CHAPTER 1

Linear Equations

OUTLINE

On the way back to college, you and a friend decide to stop off in Charlotte, North Carolina. Because you have only one full day to see the sights, you decide that renting a car is the best way to see the most. But which car rental company should you use? Naturally, the cheapest! But what is the cheapest? Is it the one with unlimited mileage or the one with a better daily rate and limited miles? The mathematics of this chapter provides the background for solving this problem. The Chapter Project at the end of the chapter will help you understand how to make the best decision.

A LOOK BACK, A LOOK FORWARD

In Appendix A, we review algebra and geometry skills from earlier courses. In this chapter we make the connection between algebra and geometry through the rectangular coordinate system. The idea of using a system of rectangular coordinates dates back to ancient times, when such a system was used for surveying and city planning. Apollonius of Perga, in 200 B.C., used a form of rectangular coordinates in his work on conics, although this use does not stand out as clearly as it does in modern treatments. Sporadic use of rectangular coordinates continued until the 1600s. By that time, algebra had developed sufficiently so that René Descartes (1596–1650) and Pierre de Fermat (1601–1665) could take the crucial step, which was the use of rectangular coordinates to translate geometry problems into algebra problems, and vice versa. This step was important for two reasons. First, it allowed both geometers and algebraists to gain critical new insights into their subjects, which previously had been regarded as separate but now were seen to be connected in many important ways. Second, the insights gained made possible the development of calculus, which greatly enlarged the number of areas in which mathematics could be applied and made possible a much deeper understanding of these areas.

1.1 Rectangular Coordinates; Lines

PREPARING FOR THIS SECTION *Before getting started, review the following:*

> Real Numbers (Section A.1, pp. A-1–A-14)

> Algebra Review (Section A.2, pp. A-15–A-25)

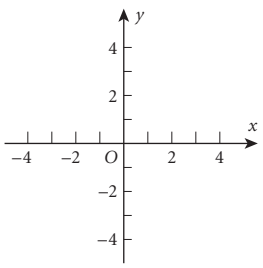
OBJECTIVES

1. Graph linear equations
2. Find the equation of a vertical line
3. Calculate and interpret the slope of a line
4. Graph a line given a point on the line and the slope
5. Use the point–slope form of a line
6. Find the equation of a horizontal line
7. Find the equation of a line given two points
8. Use the slope–intercept form of a line

We locate a point on the real number line by assigning it a single real number, called the *coordinate of the point*. For work in a two-dimensional plane, we locate points by using two numbers.

We begin with two real number lines located in the same plane: one horizontal and the other vertical. We call the horizontal line the ***x*-axis**, the vertical line the ***y*-axis**, and the point of intersection the **origin *O***. We assign coordinates to every point on these number lines as shown in Figure 1, using a convenient scale. In mathematics, we usually use the same scale on each axis; in applications, a different scale is often used on each axis.

FIGURE 1

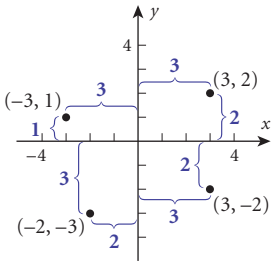


The origin O has a value of 0 on both the x-axis and the y-axis. We follow the usual convention that points on the x-axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Points on the y-axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. In Figure 1, the x-axis and y-axis are labeled as x and y, respectively, and we have used an arrow at the end of each axis to denote the positive direction.

The coordinate system described here is called a **rectangular** or **Cartesian*** **coordinate system.** The plane formed by the x-axis and y-axis is sometimes called the ***xy*-plane**, and the x-axis and y-axis are referred to as the **coordinate axes.**

Any point P in the xy-plane can then be located by using an **ordered pair** (x, y) of real numbers. Let x denote the signed distance of P from the y-axis (*signed* in the sense that, if P is to the right of the y-axis, then $x > 0$, and if P is to the left of the y-axis, then $x < 0$); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the **coordinates** of P, then gives us enough information to locate the point P in the plane.

FIGURE 2

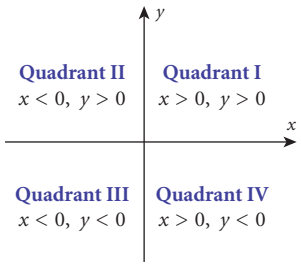


For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the x-axis to the left of O and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the x-axis has coordinates of the form $(x, 0)$, and any point on the y-axis has coordinates of the form $(0, y)$.

If (x, y) are the coordinates of a point P, then x is called the ***x*-coordinate,** or **abscissa,** of P; and y is the ***y*-coordinate,** or **ordinate,** of P. We identify the point P by its coordinates (x, y) by writing $P = (x, y)$, referring to it as "the point (x, y)," rather than "the point whose coordinates are (x, y)."

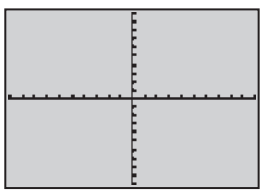
FIGURE 3



The coordinate axes divide the xy-plane into four sections, called **quadrants,** as shown in Figure 3. In quadrant I, both the x-coordinate and the y-coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.

NOW WORK PROBLEM 1.

FIGURE 4



COMMENT: On a graphing utility, you can set the scale on each axis. Once this has been done, you obtain the **viewing rectangle.** See Figure 4 for a typical viewing rectangle. You should now read Section C.1, The Viewing Rectangle, in Appendix C.

Graphs of Linear Equations in Two Variables

A **linear equation in two variables** is an equation of the form

$$Ax + By = C \tag{1}$$

where A and B are not both zero.

Examples of linear equations are

$$3x - 5y - 6 = 0 \qquad \text{This equation can be written as } 3x - 5y = 6 \qquad A = 3, B = -5, C = 6$$

*Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

$-3x = 2y - 1$	This equation can be written as $-3x - 2y = -1$ $\quad A = -3, B = -2, C = -1$ or as $3x + 2y = 1$ $\quad A = 3, B = 2, C = 1$
$y = \frac{3}{4}x - 5$	Here we can write $-\frac{3}{4}x + \ y = -5$ $\quad A = -\frac{3}{4}, B = 1, C = -5$ or $3x - 4y = 20$ $\quad A = 3, B = -4, C = 20$
$y = -5$	Here we can write $0 \cdot x + y = -5$ $\quad A = 0, B = 1, C = -5$
$x = 4$	Here we can write $x + 0 \cdot y = 4$ $\quad A = 1, B = 0, C = 4$

The **graph** of an equation is the set of all points (x, y) whose coordinates satisfy the equation. For example, $(0, 4)$ is a point on the graph of the equation $3x + 4y = 16$, because when we substitute 0 for x and 4 for y in the equation, we get

$$3 \cdot 0 + 4 \cdot 4 = 16 \qquad 3x + 4y = 16, x = 0, y = 4$$

which is a true statement.

It can be shown that if A, B, and C are real numbers, with A and B not both zero, then the graph of the equation

$$Ax + By = C$$

is a **line**. This is the reason we call it a **linear equation.**

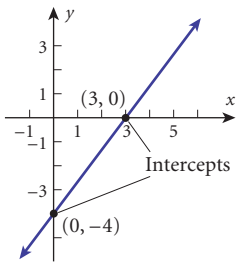
Conversely, any line is the graph of an equation of the form $Ax + By = C$.

Since any line can be written as an equation in the form $Ax + By = C$, we call this form the **general equation** of a line.

Graph linear equations 1

Given a linear equation, we can obtain its graph by plotting two points that satisfy its equation and connecting them with a line. The easiest two points to plot are the *intercepts*. For example, the line shown in Figure 5 has the intercepts $(0, -4)$ and $(3, 0)$.

FIGURE 5



Intercepts

The points at which the graph of a linear equation crosses the axes are called **intercepts**. The ***x*-intercept** is the point at which the graph crosses the x-axis; the ***y*-intercept** is the point at which the graph crosses the y-axis.

Steps for Finding the Intercepts of a Linear Equation

To find the intercepts of a linear equation $Ax + By = C$, with $A \neq 0$ or $B \neq 0$, follow these steps:

STEP 1 Let $y = 0$ and solve for x. This determines the x-intercept of the line.
STEP 2 Let $x = 0$ and solve for y. This determines the y-intercept of the line.

EXAMPLE 1 Finding the Intercepts of a Linear Equation

Find the intercepts of the equation $2x + 3y = 6$. Graph the equation.

SOLUTION **Step 1** To find the x-intercept, we need to find the number x for which $y = 0$. We let $y = 0$ in the equation and proceed to solve for x:

$$\begin{aligned} 2x + 3y &= 6 \\ 2x + 3(0) &= 6 && y = 0 \\ 2x &= 6 && \text{Simplify} \\ x &= 3 && \text{Solve for } x. \end{aligned}$$

The x-intercept is $(3, 0)$.

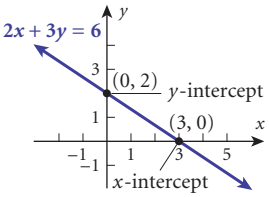
Step 2 To find the y-intercept, we let $x = 0$ in the equation and solve for y:

$$\begin{aligned} 2x + 3y &= 6 \\ 2(0) + 3y &= 6 && x = 0 \\ 3y &= 6 && \text{Simplify} \\ y &= 2 && \text{Solve for } y. \end{aligned}$$

The y-intercept is $(0, 2)$.

Since the equation is a linear equation, its graph is a line. We use the two intercepts $(3, 0)$ and $(0, 2)$ to graph it. See Figure 6.

FIGURE 6



EXAMPLE 2 Graphing a Linear Equation

Graph the equation: $y = 2x + 5$

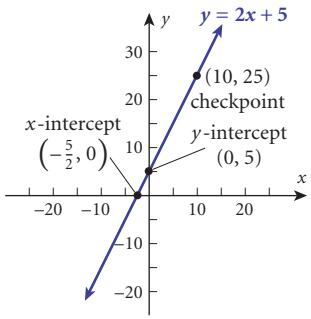
SOLUTION This equation can be written as

$$-2x + y = 5$$

This is a linear equation, so its graph is a line. The intercepts are $(0, 5)$ and $(-\frac{5}{2}, 0)$, which you should verify. For reassurance we'll find a third point. Arbitrarily, we let $x = 10$. Then $y = 2x + 5 = 2(10) + 5 = 25$, so $(10, 25)$ is a point on the graph. See Figure 7.

x	y
0	5
$-\frac{5}{2}$	0
10	25

FIGURE 7



NOW WORK PROBLEM 5.

When a line passes through the origin, it has only one intercept. To graph such lines, we need to locate an additional point on the graph.

EXAMPLE 3 Graphing a Linear Equation

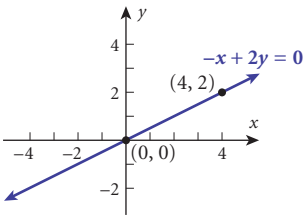
Graph the equation: $-x + 2y = 0$

SOLUTION This is a linear equation, so its graph is a line. The only intercept is $(0, 0)$. To locate another point on the graph, let $x = 4$. (This choice is arbitrary; any choice of x other than 0 could also be used). Then,

$$\begin{aligned} -4 + 2y &= 0 \quad -x + 2y = 0, x = 4 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

So, $y = 2$ when $x = 4$ and $(4, 2)$ is a point on the graph. See Figure 8.

FIGURE 8



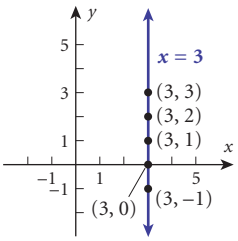
Next we discuss linear equations whose graphs are vertical lines.

EXAMPLE 4 Graphing a Linear Equation (a Vertical Line)

Graph the equation: $x = 3$

SOLUTION We are looking for all points (x, y) in the plane for which $x = 3$. Since $x = 3$, no matter what y-coordinate is used, the corresponding x-coordinate always equals 3. Consequently, the graph of the equation $x = 3$ is a vertical line with x-intercept $(3, 0)$ as shown in Figure 9.

FIGURE 9



As suggested by Example 4, we have the following result:

Equation of a Vertical Line

A vertical line is given by an equation of the form

$$\boxed{x = a}$$

where $(a, 0)$ is the x-intercept.

2 EXAMPLE 5 Finding the Equation of a Vertical Line

Find an equation for the vertical line containing the point $(-1, 6)$.

SOLUTION The x-coordinate of any point on a vertical line is always the same. Since $(-1, 6)$ is a point on the vertical line, its equation is $x = -1$.

NOW WORK PROBLEM 9(a).

Calculate and interpret the slope of a line 3

Slope of a Line

An important characteristic of a line, called its *slope*, is best defined by using rectangular coordinates.

Slope of a Line

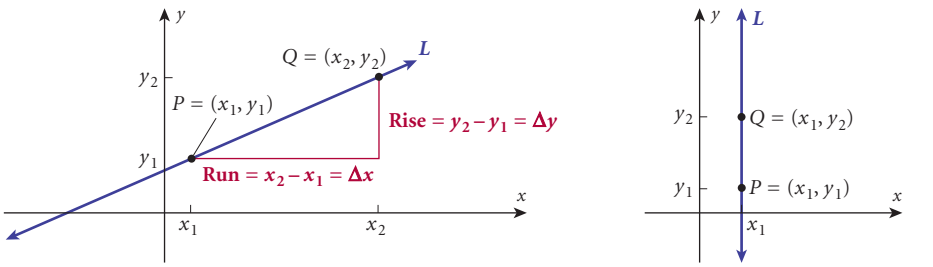
Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope** m of the nonvertical line L containing P and Q is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2 \tag{2}$$

If $x_1 = x_2$, L is a vertical line and the slope m of L is **undefined** (since this results in division by 0).

Figure 10(a) provides an illustration of the slope of a nonvertical line; Figure 10(b) illustrates a vertical line.

FIGURE 10



Slope of L is $m = \frac{y_2 - y_1}{x_2 - x_1}$ (a)

Slope is undefined; L is a vertical line (b)

As Figure 10(a) illustrates, the slope m of a nonvertical line may be given as

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Change in } y}{\text{Change in } x}$$

The change in y is usually denoted by Δy, read "delta y," and the change in x is denoted by Δx.

The slope m of a nonvertical line L measures the amount y changes, Δy, as x changes from x_1 to x_2, Δx. This is called the **average rate of change of y with respect to x.** Then, the slope m is

$$m = \frac{\Delta y}{\Delta x} = \text{average rate of change of } y \text{ with respect to } x$$

EXAMPLE 6 Finding and Interpreting the Slope of a Line

The slope m of the line containing the points $(3, -2)$ and $(1, 5)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{5 - (-2)}{1 - 3} = \frac{7}{-2} = \frac{-7}{2} = -\frac{7}{2}$$

We interpret the slope to mean that for every 2-unit change in x, y will change by -7 units. That is, if x increases by 2 units, then y decreases by 7 units. The average rate of change of y with respect to x is $-\frac{7}{2}$.

NOW WORK PROBLEMS 13 AND 17.

Two comments about computing the slope of a nonvertical line may prove helpful:

1. Any two distinct points on the line can be used to compute the slope of the line. (See Figure 11 for justification.)
2. The slope of a line may be computed from $P = (x_1, y_1)$ to $Q = (x_2, y_2)$ or from Q to P because

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

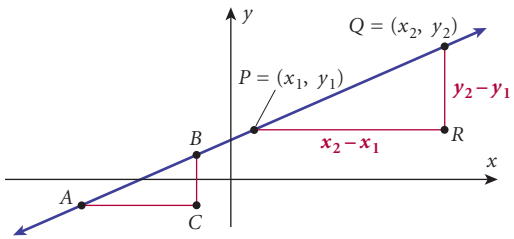


FIGURE 11 Triangles ABC and PQR are similar (they have equal angles). So ratios of corresponding sides are proportional. Then:

Slope using P and $Q = \frac{y_2 - y_1}{x_2 - x_1} =$

Slope using A and $B = \frac{d(B, C)}{d(A, C)}$

where $d(B, C)$ denotes the distance from B to C and $d(A, C)$ denotes the distance from A to C.

