_r}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right] \\ & + \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right)^2 + \frac{1}{2} \left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right)^2 \\ & + \frac{1}{2} \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right)^2 - \frac{1}{3} (\nabla \cdot \mathbf{v})^2 \end{aligned} \quad (1.45b)$$

Spherical (r, ϕ, θ):

$$\begin{aligned} \Phi = & 2 \left\{ \left[\left(\frac{\partial v_r}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} \right)^2 + \left(\frac{1}{r \sin \phi} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} + \frac{v_\phi \cot \phi}{r} \right)^2 \right] \right. \\ & + \frac{1}{2} \left[r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \phi} \right]^2 + \frac{1}{2} \left[\frac{\sin \phi}{r} \frac{\partial}{\partial \phi} \left(\frac{v_\theta}{r \sin \phi} \right) + \frac{1}{r \sin \phi} \frac{\partial v_\theta}{\partial \theta} \right]^2 \\ & \left. + \frac{1}{2} \left[\frac{1}{r \sin \phi} \frac{\partial v_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right]^2 \right\} - \frac{2}{3} (\nabla \cdot \mathbf{v})^2 \end{aligned} \quad (1.45c)$$

If the density does not vary significantly through the flow field, $\nabla \cdot \mathbf{v} = 0$ [eq. (1.6)] and the last term in each of expressions (1.45) vanishes.

It is worth reviewing the constant- ρ approximation that led to eq. (1.8) and recognizing that it differs conceptually from the “incompressible substance model” of thermodynamics. The latter is considerably more restrictive than the “nearly constant” density model, eq. (1.8). For example, a compressible substance such as air can flow in such a way that eq. (1.8) is a very good approximation of eq. (1.6).

For the restrictive class of fluids that are “incompressible” from the thermodynamic point of view, the specific heat at constant pressure c_p can be replaced by the lone specific heat of the fluid, c , on the left side of eq. (1.39). Water, liquid mercury, and engine oil are examples of fluids for which this substitution is justified. There are even convection problems in which the moving materials are actually solid (e.g., a roller and its substrate, in the zone of elastic contact). In such cases the $c_p = c$ substitution is permissible also.

Note that the specific heat at constant volume c_v does not belong on the left side of eq. (1.39). This observation is important because Fourier [4, 5], and later Poisson [6], who were the first to derive the energy equation for a convective flow, wrote c on the left side of eq. (1.39). They made this choice because their analyses were aimed specifically at *incompressible fluids* (liquids), for which c happens to have nearly the same value as c_p . Because of this choice, they did not have to account for the $P dV$ type of work done by the fluid packet as it expands or contracts in the flow field. In the modern era, however, the use of c_v instead of c_p is an error.

The prethermodynamics (caloric conservation) origins of the science of convection are also responsible for the “thermal energy equation” label that some prefer to attach to eq. (1.39) without the $\beta T DP/Dt$ term. This terminology is sometimes used to stress (incorrectly) the conservation of “thermal” energy as something distinct from “mechanical and thermal” energy. In classical thermodynamics, however, this distinction disappeared when the first law of thermodynamics was enunciated, that is, when the thermodynamic property “energy” was defined, which happened in the years 1850–1851 (see Ref. 2, pp. 28–29).

Equation (1.39) represents the first law of thermodynamics. This law proclaims the conservation of the sum of energy change (the property) and energy interactions (heat transfer *and* work transfer). The suggestion that mechanical effects (e.g., work transfer) are absent from eq. (1.39) when the $\beta T DP/Dt$ term is absent is wrong. The presence of c_p on the left side of the equation is the sign that each fluid packet expands or contracts (i.e., it does $P dV$ -type work) as it rides on the flow. The terms q''' and $\mu\Phi$ are work transfer rate terms also.

1.4 SECOND LAW OF THERMODYNAMICS

Any discussion of the basic principles of convective heat transfer must include the second law of thermodynamics, not because the second law is necessary for determining the flow and temperature field (it is not, because it is not an

equation), but because the second law is the basis for much of the engineering *motive* (objective, purpose) for formulating and solving convection problems. For example, in the development of knowhow for the heat exchanger industry, we strive for improved thermal contact (enhanced heat transfer) *and* reduced pump power loss in order to improve the *thermodynamic* efficiency of the heat exchanger. Good heat exchanger design means, ultimately, efficient thermodynamic performance, that is, *minimum generation of entropy* or minimum destruction of exergy in the power/refrigeration system incorporating the heat exchanger [7, 8].

