

PART
One

**Foundations of Risk
Management**

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CHAPTER 1

Risk Management

Financial risk management is the process by which financial risks are identified, assessed, measured, and managed in order to create economic value.

Some risks can be measured reasonably well. For those, risk can be quantified using statistical tools to generate a probability distribution of profits and losses. Other risks are not amenable to formal measurement but are nonetheless important. The function of the risk manager is to evaluate financial risks using both quantitative tools and judgment.

As financial markets have expanded over recent decades, the risk management function has become more important. Risk can never be entirely avoided. More generally, the goal is not to minimize risk; it is to take smart risks.

Risk that can be measured can be managed better. Investors assume risk only because they expect to be compensated for it in the form of higher returns. To decide how to balance risk against return, however, requires risk measurement.

Centralized risk management tools such as value at risk (VAR) were developed in the early 1990s. They combine two main ideas. The first is that risk should be measured at the top level of the institution or the portfolio. This idea is not new. It was developed by Harry Markowitz (1952), who emphasized the importance of measuring risk in a total portfolio context.¹ A centralized risk measure properly accounts for hedging and diversification effects. It also reflects the fact that equity is a common capital buffer to absorb all risks. The second idea is that risk should be measured on a forward-looking basis, using the current positions.

This chapter gives an overview of the foundations of risk management. Section 1.1 provides an introduction to the risk measurement process, using an illustration. Next, Section 1.2 discusses how to evaluate the quality of risk management processes. Section 1.3 then turns to the integration of risk measurement with business decisions, which is a portfolio construction problem. These portfolio decisions can be aggregated across investors, leading to asset pricing theories that can be used as yardsticks for performance evaluation and for judging risk management and are covered in Section 1.4. Finally, Section 1.5 discusses how risk management can add economic value.

FRM Exam Part 1 topic. In addition to the topics described in this chapter, FRM candidates should also read the GARP Code of Conduct.

¹Harry Markowitz, "Portfolio Selection," *Journal of Finance* 7 (1952): 77-91.

1.1 RISK MEASUREMENT

1.1.1 Example

The first step in risk management is the measurement of risk. To illustrate, consider a portfolio with \$100 million invested in U.S. equities. Presumably, the investor undertook the position because of an expectation for profit, or investment growth. This portfolio is also risky, however.

The key issue is whether the expected profit for this portfolio warrants the assumed risk. Thus a trade-off is involved, as in most economic problems. To help answer this question, the risk manager should construct the distribution of potential profits and losses on this investment. This shows how much the portfolio can lose, thus enabling the investor to make investment decisions.

Define ΔP as the profit or loss for the portfolio over a fixed horizon, say the coming month. This must be measured in a *risk currency*, such as the dollar. This is also the product of the initial investment value P and the future rate of return R_p . The latter is a random variable, which should be described using its **probability density function**. Using historical data over a long period, for example, the risk manager produces Figure 1.1.

This graph is based on the actual distribution of total returns on the S&P 500 index since 1925. The line is a smoothed histogram and does not assume a simplified model such as the normal distribution.

The vertical axis represents the frequency, or probability, of a gain or loss of a size indicated on the horizontal axis. The entire area under the curve covers all of the possible realizations, so should add up to a total probability of 1.

Most of the weight is in the center of the distribution. This shows that it is most likely that the return will be small, whether positive or negative. The tails have less weight, indicating that large returns are less likely. This is a typical characteristic of returns on financial assets. So far, this pattern resembles the bell-shaped curve for a normal distribution.

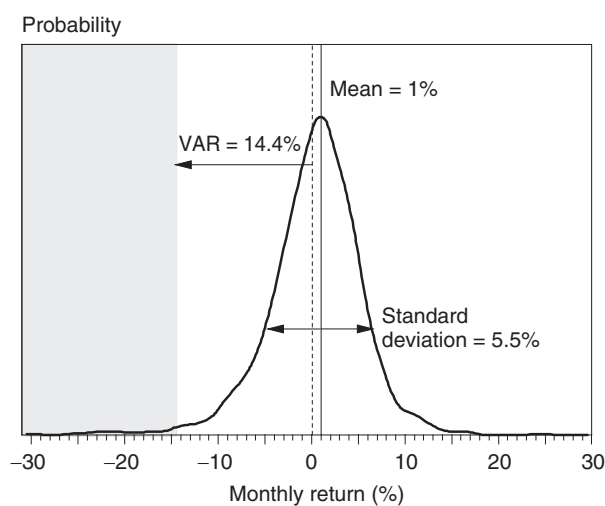


FIGURE 1.1 Distribution of Monthly Returns on U.S. Stocks

On the downside, however, there is a substantial probability of losing 10% or more in a month. This cumulative probability is 3%, meaning that in a repeated sample with 100 months, we should expect to lose 10% or more for a total of three months. This risk is worse than predicted by a normal distribution.

If this risk is too large for the investor, then some money should be allocated to cash. Of course, this comes at the expense of lower expected returns.

The distribution can be characterized in several ways. The entire shape is most informative because it could reveal a greater propensity to large losses than to gains. The distribution could be described by just a few summary statistics, keeping in mind that this is an oversimplification. Other chapters offer formal definitions of these statistics.

- The *mean*, or average return, which is approximately 1% per month. Define this as $\mu(R_P)$, or μ_P in short, or even μ when there is no other asset.
- The *standard deviation*, which is approximately 5.5%. This is often called volatility and is a measure of dispersion around the mean. Define this as σ . This is the square root of the portfolio variance, σ^2 .
- The *value at risk* (VAR), which is the cutoff point such that there is a low probability of a greater loss. This is also the percentile of the distribution. Using a 99% confidence level, for example, we find a VAR of 14.4%.

1.1.2 Absolute versus Relative Risk

So far, we have assumed that risk is measured by the dispersion of dollar returns, or in absolute terms. In some cases, however, risk should be measured relative to some **benchmark**. For example, the performance of an active manager is compared to that of an index such as the S&P 500 index for U.S. equities. Alternatively, an investor may have future liabilities, in which case the benchmark is an index of the present value of liabilities. An investor may also want to measure returns after accounting for the effect of inflation. In all of these cases, the investor is concerned with **relative risk**.

