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Introduction

In contrast with other integral transforms, such as Fourier or Laplace, the Hilbert transform (HT) is not a transform between domains. It rather assigns a complementary imaginary part to a given real part, or vice versa, by shifting each component of the signal by a quarter of a period. Thus, the HT pair provides a method for determining the instantaneous amplitude and the instantaneous frequency of a signal. Creating and applying such complementary component seems to be a simple task. Nevertheless providing explanations and justifying the HT application in vibration analysis is a rather uneasy mission. There are a number of objective reasons complicating the matter.

First, the HT mathematical definition itself was originated just 60 years ago – not as long ago as the Fourier transform (Therrien, 2002), for example. Even 30 years ago the HT was a pure theory, and then it was employed in applied researches, including vibration. Thus the most significant and interesting results have been received within the last 15 years. Presenting this material, together with the corresponding statements and judgments, requires considerable efforts.

Secondly, from the very beginning the HT approach faced numerous objections, doubts, counterexamples, paradoxes, and alternatives generating some uncertainty about the reliability and feasibility of the obtained results. Naturally, the book should contain only proved, tested, and significant results of HT applications.

Thirdly, at the same time, and in parallel with the HT, another method – the Wavelets transform – was developed in signal processing allowing us to solve similar applied problems. As numerous scientific works devoted to the evaluation of these methods are based exclusively on a comparison of empirical data, theoretical conclusions and statements have not yet been available for detailed presentation.

Fourthly, the HT itself, and the corresponding methods of signal processing, involve rather difficult theoretical and empirical constructions, while the text should be written simply enough to introduce the HT area to "just plain folks" (nonspecialist readers). We will try to make it readable for a person of first-degree level in

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Engineering Science who can understand the concept of the HT sufficiently to utilize it, or at least to determine if he or she needs to dig more deeply into the subject.

The book is divided into three main parts. The first describes the HT, the analytic signal, and the main notations, such as the envelope, the instantaneous phase, and the instantaneous frequency, as well as the analytic signal representation in the complex plane. This part also discusses the existing techniques for the HT realization in digital signal processing.

The second part describes the measured signal as a function of time, mostly vibration, which carries some important information. The HT is able to extract this time-varying information for narrow- and wideband signals. It is also capable of decomposing a multicomponent nonstationary signal into simple components or, for example, separate standing and traveling waves.

The third part is concerned with a mechanical system as a physical structure that usually takes an impulse or another input force signal and produces a vibration output signal. Use of the HT permits us to estimate the linear and nonlinear elastic and damping characteristics as instantaneous modal parameters under free and forced vibration regimes.

The book is a guide to enable you to do something with the HT, even if you are not an expert specializing in the field of modern vibration analysis or advanced signal processing. It should help you significantly (a) to reduce your literature research time, (b) to analyze vibration signals and dynamic systems more accurately, and (c) to build an effective test for monitoring, diagnosing, and identifying real constructions.

1.1 Brief history of the Hilbert transform

To place the HT subject in a historical context of mechanical vibration we will start with a very short chronology of the history of the HT. A traditional classical approach to the investigation of signals can include a spectral analysis based on the Fourier transform and also a statistical analysis based on a distribution of probabilities and other representations typical for random data. In addition to these typical spectral, correlation, and distribution characteristics, another method of representing and investigation a signal originated in the forties of the previous century (Gabor, 1946). –. This new method suggested the use of a random signal x as a product of two other independent functions: $x = A\cos\varphi$, where A is the amplitude, or envelope, and φ is the instantaneous phase. Thus, the variable x can be presented in the form of a harmonic fluctuation modulated in the amplitude and in the phase. This means of representing a function has appeared to be more descriptive and convenient for the solution of a number of theoretical and practical problems.

At that time researchers and engineers were not familiar with the HT (Therrien, 2002). However, they started to investigate the envelope and instantaneous phase by describing the signal in a $x - y$ Cartesian coordinate system (Bunimovich, 1951). In this $xy-$ plane the original signal was a first (x-axis) projection of the vector with length A and phase angle φ. The second projection in the $xy-$ plane along a vertical axis took the form $y = A\sin\varphi$. Due to the orthogonality of the bases, one obtains the following relations: $A^2 = x^2 + y^2$, $\varphi = \arctan\left(y/x\right)$. The same relations were extended to the case of a representation of a variable in the form of a Fourier series: $x = \sum (a_k \cos\varphi_k + b_k \sin\varphi_k)$, where each component of the sum means a simple harmonic. The mathematical literature (Titchmarsh, 1948) defined the second projection of the vector sum as the conjugate Fourier series $y = \sum (a_k \sin\varphi_k - b_k \cos\varphi_k)$. This started a study of the modulated signal, its envelope, instantaneous phase, and frequency based on the well-known Euler's formula for harmonic functions, according to which $e^{i\varphi} = \cos\varphi + i\sin\varphi$.