The second law of thermodynamics states that all real-life processes are irreversible: In the case of a control volume, as in Fig. 1.1, this statement is

$$\frac{\partial S_{cv}}{\partial t} \geq \sum \frac{q_i}{T_i} + \sum_{\text{inlet ports}} \dot{m}s - \sum_{\text{outlet ports}} \dot{m}s \quad (1.46)$$

where S_{cv} is the instantaneous entropy inventory of the control volume, $\dot{m}s$ represents the entropy flows (streams) into and out of the control volume, and T_i is the absolute temperature of the boundary crossed by the heat transfer q_i .^{*} The irreversibility of the process is measured by the strength of the inequality sign in eq. (1.46), or by the entropy generation rate S_{gen} , defined as

$$S_{gen} = \frac{\partial S_{cv}}{\partial t} - \sum \frac{q_i}{T_i} - \sum_{\text{inlet ports}} \dot{m}s + \sum_{\text{outlet ports}} \dot{m}s \geq 0 \quad (1.47)$$

One can show that the rate of destruction of useful work in an engineering system, W_{lost} , is directly proportional to the rate of entropy generation [2, 3, 7],

$$W_{lost} = T_0 S_{gen} \quad (1.48)$$

where T_0 is the absolute temperature of the ambient temperature reservoir ($T_0 = \text{constant}$). Equation (1.48) stresses the engineering importance of estimating the irreversibility or entropy generation rate of convective heat transfer processes: If not used wisely, these processes contribute to the waste of precious fuel resources.

Based on an analysis similar to the analyses presented for mass conservation, force balances, and the first law of thermodynamics, the second law (1.47) may be applied to a finite-size control volume $\Delta x \Delta y \Delta z$ at an arbitrary point (x, y, z) in a flow field. Thus, the rate of entropy generation per unit time and per unit volume S'''_{gen} is [2, 3, 7]

$$S'''_{gen} = \underbrace{\frac{k}{T^2}(\nabla T)^2}_{\geq 0} + \underbrace{\frac{\mu}{T}\Phi}_{\geq 0} \geq 0 \quad (1.49)$$

^{*}Defined as positive *into* the control volume.

where k and μ are assumed constant. In a two-dimensional convection situation such as in Figs. 1.1–1.3, the local entropy generation rate (1.49) yields

$$S''_{\text{gen}} = \frac{k}{T^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T} \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \geq 0 \quad (1.50)$$

In the last two equations, T represents the *absolute* temperature of the point where S''_{gen} is being evaluated. The two-dimensional expression (1.50) illustrates the competition between viscous dissipation and imperfect thermal contact (finite-temperature gradients) in the generation of entropy via convective heat transfer.

Equations (1.48) and (1.50) constitute the bridge between two research activities: fundamental convection heat transfer and thermodynamics (entropy generation minimization). Beginning with Chapter 2, we focus on the fundamental problems of determining the flow and temperature fields in a given convection heat transfer configuration. However, through eq. (1.50), we are invited to keep in mind that these fields contribute hand-in-hand to downgrading the thermodynamic merit of the engineering device that ultimately employs the convection process under consideration. The science of adjusting the convection process so that it destroys the least exergy (subject to various system constraints) is the focus of *entropy generation minimization*; this activity has been reviewed in Refs. 2, 3, and 7. The generation of flow configuration (geometry, architecture) for maximal performance under constraints is *constructal theory and design* [2, 9–14].

1.5 RULES OF SCALE ANALYSIS

This section is designed to introduce the student to the problem-solving method of *scale analysis* or *scaling*. This is necessary because scale analysis is used extensively throughout the book; in fact, scale analysis is recommended as the premier method for obtaining the most information per unit of intellectual effort. This section is also necessary because scale analysis is not discussed in the heat transfer and fluid mechanics textbooks of our time, despite the fact that it is a precondition for good analysis in dimensionless form. Scale analysis is often confused with dimensional analysis or the often arbitrary nondimensionalization of the governing equations before performing a perturbation analysis or a numerical simulation on the computer.

The object of scale analysis is to use the basic principles of convective heat transfer to produce order-of-magnitude estimates for the quantities of interest. This means that if one of the quantities of interest is the thickness of the boundary layer in forced convection, the object of scale analysis is to determine whether the boundary layer thickness is measured in millimeters or meters. Note that scale

analysis goes beyond dimensional analysis (whose objective is to determine the dimension of boundary layer thickness, namely, length). When done properly, scale analysis anticipates within a factor of order one (or within percentage points) the expensive results produced by “exact” analyses. The value of scale analysis is remarkable, particularly when we realize that the notion of “exact analysis” is as false and ephemeral as the notion of “experimental fact.”

As the first example of scale analysis, consider a problem from the field of conduction heat transfer [15]. In Fig. 1.4 we see a plate plunged at $t = 0$ into a highly conducting fluid, such that the surfaces of the plate instantaneously assume the fluid temperature $T_\infty = T_0 + \Delta T$. Suppose that we are interested in estimating the time needed by the thermal front to penetrate the plate, that is, the time until the center plane of the plate “feels” the heating imposed on the outer surfaces.