To get a better idea of the meaning of the slope m of a line L, consider the following example.

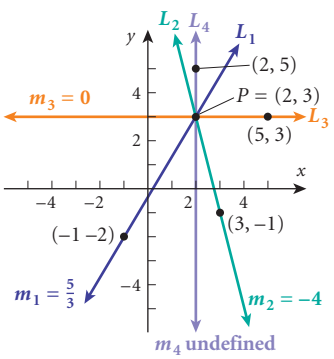
EXAMPLE 7 Finding the Slopes of Various Lines Containing the Same Point (2, 3)

Compute the slopes of the lines L_1, L_2, L_3, and L_4 containing the following pairs of points. Graph all four lines on the same set of coordinate axes.

L_1: $P = (2, 3)$ $Q_1 = (-1, -2)$
L_2: $P = (2, 3)$ $Q_2 = (3, -1)$
L_3: $P = (2, 3)$ $Q_3 = (5, 3)$
L_4: $P = (2, 3)$ $Q_4 = (2, 5)$

SOLUTION Let m_1, m_2, m_3, and m_4 denote the slopes of the lines L_1, L_2, L_3, and L_4, respectively.

FIGURE 12



Then

$$m_1 = \frac{-2-3}{-1-2} = \frac{-5}{-3} = \frac{5}{3}$$ A rise of 5 divided by a run of 3

$$m_2 = \frac{-1-3}{3-2} = \frac{-4}{1} = -4$$ A rise of -4 divided by a run of 1

$$m_3 = \frac{3-3}{5-2} = \frac{0}{3} = 0$$ A rise of 0 divided by a run of 3

m_4 is undefined The x coordinates of P and Q_4 are equal ($x_1 = x_2 = 2$)

The graphs of these lines are given in Figure 12.

As Figure 12 illustrates,

1. When the slope m of a line is positive, the line slants upward from left to right (L_1).
2. When the slope m is negative, the line slants downward from left to right (L_2).
3. When the slope m is 0, the line is horizontal (L_3).
4. When the slope m is undefined, the line is vertical (L_4).

COMMENT: Now read Section C.3, Square Screens, in Appendix C.

SEEING THE CONCEPT: On the same square screen, graph the following equations:

$$Y_1 = 0$$ Slope of line is 0.

$$Y_2 = \frac{1}{4}x$$ Slope of line is $\frac{1}{4}$.

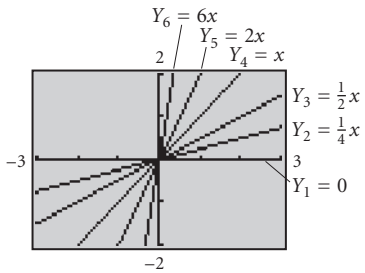
$$Y_3 = \frac{1}{2}x$$ Slope of line is $\frac{1}{2}$.

$$Y_4 = x$$ Slope of line is 1.

$$Y_5 = 2x$$ Slope of line is 2.

$$Y_6 = 6x$$ Slope of line is 6.

FIGURE 13



See Figure 13.

SEEING THE CONCEPT: On the same square screen, graph the following equations:

$$Y_1 = 0$$ Slope of line is 0.

$$Y_2 = -\frac{1}{4}x$$ Slope of line is $-\frac{1}{4}$.

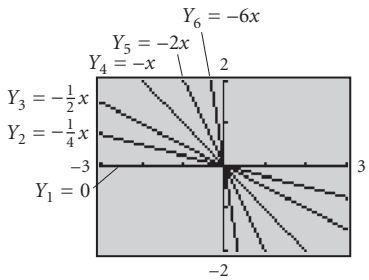
$$Y_3 = -\frac{1}{2}x$$ Slope of line is $-\frac{1}{2}$.

$$Y_4 = -x$$ Slope of line is -1.

$$Y_5 = -2x$$ Slope of line is -2.

$$Y_6 = -6x$$ Slope of line is -6.

FIGURE 14



See Figure 14.

Figures 13 and 14 illustrate that the closer the line is to the vertical position, the greater the magnitude of the slope.

The next example illustrates how the slope of a line can be used to graph the line.

EXAMPLE 8 Graphing a Line When its Slope and a Point Are Given

Draw a graph of the line that contains the point (3, 2) and has a slope of

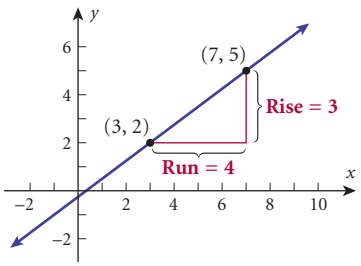
(a) $\frac{3}{4}$ **(b)** $-\frac{4}{5}$

SOLUTION **(a)** Slope $= \frac{\text{rise}}{\text{run}}$. The fact that the slope is $\frac{3}{4}$ means that for every horizontal movement (run) of 4 units to the right, there will be a vertical movement (rise) of 3 units. If we start at the given point (3, 2) and move 4 units to the right and 3 units up, we reach the point (7, 5). By drawing the line through this point and the point (3, 2), we have the graph. See Figure 15(a).

(b) The fact that the slope is $-\frac{4}{5} = \frac{-4}{5}$ means that for every horizontal movement of 5 units to the right, there will be a corresponding vertical movement of -4 units (a downward movement). If we start at the given point (3, 2) and move 5 units to the right and then 4 units down, we arrive at the point (8, -2). By drawing the line through these points, we have the graph. See Figure 15(b).

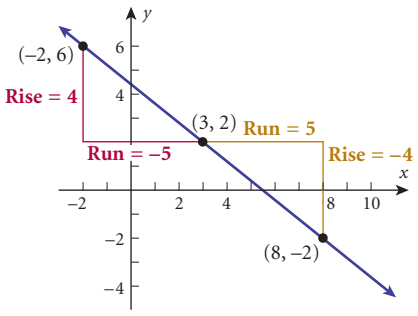
Alternatively, we can set $-\frac{4}{5} = \frac{4}{-5}$ so that for every horizontal movement of -5 units (a movement to the left), there will be a corresponding vertical movement of 4 units (upward). This approach brings us to the point $(-2, 6)$, which is also on the graph shown in Figure 15(b).

FIGURE 15



Slope $= \frac{3}{4}$

(a)

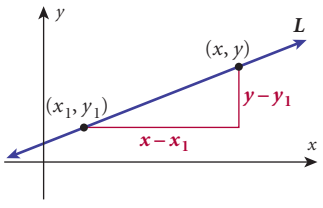


Slope $= -\frac{4}{5}$

(b)

NOW WORK PROBLEM 25.

FIGURE 16



Other Forms of the Equation of a Line

Let L be a nonvertical line with slope m and containing the point (x_1, y_1). See Figure 16. Since any two distinct points on L can be used to compute slope, for any other point (x, y) on L, we have

$$m = \frac{y - y_1}{x - x_1} \quad \text{or} \quad y - y_1 = m(x - x_1)$$

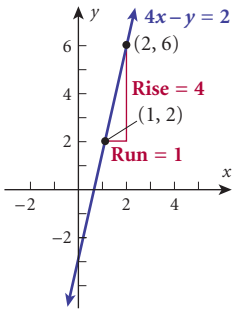
Point–Slope Form of an Equation of a Line

An equation of a nonvertical line with slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \tag{3}$$

5 EXAMPLE 9 Using the Point–Slope Form of a Line

FIGURE 17



An equation of the line with slope 4 and containing the point (1, 2) can be found by using the point-slope form with $m = 4, x_1 = 1$, and $y_1 = 2$:

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point–slope form.} \\ y - 2 &= 4(x - 1) && m = 4, x_1 = 1, y_1 = 2. \\ y - 2 &= 4x - 4 \\ 4x - y &= 2 && \text{General equation} \end{aligned}$$

See Figure 17.

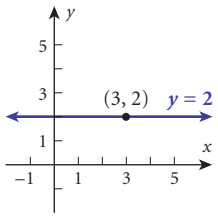
NOW WORK PROBLEM 37.

6 EXAMPLE 10 Finding the Equation of a Horizontal Line

Find an equation of the horizontal line containing the point (3, 2).

SOLUTION The slope of a horizontal line is 0. To get an equation, we use the point–slope form with $m = 0, x_1 = 3$, and $y_1 = 2$:

FIGURE 18



$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point–slope form.} \\ y - 2 &= 0 \cdot (x - 3) && m = 0, x_1 = 3, y_1 = 2. \\ y - 2 &= 0 \\ y &= 2 \end{aligned}$$

See Figure 18 for the graph.

As suggested by Example 10, we have the following result:

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b$$

where $(0, b)$ is the y-intercept.

NOW WORK PROBLEM 9(b).

7 EXAMPLE 11 Finding an Equation of a Line Given Two Points

Find an equation of the line containing the points $(2, 3)$ and $(-4, 5)$. Graph the line.

SOLUTION Since two points are given, we first compute the slope of the line:

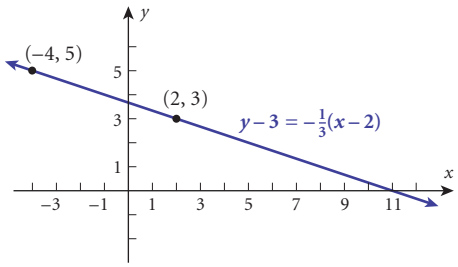
$$m = \frac{5 - 3}{-4 - 2} = \frac{2}{-6} = \frac{1}{-3} = -\frac{1}{3}$$

We use the point $(2, 3)$ and the fact that the slope $m = -\frac{1}{3}$ to get the point–slope form of the equation of the line:

$$y - 3 = -\frac{1}{3}(x - 2)$$

See Figure 19 for the graph.

FIGURE 19



NOW WORK PROBLEM 41.

In the solution to Example 11 we could have used the point $(-4, 5)$ instead of the point $(2, 3)$. The equation that results, although it looks different, is equivalent to the equation we obtained in the example. (Try it for yourself.)

The general form of the equation of the line in Example 11 can be obtained by multiplying both sides of the point–slope equation by 3 and collecting terms:

$$\begin{aligned}
y - 3 &= -\frac{1}{3}(x - 2) && \text{Point–slope equation.}\\
3(y - 3) &= 3\left(-\frac{1}{3}\right)(x - 2) && \text{Multiply by 3.}\\
3y - 9 &= -1(x - 2) && \text{Simplify}\\
3y - 9 &= -x + 2 && \text{Simplify}\\
x + 3y &= 11 && \text{General equation}
\end{aligned}$$

This is the general form of the equation of the line.

Use the slope–intercept form of a line 8

Another useful equation of a line is obtained when the slope m and y-intercept $(0, b)$ are known. In this case we know both the slope m of the line and a point $(0, b)$ on the line. Then we can use the point–slope form, Equation (3), to obtain the following equation:

$$\begin{aligned}
y - y_1 &= m(x - x_1) && \text{Point-slope form}\\
y - b &= m(x - 0) && x_1 = 0,\ y_1 = b\\
y &= mx + b && \text{Simplify and solve for } y
\end{aligned}$$

Slope–Intercept Form of an Equation of a Line

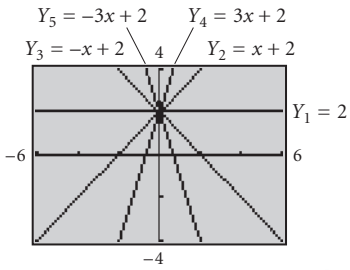
An equation of a line L with slope m and y-intercept $(0, b)$ is

$$y = mx + b \tag{4}$$

SEEING THE CONCEPT: To see the role that the slope m plays in the equation $y = mx + b$, graph the following lines on the same square screen.

$$Y_1 = 2$$
$$Y_2 = x + 2$$
$$Y_3 = -x + 2$$
$$Y_4 = 3x + 2$$
$$Y_5 = -3x + 2$$

FIGURE 20

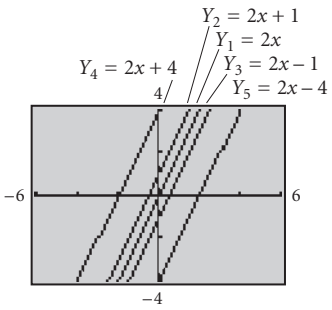


See Figure 20. What do you conclude about the lines $y = mx + 2$?

SEEING THE CONCEPT: To see the role of the y-intercept b in the equation $y = mx + b$, graph the following lines on the same square screen.

$$Y_1 = 2x$$
$$Y_2 = 2x + 1$$
$$Y_3 = 2x - 1$$
$$Y_4 = 2x + 4$$
$$Y_5 = 2x - 4$$

FIGURE 21



See Figure 21. What do you conclude about the lines $y = 2x + b$?

When an equation of a line is written in slope–intercept form, it is easy to find the slope m and y-intercept $(0, b)$ of the line. For example, suppose the equation of the line is

$$y = -2x + 3$$

Compare it to $y = mx + b$:

$$\begin{array}{l} y = -2x + 3 \\ \quad\;\uparrow \quad\uparrow \\ y = \;\; mx + b \end{array}$$

The slope of this line is -2 and its y-intercept is $(0, 3)$.

Let's look at another example.

EXAMPLE 12 Finding the Slope and y-Intercept of a Line

Find the slope m and y-intercept $(0, b)$ of the line $2x + 4y = 8$. Graph the line.

SOLUTION To obtain the slope and y-intercept, we transform the equation into its slope–intercept form. To do this, we need to solve for y:

$$\begin{aligned} 2x + 4y &= 8 \\ 4y &= -2x + 8 \\ y &= -\frac{1}{2}x + 2 \end{aligned}$$

The coefficient of x, $-\frac{1}{2}$, is the slope, and the y-intercept is $(0, 2)$.

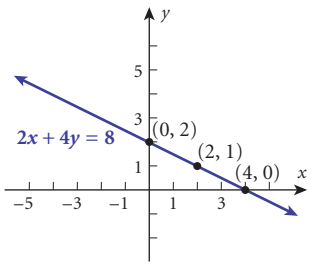
We can graph the line in either of two ways:

1. Use the fact that the y-intercept is $(0, 2)$ and the slope is $-\frac{1}{2}$. Then, starting at the point $(0, 2)$, go to the right 2 units and then down 1 unit to the point $(2, 1)$. Plot these points and draw the line containing them. See Figure 22.
2. Locate the intercepts. The y-intercept is $(0, 2)$. To obtain the x-intercept, we let $y = 0$ and solve for x. When $y = 0$, we have

$$\begin{aligned} 2x + 4 \cdot 0 &= 8 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

The intercepts are $(4, 0)$ and $(0, 2)$. Plot these points and draw the line containing them. See Figure 22.

FIGURE 22



[*Note*: The second method, locating the intercepts, only produces one point when the line passes through the origin. In this case some other point on the line must be found in order to graph the line. Refer back to Example 3.]

NOW WORK PROBLEM 59.

EXAMPLE 13 Daily Cost of Production

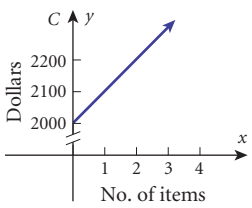
A certain factory has daily fixed overhead expenses of \$2000, while each item produced costs \$100. Find an equation that relates the daily cost C to the number x of items produced each day.

