Chapter **1**

Introduction to the Instruments

Understanding a theory means (. . .) understanding it as an attempt to solve a certain problem.

Sir Karl Popper

In this chapter, we will rapidly but formally define the instruments and present their major characteristics. It is recommended that all readers, even those knowledgeable in this area, study the following definitions, as they will provide the framework of analysis used in the book.

Derivatives

■ A **derivative** is a security whose price ultimately depends on that of another asset (called underlying). There are different categories of derivatives, ranging from something as simple as a future to something as complex as an exotic option, with all the shades in between.

The best way to look at derivatives is to separate them into two broad categories: **linear** and **nonlinear** derivatives. A linear derivative is easy to hedge and lock in completely, whereas a nonlinear one will present serious instability and require dynamic hedging.

■ A **nonlinear derivative** with respect to a parameter is one that presents a second derivative (or partial derivative with respect to that parameter) different from 0.

The option wizard presents a graphic linear or nonlinear derivative view of the concept of nonlinearity.

> ***Risk Management Rule:*** All nonlinear derivatives are time-dependent in their price.

This rule is explored in the contamination principle and will be discussed throughout this book. For now, it is enough to state that nonlinearity

Option Wizard: The Greeks

The "Greeks," as option traders call them, denote the sensitivity of the option price with respect to several parameters. The following are basic definitions for use in Part I. These terms will be explored in greater detail later in this book.

Delta Sensitivity of the option price to the change in the underlying asset price.

Gamma Sensitivity of the option delta to the change in the underlying asset price.

Vega Sensitivity of the option price to the change in implied volatility.

Theta Expected change in the option price with the passage of time assuming risk-neutral growth in the asset.

Rho Sensitivity of the option price to interest rates or dividend payout.

"Long gamma or vega" means a positive sensitivity to the Greek (a higher P/L at a higher parameter).

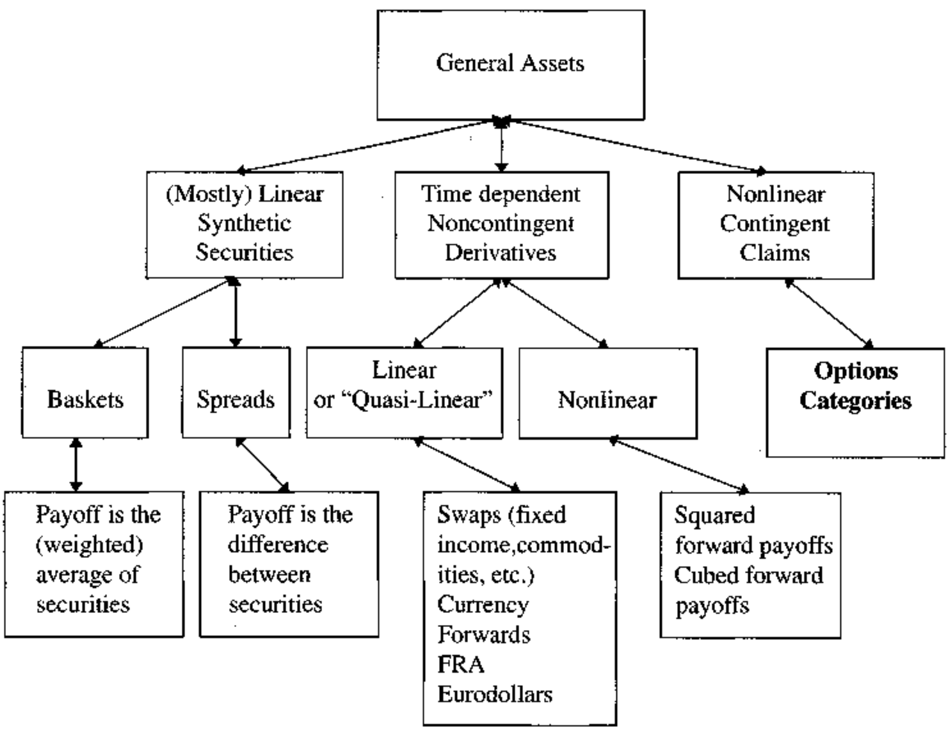

Figure 1.1 Classification of derivatives.

Option Wizard: The Hedger's Viewpoint

Throughout this book, think of a derivative in terms of its replication costs. For this purpose, the world is divided into two categories: the user and the manufacturer. Their utility function and even results will be entirely different. The user is involved in terminal value (usually), while the manufacturer engages in dynamic hedging (when he is doing his job right), which markedly alters the product.

A dynamic hedger will not be interested in whether he owns a put or a call (first-order hedges will make them identical). What matters is the strike and the expiration.

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is gamma (or more generally called convexity) and that gamma needs to be accompanied by time decay (the "rent").

Derivatives are not always linear, convex, or concave across all moves (see Figures 1.2A–D). A test of local linearity of a derivative security (that is a function of the underlying asset) between asset prices S_1 and S_2, with $0 < \lambda < 1$, will satisfy the following equality :

$$V(\lambda S_1 + (1 - \lambda)S_2) = \lambda\, V(S_1) + (1 - \lambda)V(S_2)$$

It will be convex between S_1 and S_2 if:

$$V(\lambda S_1 + (1 - \lambda)S_2) \leq \lambda\, V(S_1) + (1 - \lambda)V(S_2)$$

It will be concave if:

$$V(\lambda S_1 + (1 - \lambda)S_2) \geq \lambda\, V(S_1) + (1 - \lambda)V(S_2)$$

Option Wizard: Linear and Nonlinear Securities

Although we are initially considering linearity with respect to the underlying asset, this notion will be later extended to other parameters such as interest rates and volatility.

As the graphical representation at the top of page 12 shows, a linear security constantly behaves like a line. In option parlance, it will have a delta but no other Greeks,* and certainly no curvature. Linear securities require little or no dynamic hedging.

* The Greeks initially represented the various derivatives of the Black-Scholes-Merton formula. By extension, it became any sensitivity of a derivative security with respect to a particular market parameter.

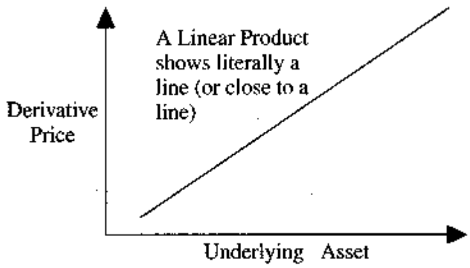


Figure 1.2A Linearity.

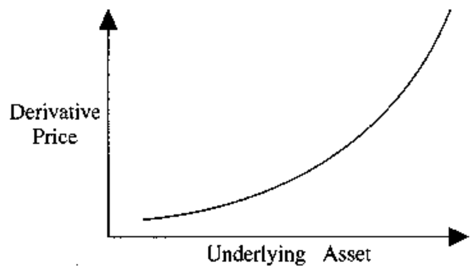


Figure 1.2B A nonlinear derivative: Convex security.

