Week 1 Multiplication and Division I

DAY 1

Before you begin the program with today's techniques, make sure you understand the basic math concepts reviewed previously. In particular, you should be able to multiply and divide by 10, 100, and so forth, as illustrated in items 5 and 6 on page 8. Today's tricks are very important because they will be used as building blocks for many of the later tricks.

Trick 1: Multiplying and Dividing with Zeroes

Strategy: The first step to performing rapid multiplication or division is to **disregard any zeroes** comprising the right-hand portion of the numbers. For example, 1,200 × 50 should be viewed as a 12 × 5 computation. Then, to complete the problem apply a "test of reasonableness." That is, ask yourself, "How many zeroes will produce an answer that makes sense?" In this case, it would seem logical to affix three zeroes to the intermediary product of 60 (12 × 5) to produce an answer of 60,000. Remember that this rule applies to rapid multiplication and division but does not apply to rapid addition and subtraction, which follow a different set of rules. Let's go over some examples, step by step.

Example #1
30 × 70

Step 1. Disregard the zeroes and think, "3 × 7."

Step 2. Multiply: 3 × 7 = 21 (intermediary product).

Step 3. Apply test of reasonableness (T of R): Because two zeroes were disregarded initially, affix two zeroes to the intermediary product, producing the answer 2,100.

Thought Process Summary

$$\begin{array}{r} 30 \\ \times\ 70 \end{array} \rightarrow \begin{array}{r} 3 \\ \times\ 7 \\ \hline 21 \end{array} \rightarrow 2{,}100$$

Example #2

4,800 ÷ 120

Step 1. Disregard the zeroes and think, "48 ÷ 12."

Step 2. Divide: 48 ÷ 12 = 4 (intermediary quotient).

Step 3. Apply T of R: As explained previously, you may cancel an equal number of right-hand zeroes when dividing. Therefore, the problem becomes 480 ÷ 12. We know that 48 ÷ 12 = 4, so 480 ÷ 12 must equal 40 (the answer).

Thought Process Summary

$$4{,}800 \div 120 \rightarrow 48 \div 12 = 4 \rightarrow 40$$

Example #3

4,500 ÷ 900

Step 1. Disregard the zeroes and think, "45 ÷ 9."

Step 2. Divide: 45 ÷ 9 = 5. (This is the answer, since an *equal* number of zeroes from the original dividend and divisor were disregarded.)

Thought Process Summary

$$4{,}500 \div 900 \rightarrow 45 \div 9 = 5$$

Number-Power Note: When you have a number such as 800.6, the two zeroes would not be disregarded, because they do not comprise the entire right-hand portion of the number.

Practice Exercises

Here are some exercises to try—remember to forget about the zeroes when starting the calculation.

1. 40 × 7 =
2. 6 × 800 =
3. 500 × 30 =
4. 60 × 900 =
5. 70 × 120 =
6. 15 × 150 =
7. 400 × 50 =
8. 24 × 400 =
9. 3,600 ÷ 900 =
10. 5,600 ÷ 7 =
11. 5,200 ÷ 130 =
12. 800 ÷ 16 =
13. 42,000 ÷ 60 =
14. 1,800 ÷ 90 =

(See solutions on page 199)

Number Potpourri #1

The number "one googol," which is 10,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000, can be easily written but couldn't be counted to in a million lifetimes. It is a number so large that it is estimated that there aren't even that many electrons in the universe. However, the number "one googolplex," which is a one followed by a googol zeroes, not only couldn't be counted to in a million lifetimes, it couldn't even be written in that same time span (except in scientific notation)!

Trick 2: Multiplying and Dividing with Decimal Points

Strategy: This is the second of two techniques to be used in conjunction with many of the later tricks. It is similar to Trick 1 and comes in handy when performing dollars and cents calculations. When you begin a rapid calculation, **disregard any decimal points.** For example, 2.4 × 1.50 should be viewed as a 24 × 15 computation. Then, to complete the problem apply a "test of reasonableness" (T of R). That is, ask yourself, "Where should I insert a decimal point within the intermediary figure to produce an answer that makes sense?" In this case, it would make sense to transform the intermediary figure of 360 (24 × 15) into 3.6. Here are some more examples:

Example #1

1.2 × 1.2

Step 1. Disregard the decimal points and think, "12 × 12."

Step 2. Multiply: 12 × 12 = 144 (intermediary product).

Step 3. Apply T of R: 144 is obviously too large to be the answer to 1.2 × 1.2. By "eyeballing" the problem, we know that the answer must be somewhere between 1 and 2.

Step 4. Insert a decimal point within the intermediary product, producing the answer 1.44.

Thought Process Summary

$$\begin{array}{r} 1.2 \\ \times 1.2 \\ \hline \end{array} \rightarrow \begin{array}{r} 12 \\ \times 12 \\ \hline 144 \end{array} \rightarrow 1.44$$

Example #2

48 ÷ 2.4

Step 1. Disregard the decimal point and think, "48 ÷ 24."

Step 2. Divide: 48 ÷ 24 = 2 (intermediary quotient).

Step 3. Apply T of R: 2 is obviously too small to be the answer to 48 ÷ 2.4. Affix one zero to the intermediary quotient, producing the answer 20.

Thought Process Summary

$48 \div 2.4 \rightarrow 48 \div 24 = 2 \rightarrow 20$

Example #3

930 ÷ 3.1

Step 1. Disregard the zero and decimal point, and think, "93 ÷ 31."

Step 2. Divide: 93 ÷ 31 = 3 (intermediary quotient).

Step 3. Apply T of R: 3 is obviously too small to be the answer to 930 ÷ 3.1; a quick estimate puts the answer around 300.

Step 4. Affix two zeroes to the intermediary quotient, producing the answer 300.

Thought Process Summary

$$930 \div 3.1 \rightarrow 93 \div 31 = 3 \rightarrow 300$$

Number-Power Note: In example #3 above, the "quick estimate" could be obtained by rounding the dividend to 900 and the divisor to 3.

Practice Exercises

Some of these exercises combine the two strategies learned to this point. Wherever possible, we will continue to build on tricks learned earlier—so be sure to do your best to remember them!

1. $80 \times 0.3 =$
2. $4.6 \times 200 =$
3. $700 \times 0.5 =$
4. $2.5 \times 300 =$
5. $3.9 \times 20 =$
6. $1.2 \times 120 =$
7. $1{,}800 \times 0.03 =$
8. $0.31 \times 30 =$
9. $720 \div 1.2 =$
10. $960 \div 3.2 =$
11. $150 \div 0.5 =$
12. $5{,}600 \div 1.4 =$
13. $81 \div 0.9 =$
14. $510 \div 1.7 =$

(See solutions on page 199)

DAY 2

Trick 3: Rapidly Multiply by 4 (or 0.4, 40, 400, etc.)

Strategy: Today's tricks are very easy to apply and very important. To multiply a number by 4, **double the number,** and then **double once again.** Remember to disregard any decimal points or zeroes when starting the calculation and to insert or affix to complete the calculation. From now on, our examples and practice exercises will be categorized as "Elementary" and "Brain Builders." In general, the elementary problems deal with the fundamentals, while the brain builders are more advanced and involve decimal points, larger numbers, and a higher degree of difficulty. Read on to see how Trick 3 is performed.

Elementary Example #1

32 × 4

Step 1. Double the 32: 32 × 2 = 64.

Step 2. Double the 64: 64 × 2 = 128 (the answer).

Thought Process Summary

32		32		64
×4	→	×2	→	×2
		64		128

Elementary Example #2

18 × 4

Step 1. Double the 18: 18 × 2 = 36.

Step 2. Double the 36: 36 × 2 = 72 (the answer).

Thought Process Summary

$$\begin{array}{r} 18 \\ \times 4 \\ \hline \\ \end{array} \rightarrow \begin{array}{r} 18 \\ \times 2 \\ \hline 36 \end{array} \rightarrow \begin{array}{r} 36 \\ \times 2 \\ \hline 72 \end{array}$$

Brain Builder #1

2.4 × 40

Step 1. Disregard the decimal point and zero, and think, "24 × 4."

Step 2. Double the 24: 24 × 2 = 48.

Step 3. Double the 48: 48 × 2 = 96 (intermediary product).