- **Absolute risk** is measured in terms of shortfall relative to the initial value of the investment, or perhaps an investment in cash. Using the standard deviation as the risk measure, absolute risk in dollar terms is

$$\sigma(\Delta P) = \sigma(\Delta P/P) \times P = \sigma(R_P) \times P \quad (1.1)$$

- **Relative risk** is measured relative to a benchmark index B . The deviation is $e = R_P - R_B$, which is also known as the **tracking error**. In dollar terms, this is $e \times P$. The risk is

$$\sigma(e)P = [\sigma(R_P - R_B)] \times P = \omega \times P \quad (1.2)$$

where ω is called **tracking error volatility** (TEV).

To compare these two approaches, take the case of an active equity portfolio manager who is given the task of beating a benchmark. In the first year, the active portfolio returns -6% but the benchmark drops by -10% . So, the excess return is positive: $e = -6\% - (-10\%) = 4\%$. In relative terms, the portfolio has done well even though the absolute performance is negative. In the second year, the portfolio returns $+6\%$, which is good using absolute measures, but not so good if the benchmark goes up by $+10\%$.

EXAMPLE 1.1: ABSOLUTE AND RELATIVE RISK

An investment manager is given the task of beating a benchmark. Hence the risk should be measured in terms of

- a. Loss relative to the initial investment
- b. Loss relative to the expected portfolio value
- c. Loss relative to the benchmark
- d. Loss attributed to the benchmark

1.2 EVALUATION OF THE RISK MEASUREMENT PROCESS

A major function of the risk measurement process is to estimate the distribution of future profits and losses. The first part of this assignment is easy. The scale of the dollar returns should be proportional to the initial investment. In other words, given the distribution in Figure 1.1, an investment of \$100 million should have a standard deviation of $\sigma(\Delta P) = \$100 \times 5.5\% = \5.5 million. Scaling the current position by a factor of 2 should increase this risk to \$11 million.

The second part of the assignment, which consists of constructing the distribution of future rates of return, is much harder. In Figure 1.1, we have taken the historical distribution and assumed that this provides a good representation of future risks. Because we have a long history of returns over many different cycles, this is a reasonable approach.

This is not always the case, however. The return may have been constant over its recent history. This does not mean that it could not change in the future. For example, the price of gold was fixed to \$35 per ounce from 1934 to 1967 by the U.S. government. As a result, using a historical distribution over the 30 years ending in 1967 would have shown no risk. Instead, gold prices started to fluctuate wildly thereafter. By 2008, gold prices had reached \$1,000. Thus, the responsibility of the risk manager is to judge whether the history is directly relevant.

How do we evaluate the quality of a risk measurement process? The occurrence of a large loss does not mean that risk management has failed. This could be simply due to bad luck. An investment in stocks would have lost 17% in October 2008. While this is a grievous loss, Figure 1.1 shows that it was not inconceivable. For

example, the stock market lost 30% in September 1931 and 22% on October 19, 1987, before recovering. So, the risk manager could have done a perfect job of forecasting the distribution of returns. How can we tell whether this loss is due to bad luck or a flaw in the risk model?

1.2.1 Known Knowns

To help answer this question, it is useful to classify risks into various categories, which we can call (1) known knowns, (2) known unknowns, and (3) unknown unknowns.² The first category consists of risks that are properly identified and measured, as in the example of the position in stocks. Losses can still occur due to a combination of bad luck and portfolio decisions.

Such losses, however, should not happen too often. Suppose that VAR at the 99% level of confidence is reported as 14.4%. Under these conditions, a string of consecutive losses of 15% or more several months in a row should be highly unusual. If this were to happen, it would be an indication of a flawed model. A later chapter will show how backtesting can be used to detect flaws in risk measurement systems.

1.2.2 Known Unknowns

The second category, called known unknowns, includes model weaknesses that are known or should be known to exist but are not properly measured by risk managers. For example, the risk manager could have ignored important known risk factors. Second, the distribution of risk factors, including volatilities and correlations, could be measured inaccurately. Third, the mapping process, which consists of replacing positions with exposures on the risk factors, could be incorrect. This is typically called **model risk**. Such risks can be evaluated using stress tests, which shock financial variables or models beyond typical ranges.

As an example, consider the \$19 billion loss suffered by UBS in 2007 alone from positions in structured credit securities backed by subprime and Alt-A mortgage-backed loans.³ UBS had invested in top-rated tranches that the bank thought were perfectly safe (yet yielded high returns). As a result, it had accumulated a position of \$90 billion in exposures to these securities, compared to \$41 billion in book equity. The bank reported that its risk measurement process relied on simplified models based on a recent period of positive growth in housing prices. As in the example of gold, the recent history gave a biased view of the true risks. In addition, UBS's risk managers overrelied on ratings provided by the credit rating agencies. Because risk management gave little indication of the downside

² Philippe Jorion, "Risk Management Lessons from the Credit Crisis," *European Financial Management* 15 (2009): 923–933.

³ See UBS, *Shareholder Report on UBS's Write-Downs* (Zurich: UBS, 2008). Loans can be classified into prime, Alt-A, and subprime, in order of decreasing credit quality. Subprime loans are loans made to consumers with low credit scores (typically below 640 out of a possible maximum of 850). Alt-A loans, short for Alternative A-paper, are the next category (typically with credit scores below 680 or for loans lacking full documentation). Subprime and Alt-A mortgage loans are expected to have higher credit risk than other (prime) loans.

risk of these investments, these losses can be viewed as a failure of risk management. Even so, the UBS report indicates that the growth strategy undertaken by top management was a “contributing factor to the buildup of UBS’s subprime positions which subsequently incurred losses.” In other words, top management was largely responsible for the losses.

Another form of known unknown is **liquidity risk**. Most risk models assume that the position can be liquidated over the selected horizon. In practice, this depends on a number of factors. First is the intrinsic liquidity of the asset. Treasury bills, for instance, are much more liquid than high-yield bonds. They trade at a lower spread and with less market impact. Second is the size of the position. This is especially a problem when the position is very large relative to normal trading activity, which would require accepting a large price drop to execute the trade.

1.2.3 Unknown Unknowns

The risks in the last category tend to be the difficult ones. They represent events totally outside the scope of most scenarios. Examples include regulatory risks such as the sudden restrictions on short sales, which can play havoc with hedging strategies, or structural changes such as the conversion of investment banks to commercial banks, which accelerated the deleveraging of the industry. Indeed, a 2010 survey reports that the top concern of risk managers is “government changing the rules.”⁴

Similarly, it is difficult to account fully for counterparty risk. It is not enough to know your counterparty; you need to know your counterparty’s counterparties, too. In other words, there are network externalities. Understanding the full consequences of Lehman’s failure, for example, would have required information on the entire topology of the financial network.⁵ Because no individual firm has access to this information, this contagion risk cannot be measured directly.

