Chapter 1

Beginning at the Very Beginning: Pre-Pre-Calculus

In This Chapter

- Brushing up on the order of operations
- Solving equalities
- Graphing equalities and inequalities
- Finding distance, midpoint, and slope

Pre-calculus is the stepping stone for calculus. It's the final hurdle after all those years of math: pre-algebra, Algebra I, geometry, and Algebra II. Now all you need is pre-calculus to get to that ultimate goal — calculus. And as you may recall from your Algebra II class, you were subjected to much of the same material you saw in algebra and even pre-algebra (just a couple steps up in terms of complexity — but really the same stuff). As the stepping stone, pre-calculus begins with certain concepts that you're expected to understand.

Therefore, we're starting here, at the very beginning, reviewing those concepts. If you feel you're already an expert at everything algebra, feel free to skip past this chapter and get the full swing of pre-calc going. If, however, you need to review, then read on.

TIP

If you don't remember some of the concepts we discuss in this chapter, or even in this book, you can pick up another *For Dummies* math book for review. The fundamentals are important. That's why they're called fundamentals. Take the time now to review and save yourself countless hours of frustration in the future!

Reviewing Order of Operations: The Fun in Fundamentals

You can't put on your sock after you put on your shoe, can you? The same concept applies to mathematical operations. There's a specific order to which operation you perform first, second, third, and so on. At this point, it should be second nature, but because the concept is so important (especially when you start doing more complex calculations), a quick review is worth it, starting with everyone's favorite mnemonic device.

TIP

Please excuse who? Oh, yeah, you remember this one — my dear Aunt Sally! The old mnemonic still stands, even as you get into more complicated problems. Please Excuse My Dear Aunt Sally is a mnemonic for the acronym PEMDAS, which stands for

- Parentheses (including absolute value, brackets, and radicals)
- Exponents
- Multiplication and Division (from left to right)
- Addition and Subtraction (from left to right)

The order in which you solve algebraic problems is very important. Always work what's in the parentheses first, then move on to the exponents, followed by the multiplication and division (from left to right), and finally, the addition and subtraction (from left to right).

PRE-CALC RULES

You should also have a good grasp on the properties of equality. If you do, you'll have an easier time simplifying expressions. Here are the properties:

- **Reflexive property: $a = a$.** For example, $4 = 4$.
- **Symmetric property: If $a = b$, then $b = a$.** For example, if $2 + 8 = 10$, then $10 = 2 + 8$.
- **Transitive property: If $a = b$ and $b = c$, then $a = c$.** For example, if $2 + 8 = 10$ and $10 = 5 \cdot 2$, then $2 + 8 = 5 \cdot 2$.
- **Commutative property of addition: $a + b = b + a$.** For example, $3 + 4 = 4 + 3$.
- **Commutative property of multiplication: $a \cdot b = b \cdot a$.** For example, $3 \cdot 4 = 4 \cdot 3$.
- **Associative property of addition: $a + (b + c) = (a + b) + c$.** For example, $3 + (4 + 5) = (3 + 4) + 5$.
- **Associative property of multiplication: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.** For example, $3 \cdot (4 \cdot 5) = (3 \cdot 4) \cdot 5$.
- **Additive identity: $a + 0 = a$.** For example, $4 + 0 = 4$.
- **Multiplicative identity: $a \cdot 1 = a$.** For example, $-18 \cdot 1 = -18$.
- **Additive inverse property: $a + (-a) = 0$.** For example, $5 + (-5) = 0$.
- **Multiplicative inverse property: $a \cdot \frac{1}{a} = 1$.** For example, $-2 \cdot (-\frac{1}{2}) = 1$.
- **Distributive property: $a(b + c) = a \cdot b + a \cdot c$.** For example, $5(3 + 4) = 5 \cdot 3 + 5 \cdot 4$.
- **Multiplicative property of zero: $a \cdot 0 = 0$.** For example, $4 \cdot 0 = 0$.
- **Zero product property: If $a \cdot b = 0$, then $a = 0$ or $b = 0$.** For example, if $x(2x - 3) = 0$, then $x = 0$ or $2x - 3 = 0$.

Following are a couple examples so you can see the order of operations and the properties of equality in action before diving into some practice questions.

Q. Simplify: $= \dfrac{6^2 - 4\left(3 - \sqrt{20+5}\right)^2}{|4-8|}$

A. The answer is 5.

Following the order of operations, simplify everything in parentheses first. (Remember that radicals and absolute value marks act like parentheses, so do operations within them first before simplifying the radicals or taking the absolute value.)

Simplify the parentheses by taking the square root of 25 and the absolute value of –4, like so:

$$\frac{6^2 - 4\left(3 - \sqrt{25}\right)^2}{|-4|} = \frac{6^2 - 4(3-5)^2}{4} = \frac{6^2 - 4(-2)^2}{4}$$

Now you can deal with the exponents by squaring the 6 and the –2: $\dfrac{36 - 4(4)}{4}$.