SOLUTION The fixed overhead expense of \$2000 represents the fixed cost, the cost incurred no matter how many items are produced. Since each item produced costs \$100, the variable cost of producing x items is $100x$. Then the total daily cost C of production is

$$C = 100x + 2000$$

The graph of this equation is given by the line in Figure 23. Notice that the fixed cost \$2000 is represented by the y-intercept, while the \$100 cost of producing each item is the slope. Also notice that, for convenience, a different scale is used on each axis.

FIGURE 23



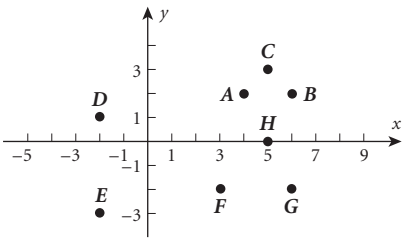
SUMMARY The graph of a linear equation, $Ax + By = C$, where A and B are not both zero, is a line. In this form it is referred to as the general equation of a line.

1. Given the general equation of a line, information can be found about the line:
   - **(a)** Place the equation in slope–intercept form $y = mx + b$ to find the slope m and y-intercept $(0, b)$.
   - **(b)** Let $x = 0$ and solve for y to find the y-intercept.
   - **(c)** Let $y = 0$ and solve for x to find the x-intercept.
2. Given information about a line, an equation of the line can be found. The form of the equation to use depends on the given information. See the table below.

Given	Use	Equation
Point (x_1, y_1), slope m	Point–slope form	$y - y_1 = m(x - x_1)$
Two points (x_1, y_1), (x_2, y_2)	If $x_1 = x_2$, the line is vertical If $x_1 \neq x_2$, find the slope m: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Then use the point–slope form	$x = x_1$ $y - y_1 = m(x - x_1)$
Slope m, y-intercept $(0, b)$	Slope–intercept form	$y = mx + b$

EXERCISE 1.1 Answers to Odd-Numbered Problems Begin on Page AN-1.

1. Give the coordinates of each point in the following figure. Assume each coordinate is an integer.



2. Plot each point in the xy-plane. Tell in which quadrant or on what coordinate axis each point lies.

(a) $A = (-3, 2)$ (b) $B = (6, 0)$
(c) $C = (-2, -2)$ (d) $D = (6, 5)$
(e) $E = (0, -3)$ (f) $F = (6, -3)$

3. Plot the points $(2, 0)$, $(2, -3)$, $(2, 4)$, $(2, 1)$, and $(2, -1)$. Describe the collection of all points of the form $(2, y)$, where y is a real number.

4. Plot the points $(0, 3)$, $(1, 3)$, $(-2, 3)$, $(5, 3)$, and $(-4, 3)$. Describe the collection of all points of the form $(x, 3)$, where x is a real number.

In Problems 5–8, use the given equations to fill in the missing values in each table. Use these points to graph each equation.

5. $y = 2x + 4$

x	0		2	−2	4	−4
y		0				

6. $y = -3x + 6$

x	0		2	−2	4	−4
y		0				

7. $2x - y = 6$

x	0		2	−2	4	−4
y		0				

8. $x + 2y = 8$

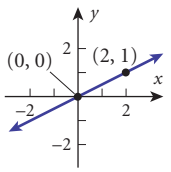
x	0		2	−2	4	−4
y		0				

In Problems 9–12: (a) find the equation of the vertical line containing the given point; (b) find the equation of the horizontal line containing the given point.

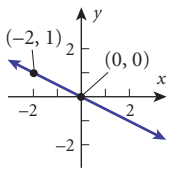
9. $(2, -3)$ **10.** $(5, 4)$ **11.** $(-4, 1)$ **12.** $(-6, -3)$

In Problems 13–16, find the slope of the line. Give an interpretation of the slope.

13.



14.



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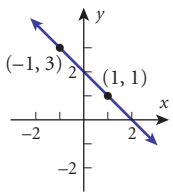

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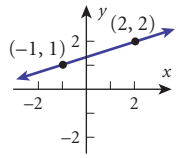


In Problems 17–24, plot each pair of points and find the slope of the line containing them. Interpret the slope and graph the line.

17. (2,3); (1,0) **18.** (1,2); (3,4) **19.** $(-2, 3); (2, 1)$ **20.** $(-1, 1); (2, 3)$

21. $(-3, -1); (2, -1)$ **22.** $(4, 2); (-5, 2)$ **23.** $(-1, 2); (-1, -2)$ **24.** $(2, 0); (2, 2)$

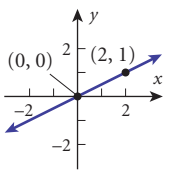
In Problems 25–32, graph the line containing the point P and having slope m.

25. $P = (1, 2); m = 2$ **26.** $P = (2, 1); m = 3$ **27.** $P = (2, 4); m = -\frac{3}{4}$ **28.** $P = (1, 3); m = -\frac{2}{3}$

29. $P = (-1, 3); m = 0$ **30.** $P = (2, -4); m = 0$ **31.** $P = (0, 3)$; slope undefined **32.** $P = (-2, 0)$; slope undefined

In Problems 33–36, write the equation of each line in the form $Ax + By = C$.

33.



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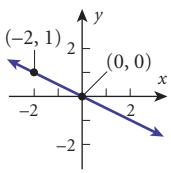
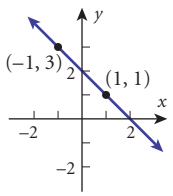
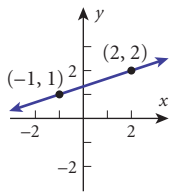


35.



36.



In Problems 37–54, write the equation of each line in the form $Ax + By = C$.

37. Slope $= 2$; containing the point $(-4, 1)$

38. Slope $= 3$; containing the point $(-3, 4)$

39. Slope $= -\frac{2}{3}$; containing the point $(1, -1)$

40. Slope $= \frac{1}{2}$; containing the point $(3, 1)$

41. Containing the points $(1, 3)$ and $(-1, 2)$

42. Containing the points $(-3, 4)$ and $(2, 5)$

43. Slope $= -2$; y-intercept $= (0, 3)$

44. Slope $= -3$; y-intercept $= (0, -2)$

45. Slope $= 3$; x-intercept $= (-4, 0)$

46. Slope $= -4$; x-intercept $= (2, 0)$

47. Slope $= \frac{4}{5}$; containing the point $(0, 0)$

48. Slope $= \frac{7}{3}$; containing the point $(0, 0)$

49. x-intercept $= (2, 0)$; y-intercept $= (0, -1)$

50. x-intercept $= (-4, 0)$; y-intercept $= (0, 4)$

51. Slope undefined; containing the point $(1, 4)$

52. Slope undefined; containing the point $(2, 1)$

53. Slope $= 0$; containing the point $(1, 4)$

54. Slope $= 0$; containing the point $(2, 1)$

In Problems 55–70, find the slope and y-intercept of each line. Graph the line.

55. $y = 2x + 3$

56. $y = -3x + 4$

57. $\frac{1}{2}y = x - 1$

58. $\frac{1}{3}x + y = 2$

59. $2x - 3y = 6$

60. $3x + 2y = 6$

61. $x + y = 1$

62. $x - y = 2$

63. $x = -4$

64. $y = -1$

65. $y = 5$

66. $x = 2$

67. $y - x = 0$

68. $x + y = 0$

69. $2y - 3x = 0$

70. $3x + 2y = 0$

---

71. Find the equation of the horizontal line containing the point $(-1, -3)$.

72. Find the equation of the vertical line containing the point $(-2, 5)$.

73. Cost of Operating a Car According to the American Automobile Association (AAA), the average cost of operating a standard-sized car, including gasoline, oil, tires, and maintenance increased to \$0.122 per mile in 2000. Write an equation that relates the average cost C of operating a standard-sized car and the number x of miles it is driven.

Source: *AAA Traveler Magazine*

74. Cost of Renting a Truck The cost of renting a truck is \$280 per week plus a charge of \$0.30 per mile driven. Write an equation that relates the cost C for a weekly rental in which the truck is driven x miles.

75. Electricity Rates in Illinois Commonwealth Edison Company supplies electricity to residential customers for a monthly customer charge of \$7.58 plus 8.275 cents per kilowatt-hour for up to 400 kilowatt-hours.

(a) Write an equation that relates the monthly charge C, in dollars, to the number x of kilowatt-hours used in a month, $0 \le x \le 400$.
(b) Graph this equation.
(c) What is the monthly charge for using 100 kilowatt-hours?
(d) What is the monthly charge for using 300 kilowatt-hours?
(e) Interpret the slope of the line.

Source: Commonwealth Edison Company, December, 2002.

76. Electricity Rates in Florida Florida Power & Light Company supplies electricity to residential customers for a monthly customer charge of \$5.25 plus 6.787 cents per kilowatt-hour for up to 750 kilowatt-hours.

(a) Write an equation that relates the monthly charge C, in dollars, to the number x of kilowatt-hours used in a month, $0 \le x \le 750$.
(b) Graph this equation.
(c) What is the monthly charge for using 200 kilowatt-hours?
(d) What is the monthly charge for using 500 kilowatt-hours?
(e) Interpret the slope of the line.

Source: Florida Power & Light Company, January 2003.

77. Weight–Height Relation in the U.S. Army Assume the recommended weight w of females aged 17-20 years in the U. S. Army is linearly related to their height h. If an Army female who is 67 inches tall should weigh 139 pounds and an Army female who is 70 inches tall should weigh 151 pounds, find an equation that expresses weight in terms of height.

Source: http://www.nutribase.com/nutrition-fwchartf.htm.

78. Wages of a Car Salesperson Dan receives \$375 per week for selling new and used cars at a car dealership in Omaha, Nebraska. In addition, he receives 5% of the profit on any sales he generates. Write an equation that relates Dan's weekly salary S when he has sales that generate a profit of x dollars.

79. Cost of Sunday Home Delivery The cost to the *Chicago Tribune* for Sunday home delivery is approximately \$0.53 per newspaper with fixed costs of \$1,070,000. Write an equation that relates the cost C and the number x of copies delivered.

Source: Chicago Tribune, 2002.

80. Disease Propagation Research indicates that in a controlled environment, the number of diseased mice will increase linearly each day after one of the mice in the cage is infected with a particular type of disease-causing germ. There were 8 diseased mice 4 days after the first exposure and 14 diseased mice after 6 days. Write an equation that will give the number of diseased mice after any given number of days. If there were 40 mice in the cage, how long will it take until they are all infected?

81. Temperature Conversion The relationship between Celsius (°C) and Fahrenheit (°F) degrees for measuring temperature is linear. Find an equation relating °C and °F if 0°C corresponds to 32°F and 100°C corresponds to 212°F. Use the equation to find the Celsius measure of 68°F.

82. Temperature Conversion The Kelvin (K) scale for measuring temperature is obtained by adding 273 to the Celsius temperature.

(a) Write an equation relating K and °C.
(b) Write an equation relating K and °F (see Problem 81).

83. Water Preservation At Harlan County Dam in Nebraska, the U.S. Bureau of Reclamation reports that the storage content of the reservoir decreased from 162,400 acre-feet (52.9 billion gallons of water) on November 8, 2002 to 161,200 acre-feet (52.5 billion gallons of water) on December 8, 2002. Suppose that the rate of loss of water remains constant.

(a) Write an equation that relates the amount A of water, in billions of gallons, to the time t, in days. Use $t = 1$ for November l, $t = 2$ for November 2, and so on.
(b) How much water was in the reservoir on November 20 ($t = 20$)?
(c) Interpret the slope.
(d) How much water will be in the reservoir on December 31, 2002 ($t = 61$)?
(e) When will the reservoir be empty?
(f) Comment on your answer to part (e).

Source: U.S. Bureau of Reclamation.

84. Product Promotion A cereal company finds that the number of people who will buy one of its products the first month it is introduced is linearly related to the amount of money it spends on advertising. If it spends $400,000 on advertising, 100,000 boxes of cereal will be sold, and if it spends $600,000, 140,000 boxes will be sold.

(a) Write an equation describing the relation between the amount spent on advertising and the number of boxes sold.
(b) How much advertising is needed to sell 200,000 boxes of cereal?
(c) Interpret the slope.

In Problems 85–92, use a graphing utility to graph each linear equation. Be sure to use a viewing rectangle that shows the intercepts. Then locate each intercept rounded to two decimal places.

85. $1.2x + 0.8y = 2$

86. $-1.3x + 2.7y = 8$

87. $21x - 15y = 53$

88. $5x - 3y = 82$

89. $\frac{4}{17}x + \frac{6}{23}y = \frac{2}{3}$

90. $\frac{9}{14}x - \frac{3}{8}y = \frac{2}{7}$

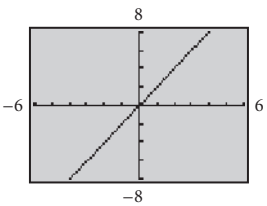
91. $\pi x - \sqrt{3}\,y = \sqrt{6}$

92. $x + \pi y = \sqrt{15}$

In Problems 93–96, match each graph with the correct equation:

(a) $y = x$; (b) $y = 2x$; (c) $y = \frac{x}{2}$; (d) $y = 4x$.

93.



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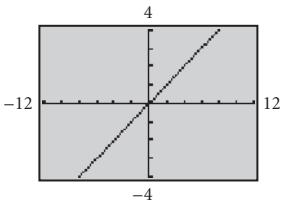

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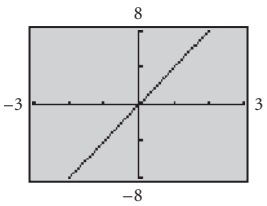

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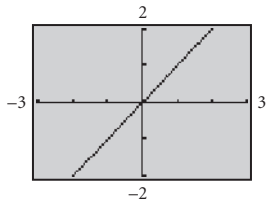
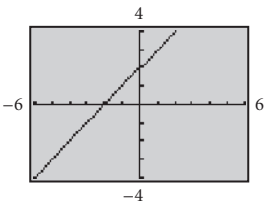


In Problems 97–100, write an equation of each line. Express your answer using either the general form or the slope-intercept form of the equation of a line, whichever you prefer.

97.



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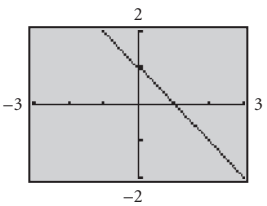
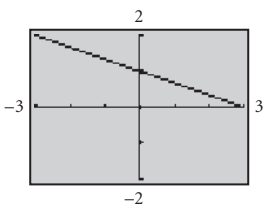
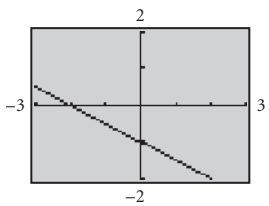


99.

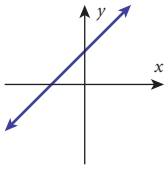


100.



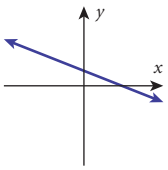
101. Which of the following equations might have the graph shown. (More than one answer is possible.)

(a) $2x + 3y = 6$
(b) $-2x + 3y = 6$
(c) $3x - 4y = -12$
(d) $x - y = 1$
(e) $x - y = -1$
(f) $y = 3x - 5$
(g) $y = 2x + 3$
(h) $y = -3x + 3$



102. Which of the following equations might have the graph shown. (More than one answer is possible.)

(a) $2x + 3y = 6$
(b) $2x - 3y = 6$
(c) $3x + 4y = 12$
(d) $x - y = 1$
(e) $x - y = -1$
(f) $y = -2x + 1$
(g) $y = -\frac{1}{2}x + 10$
(h) $y = x + 4$



103. Write the general equation of the x-axis.

104. Write the general equation of the y-axis.

105. Which form of the equation of a line do you prefer to use? Justify your position with an example that shows that your choice is better than another. Have reasons.