Similarly, some form of liquidity risk is very difficult to assess. This involves the activity and positions of similar traders, which are generally unknown. In illiquid markets, a forced sale will be much more expensive if a number of similar portfolios are sold at the same time.

This category is sometimes called Knightian **uncertainty**, a form of risk that is immeasurable. Financial institutions cannot possibly carry enough capital to withstand massive counterparty failures, or systemic risk. In such situations, the central bank or the government becomes effectively the risk manager of last resort.

1.2.4 Risk Management Failures

More generally, the role of risk management involves several tasks:

- Identifying all risks faced by the firm
- Assessing and monitoring those risks

⁴ *Risk Governance: A Benchmarking Survey* (New York: Capital Markets Risk Advisors, 2010).

⁵ A. Haldane, *Why Banks Failed the Stress Test* (London: Bank of England, 2009).

- Managing those risks if given the authority to do so
- Communicating these risks to the decision makers

A large loss is not necessarily an indication of a risk management failure. It could have been within the scope of known knowns and properly communicated to the firm, in which case it reflects bad luck. After all, the objective of risk management is not to prevent losses.

Otherwise, risk management can fail if any of these tasks has not been met. Some risks could go unrecognized. Mismeasurement of risk can occur due to model risk, due to liquidity risk, or if distributions are not adequately measured. Risk limits could not have been enforced. Finally, risk management fails when it does not communicate risks effectively.

EXAMPLE 1.2: FRM EXAM 2009—QUESTION 1-11

Based on the risk assessment of the CRO, Bank United's CEO decided to make a large investment in a levered portfolio of CDOs. The CRO had estimated that the portfolio had a 1% chance of losing \$1 billion or more over one year, a loss that would make the bank insolvent. At the end of the first year the portfolio has lost \$2 billion and the bank was closed by regulators.

Which of the following statements is correct?

- a. The outcome demonstrates a risk management failure because the bank did not eliminate the possibility of financial distress.
- b. The outcome demonstrates a risk management failure because the fact that an extremely unlikely outcome occurred means that the probability of the outcome was poorly estimated.
- c. The outcome demonstrates a risk management failure because the CRO failed to go to regulators to stop the shutdown.
- d. Based on the information provided, one cannot determine whether it was a risk management failure.

1.3 PORTFOLIO CONSTRUCTION

1.3.1 Comparing Multiple Assets

We now turn to the portfolio construction process, which involves combining expected return and risk. Assume that another choice is to invest in long-term U.S. government bonds.

Over the same period, the monthly average return of this other asset class was 0.47%. This is half that of equities. The monthly standard deviation was 2.3%,

TABLE 1.1 Risk and Expected Return on Two Assets

	Average	Volatility	Correlation
Equities	11.2%	19.2%	
Long-term bonds	5.6%	8.1%	0.13

again lower than for equities. To make the numbers more intuitive, monthly returns have been converted to annualized terms, as shown in Table 1.1.

Here, our investor is faced with a typical trade-off, which is to choose between these two alternatives. Neither dominates the other, as shown in Figure 1.2.

This graph describes a simple investment decision. More generally, it also represents more complex business decisions that involve risk. For instance, a bank must decide how much leverage to assume, as defined by the amount of assets divided by the amount of equity on its balance sheet. The horizontal axis could then represent the bank's credit rating. On one hand, higher leverage involves higher risk and accordingly a lower credit rating. In Figure 1.2, this corresponds to a move to the right. On the other hand, higher leverage means that the expected return to equity should be higher. This is because the amount of equity on the balance sheet is lower, implying that profits will be distributed to a smaller equity base. In Figure 1.2, this corresponds to a move up. Again, we observe a trade-off between higher risk and higher return. Without risk measures, deciding where to invest would be difficult.

1.3.2 Risk-Adjusted Performance Measurement

The next question is how the performance can be adjusted for risk in a single measure. The same methods apply to past performance, using historical averages, or prospective performance, using projected numbers.

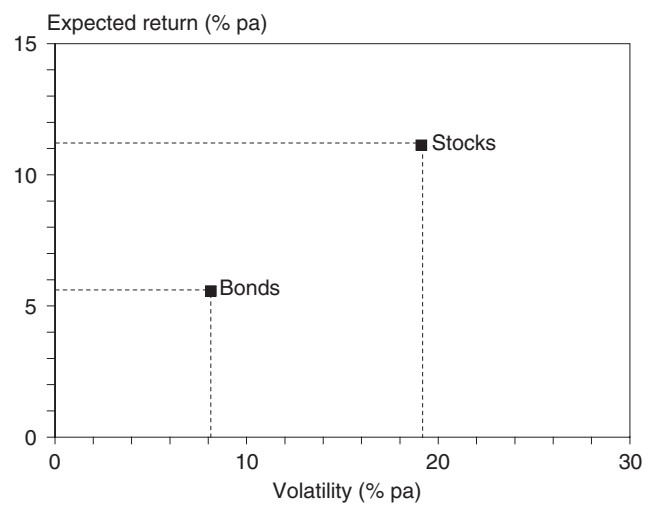


FIGURE 1.2 Comparing Risk and Expected Return

The simplest metric is the **Sharpe ratio (SR)**, which is the ratio of the average rate of return, $\mu(R_P)$, in excess of the risk-free rate R_F , to the absolute risk:

$$SR = \frac{[\mu(R_P) - R_F]}{\sigma(R_P)} \quad (1.3)$$

The Sharpe ratio focuses on total risk measured in absolute terms. This approach can be extended to include VAR in the denominator instead of the volatility of returns.

Figure 1.3 compares the SR for the two investment choices. Assume that we have a risk-free asset, cash, with a return of 3%. The SR is the slope of the line from cash to each asset. This line represents a portfolio mix between cash and each asset. In this case, stocks have a higher SR than bonds. This means that a mix of cash and stocks could be chosen with the same volatility as bonds but with higher returns.