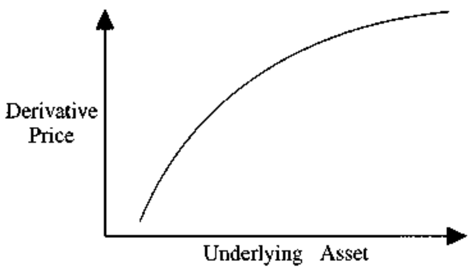


Figure 1.2C A nonlinear derivative: Concave security.

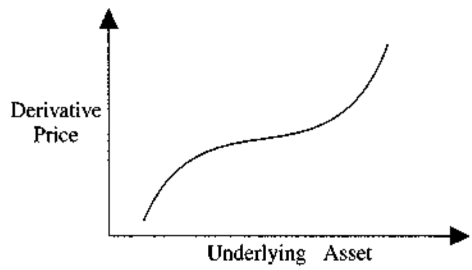


Figure 1.2D A mixed nonlinear derivative.

Many securities exhibit some linearity until the test of fire. These instruments are called "quasi-linear" securities. Convexity affects many financial instruments, even those least suspected.

The contamination principle, we will see, dictates that every nonlinear security commands time value, positive if the security is convex and negative if it is concave.

Synthetic Securities

- A **synthetic security** is a linear combination of two or more primary instruments in the markets.

A basket's price is derived from a weighted combination of existing primary instruments. For example, Standard & Poor's 500 (SP500) contract is a weighted average of the components. It can therefore be exactly replicated with a mixture of the components (for those who have the time and patience to leg the 500 stocks). A European currency unit (ECU) is another example

Option Wizard: The First Derivative Trade

The earliest option trade on record in Western literature was a bet on future crop by Thales of Miletus, which Aristotle recounted with great pride in his *Politics.** To benefit from a better than expected olive crop, Thales put a deposit on every olive press in the vicinity of Miletus. As demand for these grew, he sublet the facilities for profit, mostly to make the point that philosophers who so desire can achieve material success. The dichotomy between the "MIT-smart" and the "Brooklyn-smart," today prevalent on Wall Street, was already apparent in fifth-century Asia Minor. Thales used the first ***derivative*** instrument, actually an option on a future, a second-order derivative at that! He did not trade olives, which he would have had to sell short, but chose to buy the equivalent of a call on the olive presses, for fall delivery, with the knowledge that all he could lose was his deposit.

*See Russell (1945).

of an arithmetic average. A basket composed of 20% stock A and 80% V stock B can easily be replicated with the purchase of every component.

Synthetic securities are not always linear, but exceptions are rare enough for us not to bother with them. For example, when the average of the instruments is not arithmetic, some oddities can result. The U.S. dollar index, traded on FINEX will present some convexity owing to the geometric nature of the averaging and will therefore trade at a premium to the underlying securities for that reason. The nonlinearity that results from the convexity can cause a neutral person to benefit from a market move either way, which can make one side of the arbitrage more desirable. Convexity, which will be defined later on, usually commands a price since Wall Street rarely grants free lunches.

> ***Example:*** The Elevator Bank issues its own "mother of all baskets" and emits some notes, the payoff of which is indexed off the basket. The official reason is that the basket effectively tracks inflation or some other indicator. The true reason is that the basket will have a lower volatility than the sum of the instruments and is believed to be easy to hedge.

Time-Dependent Linear Derivatives

- **Time-dependent linear derivatives** are instruments separated from the original asset through time.

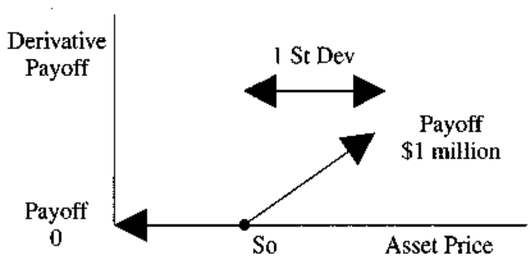
Option Wizard: The Contamination (or Convexity) Principle

The most important notion in option hedging and trading is the contamination principle: It is the fundamental principle of dynamic hedging. It means roughly that if there is a possible spot in time and space capable of bringing a profit, then the areas surrounding it need to account for that effect.

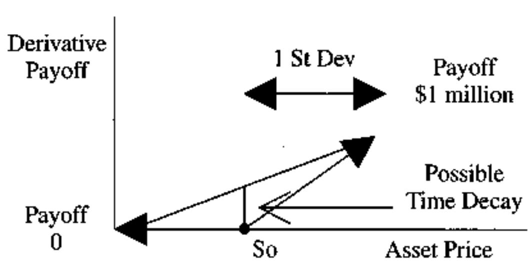
The contamination principle is similar to the notion of heat transfer.* If a spot is located near a source of heat, then its temperature will rise accordingly. If the asset price nears a level that would bring a sizable profit to the portfolio then the area surrounding it should cause a modicum of profit as well.

In the following chart, a derivative will pay $1 million should some event take place in the market. Starting at a given point, the security pays on the node to the right. It would then be unreasonable to think that such a security would be worth nothing at Point So. One standard deviation move should result in a payoff of $1,000,000 (or part of that sum).

The Contamination Principle

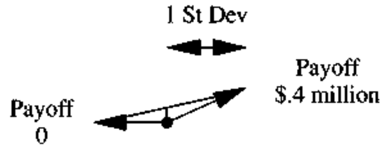


Every experienced trader, seeing the payoff and the probability attached to it, would buy the derivative security. The derivative would then be worth more than 0, and it is easy to see time value taking shape in the following chart:

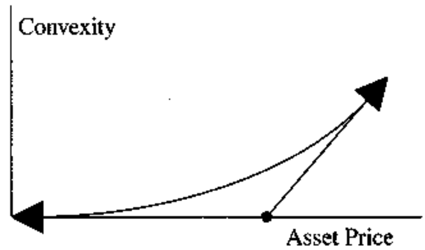


However, time value means time decay. As shown in the following chart, with everything remaining the same, the option would decrease in time value at the standard deviations as the payoff becomes less likely:

(Continued)



The final chart shows that when the points are far from each other (as in option prices), a convex line forms.



This example suggests why an option has "convexity."

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* The origin of the designation *contamination* resides in the price of an option "contaminating" those around it. When an option becomes expensive, those around it need to follow. Likewise, one can see that if an Arrow-Debreu price rises those around it need to increase as well. They are then said to contaminate each other.

A swap is a linear (or quasi-linear) derivative, as the second-order derivative with respect to the asset price is equal (or close to) zero. In trader's parlance, its hedge ratio is not supposed to change relative to movements in the underlying asset, although it can be argued that there is no such thing as a purely linear derivative.

