CHAPTER 1

FUNDAMENTALS OF ACOUSTICS, NOISE, AND VIBRATION

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1 INTRODUCTION

The vibrations in machines and structures result in oscillatory motion that propagates in air and/or water and that is known as sound. Sound can also be produced by the oscillatory motion of the fluid itself, such as in the case of the turbulent mixing of a jet with the atmosphere, in which no vibrating structure is involved. The simplest type of oscillation in vibration and sound phenomena is known as simple harmonic motion, which can be shown to be sinusoidal in time. Simple harmonic motion is of academic interest because it is easy to treat and manipulate mathematically; but it is also of practical interest. Most musical instruments make tones that are approximately periodic and simple harmonic in nature. Some machines (such as electric motors, fans, gears, etc.) vibrate and make sounds that have pure tone components. Musical instruments and machines normally produce several pure tones simultaneously. Machines also produce sound that is not simple harmonic but is random in time and is known as noise. The simplest vibration to analyze is that of a mass–spring–damper system. This elementary system is a useful model for the study of many simple vibration problems. Sound waves are composed of the oscillatory motion of air (or water) molecules. In air and water, the fluid is compressible and the motion is accompanied by a change in pressure known as sound. The simplest form of sound is one-dimensional plane wave propagation. In many practical cases (such as in enclosed spaces or outdoors in the environment) sound propagation in three dimensions must be considered.

2 DISCUSSION

In Chapter 1 we will discuss some simple theory that is useful in the control of noise and vibration. For more extensive discussions on sound and vibration fundamentals, the reader is referred to more detailed treatments available in several books.[1–7] We start off by discussing simple harmonic motion. This is because very often oscillatory motion, whether it be the vibration of a body or the propagation of a sound wave, is like this idealized case. Next, we introduce the ideas of period, frequency, phase, displacement, velocity, and acceleration. Then we study free and forced vibration of a simple mass–spring system and the influence of damping forces on the system. These vibration topics are discussed again at a more advanced level in Chapters 12, 15, and 60. In Section 5 we discuss how sound propagates in waves, and then we study *sound intensity* and *energy density*. In Section 6 we consider the use of decibels to express *sound pressure levels, sound intensity levels*, and *sound power levels.* Section 7 describes some preliminary ideas about human hearing. In Sections 8 and 9, we study frequency analysis of sound and frequency weightings and finally in Section 10 day–night and day–evening–night sound pressure levels.

In Chapter 2 we discuss some further aspects of sound propagation at a more intermediate level, including the radiation of sound from idealized spherical sources, standing waves, and the important ideas of *near, far, free,* and *reverberant* sound fields. We also study the propagation of sound in closed spaces indoors and outdoors. This has applications to industrial noise control problems in buildings and to community noise problems, respectively. Chapter 2 also serves as an introduction to some of the topics that follow in Part I of this handbook.

3 SIMPLE HARMONIC MOTION

The motion of vibrating systems such as parts of machines, and the variation of sound pressure with time is often said to be simple harmonic. Let us examine what is meant by simple harmonic motion.

Suppose a point P is revolving around an origin O with a constant angular velocity ω, as shown in Fig.1.

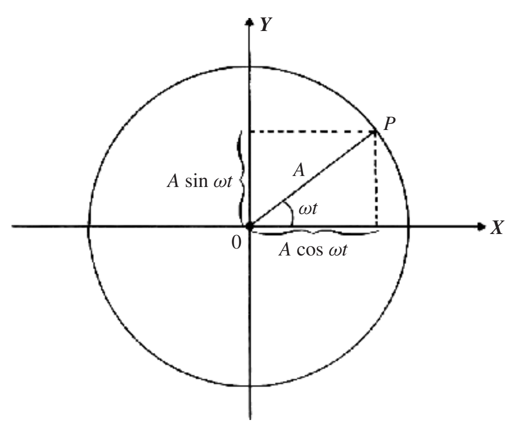

Figure 1 Representation of simple harmonic motion by projection of the rotating vector A on the X or Y axis.

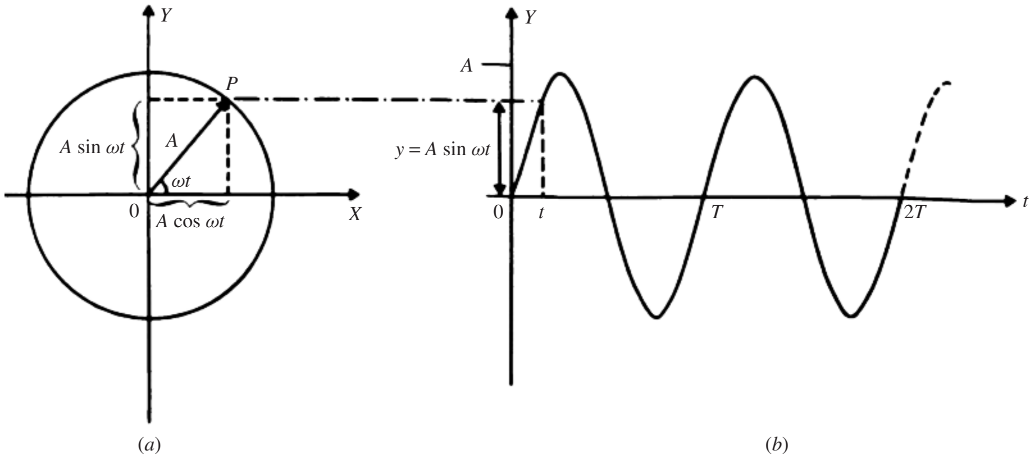

Figure 2 Simple harmonic motion.

If the vector OP is aligned in the direction OX when time $t = 0$, then after t seconds the angle between OP and OX is ωt. Suppose OP has a length A, then the projection on the X axis is $A \cos \omega t$ and on the Y axis, $A \sin \omega t$. The variation of the projected length on either the X axis or the Y axis with time is said to represent simple harmonic motion.

It is easy to generate a displacement vs. time plot with this model, as is shown in Fig. 2. The projections on the X axis and Y axis are as before. If we move the circle to the right at a constant speed, then the point P traces out a curve $y = A \sin \omega t$, horizontally. If we move the circle vertically upwards at the same speed, then the point P would trace out a curve $x = A \cos \omega t$, vertically.

3.1 Period, Frequency, and Phase

The motion is seen to repeat itself every time the vector OP rotates once (in Fig. 1) or after time T seconds (in Figs. 2 and 3). When the motion has repeated itself, the displacement y is said to have gone through one *cycle.* The number of cycles that occur per second is called the *frequency f.* Frequency may be expressed in cycles per second or, equivalently in hertz, or as abbreviated, Hz. The use of hertz or Hz is preferable because this has become internationally agreed upon as the unit of frequency. (Note cycles per second = hertz). Thus

$$f = 1/T \text{ hertz} \tag{1}$$

The time T is known as the *period* and is usually measured in seconds. From Figs. 2 and 3, we see that the motion repeats itself every time ωt increases by 2π, since $\sin 0 = \sin 2\pi = \sin 4\pi = 0$, and so on. Thus $\omega T = 2\pi$ and from Eq. 1,

$$\omega = 2\pi f \tag{2}$$

The angular frequency, ω, is expressed in radians per second (rad/s).

The motion described by the displacement y in Fig. 2 or the projection OP on the X or Y axes in Fig. 2 is said to be *simple harmonic.* We must now discuss something called the *initial phase angle*, which is sometimes just called *phase.* For the case we have chosen in Fig. 2, the phase angle is zero. If, instead, we start counting time from when the vector points in the direction OP_1, as shown in Fig. 3, and we let the angle $XOP_1 = \phi$, this is equivalent to moving the time origin t seconds to the right in Fig. 2. Time is started when P is at P_1 and thus the initial displacement is $A \sin \phi$. The *initial phase angle* is ϕ. After time t, P_1 has moved to P_2 and the displacement

$$y = A \sin(\omega t + \phi) \tag{3}$$

If the initial phase angle $\phi = 0°$, then $y = A \sin \omega t$; if the phase angle $\phi = 90°$, then $y = A \sin(\omega t + \pi/2) \equiv A \cos \omega t$. For mathematical convenience, complex exponential notation is often used. If the displacement is written as

$$y = \boldsymbol{A} e^{j\omega t}, \tag{3a}$$

and we remember that $\boldsymbol{A} e^{j\omega t} = A(\cos \omega t + j \sin \omega t)$, we see in Fig. 1 that the real part of Eq. (3a) is represented by the projection of the point P onto the x axis, $A \cos \omega t$, and of the point P onto the y or imaginary axis, $A \sin \omega t$. Simple harmonic motion, then, is often written as the real part of $\boldsymbol{A} e^{j\omega t}$, or in the more general form $Ae^{j(\omega t + \phi)}$. If the constant A is made complex, then the displacement can be written as the real part of $\boldsymbol{A} e^{j\omega t}$, where $\boldsymbol{A} = Ae^{j\phi}$.

