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The Science and Art of Structural Dynamics

What do a sport-utility vehicle traveling off-road, an airplane flying near a thunderstorm, an offshore oil platform in rough seas, and an office tower during an earthquake all have in common? One answer is that all of these are structures that are subjected to *dynamic loading*, that is, to time-varying loading. The emphasis placed on the safety, performance, and reliability of mechanical and civil structures such as these has led to the need for extensive analysis and testing to determine their response to dynamic loading. The structural dynamics techniques that are discussed in this book have even been employed to study the dynamics of snow skis and violins.

Although the topic of this book, as indicated by its title, is *structural dynamics*, some books with the word *vibrations* in their title discuss essentially the same subject matter. Powerful computer programs are invariably used to implement the modeling, analysis, and testing tasks that are discussed in this book, whether the application is one in aerospace engineering, civil engineering, mechanical engineering, electrical engineering, or even in sports or music.

1.1 INTRODUCTION TO STRUCTURAL DYNAMICS

This introductory chapter is entitled "The Science and Art of Structural Dynamics" to emphasize at the outset that by studying the principles and mathematical formulas discussed in this book you will begin to understand the *science* of structural dynamics analysis. However, structural dynamicists must also master the *art* of creating mathematical models of structures, and in many cases they must also perform dynamic tests. The cover photo depicts an automobile that is undergoing such dynamic testing. *Modeling*, *analysis*, and *testing* tasks all demand that skill and judgment be exercised in order that useful results will be obtained; and all three of these tasks are discussed in this book.

A *dynamic load* is one whose magnitude, direction, or point of application varies with time. The resulting time-varying displacements and stresses constitute the *dynamic response*. If the loading is a known function of time, the loading is said to be *prescribed loading*, and the analysis of a given structural system to a prescribed loading is called a *deterministic analysis*. If the time history of the loading is not known completely but only in a statistical sense, the loading is said to be *random*. In this book we treat only prescribed dynamic loading.

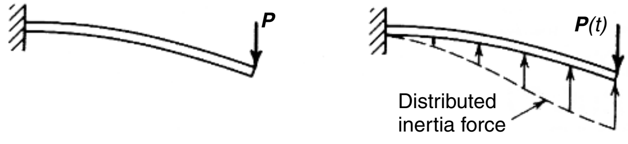


Figure 1.1 Cantilever beam under (*a*) static loading and (*b*) dynamic loading.

A structural dynamics problem differs from the corresponding static problem in two important respects. The first has been noted above: namely, the time-varying nature of the excitation. Of equal importance in a structural dynamics problem, however, is the role played by *acceleration*. Figure 1.1*a* shows a cantilever beam under static loading. The deflection and internal stresses depend directly on the static load P. On the other hand, Fig. 1.1*b* shows a similar cantilever beam subjected to a time-varying load $P(t)$. The acceleration of the beam gives rise to a distributed *inertia force*, as indicated in the figure. If the inertia force contributes significantly to the deflection of the structure and the internal stresses in the structure, a dynamical investigation is required.

Figure 1.2 shows the typical steps in a complete dynamical investigation. The three major steps, which are outlined by dashed-line boxes, are: *design*, *analysis*, and *testing*. The engineer is generally required to perform only one, or possibly two, of these steps. For example, a civil engineer might be asked to perform a dynamical analysis of an existing building and to confirm the analysis by performing specific dynamic testing of the building. The results of the analysis and testing might lead to criteria for retrofitting the building with additional bracing or damping to ensure safety against failure due to specified earthquake excitation.[1.1,1.2] Automotive engineers perform extensive analysis and vibration testing to determine the dynamical behavior of new car designs.[1.3,1.4] Results of this analysis and testing frequently lead to design changes that will improve the ride quality, economy, or safety of the vehicle.

In Section 1.2 we introduce the topic of mathematical models. In Section 1.3 we introduce the *prototype single-degree-of-freedom model* and indicate how to analyze the dynamic response of this model when it is subjected to certain simple inputs. Finally, in Section 1.4 we indicate some of the types of vibration tests that are performed on structures.

1.2 MODELING OF STRUCTURAL COMPONENTS AND SYSTEMS

Perhaps the most demanding step in any dynamical analysis is the creation of a *mathematical model* of the structure. This process is illustrated by steps 2a and 2b of Fig. 1.2. In step 2a you must contrive an idealized model of the structural system to be studied, a model essentially like the real system (which may already exist or may merely be in the design stages) but easier to analyze mathematically. This *analytical model* consists of:

1. A list of the simplifying assumptions made in reducing the real system to the analytical model
2. Drawings that depict the analytical model (e.g., see Fig. 1.3)
3. A list of the design parameters (i.e., sizes, materials, etc.)