Note: Although they're not written, parentheses are implied around the terms above and below a fraction bar. In other words, the expression $\dfrac{36-4(4)}{4}$ can also be written as $\dfrac{[36-4(4)]}{4}$. Therefore, you must simplify the numerator and denominator before dividing the terms following the order of operations, like this: $\dfrac{36-4(4)}{4} = \dfrac{36-16}{4} = \dfrac{20}{4} = 5$.

Q. Simplify: $\dfrac{\left(\frac{1}{8} + \frac{1}{3}\right) + \frac{3}{8}}{\frac{3}{18} + \frac{1}{9}}$

A. The answer is 3.

Using the associative property and the commutative property of addition, rewrite the expression to make the fractions easier to add.

$$\frac{\left(\frac{1}{8} + \frac{3}{8}\right) + \frac{1}{3}}{\frac{3}{18} + \frac{1}{9}}$$

Add the fractions with common denominators.

$$\frac{\frac{4}{8} + \frac{1}{3}}{\frac{3}{18} + \frac{1}{9}}$$

Then reduce the resulting fraction to get the following:

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{18} + \frac{1}{9}}$$

Next, find a common denominator for the fractions in the numerator and denominator.

$$\frac{\frac{3}{6} + \frac{2}{6}}{\frac{3}{18} + \frac{2}{18}}$$

Add them, like so:

$$\frac{\frac{5}{6}}{\frac{5}{18}}$$

Recognizing that this expression is a division problem, $\frac{5}{6} \div \frac{5}{18}$, multiply by the inverse and simplify:

$$\frac{5}{6} \cdot \frac{18}{5} = \frac{5 \cdot 18}{6 \cdot 5} = \frac{\cancel{5} \cdot 18}{6 \cdot \cancel{5}} = \frac{3}{1} = 3$$

1. Simplify: $\dfrac{3\sqrt{(4-6)^2+\left[2-(-1)\right]^2}}{|-3-(-1)|}$

Solve It

2. Simplify: $\dfrac{|-3|-|2|+(-1)}{|-7+2|}$

Solve It

3. Simplify: $(2^3 - 3^2)^4(-5)$

Solve It

4. Simplify: $\dfrac{|5(1-4)+6|}{3\left(-\frac{1}{6}+\frac{1}{3}\right)-\frac{1}{2}}$

Solve It

Keeping Your Balance While Solving Equalities

Just as simplifying expressions is the basis of pre-algebra, solving for variables is the basis of algebra. Why should you care? Because both are essential to the more complex concepts covered in pre-calculus.

Solving linear equations with the general format of $ax + b = c$, where a, b, and c are constants, is relatively easy using the properties of numbers. The goal, of course, is to isolate the variable, x.

REMEMBER

One type of equation you can't forget is absolute value equations. The *absolute value* is defined as the distance from 0. In other words, $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$. As such, an absolute value has two possible solutions: one where the quantity inside the absolute value bars is positive and another where it's negative. To solve these equations, you must isolate the absolute value term (find the value of the absolute value term) and then set the quantity inside the absolute value bars to the positive and negative values (see the second example question that follows).

Check out the following examples or skip ahead to the practice questions if you think you're ready to tackle them.

Q. Solve for x: $3(2x - 4) = x - 2(-2x + 3)$

A. $x = 6$

First, using the distributive property, distribute the 3 and the –2 to get $6x - 12 = x + 4x - 6$. Then combine like terms and solve using algebra, like so: $6x - 12 = 5x - 6$; $x - 12 = -6$; $x = 6$.

Q. Solve for x: $|x-3| + (-16) = -12$

A. $x = 7$ or -1

Isolate the absolute value: $|x-3| = 4$. Next, set the quantity inside the absolute value bars to the positive solution: $x - 3 = 4$. Then set the quantity inside the absolute value bars to the negative solution: $-(x - 3) = 4$. Solve both equations to find two possible solutions: $x - 3 = 4$, which gives $x = 7$; and $-(x - 3) = 4$, which yields $-x + 3 = 4$ and the solution $x = -1$.

5. Solve: $3 - 6[2 - 4x(x + 3)] = 3x(8x + 12) + 27$

Solve It

6. Solve: $\frac{x}{2} + \frac{x-2}{4} = \frac{x+4}{2}$

Solve It

7. Solve: $|x-3| + |3x+2| = 4$

Solve It

8. Solve: $3 - 4(2 - 3x) = 2(6x + 2)$

Solve It

9. Solve: $2|x-3|+12=6$

Solve It

10. Solve: $3(2x + 5) + 10 = 2(x + 10) + 4x + 5$

Solve It

When Your Image Really Counts: Graphing Equalities and Inequalities

Graphs are visual representations of mathematical equations. In pre-calculus, you'll be introduced to many new mathematical equations and then be expected to graph them. We give you lots of practice graphing these equations when we cover the more complex equations. In the meantime, it's important to practice the basics: graphing linear equalities and inequalities.