Time-dependent linear derivatives include:

- ***Forwards.*** They are agreements to swap some proceeds in the future.
- ***Floating Rate Agreements (FRAs), Eurodollars.*** They will, for the purpose of this book, be forward-forwards that can be broken down into strips of products that start period t and end period $t + 1$.
- ***Swaps.*** Whatever their end use (this is not of any concern here), they can be composed into a combination of Eurodollars or FRAs. Most of their complexity comes from being detail-heavy, but they otherwise exhibit well-behaved features.[1]

Aside from the correlation between a future price and its financing, most of the difficulties of these assets lie in the problem of interpolation between two points. The way time treats them is not always easy to ascertain. A lesser, but no small, difficulty resides in the multiplication of minute details such as the conventions on the delivery or the 360/365-day basis and other rules that no trader was ever known to memorize.

Noncontingent Time-Dependent Nonlinear Derivatives

- **Noncontingent time-dependent nonlinear derivatives** represent instruments that are convex, concave, or mixed (with respect to the underlying asset) but that are not options (i.e., noncontingent).

The infamous LIBOR-square (London Interbank Offer Rate) for example, provides a lurid example of such convexity. The LIBOR cube will have a third derivative (a convexity of the curvature) but the product does not appear very likely to sell heavily. Despite their payoffs, however, these instruments do not constitute an option since both parties are obligated to swap the proceeds. The strangeness of their payoff is that they are generally convex above a point and concave below, or vice versa. The acceleration of the positive payoffs on one side is balanced by the acceleration of the negative payoffs on the other.

> ***Example:*** The Elevator Bank sells to its customers in the area surrounding Cincinnati, Ohio, a note paying to its holder the square of the interest rate move (between inception and some predetermined time in the future). The customer, stressed out by low interest rates, would thus be compensated in an accelerated way against further rate drops. The note looks like an option, and being (on the surface) arduous to hedge, it will sell for higher than "fair" value.

Options and Other Contingent Claims

The price of options depends on contingent events. They represent the bulk of this study. They are the culprit, the topic of this book. A swap is a linear derivative while a path-dependent swap will present uncertain payoffs. In the past, operators defined these instruments as tools with "optionality," where one party had a right to choose and the other one was under an obligation.

We will study options at two levels. Level 1 is a conventional presentation of the product, as if the exotic option markets did not exist. Level 2 will go into a more generalized presentation of the option markets that would encompass both vanilla and exotics (Chapter 2).

Option Wizard: The Contamination Principle and LIBOR Square

LIBOR square is a contract with a mixed payoff: convex in one zone and concave in another. At the origin (where the contract is set), it is neither. According to the contamination principle, the contract needs to be higher than its value on the line in areas where it is long gamma and lower in areas where it is short.*

The contract pays:

$$q(x - x_0)^2 \text{ if } x > x_0$$
$$-q(x - x_0)^2 \text{ if } x < x_0$$

with q the quantity, x_0 the origin, x the present LIBOR price.

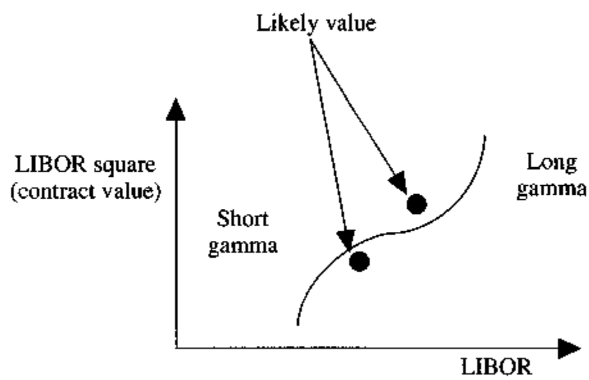
Therefore, its delta is

$$2q(x - x_0) \text{ if } x > x_0$$
$$-2q(x - x_0) \text{ if } x < x_0$$

And the gamma is

$$2q \text{ if } x > x_0$$
$$-2q \text{ if } x < x_0$$

The following chart illustrates the valuation process:



The discussion of option trading will clarify the concept of cells contamination: The arbitrage derived value of any security will depend on its delta-neutral replication.

*LIBOR square is a contract that pays to one party the square of the difference between the origin and a higher price and obligates it to pay the square of the difference between the lower price and the origin.

Simple Options

Options are contingent claims, and thus distinguish themselves from other products in being a potential asset for one party and a potential liability for another. This contingency in their value subjects them to probability theory. All option pricing consists therefore in dealing with probability.

- A **put** is the right to sell an instrument at a certain price (the **strike price**) within some time frame. A **call** is the right to buy the instrument.

With the opening of many new instruments, puts or calls can be confused. They depend on the numeraire. For a currency pair, a put on Mark/Dollar (the right to sell Marks and buy dollars) is a call on Dollar/Mark. Similarly, a put on yields is a call on bonds, a matter of some confusing importance since one illustrious exchange defined a contract on yield (thus inventing an asset), while most cash instruments are traded according to their price.

An interesting extension is that a call on the SP500 is a put on cash for someone whose P/L is computed in SP500 units (the case of indexed fund managers). More on that later.

- A **European option** can only be exercised on the last day. An **American option** can be exercised any time between its inception and the end date. A hybrid, the **Bermudan option** can be exercised on a set number of days between inception and expiration.

- **Intrinsic value,** for a call, or the in-the-money part of an option is the difference between the asset value and the strike price if that difference is positive. It is zero if the difference is negative. For a put, it is the reverse. For a European option (an option that is only exercisable on one date), the intrinsic value is typically expressed by traders (by convention) as the difference between the strike and the corresponding forward. Since the option is not exercised before expiration, the only price that matters is the term market price of the asset (for delivery on the expiration of the option).

The best way intuitively to test whether any particular instrument is an option is to see if the payoff is asymmetric and if there is a strike price (Figure 1.3). A call option is priced at expiration as the

$$\text{Max}\,(S - K, 0)$$

with S the asset price at expiration and K the strike price. It is read as the *greater of the difference between the asset price at expiration and the strike or zero.*

This simple formula means in English that the owner collects some amount (intrinsic) or nothing, whichever is greatest. $S - K$ is the difference between spot and strike. When it is negative, the operator would prefer to receive nothing.

A put option will be expressed as

$$\text{Max}\,(K - S, 0)$$

- **Forwards and futures** are contracts to unconditionally exchange an asset at a predetermined date for an agreed-on price.

They are straight claims, with assets and liabilities on both sides of the fence (as opposed to options where one party has an asset and the other a liability). They also distinguish themselves from options because the payoff does not give any party the element of choice. As will be explained later, some minor technicalities in the definition of the futures contracts (truly, very minor) such as the bond futures[2] spawned an entire cottage industry of arbitrageurs, analysts, and the like.