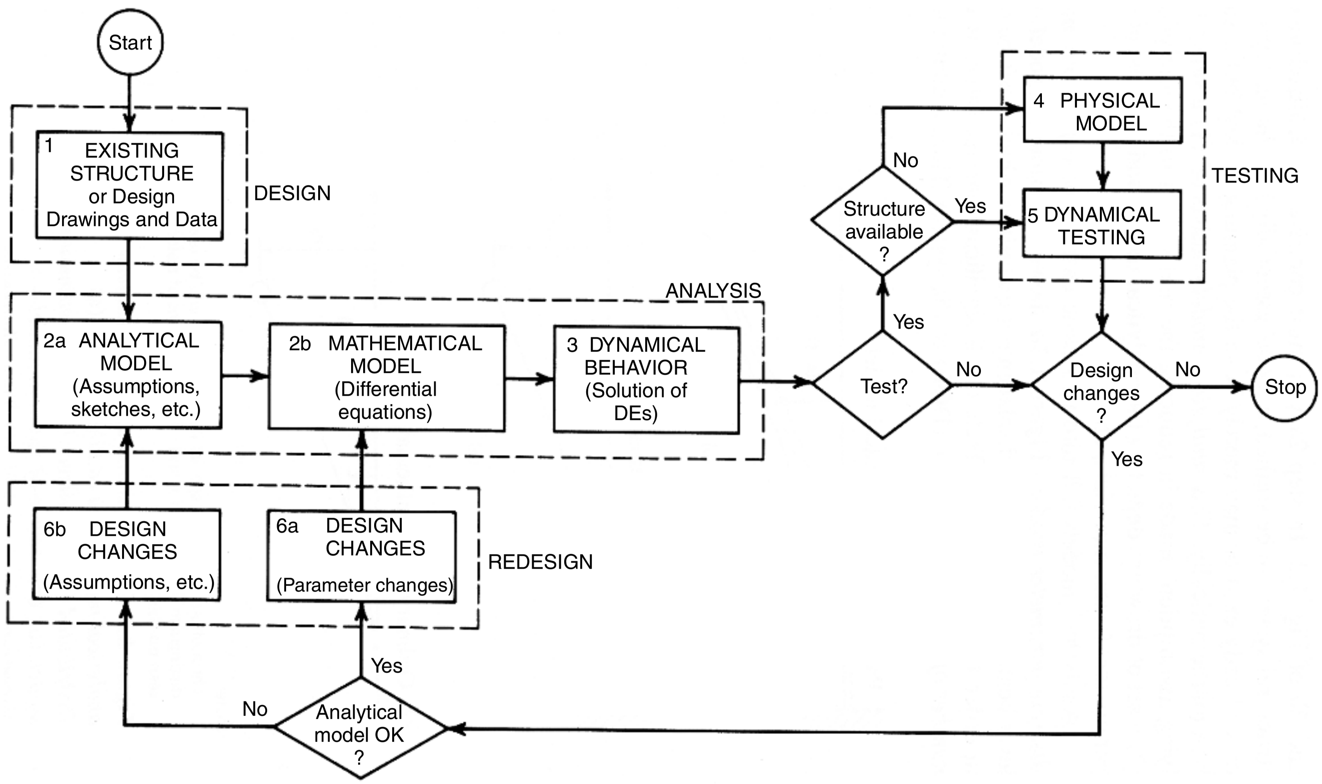


Figure 1.2 Steps in a dynamical investigation.

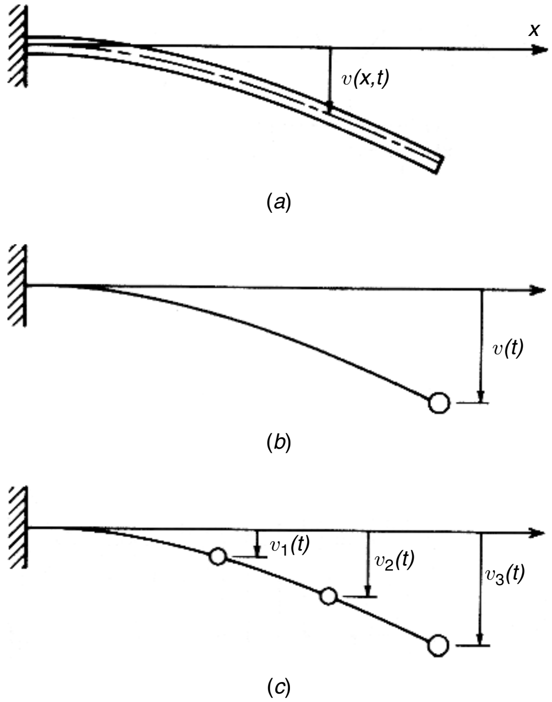


Figure 1.3 Analytical models of a cantilever beam: (*a*) distributed-mass cantilever beam, a continuous model (or distributed-parameter model); (*b*) one-degree-of-freedom model, a discrete-parameter model; (*c*) three-degree-of-freedom model, a more refined discrete-parameter model.

Analytical models fall into two basic categories: *continuous models* and *discrete-parameter models*. Figure 1.3*a* shows a continuous model of a cantilever beam. The number of displacement quantities that must be considered to represent the effects of all significant inertia forces is called the *number of degrees of freedom* (DOF) of the system. Thus, a continuous model represents an infinite-DOF system. Techniques for creating continuous models are discussed in Chapter 12. However, Fig. 1.3*b* and *c* depict finite-DOF systems. The discrete-parameter models shown here are called *lumped-mass models* because the mass of the system is assumed to be represented by a small number of point masses, or particles. Techniques for creating discrete-parameter models are discussed in Chapters 2, 8, and 14.