REMEMBER

Both graphs exist on the *Cartesian coordinate system,* which is made up of two axes: the horizontal, or x-axis, and the vertical, or y-axis. Each point on the coordinate plane is called a *Cartesian coordinate pair* and has an x coordinate and a y coordinate. So the notation for any point on the coordinate plane looks like this: (x, y). A set of these ordered pairs that can be graphed on a coordinate plane is called a *relation.* The x values of a relation are its *domain*, and the y values are its *range*. For example, the domain of the relation R = {(2, 4), (–5, 3), (1, –2)} is {2, –5, 1}, and the range is {4, 3, –2}.

You can graph a linear equation, whether it's an equality or an inequality, in two ways: by using the plug-and-chug method or by using the slope-intercept form. We review both approaches in the following sections.

Graphing with the plug-and-chug method

To graph the plug-and-chug way, start by picking domain (x) values. Plug them into the equation to solve for the range (y) values. For linear equations, after you plot these points (x, y) on the coordinate plane, you can connect the dots to make a line. The process also works if you choose range values first and then plug in to find the corresponding domain values.

There's also a helpful method for finding *intercepts,* the points that fall on the x- or y-axes. To find the x-intercept $(x, 0)$, plug in 0 for y and solve for x. To find the y-intercept $(0, y)$, plug in 0 for x and solve for y. For example, to find the intercepts of the linear equation $2x + 3y = 12$, start by plugging in 0 for y: $2x + 3(0) = 12$. Then, using properties of numbers, solve for x: $2x + 0 = 12$; $2x = 12$; $x = 6$. So the x-intercept is (6, 0). For the y-intercept, plug in 0 for x and solve for y: $2(0) + 3y = 12$; $0 + 3y = 12$; $3y = 12$; $y = 4$. Therefore, the y-intercept is (0, 4). At this point, you can plot those two points and connect them to graph the line ($2x + 3y = 12$), because, as you learned in geometry, two points make a line. See the resulting graph in Figure 1-1.

TIP

As equations become more complex, you can use the plug-and-chug method to get some key pieces of information.

Graphing by using the slope-intercept form

The slope-intercept form of a linear equation gives a great deal of helpful information in a cute little package. The equation $y = mx + b$ immediately gives you the y-intercept (b) that you worked to find in the plug-and-chug method; it also gives you the slope (m). *Slope* is a fraction that gives you the rise over the run. To change equations that aren't written in slope-intercept form, you simply solve for y. For example, if you use the linear equation $2x + 3y = 12$, you start by subtracting $2x$ from each side: $3y = -2x + 12$. Next, you divide all the terms by 3: $y = -(\frac{2}{3})x + 4$. Now that the equation is in slope-intercept form, you know that the y-intercept is 4, and you can graph this point on the coordinate plane. Then, you can use the slope to plot the second point. From the slope-intercept equation, you know that the slope is $-\frac{2}{3}$. This tells you that the rise is –2 and the run is 3. From the point (0, 4), plot the point 2 down and 3 to the right. In other words, (3, 2). Lastly, connect the two points to graph the line. The resulting graph in Figure 1-2 is identical to Figure 1-1.

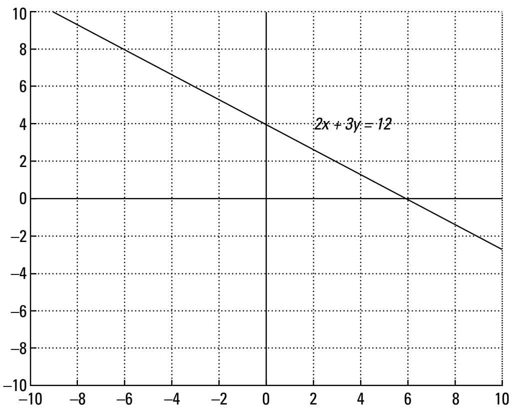


Figure 1-1: Graph of $2x + 3y = 12$.

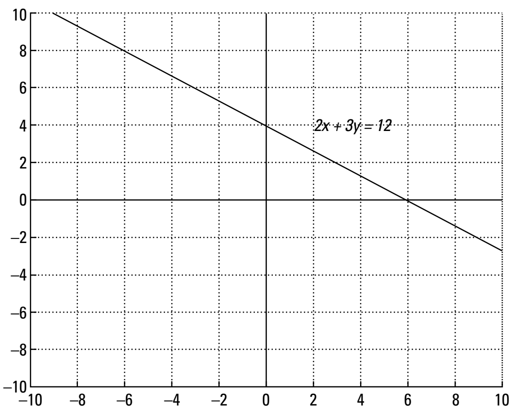


Figure 1-2: Graph of $y = -(\frac{2}{3})x + 4$.