Nevertheless, a question of how an arbitrary (but not harmonic) signal should be represented to define the envelope and other instantaneous characteristics was still open. This problem was solved by Denis Gabor in 1946 when he was the first to introduce the HT to a signal theory (Gabor, 1946). Gabor defined a generalization of the Euler formula $e^{i\varphi} = \cos(\varphi) + i\sin(\varphi)$ in a form of the complex function $Y(t) = u(t) + i\,v(t)$, where $v(t)$ is the HT of $u(t)$. In signal processing, when the independent variable is time, this associated complex function is known as an analytic signal and the projection $v(t)$ is called a quadrature (or a conjugate) of the original function $u(t)$. The HT application to the initial signal provides some additional important information on an amplitude, instantaneous phase, and frequency of vibrations.

The analytic signal theory was then progressively developed by experts in various fields, mainly in electronics, radio, and physics. Here we must mention an important result called a Bedrosian condition (identity, equality), derived in 1963 (Bedrosian, 1963). This simplifies the HT calculation of a product of functions, helps us to understand the instantaneous amplitude and frequency of signals, and provides a method of constructing basic signals in the time–frequency analysis.

The theory and the HT application progressed greatly during the following years owing to Vakman, who further developed the analytic signal theory by solving problems of nonlinear oscillation and wave separation (Vakman and Vainshtein, 1977; Vainshtein and Vakman, 1983).

Investigators of digital algorithms of the HT realization (Thrane *et al.*, 1984) made a major contribution when a "digital" revolution started, and digital computers and digital signal procedures appeared everywhere. In 1985 Bendat suggested the inclusion of the HT as a typical signal procedure to the Brüel and Kjær two-channel digital analyzer. He also wrote a B&K monograph with a cover picture of David Hilbert's face gradually rotated through 90° (Bendat, 1985). As the speed and volume of digital processors keep increasing, software and digital hardware are replacing traditional analog tools, making today's devices smarter, more reliable, less expensive, and more power efficient than ever before.

The HT and its properties have been studied extensively in fluid mechanics and geophysics for ocean and other wave analysis (Hutchinson and Wu, 1996). A detailed analysis of the HT and complex signals was made by Hahn in 1996 (Hahn, 1996a). His book covers the basic theory and practical applications of HT signal analysis and simulation in communication systems and other fields. Two volumes of the *Hilbert Transforms* recently published by King (2009) are a very definitive reference on the HTs, covering mathematical techniques for evaluating them, and their application.

In 1998 an outstanding work by Huang gave a new push to the modern research in the field of HTs (Huang *et al.*, 1998). His original technique, known as the Empirical

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Mode Decomposition (EMD), adaptively decomposes a signal into its simplest intrinsic oscillatory modes (components) at the first stage. Then, at the second stage, each decomposed component forms a corresponding instantaneous amplitude and frequency. Signal decomposition is a powerful approach; it has become extremely popular in various areas, including nonlinear and nonstationary mechanics and acoustics.

1.2 Hilbert transform in vibration analysis

In the field of radio physics and signal processing, the HT has been used for a long time as a standard procedure. The HT and its properties – as applied to the analysis of linear and nonlinear vibrations – are theoretically discussed in Vakman (1998). The HT application to the initial signal provides some additional information about the amplitude, instantaneous phase, and frequency of vibrations. The information was valid when applied to the analysis of vibration motions (Davies and Hammond, 1987). Furthermore, it became clear that the HT also could be employed for solving an inverse problem – the problem of vibration system identification (Hammond and Braun, 1986).

The first attempts to use the HT for vibration system identification were made in the frequency domain (Simon and Tomlinson, 1984; Tomlinson, 1987). The HT of the Frequency Response Function (FRF) of a linear structure reproduces the original FRF, and any departure from this (e.g., a distortion) can be attributed to nonlinear effects. It is possible to distinguish common types of nonlinearity in mechanical structures from an FRF distortion.

