UNIT ONE

Some Basic Concepts

Review Item	Ref Page	Example
1. The four fundamental operations in algebra are essentially the same as those of arithmetic: addition (+), subtraction (–), multiplication (X), and division (÷).	10	$6 \div 3 + \frac{8}{2} = \frac{6 \times 2 + 3 \times 8}{3 \times 2}$
2. Algebra differs from arithmetic in its frequent use of letters to represent numbers.	10	Arithmetic: $2 + 3 = 5$ Algebra: $a + b = c$
3. The use of letters to represent numbers makes it possible to translate long word statements into short mathematical sentences, expressions, or statements.	10	Word statement: The difference between twice a number (n) and half that number is nine. Mathematical statement: $2n - \frac{n}{2} = 9$
4. A letter used to represent a number is called a *literal number* or *variable.*	11	In the equation $t + 3 = 7$, the letter t is a literal number or variable.
5. An algebraic statement that represents two things that are equal to one another is called an *equation.*	11	$8n - 3n = 5n$

Review Item	Ref Page	Example
6. The addition symbol (+) and subtraction symbol (–) are the same in algebra as in arithmetic. In arithmetic, the multiplication symbol is the "times sign," X. In algebra there are four ways of expressing the idea of multiplication. X is seldom used.	11	We could express the idea of eight times a number in any of the following ways: 8 X n, 8 • n, 8(n), or 8n
7. Like the times sign, the division symbol (÷) is seldom used in algebra. Instead, the fraction bar or, less frequently, the colon is used.	11	For $x \div y$ we would write $\frac{x}{y}$ or $x : y$ Both mean x divided by y.
8. In arithmetic, numbers being multiplied together are called *factors*. In algebra, they are referred to as *numerical factors* if they are numbers, or *literal factors* if they are letters.	11	In the expression $2xy$, 2 is a numerical factor and x and y are literal factors.
9. Any factor or group of factors is the *coefficient* of the product of the remaining factors. If the factor is a number, it is called a *numerical coefficient*; if it is a letter, it is called a *literal coefficient*.	12	In the expression $2abc$, 2 is the numerical coefficient of abc, and a, b, and c are the literal coefficients of 2.
10. Axioms of equality:	12	
• If equals are added to equals, the sums are equal.		$4 = 6 - 2$; so, adding 2 to each side, $4 + 2 = (6 - 2) + 2$
• If equals are subtracted from equals, the differences are equal.		$6 = 4 + 2$; so, subtracting 2 from each side, $6 - 2 = (4 + 2) - 2$

Review Item	Ref Page	Example
• If equals are multiplied by equals, the products are equal.		$3 + 2 = 5$; so, multiplying both sides by 2, $2 \times (3 + 2) = 2 \times 5$
• If equals are divided by equals, the quotients are equal.		$7 \times 2 = 14$; so, dividing both sides by 2, $(7 \times 2) \div 2 = 14 \div 2$
11. Division by zero is meaningless; that is, it is an undefined operation.	12	$\frac{5}{0}$, $\frac{x}{0}$, and $\frac{7}{a}$, where $a = 0$, are meaningless expressions.
12. When adding or multiplying, the order of the numbers may be changed without affecting the result.	13	$2 + 3 = 3 + 2$ $a + d + f = f + d + a$ $2 \cdot 3 = 3 \cdot 2$ $abc = cba$
13. When subtracting or dividing, the order of the numbers may *not* be changed.	14	$3 - 2 \neq 2 - 3$ $\frac{2}{3} \neq \frac{3}{2}$ (The symbol $\neq$ means does not equal.)
14. The sum of three or more terms or the product of three or more factors is the same regardless of how they are grouped.	14	$a + (b + c) = (a + b) + c = a + b + c$ $a(bc) = (ab)c = abc$
15. The product of an expression of two or more terms multiplied by a single factor is equal to the sum of the products of each term of the expression multiplied by the single factor.	15	$a(b + c + d) = ab + ac + ad$
16. The fundamental operations should be performed in this order:	16	In the expression $6 + 3(2) - \frac{4}{2}$