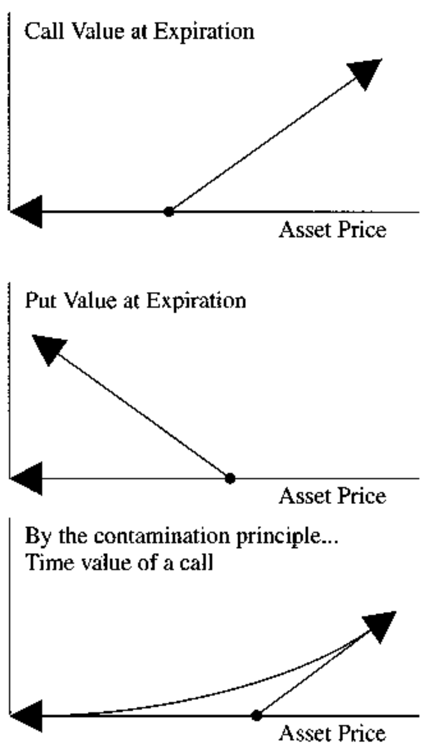


Figure 1.3 Option value.

Another intuitive way to understand an option is to think of it as half the forward or future: The long partakes of the upside (above a certain point, called the strike price) and the short partakes of the downside, for a fee. It is no wonder that being long a put and short a call replicates entirely the future (of course, there are questions of early exercise that we will ignore initially).

Options proliferate in daily life, some of which people are long (they own the choice) for some of which people are short. A chief executive officer owns, by virtue of his position, an option. The manager can partake of the upside of the company and get a bonus when the performance is acceptable. Should the industry experience a downturn and the company go bankrupt, the CEO's sole risk would be his job or some unpleasant but otherwise financially harmless castigation. He will not be asked to appear before the board with his checkbook in hand. It is easy to see that the strike price for the bonus is the required performance and that the shareholders sold (or, rather, gave) that option to the manager.[3]

Forwards, options, and futures represent the bulk of derivatives instruments. A later section in this chapter will present a more advanced version of options.

HARD AND SOFT OPTIONALITY

■ **Optionality** is a broad term used by traders to describe a nonlinearity in the payoff of an instrument. It is often applied to convex instruments or to situations like a "stop loss" or a known order in the market.

As an extension of *the contamination principle,* every item with optionality needs to trade at a premium and the shrinkage of premium with the passage of time (owing to the narrowing of the probability measure of events) will mean necessarily that every item with optionality will have time decay.

It is convenient to call hard optionality the situation where a contract has a strike price and soft optionality the situation where the contract has a built-in convexity but no real strike price. Soft optionality presents generally milder gamma and other Greeks but will present more stable features across time.

BASIC RULES OF OPTIONS EQUIVALENCE

Below are the basic rules of what traders call "option algebra."

- Put-call parity for a European option: Long call/short put = Long forward, provided they are all of the same strike.

Warning Expiration: "Pin Risks" (described in Chapter 13) cause put-call parity not to hold for listed options.

Replacing a long with a "+" sign and the short with a " − " sign allows for the following simple arithmetic:

Position equivalence:

$+C - P = F$. A long forward is equal to a long call/short put of the same strike.

Hence:

$+C = P + F$. Long a call is equivalent to long a put/long a future.

And

$+P = C - F$. Long a put is equivalent to long a call/short a future.

- For a "soft" American option (see definition later in this chapter), the put-call parity rules hold but with a weaker equivalence.
- For "hard" American options, the rule becomes more slippery. More complex rules are described in Chapter 15. It is recommended for the nonspecialist to completely ignore put-call parity.

Example: Assume that the 3-month 102 Put trades for \$1.975, that the 3-month 102 Call trades for \$2.9625, and that the forward for the exact delivery date for both trades for 101.00.

At expiration, assuming financing at 5%, the call will cost $\$1.975 \times .05 \times (90/360) = .025$ and the put will cost .0375. We will then have the P/L shown in Table 1.1.

Table 1.1 Static Put/Call Parity

Asset Price	102 Call P/L	Forward Contract P/L	102 Put P/L	Total Forward P/L + 102 Put P/L
106	2	5	−3	2
105	1	4	−3	1
104	0	3	−3	0
103	−1	2	−3	−1
102	−2	1	−3	−2
101	−2	0	−2	−2
100	−2	−1	−1	−2
99	−2	−2	0	−2
98	−2	−3	1	−2
97	−2	−4	2	−2

Table 1.1 shows the profile at expiration. If two trades are identical at expiration *everywhere* on the map of possible prices, and if they expire on the same day, then the trades will have the same risk and profit/loss profile during their life.

The following rule applies to all markets, properly rescaled, provided they are European and present liquid forwards. To make that rule acceptable for the future, proper allowance needs to be made for the "tailing" (see Chapter 7).

Mirror Image Rule

1 unit of a put + x% of the unit in forward = 1 unit of a call + $(100 - x)$% of the unit in forward, all of the same strike and expiration.

This rule is obvious: A put delta neutral at 30% is equal to $P + .3F = (C - F) + .3F = C - .7F$. If a 103 put has a forward delta of 30% the 103 call will have a forward delta of 70%. This formula uses forward delta, not cash deltas that need to be adjusted for. Most risk management systems disclose the cash, not the forward delta, as does the canonical Black-Scholes-Merton formula.

Warning: European options need to be hedged with the forward,[4] not cash. However, most commercial pricing systems tend to disclose the spot hedge instead, which often can be misleading. Traders often have recourse to cash for short-term hedges owing to the lack of liquidity in the forwards. This habit generally leads them to forget the exact conditions for adequate put/call parity.

■ A **forward delta** for a European option is the equivalent cash position with the same delivery date as the underlying asset.

For an American option, a forward delta is typically of uncertain duration. Such a duration, however, is generally calculated and called the "omega" (discussed later in this chapter), but will be too unstable for us to use for adequate equivalence.

Consequently, a straddle will be equal to two calls delta neutral or two puts delta neutral (of the same strike). Assume that the forward delta of a put is 30%:

$$\text{Straddle} = 2P + .6F = 2\,(C - F) + .6F = 2C - 2F + .6F = 2C - 1.4F$$

Consequently, a call calendar spread will have the same profile as a put calendar spread (assuming interest rates constant).

A put butterfly will have the same price as a call butterfly. Examine the 98/100/102 butterfly (buy one 98 call, sell two 100 call, buy one 102 call):

$$\begin{aligned} 1\ 98C - 2\ 100C + 1\ 102C &= 1\ (98P + F) - 2\ (100P + F) + 1\ (102P + F) \\ &= 1\ 98P - 2\ 100P + 1\ 102P + 1\ F - 2\ F + 1\ F \\ &= \text{Put Butterfly.} \end{aligned}$$

A call butterfly 98-100-102, a put butterfly 98-100-102, and a condor 98-100-102 will have the same exposure.