Step 4. Apply T of R: A quick estimate puts the answer fairly close to 100. The intermediary product of 96 is also the answer.

Thought Process Summary

$$\begin{array}{r} 2.4 \\ \times 40 \\ \hline \\ \end{array} \rightarrow \begin{array}{r} 24 \\ \times 4 \\ \hline \\ \end{array} \rightarrow \begin{array}{r} 24 \\ \times 2 \\ \hline 48 \end{array} \rightarrow \begin{array}{r} 48 \\ \times 2 \\ \hline 96 \end{array}$$

Brain Builder #2

1,900 × 0.4

Step 1. Disregard the zeroes and decimal point, and think, "19 × 4."

Step 2. Double the 19: 19 × 2 = 38.

Step 3. Double the 38: 38 × 2 = 76 (intermediary product).

Step 4. Apply T of R: Since 0.4 is slightly below $\frac{1}{2}$, the answer must be somewhat below half of 1,900.

Step 5. Affix one zero to the intermediary product, producing the answer 760.

Thought Process Summary

$$\begin{array}{r} 1,900 \\ \times 0.4 \\ \hline \\ \end{array} \rightarrow \begin{array}{r} 19 \\ \times 4 \\ \hline \\ \end{array} \rightarrow \begin{array}{r} 19 \\ \times 2 \\ \hline 38 \end{array} \rightarrow \begin{array}{r} 38 \\ \times 2 \\ \hline 76 \end{array} \rightarrow 760$$

Number-Power Note: This trick is simple and should be obvious, yet surprisingly, many people do not use it. In fact, based on informal polls I've taken, only about one-third of my college students use this trick. One final question—how could you rapidly multiply by 8? That's right—simply double three times!

Elementary Exercises

From now on, the first 16 practice exercises will be labeled "Elementary," while the last 10 will be called "Brain Builders." I challenge you to try them all!

1. $35 \times 4 =$
2. $23 \times 4 =$
3. $14 \times 4 =$
4. $85 \times 4 =$
5. $4 \times 41 =$
6. $4 \times 26 =$
7. $4 \times 55 =$
8. $4 \times 72 =$
9. $61 \times 4 =$
10. $17 \times 4 =$
11. $95 \times 4 =$
12. $48 \times 4 =$
13. $4 \times 29 =$
14. $4 \times 83 =$
15. $4 \times 65 =$
16. $4 \times 53 =$

Brain Builders

1. $54 \times 40 =$
2. $360 \times 0.4 =$
3. $7.5 \times 4 =$
4. $0.15 \times 400 =$
5. $0.4 \times 910 =$
6. $40 \times 7.9 =$
7. $4 \times 570 =$
8. $400 \times 0.44 =$
9. $2.5 \times 40 =$
10. $98 \times 4 =$

(See solutions on page 199)

Parlor Trick #1: Finding the Fifth Root

What is the fifth root of a number? It's much like the square root, only taken a few steps further. For example, $7^5 = 7 \times 7 \times 7 \times 7 \times 7$, which equals 16,807. Therefore, the fifth root of 16,807 equals 7. Similarly, $24^5 = 24 \times 24 \times 24 \times 24 \times 24 = 7{,}962{,}624$. Therefore, the fifth root of 7,962,624 equals 24.

Ask someone to multiply a whole number (from 1 to 99) by itself five times, as shown above. An 8-digit calculator will be able to accommodate up to 39^5, whereas a 10-digit calculator will be able to accommodate up to 99^5. Since most calculators accommodate only 8 digits, you may wish to purchase one with a 10-digit display to allow up to 99^5. Obviously, don't watch as your volunteer is performing the calculation, but write down the final product when the calculation has been completed. It should take you no longer than a few seconds at that point to extract the fifth root.

Strategy: Let's take a couple of examples. Suppose someone has performed the requisite calculation and obtains a product of 32,768. Within about three seconds you will know that the fifth root of that number is 8.

Here's the trick—first of all, the ones-place digit in the product will automatically become the ones-place digit of the answer. Since 8 is the ones digit of 32,768, the ones digit of the fifth root will also be 8. Next, completely ignore the next four digits to the left of the ones digit (that is, the tens, hundreds, thousands, and ten-thousands digit). It may be easier to write the product down and cross out digits with a pencil than to just ignore them because you will then concentrate solely on the digits that remain. In the above case, no digits remain after crossing out the four digits, so the answer is simply 8.

To extract a two-digit fifth root, you'll need to have the following information memorized:

If no number remains, then the answer is a one-digit number.

If the remaining number is in the 1–30 range, the tens digit is 1.

If the remaining number is in the 30–230 range, the tens digit is 2.

If the remaining number is in the 230–1,000 range, the tens digit is 3.

If the remaining number is in the 1,000–3,000 range, the tens digit is 4.

If the remaining number is in the 3,000–7,500 range, the tens digit is 5.

If the remaining number is in the 7,500–16,000 range, the tens digit is 6.

If the remaining number is in the 16,000–32,000 range, the tens digit is 7.

If the remaining number is in the 32,000–57,000 range, the tens digit is 8.

If the remaining number is in the 57,000–99,000 range, the tens digit is 9.

Now let's take a number with a two-digit fifth root. Suppose someone performs the necessary calculation and arrives at 69,343,957. What will be the ones digit of the fifth root? That's right, it will be 7. Next, ignore or cross out the next four digits (that is, the 4395). What remains is the number 693.

This number is in the 230–1,000 range; therefore, the tens digit is 3, and the answer is 37. You might be wondering what to do if the remaining number is, for example, 230. Will the tens digit be 2 or 3? You don't have to worry because the remaining number will never be any of the border numbers.

When using the ranges above, it might be easiest to count off each number with your fingers, as follows: 1–30–230–1,000–3,000–7,500–16,000–32,000–57,000, until you reach the range containing the remaining number at hand. For example, we would have counted 1–30–230 in the above example, indicating a tens digit of 3. (We stop at 230 because the next number in the series, 1,000, exceeds the remaining number, 693.)

Try another exercise: Extract the fifth root of 7,339,040,224. You know that the ones digit of the answer is 4. Crossing out the next four digits, the remaining number is 73,390. Referring to the above ranges, you can see that the tens digit is 9, and the answer is 94.

Now it's your turn to extract the fifth root of:

A.	7,776	E.	20,511,149	I.	130,691,232
B.	844,596,301	F.	371,293	J.	79,235,168
C.	3,276,800,000	G.	7,737,809,375	K.	16,807
D.	459,165,024	H.	2,887,174,368	L.	9,509,900,499

(See solutions on page 224)

Trick 4: Rapidly Divide by 4 (or 0.4, 40, 400, etc.)

Strategy: This trick, too, is simple and should be obvious, yet many people do not use it. To divide a number by 4, **halve the number,** and then **halve once again.** Let's look at a few examples.

Elementary Example #1

84 ÷ 4

Step 1. Halve the 84: 84 ÷ 2 = 42

Step 2. Halve the 42: 42 ÷ 2 = 21 (the answer).

Thought Process Summary

84 ÷ 4 → 84 ÷ 2 = 42 → 42 ÷ 2 = 21

Elementary Example #2

76 ÷ 4

Step 1. Halve the 76: 76 ÷ 2 = 38

Step 2. Halve the 38: 38 ÷ 2 = 19 (the answer).

Thought Process Summary

76 ÷ 4 → 76 ÷ 2 = 38 → 38 ÷ 2 = 19

Brain Builder #1

620 ÷ 40

Step 1. Disregard the zeroes and think, "62 ÷ 4."

Step 2. Halve the 62: 62 ÷ 2 = 31

Step 3. Halve the 31: 31 ÷ 2 = 15.5 (intermediary quotient).

Step 4. Apply T of R: When dividing, you may cancel an equal number of zeroes from the dividend and divisor. Thus, 620 ÷ 40 = 62 ÷ 4. A quick estimate indicates that the intermediary quotient of 15.5 is the answer.

Thought Process Summary

620 ÷ 40 → 62 ÷ 2 = 31 → 31 ÷ 2 = 15.5

Brain Builder #2

9.2 ÷ 4

Step 1. Disregard the decimal point, converting the problem to 92 ÷ 4

Step 2. Halve the 92: 92 ÷ 2 = 46

Step 3. Halve the 46: 46 ÷ 2 = 23 (intermediary quotient).