3.2 Velocity and Acceleration

So far we have examined the displacement y of a point. Note that, when the displacement is in the OY direction, we say it is positive; when it is in

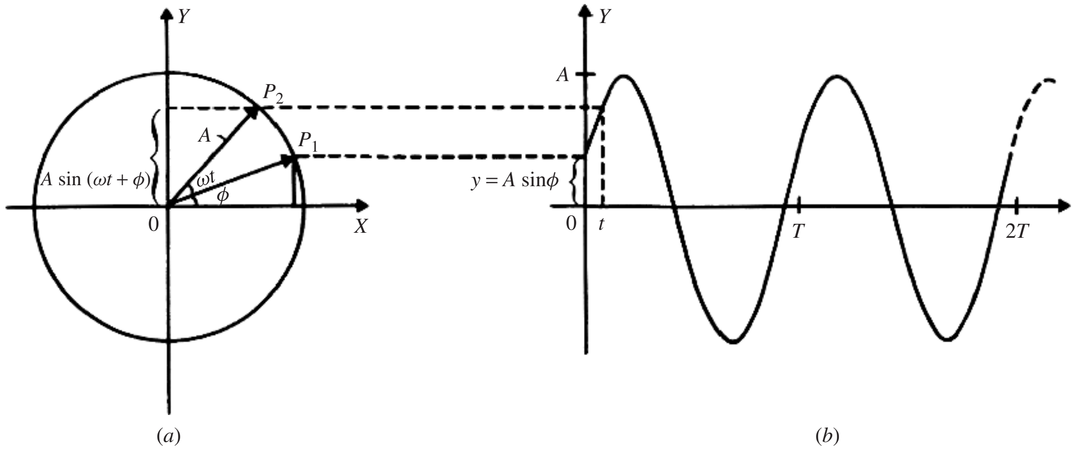

Figure 3 Simple harmonic motion with initial phase angle ϕ.

the opposite direction to OY, we say it is negative. Displacement, velocity, and acceleration are really vector quantities in mathematics; that is, they have magnitude and direction. The velocity v of a point is the rate of change of position with time of the point x in metres/second. The acceleration a is the rate of change of velocity with time. Thus, using simple calculus:

$$v = \frac{dy}{dt} = \frac{d}{dt}[A\sin(\omega t + \phi)] = A\omega\cos(\omega t + \phi) \tag{4}$$

and

$$a = \frac{dv}{dt} = \frac{d}{dt}[A\omega\cos(\omega t + \phi)] = -A\omega^2\sin(\omega t + \phi) \tag{5}$$

Equations (3), (4), and (5) are plotted in Fig. 4.

Note, by trigonometric manipulation we can rewrite Eqs. (4) and (5) as (6) and (7):

$$v = A\omega\cos(\omega t + \phi) = A\omega\sin\left(\omega t + \frac{\pi}{2} + \phi\right) \tag{6}$$

and

$$a = -A\omega^2\sin(\omega t + \phi) = +A\omega^2\sin(\omega t + \pi + \phi) \tag{7}$$

and from Eq. (3) we see that $a = -\omega^2 y$.

Equations (3), (6), and (7) tell us that for simple harmonic motion the *amplitude* of the velocity is ω or $2\pi f$ greater than the *amplitude of* the displacement, while the *amplitude* of the acceleration is ω^2 or $(2\pi f)^2$

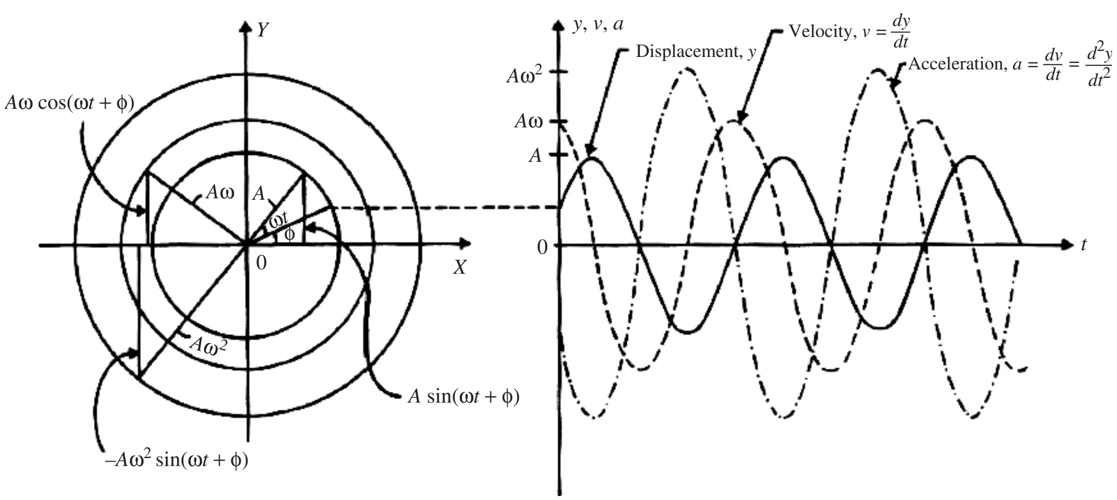

Figure 4 Displacement, velocity, and acceleration.

greater. The *phase* of the velocity is $\pi/2$ or 90° ahead of the displacement, while the acceleration is π or 180° ahead of the displacement.

Note, we could have come to the same conclusions and much more quickly if we had used the complex exponential notation. Writing

$$y = \boldsymbol{A}e^{j\omega t}$$

then

$$v = \boldsymbol{A}j\omega e^{j\omega t} = j\omega y$$

and

$$a = \boldsymbol{A}(j)^2\ \omega^2 e^{j\omega t} = -\boldsymbol{A}\omega^2 e^{j\omega t} = -\omega^2 y$$

4 VIBRATING SYSTEMS

4.1 Mass–Spring System

A. Free Vibration — Undamped Suppose a mass of M kilogram is placed on a spring of stiffness K newton-metre (see Fig. 5*a*), and the mass is allowed to sink down a distance d metres to its equilibrium position under its own weight Mg newtons, where g is the acceleration of gravity 9.81 m/s^2. Taking forces and deflections to be positive upward gives

$$-Mg = -Kd \tag{8}$$

thus the static deflection d of the mass is

$$d = Mg/K \tag{8a}$$

The distance d is normally called the *static deflection* of the mass; we define a new displacement coordinate system, where $Y = 0$ is the location of the mass after the gravity force is allowed to compress the spring.

Suppose now we displace the mass a distance y from its equilibrium position and release it; then it will oscillate about this position. We will measure the deflection from the equilibrium position of the mass (see Fig. 5*b*). Newton's law states that force is equal to mass × acceleration. Forces and deflections are again assumed positive upward, and thus

$$-Ky = M\frac{d^2y}{dt^2} \tag{9}$$

Let us assume a solution to Eq. (9) of the form $y = A\sin(\omega t + \phi)$. Then upon substitution into Eq. (9) we obtain

$$-KA\sin(\omega t + \phi) = M[-\omega^2\sin(\omega t + \phi)]$$

We see our solution satisfies Eq. (9) only if

$$\omega^2 = K/M$$

The system vibrates with free vibration at an angular frequency ω radians/second. This frequency, ω, which is generally known as the *natural* angular frequency, depends only on the stiffness K and

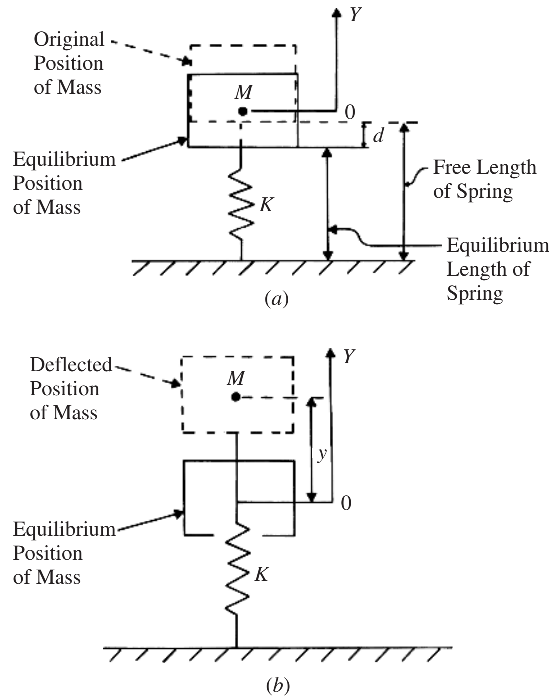

Figure 5 Movement of mass on a spring: (*a*) static deflection due to gravity and (*b*) oscillation due to initial displacement y_0.

mass M. We normally signify this so-called natural frequency with the subscript n. And so

$$\omega_n = \sqrt{K/M}$$

and from Eq. (2)

$$f_n = \frac{1}{2\pi}\sqrt{K/M} \quad \text{Hz} \tag{10}$$

The frequency, f_n hertz, is known as the *natural frequency* of the mass on the spring. This result, Eq.(10), looks physically correct since if K increases (with M constant), f_n increases. If M increases with K constant, f_n decreases. These are the results we also find in practice.