This can be extended to relative risk measures. The **information ratio (IR)** is the ratio of the average rate of return of portfolio P in excess of the benchmark B to the TEV:

$$IR = \frac{[\mu(R_P) - \mu(R_B)]}{\sigma(R_P - R_B)} \quad (1.4)$$

Table 1.2 presents an illustration. The risk-free interest rate is $R_F = 3\%$ and the portfolio average return is -6% , with volatility of 25% . Hence, the Sharpe ratio of the portfolio is $SR = [(-6\%) - (3\%)]/25\% = -0.36$. Because this is negative, the absolute performance is poor.

Assume now that the benchmark returned -10% over the same period and that the tracking error volatility was 8% . Hence, the information ratio is $IR = [(-6\%) - (-10\%)]/8\% = 0.50$, which is positive. The relative performance is good even though the absolute performance is poor.

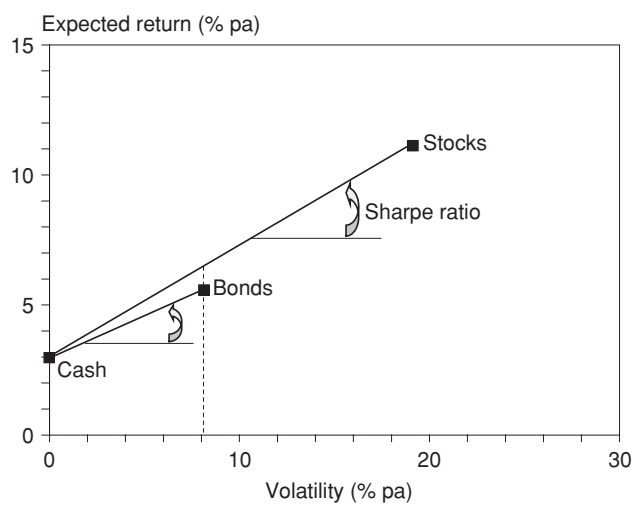


FIGURE 1.3 Comparing Sharpe Ratios

TABLE 1.2 Absolute and Relative Performance

	Average	Volatility	Performance
Cash	3%	0%	
Portfolio P	-6%	25%	$SR = -0.36$
Benchmark B	-10%	20%	$SR = -0.65$
Deviation e	4%	8%	$IR = 0.50$

The tracking error volatility can be derived from the volatilities σ_P and σ_B of the portfolio and the benchmark as well as their correlation ρ . Chapter 2 shows that the variance of a sum of random variables can be expressed in terms of the sum of the individual variances plus twice a covariance term. In terms of difference, the variance is

$$\omega^2 = \sigma_P^2 - 2\rho\sigma_P\sigma_B + \sigma_B^2 \quad (1.5)$$

In this case, if $\sigma_P = 25\%$, $\sigma_B = 20\%$, and $\rho = 0.961$, we have $\omega^2 = 25\%^2 - 2 \times 0.961 \times 25\% \times 20\% + 20\%^2 = 0.0064$, giving $\omega = 8\%$.

The IR has become commonly used to compare active managers in the same peer group. It is a pure measure of active management skill that is scaled for active risk. Consider, for example, two managers. Manager A has TEV of 2% per annum and excess return of 1%. Manager B has TEV of 6% per annum and excess return of 2%. Manager A has lower excess return but a higher information ratio, $1/2 = 0.50$, vs. $2/6 = 0.33$. As a result, it has better management skills. For example, Manager A could be asked to amplify its tracking error by a factor of 3, which would lead to an excess return of 3%, thus beating Manager B with the same level of tracking error of 6%. An information ratio of 0.50 is typical of the performance of the top 25th percentile of money managers and is considered “good.”⁶

One of the drawbacks of the information ratio is that the TEV does not adjust for average returns. For instance, a portfolio could be systematically above its benchmark by 0.10% per month. In this case, the tracking error has an average of 0.10% and a standard deviation close to zero. This leads to a very high information ratio, which is not realistic if the active risk cannot be scaled easily.

1.3.3 Mixing Assets

The analysis has so far considered a discrete choice to invest in either asset. More generally, a portfolio can be divided between the two assets. Define w_i as the weight placed on asset i . With full investment, we must have $\sum_{i=1}^N w_i = 1$, where N is the total number of assets. In other words, the portfolio weights must sum to 1.

⁶ Grinold, Richard and Ronald Kahn, *Active Portfolio Management* (New York: McGraw-Hill, 2000).

Start with a portfolio fully invested in bonds, represented by $w_1 = 1.00$, $w_2 = 0.00$. As we shift the weight to stocks, we can trace a line that represents the risk and expected return of the portfolio mix. Eventually, this moves to the stock-only position, $w_1 = 0.00$, $w_2 = 1.00$. Figure 1.4 shows the line that describes all of these portfolios. This is an example of an **asset allocation** problem, where the investor has to decide how much to allocate across asset classes.

The shape of this line depends on the correlation coefficient, ρ , which measures the extent to which the two assets comove with each other. The correlation coefficient is scaled so that it must be between -1 and $+1$. If $\rho = 1$, the two assets move perfectly proportionately to each other and the line becomes a straight line.

More generally, the line is curved. This leads to an interesting observation. The line contains a portfolio with the same level of risk as that of bonds but with a higher return. Thus, a diversified portfolio can dominate one of the assets.

To demonstrate this point, consider the computation of the expected return and volatility for a portfolio. The portfolio variance depends on the weights, the individual asset variances, as well as the correlation:

$$\sigma_P^2 = w_1^2\sigma_1^2 + 2w_1w_2(\rho\sigma_1\sigma_2) + w_2^2\sigma_2^2 \quad (1.6)$$