Review Item	Ref Page	Example
• Multiplications and divisions first, from left to right.		First multiply and divide: $6 + 6 - 2$
• Additions and subtractions next (not necessarily in order).		Then add and subtract: $6 + 6 - 2 = 10$
17. Parentheses are used:	17	
• To replace the multiplication symbol.		$3 \times 2 = (3)(2)$
• To group numbers.		$a + (b - c)$
• To show that an expression should be treated as a single number.		Double the sum of 3 and x: $2(3 + x) = 6 + 2x$
18. Parentheses can also be used to establish the order of operations when evaluating an expression.	18	In the expression $4(3 + 2)$, add the 3 and 2 in parentheses *before* multiplying by 4. Thus, $4(3 + 2) = 4 \cdot 5 = 20$
19. An *algebraic expression* is the result obtained by combining two or more numbers or letters by means of one or more of the four fundamental operations of algebra.	18	$a + b$, $2a \div bc$, $\frac{x}{y}$, $\frac{3a + 2b}{a + b}$, and $\frac{3a^2}{2bc}$ are all algebraic expressions.
20. To *evaluate* (find the value of) an expression:	19	Evaluate $2(x - y) + 3x - \frac{y}{2}$ for $x = 5$, $y = 4$.
• Substitute the given values for the letters.		$2(5 - 4) + 3(5) - \left(\frac{4}{2}\right)$
• Evaluate and combine terms inside parentheses.		$2(1) + 3(5) - \left(\frac{4}{2}\right)$

Review Item	Ref Page	Example
• Perform indicated multiplications and divisions. • Add and subtract as indicated.		$2 + 15 - 2$ $17 - 2 = 15$
21. A *monomial* is an expression that does not involve addition or subtraction.	20	a, $2ab$, and $\frac{2ab}{3bc}$ are all monomials.
22. A *multinomial* is the sum of two or more monomials. A multinomial consisting of exactly two terms is a *binomial;* one consisting of exactly three terms is a *trinomial.*	20	Binomial: $2a + 3k$ Trinomial: $a^2 + ck - 5t$ Both are multinomials.
23. Each monomial in a multinomial, together with the sign that precedes it, is called a *term* of the multinomial.	20	$2a$, $-\frac{3b}{2c}$, $\frac{a^2}{3}$, and $\frac{1}{2}b$ are terms of the multinomial $2a + \left(-\frac{3b}{2c}\right) - \frac{a^2}{3} + \frac{1}{2}b$
24. An *exponent* is a number written to the right of and slightly above another number to indicate how many times the latter number, called the *base*, is to be taken as a factor. The product of this multiplication is called the *power*.	21	base $^{\text{exponent}}$ = power $2^3 = 2 \cdot 2 \cdot 2 = 8$
25. In an algebraic expression, *like terms* or *similar terms* are those having the same literal coefficients (letters) and the same exponents. Algebraic expressions can be simplified by combining like terms.	22	In the expression $2a + a + 3b - b$, $2a$ and a, $3b$ and $-b$ are like terms. When simplified, $2a + a + 3b - b$ becomes $3a + 2b$

UNIT ONE REFERENCES

1. Algebra is simply a logical extension of arithmetic. The same four fundamental operations you learned in arithmetic are also essential in algebra: addition (+), subtraction (−), multiplication (X), and division (÷). The symbols shown are used to indicate, in mathematical shorthand, the operations to be performed. The result of addition is the sum; of subtraction, the difference or remainder; of multiplication, the product; and of division, the quotient.

2. The four operations discussed above are performed in algebra—with one major difference. In algebra *letters frequently are used to represent numbers.* Why? Because in algebra we often work with quantities without regard to their numerical values. We may need to use their numerical values eventually, but in the meantime we have to identify them in some way. So we use the letters of the alphabet.

3. How does the use of letters, numbers, and symbols make it possible to translate long word statements into brief mathematical statements? Here is an example:

Example: The sum of five times a number and two times the same number is equal to seven times the number. How can we represent this more simply?