Step 4. Apply T of R: A quick estimate puts the answer somewhere between 2 and 3

Step 5. Insert a decimal point within the intermediary quotient of 23, producing the answer 2.3

Thought Process Summary

$$\frac{9.2}{4} \rightarrow \frac{92}{4} \rightarrow \frac{92}{2} = 46 \rightarrow \frac{46}{2} = 23 \rightarrow 2.3$$

Number-Power Note: In certain instances, it really isn't necessary to disregard zeroes when starting a calculation. For example, 140 can be divided by 4 very quickly simply by just thinking, "140, 70, 35." Final note—how would you rapidly divide a number by 8? That's right, simply halve the number three times!

Elementary Exercises

Finish Day Two by "halving" yourself a ball with these:

1. 48 ÷ 4 =
2. 68 ÷ 4 =
3. 180 ÷ 4 =
4. 132 ÷ 4 =
5. 260 ÷ 4 =
6. 96 ÷ 4 =
7. 56 ÷ 4 =
8. 88 ÷ 4 =
9. 140 ÷ 4 =
10. 220 ÷ 4 =
11. 64 ÷ 4 =
12. 72 ÷ 4 =
13. 380 ÷ 4 =
14. 340 ÷ 4 =
15. 420 ÷ 4 =
16. 52 ÷ 4 =

Brain Builders

1. 440 ÷ 40 =
2. 3,600 ÷ 400 =
3. 112 ÷ 4 =
4. 94 ÷ 4 =
5. 540 ÷ 40 =
6. 17.6 ÷ 0.4 =
7. 14.4 ÷ 4 =
8. 232 ÷ 4 =
9. 81 ÷ 4 =
10. 980 ÷ 400 =

(See solutions on page 200)

DAY

3

Trick 5: Rapidly Multiply by 5 (or 0.5, 50, 500, etc.)

Strategy: This is our first trick using reciprocals. They can be very helpful in shortening a wide variety of problems—and they really do work! Reciprocals, as the term is used in this book, are two numbers that, when multiplied together, equal 10, 100, or any other power of 10. The numbers 5 and 2 are examples of reciprocals because they equal 10 when multiplied together. To multiply a number by 5, **divide the number by 2** and affix or insert any necessary zeroes or decimal point. (Be sure to disregard any decimal points or zeroes upon starting the calculation.) The assumption here is that it is easier to divide by 2 than to multiply by 5. Confused? You won't be after looking at the following examples.

Elementary Example #1

24 × 5

Step 1. Divide: $24 \div 2 = 12$ (intermediary quotient).

Step 2. Apply T of R: 12 is obviously too small to be the answer to 24×5. A quick estimate puts the answer somewhere in the 100s.

Step 3. Affix one zero to the intermediary quotient, producing the answer 120.

Thought Process Summary

$$\begin{array}{r} 24 \\ \times 5 \\ \hline \end{array} \rightarrow 24 \div 2 = 12 \rightarrow 120$$

Elementary Example #2

46 × 5

Step 1. Divide: 46 ÷ 2 = 23 (intermediary quotient).

Step 2. Apply T of R: 23 is obviously too small to be the answer to 46 × 5. A quick estimate puts the answer somewhere in the 200s.

Step 3. Affix one zero to the intermediary quotient, producing the answer 230.

Thought Process Summary

$$\begin{array}{r} 46 \\ \times 5 \\ \hline \end{array} \rightarrow 46 \div 2 = 23 \rightarrow 230$$

Brain Builder #1

35 × 50

Step 1. Disregard the zero and think, "35 × 5."

Step 2. Divide: 35 ÷ 2 = 17.5 (intermediary quotient).

Step 3. Apply T of R: 17.5 is obviously too small to be the answer to 35 × 50. A quick estimate puts the answer between 1,500 and 2,000.

Step 4. Eliminate the decimal point from the intermediary quotient and affix one zero, producing the answer 1,750.

Thought Process Summary

$$\begin{array}{r} 35 \\ \times 50 \\ \hline \end{array} \rightarrow \begin{array}{r} 35 \\ \times 5 \\ \hline \end{array} \rightarrow 35 \div 2 = 17.5 \rightarrow 1,750$$

Brain Builder #2

36.2 × 5

Step 1. Disregard the decimal point, and think, "362 × 5."

Step 2. Divide: 362 ÷ 2 = 181 (intermediary quotient).

Step 3. Apply T of R: 181 seems reasonable as the answer to 36.2 × 5, and in fact is the answer.

Thought Process Summary

$$\begin{array}{r} 36.2 \\ \times 5 \\ \hline \end{array} \rightarrow \begin{array}{r} 362 \\ \times 5 \\ \hline \end{array} \rightarrow 362 \div 2 = 181$$

Number-Power Note: This trick can also be used to multiply a number by $\frac{1}{2}$, since $\frac{1}{2}$ is the same as 0.5. However, for purposes of simplicity, any exercises in this book that involve other than whole numbers will be presented in decimal rather than fraction form.

Elementary Exercises

You won't believe how quickly you'll be able to do these exercises.

1. $16 \times 5 =$
2. $38 \times 5 =$
3. $88 \times 5 =$
4. $42 \times 5 =$
5. $5 \times 74 =$
6. $5 \times 58 =$
7. $5 \times 22 =$
8. $5 \times 76 =$
9. $62 \times 5 =$
10. $28 \times 5 =$
11. $66 \times 5 =$
12. $94 \times 5 =$
13. $5 \times 54 =$
14. $5 \times 82 =$
15. $5 \times 96 =$
16. $5 \times 44 =$

Brain Builders

1. $85 \times 5 =$
2. $49 \times 5 =$
3. $33 \times 5 =$
4. $97 \times 5 =$
5. $50 \times 55 =$
6. $0.5 \times 12.2 =$
7. $500 \times 0.79 =$
8. $5 \times 510 =$
9. $2.1 \times 50 =$
10. $6.8 \times 500 =$

(See solutions on page 200)

Trick 6: Rapidly Divide by 5 (or 0.5, 50, 500, etc.)

Strategy: Reciprocals work as effectively with division as with multiplication. To divide a number by 5, **multiply the number by 2** and affix or insert any necessary zeroes or decimal point. This technique follows the same general procedure that you learned in Trick 5 and that you will continue to apply with other multiplication and division techniques. That is, you first **operate** (multiply or divide); then you **think** (apply a test of reasonableness), and finally you **adjust,** if necessary (insert or affix a decimal point or zeroes). You'll be amazed to see how easily the following examples are operated upon.

Elementary Example #1

38 ÷ 5

Step 1. Multiply: 38 × 2 = 76 (intermediary product).

Step 2. Apply T of R: 76 is obviously too large to be the answer to 38 ÷ 5. A quick estimate puts the answer somewhere between 7 and 8.

Step 3. Insert a decimal point within the intermediary product, producing the answer 7.6.

Thought Process Summary

$$38 \div 5 \rightarrow \begin{array}{r} 38 \\ \times 2 \\ \hline 76 \end{array} \rightarrow 7.6$$

Elementary Example #2

85 ÷ 5

Step 1. Multiply: 85 × 2 = 170 (intermediary product).

Step 2. Apply T of R: 170 is obviously too large to be the answer to 85 ÷ 5. A quick estimate puts the answer somewhere between 10 and 20.

Step 3. Insert a decimal point within the intermediary product, producing the answer 17.0 (or simply 17).

Thought Process Summary

$$85 \div 5 \rightarrow \begin{array}{r} 85 \\ \times 2 \\ \hline 170 \end{array} \rightarrow 17$$

Brain Builder #1

245 ÷ 50

Step 1. Multiply: 245 × 2 = 490 (intermediary product).

Step 2. Apply T of R: 490 is obviously too large to be the answer to 245 ÷ 50. A quick estimate puts the answer just below 5.

Step 3. Insert a decimal point within the intermediary product, producing the answer 4.90 (or simply 4.9).

Thought Process Summary

$245 \div 50 \rightarrow 245 \times 2 = 490 \rightarrow 4.9$

Brain Builder #2

44.4 ÷ 5

Step 1. Disregard the decimal point, converting the problem to $444 \div 5$.