Hence the portfolio volatility, which is the square root of the variance, is a nonlinear function of the weights. In contrast, the portfolio expected return is a simple linear average:

$$\mu_P = w_1\mu_1 + w_2\mu_2 \quad (1.7)$$

Consider, for instance, the mix of 77% bonds and 23% stocks. The portfolio mean is easy to compute. Using data from Table 1.1, this is

$$\mu_P = 0.77 \times 5.6 + 0.23 \times 11.2 = 6.9\%$$

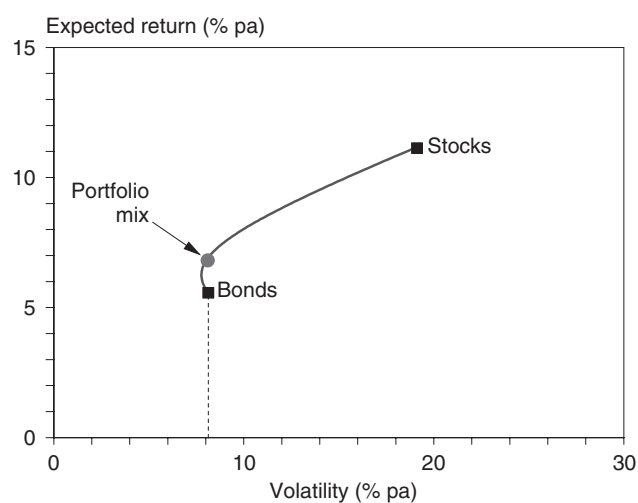


FIGURE 1.4 Mixing Two Assets

When $\rho = 1$, the portfolio variance is, using Equation (1.6):

$$\sigma_p^2 = 0.77^2 8.1^2 + 2 \times 0.77 \times 0.23(1 \times 8.1 \times 19.2) + 0.23^2 19.2^2 = 113.49$$

The portfolio volatility is 10.65. But in this case, this is also a linear average of the two volatilities $\sigma_p = 0.77 \times 8.1 + 0.23 \times 19.2 = 10.65$. Hence this point is on a straight line between the two assets.

Consider next the case where the correlation is the observed value of $\rho = 0.13$, shown in Table 1.1. The portfolio mix then has a volatility of 8.1%. This portfolio has the same volatility as bonds, yet better performance. The mix has expected return of 6.9%, against the 5.6% for bonds, which is an improvement of 1.4%. This demonstrates the power of diversification.

Finally, consider a hypothetical case where the correlation is $\rho = -1$. In this case, the variance drops to 3.32 and the volatility to 1.8%. This illustrates the important point that low correlations help to decrease portfolio risk (at least when the portfolio weights are positive).

1.3.4 Efficient Portfolios

Consider now a more general problem, which is that of diversification across a large number of stocks, for example, $N = 500$ stocks within the S&P 500 index. This seems an insurmountable problem because there are so many different combinations of these stocks. Yet Markowitz showed how the problem can be reduced to a much narrower choice.

The starting point is the assumption that all assets follow a jointly normal distribution. If so, the entire distribution of portfolio returns can be summarized by two parameters only, the mean and the variance.

To solve the diversification problem, all that is needed is the identification of the **efficient set** in this mean-variance space (more precisely, mean–standard deviation). This is the locus of points that represent portfolio mixes with the best risk–return characteristics. More formally, each portfolio, defined by a set of weights $\{w\}$, is such that, for a specified value of expected return μ_p , the risk is minimized:

$$\text{Min}_w \sigma_p^2 \quad (1.8)$$

subject to a number of conditions: (1) the portfolio return is equal to a specified value k , and (2) the portfolio weights sum to 1. Changing this specified value k traces out the efficient set.

When there are no short-sale restrictions on portfolio weights, a closed-form solution exists for the efficient set. Any portfolio is a linear combination of two portfolios. The first is the global **minimum-variance portfolio**, which has the lowest volatility across all fully invested portfolios. The second is the portfolio with the highest Sharpe ratio.

This framework can be generalized to value at risk, which is especially valuable when return distributions have fat tails. In the general case, no closed-form solution exists, however.

1.4 ASSET PRICING THEORIES

1.4.1 Capital Asset Pricing Model (CAPM)

These insights have led to the **capital asset pricing model** (CAPM), developed by William Sharpe (1964).⁷ Sharpe's first step was to simplify the covariance structure of stocks with a one-factor model. Define $R_{i,t}$ as the return on stock i during month t , $R_{F,t}$ as the risk-free rate, and $R_{M,t}$ as the market return. Then run a regression of the excess return for i on the market across time:

$$R_{i,t} - R_{F,t} = \alpha_i + \beta_i[R_{M,t} - R_{F,t}] + \epsilon_{i,t}, \quad t = 1, \dots, T \quad (1.9)$$

The slope coefficient, β_i , measures the exposure of i to the market factor and is also known as **systematic risk**. The intercept is α_i . For an actively managed portfolio α is a measure of management skill. Finally, ϵ_i is the residual, which has mean zero and is uncorrelated to R_M by construction and to all other residuals by assumption.

This is a one-factor model because any interactions between stocks are due to their exposure to the market. To simplify, ignore the risk-free rate and α , which is constant anyway. We now examine the covariance between two stocks, i and j . This is a measure of the comovements between two random variables, and is explained further in Chapter 2.

$$\begin{aligned} \text{Cov}(R_i, R_j) &= \text{Cov}(\beta_i R_M + \epsilon_i, \beta_j R_M + \epsilon_j) \\ &= \beta_i \beta_j \text{Cov}(R_M, R_M) + \beta_i \text{Cov}(R_M, \epsilon_j) + \beta_j \text{Cov}(R_M, \epsilon_i) + \text{Cov}(\epsilon_i, \epsilon_j) \\ &= \beta_i \beta_j \sigma^2(R_M) \end{aligned} \quad (1.10)$$

because all ϵ are uncorrelated with R_M and with each other. Hence, the asset-specific risk, or ϵ , is called **idiosyncratic**.

Such simplified factor structure is extremely useful because it cuts down the number of parameters. With 100 assets, for example, there are in theory $N(N-1)/2 = 4,950$ different pairwise covariances. This is too many to estimate. In contrast, the factor structure in Equation (1.10) involves only 100 parameters, the β_i , plus the variance of the market. This considerably simplifies the analysis.

This type of approximation lies at the heart of **mapping**, a widely used process in risk management. Mapping replaces individual positions by a smaller number of exposures on fundamental risk factors. It will be covered in more detail in a later chapter. Suffice it to say, this requires from the risk manager a good command of quantitative tools in addition to judgment as to whether such simplification is warranted.

Sharpe (1964) then examined the conditions for capital market equilibrium. This requires that the total demand for each asset, as derived from the investors' portfolio optimization, match exactly the outstanding supply of assets (i.e., the

⁷William Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* 19 (1964): 425-442.

issued shares of equities). The total demand can be aggregated because investors are assumed to have homogeneous expectations about the distribution of rates of return, which is also assumed normal.