To answer the question above, we focus on a half-plate of thickness $D/2$ and the energy equation for pure conduction in one direction:

$$\rho c_P \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \quad (1.51)$$

Next, we estimate the order of magnitude of each of the terms appearing in eq. (1.51). On the left-hand side we have

$$\rho c_P \frac{\partial T}{\partial t} \sim \rho c_P \frac{\Delta T}{t} \quad (1.52)$$

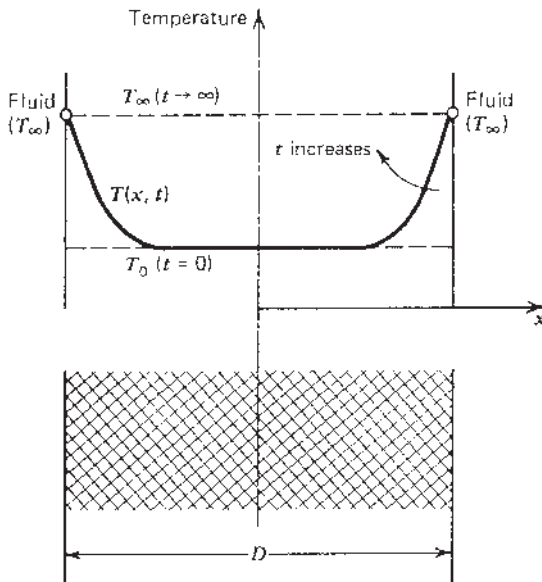


Figure 1.4 Transient heat conduction in a slab with sudden temperature change on the boundaries.

In other words, the scale of the temperature change (in the chosen space and in a time of order t) is ΔT . On the right-hand side we obtain

$$k \frac{\partial^2 T}{\partial x^2} = k \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) \sim \frac{k}{D/2} \frac{\Delta T}{D/2} = \frac{k \Delta T}{(D/2)^2} \quad (1.53)$$

Equating the two orders of magnitude (1.52) and (1.53), as required by the energy equation (1.51), we find the answer

$$t \sim \frac{(D/2)^2}{\alpha} \quad (1.54)$$

where α is the thermal diffusivity of the medium, $k/\rho c_p$. The penetration time (1.54) compares well with any order-of-magnitude interpretation of the exact solution to this classical problem [15]. However, the time and effort associated with deriving eq. (1.54) do not compare with the labor required by Fourier analysis and the graphical presentation of Fourier series.

Based on this introductory example, the following rules of scale analysis are worth teaching:

- *Rule 1.* Always define the spatial extent of the region in which you perform the scale analysis. In the example of Fig. 1.4, the size of the region of interest is $D/2$. In other problems, such as boundary layer flow, the size of the region of interest is unknown; as shown in Chapter 2, the scale analysis begins by selecting the region and by labeling the unknown thickness of this region δ . Any scale analysis of a flow or a flow region that is not uniquely defined is nonsense.
- *Rule 2.* One equation constitutes an equivalence between the scales of two dominant terms appearing in the equation. In the transient conduction example of Fig. 1.4, the left-hand side of eq. (1.51) could only be of the same order of magnitude as the right-hand side. The two terms appearing in eq. (1.51) are the dominant terms (considering that the discussion referred to pure conduction); in general, the energy equation can contain many more terms [eq. (1.39)], not all of them important. The reasoning for selecting the dominant scales from many scales is condensed in rules 3–5.
- *Rule 3.* If in the sum of two terms,

$$c = a + b \quad (1.55)$$

the order of magnitude of one term is greater than the order of magnitude of the other term,

$$O(a) > O(b) \quad (1.56)$$

then the order of magnitude of the sum is dictated by the dominant term:

$$O(c) = O(a) \quad (1.57)$$

The same conclusion holds if instead of eq. (1.55), we have the difference $c = a - b$ or $c = -a + b$.

- *Rule 4.* If in the sum of two terms, eq. (1.55), the two terms are of the same order of magnitude,

$$O(a) = O(b) \quad (1.58)$$

then the sum is also of the same order of magnitude:

$$O(c) \sim O(a) \sim O(b) \quad (1.59)$$

- *Rule 5.* In any product

$$p = ab \quad (1.60)$$

the order of magnitude of the product is equal to the product of the orders of magnitude of the two factors

$$O(p) = O(a)O(b) \quad (1.61)$$

If, instead of eq. (1.60), we have the ratio

$$r = \frac{a}{b} \quad (1.62)$$

then

$$O(r) = \frac{O(a)}{O(b)} \quad (1.63)$$

In addition to having its own set of rules, scale analysis requires special care with regard to notation. In rules 1–5, we used the following symbols:

- \sim is of the same order of magnitude as
- $O(a)$ the order of magnitude of a
- $>$ greater than, in an order-of-magnitude sense

For brevity, the scale analyses included in this book employ the language of expressions (1.56), (1.57), (1.61), and (1.63) without the repetitive potentially confusing notation $O(\cdot)$ for “order of magnitude.”

Scale analysis is now employed widely in heat transfer, and the fundamental scaling results that have been developed go beyond the first steps presented in this book. For example, Bhattacharjee and Grosshandler [16] have reported the scale analysis of a pressure-driven wall jet. Li and Djilali [17] have used scale analysis to describe the behavior of separating flows behind backward-facing steps (separation bubbles). Li [18] has reported the scaling results for jet diffusion flames. Dowell [19] applied the method of scale analysis to the study of linear and nonlinear dynamics and aerodynamics.