To create a useful analytical model, you must have clearly in mind the intended use of the analytical model, that is, the types of behavior of the real system that the model is supposed to represent faithfully. The complexity of the analytical model is determined (1) by the types and detail of behavior that it must represent, (2) by the computational analysis capability available (hardware and software), and (3) by the time and expense allowable. For example, Fig. 1.4 shows four different analytical models used in the 1960s to study the dynamical behavior of the *Apollo Saturn V* space vehicle, the vehicle that was used in landing astronauts on the surface of the moon. The 30-DOF beam-rod model was used for preliminary studies and to determine full-scale testing requirements. The 300-DOF model on the right, on the other hand, was required to give

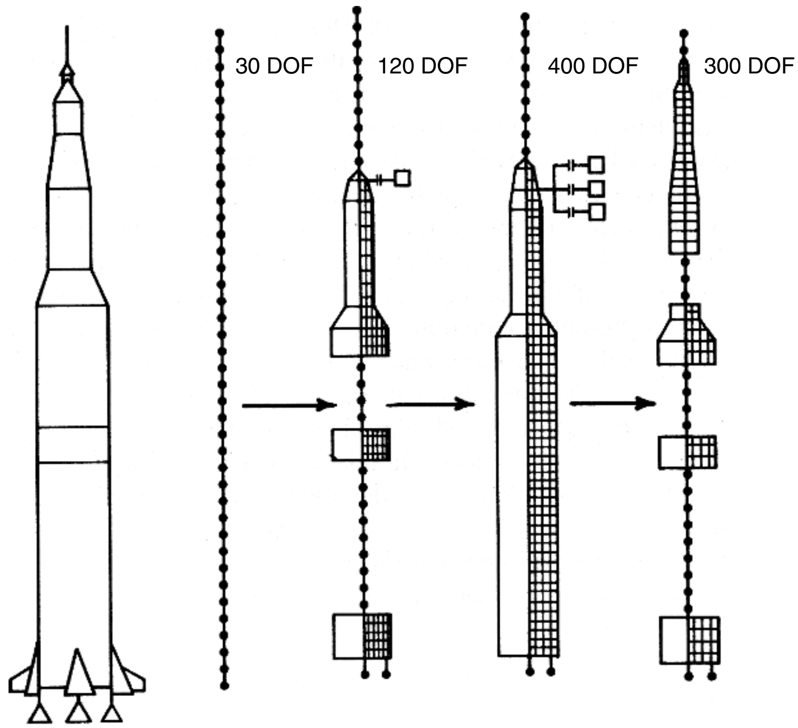
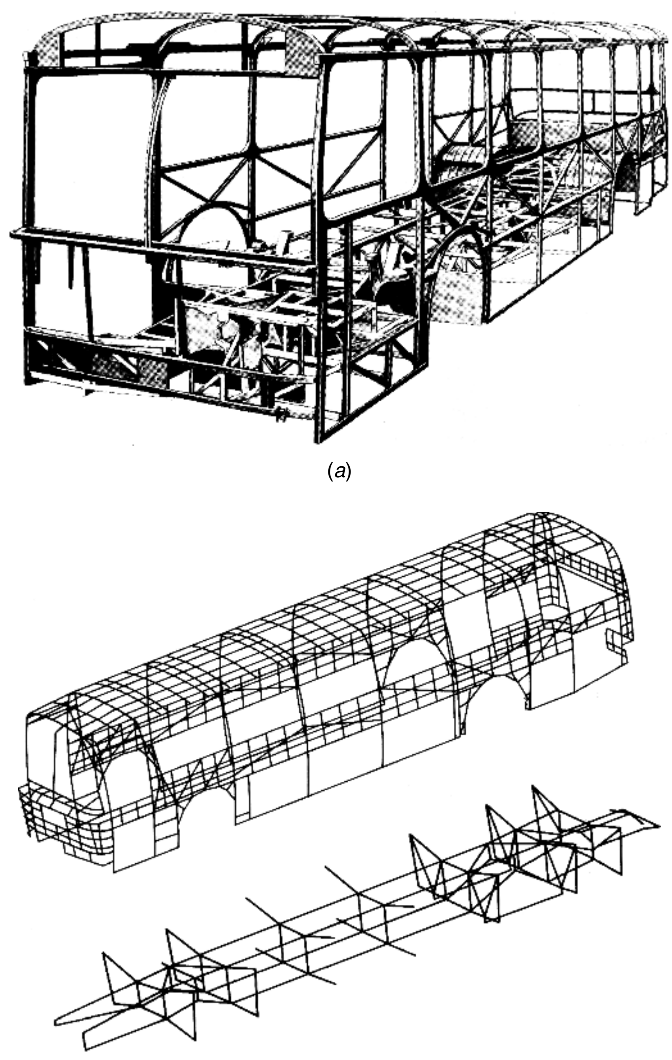


Figure 1.4 Analytical models of varying complexity used in studying the space vehicle dynamics of the *Apollo Saturn V*. (From C. E. Green et al., *Dynamic Testing for Shuttle Design Verification*, NASA, Washington, DC, 1972.)

a more accurate description of motion at the flight sensor locations. All of these *Saturn V* analytical models are extremely small compared with the multimillion-DOF models that can be analyzed now (see Section 17.8). However, supported by extensive dynamical testing, these analytical models were sufficient to ensure successful accomplishment of *Apollo V*'s moon-landing mission. Simplicity of the analytical model is very desirable as long as the model is adequate to represent the necessary behavior.

Once you have created an analytical model of the structure you wish to study, you can apply physical laws (e.g., Newton's Laws, stress–strain relationships) to obtain the differential equation(s) of motion that describe, in mathematical language, the analytical model. A continuous model leads to partial differential equations, whereas a discrete-parameter model leads to ordinary differential equations. The set of differential equations of motion so derived is called a *mathematical model* of the structure. To obtain a mathematical model, you will use methods studied in *dynamics* (e.g., Newton's Laws, Lagrange's Equations) and in *mechanics of deformable solids* (e.g., strain–displacement relations, stress–strain relations) and will combine these to obtain differential equations describing the dynamical behavior of a deformable structure.