In addition, the model assumes the existence of a risk-free asset, which can be used for borrowing or lending, at the same rate. As is usual in most economic models, capital markets are assumed perfect. That is, there are no transaction costs, securities are infinitely divisible, and short sales are allowed.

Under these conditions, Sharpe showed that the market portfolio, defined as the value-weighted average of all stocks in the portfolio, must have the highest Sharpe ratio of any feasible portfolio. Hence, it must be mean-variance efficient. Figure 1.5 shows that the line joining the risk-free asset F to the market portfolio M has the highest Sharpe ratio of any portfolio on the efficient frontier. This line is also known as the **capital market line (CML)**. It dominates any combination of cash and stock investment.

Portfolios located between F and M have a positive fraction allocated to each. Investors that are very risk-averse would hold a mix closer to F . Risk-tolerant investors would choose portfolios closer to, or even higher than, the market M . Portfolios above M represent a levered position in the market (i.e., borrowing at the risk-free rate and reinvesting the proceeds in the stock market).

Even so, all investors should hold the same relative proportion of stocks. For instance, Exxon is the largest company in the S&P 500 index, accounting for about 4% of the market. According to the CAPM, all investors should hold 4% of their stock portfolio, whether small or large, in Exxon.

Figure 1.5 demonstrates the concept of **two-fund separation**. Any efficient portfolio must be on the straight line and hence can be separated into two funds, the risk-free asset and the market.

Finally, the mean-variance efficiency of the market implies a linear relationship between expected excess returns and the systematic risk of all stocks in the market portfolio. For any stock i , we must have

$$E(R_i) = R_F + \beta_i[E(R_M) - R_F] \quad (1.11)$$

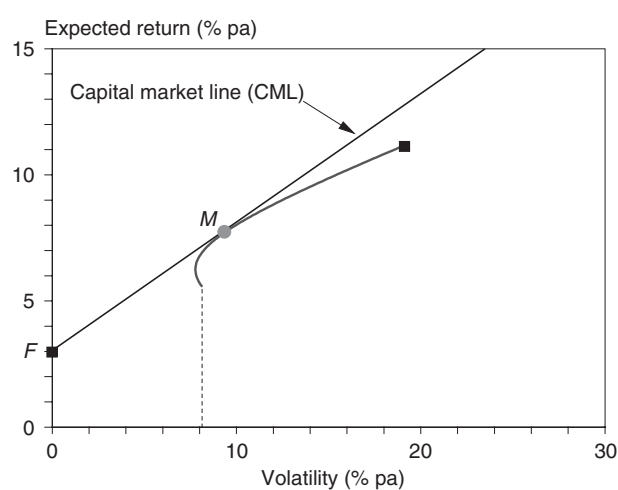


FIGURE 1.5 The Capital Market Line

This required rate of return on equity can also be viewed as the cost of equity capital. In other words, the α_i term in Equation (1.9) should all be zero. This explains why for an actively managed portfolio α is a commonly used term to measure skill.

This theory has profound consequences. It says that investors can easily diversify their portfolios. As a result, most idiosyncratic risk washes away. In contrast, systematic risk cannot be diversified. This explains why Equation (1.11) only contains β_i as a risk factor for stock i . Note that volatility σ_i never appears directly in the pricing of risk.

This leads to an alternative measure of performance, which is the **Treynor ratio** (TR), defined as

$$\text{TR} = \frac{[\mu(R_P) - R_F]}{\beta_P} \quad (1.12)$$

If the CAPM holds, this ratio should be the same for all assets. Equation (1.11) indeed shows that the ratio of $[\mu(R_i) - R_F]/\sigma(R_i)$ should be constant.

The TR penalizes for high β , as opposed to the SR, which penalizes for high σ . For an investor with a portfolio similar to the market, β measures the contribution to the risk of the portfolio. Hence, this is a better performance measure for well-diversified portfolios. In contrast, the SR can be used to adjust the performance of undiversified portfolios. Therefore, a major drawback of the SR and IR is that they do not penalize for systematic risk.

This leads to another performance measure, which directly derives from the CAPM. Suppose an active investment manager claims to add value for portfolio P . The observed average excess return is $\mu(R_P) - R_F$. Some of this, however, may be due to market beta. The proper measure of performance is then **Jensen's alpha**:

$$\alpha_P = \mu(R_P) - R_F - \beta_P[\mu(R_M) - R_F] \quad (1.13)$$

For a fixed portfolio, if all stocks were priced according to the CAPM, the portfolio alpha should be zero (actually it should be negative if one accounts for management fees and other costs).

In summary, the CAPM assumes that (1) investors have homogeneous expectations, (2) distributions are normal, (3) capital markets are perfect, and (4) markets are in equilibrium. The difficulty is that in theory, the market portfolio should contain all investible assets worldwide. As a result, it is not observable.

1.4.2 Arbitrage Pricing Theory (APT)

The CAPM specification starts from a single-factor model. This can be generalized to multiple factors. The first step is to postulate a risk structure where movements in asset returns are due to multiple sources of risk.

For instance, in the stock market, small firms behave differently from large firms. This could be a second factor in addition to the market. Other possible

factors are energy prices, interest rates, and so on. With K factors, Equation (1.9) can be generalized to

$$R_i = \alpha_i + \beta_{i1}\gamma_1 + \cdots + \beta_{iK}\gamma_K + \epsilon_i \quad (1.14)$$

Here again, the residuals ϵ are assumed to be uncorrelated with the factors by construction and with each other by assumption. The risk decomposition in Equation (1.10) can be generalized in the same fashion.

The **arbitrage pricing theory** (APT) developed by Stephen Ross (1976) relies on such a factor structure and the assumption that there is no arbitrage in financial markets.⁸ Formally, portfolios can be constructed that are well-diversified and have little risk. To prevent arbitrage opportunities, these portfolios should have zero expected returns. These conditions force a linear relationship between expected returns and the factor exposures:

$$E[R_i] = R_F + \sum_{k=1}^K \beta_{ik}\lambda_k \quad (1.15)$$

where λ_k is the market price of factor k .