Step 2. Multiply: $444 \times 2 = 888$ (intermediary product).

Step 3. Apply T of R: 888 is obviously too large to be the answer to $44.4 \div 5$. A quick estimate puts the answer just below 9.

Step 4. Insert a decimal point within the intermediary product, producing the answer 8.88.

Thought Process Summary

$44.4 \div 5 \rightarrow 444 \div 5 \rightarrow 444 \times 2 = 888 \rightarrow 8.88$

Number-Power Note: When applying a test of reasonableness to division problems, try to use the inverse of division—multiplication. For example, in Brain Builder #1 above, ask yourself, "50 times what equals 245?" With a moment's thought, you can place the answer just below 5.

Elementary Exercises

How should you approach the following exercises? Divide and conquer, of course!

1. $27 \div 5 =$
2. $53 \div 5 =$
3. $72 \div 5 =$
4. $67 \div 5 =$
5. $118 \div 5 =$
6. $95 \div 5 =$
7. $41 \div 5 =$
8. $49 \div 5 =$
9. $122 \div 5 =$
10. $14 \div 5 =$
11. $76 \div 5 =$
12. $81 \div 5 =$

13. $33 \div 5 =$
14. $58 \div 5 =$
15. $98 \div 5 =$
16. $64 \div 5 =$

Brain Builders

1. $230 \div 50 =$
2. $18.5 \div 5 =$
3. $8,300 \div 500 =$
4. $33.3 \div 5 =$
5. $190 \div 50 =$
6. $4.2 \div 5 =$
7. $43.5 \div 0.5 =$
8. $920 \div 50 =$
9. $610 \div 500 =$
10. $4.6 \div 5 =$

(See solutions on page 200)

DAY 4

Trick 7: Rapidly Square Any Number Ending in 5

Strategy: This trick is one of the oldest in the book, and one of the best! To square a number that ends in 5, first **multiply the tens digit by the next whole number**. To that product, **affix the number 25**. The number to affix (25) is easy to remember, because $5^2 = 25$. Although a calculation such as 7.5×750 is technically not a square, it too can be solved using this technique. This trick will also work for numbers with more than two digits. Read on to see how this marvelous trick works.

Elementary Example #1
15^2

Step 1. Multiply: $1 \times 2 = 2$.

Step 2. Affix 25: 225 (the answer).

Thought Process Summary

$$\begin{array}{r} 15 \\ \times 15 \\ \hline \end{array} \rightarrow \begin{array}{r} 1 \\ \times 2 \\ \hline 2 \end{array} \rightarrow 225$$

Elementary Example #2
65^2

Step 1. Multiply: $6 \times 7 = 42$.

Step 2. Affix 25: 4,225 (the answer).

Thought Process Summary

$$\begin{array}{r} 65 \\ \times 65 \\ \hline \end{array} \rightarrow \begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array} \rightarrow 4,225$$

Brain Builder #1

450^2

Step 1. Disregard the zero and think, "45 squared."

Step 2. Multiply: 4 × 5 = 20.

Step 3. Affix 25: 2,025 (intermediary product).

Step 4. Apply T of R: For each zero initially disregarded in a squaring problem, two must eventually be affixed to obtain the product.

Step 5. Affix two zeroes to the intermediary product, producing the answer 202,500.

Thought Process Summary

$$\begin{array}{r} 450 \\ \times 450 \\ \hline \end{array} \rightarrow \begin{array}{r} 45 \\ \times 45 \\ \hline \end{array} \rightarrow \begin{array}{r} 4 \\ \times 5 \\ \hline 20 \end{array} \rightarrow 2,025 \rightarrow 202,500$$

Brain Builder #2

7.5 × 750

Step 1. Disregard the decimal point and zero, and think, "75 squared."

Step 2. Multiply: 7 × 8 = 56.

Step 3. Affix 25: 5,625 (intermediary product).

Step 4. Apply T of R: A quick estimate puts the answer in the 5,000s. The intermediary product of 5,625 is therefore the answer.

Thought Process Summary

$$\begin{array}{r} 7.5 \\ \times 750 \\ \hline \end{array} \rightarrow \begin{array}{r} 75 \\ \times 75 \\ \hline \end{array} \rightarrow \begin{array}{r} 7 \\ \times 8 \\ \hline 56 \end{array} \rightarrow 5,625$$

Brain Builder #3

115^2

Step 1. Multiply: $11 \times 12 = 132$.

Step 2. Affix 25: 13,225 (the answer).

Thought Process Summary

$$\begin{array}{r} 115 \\ \times 115 \\ \hline \end{array} \rightarrow \begin{array}{r} 11 \\ \times 12 \\ \hline 132 \end{array} \rightarrow 13{,}225$$

Number-Power Notes: Trick 23 involves rapidly squaring any two-digit number **beginning** in 5, and Trick 20 is a variation on Trick 7.

Elementary Exercises

Don't be confused by the two ways these squaring exercises are presented.

1. $35 \times 35 =$
2. $85 \times 85 =$
3. $95 \times 95 =$
4. $25 \times 25 =$
5. $55 \times 55 =$
6. $75 \times 75 =$
7. $45 \times 45 =$
8. $15 \times 15 =$
9. $65^2 =$
10. $95^2 =$
11. $85^2 =$
12. $35^2 =$
13. $25^2 =$
14. $55^2 =$
15. $75^2 =$
16. $45^2 =$

Brain Builders

1. $105 \times 105 =$
2. $3.5 \times 350 =$
3. $750^2 =$
4. $0.85 \times 85 =$
5. $65 \times 6.5 =$
6. $150 \times 15 =$
7. $11.5^2 =$
8. $5.5 \times 550 =$
9. $0.45 \times 0.45 =$
10. $950 \times 9.5 =$

(See solutions on page 201)

Mathematical Curiosity #1

$12{,}345{,}679 \times 9 = 111{,}111{,}111$	$12{,}345{,}679 \times 54 = 666{,}666{,}666$
$12{,}345{,}679 \times 18 = 222{,}222{,}222$	$12{,}345{,}679 \times 63 = 777{,}777{,}777$
$12{,}345{,}679 \times 27 = 333{,}333{,}333$	$12{,}345{,}679 \times 72 = 888{,}888{,}888$
$12{,}345{,}679 \times 36 = 444{,}444{,}444$	$12{,}345{,}679 \times 81 = 999{,}999{,}999$
$12{,}345{,}679 \times 45 = 555{,}555{,}555$	$12{,}345{,}679 \times 999{,}999{,}999 = 12{,}345{,}678{,}987{,}654{,}321$

Trick 8: Rapidly Multiply Any Two-Digit Number by 11 (or 0.11, 1.1, 110, etc.)

Strategy: The "11 trick" is the most popular trick of them all—and one of the most useful. To multiply a two-digit number by 11, first **write the number**, leaving some space between the two digits. Then **insert the sum of the number's two digits** in between the two digits themselves. You will have to carry when the sum of the digits exceeds 9. This trick is almost too good to be true, as you'll see in the following examples.

Elementary Example #1

24 × 11

Step 1. Write the multiplicand in the answer space, leaving some room between the two digits: 2 ? 4 (intermediary product).

Step 2. Add the two digits of the multiplicand: $2 + 4 = 6$.

Step 3. Insert the 6 within the intermediary product, producing the answer 264.

Thought Process Summary

$24 \times 11 \rightarrow 2\ ?\ 4 \rightarrow 2 + 4 = 6 \rightarrow 264$

Elementary Example #2

76 × 11

Step 1. Write the multiplicand in the answer space, leaving some room between the two digits: 7 ? 6 (intermediary product).

Step 2. Add the two digits of the multiplicand: $7 + 6 = 13$.

Step 3. Insert the ones digit of the 13 within the intermediary product, producing a second intermediary product of 736.

Step 4. Because the two digits of the multiplicand total more than 9, one must "carry," converting 736 to the answer, 836.

Thought Process Summary

$$\begin{array}{r}76\\ \times 11\end{array} \rightarrow 7\,\underline{?}\,6 \rightarrow \begin{array}{r}7\\ +6\\ \hline 13\end{array} \rightarrow 736 \rightarrow 836$$

Brain Builder #1

53 × 110

Step 1. Disregard the zero, and think, "53 × 11."