106. Can every line be written in slope–intercept form? Explain.

107. Does every line have two distinct intercepts? Explain. Are there lines that have no intercepts? Explain.

108. What can you say about two lines that have equal slopes and equal y-intercepts?

109. What can you say about two lines with the same x-intercept and the same y-intercept? Assume that the x-intercept is not $(0, 0)$.

110. If two lines have the same slope, but different x-intercepts, can they have the same y-intercept?

111. If two lines have the same y-intercept, but different slopes, can they have the same x-intercept? What is the only way that this can happen?

112. The accepted symbol used to denote the slope of a line is the letter m. Investigate the origin of this symbolism. Begin by consulting a French dictionary and looking up the French word *monter*. Write a brief essay on your findings.

1.2 Pairs of Lines

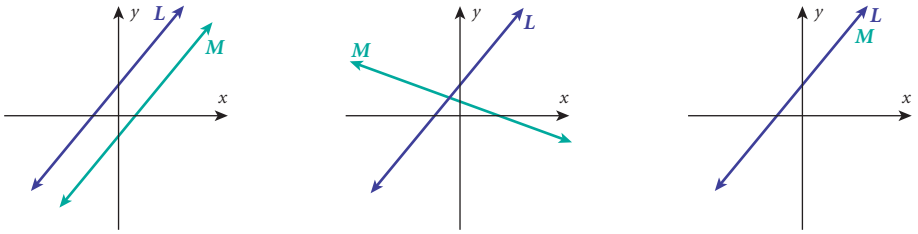
OBJECTIVES
1. Show that two lines are coincident
2. Show that two lines are parallel
3. Show that two lines intersect
4. Find the point of intersection of two intersecting lines
5. Show that two lines are perpendicular

Consider two lines L and M in the same plane. Then exactly one of the following three circumstances must hold:

1. They have no points in common, in which case they are **parallel.**
2. They have one point in common, in which case they **intersect.**
3. They have two points in common, in which case they are **coincident** or **identical,** and all the points on L are the same as the points on M.

Figure 24 illustrates the three circumstances.

FIGURE 24



Parallel Lines: No points in common

(a)

Intersecting Lines: One point in common

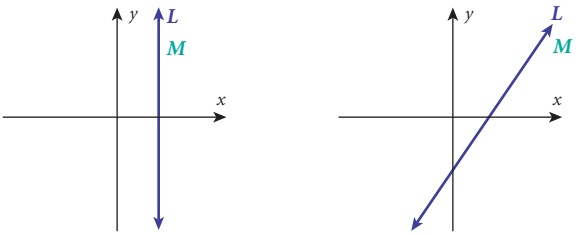
(b)

Coincident Lines: All points on L are the same as the points on M.

(c)

Figure 25 illustrates coincident lines that are vertical and nonvertical.

FIGURE 25



Coincident lines that are vertical

(a)

Coincident lines that are nonvertical

(b)

We are led to the following result.

Coincident Lines

Coincident lines that are vertical have undefined slope and the same x-intercept.
Coincident lines that are nonvertical have the same slope and the same intercepts.

Show that two lines are coincident 1

To show that two nonvertical lines are coincident only requires that you show they have the same slope and the same y-intercept. Do you see why?

EXAMPLE 1 **Showing that Two Lines Are Coincident**

Show that the lines given by the equations below are coincident.

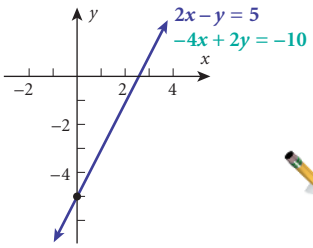
$$L\colon\ 2x - y = 5 \qquad M\colon\ -4x + 2y = -10$$

SOLUTION We put each equation into slope–intercept form:

$$\begin{aligned} L:\ 2x - y &= 5 & \qquad M:\ -4x + 2y &= -10 \\ -y &= -2x + 5 & 2y &= 4x - 10 \\ y &= 2x - 5 & y &= 2x - 5 \end{aligned}$$

The lines L and M have the same slope 2 and the same y-intercept $(0, -5)$ so they are coincident. See Figure 26.

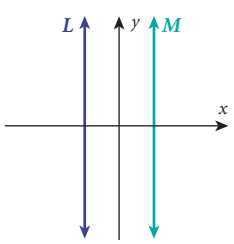
FIGURE 26



NOW WORK PROBLEM 5.

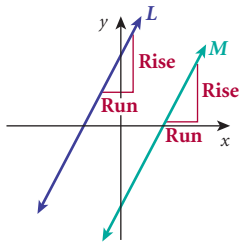
Next, consider the parallel lines in Figure 27.

FIGURE 27



Parallel lines that are vertical

(a)



Parallel lines that are nonvertical

(b)

We see in Figure 27(a) that the two vertical parallel lines have different x-intercepts. For the two nonvertical parallel lines in Figure 27(b), equal runs result in equal rises. As a result, nonvertical parallel lines have the same slope. Since they also have no points in common, they will have different x- and y-intercepts.

Parallel Lines

Parallel lines that are vertical have undefined slope and different x-intercepts.
Parallel lines that are nonvertical have the same slope and different intercepts.

Show that two lines are parallel 2

To show that two nonvertical lines are parallel only requires that you show they have the same slope and different y-intercepts. Do you see why?

EXAMPLE 2 **Showing that Two Lines Are Parallel**

Show that the lines given by the equations below are parallel.

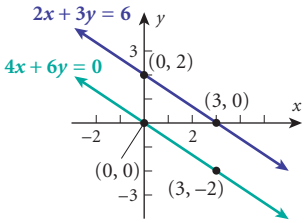
$$L:\ 2x + 3y = 6 \qquad M:\ 4x + 6y = 0$$

SOLUTION To see if these lines have equal slopes, we put each equation into slope–intercept form:

$$L:\ 2x + 3y = 6 \qquad M:\ 4x + 6y = 0$$
$$3y = -2x + 6 \qquad 6y = -4x$$
$$y = -\frac{2}{3}x + 2 \qquad y = -\frac{2}{3}x$$
$$\text{Slope} = -\frac{2}{3} \qquad \text{Slope} = -\frac{2}{3}$$
$$y\text{-intercept} = (0, 2) \qquad y\text{-intercept} = (0, 0)$$

Since each has slope $-\frac{2}{3}$ but different y-intercepts, the lines are parallel. See Figure 28.

FIGURE 28



NOW WORK PROBLEM 1.

If two lines L and M have exactly one point in common, the common point is called the **point of intersection.** The slopes of intersecting lines are unequal. Do you see why?

Intersecting Lines

Intersecting lines have different slopes.

3 EXAMPLE 3 Showing that Two Lines Intersect

Show that the lines given by the equations below intersect.

$$L:\ 2x - y = 5 \qquad M:\ x + y = 4$$

SOLUTION We put each equation into slope–intercept form.

$$L:\ 2x - y = 5 \qquad M:\ x + y = 4$$
$$-y = -2x + 5 \qquad y = -x + 4$$
$$y = 2x - 5$$

The slope of L is 2 and the slope of M is -1, so the lines intersect.

NOW WORK PROBLEM 3.

4 EXAMPLE 4 Finding the Point of Intersection

Find the point of intersection of the two lines

$$L:\ 2x - y = 5 \qquad M:\ x + y = 4$$

SOLUTION The slope–intercept form of each line, as found in Example 3, is

$$L:\ y = 2x - 5 \qquad M:\ y = -x + 4$$

If (x_0, y_0) denotes the point of intersection, then (x_0, y_0) is a point on both L and M. As a result, we must have

$$y_0 = 2x_0 - 5 \qquad \text{and} \qquad y_0 = -x_0 + 4$$

We set these equal and solve for x_0.

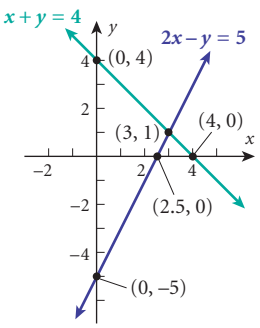
$$2x_0 - 5 = -x_0 + 4$$
$$3x_0 = 9$$
$$x_0 = 3$$

Substituting $x_0 = 3$ in $y_0 = 2x_0 - 5$ (or in $y_0 = -x_0 + 4$), we find

$$y_0 = 2x_0 - 5 = 2(3) - 5 = 1$$

The point of intersection of L and M is $(3, 1)$. See Figure 29.

FIGURE 29



Check: We verify that the point $(3, 1)$ is on both L and M.

$$L\text{: } 2x - y = 2(3) - 1 = 6 - 1 = 5 \qquad M\text{: } x + y = 3 + 1 = 4$$

NOW WORK PROBLEM 13.

Perpendicular Lines

When two lines intersect and form a right angle, they are said to be **perpendicular.** For example, a vertical line and a horizontal line are perpendicular.

Perpendicular Lines

Two nonvertical lines L and M with slopes m_1 and m_2, respectively, are perpendicular if and only if $m_1 m_2 = -1$, that is, if and only if the product of their slopes is -1.

EXAMPLE 5 **Showing that Two Lines Are Perpendicular**

Show that the lines given by the equations below are perpendicular.

$$L\text{: } x - 2y = 6 \qquad M\text{: } 2x + y = 1$$

SOLUTION To see if these lines are perpendicular, find the slope of each:

$$L\text{: } x - 2y = 6 \qquad M\text{: } 2x + y = 1$$
$$-2y = -x + 6 \qquad y = -2x + 1$$
$$y = \frac{1}{2}x - 3$$

$$\text{Slope} = m_1 = \frac{1}{2} \qquad \text{Slope} = m_2 = -2$$

Since $m_1 m_2 = \frac{1}{2} \cdot (-2) = -1$, the lines are perpendicular.

NOW WORK PROBLEM 25.

EXAMPLE 6 **Finding the Equation of Two Lines: One Parallel to and the Other Perpendicular to a Given Line**

Given the line $x - 4y = 8$, find an equation for the line that contains the point $(2, 1)$ and is

(a) Parallel to the given line **(b)** Perpendicular to the given line

SOLUTION First find the slope of the line $x - 4y = 8$ by putting it in slope–intercept form $y = mx + b$:

$$\begin{aligned} x - 4y &= 8 \\ -4y &= -x + 8 \\ y &= \frac{1}{4}x - 2 \qquad y = mx + b;\ m = \frac{1}{4};\ b = -2 \end{aligned}$$

The slope of the line is $\frac{1}{4}$.

(a) We seek a line parallel to the given line that contains the point $(2, 1)$. The slope of this line must be $\frac{1}{4}$. (Do you see why?) Using the point–slope form of the equation of a line, we have

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= \frac{1}{4}(x - 2) \qquad m = \frac{1}{4}, x_1 = 2, y_1 = 1 \\ y &= \frac{1}{4}x + \frac{1}{2} \end{aligned}$$

(b) We seek a line perpendicular to the given line, whose slope is $\frac{1}{4}$. The slope m of this line obeys

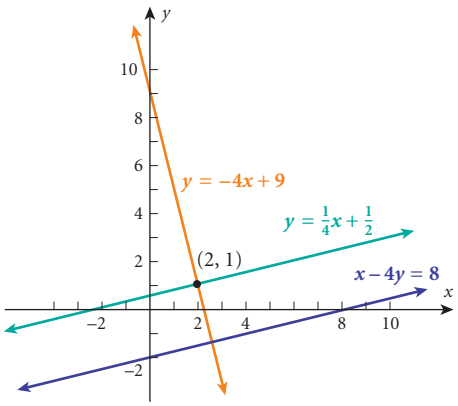
$$\begin{aligned} m \cdot \frac{1}{4} &= -1 \qquad \text{Product of the slopes is } -1 \\ m &= -4 \qquad \text{Solve for } m. \end{aligned}$$

Since the line we seek contains the point $(2, 1)$, we have

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= -4(x - 2) \qquad m = -4; x_1 = 2, y_1 = 1 \\ y &= -4x + 9 \end{aligned}$$

Figure 30 illustrates these solutions.

FIGURE 30



NOW WORK PROBLEMS 35 AND 41.

SUMMARY

Pair of Lines	*Conclusion: The lines are*
Both vertical	(a) Coincident, if they have the same x-intercept (b) Parallel, if they have different x-intercepts

One vertical, one nonvertical	(a) Intersecting (b) Perpendicular, if the nonvertical line is horizontal
Neither vertical	Write the equation of each line in slope–intercept form: $y = m_1x + b_1, \quad y = m_2x + b_2$ (a) Coincident, if $m_1 = m_2, b_1 = b_2$ (b) Parallel, if $m_1 = m_2, b_1 \neq b_2$ (c) Intersecting, if $m_1 \neq m_2$ (d) Perpendicular, if $m_1m_2 = -1$

EXERCISE 1.2 Answers to Odd-Numbered Problems Begin on Page AN-3.

In Problems 1–12, determine whether the given pairs of lines are parallel, coincident, or intersecting.

1. $L: \ x + y = 10$
$M: \ 3x + 3y = 6$

2. $L: \ x - y = 5$
$M: \ -2x + 2y = 8$

3. $L: \ 2x + y = 4$
$M: \ 2x - y = 8$

4. $L: \ 2x + y = 8$
$M: \ 2x - y = -4$

5. $L: \ -x + y = 2$
$M: \ 2x - 2y = -4$

6. $L: \ x + y = -4$
$M: \ 3x + 3y = -12$

7. $L: \ 2x - 3y = -8$
$M: \ 6x - 9y = -2$

8. $L: \ 4x - 2y = -7$
$M: \ -2x + y = -2$

9. $L: \ 3x - 4y = 1$
$M: \ x - 2y = -4$

10. $L: \ 4x + 3y = 2$
$M: \ 2x - y = -1$

11. $L: \ x = 3$
$M: \ y = -2$

12. $L: \ x = 4$
$M: \ x = -2$

In Problems 13–24, the given pairs of lines intersect. Find the point of intersection. Graph each pair of lines.

13. $L: \ x + y = 5$
$M: \ 3x - y = 7$

14. $L: \ 2x + y = 7$
$M: \ x - y = -4$

15. $L: \ x - y = 2$
$M: \ 2x + y = 7$

16. $L: \ 2x - y = -1$
$M: \ x + y = 4$

17. $L: \ 4x + 2y = 4$
$M: \ 4x - 2y = 4$

18. $L: \ 4x - 2y = 8$
$M: \ 6x + 3y = 0$

19. $L: \ 3x - 4y = 2$
$M: \ x + 2y = 4$

20. $L: \ 4x + 3y = 2$
$M: \ 2x - y = 1$

21. $L: \ 3x - 2y = -5$
$M: \ 3x + y = -2$

22. $L: \ 4x + y = 6$
$M: \ 4x - 2y = 0$

23. $L: \ x = 4$
$M: \ y = -2$

24. $L: \ x = 0$
$M: \ y = 0$

In Problems 25–30, show that the lines are perpendicular.

25. $-x + 3y = 2$
$6x + 2y = 5$

26. $2x + 3y = 4$
$9x - 6y = 1$

27. $x + 2y = 7$
$-2x + y = 15$

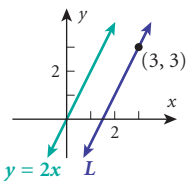
28. $4x + 12y = 3$
$15x - 5y = -1$

29. $3x + 12y = 2$
$4x - y = -2$

30. $20x - 2y = -7$
$x + 10y = 8$

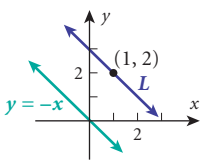
In Problems 31–34, find an equation for the line L. Express the answer using the general form or the slope–intercept form, whichever you prefer.

31.