We have seen that a solution to Eq. (9) is $y = A\sin(\omega t + \phi)$ or the same as Eq. (3). Hence we know that *any system that has a restoring force that is proportional to the displacement* will have a displacement that is *simple harmonic*. This is an alternative definition to that given in Section 3 for *simple harmonic motion.*

B. Free Vibration — Damped Many mechanical systems can be adequately described by the simple mass–spring system just discussed above. However, for some purposes it is necessary to include the effects of losses (sometimes called damping). This

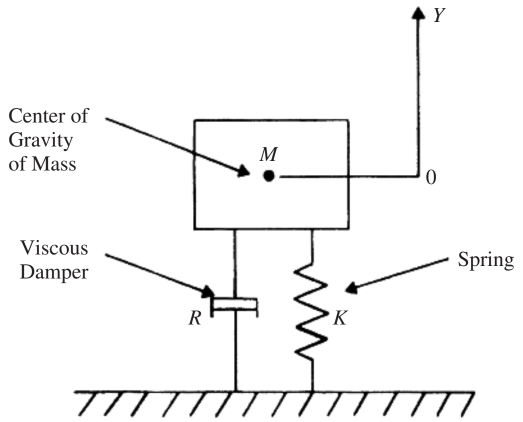

Figure 6 Movement of damped simple system.

is normally done by including a viscous damper in the system (see Fig. 6). See Chapters 15 and 60 for further discussion on passive damping. With viscous or "coulomb" damping the friction or damping force F_d is assumed to be proportional to the velocity, dy/dt. If the constant of proportionality is R, then the damping force F_d on the mass is

$$F_d = -R\frac{dy}{dt} \tag{11}$$

and Eq. (9) becomes

$$-R\frac{dy}{dt} - Ky = M\frac{d^2y}{dt^2} \tag{12}$$

or equivalently

$$M\ddot{y} + R\dot{y} + Ky = 0 \tag{13}$$

where the dots represent single and double differentiation with respect to time.

The solution of Eq. (13) is most conveniently found by assuming a solution of the form: y is the real part of $\boldsymbol{A}^{j\lambda t}$ where $\boldsymbol{A}$ is a complex number and λ is an arbitrary constant to be determined. By substituting $y = \boldsymbol{A}^{j\lambda t}$ into Eq. (13) and assuming that the damping constant R is small, $R < (4MK)^{1/2}$ (which is true in most engineering applications), the solution is found that:

$$y = \boldsymbol{A}e^{-(R/2M)t}\sin(\omega_d t + \phi) \tag{14}$$

Here ω_d is known as the damped "natural" angular frequency:

$$\omega_d = \omega_n[1 - (R/2M)^2]^{1/2} \tag{15}$$

where ω_n is the undamped natural frequency $\sqrt{K/M}$. The motion described by Eq. (14) is plotted in Fig.7.

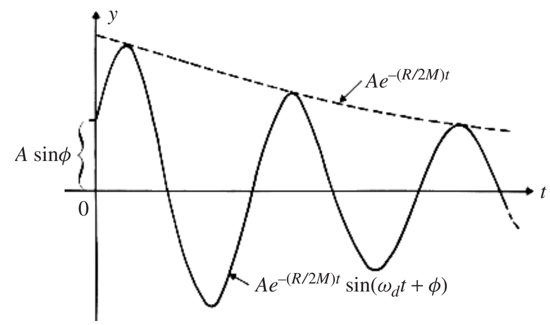

Figure 7 Motion of a damped mass–spring system, $R < (4MK)^{1/2}$.

The amplitude of the motion decreases with time unlike that for undamped motion (Fig. 3). If the damping is increased until R equals $(4MK)^{1/2}$, the damping is then called critical, $R_{\text{crit}} = (4MK)^{1/2}$. In this case, if the mass in Fig. 6 is displaced, it gradually returns to its equilibrium position and the displacement never becomes negative. In other words, there is no oscillation or vibration. If $R > (4MK)^{1/2}$, the system is said to be overdamped.

The ratio of the damping constant R to the critical damping constant R_{crit} is called the damping ratio δ:

$$\delta = R/R_{\text{crit}} = R/(2M\omega) \tag{16a}$$

In most engineering cases the damping ratio, δ, in a structure is hard to predict and is of the order of 0.01 to 0.1. There are, however, several ways to measure damping experimentally. (See Chapters 15 and 60.)

C. Forced Vibration — Damped If a damped spring–mass system is excited by a simple harmonic force at some arbitrary angular forcing frequency ω (Fig. 8), we now obtain the equation of motion (16b):

$$M\ddot{y} + R\dot{y} + Ky = \boldsymbol{F}e^{j(\omega t)} = |\boldsymbol{F}|e^{j(\omega t+\phi)} \tag{16b}$$

The force $\boldsymbol{F}$ is normally written in the complex form for mathematical convenience. The real force acting is, of course, the real part of $\boldsymbol{F}$ or $|\boldsymbol{F}|\cos(\omega t)$, where $|\boldsymbol{F}|$ is the force amplitude.

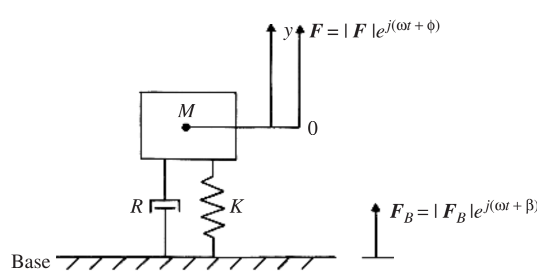

Figure 8 Forced vibration of damped simple system.

If we assume a solution of the form $y = \mathbf{A}e^{j\omega t}$ then we obtain from Eq. (16b):

$$\boldsymbol{A} = \frac{|\boldsymbol{F}|}{j\omega R + K - M\omega^2} \tag{17}$$

We can write $\boldsymbol{A} = |\boldsymbol{A}|e^{j\alpha}$, where α is the phase angle between force and displacement. The phase, α, is not normally of much interest, but the amplitude of motion $|\boldsymbol{A}|$ of the mass is. The amplitude of the displacement is

$$|\boldsymbol{A}| = \frac{|\boldsymbol{F}|}{[\omega^2 R^2 + (K - M\omega^2)^2]^{1/2}} \tag{18}$$

This can be expressed in alternative form:

$$\frac{|\boldsymbol{A}|}{|\boldsymbol{F}|/K} = \frac{1}{[4\delta^2(\omega/\omega_n)^2 + (1 - (\omega/\omega_n)^2)^2]^{1/2}} \tag{19}$$

Equation (19) is plotted in Fig. 9. It is observed that if the forcing frequency ω is equal to the natural