Graphing inequalities by either method

Similar to graphing equalities, graphing inequalities begins with plotting two points by either method. However, because *inequalities* are used for comparisons — greater than, less than, or equal to — you have two more questions to answer after finding two points:

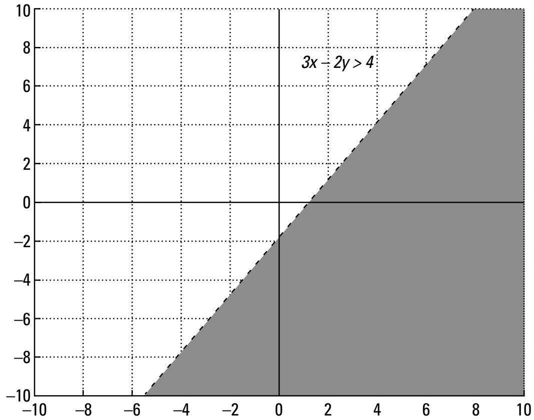
- Is the line dashed (< or >) or solid (≤ or ≥)?
- Do you shade under the line ($y <$ or $y \leq$) or above the line ($y >$ or $y \geq$)?

Here's an example of an inequality followed by a few practice questions.

Q. Sketch the graph of the inequality: $3x - 2y > 4$

A. Put the inequality into slope-intercept form by subtracting $3x$ from each side of the equation to get $-2y > -3x + 4$ and dividing each term by -2 to get $y < (\frac{3}{2})x - 2$. (***Remember:*** When you multiply or divide an inequality by a negative, you need to reverse the inequality.) From the resulting equation, you can find the y-intercept, -2, and the slope, $(\frac{3}{2})$. Use this information to graph two points by using the slope-intercept form. Next, decide the nature of the line (solid or dashed). Because the inequality is strict, the line is dashed. Graph the dashed line so you can decide where to shade. Because $y < (\frac{3}{2})x - 2$ is a less-than inequality, shade below the dashed line, as shown in the following figure.

This method works only if the boundary line is first converted to slope-intercept form. An alternative is to graph the boundary line using any method and then use a sample point (such as (0,0)) to determine which half-plane to shade.



11. Sketch the graph of $\frac{1}{3}(6x + 2y) = 16$.

Solve It

12. Sketch the graph of $\frac{5x+4y}{2} \geq 6$.

Solve It

13. Sketch the graph of $4x + 5y \geq 2(3y + 2x + 4)$.

Solve It

14. Sketch the graph of $x - 3y = 4 - 2y - y$.

Solve It

Using Graphs to Find Distance, Midpoint, and Slope

Graphs are more than just pretty pictures. From a graph, it's possible to determine two points. From these points, you can figure out the distance between them, the midpoint of the segment connecting them, and the slope of the line connecting them. As graphs become more complex in both pre-calculus and calculus, you're asked to find and use all three of these pieces of information. Aren't you lucky?

Finding the distance

PRE-CALC RULES

Distance refers to how far apart two things are. In this case, you're finding the distance between two points. Knowing how to calculate distance is helpful for when you get to conics (see Chapter 12). To find the distance between two points (x_1, y_1) and (x_2, y_2), use the following formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Calculating the midpoint

PRE-CALC RULES

The *midpoint* is the middle of a segment. This concept also comes up in conics (see Chapter 12) and is ever so useful for all sorts of other pre-calculus calculations. To find the midpoint, M, of the points (x_1, y_1) and (x_2, y_2), you just need to average the x and y values and express them as an ordered pair, like so: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

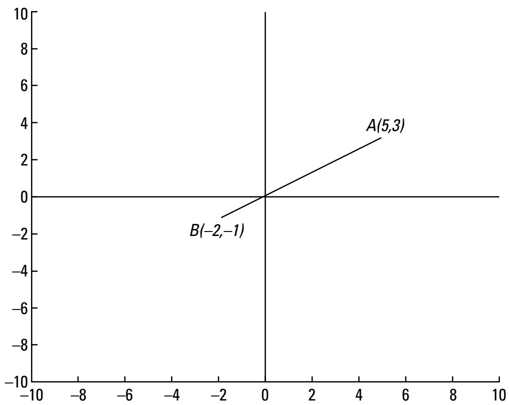
Discovering the slope

Slope is a key concept for linear equations, but it also has applications for trigonometric functions and is essential for differential calculus. *Slope* describes the steepness of a line on the coordinate plane (think of a ski slope). Use this formula to find the slope, m, of the line (or segment) connecting the two points (x_1, y_1) and (x_2, y_2): $m = \frac{y_2 - y_1}{x_2 - x_1}$

Note: Positive slopes move up and to the right (+/+) or down and to the left (–/–). Negative slopes move down and to the right (–/+) or up and to the left (+/–). Horizontal lines have a slope of 0, and vertical lines have an undefined slope.

Following is an example question for your reviewing pleasure. Look it over and then try your hand at the practice questions.