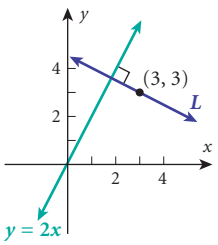
L is parallel to $y = 2x$

32.



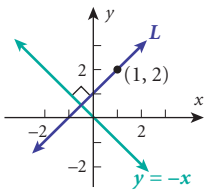
L is parallel to $y = -x$

33.



L is perpendicular to $y = 2x$

34.



L is perpendicular to $y = -x$

In Problems 35–44, find an equation for the line with the given properties. Express the answer using the general form or the slope–intercept form, whichever you prefer.

35. Parallel to the line $y = 4x$; containing the point $(-1, 2)$

36. Parallel to the line $y = -3x$; containing the point $(-1, 2)$

37. Parallel to the line $2x - y = -2$; containing the point $(0, 0)$

38. Parallel to the line $x - 2y = -5$; containing the point $(0, 0)$

39. Parallel to the line $x = 3$; containing the point $(4, 2)$

40. Parallel to the line $y = 3$; containing the point $(4, 2)$

41. Perpendicular to the line $y = 2x - 5$; containing the point $(-1, -2)$

42. Perpendicular to the line $6x - 2y = -5$; containing the point $(-1, -2)$

43. Perpendicular to the line $y = 2x - 5$; containing the point $\left(-\frac{1}{3}, \frac{4}{5}\right)$

44. Perpendicular to the line $y = 3x - 15$; containing the point $\left(-\frac{2}{3}, \frac{3}{5}\right)$

45. Find t so that $tx - 4y = -3$ is perpendicular to the line $2x + 2y = 5$.

46. Find t so that $(1, 2)$ is a point on the line $tx - 3y = -4$.

47. Find the equation of the line containing the point $(-2, -5)$ and perpendicular to the line containing the points $(-2, 9)$ and $(3, -10)$.

48. Find the equation of the line containing the point $(-2, -5)$ and perpendicular to the line containing the points $(-4, 5)$ and $(2, -1)$.

49. The figure below shows the graph of two parallel lines. Which of the following pairs of equations might have such a graph?

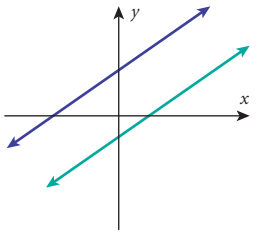
(a) $x - 2y = 3$
$x + 2y = 7$

(b) $x + y = 2$
$x + y = -1$

(c) $x - y = -2$
$x - y = 1$

(d) $x - y = -2$
$2x - 2y = -4$

(e) $x + 2y = 2$
$x + 2y = -1$



50. The figure below shows the graph of two perpendicular lines. Which of the following pairs of equations might have such a graph?

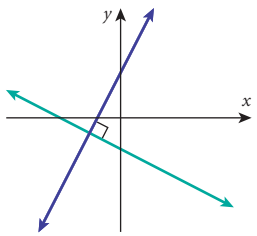
(a) $y - 2x = 2$
$y + 2x = -1$

(b) $y - 2x = 0$
$2y + x = 0$

(c) $2y - x = 2$
$2y + x = -2$

(d) $y - 2x = 2$
$x + 2y = -1$

(e) $2x + y = -2$
$2y + x = -2$



1.3 Applications: Prediction; Break-Even Point; Mixture Problems; Economics

OBJECTIVE 1 Solve applied problems

Solve applied problems 1

In this section we present four applied situations in which linear equations are used. These subsections are independent of each other.

Prediction

Linear equations are sometimes used as predictors of future results. Let's look at an example.

EXAMPLE 1 Predicting the Cost of a Home

In 1999 the cost of an average home in Chicago was \$170,100. In 2000 the cost was \$173,600. Assuming that the relationship between time and cost is linear, develop a formula for predicting the cost of an average home in 2004.

Source: National Association of Realtors.

SOLUTION We agree to let x represent the year and y represent the cost. We seek a relationship between x and y. Two points on the graph of the equation relating x and y are

$$(1999, 170100) \quad \text{and} \quad (2000, 173600)$$

The assumption is that the equation relating x and y is linear. The slope of this line is

$$\frac{173{,}600 - 170{,}100}{2000 - 1999} = 3500$$

Using this fact and the point (1999, 170100), the point–slope form of the equation of the line is

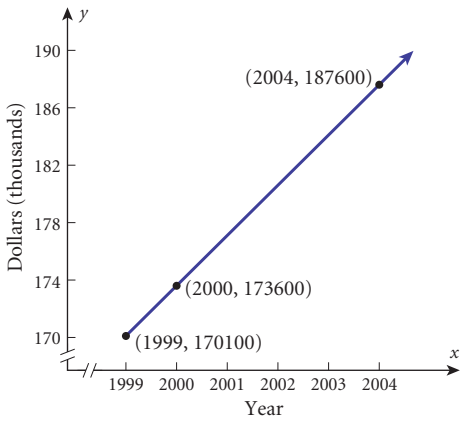
$$y - 170{,}100 = 3500(x - 1999)$$
$$y = 170{,}100 + 3500(x - 1999)$$

For $x = 2004$ we predict the cost of an average home to be

$$\begin{aligned} y &= 170{,}100 + 3500(x - 1999) \\ &= 170{,}100 + 3500(2004 - 1999) \\ &= 170{,}100 + 3500(5) \\ &= \$187{,}600 \end{aligned}$$

Figure 31 illustrates the situation.

FIGURE 31



This prediction of future cost is based on the assumption that annual increases remain the same. In this example, the assumption is that each year the cost of a house will go up \$3500 (the slope of the line). Of course, if this assumption is not correct, the predicted cost will also not be correct. ◗

NOW WORK PROBLEM 1.

Break-Even Point

In many businesses the cost C of production and the number x of items produced can be expressed as a linear equation. Similarly, sometimes the revenue R obtained from sales and the number x of items produced can be expressed as a linear equation. When the cost C of production exceeds the revenue R from the sales, the business is operating at a loss; when the revenue R exceeds the cost C, there is a profit; and when the revenue R and the cost C are equal, there is no profit or loss. The point at which $R = C$, that is, the point of intersection of the two lines, is usually referred to as the **break-even point** in business.

EXAMPLE 2 Finding the Break-Even Point

Sweet Delight Candies, Inc., has daily fixed costs from salaries and building operations of \$300. Each pound of candy produced costs \$1 and is sold for \$2.

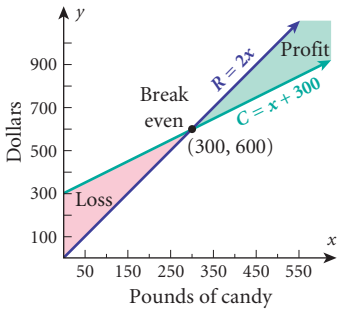
(a) Find the cost C of production for x pounds of candy.
(b) Find the revenue R from selling x pounds of candy.
(c) What is the break-even point? That is, how many pounds of candy must be sold daily to guarantee no loss and no profit?
(d) Graph C and R and label the break-even point.

SOLUTION

(a) The cost C of production is the fixed cost of \$300 plus the variable cost of producing x pounds of candy at \$1 per pound. That is,

$$C = \$1 \cdot x + \$300 = x + 300$$

FIGURE 32



(b) The revenue R realized from the sale of x pounds of candy at \$2 per pound is

$$R = \$2 \cdot x = 2x$$

(c) The break-even point is the point where $R = C$. Setting $R = C$, we find

$$2x = x + 300$$
$$x = 300$$

That is, 300 pounds of candy must be sold to break even.

(d) Figure 32 shows the graphs of C and R and the break-even point. Notice that for $x > 300$, the revenue R always exceeds the cost C so that a profit results. Similarly, for $x < 300$, the cost exceeds the revenue, resulting in a loss.

EXAMPLE 3 Analyzing Break-Even Points

After negotiations with employees of Sweet Delight Candies and an increase in the price of sugar, the daily cost C of production for x pounds of candy is

$$C = \$1.05x + \$330$$

(a) If each pound of candy is sold for \$2.00, how many pounds must be sold daily to make a profit?
(b) If the selling price is increased to \$2.25 per pound, what is the break-even point?

(c) If it is known that 325 pounds of candy can be sold daily, what price should be charged per pound to guarantee no loss?

SOLUTION **(a)** If each pound is sold for \$2.00, the revenue R from sales is

$$R = \$2x$$

where x represents the number of pounds sold. When we set $R = C$, we find that the break-even point is the solution of the equation

$$\begin{aligned} 2x &= 1.05x + 330 \\ 0.95x &= 330 \\ x &= \frac{330}{0.95} = 347.37 \end{aligned}$$

If 347 pounds or less of candy are sold, a loss is incurred; if 348 pounds or more are sold, a profit results.

(b) If the selling price is increased to \$2.25 per pound, the revenue R from sales is

$$R = \$2.25x$$

The break-even point is the solution of the equation

$$\begin{aligned} 2.25x &= 1.05x + 330 \\ 1.2x &= 330 \\ x &= \frac{330}{1.2} = 275 \end{aligned}$$

With the new selling price, the break-even point is 275 pounds.

(c) If we know that 325 pounds of candy will be sold daily, the price per pound p needed to guarantee no loss (that is, to guarantee at worst a break-even point) is the solution of the equation

$$\begin{aligned} R &= C \\ 325p &= (1.05)(325) + 330 \\ 325p &= 671.25 \\ p &= \$2.07 \end{aligned}$$

We should charge \$2.07 per pound to guarantee no loss, provided at least 325 pounds will be sold. ◗

EXAMPLE 4 Analyzing Break-Even Points

A producer sells items for \$0.30 each.

(a) Determine the revenue R from selling x items.

(b) If the cost for production is

$$C_1 = \$0.15x + \$105$$

where x is the number of items sold, find the break-even point.

(c) If the cost can be changed to

$$C_2 = \$0.12x + \$110$$

would it be advantageous?

(d) Graph R, C_1, and C_2 together.

SOLUTION **(a)** The revenue R from selling x items is

$$R = \$0.30x$$

(b) If the cost for production is $C_1 = \$0.15x + \$105$, then the break-even point is the solution of the equation

$$\begin{aligned} 0.3x &= 0.15x + 105 \quad R = C_1 \\ 0.15x &= 105 \\ x &= 700 \end{aligned}$$

The break-even point is 700 items.

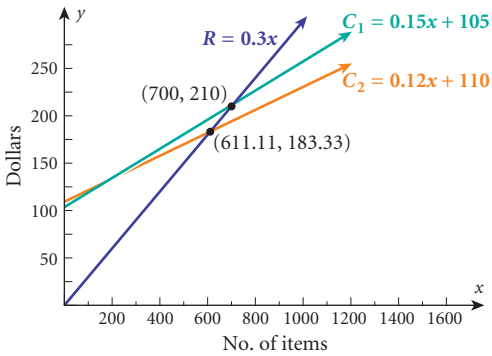
(c) If the revenue received remains at $R = \$0.3x$, but the cost for production changes to $C_2 = \$0.12x + \$110$, then the break-even point is the solution of the equation

$$\begin{aligned} 0.3x &= 0.12x + 110 \quad R = C_2. \\ 0.18x &= 110 \\ x &= 611.11 \end{aligned}$$

The break-even point for the cost in (a) was 700 items. Since the cost in (b) will require fewer items to be sold in order to break even, management should probably change over to the new cost.

(d) Figure 33 shows the graphs of R, C_1, and C_2.

FIGURE 33



NOW WORK PROBLEM 15.

Mixture Problems

Oil refineries sometimes produce gasoline that is a blend of two other fuels; special blend coffees are created by mixing two other flavors of coffee. These problems are referred to as **mixture problems** because they combine two or more quantities to form a mixture.

EXAMPLE 5 **Mixing Peanuts**

A store that specializes in selling nuts sells cashews for \$5 per pound and peanuts for \$2 per pound. At the end of the month the manager finds that the peanuts are not selling

well. In order to sell 30 pounds of peanuts more quickly, the manager decides to mix the 30 pounds of peanuts with some cashews and sell the mixture of peanuts and cashews for \$3 a pound. How many pounds of cashews should be mixed with the peanuts so that the revenue remains the same?

SOLUTION There are two unknowns: the number of pounds of cashews (call this x) and the number of pounds of the mixture (call this y). Since we know that the number of pounds of cashews plus 30 pounds of peanuts equals the number of pounds of the mixture, we can write

$$y = x + 30$$

Also, in order to keep revenue the same, we must have

$$\begin{pmatrix}\text{Price}\\ \text{per}\\ \text{pound}\\ \$5\end{pmatrix}\cdot\begin{pmatrix}\text{Pounds}\\ \text{of}\\ \text{cashews}\\ x\end{pmatrix}+\begin{pmatrix}\text{Price}\\ \text{per}\\ \text{pound}\\ \$2\end{pmatrix}\cdot\begin{pmatrix}\text{Pounds}\\ \text{of}\\ \text{peanuts}\\ 30\end{pmatrix}=\begin{pmatrix}\text{Price}\\ \text{per}\\ \text{pound}\\ \$3\end{pmatrix}\cdot\begin{pmatrix}\text{Pounds}\\ \text{of}\\ \text{mixture}\\ y\end{pmatrix}$$

That is,

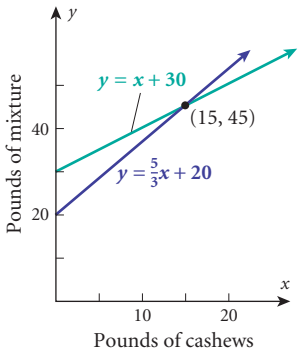
$$5x + 2(30) = 3y$$

$$\frac{5}{3}x + 20 = y \qquad \text{Divide both sides by 3}$$

We now have two equations

$$y = x + 30 \qquad \text{and} \qquad y = \frac{5}{3}x + 20$$

FIGURE 34



Since the number of pounds of the mixture is the same in each case, we have

$$\frac{5}{3}x + 20 = x + 30$$

$$\frac{2}{3}x = 10$$

$$x = 15$$

The manager should mix 15 pounds of cashews with 30 pounds of peanuts. See Figure 34. Notice that the point of intersection (15, 45) represents the pounds of cashews (15) in the mixture (45 pounds).

NOW WORK PROBLEM 17.

Certain types of problems that involve investments in two interest-bearing accounts can be solved as a mixture problem.

EXAMPLE 6 Financial Planning

Kathleen has \$80,000 to invest and requires an overall rate of return of 5% per year. She can invest in a safe, government-insured Certificate of Deposit, but it only pays 3% per year. To obtain 5%, she agrees to invest some of her money in noninsured corporate bonds paying 6% per year. How much should be placed in each investment to achieve her goals?

SOLUTION The question is asking how Kathleen should split her money between the two investments in order for her to realize her goal of earning a 5% rate of return on her money.

We need two dollar amounts: the amount (called the principal) to invest in corporate bonds (call this x) and the principal to invest in the Certificate of Deposit (call this y). Since we know that the amount she invests in corporate bonds plus the amount she invests in the Certificate of Deposit equals \$80,000, we can write

$$x + y = \$80{,}000.$$

Solving this for y, we get

$$y = \$80{,}000 - x$$

which is the amount that will be invested in the Certificate of Deposit. See Table 1.