Solution: If we let n represent the number we are talking about, we can say the same thing with this short algebraic sentence: $5n + 2n = 7n$.

Try this one: Three times a number subtracted from eight times the same number equals five times the number.

Solution: $8n - 3n = 5n$

Use letters and symbols to change these word statements into algebraic expressions:

(a) The sum of one-half x and one x equals 12. ____________________

(b) Twice d plus half of b added to 3 equals nine. ____________________

(c) Ten times a number (n) minus three times the number equals 7 more than four times the number. ____________________

(d) The area (A) of a triangle is equal to one-half the base (b) times the height (h). ____________________

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(a) $\frac{x}{2} + x = 12$; (b) $2d + \frac{b}{2} + 3 = 9$; (c) $10n - 3n = 4n + 7$;

(d) $A = \frac{1}{2}bh$ or $\frac{bh}{2}$

4. The word *literal* means having to do with a letter (of the alphabet). In algebra, we have a special name for a letter that is used to represent a number. It is called a *literal number* or a *variable.*

5. The word statements which you translated into algebraic expressions in reference item 3 are examples of equations since, in each case, one quantity was equal to another. Bear in mind that an algebraic expression is not necessarily an equation, unless there is an equality involved. For example, $ax + by + c$ is an algebraic expression; $ax + by + c = 0$ is an algebraic expression in the form of an equation. An equation will always contain an equal sign (=).

6. The "times sign," X, is seldom used in algebra to indicate multiplication. One reason for this is the possibility of confusing it with the letter x of the alphabet, which *does* appear frequently in algebra as a variable. As shown in the example, there are other ways of indicating multiplication. Both the dot and the use of parentheses are acceptable. Omission of the multiplication sign, as in $8n$, is preferred where either or both of the factors is a letter. Express the product of the following without using multiplication signs.

(a) $a \times b \times c$ ________ (d) $0.5 \times 40 \times t$ ________

(b) $3 \times c \times d$ ________ (e) $3 \times 4 \times dy$ ________

(c) $\frac{2}{5} \times 15 \times q$ ________

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(a) abc; (b) $3cd$; (c) $6q$; (d) $20t$; (e) $12dy$

7. The division symbol (÷) is used occasionally but not commonly. More frequently, the fraction bar is used to indicate division, and sometimes the colon (:). Thus, for $k^2 \div p$ we usually would write $\frac{k^2}{p}$ or $k^2{:}p$, both of which mean k^2 divided by p.

8. Factors, in algebra as in arithmetic, are simply numbers that are being multiplied together. If the factor is a letter, we refer to it as a *literal factor*; if it is a number, we call it a *numerical factor.* This makes it easier to talk about the various parts of an algebraic expression. For example:

Expression	Literal factors	Numerical factors
$3axy$	a, x, and y	3
$2a$	a	2
$4z(7\ kp)$	z, k, and p	4 and 7
$ab \cdot 9ry$	a, b, r, and y	9

9. From review item 8 you know that, for example, in the expression $3xyz$, x, y, and z would be called the literal factors, and 3 would be called the numerical factor. Similarly, 3 and $(c + d)$ are factors of the expression $3(c + d)$. Now we are introducing some new terminology that will prove highly useful in the future in identifying the components of a group of factors.

Any factor or group of factors is called the *coefficient* of the product of the remaining factors. Thus, in the product $3 \cdot 5$, the number 3 is the coefficient of 5, and 5 is the coefficient of 3. In the product $4ab$, 4 is the *numerical coefficient* of ab, and ab is the *literal coefficient* of 4. If a letter does not have a coefficient written before it, the coefficient is understood to be 1. Thus, a means $1a$, x means $1x$, k means $1k$, and so on.

As we said before, our bottom line is solving math problems correctly. Although all these terms are important, just think of them as part of your vocabulary. You don't have to know their exact definitions as long as you can use them in problem solving.