1.6 HEATLINES FOR VISUALIZING CONVECTION

The opportunity to actually “see” the solution to a problem is essential to a problem solver’s ability to learn from experience and in this way to improve his or her technique. In convection problems it is important to visualize the flow of fluid and, riding on this, the flow of energy. For example, in the two-dimensional Cartesian configuration of Fig. 1.1, it has been common practice to define a streamfunction $\psi(x, y)$ as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (1.64)$$

such that the mass continuity equation for incompressible flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.65)$$

is satisfied identically. It is easy to verify that the actual flow is locally parallel to the $\psi = \text{constant}$ line passing through the point of interest. Therefore, although there are no substitutes for u and v as bearers of information regarding the local flow, the family of $\psi = \text{constant}$ streamlines provides a bird’s-eye view of the entire flow field and its main characteristics.

In convection, the transport of energy through the flow field is a combination of both thermal diffusion and enthalpy flow [cf. eq. (1.42)]. For any such field, Kimura and Bejan [20] and the 1984 edition of this book defined a new function $H(x, y)$ such that the net flow of energy (thermal diffusion and enthalpy flow) is zero across each $H = \text{constant}$ line. The mathematical definition of the *heatfunction* H follows in the steps of eqs. (1.64) with the aim of satisfying the energy equation. For steady-state two-dimensional convection through a constant-property homogeneous fluid, eq. (1.42) becomes

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (1.66)$$

or

$$\frac{\partial}{\partial x} \left(\rho c_P u T - k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\rho c_P v T - k \frac{\partial T}{\partial y} \right) = 0 \quad (1.67)$$

The heatfunction is defined as follows:

Net energy flow in the x direction:

$$\frac{\partial H}{\partial y} = \rho c_P u (T - T_{\text{ref}}) - k \frac{\partial T}{\partial x} \quad (1.68)$$

Net energy flow in the y direction:

$$-\frac{\partial H}{\partial x} = \rho c_P v (T - T_{\text{ref}}) - k \frac{\partial T}{\partial y} \quad (1.69)$$

so that the heatfunction $H(x, y)$ satisfies eq. (1.66) identically. Note that the definition above also applies to convection through a fluid-saturated porous medium, where eq. (1.66) accounts for energy conservation (Chapter 12).

The reference temperature T_{ref} is, in principle, an arbitrary constant that can be selected based on convention. Patterns of $H = \text{constant}$ *heatlines* are instructive when T_{ref} is the lowest temperature that occurs in the heat transfer configuration. For example, if the wall shown in Fig. 2.1 is warmer than the free stream, $T_0 > T_\infty$, the choice of reference temperature is $T_{\text{ref}} = T_\infty$. For a meaningful comparison of the heatlines of one flow with the heatlines of another flow, I proposed that T_{ref} always be set equal to the lowest temperature of the flow field.

If the fluid flow subsides ($u = v = 0$), the heatlines become identical to the *heat flux lines* employed frequently in the study of conduction phenomena. Therefore, as a heat transfer visualization technique, the use of heatlines is the convection counterpart or generalization of a standard technique (heat flux lines) used in conduction. Note that the contemporary use of $T = \text{constant}$ lines is not a proper way to visualize heat transfer in the field of convection; isotherms are a proper heat transfer visualization tool only in the field of conduction (where, in fact, they have been invented) because only there are they locally orthogonal to the true direction of energy flow. The use of $T = \text{constant}$ lines to visualize convection heat transfer makes as much sense as using $P = \text{constant}$ lines to visualize fluid flow.

The heatline method for the visualization of convective heat transfer was proposed in the first edition of this book (1984), along with a first application to natural convection in an enclosure heated from the side [20]. The method has since been adopted and extended in many ways in the post-1984 heat transfer literature [21–55].

REFERENCES

1. D. P. Simpson, *Cassell's Latin Dictionary*, Macmillan, New York, 1978, p. 150.
2. A. Bejan, *Advanced Engineering Thermodynamics*, 3rd ed., Wiley, Hoboken, 2006, p. 23.
3. A. Bejan, *Entropy Generation through Heat and Fluid Flow*, Wiley, New York, 1982.
4. J. B. J. Fourier, Mémoire d'analyse sur le mouvement de la chaleur dans les fluides, in *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, Didot, Paris, 1833, pp. 507–530 (presented on Sept. 4, 1820).
5. J. B. J. Fourier, *Oeuvres de Fourier*, G. Darboux, ed., Vol. 2, Gauthier-Villars, Paris, 1890, pp. 595–614.
6. S. D. Poisson, *Théorie Mathématique de la Chaleur*, Paris, 1835, Chapter 4, p. 86.
7. A. Bejan, *Entropy Generation Minimization*, CRC Press, Boca Raton, FL, 1996.
8. A. Bejan, G. Tsatsaronis, and M. Moran, *Thermal Design and Optimization*, Wiley, New York, 1996.