Other approaches (Feldman, 1985) were devoted to the HT application in the time domain, where the simplest natural vibration system, having a mass and a linear stiffness element, initiates a pure harmonic motion. A real vibration always gradually decreases in amplitude owing to energy losses from the system. If the system has nonlinear elastic forces, the natural frequency will depend decisively on the vibration amplitude. Energy dissipation lowers the instantaneous amplitude according to a nonlinear dissipative function. As nonlinear dissipative and elastic forces have totally different effects on free vibrations, the HT identification methodology enables us to determine some aspects of the behavior of these forces. For this identification in the time domain, it was suggested that relationships should be formed between the damping coefficient (or decrement) as a function of amplitude and between the instantaneous frequency and the amplitude. Lately, it was suggested that the linear damping coefficient could be formed by extracting the slope of the vibration envelope (Hammond and Braun, 1986; Agneni and Balis-Crema, 1989).

Some studies (Feldman, 1994a, 1994b), provide the reader with a comprehensive concept for dealing with a free and forced response data involving the HT identification of SDOF nonlinear systems under free or forced vibration conditions. These methods, being strictly nonparametric, were recommended for the identification of an instantaneous modal parameter, and for the determination of a system backbone and damping curve.

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A recent development of the HT-based methods for analyzing and identifying single- and multi-DOF systems, with linear and nonlinear characteristics, is attributable to J.K Hammond, G.R. Tomlinson, K. Worden, A.F. Vakakis, G. Kerschen, F. Paia, A. R. Messina, and others who explored this subject much further. Since the HT application in the vibration analysis was reported only 25 years ago, it has not been well perceived in spite of its advantages in some practical applications. At present, there is still a lot to be done for both a theoretical development and practical computations to provide many of various practical requirements.

1.3 Organization of the book

This book proceeds with three parts and twelve chapters.

Part I "Hilbert Transform and Analytic Signal," contains three chapters. Chapter 1 gives a general introduction and key definition, and mentions concisely some of the HT history together with its key properties. Chapter 2, which includes a review of some relevant background mathematics, focuses on a rigorous derivation of the HT envelope and the instantaneous frequency, including the problem of their possible negative values. Chapter 3 deals with two demodulation techniques: the envelope and instantaneous frequency extraction, and the synchronous signal detection. It describes a realization of the Hilbert transformers in the frequency and time domains. The sources and characteristics of possible distortions, errors, and end-effects are discussed.

Part II, "Hilbert Transform and Vibration Signals," contains four chapters. Chapter 4 introduces typical examples of vibration signals such as random, sweeping, modulated, and composed vibration. It explains the derivatives, the integral, and the frequency content of the signal. Chapter 5 covers some new ideas related to the mono- and multicomponent vibration signal. Material that has important practical applications in signal analysis is treated, and some topics – especially the congruent envelope of the envelope – that have the potential for important practical applications are covered. Chapter 6 is devoted to examining the behavior of local extrema and the envelope function. Material in this chapter has an application to the explanation of the well-known Empirical Mode Decomposition (EMD). It also describes a relatively new technique called the Hilbert Vibration Decomposition (HVD) for the separation of nonstationary vibration into simple components. The chapter illustrates some limitations of the technique including the poor frequency resolution of the EMD. Most of key properties of these two decomposition methods are covered, and the most important application for typical signals is treated. Chapter 7 provides examples of HT applications to structural health monitoring, the real-time kinematic separation of nonstationary traveling and standing waves, the estimation of echo signals, a description of phase synchronization, and the analysis of motion trajectory.

Part III, "Hilbert Transform and Vibration Systems," contains five chapters. Chapter 8 gives some introductory material on quadrature methods, when the real and imaginary parts of a complex frequency function are integrally linked together by the HT. The chapter explains the important Kramers–Kronig formulas, used widely in applications. It covers some solutions of the frequency response function that can

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be used for the detection of nonlinearity. This chapter links both the initial nonlinear spring and the initial nonlinear friction elements and analytic vibration behavior. Both simple and mathematically rigorous derivations are presented. The chapter also covers some typical nonlinear stiffness and damping examples. Chapter 9 describes the foundation for the identification methods that are treated in the next chapter. The important sum rules that come directly from the HT relations – such as skeleton and damping curves, static force characteristics, and nonlinear output frequency response functions – are discussed in detail. Chapter 10 presents FREEVIB and FORCEVIB methods as a summary of all the key properties of the HT for practical implementation in dynamic testing. The skeleton and damping curves are treated together with the reconstructed initial nonlinear static forces. Chapter 11 treats the case of precise nonlinear vibration identification. The special difficulties that arise for the significant role of the large number of high-order superharmonics are analyzed in detail. Applications of some results developed in Chapter 9 for the identification of multi-degree-of-freedom (MDOF) systems are illustrated. Chapter 12, the final chapter, considers industrial applications in a number of different areas. To conclude the book, this chapter provides references to HT examples of a successful realization of the parametric and nonparametric identification of nonlinear mechanical vibration systems.