10. Both arithmetic and algebra make use of the axioms of equality. An *axiom*, as you may remember, is a basic assumption that is accepted as true without proof. Axioms are considered self-evident. They are, in effect, the building blocks of mathematics. In addition to the four axioms in review item 10, here is another very important one:

> *Things equal to the same thing are equal to each other.* Thus, if $a = 4$ and $b = 4$, then $a = b$. Test your understanding of these axioms by completing the following:

(a) If $x = 21$ and $y = 21$, then $x =$ __________

(b) If $x = y$ and $y = z$, then $x =$ __________

(c) If $r = 15$ and $s = 15$, then $r =$ __________

(a) y; (b) z; (c) s

11. Since, as you are now well aware, letters often are used in algebra to represent numbers, it is important to be alert to one special situation that could get you into trouble. That is the situation in which a letter stands for zero.

From arithmetic, we know that the result of adding zero to or subtracting zero from another number is the same as the original number ($x + 0 = x$; $x - 0 = x$). Nothing has changed. You probably recall also that multiplying a number by zero — or multiplying zero by a number — gives zero as a result ($x \cdot 0 = 0$). But what happens when we try to *divide* by zero?

Division by zero is an impossible operation. As shown in the example, such fractions as $\frac{8}{0}$ or $\frac{x-5}{0}$ are meaningless. This is easy to recognize when you actually see zero as the denominator (that is, the lower half of a fraction). But when the denominator contains one or more literal factors, you must be very careful that one of these letters doesn't stand for zero, or that the value assigned to the letter doesn't cause the denominator to *become* zero.

To see how this might happen, indicate in the fractions below which value of the letters in each of the denominators would result in an impossible division (that is, a zero denominator).

(a) $\frac{8}{c}$ ______; (b) $\frac{a}{2b}$ ______; (c) $\frac{6}{x-3}$ ______; (d) $\frac{8}{xy}$ ______;
(e) $\frac{3}{y-5}$ ______; (f) $\frac{2a}{3b}$ ______; (g) $\frac{t}{x-y}$ ______; (h) $\frac{0.7ax}{9.2ky}$ ______.

(a) $c = 0$; (b) $b = 0$; (c) $x = 3$; (d) x or $y = 0$; (e) $y = 5$; (f) $b = 0$; (g) $y = x$; (h) k or $y = 0$

12. When adding, subtracting, multiplying, or dividing, the order of the numbers in an algebraic expression can sometimes be changed without affecting the result—but not in every case. If the numbers *can* be interchanged without affecting the result, the operation is said to be *commutative.* A little investigation shows that only two of the fundamental operations are commutative: addition and multiplication. This gives us the following two laws:

> *The sum of two quantities is the same whatever the order of addition.*
>
> *The product of two quantities is the same whatever the order of multiplication.*

Notice that these laws apply only to *pairs* of numbers, not to triples.

Indicate by the words *true* and *false* which of the following are correct examples of the commutative laws for addition and multiplication.

(a) $p + k = k + p$ ______ (e) $42 + 13 = 13 + 42$ ______

(b) $6 + 3 = 3 + 6$ ______ (f) $9b = b9$ ______

(c) $xy = yx$ ______ (g) $\frac{a}{5} = \frac{5}{a}$ ______

(d) $7 - d = d - 7$ ______ (h) $abc = cba$ ______

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(a) true; (b) true; (c) true; (d) false; (e) true; (f) true; (g) false; (h) true (but for a reason we will discuss later, the commutative law for multiplication does not apply to *three* terms).

13. Practice problems (d) and (g) in reference item 12 were false, because the commutative laws for addition and multiplication do not hold for subtraction or division. To make this clearer, suppose in problem (d) we allowed the letter d to represent the numerical value 3. We would then have

$$7 - d = d - 7 \text{ or } 7 - 3 = 3 - 7$$

which obviously is untrue.

Similarly, if in problem (g) we let the letter a represent the value 3, this would give us

$$\frac{a}{5} = \frac{5}{a} \text{ or } \frac{3}{5} = \frac{5}{3}$$

which is also obviously not true.