Step 2. Follow steps 1, 2, and 3 of the preceding Elementary Examples, producing an intermediary product of 583.

Step 3. Apply T of R: The zero that was disregarded must be reaffixed, producing the answer 5,830.

Thought Process Summary

$$\begin{array}{r}53\\ \times 110\\ \hline\end{array} \rightarrow \begin{array}{r}53\\ \times 11\\ \hline\end{array} \rightarrow 5\,\underline{?}\,3 \rightarrow \begin{array}{r}5\\ +3\\ \hline 8\end{array} \rightarrow 583 \rightarrow 5,830$$

Brain Builder #2

8.7 × 1.1

Step 1. Disregard the decimal points and think, "87 × 11."

Step 2. Follow steps 1, 2, and 3 of the preceding Elementary Examples, producing an intermediary product of 957.

Step 3. Apply T of R: A quick estimate puts the answer at about 9 or 10.

Step 4. Insert a decimal point within the intermediary product, producing the answer 9.57.

Thought Process Summary

$$\begin{array}{r}8.7\\ \times 1.1\\ \hline\end{array} \rightarrow \begin{array}{r}87\\ \times 11\\ \hline\end{array} \rightarrow 8\,\underline{?}\,7 \rightarrow \begin{array}{r}8\\ +7\\ \hline 15\end{array} \rightarrow 857 \rightarrow 957 \rightarrow 9.57$$

Number-Power Note: Trick 59 involves rapidly multiplying a three-digit or larger number by 11. An alternative but less efficient method to multiply by 11 is to multiply the number by 10 and then add the number itself. For example, in Elementary Example #1 above, 24 × 11 = (24 × 10) + 24 = 264.

Elementary Exercises

You should be able to float through these exercises with the greatest of ease.

1. 62 × 11 =
2. 18 × 11 =
3. 35 × 11 =
4. 81 × 11 =
5. 11 × 26 =
6. 11 × 44 =
7. 11 × 58 =
8. 11 × 92 =
9. 17 × 11 =
10. 69 × 11 =
11. 31 × 11 =
12. 74 × 11 =
13. 11 × 96 =
14. 11 × 39 =
15. 11 × 47 =
16. 11 × 99 =

Brain Builders

1. 2.7 × 110 =
2. 65 × 1.1 =
3. 8.3 × 1.1 =
4. 0.56 × 1,100 =
5. 1.1 × 130 =
6. 110 × 330 =
7. 110 × 7.8 =
8. 1.1 × 2.2 =
9. 940 × 11 =
10. 4.9 × 110 =

(See solutions on page 201)

DAY

5

Trick 9: Rapidly Multiply by 25 (or 0.25, 2.5, 250, etc.)

Strategy: Let's begin day five with another reciprocal technique that will come in handy time and time again. To multiply a number by 25, **divide the number by 4** and affix or insert any necessary zeroes or decimal point. Here are some examples that illustrate this very practical trick.

Elementary Example #1

28 × 25

Step 1. Divide: 28 ÷ 4 = 7 (intermediary quotient).

Step 2. Apply T of R: 7 and 70 both seem too small to be the answer to 28 × 25.

Step 3. Affix two zeroes to the intermediary quotient, producing the answer 700.

Thought Process Summary

$$\begin{array}{r} 28 \\ \times 25 \\ \hline \end{array} \rightarrow \quad 28 \div 4 = 7 \quad \rightarrow \quad 700$$

Elementary Example #2

76 × 25

Step 1. Divide: 76 ÷ 4 = 19 (intermediary quotient).

Step 2. Apply T of R: 19 and 190 both seem too small to be the answer to 76 × 25.

Step 3. Affix two zeroes to the intermediary quotient, producing the answer 1,900.

Thought Process Summary

$$\begin{array}{r} 76 \\ \times 25 \\ \hline \end{array} \rightarrow 76 \div 4 = 19 \rightarrow 1,900$$

Brain Builder #1

36 × 250

Step 1. Divide: 36 ÷ 4 = 9 (intermediary quotient).

Step 2. Apply T of R: 9, 90, and 900 all seem too small to be the answer to 36 × 250.

Step 3. Affix three zeroes to the intermediary quotient, producing the answer 9,000.

Thought Process Summary

$$\begin{array}{r} 36 \\ \times 250 \\ \hline \end{array} \rightarrow 36 \div 4 = 9 \rightarrow 9,000$$

Brain Builder #2

420 × 2.5

Step 1. Disregard the zero and decimal point, and think, "42 × 25."

Step 2. Divide: 42 ÷ 4 = 10.5 (intermediary quotient).

Step 3. Apply T of R: A quick estimate puts the answer around 1,000.

Step 4. Eliminate the decimal point from the intermediary quotient and affix one zero to produce the answer 1,050.

Thought Process Summary

$$\begin{array}{r} 420 \\ \times 2.5 \\ \hline \end{array} \rightarrow \begin{array}{r} 42 \\ \times 25 \\ \hline \end{array} \rightarrow 42 \div 4 = 10.5 \rightarrow 1,050$$

Number-Power Note: Remember that Trick 4 recommends that you divide by 4 by halving, and then halving again.

Elementary Exercises

You can combine Tricks 4 and 9 when working these exercises.

1. $12 \times 25 =$	9. $92 \times 25 =$
2. $44 \times 25 =$	10. $68 \times 25 =$
3. $52 \times 25 =$	11. $48 \times 25 =$
4. $16 \times 25 =$	12. $80 \times 25 =$
5. $25 \times 64 =$	13. $25 \times 32 =$
6. $25 \times 88 =$	14. $25 \times 84 =$
7. $25 \times 24 =$	15. $25 \times 72 =$
8. $25 \times 56 =$	16. $25 \times 96 =$

Brain Builders

1. $34 \times 25 =$	6. $250 \times 28 =$
2. $78 \times 25 =$	7. $2.5 \times 86 =$
3. $58 \times 25 =$	8. $0.25 \times 600 =$
4. $14 \times 25 =$	9. $25 \times 8.4 =$
5. $25 \times 7.4 =$	10. $250 \times 18 =$

(See solutions on page 201)

Trick 10: Rapidly Divide by 25 (or 0.25, 2.5, 250, etc.)

Strategy: Perhaps you've already guessed this trick's shortcut. To divide a number by 25, **multiply the number by 4**, and affix or insert any necessary zeroes or decimal point. Dividing by 25 is even easier than multiplying by 25, as you'll see in the following examples.

Elementary Example #1

700 ÷ 25

Step 1. Disregard the zeroes and multiply: $7 \times 4 = 28$ (intermediary product).

Step 2. Apply T of R: 28 is not only in the ballpark, it is the answer. (Refer to the Number-Power Note for further explanation.)

Thought Process Summary

$$700 \div 25 \rightarrow 7 \div 25 \rightarrow 7 \times 4 = 28$$

Elementary Example #2

210 ÷ 25

Step 1. Disregard the zero and multiply: 21 × 4 = 84 (intermediary product).

Step 2. Apply T of R: A quick estimate puts the answer between 8 and 9.

Step 3. Insert a decimal point within the intermediary product, producing the answer 8.4.

Thought Process Summary

$$210 \div 25 \rightarrow 21 \div 25 \rightarrow \begin{array}{r} 21 \\ \times 4 \\ \hline 84 \end{array} \rightarrow 8.4$$

Brain Builder #1

450 ÷ 2.5

Step 1. Disregard the zero and decimal point, and multiply: 45 × 4 = 180 (intermediary product).

Step 2. Apply T of R: A quick estimate puts the answer somewhere between 100 and 200. Thus, the intermediary product (180) is the answer.

Thought Process Summary

$$450 \div 2.5 \rightarrow 45 \div 25 \rightarrow \begin{array}{r} 45 \\ \times 4 \\ \hline 180 \end{array}$$

Brain Builder #2

5,200 ÷ 250

Step 1. Disregard the zeroes and multiply: 52 × 4 = 208 (intermediary product).

Step 2. Apply T of R: Remove one zero each from the original dividend and divisor, and try to arrive at an estimate for 520 ÷ 25. It appears to be around 20.