A 98-100-102 condor is defined as long a 98 put, long a 102 call, and short the 100 straddle:

$$\begin{aligned} &= 98P - 100P - 100C + 102C \\ &= 98P - 100P - (100P + F) + (102P + F) \\ &= 98P - 100P - 100P + 102P - F + F \\ &= 98P - 2\ 100P + 102P \text{ a put butterfly, which we established is equivalent to the call butterfly.} \end{aligned}$$

To gain an intuitive feel the reader can verify that Figure 1.4 shows the same P/L profile for the following:

Long 98, long 102, short twice the 100, all calls or all puts.
Long 98 puts, long 102 calls, short the 100 straddle.
Long the 98 calls, long the 102 puts, short the 100 straddle.

A result of these rules is that the volatility of an out-of-the-money put should be exactly equal to that of a corresponding in-the-money call of the same strike.

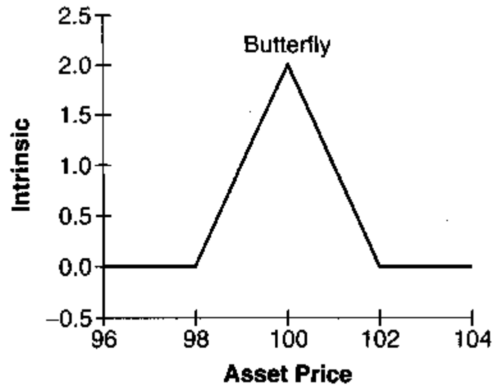


Figure 1.4 Butterfly profile.

> ***Risk Management Rules:***
>
> Traders should never carry put-call parity rules outside of a strike.
>
> Some of the preceding rules can be used with soft American options, except when the delta of the option becomes too high.

American Options, Early Exercise, and Other Headaches (Advanced Topic)

An American option poses more problems than European options because the path followed by the underlying asset can lead to possible early exercise. With a European option, pricing is a simple matter: One can just discount the final payoffs on expiration day.

Without getting immersed in the pricing complications of American options, it is safe to say that the complexity arises because uncertainty about the date of occurrence of the early exercise makes it difficult to model.[5] The rules depend on time and the amount of intrinsic value, which makes the early exercise rules too uncertain.

Such early exercise is generally determined two ways: the soft (or easy) rule and the hard rule.

Soft American Options

- A **soft American option** (also called a pseudo-European option) is only subjected to early exercise from the standpoint of the financing of the intrinsic value.

An **extension** of this definition is that only one interest rate, that affecting the financing of the premium for the operator, impacts the decision to early exercise.

For risk management and trading purposes, soft American options will be largely similar to the European options, except when interest rates become very high relative to volatility. The reason they are often called pseudo-European options is that they behave in general like European options, except when very deep in the money. The test of early exercise is whether the total option value is less than the time value of the money between the time of consideration and expiration.

Example: Assume that an asset trades at \$100, with interest rates at 6% (annualized) and volatility at 15.7%. Assume also that the 3-month 80 call is worth \$20, at least if it is American. Forgoing early exercise would create an opportunity cost of $20 \times 90/360 \times .06 = .30$ cents, the financing of the \$20 premium for 3 months. The time value of the equivalent put is close to zero (by put-call parity), so the intelligent operator can swap the call into the underlying asset and buy the put to replicate the same initial structure at a better cost. He would end up long the put and long the underlying asset.

Hard American Options

■ A **hard American option** is an option subjected to early exercise tests from the standpoint of both the financing of the intrinsic value and the carry costs of the underlying asset until the nominal expiration.

By extension, two rates, that of the premium for the operator, and that of the carry of the underlying asset impact the decision to early exercise.

An early exercise can thus be attributed, in addition to the soft American rule, to the yield benefits of an early position in the asset. The following filter needs to be taken into account at all times during the life of the option: Would the operator do better if he owned an interest-bearing asset than if he owned the equivalent position through the options?

> ***Example:*** The operator owns the same call as earlier, but the underlying asset is a currency that pays 20% interest while domestic rates are 6%. The option trader has the additional benefit of exercising the call because, on top of the financing, he can own the currency that pays the high interest rates against his home currency, which costs him only 6% to short, therefore earning approximately 14% annualized on the same value. The benefits of early exercise from the asset ownership are much higher than in the previous case: $.14 \times 100$ (face value) $\times 90/360 =$ \$3.5. In addition to that, the trader has the value of getting the cash much earlier, which was computed in the previous case at \$.30. The operator should perhaps have exercised the option much earlier to benefit from this extra kicker.
>
> Likewise the put on the high-yielding currency will not be subjected to early exercise. Take the following rule: A market trades at 100. The 120 put with \$20 of intrinsic value will be better held till expiration since the operator would have to pay \$3.50 to hold the equivalent short position in financing differential. True, the 30 cents of financing the premium would be deducted from that value in the cost-benefits analysis. So the put would not be early exercised by an optimal operator. It will trade and will be considered for all intents and purposes as a European instrument.[6]

These rules of hard early exercise extend to the following instruments:

- ***Bonds with a Positive Carry.*** American calls are early exercisable when the bond is financed more cheaply than its yield. American puts will therefore be similar to (but not quite the same as) European puts.
- ***Bonds with a Negative Carry.*** The reverse is true.
- ***Equities.*** When the equities are negative carry (and the dividend payout is known), the calls resemble European ones and vice versa.

Never trust the price for an American option[7] on a cash instrument. There may be changes in the parameters throughout the life of the instrument (nondividend paying stocks starting to pay, changes in interest rates, reversal of the interest rate differential) that can affect its future value. Parameters, unfortunately, are not frozen.

Options on futures, therefore, will be considered subject to the soft exercise rules. We will generally fold them within the category of European instruments for the purpose of our analysis. At all times, the test will be that of the financing of the premium; differences only become pertinent between European and American options for deep in-the-money instruments.

European instruments tend to prevail where there are differences in the pricing mechanism. The market generally seems to go to the most liquid instrument, and operators—by shying away from complicated options—make them less attractive to trade. European options dominate with the currencies, and represent close to 99% of the volume of those with a high differential.

Option Wizard: The Simplicity Rule

A major rule should be taken into account: **The market always tends to flow to simplicity.** Complexity is generally costlier to both monitor and produce, and somehow in the long run, demand moves away from the complex in favor of the simple. New, elaborate contracts certainly attract people, but typically the novelty wanes. Operators will then try to satisfy their needs for protection by seeking the cheapest possible way.

Complex products cost more to replicate. An optimal operator becomes cost conscious and avoids enriching the financial institutions when he can satisfy his interests more economically. This becomes noticeable with the life or death of listed financial instruments. It is the recipe for the survival of exchange contracts. That rule will be discussed in the study of exotic options.

