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Introduction

1.1 A General View

Newton's law of universal gravitation describes the interaction between discrete point masses at distances apart. The nature of gravitational forces and the idea of an object interacting on a distant object of finite sizes are easier to visualize when a model of force fields is employed. This idea of a field model came about because of the conceptual problems inherent in Newton's law of universal gravitation interpreted as action at a distance. The idea lends itself to pictorial description and explains how one object knows the existence of another object in a field model. With the introduction of concept of force transmission and concepts of fields and potentials, we think of the space surrounding the earth as permeated by a gravitational field of force created by the earth. Any mass placed in the field experiences a force due to this field. Gravitational field is one of the physical fields in nature that exerts a force at a mass center point in space with a specific magnitude and direction towards the earth center, giving rise to a field line (line of force). Gravitational fields are represented by vector diagrams using a system of line of forces radiating from the earth center like a pencil of rays. It is then said there exists a first order tensor field. Tensors are multivariate data embedded with complex information in time and space. A vivid pictorial representation of tensor field by field line distributions is essential for an understanding and perception of field characteristics and behaviors. It adds significantly to our ability to interpret physical phenomena. An accurate graphical representation of a first order tensor field must convey both magnitudes and directions of the fields in space. In addition, the point or line of action is also essential. Several graphical display methods are in common use. In a

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vector diagram, the length or thickness of each vector line segment at selected points is drawn proportional to its magnitude with arrows pointing in the field directions. These discrete geometric vectors making up a field of arrows are regarded as position vectors emanating from the points of interest. They generally cause cluster. In this respect, field line trajectories are commonly used to compliment a vector diagram with continuity. A tangent along a field line trajectory defines the direction of vector fields at a point. Field line trajectories are a graphical representation of lines of action continuously throughout the field domains. They are a system of non-intersecting curves in space except at a singular/critical point. This field trajectory method is generally applicable to any vector fields regardless of their nature, source or domain dimension.

The other method employs a family of flux lines (streamlines) generated from the corresponding vector potential (stream function) when vector fields are steady, two dimensional and solenoidal. Similarly, tangents along a flux line trajectory define the vector field directions. Flux line trajectories are spaced in such a way that the number of lines crossing a unit area placed perpendicular to the field at some points is proportional to the magnitude of the field at that point. When flux lines are displayed at equal contour intervals in a physical plane, the resulting flux density is a measure of field strength since flux bounded between flux lines is invariant in solenoidal fields. Flux lines are crowded closely where the field is strong or conversely. They obviate the cumbersome use of arrow segments having various length or thickness. Flux line trajectories are a contour representation of stream function, which assumes a constant different from one streamline to another. The engagement with stream function gives flux line trajectories the advantage over field line trajectories. Stream function ensures equation of continuity is satisfied everywhere. It is able to determine vector field directions, magnitudes and property transport quantitatively.

In irrotational fields, another set of lines referred to as equipotentials is produced to provide another means of vector field visualization. They are derived from the corresponding scalar potential (or potential function) which describes field transport capability quantitatively in terms of potential levels with respect to a reference datum. Potential gradients indicate vector field magnitudes and directions, which correspond to the tangents along flux lines. Along these directions maximum spatial rate of potential change occurs. In two-dimensional fields, equipotentials are also spaced at constant contour intervals such that equipotential density is a measure of field strength. Equipotential contours are crowded closely where the field is strong or conversely.

For harmonic fields, both equipotentials and streamlines exist and are mapped together to produce a field map in a physical plane as shown in Figure 1.1. These two sets of curves cross each other orthogonally forming a

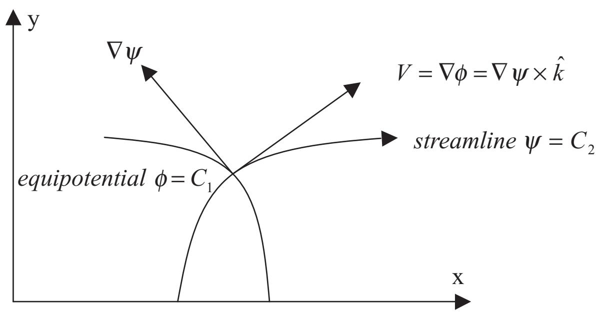

Figure 1.1 Field map.

network of curvilinear squares. In particular, when field maps are produced with a constant ratio of contour intervals equal to curvilinear square aspect ratio of unity, they are an effective means for graphical representation of vector fields. Thus, scalar potentials and vector potentials play a central role in forming the platform for vector field visualization and analysis. In the present study, they are also employed to construct complex potentials paving the way for mapping fields between complex planes.