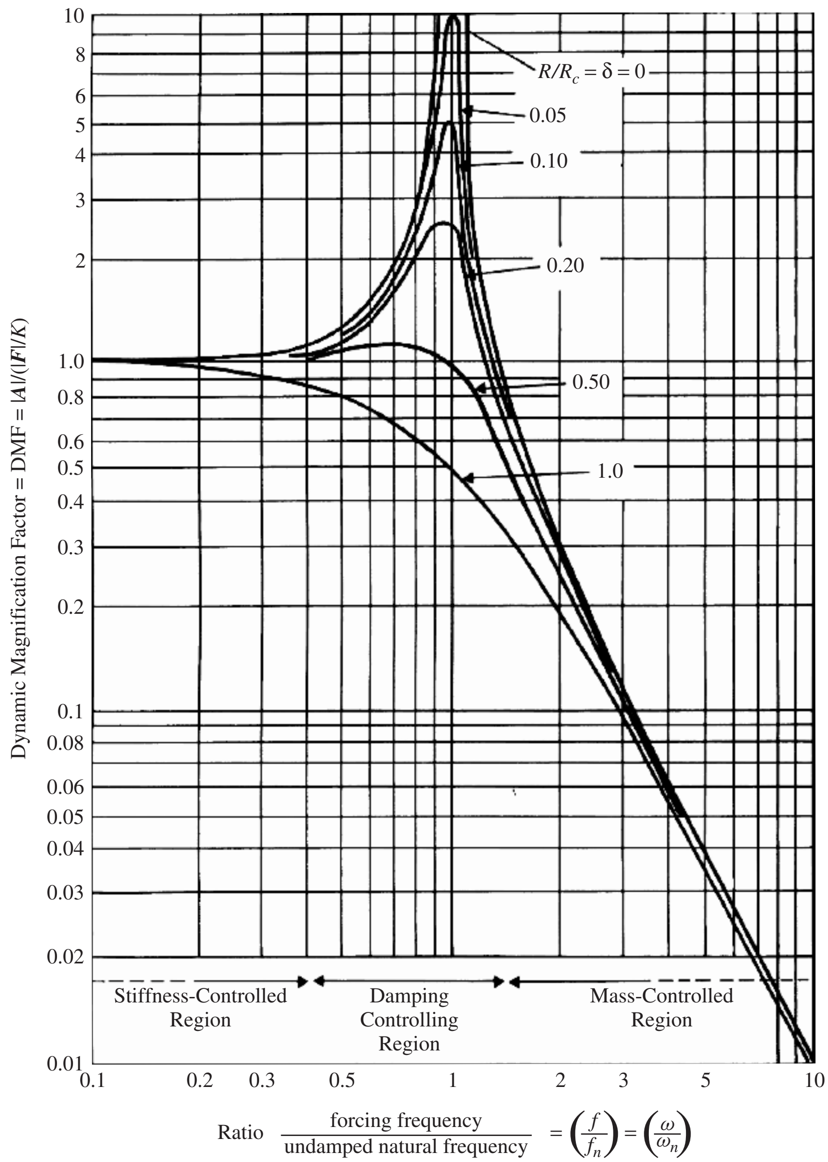

Figure 9 Dynamic modification factor (DMF) for a damped simple system.

frequency of the structure, ω_n, or equivalently $f = f_n$, a condition called resonance, then the amplitude of the motion is proportional to $1/(2\delta)$. The ratio $|\boldsymbol{A}|/(|\boldsymbol{F}|/K)$ is sometimes called the *dynamic magnification factor* (DMF). The number $|\boldsymbol{F}|/K$ is the *static deflection* the mass would assume if exposed to a *constant* nonfluctuating force $|\boldsymbol{F}|$. If the damping ratio, δ, is small, the displacement amplitude $\boldsymbol{A}$ of a structure excited at its *natural* or *resonance* frequency is very high. For example, if a simple system has a damping ratio, δ, of 0.01, then its dynamic displacement amplitude is 50 times (when exposed to an oscillating force of $|\boldsymbol{F}|$ N) its static deflection (when exposed to a static force of amplitude $|\boldsymbol{F}|$ N), that is, DMF = 50. Situations such as this should be avoided in practice, wherever possible. For instance, if an oscillating force is present in some machine or structure, the frequency of the force should be moved away from the natural frequencies of the machine or structure, if possible, so that resonance is avoided. If the forcing frequency f is close to or coincides with a natural frequency f_n, large amplitude vibrations can occur with consequent vibration and noise problems and the potential of serious damage and machine malfunction.

The force on the idealized damped simple system will create a force on the base $\boldsymbol{F}_B = R\dot{y} + Ky$. Substituting this into Eq. (16) and rearranging and finally comparing the amplitudes of the imposed force $|\boldsymbol{F}|$ with the force transmitted to the base $|\boldsymbol{F}_B|$ gives

$$\frac{|\boldsymbol{F}_B|}{|\boldsymbol{F}|} = \left[\frac{1 + 4\delta^2(\omega/\omega_n)^2}{4\delta^2(\omega/\omega_n)^2 + (1 - (\omega/\omega_n)^2)^2}\right]^{1/2} \quad (20)$$

Equation (20) is plotted in Fig. 10. The ratio $|\boldsymbol{F}_B|/|\boldsymbol{F}|$ is sometimes called the force transmissibility T_F. The force amplitude transmitted to the machine support base, F_B, is seen to be much greater than one, if the exciting frequency is at the system resonance frequency. The results in Eq. (20) and Fig. 10 have important applications to machinery noise problems that will be discussed again in detail in Chapter 54. Briefly, we can observe that these results can be utilized in designing vibration isolators for a machine. The natural frequency ω_n of a machine of mass M resting on its isolators of stiffness K and damping constant R must be made much less than the forcing frequency ω. Otherwise, large force amplitudes will be transmitted to the machine base. Transmitted forces will excite vibrations in machine supports and floors and walls of buildings, and the like, giving rise to additional noise radiation from these other areas. Chapter 59 gives a more complete discussion on vibration isolation.

5 PROPAGATION OF SOUND

5.1 Plane Sound Waves

The propagation of sound may be illustrated by considering gas in a tube with rigid walls and having a rigid piston at one end. The tube is assumed to be infinitely long in the direction away from the piston. We shall assume that the piston is vibrating with simple harmonic motion at the left-hand side of the tube (see Fig. 11) and that it has been oscillating back and forth for some time. We shall only consider the piston motion and the oscillatory motion it induces in the fluid from when we start our clock. Let us choose to start our clock when the piston is moving with its maximum velocity to the right through its normal equilibrium position at $x = 0$. See the top of Fig. 11, at $t = 0$. As time increases from $t = 0$, the piston straight away starts slowing down with simple harmonic motion, so that it stops moving at $t = T/4$ at its maximum excursion to the right. The piston then starts moving to the left in its cycle of oscillation, and at $t = T/2$ it has reached its equilibrium position again and has a maximum velocity (the same as at $t = 0$) but now in the negative x direction. At $t = 3T/4$, the piston comes to rest again at its maximum excursion to the left. Finally at $t = T$ the piston reaches its equilibrium position at $x = 0$ with the same maximum velocity we imposed on it at $t = 0$. During the time T, the piston has undergone one complete cycle of oscillation. We assume that the piston continues vibrating and makes f oscillations each second, so that its frequency $f = 1/T$ (Hz).

As the piston moves backward and forward, the gas in front of the piston is set into motion. As we all know, the gas has mass and thus inertia and it is also compressible. If the gas is compressed into a smaller volume, its pressure increases. As the piston moves to the right, it compresses the gas in front of it, and as it moves to the left, the gas in front of it becomes rarified. When the gas is compressed, its pressure increases above atmospheric pressure, and, when it is rarified, its pressure decreases below atmospheric pressure. The pressure difference above or below the atmospheric pressure, p_0, is known as the sound pressure, p, in the gas. Thus the sound pressure $p = p_{\text{tot}} - p_0$, where p_{tot} is the total pressure in the gas. If these pressure changes occurred at constant temperature, the fluid pressure would be directly proportional to its density, ρ, and so $p/\rho =$ constant. This simple assumption was made by Sir Isaac Newton, who in 1660 was the first to try to predict the speed of sound. But we find that, in practice, regions of high and low pressure are sufficiently separated in space in the gas (see Fig. 11) so that heat cannot easily flow from one region to the other and that the adiabatic law, $p/\rho^{\gamma} =$ constant, is more closely followed in nature.

As the piston moves to the right with maximum velocity at $t = 0$, the gas ahead receives maximum compression and maximum increase in density, and this simultaneously results in a maximum pressure increase. At the instant the piston is moving to the left with maximum negative velocity at $t = T/2$, the gas behind the piston, to the right, receives maximum rarefaction, which results in a maximum density and pressure decrease. These piston displacement and velocity perturbations are superimposed on the much greater random motion of the gas molecules (known as the Brownian motion). The mean speed of the molecular random motion in the gas depends on its absolute