9. A. Bejan, *Shape and Structure, from Engineering to Nature*, Cambridge University Press, Cambridge, 2000.
10. R. N. Rosa, A. H. Reis, and A. F. Miguel, eds., *Bejan's Constructal Theory of Shape and Structure*, Évora Geophysics Center, University of Évora, Portugal, 2004.
11. H. Poirier, Une théorie explique l'intelligence de la nature, *Science & Vie*, No. 1034, November 2003, pp. 44–63.
12. A. Bejan and S. Lorente, Constructal theory of generation of configuration in nature and engineering, *J. Appl. Phys.*, Vol. 100, 2006, 041301.
13. A. Bejan and S. Lorente, The constructal law of design and evolution in nature, *Phil. Trans. R. Soc. B*, Vol. 365, 2010, pp. 1335–1347.
14. A. Bejan and S. Lorente, *Design with Constructal Theory*, Wiley, Hoboken, 2008.
15. A. Bejan, *Heat Transfer*, Wiley, New York, 1993, Chapter 4.
16. S. Bhattacharjee and W. L. Grosshandler, The formation of a wall jet near a high temperature wall under microgravity environment, *ASME HTD*, Vol. 96, 1988, pp. 711–716.
17. X. Li and N. Djilali, On the scaling of separation bubbles, *JSME Int. J., Ser. B*, Vol. 38, No. 4, 1995, pp. 541–548.
18. X. Li, On the scaling of the visible lengths of jet diffusion flames, *J. Energy Resour. Technol.*, Vol. 118, 1996, pp. 128–133.
19. E. H. Dowell, Transonic unsteady potential flow: Scaling analysis of linear and nonlinear dynamics, *AIAA J.*, Vol. 48, 2010, pp. 1017–1019.
20. S. Kimura and A. Bejan, The “heatline” visualization of convective heat transfer, *J. Heat Transfer*, Vol. 105, 1983, pp. 916–919.
21. D. Littlefield and P. Desai, Buoyant laminar convection in a vertical cylindrical annulus, *J. Heat Transfer*, Vol. 108, 1986, pp. 814–821.
22. O. V. Trevisan and A. Bejan, Combined heat and mass transfer by natural convection in a vertical enclosure, *J. Heat Transfer*, Vol. 109, 1987, pp. 104–109.
23. F. L. Bello-Ochende, Analysis of heat transfer by free convection in tilted rectangular cavities using the energy analogue of the stream function, *Int. J. Mech. Eng. Ed.*, Vol. 15, 1987, pp. 91–98.
24. F. L. Bello-Ochende, A heat function formulation for thermal convection in a square cavity, *Int. Comm. Heat Mass Transfer*, Vol. 15, 1988, pp. 193–202.
25. A. M. Morega, The heat function approach to the thermo-magnetic convection of electroconductive melts, *Rev. Roum. Sci. Tech. Ser. Electrotech. Energ.*, Vol. 33, 1988, pp. 33–39.
26. S. K. Aggarwal and A. Manhapra, Use of heatlines for unsteady buoyancy-driven flow in a cylindrical enclosure, *J. Heat Transfer*, Vol. 111, 1989, pp. 576–578.
27. S. K. Aggarwal and A. Manhapra, Transient natural convection in a cylindrical enclosure nonuniformly heated at the top wall, *Numer. Heat Transfer, Part A*, Vol. 15, 1989, pp. 341–356.
28. C. J. Ho, Y. H. Lin, and T. C. Chen, A numerical study of natural convection in concentric and eccentric horizontal cylindrical annuli with mixed boundary conditions, *Int. J. Heat Fluid Flow*, Vol. 10, 1989, pp. 40–47.