Graphical representations of second order tensors remain challenging. Traditionally they are visualized through their principal states. In the present study, the tensor rank reduction method is applied to extract tensor vectors on a reference plane or at a point by contraction. The method is extended to include the use of tensor invariants. Last but not least, the hybrid method of displaying evolving tensor ellipse icons along principal axes trajectories is effective for second order tensor field visualization.

1.2 Historical Development and Progress in Visual Science

Graphical representation and visualization of fields and transport phenomena go back centuries. Ancient Babylonians/Egyptians and historians of architecture/artists developed the concept of measurement and descriptive geometry to describe patterns and beauty of shapes. Leonardo da Vinci, a great scientist and artist in the Renaissance period described movement of water, vortices and floating objects and presented sketches of botany and streamline/shapes. Regular polygons/polyhedra were subjects of special study by ancient Greeks. They led to visual conceptual graphics in the field of astronomy reflected in the work of Kepler. In his scientific exploration of the universe, Kepler succeeded in describing the planet systems and orbits by arranging a set of regular polyhedral in three-dimensions. As the arts of

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visual representation of geometry continue to evolve, an abstract geometry of several dimensions or hypergraphics was then systematically developed. The combination of arts and sciences has led to unprecedented technological advancements and revolutionary developments. From classical concept of action-at-a-distance to the theory of contiguous action and modern concept of fields in field theory, the development has made significant contributions to the pool of knowledge in visual sciences. Historically, R. Norman as a navigational instrument maker about Kepler's time put the idea of a magnetic field forward. Gilbert continued his work to visualize magnetic effects with the birth of field conception coined orbis virtutis. Until then the concept of field lines was originated from Faraday in his study of electrical sciences in an attempt to describe electric field structure. Field lines have become one of the basic techniques of visual representations to interpret field phenomena ever since. They not only lay the foundation ideas of field visualization but also opened the window of applications to a wide range of field phenomena in physical sciences. Field lines are regarded as the structural elements of a field and are represented by straight lines perpendicular to the charge surface. Faraday was convinced on experimental grounds that the physical nature of line of force was action propagating between contiguous particles through a medium. The physical reality to these field lines was then reckoned as material lines consisting of a chain of polarized particles subject to longitudinal strain. This gives rise to an electric phenomenon referred to as the arrangement of electric dipoles. As the ideas took shape, they made significant influence on modern field theory development. Maxwell developed Faraday's ideas into mathematical forms of a theory of contiguous action. He conveyed the importance and physical reality of fields as stress-transmitting media through which electromagnetic waves propagate in specific directions at some finite speeds. The theory integrated forces in nature and exhibited the underlying similarity of field phenomena with a unified view, which remains hidden from direct perception. At about the same time, the concept of continuum and notion of a field theory began to emerge in the work of Euler in hydrodynamics and in the theory of sounds. He laid the foundation for Eulerian description of fluid motions and presented mathematical models to describe the field of motion of inviscid fluids in an inertia frame. The law of motion was applied to point masses in the differential form of conservation of linear momentum for continuous media, bridging up with Newtonian mechanics.

In hydrodynamics, D'Alembert was credited with the introduction of stream function concept for flow visualization. In the study of elasticity, the introduction of Airy stress function and/or strain potential function makes it possible to describe the state of a second order tensor in terms of a single scalar function. Prandtl then developed another version of stress function to obtain stress trajectories produced by torsion. It plays the same role as stream functions introduced by Lagrange and Stokes who laid the foundation work of graphical representation of vector fields in hydrodynamics. Bejan and his co-workers (Kimura and Bejan 1983, 1985; Morega and Bejan 1993; Trevisan and Bejan 1987) then extended streamline technique to transport phenomena involving heat and mass. They are known as heatfunction and massfunction.