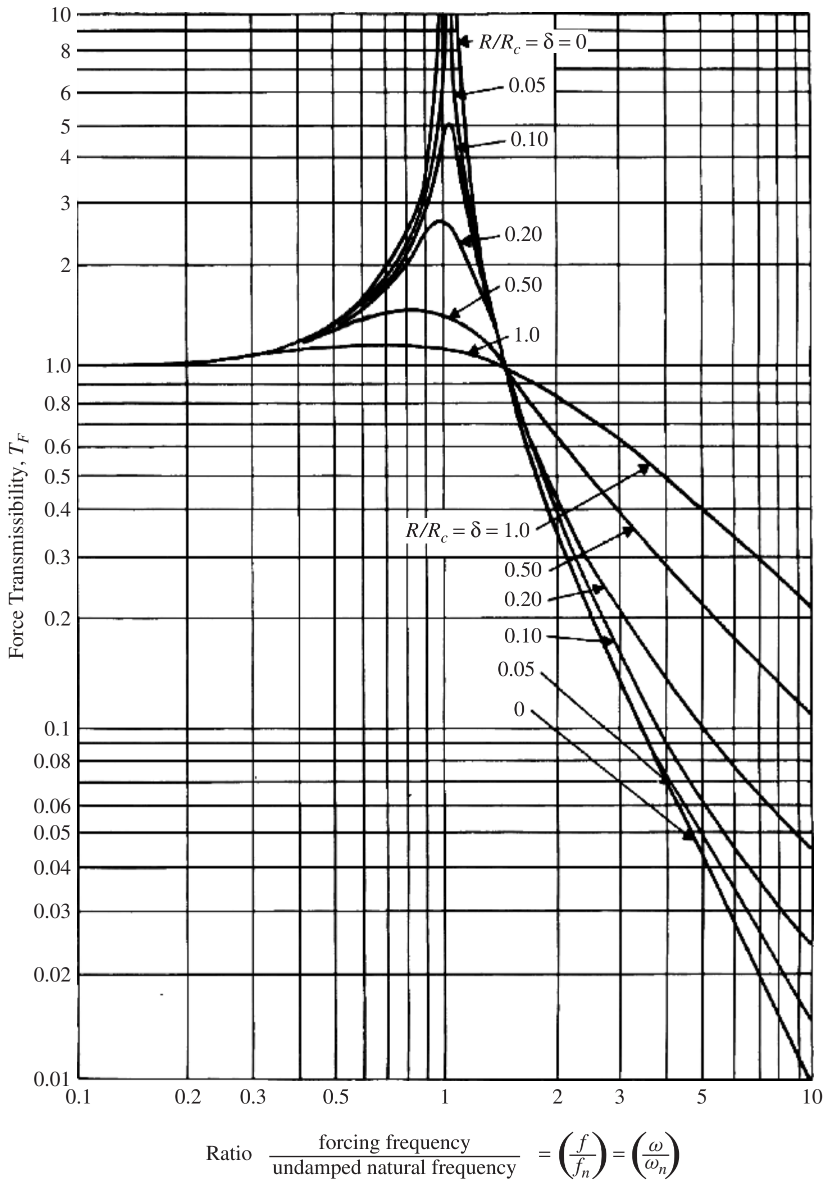

Figure 10 Force transmissibility, T_F, for a damped simple system.

temperature. The disturbances induced in the gas are known as acoustic (or sound) disturbances. It is found that momentum and energy pulsations are transmitted from the piston throughout the whole region of the gas in the tube through molecular interactions (sometimes simply termed molecular collisions). The rate at which the motion is transmitted throughout the fluid depends upon its absolute temperature. The speed of transmission is known as the speed of sound, c_0:

$$c_0 = (\gamma RT)^{1/2} \quad \text{metres/second}$$

where γ is the ratio of specific heats, R is the gas constant of the fluid in the tube, and T is the absolute temperature (K). A small region of fluid instantaneously enclosing a large number of gas molecules is known as a particle. The motion of the gas particles "mimics" the piston motion as it moves back and forth. The velocity of the gas particles (superimposed on the random Brownian motion of the molecules) depends upon the velocity of the piston as it moves back and forth and is completely unrelated to the speed of the sound propagation c_0. For a given amplitude of vibration of

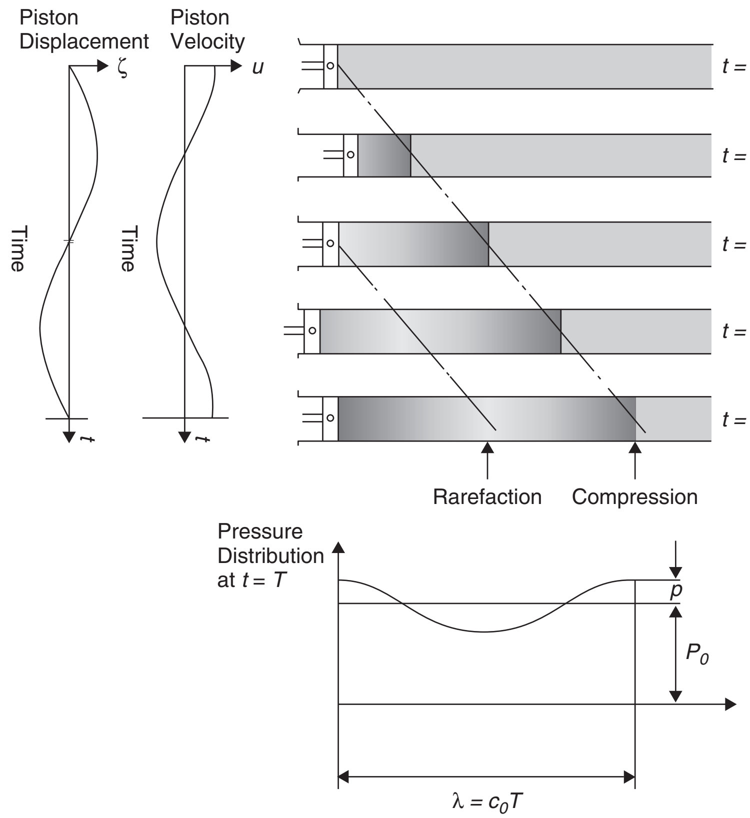

Figure 11 Schematic illustration of the sound pressure distribution created in a tube by a piston undergoing one complete simple harmonic cycle of operation in period T seconds.

the piston, A, we know from Eq. (4) that the velocity amplitude is ωA, which increases with frequency, and thus the piston only has a high-velocity amplitude if it is vibrated at high frequency.

Figure 11 shows the way that sound disturbances propagate along the tube from the oscillating piston. Dark regions in the tube indicate regions of high gas compression and high positive sound pressure. Light regions in the tube indicate regions of rarefaction and low negative sound pressure. Since the motion in the fluid is completely repeated periodically at one location and also is identically repeated spatially along the tube, we call the motion wave motion. At time $t = T$, the fluid disturbance, which was caused by the piston beginning at $t = 0$, will only have reached a distance c_0T along the tube. We call this location, the location of the *wave front* at the time T. Figure 11 shows that at distance c_0T along the tube, at which the motion starts to repeat itself. The distance c_0T is known as the wavelength λ (metres), and thus

$$\lambda = c_0 T \quad \text{metres}$$

Figure 11 shows the location of the wave front for different times and the sound pressure distribution in the tube at $t = T$. The sound pressure distribution at some instant t is given by

$$p = P\cos(2\pi x/\lambda)$$

where P is the sound pressure amplitude (N/m^2). Since the piston is assumed to vibrate with simple harmonic motion with period T, its frequency of oscillation $f = 1/T$. Thus the wavelength λ (m) can be written

$$\lambda = c_0/f$$

The sound pressure distribution, p (N/m^2), in the tube at any time t (s) can thus be written

$$p = P\cos[2\pi(x/\lambda - t/T)]$$

or

$$p = P\cos[(kx - \omega t)]$$

where $k = 2\pi/\lambda = \omega/c_0$ and $\omega = 2\pi f$. The parameter, k, is commonly known as the wavenumber, although the term wavelength parameter is better, since k has the dimensions of 1/m.

5.2 Sound Pressure

With sound waves in a fluid such as air, the sound pressure at any point is the difference between the total pressure and normal atmospheric pressure. The sound pressure fluctuates with time and can be positive or negative with respect to the normal atmospheric pressure.

Sound varies in magnitude and frequency and it is normally convenient to give a single number measure of the sound by determining its time-averaged value. The time average of the sound pressure at any point in space, over a sufficiently long time, is zero and is of no interest or use. The time average of the square of the sound pressure, known as the mean square pressure, however, is not zero. If the sound pressure at any instant t is $p(t)$, then the mean square pressure, $\langle p^2(t)\rangle_t$, is the time average of the square of the sound pressure over the time interval T:

$$\langle p^2(t)\rangle_t = \frac{1}{T}\int_0^T p^2(t)\ dt \tag{21}$$

where $\langle\rangle_t$ denotes a time average.

It is usually convenient to use the square root of the mean square pressure:

$$p_{\text{rms}} = \sqrt{\langle p^2(t)\rangle_t} = \sqrt{\frac{1}{T}\int_0^T p^2(t)\ dt}$$

which is known as the root mean square (rms) sound pressure. This result is true for all cases of continuous sound time histories including noise and pure tones. For the special case of a pure tone sound, which is simple harmonic in time, given by $p = P\cos(\omega t)$, the root mean square sound pressure is

$$p_{\text{rms}} = P/\sqrt{2} \tag{22}$$

where P is the sound pressure amplitude.