29. C. J. Ho and Y. H. Lin, Thermal convection heat transfer of air/water layers enclosed in horizontal annuli with mixed boundary conditions, *Wärme Stoffübertrag.*, Vol. 24, 1989, pp. 211–224.
30. C. J. Ho and Y. H. Lin, Natural convection of cold water in a vertical annulus with constant heat flux on the inner wall, *J. Heat Transfer*, Vol. 112, 1990, pp. 117–123.
31. A. M. Morega and A. Bejan, Heatline visualization of forced convection boundary layers, *Int. J. Heat Mass Transfer*, Vol. 36, 1993, pp. 3957–3966.
32. A. M. Morega and A. Bejan, Heatline visualization of forced convection in porous media, *Int. J. Heat Fluid Flow*, Vol. 15, 1994, pp. 42–47.
33. V. A. F. Costa, Double diffusive natural convection in a square enclosure with heat and mass diffusive walls, *Int. J. Heat Mass Transfer*, Vol. 40, 1997, pp. 4061–4071.
34. V. A. F. Costa, Double diffusive natural convection in enclosures with heat and mass diffusive walls, in G. De Vahl Davis and E. Leonardi., eds., *Proceedings of the International Symposium on Advances in Computational Heat Transfer (CHT'97)*, Begell House, New York, 1998, pp. 338–344.
35. H. Y. Wang, F. Penot, and J. B. Saulnier, Numerical study of a buoyancy-induced flow along a vertical plate with discretely heated integrated circuit packages, *Int. J. Heat Mass Transfer*, Vol. 40, 1997, pp. 1509–1520.
36. V. A. F. Costa, Unification of the streamline, heatline and massline methods for the visualization of two-dimensional transport phenomena, *Int. J. Heat Mass Transfer*, Vol. 42, 1999, pp. 27–33.
37. S. J. Kim and S. P. Jang, Experimental and numerical analysis of heat transfer phenomena in a sensor tube of a mass flow controller, *Int. J. Heat Mass Transfer*, Vol. 44, 2001, pp. 1711–1724.
38. Q.-H. Deng and G.-F. Tang, Numerical visualization of mass and heat transport for conjugate natural convection/heat conduction by streamline and heatline, *Int. J. Heat Mass Transfer*, Vol. 45, 2002, pp. 2375–2385.
39. Q.-H. Deng and G.-F. Tang, Numerical visualization of mass and heat transport for mixed convective heat transfer by streamline and heatline, *Int. J. Heat Mass Transfer*, Vol. 45, 2002, pp. 2387–2396.
40. A. Mukhopadhyay, X. Qin, S. K. Aggarwal, and I. K. Puri, On extension of ‘‘heatline’’ and ‘‘massline’’ concepts to reacting flows through the use of conserved scalars, *J. Heat Transfer*, Vol. 124, 2002, pp. 791–799.
41. V. A. F. Costa, Comment on the paper by Qi-Hong Deng and Guang-Fa Tang, Numerical visualization of mass and heat transport for conjugate natural convection/heat conduction by streamline and heatline, *Int. J. Heat Mass Transfer*, Vol. 46, 2003, pp. 185–187.
42. V. A. F. Costa, Unified streamline, heatline and massline methods for the visualization of two-dimensional heat and mass transfer in anisotropic media, *Int. J. Heat Mass Transfer*, Vol. 46, 2003, pp. 1309–1320.
43. A. Mukhopadhyay, X. Qin, I. K. Puri, and S. K. Aggarwal, Visualization of scalar transport in nonreacting and reacting jets through a unified ‘‘heatline’’ and ‘‘massline’’ formulation, *Numerical Heat Transfer, Part A*, Vol. 44, 2003, pp. 683–704.
44. A. Dalal and M. K. Das, Heatline method for the visualization of natural convection in a complicated cavity, *Int. J. Heat Mass Transfer*, Vol. 51, 2008, pp. 263–272.

45. F.-Y. Zhao, D. Liu, and G.-F. Tang, Application issues of the streamline, heatline and massline for conjugate heat and mass transfer, *Int. J. Heat Mass Transfer*, Vol. 50, 2007, pp. 320–334.
46. T. Basak and S. Roy, Role of “Bejan’s heatlines” in heat flow visualization and optimal thermal mixing for differentially heated square enclosures, *Int. J. Heat Mass Transfer*, Vol. 51, 2008, pp. 3486–3503.
47. E. Hakyemez, M. Mobedi, and H. F. Oztop, Effects of wall-located heat barrier on conjugate conduction/natural-convection heat transfer and fluid flow in enclosures, *Numer. Heat Transfer Part A*, Vol. 54, 2008, pp. 197–220.
48. S. Mahmud and R. A. Fraser, Visualizing energy flows through energy streamlines and pathlines, *Int. J. Heat Mass Transfer*, Vol. 50, 2007, pp. 3990–4002.
49. T. Basak, S. Roy, and I. Pop, Heat flow analysis for natural convection within trapezoidal enclosures based on heatline concept, *Int. J. Heat Mass Transfer*, Vol. 52, 2009, pp. 2471–2483.
50. T. Basak, G. Aravind, and S. Roy, Visualization of heat flow due to natural convection within triangular cavities using Bejan’s heatline concept, *Int. J. Heat Mass Transfer*, Vol. 52, 2009, pp. 2824–2833.
51. R. S. Kaluri, T. Basak, and S. Roy, Bejan’s heatlines and numerical visualization of heat flow and thermal mixing in various differentially heated porous square cavities, *Numer. Heat Transfer Part A*, Vol. 55, 2009, pp. 487–516.
52. M. A. Waheed, Heatfunction formulation of thermal convection in rectangular enclosures filled with porous media, *Numer. Heat Transfer Part A*, Vol. 55, 2009, pp. 185–204.
53. Y. Varol, H. F. Oztop, M. Mobedi, and I. Pop, Visualization of natural convection heat transport using heatline method in porous non-isothermally heated triangular cavity, *Int. J. Heat Mass Transfer*, Vol. 51, 2008, pp. 5040–5051.
54. T. Basak, S. Roy, and G. Aravind, Analysis of heat recovery and thermal transport within entrapped fluid based on heatline approach, *Chem. Eng. Sci.*, Vol. 64, 2009, pp. 1673–1686.
55. R. S. Kaluri, T. Basak, and S. Roy, Heatline approach for visualization of heat flow and efficient thermal mixing with discrete heat sources, *Int. J. Heat Mass Transfer*, Vol. 53, 2010, pp. 3241–3261.