A Brief Warning about Early Exercise Tests

Most operators have their risk management system flag an early exercise and routinely terminate the option. This is often less than optimal (and downright dangerous) for the following reasons:

- One should refine the test by using a proper volatility for the corresponding out-of-the-money option of the same strike. Most risk management systems do not apply the proper volatility smiles correctly. For example, if the option early exercise test is done at a volatility of 16% and the strike price concerned trades at 20%, the early exercise flagged by the system would be erroneous.
- An additional refinement should be the use of a volatility term structure by retesting with a longer or shorter duration, inputting the highest possible volatility between time zero and the expiration of the option.
- When dealing with a large-size position, dealers, upon exercise, synthetically become short an out-of-the-money option. It is recommended to test for the liquidity of such strike in the market and compute the replacement costs.
- A war story: The day before the stock market crash (of 1987) X, a market maker in the Eurodollar options, found a deep-in-the-money "reversal" in his books (*a reversal means that the trader is long a put, short a call, short the future*). According to his risk management system, the put was early exercisable. He exercised the put staying naked the calls. The following day, the market experienced a 10 standard deviation rally on the open, putting him out of business and, worse, causing his story to become a legend. Ironically, X belonged to the category of "wings" buyers (people who always own out-of-the-money options). He never shorted "wing" options and ended up hurt by a synthetic (and entirely accidental) short.

Consequence: Smile-Calendars (Advanced Topic). The volatility of options of the same strike needs to be equal to allow for put-call parity equivalence. While this rule applies unconditionally for European options, many operators mistakenly apply it to the American variety. In some cases, the rules can be applied, and in other cases they need to be lifted.

When a strong skew is impacting the market, the put-call parity can be weakened considerably by the following:

- A rising volatility curve could separate the put and calls from each other because the nonexercisable leg would follow the nominal maturity, whereas the exercisable one would have a considerably shorter expected life.

Example: Assume that 3-month options trade at 15.7% volatility but that the 1-day options traded at 13%. An exercisable 80 call would be priced at 13%, whereas the nonexercisable leg (the 80 put) would trade at 20%.

- A strong smile (defined as an implied volatility that is a function of the strike price and time to maturity) can present worse results.

Example: As shown in Figure 1.5, assume that the 80 strike for the 3-month option trades at 20% volatility (point A), while the at-the-money trades at 15.7 for 3-month (point B) and 13% for 1-day options (point C). The 3-month 80 call will trade at 13% volatility (point C), while the 80 put would trade at 20%.

> ***Risk Management Rule:*** Hard American options are more valuable than European options of the same expiration, because they harbor a compound option on interest rates or volatility. The difference between them increases with either the *volatility of volatility* or the level and volatility of interest (or carry) rates.

The rule is easy to explain to traders who had to live in highly unstable interest rates or fluctuating implied volatility (vvol). The American option gives the owner the right to pay the carry to "extend" the option one additional day and therefore make a bet that the option may no longer be

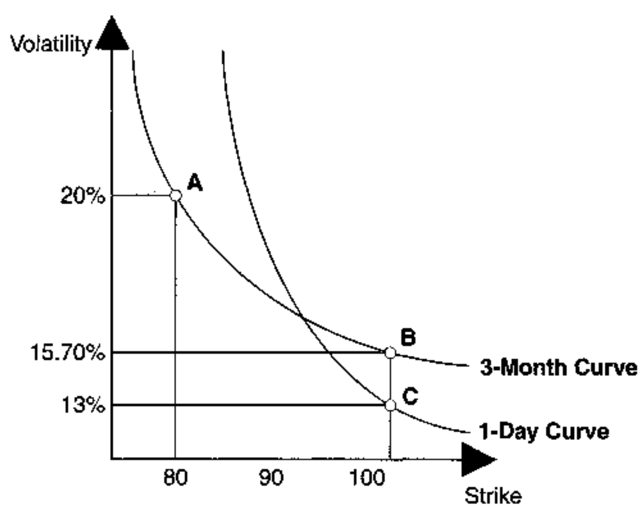


Figure 1.5 Smile curves.

exercisable. As such it becomes an extendible option (for a discussion of compound options see Chapter 21).

American Options Nobody Ever Exercises. Some contracts for American options do not present any early exercise value. They are the options on futures where there is a marks-to-market daily. For example: London Interbank Financial Future Exchange (LIFFE) option products are margined as a future. The profits can be taken out daily and can thus earn interest. Buying a deep-in-the-money option requires a much smaller outlay of cash than other markets.

FORWARDS, FUTURES, AND FORWARD-FORWARDS (ADVANCED TOPIC)

- A **forward contract** between two parties obligates both of them to buy and sell a given asset on a specified date, or to exchange payments according to a formula.

- A **future** is a standardized forward contract listed on an exchange with set maturities where the exchange clearing house is the counterparty. Trades take place in an open outcry system and the liquidity is improved by the standardization and the interchangeability between contracts.

There is a difference in hedge ratios between the forward and the future. In the forward, the exchange of payments takes place at a terminal date, while with the future there is a variation margin system of pay-as-you-go. The parties need to exchange payments that correspond to their daily profit or loss. A winner can thus earn interest on the profits. Such difference can be substantial for long-dated instruments.

To hedge a forward with a future, the trader needs to "tail,"[8] to adjust for the difference between a present value claim and a cash one. The hedge (number of units of futures for one unit of forward) is:

$$h = e^{-rt}$$

where t is the number of years and r is the zero-rate until the forward delivery.[9]

> ***Example:*** A 3-month listed future on the Chicago Merchantile Exchange (CME) in the Deutsche Mark (DEM) against the U.S. Dollar (USD) trades at .70 cents per dollar (one whose expiration happens to fall exactly 90 days from today). The 90-day forward, by arbitrage, also trades at .70 (expressed in spot at 1.4286).

What is the equivalent Deutsche mark (DEM) amount to 100 futures (a future on CME represents 125,000 DEM)? Assume 90-day interest rates are 6%.

Total face value is DEM 12,500,000. The hedge ratio h is Exp $[-.06 \times .25] = .985$, number of futures for one forward. The hedge in the forward is therefore: 12,500,000/.985 = 1,269,000.

If the DEM future immediately moves to .71 the profit on the future leg will be $125,000 deliverable immediately. The profits in the forward will be $126,900 but will be only a mark-to-market realizable in 90 days.

Credit

Credit is another difference between futures and the forwards. Generic swaps, caps, and floors (with futures equivalent dates) are easy to hedge with strips and Euro options. Going to the over-the-counter market would create a mismatch that consumes two-way credit facilities. If Credit Syldavia engages in a swap with Banca Nazionale del Lavoro, both parties will reduce their credit lines to each other and swell their contingent balance sheets. Should both unwind their side of the trade at different times with different counterparties who in turn trade with each other, the numbers would multiply: Each of the four parties would reduce the size of its books and have its lines to the rest of the world reduced as well. Had all the trades taken place on a standardized exchange, everyone would be flat.