Poincare established connections between vector field topology and differential equations. His work laid the foundations for phase-space technique in the study of vector spaces and dynamic systems. This is followed by the development of theory in two-dimensions in the middle of the twentieth century. Mapping of fields has added a new horizon in visual science. Needham (1997) presents a vivid example of electromagnetic fields in television/radio, which is described completely by mapping of electric fields and magnetic fields. Nevertheless, all these pioneer works show the vital importance of tensor field representations and visualizations in a vast range of physical science applications. They have set the stage for modern development and practice in the arts of visual representation.

Scientific visualization developments have gone beyond and across various disciplines. A brief review of current state-of-the-art scientific visualization is a useful starting point to follow its progress, research and directions of development. Graphical representation of data and object has appeared in the work of statistics, botany and earth sciences in the last couple of decades. Since then, computer technology has changed the face of the world and dominated human activities in all trades. Scientific visualization and image processing research have been active among computer scientists and engineers. Computer scientists have keen interests in the visualization of tensor field topological structure and in the development of visual icons ranging from point, line, surface and volume domains at various information levels. Researchers who presented investigations on the topology of simulated tensor fields have suggested the methods of field topology classification and representation based on analysis of critical points (Helman and Hesselink, 1989a, 1989b, 1990, 1991). This data compression/extraction technique was applied to the visualization of higher order critical points.

Computer graphics based iconographic techniques play an important role in the development of visual icons. They are effective visual attributes useful in general applications. In particular, computer graphics techniques make use of color/texture encoding with appealing effects. Elementary vector icons such as arrows (point icon), streamlines (line icon) and stream surfaces/stream ribbons (surface icon) have been in use for sometime. Elementary tensor icons such as Lame stress ellipse and Cauchy stress quadric were introduced in the theory of elasticity at an earlier date. Other areas of development include for instance, ellipsoids, tensor glyphs and probes. These

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are tensor point icons showing the state of a tensor field at a point. Tensor line icons (lines of principal stress) were introduced formally by Woods (1903), Dickinson (1989, 1991) in elasticity referred to as tensor field trajectories based on principal axes transformation. These replace the point-wise glyphs, which lack data continuity and cause cluster. A series of such trajectories is used to display the continuous distributions of principal directions, which describe load transmissions and/or transport paths in a system. The corresponding principal values are usually presented in contour forms to depict field strength. Hyperstreamlines are then presented by researchers (Delmarcelle and Hesselink, 1992, 1993, 1994) as an extension of tensor field trajectories. Hyperstreamlines sample tensor fields along their principal trajectories and integrate a continuous distribution of point icons. These topological skeletons embed complex information through colors and textures.

Following the development of phase-space technique in data visualization, a number of methods has since then sprung-off. Research work in unsteady field visualization was in progress in a great number of research centers around the globe. Graphical line integral convolution method was introduced to represent the texture of vector fields. This improved rendering technique converted data into images and was free of seeding problems. Development in feature-based visualization includes image based flow visualization. Three dimensional visualization techniques are also in the pipeline of research and development. Three-dimensional icons were then developed to produce three-dimensional texture from two-dimensional domains. Work is also in progress in combining experimental and computational visualization methods in complex environments such as combustions. Comparison studies on vector field visualization methods, classifications and research issues have been presented in literature.

In parallel to these research activities, engineers are also actively engaged in the development of visualization techniques in transport processes. A novel approach to heat transfer visualization based on the concept of heatlines was first introduced by Kimura and Bejan (1983, 1985) in their study of natural convection in enclosure heated from the side. It is an energy analog of streamlines in hydrodynamics. The analogy carried over to the introduction of heatfunction, which plays the same role as stream function. A useful feature of heatlines is the depiction of transport paths, which facilitate heat transfer analysis. The techniques were then applied extensively in a wide spectrum of transport processes in anisotropic medium, in porous medium, in unsteady buoyancy-driven flow, and in turbulent flows. Trevisan and Bejan (1987) then introduced the mass transfer analog of the same idea referred to as masslines. Other researchers presented a unified formulation of the two systems of lines in non-reacting and reacting jets. They also introduce the concept of conserved scalars to extend heatlines and masslines to fields with sources in post-processing visualization.