Step 3. Insert a decimal point within the intermediary product, producing the answer 20.8.

Thought Process Summary

$$5{,}200 \div 250 \rightarrow 52 \div 25 \rightarrow \begin{array}{r} 52 \\ \times 4 \\ \hline 208 \end{array} \rightarrow 20.8$$

Number-Power Notes: Remember that Trick 3 recommends that you multiply by 4 by doubling, and then doubling again. Also, to obtain an estimate for 700 ÷ 25 in Elementary Example #1, you could work backwards and think, "25 goes into 100 four times, and 100 goes into 700 seven times; 4 times 7 equals 28."

Elementary Exercises

You can combine Tricks 3 and 10 when working through these exercises.

1. 80 ÷ 25 =
2. 300 ÷ 25 =
3. 180 ÷ 25 =
4. 2,400 ÷ 25 =
5. 320 ÷ 25 =
6. 650 ÷ 25 =
7. 1,200 ÷ 25 =
8. 850 ÷ 25 =
9. 95 ÷ 25 =
10. 400 ÷ 25 =
11. 9,000 ÷ 25 =
12. 1,500 ÷ 25 =
13. 500 ÷ 25 =
14. 2,200 ÷ 25 =
15. 750 ÷ 25 =
16. 270 ÷ 25 =

Brain Builders

1. 350 ÷ 2.5 =
2. 111 ÷ 25 =
3. 2.3 ÷ 2.5 =
4. 550 ÷ 25 =
5. 1,700 ÷ 250 =
6. 4.2 ÷ 0.25 =
7. 222 ÷ 25 =
8. 820 ÷ 2.5 =
9. 130 ÷ 25 =
10. 3,300 ÷ 250 =

(See solutions on page 202)

DAY 6

Trick 11: Rapidly Multiply Any One- or Two-Digit Number by 99 (or 0.99, 9.9, 990, etc.)

Strategy: Nine is by far the most mysterious and magical number. Here is a trick involving two 9s. To multiply a one- or two-digit number by 99, first **subtract 1 from the number** to obtain the **left-hand portion** of the answer. Then, **subtract the number from 100** to obtain the **right-hand portion**. (Hint: You will learn from Trick 29 that it is faster to subtract by adding. For example, to subtract 88 from 100, ask yourself, "88 plus what equals 100?" to obtain the answer, 12.) When you've reviewed the examples below, you'll see that this trick is a lot easier to apply than it sounds.

Elementary Example #1

15 × 99

Step 1. Subtract: 15 − 1 = 14 (left-hand portion of the answer).

Step 2. Subtract: 100 − 15 = 85 (right-hand portion of the answer).

Step 3. Combine: 1,485 is the answer.

Thought Process Summary

15		15		100		
×99	→	−1	→	−15	→	1,485
		14		85		

Elementary Example #2

7 × 99

Step 1. Subtract: 7 − 1 = 6 (left-hand portion of the answer).

Step 2. Subtract: 100 − 7 = 93 (right-hand portion of the answer).

Step 3. Combine: 693 is the answer.

Thought Process Summary

$$\begin{array}{r} 7 \\ \times 99 \\ \hline \end{array} \rightarrow \begin{array}{r} 7 \\ -1 \\ \hline 6 \end{array} \rightarrow \begin{array}{r} 100 \\ -7 \\ \hline 93 \end{array} \rightarrow 693$$

Brain Builder #1

2.8 × 9.9

Step 1. Disregard decimal points and subtract: 28 − 1 = 27 (left-hand portion of the answer).

Step 2. Subtract: 100 − 28 = 72 (right-hand portion of the answer).

Step 3. Combine: 2,772 (intermediary product).

Step 4. Apply T of R: 9.9 is almost 10, so 2.8 × 9.9 must equal just under 28. Insert a decimal point within the intermediary product to obtain the answer, 27.72.

Thought Process Summary

$$\begin{array}{r} 2.8 \\ \times 9.9 \\ \hline \end{array} \rightarrow \begin{array}{r} 28 \\ -1 \\ \hline 27 \end{array} \rightarrow \begin{array}{r} 100 \\ -28 \\ \hline 72 \end{array} \rightarrow 2,772 \rightarrow 27.72$$

Brain Builder #2

430 × 0.99

Step 1. Disregard the zero and decimal point, and subtract: 43 − 1 = 42 (left-hand portion of the answer).

Step 2. Subtract: 100 − 43 = 57 (right-hand portion of the answer).

Step 3. Combine: 4,257 (intermediary product).

Step 4. Apply T of R: 0.99 is almost 1, so 430 × 0.99 must equal just under 430. Insert a decimal point within the intermediary product to obtain the answer, 425.7.

Thought Process Summary

$$\begin{array}{r} 430 \\ \times 0.99 \\ \hline \end{array} \rightarrow \begin{array}{r} 43 \\ -1 \\ \hline 42 \end{array} \rightarrow \begin{array}{r} 100 \\ -43 \\ \hline 57 \end{array} \rightarrow 4,257 \rightarrow 425.7$$

Number-Power Note: Trick 60 shows how to rapidly **divide** by 9, 99, and so forth.

Elementary Exercises

In doing these exercises, remember to obtain the left-hand portion of the answer before computing the right-hand portion.

1. 60 × 99 =
2. 75 × 99 =
3. 9 × 99 =
4. 88 × 99 =
5. 99 × 35 =
6. 99 × 61 =
7. 99 × 66 =
8. 99 × 48 =
9. 80 × 99 =
10. 22 × 99 =
11. 4 × 99 =
12. 54 × 99 =
13. 99 × 83 =
14. 99 × 39 =
15. 99 × 97 =
16. 99 × 11 =

Brain Builders

1. 5.2 × 990 =
2. 91 × 9.9 =
3. 0.77 × 99 =
4. 260 × 0.99 =
5. 9.9 × 200 =
6. 990 × 330 =
7. 99 × 7.2 =
8. 0.99 × 440 =
9. 0.57 × 9.9 =
10. 3 × 990 =

(See solutions on page 202)

Mathematical Curiosity #2

987,654,321 × 9 = 8,888,888,889
987,654,321 × 18 = 17,777,777,778
987,654,321 × 27 = 26,666,666,667
987,654,321 × 36 = 35,555,555,556
987,654,321 × 45 = 44,444,444,445
987,654,321 × 54 = 53,333,333,334
987,654,321 × 63 = 62,222,222,223
987,654,321 × 72 = 71,111,111,112
987,654,321 × 81 = 80,000,000,001

(Also note: In each case, the first and last digit of the product is the same as the multiplier.)

Trick 12: Rapidly Multiply Any One- or Two-Digit Number by 101 (or 1.01, 10.1, 1,010, etc.)

Strategy: Day six ends with the easiest trick of them all. To multiply a one-digit number by 101, **write down the one-digit number twice,** and insert a zero in between. For example, $101 \times 7 = 707$. To multiply a two-digit number by 101, **write down the two-digit number twice**, and you have the answer! For example, $36 \times 101 = 3,636$. You probably don't need more examples to understand this trick, but here are some more, anyway.

Elementary Example #1

4 × 101

Step 1. Write down the 4 twice, and insert a zero: 404 (the answer).

Thought Process Summary

$$\begin{array}{r} 4 \\ \underline{\times 101} \end{array} \rightarrow 404$$

Elementary Example #2

27 × 101

Step 1. Write down the 27 twice: 2,727 (the answer).

Thought Process Summary

$$\begin{array}{r} 27 \\ \underline{\times 101} \end{array} \rightarrow 2,727$$

Brain Builder #1

56 × 1.01

Step 1. Write down the 56 twice: 5,656 (intermediary product).

Step 2. Apply T of R: A quick estimate puts the answer just above 56.

Step 3. Insert a decimal point within the intermediary product, producing the answer 56.56.

Thought Process Summary

$56 \times 1.01 \rightarrow 5,656 \rightarrow 56.56$

Brain Builder #2

740 × 10.1

Step 1. Disregard the zero, and write down the 74 twice: 7,474 (intermediary product).

Step 2. Apply T of R: A quick estimate puts the answer just above 7,400. Accordingly, 7,474 appears to be the answer.