Consider, for example, the simplest case where there is one factor only. We have three stocks, A, B, and C, with betas of 0.5, 1.0, and 1.5, respectively. Assume now that their expected returns are 6%, 8%, and 12%. We can then construct a portfolio long 50% in A, long 50% in C, and short 100% in B. The portfolio beta is $50\% \times 0.5 + 50\% \times 1.5 - 100\% \times 1.0 = 0$. This portfolio has no initial investment but no risk and therefore should have expected return of zero. Computing the expected return gives $50\% \times 6\% + 50\% \times 12\% - 100\% \times 8\% = +1\%$, however. This would create an arbitrage opportunity, which must be ruled out. These three expected returns are inconsistent with the APT. From A and C, we have $R_F = 3\%$ and $\lambda_1 = 6\%$. As a result, the APT expected return on B should be, by Equation (1.15), $E[R_i] = R_F + \beta_{i1}\lambda_1 = 3\% + 1.0 \times 6\% = 9\%$.

Note that the APT expected return is very similar to the CAPM, Equation (1.11), in the case of a single factor. The interpretation, however, is totally different. The APT does not rely on equilibrium but simply on the assumption that there should be no arbitrage opportunities in capital markets, a much weaker requirement. It does not even need the factor model to hold strictly. Instead, it requires only that the residual risk is very small. This must be the case if a sufficient number of common factors is identified and in a well-diversified portfolio, also called **highly granular**.

The APT model does not require the market to be identified, which is an advantage. Unfortunately, tests of this model are ambiguous because the theory provides no guidance as to what the factors should be.

Several approaches to the selection of factors are possible. The first, a structural approach, is to prespecify factors from economic intuition or experience. For

⁸ Stephen Ross, "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory* 13 (1976): 341–360.

example, factors such as value, size, and momentum have been widely used to explain expected stock returns. The second is a statistical approach that extracts the factors from the observed data. For example, **principal component analysis (PCA)** is a technique that provides the best fit to the correlation matrix of asset returns. The first PC is a linear combination of asset returns that provides the best approximation to the diagonal of this matrix. The second PC is a linear combination of asset returns that is orthogonal to the first and provides the next best approximation. The analysis can continue until the remaining factors are no longer significant.

Such an approach is very important for risk management when it is a large-scale problem, as for a large portfolio. For example, the risk manager might be able to reduce the dimensionality of a portfolio of 100 stocks using perhaps 10 risk factors. This reduction is particularly important for Monte Carlo simulation, as it cuts down the computing time.

EXAMPLE 1.3: FRM EXAM 2009—QUESTION 1-4

An analyst at CAPM Research Inc. is projecting a return of 21% on Portfolio A. The market risk premium is 11%, the volatility of the market portfolio is 14%, and the risk-free rate is 4.5%. Portfolio A has a beta of 1.5. According to the capital asset pricing model, which of the following statements is true?

- a. The expected return of Portfolio A is greater than the expected return of the market portfolio.
- b. The expected return of Portfolio A is less than the expected return of the market portfolio.
- c. The return of Portfolio A has lower volatility than the market portfolio.
- d. The expected return of Portfolio A is equal to the expected return of the market portfolio.

EXAMPLE 1.4: FRM EXAM 2009—QUESTION 1-6

Suppose Portfolio A has an expected return of 8%, volatility of 20%, and beta of 0.5. Suppose the market has an expected return of 10% and volatility of 25%. Finally, suppose the risk-free rate is 5%. What is Jensen's alpha for Portfolio A?

- a. 10.0%
- b. 1.0%
- c. 0.5%
- d. 15%

EXAMPLE 1.5: FRM EXAM 2007—QUESTION 132

Which of the following statements about the Sharpe ratio is *false*?

- a. The Sharpe ratio considers both the systematic and unsystematic risks of a portfolio.
- b. The Sharpe ratio is equal to the excess return of a portfolio over the risk-free rate divided by the total risk of the portfolio.
- c. The Sharpe ratio cannot be used to evaluate relative performance of undiversified portfolios.
- d. The Sharpe ratio is derived from the capital market line.

EXAMPLE 1.6: SHARPE AND INFORMATION RATIOS

A portfolio manager returns 10% with a volatility of 20%. The benchmark returns 8% with risk of 14%. The correlation between the two is 0.98. The risk-free rate is 3%. Which of the following statements is *correct*?

- a. The portfolio has higher SR than the benchmark.
- b. The portfolio has negative IR.
- c. The IR is 0.35.
- d. The IR is 0.29.

1.5 VALUE OF RISK MANAGEMENT

The previous sections have shown that most investment or business decisions require information about the risks of alternative choices. The asset pricing theories also give us a frame of reference to think about how risk management can increase economic value.

1.5.1 Irrelevance of Risk Management

The CAPM emphasizes that investors dislike systematic risk. They can diversify other, nonsystematic risks on their own. Hence, systematic risk is the only risk that needs to be priced.

Companies could use financial derivatives to hedge their volatility. If this changes the volatility but not the market beta, however, the company's cost of capital and hence valuation cannot be affected. Such a result holds only under the perfect capital market assumptions that underlie the CAPM.

As an example, suppose that a company hedges some financial risk that investors could hedge on their own anyway. For example, an oil company could hedge its oil exposure. It would be easy, however, for investors in the company's stock to hedge such oil exposure on their own if they so desired. For instance, they could use oil futures, which can be easily traded. In this situation, the risk management practice by this company should not add economic value.

In this abstract world, risk management is irrelevant. This is an application of the classic Modigliani and Miller (MM) theorem, which says that the value of a firm cannot depend on its financial policies. The intuition for this result is that any financing action undertaken by a corporation that its investors could easily undertake on their own should not add value. Worse, if risk management practices are costly, they could damage the firm's economic value.

1.5.2 Relevance of Risk Management

The MM theorem, however, is based on a number of assumptions: There are no frictions such as financial distress costs, taxes, and access to capital markets; there is no asymmetry of information between financial market participants.

In practice, risk management can add value if some of these assumptions do not hold true.