5.3 Particle Velocity

As the piston vibrates, the gas immediately next to the piston must have the same velocity as the piston. A small element of fluid is known as a *particle*, and its velocity, which can be positive or negative, is known as the *particle velocity*. For waves traveling away from the piston in the positive x direction, it can be shown that the particle velocity, u, is given by

$$u = p/\rho c_0 \tag{23}$$

where ρ = fluid density (kg/m^3) and c_0 = speed of sound (m/s).

If a wave is reflected by an obstacle, so that it is traveling in the negative x direction, then

$$u = -p/\rho c_0 \tag{24}$$

The negative sign results from the fact that the sound pressure is a scalar quantity, while the particle velocity is a vector quantity. These results are true for any type of plane sound waves, not only for sinusoidal waves.

5.4 Sound Intensity

The intensity of sound, I, is the time-averaged sound energy that passes through unit cross-sectional area in unit time. For a plane progressive wave, or far from any source of sound (in the absence of reflections):

$$I = p_{\text{rms}}^2/\rho c_0 \tag{25}$$

where ρ = the fluid density (kg/m^3) and c_0 = speed of sound (m/s).

In the general case of sound propagation in a three-dimensional field, the sound intensity is the (net) flow of sound energy in unit time flowing through unit cross-sectional area. The intensity has magnitude and direction

$$I = pu_r = \langle p \cdot u_r\rangle_t = \frac{1}{T}\int_0^T p \cdot u_r\ dt \tag{26}$$

where p is the total fluctuating sound pressure, and u_r is the total fluctuating sound particle velocity in the r direction at the measurement point. The total sound pressure p and particle velocity u_r include the effects of incident and reflected sound waves.

5.5 Energy Density

Consider the case again of the oscillating piston in Fig. 11. We shall consider the sound energy that is produced by the oscillating piston, as it flows along the tube from the piston. We observe that the wavefront and the sound energy travel along the tube with velocity c_0 metres/second. Thus after 1 s, a column of fluid of length c_0 m contains all of the sound energy provided by the piston during the previous second. The total amount of energy E in this column equals the time-averaged sound intensity multiplied by the cross-sectional area S, which is from Eq. (22):

$$E = SI = Sp_{\text{rms}}^2/\rho c_0 \tag{27}$$

The sound per energy unit volume is known as the energy density ε,

$$\varepsilon = \lfloor Sp_{\text{rms}}^2/\rho c_0\rfloor/c_0 S = p_{\text{rms}}^2/\rho c_0^2 \tag{28}$$

This result in Eq. (28) can also be shown to be true for other acoustic fields as well, as long as the total sound pressure is used in Eq. (28), and provided the location is not very close to a sound source.

5.6 Sound Power

Again in the case of the oscillating piston, we will consider the sound power radiated by the piston into the tube. The sound power radiated by the piston, W, is

$$W = SI \tag{29}$$

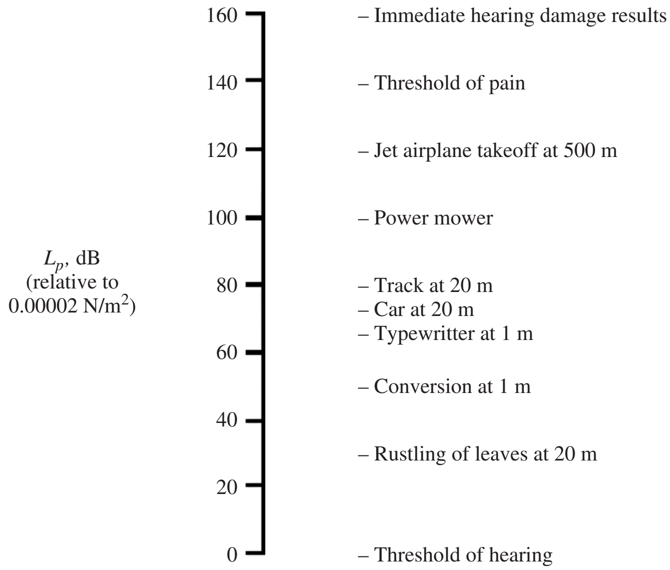

Figure 12 Some typical sound pressure levels, L_p.

But from Eqs. (23) and (25) the power is

$$W = S(p_{\text{rms}} u_{\text{rms}}) \tag{29a}$$

and close to the piston, the rms particle velocity, u_{rms}, must be equal to the rms piston velocity.
From Eq. (29a), we can write

$$W = S\rho c_0 v_{\text{rms}}^2 = 4\pi r^2 \rho c_0 v_{\text{rms}}^2 \tag{30}$$

where r is the piston and duct radius, and v_{rms} is the rms velocity of the piston.

6 DECIBELS AND LEVELS

The range of sound pressure magnitudes and sound powers of sources experienced in practice is very large. Thus, logarithmic rather than linear measures are often used for sound pressure and sound power. The most common measure of sound is the *decibel.* Decibels are also used to measure vibration, which can have a similar large range of magnitudes. The decibel represents a relative measurement or ratio. Each quantity in decibels is expressed as a ratio relative to a *reference sound pressure, sound power,* or *sound intensity*, or in the case of vibration relative to a *reference displacement, velocity,* or *acceleration.* Whenever a quantity is expressed in decibels, the result is known as a *level.*

The decibel (dB) is the ratio R_1 given by

$$\log_{10} R_1 = 0.1 \qquad 10 \log_{10} R_1 = 1 \text{ dB} \tag{31}$$

Thus, $R_1 = 10^{0.1} = 1.26$. The decibel is seen to represent the ratio 1.26. A larger ratio, the *bel*, is sometimes used. The bel is the ratio R_2 given by $\log_{10} R_2 = 1$. Thus, $R_2 = 10^1 = 10$. The bel represents the ratio 10 and is thus much larger than a decibel.

6.1 Sound Pressure Level

The sound pressure level L_p is given by

$$L_p = 10 \log_{10}\left(\frac{\langle p^2\rangle_t}{p_{\text{ref}}^2}\right) = 10 \log_{10}\left(\frac{p_{\text{rms}}^2}{p_{\text{ref}}^2}\right) = 20 \log_{10}\left(\frac{p_{\text{rms}}}{p_{\text{ref}}}\right) \text{dB} \tag{32}$$

where p_{ref} is the reference pressure, $p_{\text{ref}} = 20\ \mu\text{Pa} = 0.00002$ N/m^2 ($= 0.0002\ \mu$bar) for air. This reference pressure was originally chosen to correspond to the quietest sound (at 1000 Hz) that the average young person can hear. The sound pressure level is often abbreviated as SPL. Figure 12 shows some sound pressure levels of typical sounds.

6.2 Sound Power Level

The sound power level of a source, L_W, is given by

$$L_W = 10 \log_{10}\left(\frac{W}{W_{\text{ref}}}\right) \text{dB} \tag{33}$$

where W is the sound power of a source and $W_{\text{ref}} = 10^{-12}$ W is the reference sound power.

Some typical sound power levels are given in Fig. 13.

6.3 Sound Intensity Level

The sound intensity level L_I is given by

$$L_I = 10 \log_{10}\left(\frac{I}{I_{\text{ref}}}\right) \text{dB} \tag{34}$$

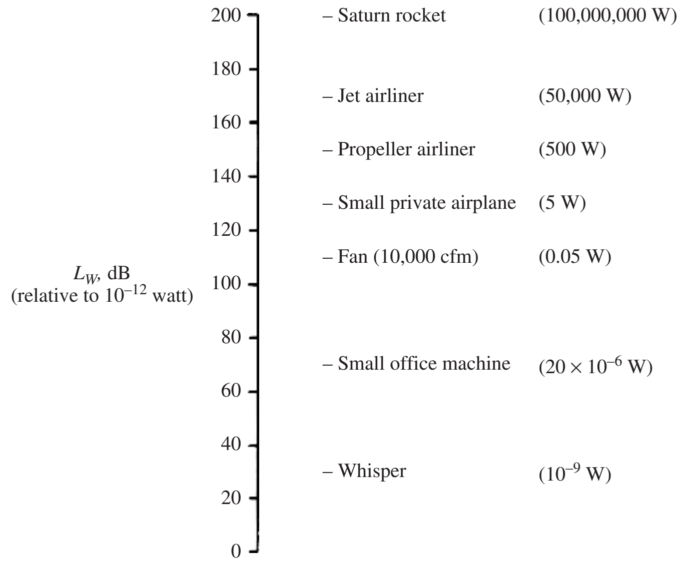

Figure 13 Some typical sound power levels, L_W.

where I is the component of the sound intensity in a given direction and $I_{\text{ref}} = 10^{-12}$ W/m^2 is the reference sound intensity.