In practice you will find that the entire process of creating first an analytical model and then a mathematical model may be referred to simply as *mathematical modeling*. In using a finite element computer program such as ABAQUS[1.5], ANSYS[1.6], MSC-Nastran[1.7], OpenFEM[1.8], SAP2000[1.9], or another computer program to carry out a structural dynamics analysis, your major modeling task will be to simplify the system and provide input data on dimensions, material properties, loads, and so on. This is



(*a*)

(*b*)

Figure 1.5 (*a*) Actual bus body and frame structure; (*b*) finite element models of the body and frame. (From D. Radaj et al., *Finite Element Analysis: An Automobile Engineer's Tool*, Society of Automotive Engineers, 1974. Used with permission of the Society of Automotive Engineers, Inc. Copyright © 1974 SAE.)

where the "art" of structural dynamics comes into play. On the other hand, actual creation and solution of the differential equations is done by the computer program. Figure 1.5 shows a picture of an actual bus body and a computer-generated plot of the idealized structure, that is, analytical model, which was input to a computer. Computer graphics software (e.g., MSC-Patran[1.7]) has become an invaluable tool for use in creating mathematical models of structures and in displaying the results of the analyses that are performed by computers.

Once a mathematical model has been formulated, the next step in a dynamical analysis is to solve the differential equation(s) to obtain the dynamical response that is predicted. (*Note:* The terms *dynamical response* and *vibration* are used interchangeably.) The two types of dynamical behavior that are of primary importance in structural applications are *free vibration* and *forced vibration* (or *forced response*), the former being the motion resulting from specified initial conditions, and the latter being the motion resulting directly from specified inputs to the system from external sources. Thus, you solve the differential equations of motion subject to specified initial conditions and to specified inputs from external sources, and you obtain the resulting time histories of the motion of the structure and stresses within the structure. This constitutes the behavior predicted for the (real) structure, or the *response*.

The analysis phase of a dynamical investigation consists of the three steps just described: defining the *analytical model*, deriving the corresponding *mathematical model*, and solving for the *dynamical response*. This book deals primarily with the second and third steps in the analysis phase of a structural dynamics investigation. Section 1.3 illustrates these steps for the simplest analytical model, a lumped-mass single-DOF model. Section 1.4 provides a brief discussion of dynamical testing.

1.3 PROTOTYPE SPRING–MASS MODEL

Before proceeding with the details of how to model complex structures and analyze their dynamical behavior, let us consider the simplest structure undergoing the simplest forms of vibration. The structure must have an *elastic component*, which can store and release potential energy; and it must have *mass*, which can store and release kinetic energy. The simplest model, therefore, is the *spring–mass oscillator*, shown in Fig. 1.6*a*.

1.3.1 Simplifying Assumptions: Analytical Model

The simplifying assumptions that define this *prototype analytical model* are:

1. The mass is a point mass that is confined to move along one horizontal direction on a frictionless plane. The displacement of the mass in the x direction from the position where the spring is undeformed is designated by the displacement variable $u(t)$.
2. The mass is connected to a fixed base by an idealized massless, linear spring. The fixed base serves as an inertial reference frame. Figure 1.6*b* shows the linear relationship between the *elongation* (u positive) and *contraction* (u negative) of the spring and the force $f_S(t)$ that the spring exerts on the mass. When the spring is in tension, f_S is positive; when the spring is in compression, f_S is negative.
3. A specified external force $p(t)$ acts on the mass, as shown in Fig. 1.6*a*.

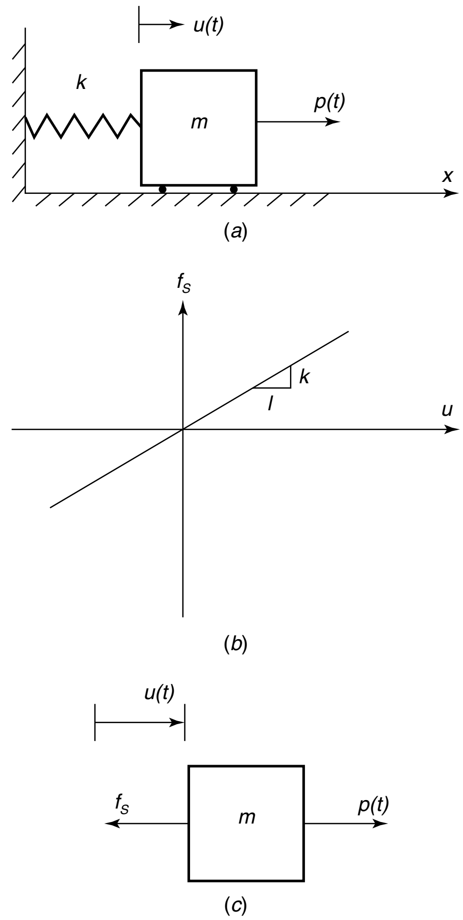


Figure 1.6 (*a*) Spring–mass oscillator; (*b*) force–elongation behavior of a linear spring; (*c*) free-body diagram of the spring–mass oscillator.

Since it takes only one variable [e.g., $u(t)$] to specify the instantaneous position of the mass, this is called a *single-degree-of-freedom* (SDOF) *system*.