TABLE 1

	Principal \$	Rate	Time yr	Interest \$
Bonds	x	6% = 0.06	1	$0.06x$
Certificate	$80{,}000 - x$	3% = 0.03	1	$0.03(80{,}000 - x)$
Total	80,000	5% = 0.05	1	$0.05(80{,}000) = 4000$

Since the total interest from the investments is equal to $0.05(\$80{,}000) = \$4000$, the equation relating the interest earned on the accounts is given as:

interest earned on the bonds + interest earned on the certificate = total interest earned

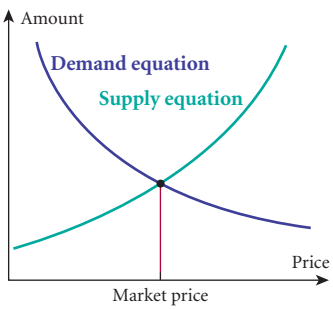
$$0.06x + 0.03(80{,}000 - x) = 4000$$

(Note that the units are consistent: the unit is dollars on both sides of the equation.) Now we solve the equation for x, the amount invested in corporate bonds.

$$\begin{aligned} 0.06x + 2400 - 0.03x &= 4000 \\ 0.03x + 2400 &= 4000 \\ 0.03x &= 1600 \\ x &= 53{,}333.33 \end{aligned}$$

Kathleen should invest \$53,333.33 in corporate bonds and \$80,000 − \$53,333.33 = \$26,666.67 in the Certificate of Deposit.

FIGURE 35



Economics

The **supply equation** in economics is used to specify the amount of a particular commodity that sellers are willing to offer in the market at various prices. The **demand equation** specifies the amount of a particular commodity that buyers are willing to purchase at various prices.

An increase in price p usually causes an increase in the supply S and a decrease in demand D. On the other hand, a decrease in price brings about a decrease in supply and an increase in demand. The **market price** is defined as the price at which supply and demand are equal (the point of intersection).

Figure 35 illustrates a typical supply/demand situation.

EXAMPLE 7 Supply and Demand

The supply and demand for flour have been estimated as being given by the equations

$$S = 0.8p + 0.5 \qquad D = -0.4p + 1.5$$

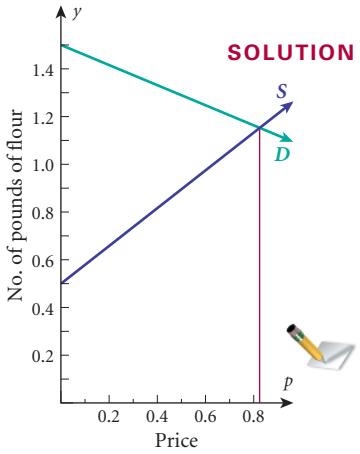
where p is measured in dollars and S and D are measured in pound units of flour. Find the market price and graph the supply and demand equations.

SOLUTION The market price p is the solution of the equation

$$S = D$$
$$0.8p + 0.5 = -0.4p + 1.5$$
$$1.2p = 1$$
$$p = 0.83$$

At a price of \$0.83 per pound, supply and demand for flour are equal. The graphs are shown in Figure 36.

FIGURE 36



NOW WORK PROBLEM 27.

EXERCISE 1.3 Answers to Odd-Numbered Problems Begin on Page AN-4.

Problems 1–8 involve prediction.

1. **Predicting Sales** Suppose the sales of a company are given by
$$S = \$5000x + \$80{,}000$$
where x is measured in years and $x = 0$ corresponds to the year 2002.
   (a) Find S when $x = 0$.
   (b) Find S when $x = 3$.
   (c) Find the predicted sales in 2007, assuming this trend continues.
   (d) Find the predicted sales in 2010, assuming this trend continues.

2. **Predicting Sales** Rework Problem 1 if the sales of the company are given by
$$S = \$3000x + \$60{,}000$$

3. **Predicting the Cost of a Compact Car** In 2000, the cost of a compact car averaged \$12,500. In 2003, the cost of a compact car averaged \$14,450. Assuming that the relationship between time and cost is linear, develop a formula for predicting the average cost of a compact car in the future. What do you predict the average cost of a compact car will be in 2005? Interpret the slope.

4. **Oil Depletion** The Yates Oil Field, discovered in 1926 in Texas, through 1999 has produced 1.4 billion barrels of oil. In 1999, the field had an estimated reserve of 600 million barrels. Based on previous production levels, in 2002 the field should have had an estimated reserve of 540 million barrels. Now assume the rate of depletion is constant.
   (a) Write an equation that relates the amount A, in millions of barrels, of oil left in the field at any time t, where t is the year.
   (b) If the trend continues, when will the oil well dry out?
   (c) Interpret the slope.

   Source: Energy Information Administration, Department of Energy.

5. **SAT Scores** The average score on the mathematics portion of the SAT has been steadily increasing for South Carolina students over the past 15 years. In 1987 the average SAT mathematics score was 468, while in 2001 the average SAT mathematics score was 488. Assume the rate of increase is constant.
   (a) Write an equation that relates the average SAT mathematics score S at any time t, where t is the year.
   (b) If the trend continues, what will the average SAT mathematics score of South Carolina students be in 2004?

   Source: Digest of Education Statistics, 2001, National Center for Education Statistics.

6. **Financial Statements** In March 2001, Kellogg Co. purchased Keebler Foods Co. After the purchase, revenue increased from \$1.99 billion at the end of the first financial quarter in 2002 to \$2.13 billion at the end of the second financial quarter in 2002. Now assume the rate of increase is constant.
   (a) Write an equation that relates the revenue R, in billions of dollars, and the time t. Use $t = 1$ for the first quarter of 2002, $t = 2$ for the second quarter, and so on.
   (b) What is the projected revenue for the fourth quarter?
   (c) Interpret the slope.

   Source: Kellogg Company Annual Report, 2002.

7. **Percent of Population with Bachelor's Degrees** In 1990 the percent of people over 25 years old who had a bachelor's degree or higher was 20.3%. By 2000, the percent of people over 25 with a bachelor's degree or higher was 25.6%. Assume the rate of increase is constant.

   (a) Write an equation that relates the percent P of people over 25 with a bachelor's degree or higher at any time t, where t is the year.
   (b) If the trend continues, estimate the percentage of people over 25 who will have a bachelor's degree or higher by 2004.
   (c) Interpret the slope.

   Source: Digest of Education Statistics, 2001, National Center for Education Statistics.

8. **College Degrees** In 1993 1,159,931 bachelor's degrees were conferred by colleges and universities in the United States. In 1997 1,168,023 bachelor's degrees were awarded. Suppose we assume the relationship between time and degrees conferred is linear.

   (a) Write an equation that relates the number N of bachelor's degrees awarded to the year t.
   (b) If the trend continues, estimate the number of bachelor's degrees that were awarded in 2003.
   (c) Interpret the slope.

   Source: Digest of Education Statistics, 1999. National Center for Education Statistics.

Problems 9–16 involve break-even points.
In Problems 9–12, find the break-even point for the cost C of production and the revenue R. Graph each result. Indicate the break-even point and where a profit results and where a loss results.

9. $C = \$10x + \$600$ $R = \$30x$
10. $C = \$5x + \$200$ $R = \$8x$
11. $C = \$0.20x + \$50$ $R = \$0.3x$
12. $C = \$1800x + \$3000$ $R = \$2500x$

13. **Break-Even Point** A manufacturer produces items at a daily cost of \$0.75 per item and sells them for \$1 per item. The daily operational overhead is \$300. What is the break-even point? Graph your result.

14. **Break-Even Point** If the manufacturer in Problem 13 is able to reduce the cost per item to \$0.65, but with a resultant increase to \$350 in operational overhead, is it advantageous to do so? Give reasons. Graph your result.

15. **Profit from Sunday Home Delivery** For \$1.79 per copy, the Chicago Tribune will deliver the Sunday newspaper to your front door. The cost to the Tribune for Sunday home delivery is approximately \$0.53 per newspaper with fixed costs of \$1,070,000.

   (a) Determine the revenue R from delivering x newspapers.
   (b) Determine the cost C of delivering x newspapers.
   (c) Determine the profit P of delivering x newspapers.
   (d) Determine the break-even point.
   (e) Graph R and C together and label the break-even point.
   (f) Graph P and label its x-intercept.
   (g) Comment on the relationship between the break-even point and the x-intercept.

   Source: Chicago Tribune, 2002.

16. **Wages of a Car Salesperson** In 2000 median earnings, including commissions, for car salespersons was \$17.81 per hour or \$712.40 per week. Dan receives \$375 per week for selling new and used cars. In addition, he receives 5% of the profit on any sales he generates.

   (a) Write an equation that relates Dan's weekly salary S when he has sales that generate a profit of x dollars.
   (b) If Dan has sales that generate a profit of \$4000, what are his weekly earnings?
   (c) To equal the median earnings of a car salesperson, Dan would have to have sales that generate a profit of how many dollars?

   Source: U.S. Department of Labor, Occupational Outlook Handbook 2002–2003 Edition.

Problems 17–24, involve mixture problems.

17. **Mixture Problem** Sweet Delight Candies sells boxes of candy consisting of creams and caramels. Each box sells for \$8.00 and holds 50 pieces of candy (all pieces are the same size). If the caramels cost \$0.10 to produce and the creams cost \$0.20 to produce, how many caramels and creams should be in each box for no profit or loss? Would you increase or decrease the number of caramels in order to obtain a profit?

18. **Mixture Problem** The manager of Nutt's Nuts regularly sells cashews for \$6.50 per pound, pecans for \$7.50 per pound, and peanuts for \$2.00 per pound. How many pounds of cashews and pecans should be mixed with 40 pounds of peanuts to obtain a mixture of 100 pounds that will sell for \$4.89 a pound so that the revenue is unchanged?

19. **Investment Problem** Mr. Nicholson has just retired and needs \$10,000 per year in supplementary income. He has \$150,000 to invest and can invest in AA bonds at 10% annual interest or in Savings and Loan Certificates at 5% interest per year. How much money should be invested in each so that he realizes exactly \$10,000 in extra income per year?

20. **Investment Problem** Mr. Nicholson finds after 2 years that because of inflation he now needs \$12,000 per year in supplementary income. How should he transfer his funds to achieve this amount? (Use the data from Problem 19.)

21. Mixture Problem California Coffee Roasters sells Kona coffee for $22.95 per pound and Colombian coffee for $6.75 per pound. Suppose they offer a blend of those two coffees for a price of $10.80 per pound. What amounts of Kona and Colombian coffees should be blended to obtain the desired mixture? *Hint:* Assume that the total weight of the blend is 100 pounds.

Source: California Coffee Roasters.

22. Theater Attendance The Star Theater wants to know whether the majority of its patrons are adults or children. During a week in July 2600 tickets were sold, and the receipts totaled $16,440. The adult admission is $8 and the children's admission is $4. How many adult patrons were there?

23. Mixture Problem One solution is 15% acid and another is 5% acid. How many cubic centimeters of each should be mixed to obtain 100 cubic centimeters of a solution that is 8% acid?

24. Investment Problem A bank loaned $10,000, some at an annual rate of 8% and some at an annual rate of 12%. If the income from these loans was $1000, how much was loaned at 8%? How much at 12%?

Problems 25–32 involve economics.
In Problems 25–28, find the market price for each pair of supply and demand equations.

25. $S = p + 1$ $D = 3 - p$

26. $S = 2p + 3$ $D = 6 - p$

27. $S = 20p + 500$ $D = 1000 - 30p$

28. $S = 40p + 300$ $D = 1000 - 30p$

29. Market Price of Sugar The supply and demand equations for sugar have been estimated to be given by the equations

$$S = 0.7p + 0.4 \qquad D = -0.5p + 1.6$$

(a) Find the market price.
(b) What quantity of supply is demanded at this market price?
(c) Graph both the supply and demand equations.
(d) Interpret the point of intersection of the two lines.

30. Supply and Demand Problem The market price for a certain product is $5.00 per unit and occurs when 14,000 units are produced. At a price of $1, no units are manufactured and, at a price of $19.00, no units will be purchased. Find the supply and demand equations, assuming they are linear.

31. Supply and Demand Problem For a certain commodity the supply equation is given by

$$S = 2p + 5$$

At a price of $1, there is a demand for 19 units of the commodity. If the demand equation is linear and the market price is $3, find the demand equation.

32. Supply and Demand Problem For a certain commodity the demand equation is given by

$$D = -3p + 20$$

At a price of $1, four units of the commodity are supplied. If the supply equation is linear and the market price is $4, find the supply equation.

1.4 Scatter Diagrams; Linear Curve Fitting

OBJECTIVES
1 Draw and interpret scatter diagrams
2 Distinguish between linear and nonlinear relations
3 Use a graphing utility to find the line of best fit

Scatter Diagrams

Draw and interpret scatter diagrams 1

A **relation** is a correspondence between two sets. If x and y are two elements in these sets and if a relation exists between x and y, then we say that x **corresponds to** y or that y **depends on** x and write $x \longrightarrow y$. We may also write $x \longrightarrow y$ as the ordered pair (x, y). Here, y is referred to as the **dependent** variable and x is called the **independent** variable.

Often we are interested in specifying the type of relation (such as an equation) that might exist between two variables. The first step in finding this relation is to plot the ordered pairs using rectangular coordinates. The resulting graph is called a **scatter diagram.**

EXAMPLE 1 Drawing a Scatter Diagram

The data listed in Table 2, represent the apparent temperature versus the relative humidity in a room whose actual temperature is 72° Fahrenheit.

TABLE 2

Relative Humidity (%), x	Apparent Temperature, y	(x, y)	Relative Humidity (%), x	Apparent Temperature, y	(x, y)
0	64	(0, 64)	60	72	(60, 72)
10	65	(10, 65)	70	73	(70, 73)
20	67	(20, 67)	80	74	(80, 74)
30	68	(30, 68)	90	75	(90, 75)
40	70	(40, 70)	100	76	(100, 76)
50	71	(50, 71)			

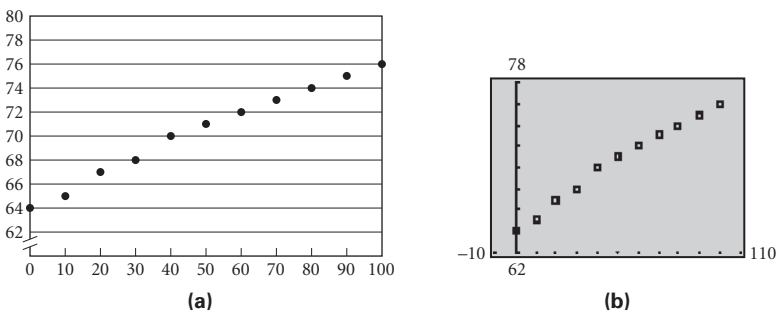
(a) Draw a scatter diagram by hand.
(b) Use a graphing utility to draw a scatter diagram.*
(c) Describe what happens to the apparent temperature as the relative humidity increases.

SOLUTION

(a) To draw a scatter diagram by hand, we plot the ordered pairs listed in Table 2, with the relative humidity as the x-coordinate and the apparent temperature as the y-coordinate. See Figure 36(a). Notice that the points in a scatter diagram are not connected.

(b) Figure 36(b) shows the scatter diagram using a graphing utility.

FIGURE 36



(c) We see from the scatter diagrams that, as the relative humidity increases, the apparent temperature increases. ◗

NOW WORK PROBLEM 7(a).

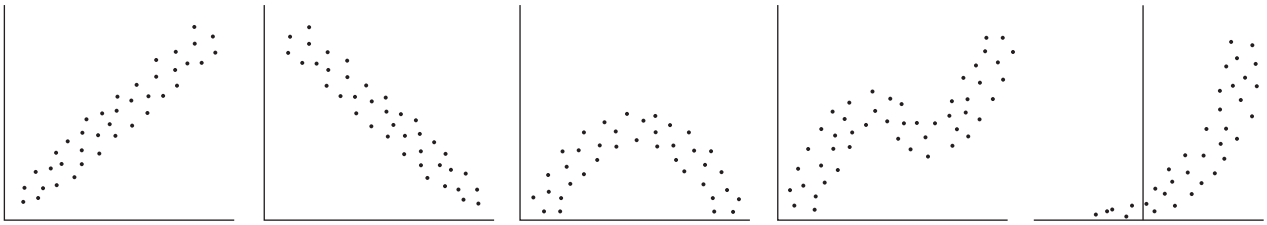
Consult your owner's manual for the proper keystrokes.