Q. Find the distance, slope, and midpoint of $\overline{AB}$.



A. The distance is $\sqrt{65}$, the slope is $\frac{4}{7}$, and the midpoint is $m = (\frac{3}{2}, 1)$

Plug the x and y values into the distance formula and, following the order of operations, simplify the terms under the radical (keeping in mind the implied parentheses of the radical itself).

$$d = \sqrt{[5-(-2)]^2 + [3-(-1)]^2} =$$
$$\sqrt{(5+2)^2 + (3+1)^2} = \sqrt{(7)^2 + (4)^2} =$$
$$\sqrt{49+16} = \sqrt{65}$$

Because 65 doesn't contain any perfect squares as factors, this is as simple as you can get. To find the midpoint, plug the points into the midpoint equation and simplify using the order of operations.

$$M = \left(\frac{5+(-2)}{2}, \frac{3+(-1)}{2}\right) = \left(\frac{3}{2}, \frac{2}{2}\right) = \left(\frac{3}{2}, 1\right)$$

To find the slope, use the formula, plug in your x and y values, and use the order of operations to simplify.

$$m = \frac{-1-3}{-2-5} = \frac{-4}{-7} = \frac{4}{7}$$

15. Find the length of segment CD, where C is (–2, 4) and D is (3, –1).

Solve It

16. Find the midpoint of segment EF, where E is (3, –5) and F is (7, 5).

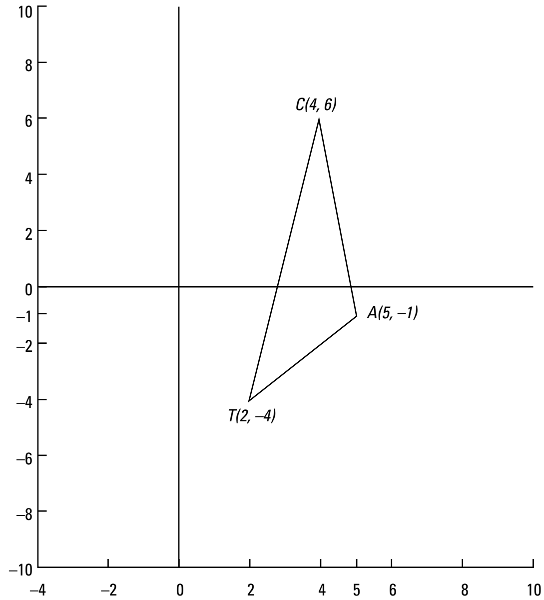
Solve It

17. Find the slope of line *GH*, where *G* is (–3, –5) and *H* is (–3, 4).

Solve It

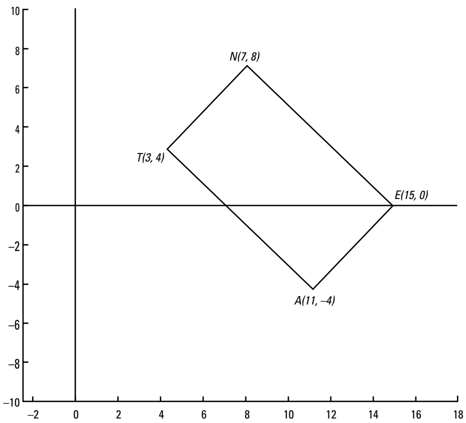
18. Find the perimeter of triangle *CAT*.

Solve It



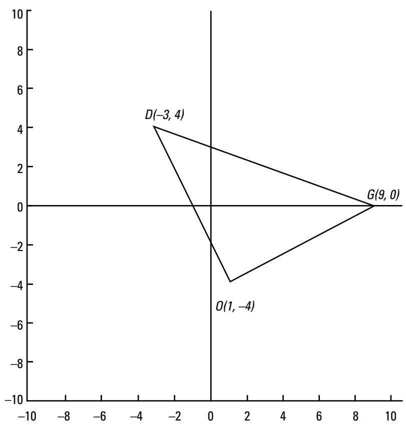
19. Find the center of the rectangle *NEAT*.

Solve It



20. Determine whether triangle *DOG* is a right triangle.

Solve It



Answers to Problems on Fundamentals

Following are the answers to questions dealing with pre-calculus fundamentals. We also provide guidance on getting the answers if you need to review where you went wrong.

1 Simplify $\frac{3\sqrt{(4-6)^2+[2-(-1)]^2}}{|-3-(-1)|}$. The answer is $=\frac{3\sqrt{13}}{2}$.

Start by simplifying everything in the parentheses. Next, simplify the exponents. Finally, add the remaining terms. Here's what your math should look like:

$$\frac{3\sqrt{(4-6)^2+[2-(-1)]^2}}{|-3-(-1)|}=\frac{3\sqrt{(-2)^2+(2+1)^2}}{|-3+1|}=\frac{3\sqrt{4+(3)^2}}{|-2|}=\frac{3\sqrt{4+9}}{2}=\frac{3\sqrt{13}}{2}$$

2 Simplify $\frac{|-3|-|2|+(-1)}{|-7+2|}$. The answer is 0.

Recognizing that the absolute value in the denominator acts as parentheses, add the –7 and 2 inside there first. Then rewrite the absolute value of each. Next, add the terms in the numerator. Finally, recognize that $\frac{0}{5}$ equals 0.