1.3 Scientific Visualization Philosophy, Techniques and Challenges

Computer technology leads to the generation of an increasingly huge amount of multivariate data of great complexity. The phenomenon of knowledge growth and flow of information is on an incredible scale. The database is not useful unless its information can be conveyed to the users effectively. There is a need to develop scientific visual technology blended with human visual systems. The work on scientific visualizations generally involves data management, data mining, graphical display, transformation, analysis and integration/correlation. The input effort has resulted in the production of various visualization systems; each has its merits or limitations. Scientific visualization technology aims at extracting meaningful representations from multivariate data sources suitable for visual communication and comprehension systems. Data mining methods are used to meet with a variety of goals. It includes searching for patterns, discovering useful data structure and exploring field properties and behaviors. Graphs/icons are an effective means of serving such a purpose to give a visual impression of information at a glance. The general philosophy is to integrate simplicity of graphical representations with continuity of the fields and analysis capability in quantitative terms. Implicit representation of curves/surfaces has played a key role in forging the basic ideas and techniques in field visualization. Implicit function makes it possible to represent relations as the graph of a function and has certain advantages over explicit expressions. This is because an explicit expression from an implicit expression may not exist. Implicit representation offers more flexibility and renders two-dimensional contour curves easily back to three-dimensional surface with the constant assuming the vertical axis. It is also ideal to test graphs/functions for possible symmetries. Visualization and identification of curves/surfaces are expedited in accordance with the value of parameter constant specified by the implicit function. Alternatively, parametric representation is capable of describing complicated curves/surfaces directly by considering a system of equations simultaneously as a function of a third independent variable. Certain graph visualization techniques are effective in displaying the characteristics and behaviors of fields based on the function itself. Plane/line-symmetry, point-symmetry or rotation symmetry, if it exists is useful for displaying field patterns as mirror reflection images in the same or opposite fashion. By observing and taking advantage of symmetry or anti-symmetry of any kind, it is possible to limit the study/visualization of field patterns in certain domains or regions. Similarly, periodicity is to be observed. Asymptotes including dividing streamlines are suitable for shaping the graph of a function in general and to separate the fields into distinct regions of behavior in particular. Asymptotes help to study the behavior of function at infinity. Critical/singular points are important visual elements as they contribute significantly to the nature of fields. Distributions of critical points or singular points provide key information about field behavior since they possess unique topological skeletons or field patterns, which unveil the nature of these points and the fields as well. The displays of field topology in the neighborhood of critical points not only reveal local phenomena but also have connections to global behaviors. The way dividing streamlines split at a critical point sheds light on the interaction between dynamic aspects of fields and boundary geometry/conditions. By extracting salient views and tensor topological structure, they can be blended to provide a comprehensive understanding of the original fields.

The method of transformation is of great value in the analysis and visualization of field behaviors. Mapping of field patterns from a given plane onto another with a great variety of transformation complements each other and produces new information from old. Gradient mapping takes a point in the domain of a scalar field onto a gradient vector pointing in the direction of maximum spatial rate of property change at that point. The gradient system is a vivid display of "runoff" of water flowing downhill towards local minima or away from local maxima. Mapping of singular points to points at infinity in some planes makes it possible to evaluate field properties such as shape factor by graphical means and to interpret the concept of infinity in a geometric setting. Other special points of interest include, but are not limited to, multiple points, higher order critical points, degenerate points of higher multiplicity, inflection points, cuspidal points, points of contact, vertices and so on. These points are invariant in coordinate transformation and help to shape the trajectories in one way or another. Analytic functions of complex variables open the window of exploration for new complex potentials through synthesizing or mapping (composite/differentiation/integration) in the search for target field models. We could study field patterns on different domain boundaries or produce distortion/warping effects on images by conformal transformation. Hodograph method and mapping make it possible to visualize the free boundary of unknown shapes in the physical plane or the inverse images on other complex planes. Hodograph representation of trajectories or field patterns compliments the position hodograph representation and is useful for interpretation of field phenomena. This includes visualization and mapping of trajectories/tangents, straight boundaries (impervious/equipotential/seepage) or radial lines/circles about origin from hodograph plane to physical plane and conversely. Critical points and dividing trajectories are mapped to origin and curves passing through on the hodograph plane. By mapping circles about origin from the hodograph plane, we could determine locus of constant speed or kinetic energy contours on the physical plane. Likewise, by mapping straight lines passing through origin from hodograph plane to physical plane, we could visualize locus of isoclines that is the constant directions of vector fields equal to the slope of the straight lines.