1.3.2 Mathematical Model: Equation of Motion

Newton's Second Law To obtain a mathematical model describing the behavior of the spring–mass oscillator, we start by drawing a *free-body diagram* of the mass (Fig. 1.6*c*) and applying *Newton's Second Law*,

$$\sum F_x = ma_x \tag{1.1}$$

where m is the mass and a_x is the acceleration of the mass, taken as positive in the $+x$ direction. Acceleration a_x is given by the second derivative of the displacement, that is, $a_x = \ddot{u}(t)$; similarly, the velocity is given by $\dot{u}(t)$. By assuming that the mass is displaced u to the right of the position where the spring force is zero, we can say that the spring will be in tension, so the spring force will act to the left on the mass, as shown on the free-body diagram. Thus, Eq. 1.1 becomes

$$-f_S + p(t) = m\ddot{u} \tag{1.2}$$

Force–Displacement Relationship As indicated in Fig. 1.6*b*, there is assumed to be a linear relationship between the force in the spring and its elongation u, so

$$f_S = ku \tag{1.3}$$

where k is the *stiffness* of the spring.

Equation of Motion Finally, by combining Eqs. 1.2 and 1.3 and rearranging to place all u-terms on the left, we obtain the *equation of motion* for the prototype undamped SDOF model:

$$\boxed{m\ddot{u} + ku = p(t)} \tag{1.4}$$

This equation of motion is a linear second-order ordinary differential equation. It is the *mathematical model* of this simple SDOF system.

Having Eq. 1.4, the equation of motion that governs the motion of the SDOF spring–mass oscillator in Fig. 1.6*a*, we now examine the dynamic response of this prototype system. The response of the system is determined by its *initial conditions*, that is, by the values of its displacement and velocity at time $t = 0$:

$$u(0) = u_0 = \text{initial displacement}, \qquad \dot{u}(0) = v_0 = \text{initial velocity} \tag{1.5}$$

and by $p(t)$, the *external force* acting on the system. Here we consider two simple examples of vibration of the spring–mass oscillator; a more general discussion of SDOF systems follows in Chapters 3 through 7.

1.3.3 Free Vibration Example

The spring–mass oscillator is said to undergo *free vibration* if $p(t) \equiv 0$, but the mass has nonzero initial displacement u_0 and/or nonzero initial velocity v_0. Therefore, the equation of motion for free vibration is the homogeneous second-order differential equation

$$m\ddot{u} + ku = 0 \tag{1.6}$$

The general solution of this well-known simple differential equation is

$$u = A_1 \cos \omega_n t + A_2 \sin \omega_n t \tag{1.7}$$

where ω_n is the *undamped circular natural frequency*, defined by

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}} \tag{1.8}$$

The units of ω_n are radians per second (rad/s).

The constants A_1 and A_2 in Eq. 1.7 are chosen so that the two initial conditions, Eqs. 1.5, will be satisfied. Thus, *free vibration of an undamped spring–mass oscillator* is characterized by the time-dependent displacement

$$\boxed{u(t) = u_0 \cos \omega_n t + \frac{v_0}{\omega_n} \sin \omega_n t} \tag{1.9}$$

It is easy to show that this solution satisfies the differential equation, Eq. 1.6, and the two initial conditions, Eqs. 1.5.

Figure 1.7 depicts the response of a spring–mass oscillator released from rest from an initial displacement of u_0. Thus, the motion depicted in Fig. 1.7 is given by

$$u(t) = u_0 \cos \omega_n t = u_0 \cos \frac{2\pi t}{T_n} \tag{1.10}$$

From Eq. 1.10 and Fig. 1.7, free vibration of an undamped SDOF system consists of harmonic (sinusoidal) motion that repeats itself with a *period* (in seconds) given by

$$\boxed{T_n = \frac{2\pi}{\omega_n}} \tag{1.11}$$

as illustrated in Fig. 1.7. The *amplitude* of the vibration is defined as the maximum displacement that is experienced by the mass. For the free vibration depicted in Fig. 1.7, the amplitude is equal to the initial displacement u_0.

Free vibration is discussed further in Chapter 3.

1.3.4 Forced Response Example

The spring–mass oscillator is said to undergo *forced vibration* if $p(t) \neq 0$ in Eq. 1.4. Solution of the differential equation of motion for this case, Eq. 1.4, requires both a *complementary solution* u_c and a *particular solution* u_p. Thus,

$$u(t) = u_c(t) + u_p(t) \tag{1.12}$$

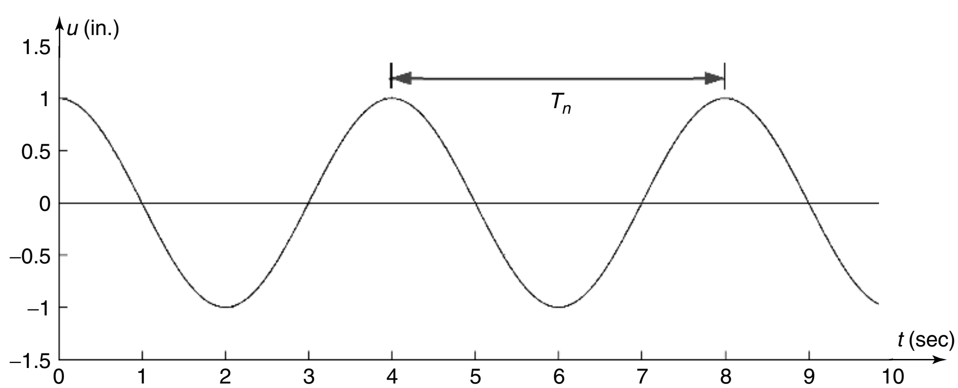


Figure 1.7 Free vibration of a spring-mass oscillator with $u_0 = 1.0$ in., $v_0 = 0$, and $T_n = 4.0$ sec.