Curve Fitting

Distinguish between linear and nonlinear relations 2

Scatter diagrams are used to help us see the type of relation that may exist between two variables. In this text, we concentrate on distinguishing between linear and nonlinear relations. See Figure 37.

FIGURE 37

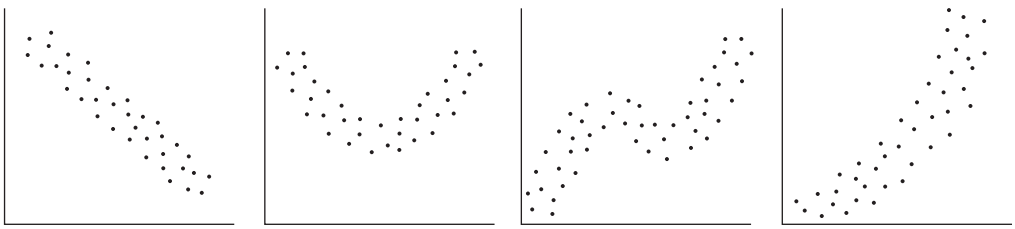


(a) Linear $y = mx + b, m > 0$ **(b)** Linear $y = mx + b, m < 0$ **(c)** Nonlinear **(d)** Nonlinear **(e)** Nonlinear

EXAMPLE 2 **Distinguishing between Linear and Nonlinear Relations**

Determine whether the relation between the two variables in Figure 38 is linear or nonlinear.

FIGURE 38



(a) **(b)** **(c)** **(d)**

SOLUTION (a) Linear (b) Nonlinear (c) Nonlinear (d) Nonlinear

NOW WORK PROBLEM 1.

In this section we will study data whose scatter diagrams imply that a linear relation exists between the two variables.

Suppose that the scatter diagram of a set of data appears to be linearly related as in Figure 37(a) or (b). We might wish to find an equation of a line that relates the two variables. One way to obtain an equation for such data is to draw a line through two points on the scatter diagram and find the equation of the line.

EXAMPLE 3 **Find an Equation for Linearly Related Data**

Using the data in Table 2 from Example 1, select two points from the data and find an equation of the line containing the points.

(a) Graph the line on the scatter diagram obtained in Example 1(a).
(b) Graph the line on the scatter diagram obtained in Example 1(b).

SOLUTION Select two points, say (10, 65) and (70, 73). (You should select your own two points and complete the solution.) The slope of the line joining the points (10, 65) and (70, 73) is

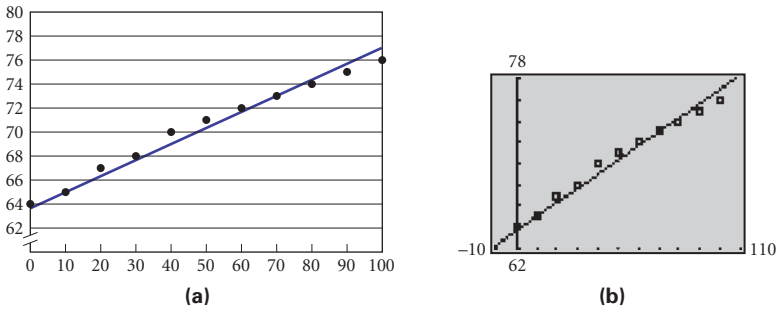
$$m = \frac{73 - 65}{70 - 10} = \frac{8}{60} = \frac{2}{15}$$

The equation of the line with slope $\frac{2}{15}$ and passing through (10, 65) is found using the point–slope form with $m = \frac{2}{15}$, $x_1 = 10$, and $y_1 = 65$.

$$y - y_1 = m(x - x_1)$$
$$y - 65 = \frac{2}{15}(x - 10)$$
$$y = \frac{2}{15}x + \frac{191}{3}$$

(a) Figure 39(a) shows the scatter diagram with the graph of the line drawn by hand.
(b) Figure 39(b) shows the scatter diagram with the graph of the line using a graphing utility.

FIGURE 39



NOW WORK PROBLEMS 7(b) AND (c).

Use a graphing utility to find the line of best fit

LINE OF BEST FIT

The line obtained in Example 3 depends on the selection of points, which will vary from person to person. So the line that we found might be different from the line that you found. Although the line that we found in Example 3 appears to "fit" the data well, there may be a line that "fits better." Do you think your line fits the data better? Is there a line of *best fit*? As it turns out, there is a method for finding the line that best fits linearly related data (called the *line of best fit*).

EXAMPLE 4 Finding the Line of Best Fit

Using the data in Table 2 from Example 1:

(a) Find the line of best fit using a graphing utility.
(b) Graph the line of best fit on the scatter diagram obtained in Example 1(b).
(c) Interpret the slope of the line of best fit.
(d) Use the line of best fit to predict the apparent temperature of a room whose actual temperature is 72°F and relative humidity is 45%.

SOLUTION

(a) Graphing utilities contain built-in programs that find the line of best fit for a collection of points in a scatter diagram. (Look in your owner's manual under Linear Regression or Line of Best Fit for details on how to execute the program.) Upon executing the LINear REGression program on a TI-83 Plus we obtain the results shown in Figure 40. The output that the utility provides shows us the equation $y = ax + b$, where a is the slope of the line and b is the y-intercept. The line of best fit that relates relative humidity to apparent temperature may be expressed as the line $y = 0.121x + 64.409$.

(b) Figure 41 shows the graph of the line of best fit, along with the scatter diagram.

FIGURE 40

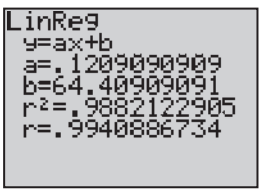
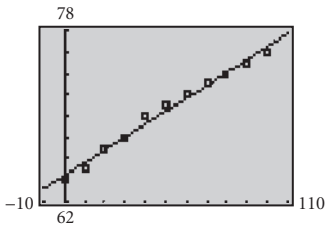


FIGURE 41



(c) The slope of the line of best fit is 0.121, which means that, for every 1% increase in the relative humidity, apparent room temperature increases 0.121°F.

(d) Letting $x = 45$ in the equation of the line of best fit, we obtain $y = 0.121(45) + 64.409 \approx 70$°F, which is the apparent temperature in the room.

NOW WORK PROBLEMS 7(d), (e) AND (f).

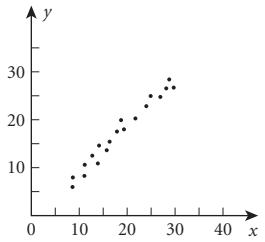
Does the line of best fit appear to be a good fit? In other words, does the line appear to accurately describe the relation between temperature and relative humidity?

And just how "good" is this line of best fit? The answers are given by what is called the *correlation coefficient*. Look again at Figure 40. The last line of output is $r = 0.994$. This number, called the **correlation coefficient, r**, $-1 \le r \le 1$, is a measure of the strength of the *linear relation* that exists between two variables. The closer that $|r|$ is to 1, the more perfect the linear relationship is. If r is close to 0, there is little or no *linear* relationship between the variables. A negative value of r, $r < 0$, indicates that as x increases y decreases; a positive value of r, $r > 0$, indicates that as x increases y does also. The data given in Example 1, having a correlation coefficient of 0.994, are indicative of a strong linear relationship with positive slope.

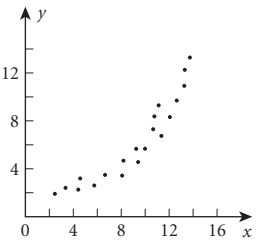
EXERCISE 1.4 Answers to Odd-Numbered Problems Begin on Page AN-5.

In Problems 1–6, examine the scatter diagram and determine whether the type of relation, if any, that may exist is linear or nonlinear.

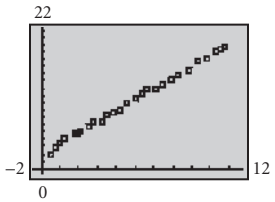
1.



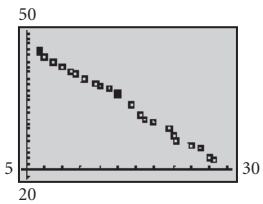
2.



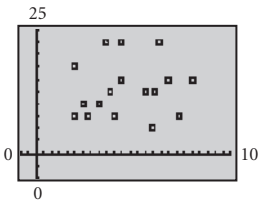
3.



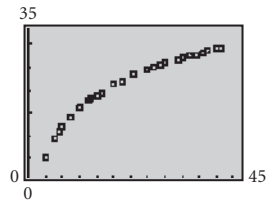
4.



5.



6.



In Problems 7–14:

(a) *Draw a scatter diagram by hand.*
(b) *Select two points from the scatter diagram and find the equation of the line containing the points selected.**
(c) *Graph the line found in part (b) on the scatter diagram.*
(d) *Use a graphing utility to draw a scatter diagram.*
(e) *Use a graphing utility to find the line of best fit.*
(f) *Use a graphing utility to graph the line of best fit on the scatter diagram.*

7.

x	3	4	5	6	7	8	9
y	4	6	7	10	12	14	16

8.

x	3	5	7	9	11	13
y	0	2	3	6	9	11

9.

x	−2	−1	0	1	2
y	−4	0	1	4	5

10.

x	−2	−1	0	1	2
y	7	6	3	2	0

11.

x	20	30	40	50	60
y	100	95	91	83	70

12.

x	5	10	15	20	25
y	2	4	7	9	11

13.

x	−20	−17	−15	−14	−10
y	100	120	118	130	140

14.

x	−30	−27	−25	−20	−14
y	10	12	13	13	18

15. Consumption and Disposable Income An economist wishes to estimate a line that relates personal consumption expenditures C and disposable income I. Both C and I are in thousands of dollars. She interviews eight heads of households for families of size 3 and obtains the data below.

I(000)	20	20	18	27	36	37	45	50
C(000)	16	18	13	21	27	26	36	39

Let I represent the independent variable and C the dependent variable.

(a) Draw a scatter diagram by hand.
(b) Find a line that fits the data.*
(c) Interpret the slope. The slope of this line is called the **marginal propensity to consume.**
(d) Predict the consumption of a family whose disposable income is $42,000.
(e) Use a graphing utility to find the line of best fit to the data.

**Answers will vary. We will use the first and last data points in the answer section.*

16. Marginal Propensity to Save The same economist as in Problem 15 wants to estimate a line that relates savings S and disposable income I. Let $S = I - C$ be the dependent variable and I the independent variable.

(a) Draw a scatter diagram by hand.
(b) Find a line that fits the data.
(c) Interpret the slope. The slope of this line is called the **marginal propensity to save.**
(d) Predict the savings of a family whose income is $42,000.
(e) Use a graphing utility to find the line of best fit.

17. Mortgage Qualification The amount of money that a lending institution will allow you to borrow mainly depends on the interest rate and your annual income. The following data represent the annual income, I, required by a bank in order to lend L dollars at an interest rate of 7.5% for 30 years.

Annual Income, I($)	Loan Amount, L($)
15,000	44,600
20,000	59,500
25,000	74,500
30,000	89,400
35,000	104,300
40,000	119,200
45,000	134,100
50,000	149,000
55,000	163,900
60,000	178,800
65,000	193,700
70,000	208,600

Source: Information Please Almanac, 1999

Let I represent the independent variable and L the dependent variable.

(a) Use a graphing utility to draw a scatter diagram of the data.
(b) Use a graphing utility to find the line of best fit to the data.
(c) Graph the line of best fit on the scatter diagram drawn in part (a).
(d) Interpret the slope of the line of best fit.
(e) Determine the loan amount that an individual would qualify for if her income is $42,000.

18. Mortgage Qualification The amount of money that a lending institution will allow you to borrow mainly depends on the interest rate and your annual income. The following data represent the annual income, I, required by a bank in order to lend L dollars at an interest rate of 8.5% for 30 years.

Annual Income, I($)	Loan Amount, L($)
15,000	40,600
20,000	54,100
25,000	67,700
30,000	81,200
35,000	94,800
40,000	108,300
45,000	121,900
50,000	135,400
55,000	149,000
60,000	162,500
65,000	176,100
70,000	189,600

Source: Information Please Almanac, 1999

Let I represent the independent variable and L the dependent variable.

(a) Use a graphing utility to draw a scatter diagram of the data.
(b) Use a graphing utility to find the line of best fit to the data.
(c) Graph the line of best fit on the scatter diagram drawn in part (a).
(d) Interpret the slope of the line of best fit.
(e) Determine the loan amount that an individual would qualify for if her income is $42,000.

19. Apparent Room Temperature The data on page 42 represent the apparent temperature versus the relative humidity in a room whose actual temperature is 65° Fahrenheit. Let h represent the independent variable and T the dependent variable.

(a) Use a graphing utility to draw a scatter diagram of the data.
(b) Use a graphing utility to find the line of best fit to the data.
(c) Graph the line of best fit on the scatter diagram drawn in part (a).
(d) Interpret the slope of the line of best fit.
(e) Determine the apparent temperature of a room whose actual temperature is 65°F if the relative humidity is 75%.

Relative Humidity, h(%)	Apparent Temperature, T (°F)
0	59
10	60
20	61
30	61
40	62
50	63
60	64
70	65
80	65
90	66
100	67

Source: National Oceanic and Atmospheric Administration

Relative Humidity, h(%)	Apparent Temperature, T (°F)
0	68
10	69
20	71
30	72
40	74
50	75
60	76
70	76
80	77
90	78
100	79

Source: National Oceanic and Atmospheric Administration

20. Apparent Room Temperature The following data represent the apparent temperature versus the relative humidity in a room whose actual temperature is 75° Fahrenheit. Let h represent the independent variable and let T be the dependent variable.

(a) Use a graphing utility to draw a scatter diagram of the data.

(b) Use a graphing utility to find the line of best fit to the data.

(c) Graph the line of best fit on the scatter diagram drawn in part (a).

(d) Interpret the slope of the line of best fit.

(e) Determine the apparent temperature of a room whose actual temperature is 75°F if the relative humidity is 75%.

Chapter 1 Review

OBJECTIVES

Section	You should be able to	Review Exercises
1.1	1 Graph linear equations	1–8, 23–26
	2 Find the equation of a vertical line	12, 13
	3 Calculate and interpret the slope of a line	5–8(a), 45(b–e, g, h), 46(b–e, h), 47(c, d)
	4 Graph a line given a point on the line and the slope	9–11
	5 Use the point–slope form of a line	9–11
	6 Find the equation of a horizontal line	11, 14
	7 Find the equation of a line given two points	5–8(b), 15–18
	8 Use the slope–intercept form of a line	23–26
1.2	1 Show that two lines are coincident	28, 31
	2 Show that two lines are parallel	19, 20, 27, 32
	3 Show that two lines intersect	29, 30

	4	Find the point of intersection of two intersecting lines	33–38
	5	Show that two lines are perpendicular	21, 22
1.3	1	Solve applied problems	39–42
1.4	1	Draw and interpret scatter diagrams	43, 44, 45(a), 46(a), 47(a)
	2	Distinguish between linear and nonlinear relations	43, 44
	3	Use a graphing utility to find the line of best fit	45(f), 46(f), 47(e)

IMPORTANT FORMULAS

Linear Equation, General Form (p. 3)
$Ax + By = C$ A, B not both zero

Vertical Line (p. 6):
$x = a$ $(a, 0)$ is the x-intercept

Slope of a Line (p. 7)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ if } x_1 \neq x_2; \quad \text{undefined if } x_1 = x_2$$

Point–Slope Form of the Equation of a Line (p. 11)
$y - y_1 = m(x - x_1)$ m is the slope of the line; (x_1, y_1) is a point on the line

Horizontal Line (p. 11):
$y = b$ $(0, b)$ is the y-intercept

Slope–Intercept Form of the Equation of a Line (p. 13)
$y = mx + b$ m is the slope of the line; $(0, b)$ is the y-intercept

Pair of Lines (pp. 19–25)	*Conclusion: The lines are*
Both vertical	(a) Coincident, if they have the same x-intercept (b) Parallel, if they have different x-intercepts
One vertical, one nonvertical	(a) Intersecting (b) Perpendicular, if the nonvertical line is horizontal
Neither vertical	Write the equation of each line in slope–intercept form: $y = m_1x + b_1, y = m_2x + b_2$ (a) Coincident, if $m_1 = m_2, b_1 = b_2$ (b) Parallel, if $m_1 = m_2, b_1 \neq b_2$ (c) Intersecting, if $m_1 \neq m_2$ (d) Perpendicular, if $m_1m_2 = -1$

TRUE–FALSE ITEMS Answers are on page AN-7.