Thought Process Summary

$740 \times 10.1 \rightarrow 7,474$

Number-Power Note: To multiply a two-digit number by 1,001, perform the same steps as for 101, but insert a zero in the middle. For example, $49 \times 1,001 = 49,049$. However, to multiply a three-digit number by 1,001, simply write down the three-digit number twice! For example, $417 \times 1,001 = 417,417$.

Elementary Exercises

You'll be able to work through these exercises as quickly as you can write the answers!

1. $15 \times 101 =$
2. $62 \times 101 =$
3. $39 \times 101 =$
4. $8 \times 101 =$
5. $101 \times 93 =$
6. $101 \times 41 =$
7. $101 \times 87 =$
8. $101 \times 70 =$
9. $12 \times 101 =$
10. $45 \times 101 =$
11. $81 \times 101 =$
12. $23 \times 101 =$
13. $101 \times 6 =$
14. $101 \times 78 =$
15. $101 \times 32 =$
16. $101 \times 99 =$

Brain Builders

1. $4.8 \times 1.01 =$
2. $630 \times 0.101 =$
3. $0.36 \times 1,010 =$
4. $8.5 \times 10.1 =$
5. $101 \times 110 =$
6. $1.01 \times 920 =$
7. $10.1 \times 30 =$
8. $1,010 \times 1.9 =$
9. $890 \times 1.01 =$
10. $5.1 \times 101 =$

(See solutions on page 202)

DAY 7

Trick 13: Rapidly Multiply Two Numbers Whose Difference Is 2

Strategy: Before learning this trick, you should review the squares table on page 11. To multiply two numbers whose difference is 2, first **square the number between the two.** Then, **subtract 1** from the product to obtain the answer. Let's look at some examples of this trick.

Elementary Example #1

11 × 13

Step 1. Square the number in the middle: $12^2 = 144$ (intermediary product).

Step 2. Subtract 1 from the intermediary product: $144 - 1 = 143$, which is the answer. (Note: Why not use Trick 8—the "11 trick"—to check your answer?)

Thought Process Summary

11		12		144
×13	→	×12	→	−1
		144		143

Elementary Example #2

24 × 26

Step 1. Square the number in the middle: $25^2 = 625$ (intermediary product). (Did you remember to use Trick 7 to square the 25?)

Step 2. Subtract 1 from the intermediary product: $625 - 1 = 624$, which is the answer.

Thought Process Summary

$$\begin{array}{r} 24 \\ \times 26 \\ \hline \end{array} \rightarrow \begin{array}{r} 25 \\ \times 25 \\ \hline 625 \end{array} \rightarrow \begin{array}{r} 625 \\ -1 \\ \hline 624 \end{array}$$

Brain Builder #1

1.9 × 210

Step 1. Disregard the decimal point and zero, and think, "19 × 21."

Step 2. Square the number in the middle: $20^2 = 400$ (intermediary product).

Step 3. Subtract 1 from the intermediary product: $400 - 1 = 399$ (revised intermediary product).

Step 4. Apply T of R: A quick estimate puts the answer around 400, so the revised intermediary product of 399 is the answer.

Thought Process Summary

$$\begin{array}{r} 1.9 \\ \times 210 \\ \hline \end{array} \rightarrow \begin{array}{r} 19 \\ \times 21 \\ \hline \end{array} \rightarrow \begin{array}{r} 20 \\ \times 20 \\ \hline 400 \end{array} \rightarrow \begin{array}{r} 400 \\ -1 \\ \hline 399 \end{array}$$

Brain Builder #2

1.4 × 1.2

Step 1. Disregard the decimal points and think, "14 × 12."

Step 2. Square the number in the middle: $13^2 = 169$ (intermediary product).

Step 3. Subtract 1 from the intermediary product: $169 - 1 = 168$ (revised intermediary product).

Step 4. Apply T of R: A quick estimate puts the answer somewhere between 1 and 2.

Step 5. Insert a decimal point within the revised intermediary product, producing the answer 1.68.

Thought Process Summary

$$\begin{array}{r} 1.4 \\ \times 1.2 \\ \hline \end{array} \rightarrow \begin{array}{r} 14 \\ \times 12 \\ \hline \end{array} \rightarrow \begin{array}{r} 13 \\ \times 13 \\ \hline 169 \end{array} \rightarrow \begin{array}{r} 169 \\ -1 \\ \hline 168 \end{array} \rightarrow 1.68$$

Number-Power Note: Trick 26 is a variation on this technique.

Elementary Exercises

The better you are at squaring, the easier you'll find these exercises.

1. $15 \times 13 =$
2. $17 \times 15 =$
3. $29 \times 31 =$
4. $14 \times 16 =$
5. $36 \times 34 =$
6. $79 \times 81 =$
7. $99 \times 101 =$
8. $54 \times 56 =$
9. $13 \times 11 =$
10. $12 \times 14 =$
11. $21 \times 19 =$
12. $16 \times 18 =$
13. $59 \times 61 =$
14. $26 \times 24 =$
15. $19 \times 17 =$
16. $84 \times 86 =$

Brain Builders

1. $2.1 \times 19 =$
2. $74 \times 7.6 =$
3. $110 \times 1.3 =$
4. $1.7 \times 1.5 =$
5. $0.44 \times 460 =$
6. $9.9 \times 101 =$
7. $140 \times 120 =$
8. $49 \times 0.51 =$
9. $1.9 \times 17 =$
10. $640 \times 6.6 =$

(See solutions on page 203)

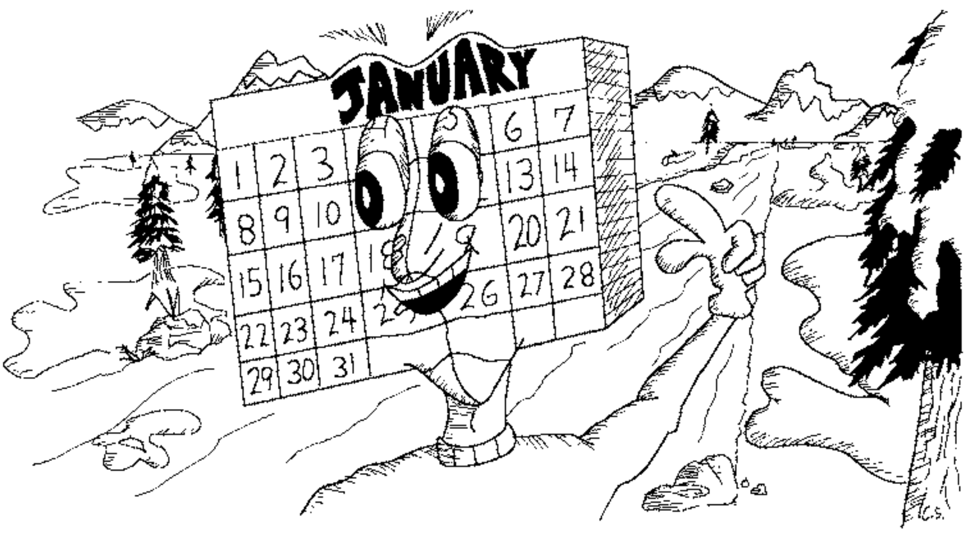
Parlor Trick #2: The Perpetual Calendar

This trick will enable you to determine, within about 15 or 30 seconds, the day of the week, given any date in the twentieth century.

At first, the amount of information to memorize may seem rather overwhelming. However, with some patience, practice, and concentration, you will be able to master this excellent parlor trick. You may even find it useful on a day-to-day basis!

Strategy: You will need to memorize the perpetual calendar formula:

$$\frac{\text{Year} + \text{Year}/4 + \text{Date of month} + \text{Factor for month}}{7}$$



What matters is not the quotient of this division, but the remainder, as will be explained.

The Year in the equation above consists of the last two digits. For example, the last two digits of 1935 are 35. When calculating Year/4, divide 4 into the last two digits of the year. It's important that you remember to **round down** to the nearest whole number. For example, if the year under consideration is 1935, take 35 and divide it by 4, producing 8 (rounded down).