$$\frac{|-3|-|2|+(-1)}{|-7+2|}=\frac{|-3|-|2|+(-1)}{|-5|}=\frac{3-2+(-1)}{5}=\frac{0}{5}=0$$

3 Simplify $(2^3-3^2)^4(-5)$. The answer is –5.

Begin by simplifying the exponents in the parentheses. Next, simplify the parentheses by subtracting 9 from 8. Simplify the resulting exponent and multiply the result, 1, by –5.

$$(2^3-3^2)^4(-5)=(8-9)^4(-5)=(-1)^4(-5)=1(-5)=-5$$

4 Simplify $\frac{|5(1-4)+6|}{3\left(-\frac{1}{6}+\frac{1}{3}\right)-\frac{1}{2}}$. The answer is undefined.

Start by simplifying the parentheses. To do this, subtract 4 from 1 in the numerator and find a common denominator for the fractions in the denominator in order to add them. Next, multiply the terms in the numerator and denominator. Then add the terms in the absolute value bars in the numerator and subtract the terms in the denominator. Take the absolute value of –9 to simplify the numerator. Finally, remember that you can't have 0 in the denominator; therefore, the resulting fraction is undefined.

$$\frac{|5(1-4)+6|}{3\left(-\frac{1}{6}+\frac{1}{3}\right)-\frac{1}{2}}=\frac{|5(-3)+6|}{3\left(\frac{1}{6}\right)-\frac{1}{2}}=\frac{|-15+6|}{\frac{1}{2}-\frac{1}{2}}=\frac{|-9|}{0}$$, which is undefined.

5 Solve $3-6[2-4x(x+3)]=3x(8x+12)+27$. The answer is $x=1$.

Lots of parentheses in this one! Get rid of them by distributing terms. Start by distributing the $-4x$ on the left side over $(x+3)$ and the $3x$ on the right side over $(8x+12)$. Doing so gives you $3-6[2-4x(x+3)]=3-6[2-4x^2-12x]$ and $3x(8x+12)+27=24x^2+36x+27$. Next, distribute the –6 over the remaining parentheses on the left side of the equation to get $3-12+24x^2+72x=24x^2+72x-9$. Combine like terms on the left side: $-9+24x^2+72x=24x^2+36x+27$. To isolate x onto one side, subtract $24x^2$ from both sides to get $-9+72x=36x+27$. Subtracting $36x$ from each side gives you $-9+36x=27$. Adding 9 to both sides results in $36x=36$. Finally, dividing both sides by 36 leaves you with your solution: $x=1$.

6 Solve $\frac{x}{2}+\frac{x-2}{4}=\frac{x+4}{2}$. The answer is $x = 10$.

Don't let those fractions intimidate you. Multiply through by the common denominator, 4, to eliminate the fractions altogether. Then solve like normal by combining like terms and isolating x. Here's the math:

$$\frac{x}{2}+\frac{x-2}{4}=\frac{x+4}{2};\ 4\left[\frac{x}{2}+\frac{x-2}{4}=\frac{x+4}{2}\right];\ 2x+x-2=2x+8;\ 3x-2=2x+8;\ 3x=2x+10;\ x=10$$

7 Solve $|x-3|+|3x+2|=4$. The answer is $x = -3/4, -1/2$.

So you have two absolute value terms? Just relax and remember that absolute value means the distance from 0, so you have to consider all the possibilities to solve this problem. In other words, you have to consider and try four different possibilities: both absolute values are positive, both are negative, the first is positive and the second is negative, and the first is negative and the second is positive.

TIP

When you have multiple absolute value terms in a problem, not all the possibilities will work. As you calculate the possibilities, you may create what math people call *extraneous solutions*. These are actually false solutions that don't work in the original equation. You create extraneous solutions when you change the format of an equation. To be sure a solution is real and not extraneous, you need to plug your answer into the original equation to check it. Time to try the possibilities:

- **Positive/positive:** $(x-3)+(3x+2)=4$, $4x-1=4$, $4x=5$, $x=5/4$. Plugging this answer back into the original equation, you get $30/4 = 4$. Nope! You have an extraneous solution.
- **Negative/negative:** $-(x-3)+-(3x+2)=4$, $-x+3-3x-2=4$, $-4x+1=4$, $-4x=3$, $x=-3/4$. Plug that into the original equation and you get $4 = 4$. Voilà! Your first solution.
- **Positive/negative:** $(x-3)+-(3x+2)=4$, $x-3-3x-2=4$, $-2x-5=4$, $-2x=9$, $x=-9/2$. Put it back into the original equation, and you get $19 = 4$. Nope, again — that's another extraneous solution.
- **Negative/positive:** $-(x-3)+(3x+2)=4$, $-x+3+3x+2=4$, $2x+5=4$, $2x=-1$, $x=-1/2$. Into the original equation it goes, and you get $4 = 4$. Tada! Your second solution.