As a simple illustration of forced vibration we consider *ramp response*, the response of the spring–mass oscillator to the linearly varying ramp excitation force given by

$$p(t) = p_0 \frac{t}{t_0}, \qquad t > 0 \tag{1.13}$$

and illustrated in Fig. 1.8a. (The time t_0 is the time at which the force reaches the value p_0.) The particular solution, like the excitation, varies linearly with time. The complementary solution has the same form as given in Eq. 1.7, so the total response has the form

$$u(t) = A_1 \cos \omega_n t + A_2 \sin \omega_n t + \frac{p_0}{k} \frac{t}{t_0} \tag{1.14}$$

where the constants A_1 and A_2 must be selected so that the initial conditions $u(0)$ and $\dot{u}(0)$ will be satisfied.

Figure 1.8b depicts the response of a spring–mass oscillator that is initially at rest at the origin, so the initial conditions are $u(0) = \dot{u}(0) = 0$. The corresponding *ramp response* is thus given by

$$\boxed{u(t) = \frac{p_0}{k} \left(\frac{t}{t_0} - \frac{1}{\omega_n t_0} \sin \omega_n t \right)} \tag{1.15}$$

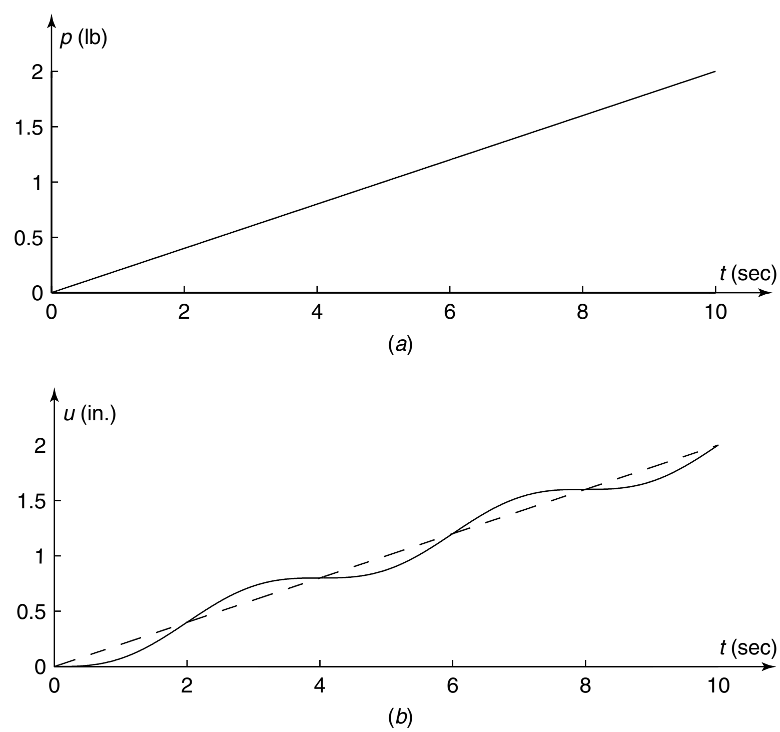


Figure 1.8 (a) Ramp excitation $p(t) = p_0(t/t_0)$ for $t > 0$, with $p_0 = 2$ lb, $t_0 = 10$ sec; (b) response of a spring–mass oscillator to ramp excitation. For (b), $k = 1$ lb/in. and $T_n = 4$ sec.

Clearly evident in this example of forced response are two components: (1) a linearly time-varying displacement (dashed curve), which is due directly to the linearly time-varying ramp excitation, and (2) an induced oscillatory motion at the undamped natural frequency ω_n. Of course, this ramp response is only valid as long as the spring remains within its linearly elastic range.

1.3.5 Conclusions

In this section we have taken a preliminary look at several characteristics that are typical of the response of structures to nonzero initial conditions and/or to time-varying excitation. We have especially noted the oscillatory nature of the response. In Chapters 3 through 7, we consider many additional examples of free and forced vibration of SDOF systems, including systems with damping.

1.4 VIBRATION TESTING OF STRUCTURES

A primary purpose of dynamical testing is to confirm a mathematical model and, in many instances, to obtain important information on loads, on damping, and on other quantities that may be required in the dynamical analysis. In some instances these tests are conducted on reduced-scale *physical models*: for example, wind tunnel tests of airplane models. In other cases, when a full-scale structure (e.g., an automobile) is available, the tests may be conducted on it.

Aerospace vehicles (i.e., airplanes, spacecraft, etc.) must be subjected to extensive static and dynamic testing on the ground prior to actual flight of the vehicle. Figure 1.9*a* shows a *ground vibration test* in progress on a Boeing 767 airplane. Note the electrodynamic shaker in place under each wingtip and the special soft support under the nose landing gear.

Dynamical testing of physical models may be employed for determining qualitatively and quantitatively the dynamical behavior characteristics of a particular class of structures. For example, Fig. 1.9*b* shows an aeroelastic model of a Boeing 777 airplane undergoing ground vibration testing in preparation for testing in a wind tunnel to aid in predicting the dynamics of the full-scale airplane in flight. Note the soft bungee-cord distributed support of the model and the two electrodynamic shakers that are attached by stingers to the engine nacelles. Figure 1.10 shows a fluid-filled cylindrical tank structure in place on a shake table in a university laboratory. The shake table is used to simulate earthquake excitation at the base of the tank structure.