PROBLEMS

- 1.1. Consider the unsteady mass conservation equation (1.5) as it might describe the flow accelerating through a duct with a variable cross section. If the largest velocity gradient measured locally is du/dx and the largest density gradient is $d\rho/dx$, what order-of-magnitude relationship must exist between du/dx and $d\rho/dx$ for the simplified equation (1.8) to be applicable?
- 1.2. Derive the mass conservation equation in cylindrical coordinates [eq. (1.9)] by applying the general principle (1.1) to an elementary control volume of size $\Delta r(r \Delta\theta) \Delta z$ in Fig. 1.1b (assume that $\rho = \text{constant}$).

- 1.3.** Derive the mass conservation statement for spherical coordinates [eq. (1.10)] by writing eq. (1.1) for the elementary control volume $(\Delta r)(r \sin \phi \Delta \theta)(r \Delta \phi)$ around point (r, θ, ϕ) in Fig. 1.1c (assume that $\rho = \text{constant}$).
- 1.4.** Consider a flow in which ρ and μ may be regarded as constant. Show that the x momentum equation (1.18) follows from eq. (1.17) through proper use of the mass conservation principle.
- 1.5.** Imagine a flow described by eqs. (1.9) and (1.21) in cylindrical coordinates. If the flow is situated on one side of and infinitely far from the $r = 0$ origin of the coordinate system, the local three-directional increments Δr , $r \Delta \theta$, Δz become analogous to three Cartesian increments Δx , Δy , Δz measured away from the local point (r, θ, z) in the flow field. Show that in the limit $r \rightarrow \infty$, the transformation $\Delta r \rightarrow \Delta x$, $r \Delta \theta \rightarrow \Delta y$, $\Delta z \rightarrow \Delta z$ leads to the collapse of eqs. (1.9) and (1.21) into their (x, y, z) Cartesian equivalents [eqs. (1.8) and (1.19)].
- 1.6.** Consider the conservation of mass and the three force balances in spherical coordinates [eqs. (1.10) and (1.22)]. If the flow described by these equations is situated infinitely far from the $r = 0$ origin of the spherical system, the following transformation is applicable (Fig. 1.1): $\Delta r \rightarrow \Delta x$, $r \sin \phi \Delta \theta \rightarrow \Delta y$, $r \Delta \phi \rightarrow \Delta z$. Show that through this transformation, in the limit $r \rightarrow \infty$, eqs. (1.10) and (1.22) become the same as eqs. (1.8) and (1.19).
- 1.7.** Implicit in the derivation of the energy equation (1.39) is the assumption that changes in kinetic energy $V^2/2$ are negligible relative to changes in internal energy e [see expressions (1.25), where e should, in general, be replaced by $e + V^2/2$]. Retrace the path leading to eq. (1.39) by taking into account changes in kinetic energy; show that the result of this more rigorous analysis is identical to eq. (1.39).
- 1.8.** Demonstrate that lost work is always proportional to entropy generation [eq. (1.48)], where $W_{\text{lost}} = W_{\text{maximum}} - W_{\text{actual}}$, and where W_{maximum} corresponds to the reversible limit ($S_{\text{gen}} = 0$). Write the first law of thermodynamics for a control volume, first for the actual (real) process and then for the reversible process. Then use the definition of W_{lost} and S_{gen} to prove eq. (1.48).
- 1.9.** Derive the formula for the local rate of entropy generation [eq. (1.49)]. Begin with translating the general statement (1.47) into the language of the two-dimensional control volume $\Delta x \Delta y$. Combine the resulting expression with the first law of thermodynamics as given by eq. (1.26), plus the canonical relation for internal energy (Table 1.1).
- 1.10.** Consider the Couette flow between two parallel plates separated by a gap of width D and moving relative to one another with a speed U . The temperature difference ΔT is imposed between the two plates. Estimate the rate of entropy generation per unit volume in this flow. What relationship

must exist between D , U , ΔT and the fluid properties μ and k for S'''_{gen} to be dominated by the irreversibility due to fluid friction?

- 1.11. According to the one-dimensional (longitudinal) conduction model of a fin, the temperature distribution along the fin, $T(x)$, obeys the energy equation [15]

$$\underbrace{kA \frac{d^2 T}{dx^2}}_{\text{Longitudinal conduction}} - \underbrace{hP(T - T_0)}_{\text{Lateral convection}} + \underbrace{q''' A}_{\text{Internal heat generation}} = 0$$

where A , h , P , and q''' are the fin cross-sectional area, fin-fluid heat transfer coefficient, perimeter of the fin cross section (called the *wetted perimeter*), and volumetric rate of heat generation. Consider the semi-infinite fin that, as shown in Fig. P1.11, is bathed by a fluid of temperature T_0 and is attached to a solid wall of temperature T_0 . The heat generated by the fin is absorbed by either the fluid or the solid wall.