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Icons, colors and textures are useful visual attributes to take advantage of human visual systems. As the amount of rendering needs to be economized, there appears to be no need to render every piece of data by a glyph or some other means. It is, however, essential to deliver quantitative and qualitative information at selected points of interest and to display salient features of the fields with continuity. The objectives are to capture field structure in a subset of data and to zoom onto regions of interest through integration and correlation of information from different perspectives. Displays of both velocity arrow icons and acceleration arrow icons at a point are effective at describing the state of motions. By decomposing the acceleration vector into streamline tangential and normal directions through the law of parallelogram, we could determine whether flow is accelerating, decelerating or changing direction. Speed will increase or decrease when the tangential component is in the same or in the opposite velocity direction. Motion is at a constant speed if this component vanishes or the acceleration vector and velocity are mutually perpendicular. Alternatively, we could arrive at the same findings based on the values (positive, negative or zero) of the dot product of the two vectors. The normal component of acceleration vector assumes the role of centripetal acceleration always points towards the center of curvature indicating direction changes. If this component vanishes, it may be a point of inflection or a straight path. The same idea could be used in other vector fields such as mechanical energy flux or heat flux. The displays of mechanical energy flux vector, heat flux vector and velocity at a point allow us to assess the energy transfer processes. To maintain data continuity, we focus on computer graphic techniques of visualization using line icons such as streamlines (flux line trajectories), field line trajectories and potential contours. In this connection, vector potentials including, but are not limited to mechanical energy function, vorticity function and warping flux function, are used to visualize transport trajectories analogous to stream function and streamlines.

Because of the limitations of complex functions in two-dimensions and the complexity of flux (dual) functions in three-dimensions, the present work focuses on a two-dimensional approach. The cross section method is effective to present graphs from multi-dimensions on coordinate planes by holding the spatial variables constant systematically. A pseudo three-dimensional field could be constructed by staggering up a combination of coordinate planes. With regard to second order tensor fields, we apply tensor rank reduction techniques. These include contractions, tensor invariants, tensor icons and principal axes trajectories. The latter is connected to the idea of independently moving graphical icons leading to two-dimensional ellipse-based principal axes trajectories (Kerlick, 1990). The hybrid method is able to maintain data continuity and convey decoding information visually by a sequence of evolving tensor ellipses animated along a transport path. The connections between vector field visualization, mapping and those in tensor fields lay the platform for integration and correlation in mixed field phenomena.

Almost half a century has elapsed since the birth of the first two-dimensional television set. Today a three-dimensional television set remains to be seen. The difficulty and cost of developing three-dimensional visual technology and high order tensor representations present a compelling challenge. Simulation of three-dimensional scenes/images is possible in certain fields but has not yet become widespread. The development in stereoscopic technique, virtual reality, two-dimensional data integration with an elevation model, computer-aided design, optical images, graphical simulation and visual comprehension are encouraging and have attracted a great deal of research interest. Intriguing work has been presented in the art of hypergraphics for multi-dimension geometry. It appears that the inherent problems of presenting three-dimensional graphs on two-dimensional views/papers will remain with us and impose restrictions on our visual ability. The problems inevitably demand three-dimensional visual technology and viewing hardware. Until then multiple views and transformation will be and always will be in demand in a visualization system. To present multiple levels of complexity of fields and models, there is a need to develop visualization of multiple tensor fields simultaneously to study the structure and interaction of one field in the context of another or more. This calls for multivariate visualization methods in an integrated system. Visual representations become more complex when fields are unsteady. There are a number of hurdles in the quest for knowledge and information visualization. The missions are expected to add new dimensions in scientific visualizations and in modern theory development.