Chapter 18 provides an introduction to *Experimental Modal Analysis*, a very important structural dynamics test procedure that is used extensively in the automotive and aerospace industries and is also used to test buildings, bridges, and other civil structures.

1.5 SCOPE OF THE BOOK

Part I, encompassing Chapters 2 through 7, treats single-degree-of-freedom (SDOF) systems. In Chapter 2 procedures are described for developing SDOF mathematical models; both Newton's Laws and the Principle of Virtual Displacements are employed. The free vibration of undamped and damped systems is the topic of Chapter 3, and

(a)

(b)

Figure 1.9 (*a*) Ground vibration testing of the Boeing 767 airplane; (*b*) aeroelastic wind-tunnel model of the Boeing 777 airplane. (Courtesy of Boeing Commercial Airplane Company.)

in Chapter 4 you will learn about the response of SDOF systems to harmonic (i.e., sinusoidal) excitation. This is, perhaps, the most important topic in the entire book, because it describes the fundamental characteristics of the dynamic response of flexible structures. Chapters 5 and 6 continue the discussion of dynamic response of SDOF systems. In Chapter 5 we treat closed-form methods for evaluating the response of SDOF systems to nonperiodic inputs, and Chapter 6 deals with numerical techniques. Part I concludes with Chapter 7, where the response of SDOF systems to periodic excitation is considered. Frequency-domain techniques, which are currently used widely

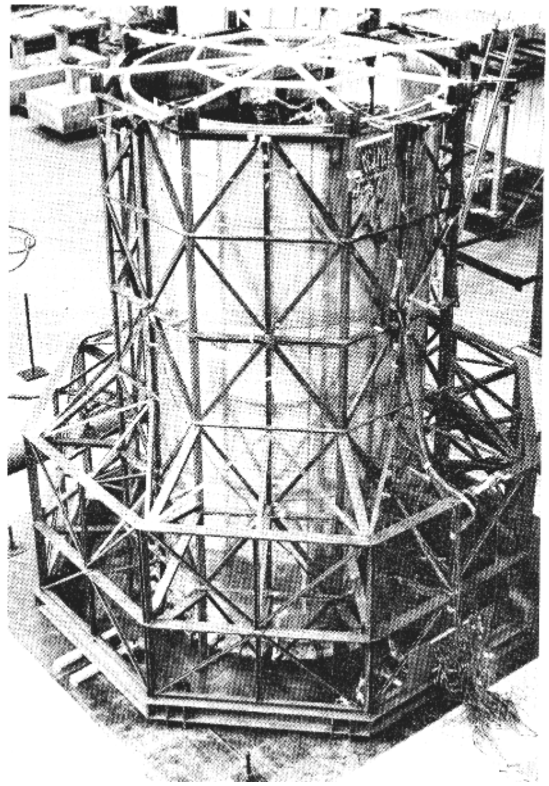


Figure 1.10 Fluid-filled tank subjected to simulated earthquake excitation. (Courtesy of R. W. Clough.)

in dynamic testing of structures, are introduced as a natural extension of the discussion of periodic excitation.

Analysis of the dynamical behavior of structures of even moderate complexity requires the use of multiple-degree-of-freedom (MDOF) models. Part II, which consists of Chapters 8 through 11, discusses procedures for obtaining MDOF mathematical models and for analyzing their free- and forced-vibration response. Mathematical modeling is treated in Chapter 8. While Newton's Laws can be used to derive mathematical models of some MDOF systems, the primary procedure employed for deriving such models is the use of Lagrange's Equations. Continuous systems are approximated by MDOF models through the use of the Assumed-Modes Method. Chapter 9, which treats vibration of undamped two-degree-of-freedom (2-DOF) systems, introduces many of the concepts that are important to a study of the response of MDOF systems: for example, natural frequencies, normal modes, and mode superposition. In Chapter 10 we discuss many of the mathematical properties related to modes and frequencies of both undamped and damped MDOF systems and introduce several popular schemes for representing damping in structures. In addition, the subject of complex modes is introduced in Chapter 10. Finally, in Chapter 11 we discuss mode superposition, the most widely used procedure for determining the response of MDOF systems.

Part III treats structures that are modeled as continuous systems. Although important topics such as the determination of partial differential equation mathematical models

(Chapter 12) and determination of modes and frequencies (Chapter 13) are discussed, the primary purpose of Part III is to provide several "exact solutions" that can be used for evaluating the accuracy of the approximate MDOF models analyzed in Parts II and IV.

Part IV presents computational techniques for handling structural dynamics applications in engineering by the methods that are widely used for analyzing and testing the behavior of complex structures (e.g., airplanes, automobiles, high-rise buildings). In Chapter 14, a continuation of the mathematical modeling topics in Chapter 8, you are introduced to the important Finite Element Method (FEM) for creating MDOF mathematical models. Chapter 14 also includes several FEM examples involving mode shapes and natural frequencies, extending the discussion of those topics in Chapters 9 and 10. Prior study of the Finite Element Method, or of Matrix Structural Analysis, is not a prerequisite for Chapter 14. Chapters 15 through 17 treat advanced numerical methods for solving structural dynamics problems: eigensolvers for determining modes and frequencies of complex structures (Chapter 15), direct integration methods for computing dynamic response (Chapter 16), and substructuring (Chapter 17).

Finally, in Part V, Chapters 18 through 20, we introduce advanced structural dynamics applications: experimental modal analysis, or modal testing (Chapter 18); structures containing piezoelectric members, or "active structures" (Chapter 19); and earthquake response of structures (Chapter 20).