T F **1.** In the slope–intercept equation of a line, $y = mx + b$, m is the slope and $(0, b)$ is the x-intercept.

T F **2.** The graph of the equation $Ax + By = C$, where A, B, C are real numbers and A, B are not both zero, is a line.

T F **3.** The y-intercept of the line $2x - 3y = -6$ is $(0, 2)$.

T F **4.** The slope of the line $2x - 4y = -7$ is $-\frac{1}{2}$.

T F **5.** Parallel lines always have the same intercepts.

T F **6.** Intersecting lines have different slopes.

T F **7.** Perpendicular lines have slopes that are reciprocals of each other.

T F **8.** A linear relation between two variables can always be graphed as a line.

T F **9.** All lines with equal slopes are distinct.

T F **10.** All vertical lines have positive slope.

FILL IN THE BLANKS Answers are on page AN-7.

1. If (x, y) are rectangular coordinates of a point, the number x is called the _____ and y is called the _____.

2. The slope of a vertical line is _____; the slope of a horizontal line is _____.

3. If a line slants downward from left to right, its slope will be a _____ number.

4. If two lines have the same slope but different y-intercepts, they are _____.

5. If two lines have the same slope and the same y-intercept, they are said to be _____.

6. Lines that intersect at right angles are said to be _____ to each other.

7. Distinct lines that have different slopes are _____ lines.

REVIEW EXERCISES

Answers to odd-numbered problems begin on page AN-7.
Blue Problem numbers indicate the author's suggestions for use in a practice test.

In Problems 1–4, graph each equation.

1. $y = -2x + 3$
2. $y = 6x - 2$
3. $2y = 3x + 6$
4. $3y = 2x + 6$

In Problems 5–8, (a) calculate and interpret the slope of the line containing each pair of points; (b) find an equation for the line containing each pair of points. Write the equation using the general form or the slope–intercept form, whichever you prefer.
(c) Graph each line.

5. $P = (1, 2)$ $Q = (-3, 4)$
6. $P = (-1, 3)$ $Q = (1, 1)$
7. $P = (-1, 5)$ $Q = (-2, 3)$
8. $P = (-2, 3)$ $Q = (0, 0)$

In Problems 9–22, find an equation of the line having the given characteristics. Write the equation using the general form or the slope–intercept form, whichever you prefer. Graph each line.

9. Slope $= -3$; containing the point $(2, -1)$
10. Slope $= 4$; containing the point $(-1, -3)$
11. Slope $= 0$; containing the point $(-3, 4)$
12. Slope undefined; containing the point $(-3, 4)$
13. Vertical; containing the point $(8, 5)$
14. Horizontal; containing the point $(5, 8)$
15. x-intercept $= (2, 0)$; containing the point $(4, -5)$
16. y-intercept $= (0, -2)$; containing the point $(5, -3)$
17. x-intercept $= (-3, 0)$; y-intercept $= (0, -4)$
18. Containing the points $(3, -4)$ and $(2, 1)$
19. Parallel to the line $2x + 3y = -4$; containing the point $(-5, 3)$
20. Parallel to the line $x + y = 2$; containing the point $(1, -3)$
21. Perpendicular to the line $2x + 3y = -4$; containing the point $(-5, 3)$
22. Perpendicular to the line $x + y = 2$; containing the point $(1, -3)$

In Problems 23–26 find the slope and y-intercept of each line. Graph each line.

23. $9x + 2y = 18$
24. $4x + 5y = 20$
25. $4x + 2y = 9$
26. $3x + 2y = 8$

In Problems 27–32 determine whether the two lines are parallel, coincident, or intersecting.

27. $3x - 4y = -12$
    $6x - 8y = -9$
28. $2x + 3y = -5$
    $4x + 6y = -10$
29. $x - y = -2$
    $3x - 4y = -12$
30. $2x + 3y = 5$
    $x + y = 2$
31. $4x + 6y = -12$
    $2x + 3y = -6$
32. $-3x + y = 0$
    $6x - 2y = -5$

In Problems 33–38, the given pair of lines intersect. Find the point of intersection. Graph the lines.

33. L: $x - y = 4$
    M: $x + 2y = 7$
34. L: $x + y = 4$
    M: $x - 2y = 1$
35. L: $x - y = -2$
    M: $x + 2y = 7$

36. L: $2x + 4y = 4$
M: $2x - 4y = 8$

37. L: $2x - 4y = -8$
M: $3x + 6y = 0$

38. L: $3x + 4y = 2$
M: $x - 2y = 1$

39. Investment Problem Mr. and Mrs. Byrd have just retired and find that they need \$10,000 per year to live on. Fortunately, they have a nest egg of \$90,000, which they can invest in somewhat risky B-rated bonds at 12% interest per year or in a well-known bank at 5% per year. How much money should they invest in each so that they realize exactly \$10,000 in interest income each year?

40. Mixture Problem One solution is 20% acid and another is 12% acid. How many cubic centimeters of each solution should be mixed to obtain 100 cubic centimeters of a solution that is 15% acid?

41. Attendance at a Dance A church group is planning a dance in the school auditorium to raise money for its school. The band they will hire charges \$500; the advertising costs are estimated at \$100; and food will be supplied at the rate of \$5 per person. The church group would like to clear at least \$900 after expenses.

(a) Determine how many people need to attend the dance for the group to break even if tickets are sold at \$10 each.
(b) Determine how many people need to attend in order to achieve the desired profit if tickets are sold for \$10 each.
(c) Answer the above two questions if the tickets are sold for \$12 each.

42. Mixture Problem A coffee manufacturer wants to market a new blend of coffee that will cost \$6.00 per pound by mixing \$5.00 per pound coffee and \$7.50 per pound coffee. What amounts of the \$5.00 per pound coffee and \$7.50 per pound coffee should be blended to obtain the desired mixture? [*Hint:* Assume the total weight of the desired blend is 100 pounds.]

In Problems 43 and 44, draw a scatter diagram for each set of data. Then determine whether the relation, if any, that may exist is linear or non-linear.

43.

x	0	1	2	3	4	5	6
y	90	45	21	12	5	3	2

44.

x	3	5	7	9	11	13
y	74	70	67	58	55	51

45. Concentration of Carbon Monoxide in the Air The following data represent the average concentration of carbon monoxide in parts per million (ppm) in the air for 1987–1993.

Year	Concentration of Carbon Monoxide (ppm)
1987	6.69
1988	6.38
1989	6.34
1990	5.87
1991	5.55
1992	5.18
1993	4.88

Source: U.S. Environmental Protection Agency.

(a) Treating the year as the x-coordinate and the average level of carbon monoxide as the y-coordinate, draw a scatter diagram of the data.
(b) What is the slope of the line joining the points (1987, 6.69) and (1990, 5.87)?
(c) Interpret this slope.
(d) What is the slope of the line joining the points (1990, 5.87) and (1993, 4.88)?
(e) Interpret this slope.
(f) Use a graphing utility to find the slope of the line of best fit for these data.
(g) Interpret this slope.
(h) How do you explain the differences among the three slopes obtained?
(i) What is the trend in the data? In other words, as time passes, what is happening to the average level of carbon monoxide in the air? Why do you think this is happening?

46. Housing Costs The data on page 46 represent the average price of houses sold in the United States for 1994–2001.

(a) Treating the year as the x-coordinate and the price of the houses as the y-coordinate, draw a scatter diagram of the data.
(b) What is the slope of the line joining the points (1994, 154500) and (1998, 181900)?
(c) Interpret this slope.
(d) What is the slope of the line joining the points (1998, 181900) and (2001, 213200)?
(e) Interpret this slope.
(f) Use a graphing utility to find the slope of the line of best fit for this data.

Year	Price (Dollars)
1994	154,500
1995	158,700
1996	166,400
1997	176,200
1998	181,900
1999	195,600
2000	207,000
2001	213,200

Source: U.S. Census Bureau, 2002.

(g) Interpret this slope.

(h) How do you explain the differences among the three slopes obtained?

(i) What is the trend in the data? In other words, what is happening to the average price of a home in the United States? Why do you think this is happening?

47. Value of a Portfolio The following data represent the value of the Vanguard 500 Index Fund for 1996–1999.

Year	Value per Share
1996	$ 69.17
1997	90.07
1998	113.95
1999	135.33

Source: Vanguard 500 Index Fund, Annual Report 2001.

(a) Treating the year as the x-coordinate and the value of the Vanguard 500 Index Fund as the y-coordinate, draw a scatter diagram of the data.

(b) Do the data appear to be linearly related?

(c) What is the slope of the line connecting (1996, 69.17) and (1999, 135.33)?

(d) Interpret the slope.

(e) Use a graphing utility to find the line of best fit for these data.

(f) Assuming the line of best fit truly represents the trend in the data, predict the value of a share of Vanguard 500 Index Fund in the year 2001.

48. Value of a Portfolio Investment ads warn that "past performance is no guarantee of future performance." Vanguard's Annual Report shows the value of a share was $121.86 in 2000 and $105.89 in the year 2001.

(a) Add these data to the chart in Problem 47 and draw a revised scatter diagram representing the years 1996–2001.

(b) Do the data appear to be linearly related?

(c) What would you say about the prediction you made in Exercise 47 (f)?

49. Make up four problems that you might be asked to do given the two points $(-3, 4)$ and $(6, 1)$. Each problem should involve a different concept. Be sure that your directions are clearly stated.

50. Describe each of the following graphs in the xy-plane. Give justification.

(a) $x = 0$

(b) $y = 0$

(c) $x + y = 0$

Chapter 1 Project

CHOOSING A RENTAL CAR COMPANY*

You have a decision to make. You will be flying to Charlotte, North Carolina for a one-day stay. You will need to rent a midsize car while you are there, and you want to do this in the most economical way possible. The cost to rent a car sometimes depends on the number of miles it is driven, but sometimes the cost includes unlimited mileage. You need to figure out which rental company gives you the best deal. As it turns out, the equations involved are linear, so the problem can be analyzed using techniques discussed in this chapter.

**All rates quoted have been taken from quotes provided by car rental company web sites in February 2003. There are other fees and taxes which are later added to each quote; these fees and taxes are ignored in our analysis.*

You begin by contacting two car rental agencies: Avis and Enterprise. Avis offers a midsize car for $64.99 per day with unlimited mileage. This means that for this rental the number of miles driven does not impact on the cost. Enterprise offers a midsize car for $45.87 per day with 150 miles free. But each mile beyond 150 that the car is driven costs $0.25. It is clear that Enterprise offers the better deal if the car is driven fewer than 150 miles. But what if the car is driven more than 150 miles? At what point will the Avis rental become a better deal?

Let's analyze the situation. Let x denote the number of miles the car is driven.

1. Suppose A is the cost of renting at Avis. Find a linear equation involving A and x.
2. Now let E be the cost of renting at Enterprise. Find a linear equation involving E and x, if $x \leq 150$. Find a linear equation involving E and x if $x > 150$.
3. Graph the linear equations found in parts (1) and (2) on the same set of coordinate axes. Be careful about the restrictions on x for the equations found in part (2).
4. Find the mileage beyond which the Avis rental is more economical by finding the point of intersection of the two graphs. Label this point on your graph.
5. Explain how you can use the solution to part (4) to decide on which car rental is more economical.
6. In an effort to find an even better deal, you contact AutoSaveRental. They offer a midsize car for $36.99 per day with 100 miles free. Each mile driven over 100 miles costs $0.25. Find equation(s) that involve the cost S at AutoSave and the miles x driven.
7. Graph this equation on the same graph found in part (3).
8. It should be clear that AutoSave is the best choice among the three if you are driving less than 100 miles. For what values of x, if any, do Avis or Enterprise offer a better deal?
9. Still not sure you have the best deal, you call Usave Car Rental. They offer a midsize car for $35.99 per day with 200 free miles. Each mile over 200 costs $0.25. Find an equation(s) for the cost U at Usave and graph it on the same graph found in part (7). For what values of x, if any, is Usave the best deal?
10. Discuss your choice of car rental companies if you think you will drive somewhere between 125 and 175 miles.

MATHEMATICAL QUESTIONS FROM PROFESSIONAL EXAMS*

1. **CPA Exam** The Oliver Company plans to market a new product. Based on its market studies, Oliver estimates that it can sell 5500 units in 1992. The selling price will be $2 per unit. Variable costs are estimated to be 40% of the selling price. Fixed costs are estimated to be $6000. What is the break-even point?

   (a) 3750 units (c) 5500 units
   (b) 5000 units (d) 7500 units

2. **CPA Exam** The Breiden Company sells rodaks for $6 per unit. Variable costs are $2 per unit. Fixed costs are $37,500. How many rodaks must be sold to realize a profit before income taxes of 15% of sales?

   (a) 9375 units (c) 11,029 units
   (b) 9740 units (d) 12,097 units

3. **CPA Exam** Given the following notations, what is the break-even sales level in units?

   SP = Selling price per unit VC = Variable cost per unit
   FC = Total fixed cost

   (a) $\dfrac{SP}{FC \div VC}$ (c) $\dfrac{VC}{SP - FC}$
   (b) $\dfrac{FC}{VC \div SP}$ (d) $\dfrac{FC}{SP - VC}$

4. **CPA Exam** At a break-even point of 400 units sold, the variable costs were $400 and the fixed costs were $200. What will the 401st unit sold contribute to profit before income taxes?

   (a) $0 (c) $1.00
   (b) $0.50 (d) $1.50

5. **CPA Exam** A graph is set up with "depreciation expense" on the vertical axis and "time" on the horizontal axis. Assuming linear relationships, how would the graphs for straight-line and sum-of-the-year's-digits depreciation, respectively, be drawn?

   (a) Vertically and sloping down to the right
   (b) Vertically and sloping up to the right
   (c) Horizontally and sloping down to the right
   (d) Horizontally and sloping up to the right

The following statement applies to Questions 6–8:
In analyzing the relationship of total factory overhead with changes in direct labor hours, the following relationship was found to exist: $Y = \$1000 + \$2X$.

6. **CPA Exam** The relationship as shown above is

   (a) Parabolic (d) Probabilistic
   (b) Curvilinear (e) None of the above
   (c) Linear

7. **CPA Exam** Y in the above equation is an estimate of

   (a) Total variable costs (d) Total direct labor hours
   (b) Total factory overhead (e) None of the above
   (c) Total fixed costs

8. **CPA Exam** The $2 in the equation is an estimate of

   (a) Total fixed costs
   (b) Variable costs per direct labor hour
   (c) Total variable costs
   (d) Fixed costs per direct labor hour
   (e) None of the above