6.4 Combination of Decibels

If the sound pressures p_1 and p_2 at a point produced by two independent sources are combined, the mean square pressure is

$$p_{\text{rms}}^2 = \frac{1}{T}\int_0^T (p_1 + p_2)^2\,dt = \langle p_1^2 + 2p_1p_2 + p_2^2\rangle_t$$
$$= \langle p_1^2\rangle_t + \langle p_2^2\rangle_t + 2\langle p_1p_2\rangle_t \equiv \overline{p_1^2} + \overline{p_2^2} + 2\overline{p_1p_2}, \qquad (35)$$

where $\langle\ \ \rangle_t$ and the overbar indicate the time average $\frac{1}{T}\int(\ )dt$.

Except for some special cases, such as two pure tones of the same frequency or the sounds from two correlated sound sources, the cross term $2\langle p_1p_2\rangle_t$ disappears if $T \to \infty$. Then in such cases, the mean square sound pressures $\overline{p_1^2}$ and $\overline{p_2^2}$ are additive, and the total mean square sound pressure at some point in space, if they are completely independent noise sources, may be determined using Eq. (35a).

$$p_{\text{rms}}^2 = \overline{p_1^2} + \overline{p_2^2} \qquad (35a)$$

Let the two mean square pressure contributions to the total noise be $p_{\text{rms}1}^2$ and $p_{\text{rms}2}^2$ corresponding to sound pressure levels L_{p1} and L_{p2}, where $L_{p2} = L_{p1} - \Delta$. The total sound pressure level is given by the sum of the individual contributions in the case of uncorrelated sources, and the total sound pressure level is given by forming the total sound pressure level by taking logarithms of Eq. (35a)

$$\begin{aligned} L_{pt} &= 10\log[(p_{\text{rms}1}^2 + p_{\text{rms}2}^2)/p_{\text{ref}}^2] \\ &= 10\log(10^{L_{p1}/10} + 10^{L_{p2}/10}) \\ &= 10\log(10^{L_{p1}/10)} + 10^{(L_{p1}-\Delta)/10} \\ &= 10\log[10^{L_{p1}/10}(1 + 10^{-\Delta/10})] \\ &= L_{p1} + 10\log(1 + 10^{-\Delta/10}) \qquad (35b) \end{aligned}$$

where, L_{pT} = combined sound pressure level due to both sources
L_{p1} = greater of the two sound pressure level contributions
Δ = difference between the two contributions, all in dB

Equation (35b) is presented in Fig. 14.

Example 1 If two independent noise sources each create sound pressure levels operating on their own of 80 dB, at a certain point, what is the total sound pressure level? *Answer:* The difference in levels is 0 dB; thus the total sound pressure level is $80 + 3 = 83$ dB.

Example 2 If two independent noise sources have sound power levels of 70 and 73 dB, what is the total level? *Answer:* The difference in levels is 3 dB; thus the total sound power level is $73 + 1.8 = 74.8$ dB.

Figure 14 and these two examples do *not* apply to the case of two pure tones of the same frequency.

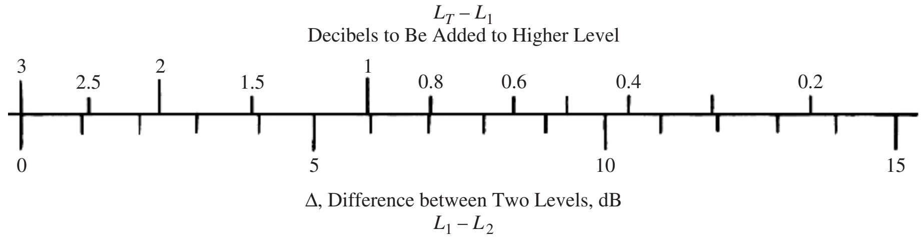

Figure 14 Diagram for combination of two sound pressure levels or two sound power levels of uncorrelated sources.

Note: For the special case of two pure tones of the same amplitude and frequency, if $p_1 = p_2$ (and the sound pressures are in phase at the point in space of the measurement):

$$L_{p_{\text{total}}} = 10 \log \left[\frac{1}{T} \int_0^T (p_1 + p_2)^2 dt \right] = L_{p_1} + 10 \log 4 \equiv L_{p_2} + 6 \text{ dB} \quad (36)$$

Example 3 If $p_1 = p_2 = 1$ Pa and the two sound pressures are of the same amplitude and frequency and in phase with each other, then the total sound pressure level

$$L_p(\text{total}) = 20 \log \left[\frac{2}{20 \times 10^{-6}} \right] = 100 \text{ dB}$$

Example 4 If $p_1 = p_2 = 1$ Pa and the two sound pressures are of the same amplitude and frequency, but in opposite phase with each other, then the total sound pressure level

$$L_p(\text{total}) = 20 \log \left[\frac{0}{20 \times 10^{-6}} \right] = -\infty \text{ dB}$$

For such a case as in Example 1 above, for pure-tone sounds, instead of 83 dB, the total sound pressure level can range anywhere between 86 dB (for in-phase sound pressures) and $-\infty$ dB (for out-of-phase sound pressures). For the Example 2 above, the total sound power radiated by the two pure-tone sources depends on the phasing and separation distance.

7 HUMAN HEARING

Human hearing is most sensitive at about 4000 Hz. We can hear sound down to a frequency of about 15 or 16 Hz and up to about 15,000 to 16,000 Hz. However, at low frequency below about 200 Hz, we cannot hear sound at all well, unless the sound pressure level is quite high. See Chapters 19 and 20 for more details. Normal speech is in the range of about 100 to 4000 Hz with vowels mostly in the low- to medium-frequency range and consonants mostly in the high-frequency range. See Chapter 22. Music has a larger frequency range and can be at much higher sound pressure levels than the human voice. Figure 15 gives an idea of the approximate frequency and sound pressure level boundaries of speech, music, and the audible range of human hearing. The lower boundary in Fig. 15 is called the threshold of hearing since sounds below this level cannot be heard by the average young person. The upper boundary is called the threshold of feeling since sounds much above this boundary can cause unpleasant sensations in the ear and even pain and, at high enough sound pressure levels, immediate damage to the hearing mechanism. See Chapter 21.

8 FREQUENCY ANALYSIS

Sound signals can be combined, but they can also be broken down into frequency components as shown by Fourier over 200 years ago. The ear seems to work as a frequency analyzer. We also can make instruments to analyze sound signals into frequency components.

Frequency analysis is commonly carried out using (a) constant frequency band filters and (b) constant percentage filters. The constant percentage filter (usually one-octave or one-third-octave band types) most parallels the way the human auditory system analyzes sound and, although digital processing has mostly overtaken analog processing of signals, it is still frequently used. See Chapters 40, 41, and 42 for more details about filters, signal processing, and data analysis.

The following symbol notation is used in Sections 8.1 and 8.2: f_L and f_U are the lower and upper cutoff frequencies, and f_C and Δf are the band center frequency and the frequency bandwidth, respectively. Thus $\Delta f = f_U - f_L$. See Fig. 16.

8.1 One-Octave Bands

For one-octave bands, the cutoff frequencies f_L and f_U are defined as follows:

$$f_L = f_C/\sqrt{2}$$
$$f_U = \sqrt{2} f_C$$

The center frequency (or geometric mean) is

$$f_C = \sqrt{f_L f_U}$$

Thus

$$f_U/f_L = 2$$

The bandwidth Δf is given by

$$\Delta f = f_U - f_L = f_C(\sqrt{2} - 1/\sqrt{2}) = f_C/\sqrt{2}$$

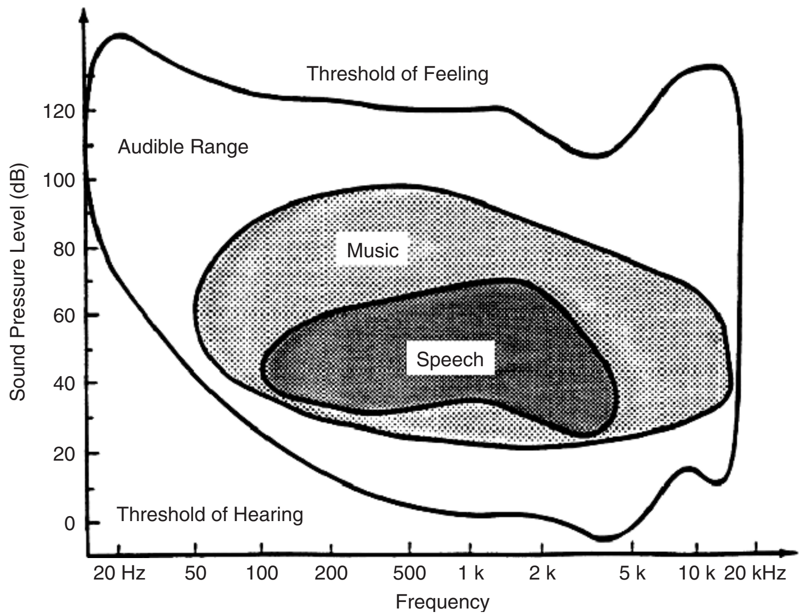

Figure 15 Sound pressure level versus frequency for the audible range, typical music range, and range of speech.