A reader who merely wishes to attain an introductory level of understanding of current structural dynamics analysis techniques need only study Chapters 2 through 11 and Chapter 14.

1.6 COMPUTER SIMULATIONS; SUPPLEMENTARY MATERIAL ON THE WEBSITE

The objective of this book is to introduce you to computer methods in structural dynamics. Therefore, most chapters contain computer plots of one or more of the following: mode shapes, response time histories, frequency-response functions, and so on. Some of these are plots of closed-form mathematical expressions; others are generated by algorithms that are presented in the text. Also, a number of homework problems ask for similar computer-generated results.

To facilitate your understanding of the procedures discussed in the book, a number of supplementary materials are made available to you on the book's website www.wiley.com/college/craig. Included are the following:

- MATLAB[1] .m-files that were used to create the plots included in this book, and MATLAB .m-files for many of the numerical analysis algorithms presented in Chapters 6, 7, 15, and 16.
- Line drawings and plots in the form of .eps files.
- *ISMIS*, a MATLAB-based matrix structural analysis computer program. Included are the MATLAB .m-files, a .pdf file entitled *Special ISMIS Operations for Structural Analysis*, and two *ISMIS* examples. This software supplements Chapter 14.

[1]MATLAB is the most widely used computer software for solving dynamics problems, including structural dynamics problems of the size and scope of those in this book[1.10]. If you are inexperienced in the use of MATLAB, you should read Appendix E, "Introduction to the Use of MATLAB," in its entirety.

- *TUTORIAL NOTES: Structural Dynamics and Experimental Modal Analysis* by Peter Avitabile. These short course notes supplement Chapter 18.
- "Airplane Ground Vibration Testing—Nominal Modal Model Correlation," by Charles Pickrel. This technical article supplements Chapter 18.
- Other selected items.

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[1.8] OpenFEM, http://www-rocq.inria.fr/OpenFEM.

[1.9] SAP2000, http://www.csiberkeley.com.

[1.10] MATLAB, http://www.mathworks.com.

PROBLEMS

Problem Set 1.3[2]

1.1 (**a**) What is the natural frequency in hertz of the spring–mass system in Fig. 1.6*a* if $k = 40$ N/m and the mass is $m = 2.0$ kg? (**b**) What is the natural frequency if $k = 100$ lb/in. and the mass weighs $W = 50$ lb? Recall that $m = W/g$, where the value of g must be given in the proper units.

> Use Newton's Laws to determine the equations of motion of the SDOF spring–mass systems in Problems 1.2 and 1.3. Show all necessary free-body diagrams and deformation diagrams.

1.2 As shown in Fig. P1.2, a mass m is connected to rigid walls on both sides by identical massless, linear springs, each having stiffness k. (**a**) Following the steps in Section 1.3.2, determine the equation of motion of mass m. (**b**) Determine expressions for the undamped circular natural frequency ω_n and the period T_n of this spring–mass system. (**c**) If $k = 40$ lb/in. and the mass weighs $W = 20$ lb, what is the resulting natural frequency of this system in hertz? Recall that $m = W/g$, where the value of g must be given in the proper units.

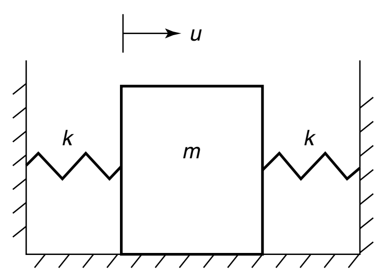

Figure P1.2

1.3 As shown in Fig. P1.3, a mass m is suspended from a rigid ceiling by a massless linear spring of stiffness k. (**a**) Following the steps in Section 1.3.2, determine the equation of motion of mass m. Measure the displacement u of the mass vertically downward from the position where the spring is unstretched. Write the equation of

[2]Problem Set headings refer to the text section to which the problem set pertains.

motion in the form given by Eq. 1.4. (**b**) Determine expressions for the undamped circular natural frequency ω_n and the period T_n of this spring–mass system. (**c**) If $k = 1.2$ N/m and the mass is $m = 0.8$ kg, what is the resulting natural frequency of this system in hertz?

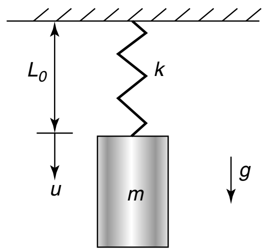

Figure P1.3

1.4 (**a**) Determine an expression for the *free vibration* of the undamped SDOF spring–mass system in Fig. 1.6*a* if $T_n = 4$ sec, $u_0 = 0$, and $v_0 = 10$ in./sec. (**b**) What is the maximum displacement of the mass as it vibrates?

1.5 Determine an expression for the *free vibration* of the undamped SDOF spring–mass system in part (**c**) of Problem 1.2 if $u_0 = 2$ in. and $v_0 = 0$.

For problems whose number is preceded by a **C**, you are to write a computer program and use it to produce the plot(s) requested. *Note*: MATLAB .m-files for many of the plots in this book may be found on the book's website.

C1.6 (**a**) Determine an expression for the *free vibration* of the undamped SDOF system in Fig. 1.6*a* if $T_n = 2$ sec, $u_0 = 0$, and $v_0 = 10$ in./sec. (**b**) Using your answer to part (a), write a computer program (e.g., a MATLAB .m-file), and generate a plot similar to Fig. 1.7.