Typically, the way market makers operate is to use the exchange when initiating a trade (when they act as a customer) and to trade a forward when they are acting as a market maker, with other parties calling them and dealing on their prices.

Marks-to-Market Differences

The marks-to-market rules create non-trivial differences between the forwards (and similar instruments) and the futures (see Table 1.2). Forwards are self-marked by operators according to some convention. A few points are updated by the trader and the back office and all the intermediate points on the curve are computed by whatever algorithm is used by the system.

The future, on the other hand, is marked by the exchange through some well-defined rules. The Eurodollar curve, about 40 contracts, is not generated through computer algorithms but with the last trade or bid/offer in mind, even if the resulting curve becomes jagged. An operator who marks the same curve with fewer points will generally have a smoother result.

Another issue lies in the timing. It is called the *nonsynchronous* marked-to-market problem. Many operators carry positions on one exchange offset against positions on another. Exchanges do not have the same settlement time. A future marks-to-market would not accurately portray the resulting P/L.

Table 1.2 Differences between Forward and Futures

	Forward	Future
Marks to Market	Institutional, self-created	Official settlement
Variation Margins	None	Daily
Credit Risks	Depends on the institution	Almost nonexistent:* systemic rather than limited to one institution
Instrument Hedge	Delta is present valued (see Greeks)	Delta is "raw," not present valued
Trading Risks	Higher exposure to illiquidity as the contracts rapidly move away from the liquid maturities	Higher liquidity but fewer "pillars" of trading
Trading costs	Direct costs are generally low Indirect costs are higher, as the bid/offer spread generally larger	Direct costs are high: commission, clearing charges, exchange fees Indirect costs are lower: "spread" is tighter
Sensitivity to Financing	Nonsensitive to the correlation with the financing rate	Sensitive to the correlation between price and the financing rate

* Except, of course, in "emerging" markets.

Example: An arbitrageur plays the forwards in USD-JPY (Japanese yen) by trading forwards in the over-the-counter market against EuroYen futures in Singapore and the Eurodollars in the United States. The bank marks the positions at 4:30 P.M. New York time, while the Singapore futures are marked before he starts his day in New York and the Eurodollar at 3:00 P.M. New York time. His resultant P/L will never reflect the accurate liquidating value of his position.

The Correlation between the Future and the Financing (Advanced Issue)

In the preceding situations, there is independence between the financing rate of the P/L stemming from the future and the expected moves in the future. A correlation between the financing and the future will translate into a convexity or concavity of the future compared with the forward.

> ***Risk Management Rule:*** When there is a positive correlation between the financing rate r and a future contract F[10] (on any possible underlying asset) subjected to marks-to-market rules, the future will be convex and will trade above the forward of the same delivery.
>
> **Converse:** Whenever there is a negative correlation between the financing rate r and a future contract F, the future will be concave and will then need to trade at a lower rate than a corresponding forward.

An illustration of the rule in provided in Chapter 10.

Forward-Forward

- A **forward-forward** is a contract to exchange an asset at one period against the reverse trade at a later period.

For quasi-linear derivatives, such as fixed-income instruments, it is the price ratio between two forwards.

$$FF(t1, t2) = F(t2)/F(t1)$$

The forward-forward rate is determined by the existing rates in the market interpolated to solve for the break-even rate for the period between $t1$ and $t2$.

The Eurodollar futures are forward-forward deposits, and ironically, the forward-forward often sets the spot (the tail wagging the dog, as often repeated in future circles).

For options, the forward-forward is computed with the nonlinearity of time in mind. This will be dealt with in Chapter 9.

Core Risk Management: Distinction between Primary and Secondary Risks

Market risks can be primary or secondary, but sometimes the distinction can be counterintuitive; some instruments and markets present more danger in the fringe risks than in their primary exposures.

A primary risk is one that constitutes the bulk of the variance of profits and losses (the P/L), and is where most hedging efforts should be concentrated. Since markets move rapidly, it is easy to see the primary risks as those a trader would cover first, and the residual risks are those that can usually wait until the end of the trading session.

The following classification of risks excludes the party-spoiling correlation-based products (like options of one of two instruments and other goodies).

- **For an equity derivatives portfolio,** market risks are almost entirely directional, with all the possible permutations concerning the derivatives of the effect of the underlying equities. Matters pertaining to interest rates are deemed secondary, as the thrust of the positioning is not at the level of these parameters. Someone can be hurt from the indirect effects of an interest rate change on a long dated stock position (through the effect on the pricing of the forwards), but such variance would be insignificant compared with that caused by the price action and the changes in volatility. Such analysis does not reflect the possible effect of the interest rates move on the equity markets, just their effect on the time structure or equity prices. The health risks of passive smoking are not seriously significant to a man already diagnosed with cancer.
- **For a fixed income derivatives book,** however, both the underlying asset (that is the cash flow schedules as discounted by the interest rates) and the general structure of the interest rate curve need to be taken into account as primary risks. This is mostly attributable to the consideration that fixed-income positions react to a schedule of prices spread over time, not just to one price like an equity or a currency. Every coupon paying fixed income security is a simple sum of smaller zero-coupon securities with different expirations. A 10-year swap's price will principally be affected by the 10-year rates. It will also be affected by the 4-year rates, everything else being equal, because of the effect on the reinvestment over the period. A term structure becomes an integral part of the risks and rewards of every instrument and should therefore be considered a primary risk.
- **For a currency book,** the exchange rate of the currency pair and the volatility are the primary risks. But both the interest rate differentials and the term structures of interest rates are preeminent; they become part of the primary risks with some categories of developing markets that experience a high interest rate volatility.

Such classifications need to adapt to most possible instruments: Those who have tried to come up with a generalized theory of risk management have so far failed in their laborious attempts. No two instruments will be equal. Traders, therefore, must search initially for common points between instruments.

Tables 1.3 and 1.4 will apply to the SDF (smallest decomposable fragment).

Liquidity is preeminent as a risk (the *invisible risk* as traders concede). It will be examined in this book at all levels without fitting it in any category. Liquidity is the source and the cause of everything in trading and it should remain in the back of every risk manager's mind.

Since all options are basically alike this text adheres, whenever possible, to a standardized example. Most examples in the book depict option risks for a generic asset with a flat forward. This approach simplifies the effect of the projection into the future and isolates the pure option risks of the instruments.

In other words we rescale everything to a numeraire. Hence the forward will trade at 100% of cash regardless of the expiration. Such simplification allows a focus on the option risks, without invalidating the real issues. Where the simplification leads to inaccuracies, a tradable instrument will be used taking into account its particularities. The drift and the notion of Girsanov change of probability will be introduced in cases where it affects the hedging technique.