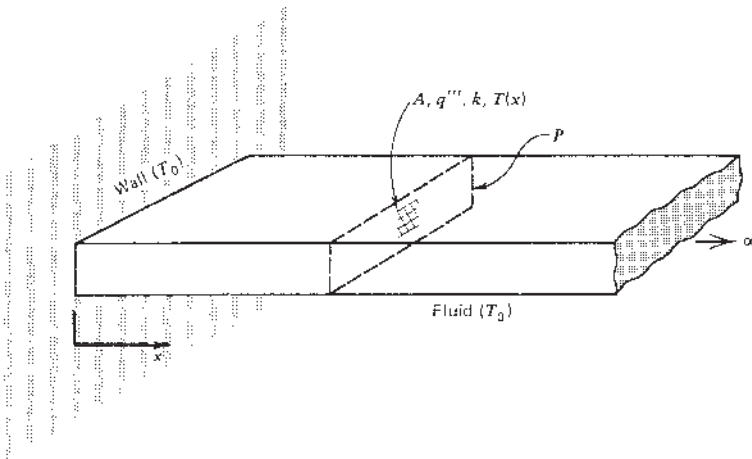


Figure P1.11

- (a) As a system for scale analysis, select the fin section of length x , where x is measured away from the wall. Let T_∞ be the fin temperature sufficiently far from the wall. Show that if x is large enough, the longitudinal conduction term becomes negligible in the energy equation.
- (b) Invoking the balance between lateral convection and internal heat generation, determine the fin temperature sufficiently far from the wall, T_∞ .

- (c) Determine the fin section of length δ near the wall where the heat transfer is ruled by the balance between longitudinal conduction and internal heat generation.
 - (d) Determine the heat transfer rate into the wall through the base of the fin.
- 1.12.** Consider the laminar flow near a flat, solid wall, as illustrated in Fig. 2.1. The momentum equation for this flow involves the competition among three effects: inertia, pressure gradient, and friction [see eq. (2.26)]. For the purpose of scale analysis, consider a flow region of length L and thickness L . Show that in this region, the ratio of inertia to friction is of order Re_L , where Re_L is the Reynolds number based on wall length. Note that the region selected for analysis is not the boundary layer region discussed in Chapter 2. In a certain flow, the value of Re_L is 10^3 . What force balance rules the $L \times L$ region: inertia \sim pressure, inertia \sim friction, or pressure \sim friction?
- 1.13.** The hot components of a power plant must be fitted with thermal insulation so that they do not leak heat excessively to the ambient (T_0). The thermal conductivity of the insulation is known (k). The total volume of the insulation (V) is fixed.

A simple model of the hot components is the two-chamber model shown in Fig. P1.13. The hottest is the furnace, which is enclosed by a surface of area A_H and high temperature T_H . The thickness of the insulation mounted on A_H is t_H . This thickness is sufficiently small so that the volume of the insulation on A_H is $A_H t_H$.

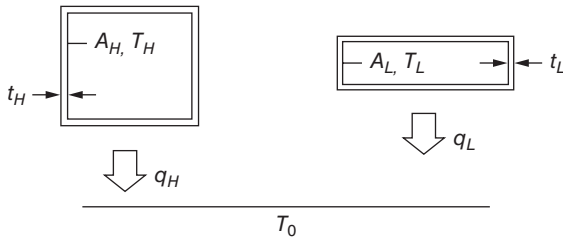


Figure P1.13

The rest of the hot components (pipes, feed water heaters, turbines) are not as hot as the furnace. They are inside an enclosure with area A_L , temperature T_L , insulation thickness t_L , and insulation volume $A_L t_L$.

The heat leaks q_H and q_L are by pure conduction and are driven by the temperature differences $\Delta T_H = T_H - T_0$ and $\Delta T_L = T_L - T_0$, respectively. These temperature differences are known. The problem is to determine t_H/t_L , that is, how to distribute the available insulation on A_H and A_L .

1. Minimize the total heat leak from A_H and A_L to the ambient, namely $q_H + q_L$, and determine the optimal ratio t_H/t_L as a function of other parameters of the two-chamber model. Does t_H/t_L depend on A_H/A_L ?
2. Explain why 1 W of heat leak from T_H is not the same as 1 W of heat leak from T_L . Which do you think is more damaging to the performance of the power plant?
3. Imagine that q_H can be intercepted outside A_H and used to run a Carnot engine between T_H and T_0 . The power producible in this way (W_H) is lost because q_H is dumped straight into the ambient. Imagine the equivalent scenario for q_L , and derive a formula for the Carnot power W_L that is lost because of this second heat leak.
4. Minimize the total loss of power ($W_H + W_L$) and determine the optimal ratio t_H/t_L .
5. Compare the t_H/t_L results obtained at sections 1 and 4. Which ratio is larger? Which is more relevant for actual implementation?