8 Solve $3-4(2-3x)=2(6x+2)$. The answer is no solution.

Distribute over the parentheses on each side: $3-8+12x=12x+4$. Combine like terms to get $-5+12x=12x+4$ and subtract $12x$ from each side. The result is $-5 = 4$, which is false. Consequently, there is no solution.

9 Solve $2|x-3|+12=6$. The answer is no solution.

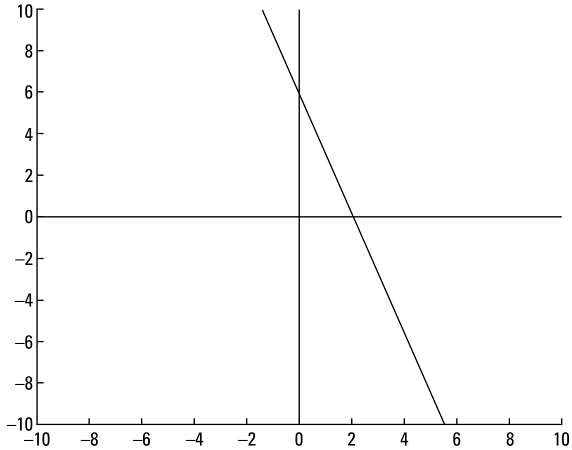
Start by isolating the absolute value: $2|x-3|+12=6;\ 2|x-3|=-6;\ |x-3|=-3$. Because an absolute value must be positive or zero, there's no solution to satisfy this equation.

10 Solve $3(2x+5)+10=2(x+10)+4x+5$. The answer is all real numbers.

Begin by distributing over the parentheses on each side: $3(2x+5)+10=2(x+10)+4x+5$, $6x+15+10=2x+20+4x+5$. Next, combine like terms on each side: $6x+25=6x+25$. Subtracting $6x$ from each side gives you $25 = 25$. This is a true statement. Because all these steps are reversible, we see that all real numbers would satisfy this equation.

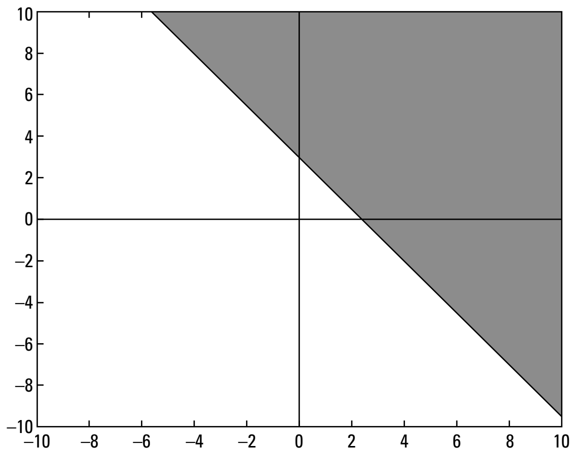
11 Sketch the graph of $\frac{4}{3}(6x + 2y) = 16$. See the graph for the answer.

Using slope-intercept form, multiply both sides of the equation by the reciprocal of $\frac{4}{3}$, which is $\frac{3}{4}$: $\frac{3}{4} \cdot \frac{4}{3}(6x + 2y) = \frac{3}{4} \cdot 16$. Doing so leaves you with $6x + 2y = 12$. Next, solve for y by subtracting $6x$ from each side and dividing by 2: $2y = -6x + 12$; $y = -3x + 6$. Now, because $y = -3x + 6$ is in slope-intercept form, you can identify the slope (–3) and y-intercept (6). Use these to graph the equation. Start at the y-intercept (0, 6) and move down 3 units and to the right 1 unit. Finally, connect the two points to graph the line.



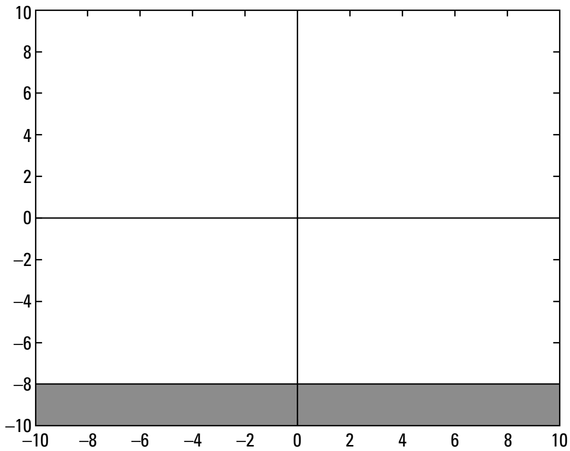
12 Sketch the graph of $\frac{5x+4y}{2} \geq 6$. See the graph for the answer.

Start by multiplying both sides of the equation by 2: $5x + 4y \geq 12$. Next, isolate y by subtracting $5x$ from each side and dividing by 4, like so: $4y \geq -5x + 12$; $y \geq -(\frac{5}{4})x + 3$. Now that $y \geq -(\frac{5}{4})x + 3$ is in slope-intercept form, you can graph the line $y = -(\frac{5}{4})x + 3$. Because $y \geq -(\frac{5}{4})x + 3$ is a greater-than-or-equal-to inequality, draw a solid line and shade above the line.