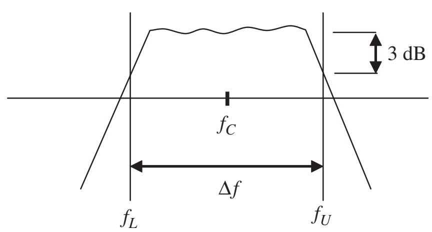

Figure 16 Typical frequency response of a filter of center frequency f_C and upper and lower cutoff frequencies, f_U and f_L.

so

$$\Delta f \approx 70\%(f_C)$$

8.2 One-Third-Octave Bands

For one-third-octave bands the cutoff frequencies, f_L and f_U, are defined as follows:

$$f_L = f_C/\sqrt[6]{2} = f_C/2^{1/6}$$
$$f_U = f_C 2^{1/6}$$

The center frequency (geometric mean) is given by

$$f_C = \sqrt{f_L f_U}$$

Thus

$$f_U/f_L = 2^{1/3}$$

The bandwidth Δf is given by

$$\Delta f = f_U - f_L = f_C(2^{1/6} - 2^{-1/6})$$

so

$$\Delta f \approx 23\%(f_C)$$

NOTE

1. The center frequencies of one-octave bands are related by 2, and 10 frequency bands are used to cover the human hearing range. They have center frequencies of 31.5, 63, 125, 250, 500, 1000, 2000, 4000, 8000, 16,000 Hz.
2. The center frequencies of one-third octave bands are related by $2^{1/3}$ and 10 cover a decade of frequency, and thus 30 frequency bands are used to cover the human hearing range: 20, 25, 31.5, 40, 50, 63, 80, 100, 125, 160,… 16,000 Hz.

9 FREQUENCY WEIGHTING (A, B, C, D)

Other filters are often used to simulate the hearing system of humans. The relative responses of A-, B-, C-, and D-weighting filters are shown in Fig. 17. The most commonly used is the A-weighting filter. These filter weightings are related to human response to pure tone sounds, although they are often used to give an approximate evaluation of the loudness of noise as well. Chapter 21 discusses the loudness of sound in more detail.

10 EQUIVALENT SOUND PRESSURE LEVEL (L_{eq})

The equivalent sound pressure level, L_{eq}, has become very frequently used in many countries in the last 20 to 25 years to evaluate industrial noise, community noise near airports, railroads, and highways. See Chapter 34

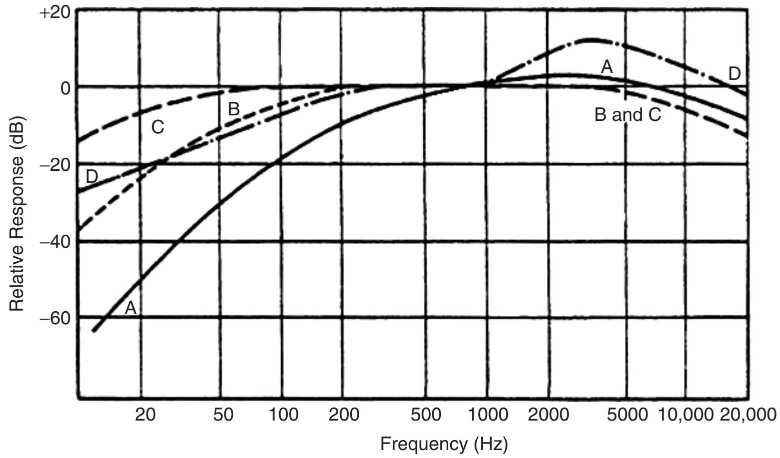

Figure 17 Frequency weightings.

for more details. The equivalent sound pressure level is defined by

$$L_{\text{eq}} = 10\log\left[\frac{\overline{p_{\text{rms}}^2}}{p_{\text{ref}}^2}\right] = 10\log\frac{1}{T}\int_0^T 10^{L(t)/10}dt$$

$$= 10\log\frac{1}{N}\sum_{i=1}^{N}10^{L_i 10} \qquad (37)$$

The averaging time T can be, for example, 1 h, 8 h, 1 day, 1 week, 1 month, and so forth. $L(t)$ is the short-time average. See Fig. 18*a*. L_i can be a set of short-time averages for L_p over set periods. If the sound pressure levels, L_i, are values averaged over constant time periods such as one hour, then they can be summed as in Eq. (37). See Fig. 18*b*. The sound pressure signal is normally filtered with an A-weighting filter.

11 DAY–NIGHT SOUND PRESSURE LEVEL(L_{dn})

In some countries, penalties are made for noise made at night. For instance in the United States the so-called day–night level is defined by

$$L_{dn} = 10\log\left[\frac{1}{24}\left\{\int_{07:00}^{22:00}10^{L_p/10}dt + \int_{22:00}^{07:00}10^{(L_p+10)/10}dt\right\}\right] \qquad (38)$$

The day–night level L_{dn} has a 10-dB penalty applied between the hours of 22:00 and 07:00. See Eq. (38).

The sound pressure level L_p readings (short time) used in Eq. (38) are normally A-weighted. The day–night descriptor can also be written

$$L_{dn} = 10\log\left[\frac{1}{24}\{15\times 10^{L_{\text{eq}d}/10} + 9\times 10^{(L_{\text{eq}n}+10)/10}\}\right] \qquad (39)$$

where $L_{\text{eq}d}$ is the A-weighted daytime equivalent sound pressure level (from 07:00 to 22:00) and $L_{\text{eq}n}$ is the night-time A-weighted sound pressure level (from 22:00 to 07:00).

12 DAY–EVENING–NIGHT SOUND PRESSURE LEVEL(L_{den})

In some countries, separate penalties are made for noise made during evening and night periods. For instance, the so-called day–evening–night level is defined by

$$L_{den} = 10\log\left[\frac{1}{24}\left\{\int_{07:00}^{19:00}10^{L_p/10}dt + \int_{19:00}^{22:00}10^{(L_p+5)/10}dt + \int_{22:00}^{07:00}10^{(L_p+10)/10}dt\right\}\right] \qquad (40)$$

The day–evening–night level L_{den} has a 5-dB penalty applied during the evening hours (here shown as 19:00 to 22:00) and a 10-dB penalty applied between the hours of 22:00 and 07:00. See Eq. (40). Local jurisdictions can set the evening period to be different

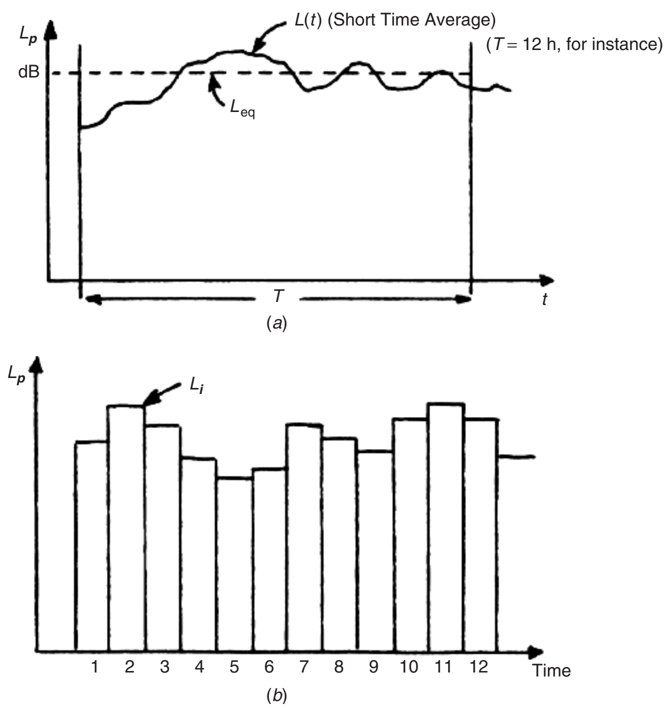

Figure 18 Equivalent sound pressure level.

from 19:00 to 22:00, if they wish to do so for their community.

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