Initially, readers will measure the risks of the derivatives, with simple cases where a "pure" portfolio is created. The basic "Greeks" (vega, gamma,

Table 1.3 Primary and Secondary Risks by Market

Market	Primary Market Risks	Secondary Market Risks
Equities	The underlying equity	The domestic interest rate
	Volatility	Dividend payout
		Volatility term structure
Fixed Income	Rates	Higher order derivatives
	Term structure of rates	Risks of pricing formula
	Volatility	Volatility term structure*
		Stability of the covariances between maturities
Currencies	Price	Rates in each currency
	Volatility	Volatility term structure
Commodities	Price	The domestic interest rate
	Volatility	Storage costs
	Term structure of prices	Volatility term structure

* It is assumed that a cap or a floor are decomposed into caplets and floorlets.

Table 1.4 Primary and Secondary Risks by Instrument

Instrument	Additional Primary Market Risks	Additional Secondary Market Risks
Barrier Options	Term structure of rates	Skew
	Term structure of volatility	Volatility of volatility
	Liquidity at the barrier	
Asian Options	Term structure of rates	Model Risk
	Term structure of volatility	Skew
	Variance ratio	
Lookback Options	Mean reversion	Volatility of volatility
		Arcsine law
		"Driftwood" effect
Compound Options	Volatility of volatility	Mean reversion
Multidimensional Options	Correlation measure	
	Volatility of volatility (hence correlations)	

time decay) will be examined before readers modify their measures as practitioners do routinely, whether they are conscious of these transformations or not. This will be followed by a thorough presentation of the modified delta, the modified vega, the modified theta, and the modified gamma. This blanket method should be possible to apply to all instruments.

The generating blocks of some exotic options—bets, barriers, and correlation—will be the final objects of study. Readers will be shown a road map for extending these principles to other, complex instruments. To apply these techniques to a book of specific instruments, it is important to learn to decompose the option risk from the residual risks proper to the instrument. This is best done through the decomposition of the instrument into liquid traded segments (see Figure 1.7).

Applying the Framework to Specific Instruments

A swap is simply a multiasset instrument composed of correlated segments often entirely decomposable into very liquid Eurodollar strips. The framework on multiasset options and the techniques of stacking with correlation matrices should be sufficient for the trader to adapt the principles of dynamic hedging to swaps. As for index-amortizing swaps, the use of

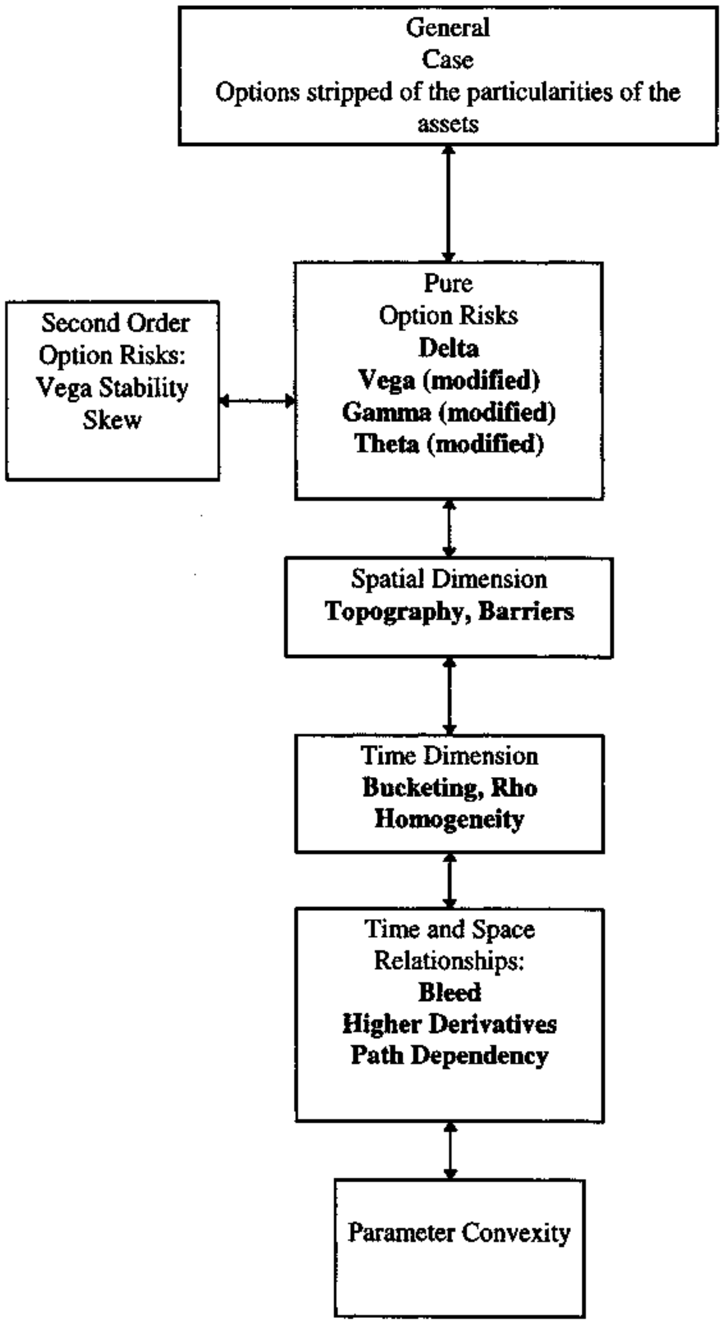


Figure 1.6 Options risks defined.

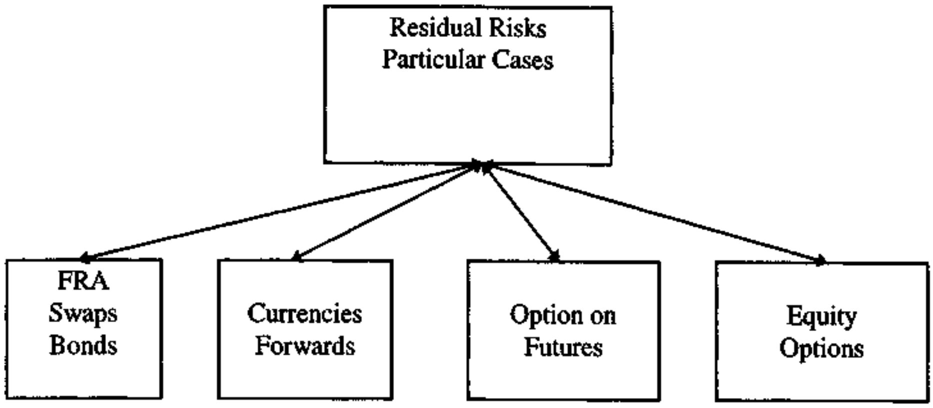


Figure 1.7 An analysis of residual risk by groups of instruments.

American digital options should perform the task, mixed with some knowledge of compound options where applicable. The problem with most fixed-income traders is that they tend to limit their knowledge to the intricacies of the instrument rather than focus on the generating blocks. This method is considerably easier than the holistic Heath-Jarrow-Morton approach.