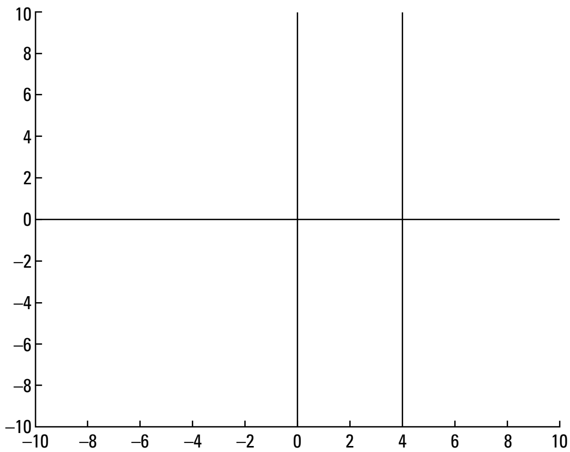
13 Sketch the graph of $4x + 5y \geq 2(3y + 2x + 4)$. See the graph for the answer.

First things first: Get the equation into slope-intercept form by distributing the 2 on the left side. Next, isolate y by subtracting $4x$ from each side, subtracting y from each side, and then dividing by –1 (don't forget to switch your inequality sign, though). Here's what your equation string should look like: $4x + 5y \geq 2(3y + 2x + 4)$; $4x + 5y \geq 6y + 4x + 8$; $5y \geq 6y + 8$; $-y \geq 8$; $y \leq -8$. Because there's no x term, you can think of this as $0x$, which tells you that the slope is 0. Therefore, the resulting line is a horizontal line at –8. Because the inequality is less than or equal to, you shade below the line.



14 Sketch the graph of $x - 3y = 4 - 2y - y$. See the graph for the answer.

Simplify the equation to put it in slope-intercept form. Combine like terms and add $3y$ to each side: $x - 3y = 4 - 2y - y$; $x - 3y = 4 - 3y$; $x = 4$. Here, the resulting line is a vertical line at 4.



15 Find the length of segment *CD*, where *C* is (–2, 4) and *D* is (3, –1). The answer is $d = 5\sqrt{2}$.

Using the distance formula, plug in the *x* and *y* values: $d = \sqrt{(-2-3)^2 + [4-(-1)]^2}$. Then, simplify using the order of operations: $d = \sqrt{(-5)^2 + (5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$.

16 Find the midpoint of segment *EF*, where *E* is (3, –5) and *F* is (7, 5). The answer is *M* = (5, 0).

Using the midpoint formula, you get $M = \left(\frac{3+7}{2}, \frac{-5+5}{2}\right)$. Simplify from there to find that $M = (10/2, 0/2)$, *M* = (5, 0).

17 Find the slope of line *GH*, where *G* is (–3, –5) and *H* is (–3, 4). The answer is the slope is undefined.

Using the formula for slope, plug in the *x* and *y* values for the two points: $m = \frac{-5-4}{-3-(-3)}$. This equation simplifies to –9/0, which is undefined.

18 Find the perimeter of triangle *CAT*. The answer is $8\sqrt{2} + 2\sqrt{26}$.

To find the perimeter, you need to calculate the distance on each side of the triangle, which means you have to find the lengths of *CA*, *AT*, and *TC*. Plugging the values of *x* and *y* for each point into the distance formula, you find that the distances are as follows: $CA = 5\sqrt{2}$, $AT = 3\sqrt{2}$, and $TC = 2\sqrt{26}$. Adding like terms gives you the perimeter of $8\sqrt{2} + 2\sqrt{26}$.

19 Find the center of the rectangle *NEAT*. The answer is (9, 2).

The trick to this one is to realize that if you can find the midpoint of one of the rectangle's diagonals, you can identify the center of it. Easy, huh? So, by using the diagonal *NA*, you can find the midpoint and thus the center: $M = \left(\frac{7+11}{2}, \frac{8+(-4)}{2}\right)$. That simplifies to *M* = (9, 2).

20 Determine whether triangle *DOG* is a right triangle. The answer is yes.

This problem forces you to recall that right triangles have one set of perpendicular lines (which form that right angle) and that perpendicular lines have negative reciprocal slopes. In other words, if you multiply their slopes together, you get –1. With that information in your head, all you have to do is find the slopes of the lines that appear to be perpendicular. If they multiply to equal –1, then you know you have a right triangle.

Start by finding the slope of *DO*: $m = \frac{4-(-4)}{-3-1} = -\frac{8}{4} = -\frac{2}{1}$. Next, find the slope of *OG*: $m = \frac{-4-0}{1-9} = \frac{-4}{-8} = \frac{1}{2}$.

By multiplying the two slopes together, you find that they equal –1, indicating that you have perpendicular lines: $(-2)(1/2) = -1$. Therefore, triangle *DOG* is a right triangle.