The Date of month is self-explanatory. However, the Factor for month is an arbitrary number that you must memorize for each month, as shown below:

January:	1	(0 if leap year)	July:	0
February:	4	(3 if leap year)	August:	3
March:	4		September:	6
April:	0		October:	1
May:	2		November:	4
June:	5		December:	6

When you perform the division shown on the previous page, the day of the week is determined by the remainder, as shown below:

Remainder of 1 = Sunday
Remainder of 2 = Monday
Remainder of 3 = Tuesday
Remainder of 4 = Wednesday
Remainder of 5 = Thursday
Remainder of 6 = Friday
Remainder of 0 = Saturday

You may use these factors to determine the day of the week for any date, from 1900–1999. For any date within the years 2000–2099, you may apply the same information and formula, but you'll need to subtract 1 from the numerator of the formula.

You'll note above that the factors for January and February are diminished by 1 during leap years. During the twentieth century, the leap years were, or are, 1904, 1908, 1912, 1916, 1920, 1924, 1928, 1932, 1936, 1940, 1944, 1948, 1952, 1956, 1960, 1964, 1968, 1972, 1976, 1980, 1984, 1988, 1992, and 1996. The year 1900 was not a leap year. The year 2000 *is* a leap year, however, followed by leap years every four years throughout the twenty-first century.

Let's take an example—November 22, 1963. Apply the formula and you get $(63 + \frac{63}{4} + 22 + 4)/7$. The second number, $\frac{63}{4}$, is rounded down to 15, so the numerator totals 104. Now divide 7 into 104, and obtain 14 Remainder 6. All that matters is the remainder, so (according to the equivalencies above) November 22, 1963 fell on a Friday.

See if you can determine each day of the week in under 30 seconds:

A. May 14, 1952	E. Apr. 27, 1918	I. Mar. 9, 1986
B. Aug. 3, 1920	F. July 20, 1939	J. Nov. 30, 1967
C. Dec. 8, 1943	G. Feb. 11, 1977	K. June 12, 1900
D. Jan. 1, 1996	H. Oct. 18, 1909	L. Sept. 29, 1954

(See solutions on page 224)

Trick 14: Rapidly Check Multiplication and Division

Strategy: Let's finish week one by learning how to check multiplication and division with a technique called "casting out nines." When a calculation has been performed correctly, this trick will indicate as such. When the wrong answer has been obtained, this method will *probably*, but not definitely, uncover the error. To check multiplication, the basic idea is to first obtain "digit sums" for the multiplicand and multiplier. For example, the digit sum of 25 is 7 (2 + 5). **Only a one-digit digit sum can be used.** Therefore, whenever a digit sum exceeds nine, simply perform another digit-sum calculation. For example, the digit sum of 683 is 17 (6 + 8 + 3). However, we must then compute the digit sum of 17, which is 8 (1 + 7). The second step is to multiply the digit sums of the multiplicand and multiplier together, to obtain a third digit sum. If this third digit sum matches the digit sum of the computed answer, chances are that the answer is correct. If they do not match, then the answer is definitely incorrect. To save even more time, disregard totals of "9" along the way. The following examples will clarify this fascinating technique.

Elementary Example #1

31 × 11 = 341

$$\begin{array}{rl} 31 & (\text{digit sum} = 4) \\ \times 11 & (\text{digit sum} = 2) \\ \hline & (\text{multiply: } 4 \times 2 = 8) \\ 341 & (\text{digit sum} = 8) \end{array}$$

Because the third and fourth digit sums are identical, the answer is probably correct.

Elementary Example #2

82 × 69 = 5,737

$$\begin{array}{rl} 83 & (\text{digit sum} = 11;\ \text{digit sum of } 11 = 2) \\ \times 69 & (\text{digit sum} = 15;\ \text{digit sum of } 15 = 6) \\ \hline & (\text{multiply: } 2 \times 6 = 12;\ \text{digit sum of } 12 = 3) \\ 5,737 & (\text{digit sum} = 22;\ \text{digit sum of } 22 = 4) \end{array}$$

Because the third and fourth digit sums (3 and 4) do not agree, the answer is definitely incorrect. (Note: To save time, the 9 of the multiplier could have been disregarded, to obtain the same digit sum of 6.)

Brain Builder #1

836 × 794 = 663,784

$$\begin{array}{rl} 836 & (\text{digit sum} = 17;\ \text{digit sum of } 17 = 8) \\ \times 794 & (\text{digit sum} = 20;\ \text{digit sum of } 20 = 2) \\ \hline & (\text{multiply: } 8 \times 2 = 16;\ \text{digit sum of } 16 = 7) \\ 663,784 & (\text{digit sum} = 34;\ \text{digit sum of } 34 = 7) \end{array}$$

Because the third and fourth digit sums are identical, the answer is probably correct.

Number-Power Note: This trick has been illustrated for multiplication only. However, because division is the inverse of multiplication, to apply the technique, simply convert the division into a multiplication problem. For example, to test 884 ÷ 26 = 34, look at it as 34 × 26 = 884, and go from there. This trick will also work with decimal points and zeroes—just disregard them. (They have been omitted here to simplify the explanation.) Trick 42 uses the same method, with a slight variation, to check addition and subtraction.

Remember, it is possible for the digit sums to agree but for the answer to be wrong. For example, in Elementary Example #1, an incorrect answer of 431 would produce the same digit sum, 8.

Elementary Exercises

Check the calculations below for accuracy, using the "casting out nines" method. For each one, indicate "probably correct" or "definitely incorrect." Reread the Number-Power Note to see how to check division problems for accuracy.

1. $53 \times 27 = 1,441$
2. $77 \times 22 = 1,694$
3. $96 \times 18 = 1,728$
4. $45 \times 600 = 277,000$
5. $14 \times 62 = 858$
6. $88 \times 33 = 2,904$
7. $71 \times 49 = 3,459$
8. $65 \times 24 = 1,560$
9. $38 \times 92 = 3,496$
10. $42 \times 56 = 2,352$
11. $5,846 \div 79 = 84$
12. $1,360 \div 85 = 16$
13. $3,149 \div 47 = 77$
14. $5,684 \div 98 = 58$
15. $2,349 \div 29 = 71$
16. $988 \div 13 = 76$

Brain Builders

1. $364 \times 826 = 300{,}664$
2. $555 \times 444 = 247{,}420$
3. $797 \times 51 = 40{,}647$
4. $286 \times 972 = 277{,}992$
5. $319 \times 634 = 202{,}276$
6. $740 \times 561 = 414{,}140$
7. $493 \times 168 = 82{,}824$
8. $666 \times 425 = 283{,}050$
9. $2,691{,}837 \div 857 = 3{,}141$
10. $992{,}070 \div 365 = 2{,}618$
11. $877{,}982 \div 217 = 4{,}046$
12. $689{,}976 \div 777 = 888$

(See solutions on page 203)

Week 1 Quick Quiz

Let's see how many tricks from week one you can remember and apply by taking this brief test. There's no time limit, but try to work through these items as rapidly as possible. Before you begin, glance at the computations and try to identify the trick that you could use. When you flip ahead to the solutions, you will see which trick was intended.

Elementary Examples

1. $45 \times 4 =$
2. $44 \div 5 =$
3. $11 \times 36 =$
4. $1{,}800 \div 60 =$
5. $900 \times 60 =$
6. $72 \times 25 =$
7. $99 \times 65 =$
8. $65^2 =$
9. $31 \times 29 =$
10. $76 \div 4 =$
11. $52 \times 5 =$
12. $93 \times 101 =$
13. $0.3 \times 700 =$
14. $640 \div 1.6 =$
15. $700 \div 25 =$
16. Using the "casting out nines" method, indicate whether this calculation is probably correct or definitely incorrect:
    $14 \times 87 = 1,318$

Brain Builders

1. $500 \times 4.7 =$
2. $350 \times 3.5 =$
3. $110 \times 5.8 =$
4. $580 \div 40 =$
5. $710 \div 500 =$
6. $211 \div 2.5 =$
7. $52 \times 40 =$
8. $3.9 \times 41 =$
9. $50 \times 9.4 =$
10. $7.5 \times 75 =$
11. $2.5 \times 36 =$
12. $1,010 \times 0.37 =$
13. $9.9 \times 0.85 =$
14. Using the "casting out nines" method, indicate whether this calculation is probably correct or definitely incorrect:
    $364,231 \div 853 = 427$

(See solutions on page 221)