- Hedging should increase value if it helps avoid a large *cost of financial distress*. Companies that go bankrupt often experience a large drop in value due to forced sales or the legal costs of the bankruptcy process. Consider, for example, the Lehman Brothers bankruptcy. Lehman had around \$130 billion of outstanding bonds. The value of these bonds dropped to 9.75 cents on the dollar after September 2008. This implied a huge drop in the firm value due to the bankruptcy event.
- Corporate income *taxes* can be viewed as a form of friction. Assuming no carry-over provisions, the income tax in a very good year is high and is not offset by a tax refund in a year with losses. A tax is paid on income if positive, which is akin to a long position in an option, with a similar convexity effect. Therefore, by stabilizing earnings, corporations reduce the average tax payment over time, which should increase their value.
- Other frictions arise when *external financing* is more costly than internally generated funds. A company could decide not to hedge its financial risks, which leads to more volatile earnings. In good years, projects are financed internally. In bad years, it is always possible to finance projects by borrowing from capital markets. If the external borrowing cost is too high, however, some worthy projects will not be funded in bad years. Hedging helps avoid this **underinvestment problem**, which should increase firm value.
- A form of information asymmetry is due to *agency costs of managerial discretion*. Investors hire managers to serve as their agents, and give them discretion to run the company. Good and bad managers, however, are not always easy to identify. Without hedging, earnings fluctuate due to outside forces. This

makes it difficult to identify the performance of management. With hedging, there is less room for excuses. Bad managers can be identified more easily and fired, which should increase firm value.

- Another form of information asymmetry arises when a large shareholder has *expertise in the firm's business*. This expertise, in addition to the ability to monitor management more effectively than others, may increase firm value. Typically, such investors have a large fraction of their wealth invested in this firm. Because they are not diversified, they may be more willing to invest in the company and hence add value when the company lowers its risk.

In practice, there is empirical evidence that firms that engage in risk management programs tend to have higher valuation than others. This type of analysis is fraught with difficulties, however. Researchers do not have access to identical firms that solely differ by their hedging program. Other confounding effects might be at work. Hedging might be correlated with the quality of management, which does have an effect on firm value. For example, firms with risk management programs are more likely to employ financial risk managers (FRMs).

EXAMPLE 1.7: FRM EXAM 2009—QUESTION 1-8

In perfect markets, risk management expenditures aimed at reducing a firm's diversifiable risk serve to

- a. Make the firm more attractive to shareholders as long as costs of risk management are reasonable
- b. Increase the firm's value by lowering its cost of equity
- c. Decrease the firm's value whenever the costs of such risk management are positive
- d. Has no impact on firm value

EXAMPLE 1.8: FRM EXAM 2009—QUESTION 1-2

By reducing the risk of financial distress and bankruptcy, a firm's use of derivatives contracts to hedge its cash flow uncertainty will

- a. Lower its value due to the transaction costs of derivatives trading
- b. Enhance its value since investors cannot hedge such risks by themselves
- c. Have no impact on its value as investors can costlessly diversify this risk
- d. Have no impact as only systematic risks can be hedged with derivatives

1.6 IMPORTANT FORMULAS

Absolute risk: $\sigma(\Delta P) = \sigma(\Delta P/P) \times P = \sigma(R_P) \times P$

Relative risk: $\sigma(e)P = [\sigma(R_P - R_B)] \times P = \omega \times P$

Tracking error volatility (TEV): $\omega = \sigma(R_P - R_B)$

Sharpe ratio (SR): $SR = [\mu(R_P) - R_F]/\sigma(R_P)$

Information ratio (IR): $IR = [\mu(R_P) - \mu(R_B)]/\omega$

Treynor ratio (TR): $TR = [\mu(R_P) - R_F]/\beta_P$

Jensen's alpha: $\alpha_P = \mu(R_P) - R_F - \beta_P[\mu(R_M) - R_F]$

Multiple factor model: $R_i = \alpha_i + \beta_{i1}\gamma_1 + \dots + \beta_{iK}\gamma_K + \epsilon_i$

CAPM expected returns: $E(R_i) = R_F + \beta_i[E(R_M) - R_F]$

APT expected returns: $E[R_i] = R_F + \sum_{k=1}^K \beta_{ik}\lambda_k$

1.7 ANSWERS TO CHAPTER EXAMPLES

Example 1.1: Absolute and Relative Risk

c. This is an example of risk measured in terms of deviations of the active portfolio relative to the benchmark. Answers a. and b. are incorrect because they refer to absolute risk. Answer d. is also incorrect because it refers to the absolute risk of the benchmark.

Example 1.2: FRM Exam 2009—Question 1-11

d. It is the role of the CEO to decide on such investments, not the CRO. The CRO had correctly estimated that there was some chance of losing \$1 billion or more. In addition, there is no information on the distribution beyond VAR. So, this could have been bad luck. A risk management failure could have occurred if the CRO had stated that this probability was zero.

Example 1.3: FRM Exam 2009—Question 1-4

a. According to the CAPM, the required return on Portfolio A is $R_F + \beta[E(R_M) - R_F] = 4.5 + 1.5[11] = 21\%$ indeed. Because the beta is greater than 1, it must be greater than the expected return on the market, which is 15.5%. Note that the question has a lot of extraneous information.

Example 1.4: FRM Exam 2009—Question 1-6

c. This is the reverse problem. The CAPM return is $R_F + \beta[E(R_M) - R_F] = 5 + 0.5[10 - 5] = 7.5\%$. Hence the alpha is $8 - 7.5 = 0.5\%$.

Example 1.5: FRM Exam 2007—Question 132

c. The SR considers total risk, which includes systematic and unsystematic risks, so a. and b. are correct statements, and incorrect answers. Similarly, the SR is derived from the CML, which states that the market is mean-variance efficient and hence has the highest Sharpe ratio of any feasible portfolio. Finally, the SR can be used to evaluate undiversified portfolios, precisely because it includes idiosyncratic risk.

Example 1.6: Sharpe and Information Ratios

d. The Sharpe ratios of the portfolio and benchmark are $(10\% - 3\%)/20\% = 0.35$ and $(8\% - 3\%)/14\% = 0.36$, respectively. So the SR of the portfolio is lower than that of the benchmark; answer a. is incorrect. The TEV is the square root of $20\%^2 + 14\%^2 - 2 \times 0.98 \times 20\% \times 14\%$, which is $\sqrt{0.00472} = 6.87\%$. So, the IR of the portfolio is $(10\% - 8\%)/6.87\% = 0.29$. This is positive, so answer b. is incorrect. Answer c. is the SR of the portfolio, not the IR, so it is incorrect.

Example 1.7: FRM Exam 2009—Question 1-8

c. In perfect markets, risk management actions that lower the firm's diversifiable risk should not affect its cost of capital, and hence will not increase value. Further, if these activities are costly, the firm value should decrease.

Example 1.8: FRM Exam 2009—Question 1-2

b. The cost of financial distress is a market imperfection, or deadweight cost. By hedging, firms will lower this cost, which should increase the economic value of the firm.