14. So far, in review items 12 and 13, we have considered the laws for interchanging numbers only as they relate to *pairs* of numbers. What if there are three numbers? Adding three numbers is slightly more involved. For example, if we wish to add $2 + 5 + 8$, we might first add $2 + 5 = 7$, then add $7 + 8 = 15$. But we could just as well add $5 + 8 = 13$ and then $2 + 13 = 15$. The result is the same; that is, $(2 + 5) + 8 = 2 + (5 + 8)$. To describe this property, we say that addition is *associative*. The associative law for addition states:

> *The sum of three quantities is the same regardless of the manner in which the partial sums are grouped.*

Here are some further examples:

$$a + 2 + 3 = (a + 2) + 3 = a + (2 + 3)$$
$$c + d + a = (c + d) + a = c + (d + a)$$
$$x + y + z = z + x + y = z + y + x$$

The last example combines the commutative and associative laws and illustrates the somewhat more general rule:

> *The sum of three or more numbers is the same regardless of the order in which the addition is performed.*

Similarly, if we have three *factors*, then $a \cdot b \cdot c = a(b \cdot c) = (a \cdot b)c$. This is known as the associative law for multiplication:

The product of three or more numbers is the same regardless of the order in which the multiplication is performed.

Here are some examples:

$2 \cdot 3 \cdot 4 = 2(3 \cdot 4) = (2 \cdot 3)4 = 24$
$c \cdot d \cdot f = c(d \cdot f) = (c \cdot d)f = cdf$ or, simply,
$cdf = c(df) = (cd)f$

Based on what we have covered so far about the commutative and associative laws, determine whether the following statements are *true* or *false*.

(a) $7xy = yx7$ ____________

(b) $k + r + t = r + k + t$ ____________

(c) $pt(z) = (p)zt$ ____________

(d) $a + b - c = b + a - c$ ____________

(e) $2x \div y = 2y \div x$ ____________

(f) $k + m - n = n + k - m$ ____________

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(a) true; (b) true; (c) true; (d) true (because the position of the number being subtracted was not changed); (e) false; (f) false (because the position of the number being subtracted was changed)

Once more, then:

- When *adding*, you *may* change the order of the numbers.
- When *subtracting*, you *may not* change the order of the numbers.
- When *multiplying*, you *may* change the order of the numbers.
- When *dividing*, you *may not* change the order of the numbers.

15. In addition to the commutative and associative laws, there is a third law known as the *distributive* law for multiplication. This law states:

The product of an expression of two or more terms by a single factor is equal to the sum of the products of each term of the expression by the single factor.

In simpler mathematical language this law says that

$a(b + c) = ab + ac$

or, using numbers instead of letters,

$$2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$$

Before considering further applications of the distributive law, you need to recall that if a number, such as a, is multiplied by itself, we write $a \cdot a = a^2$. Similarly, $a \cdot a \cdot a = a^3$. The exponent (that is, the number written to the right and a little above the number being multiplied by itself, in this case the letter a) indicates the number of times the quantity a is used as a factor. (See reference item 24 below.)

Here are a few more examples of the distributive law:

$$a(a + b) = a^2 + ab$$
$$2b(ab + bc) = 2ab^2 + 2b^2c$$

For more than two terms we use the *extended distributive* law:

$$2a(2a + 3b - 4ad) = 4a^2 + 6ab - 8a^2d$$
$$3ab(a^2 - 2ad + b) = 3a^3b - 6a^2bd + 3ab^2$$

Apply the extended distributive law to the following multiplication problems.

(a) $b(c + d + e) =$ ____________

(b) $bc(c + d - 2) =$ ____________

(c) $3x(2 - xy + z) =$ ____________

(d) $4ab(2ab + 3ac - cd) =$ ____________

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(a) $bc + bd + be$; (b) $bc^2 + bcd - 2bc$; (c) $6x - 3x^2y + 3xz$;
(d) $8a^2b^2 + 12a^2bc - 4abcd$

16. When more than one algebraic operation is indicated in an expression, the various operations should be performed in the right order for best results. The rule is:

Do multiplications and divisions first, in order from left to right.

Do additions and subtractions second.

For example, in the expression $6 + 3(2) - \frac{4}{2}$, performing the multiplication gives us $6 + 6 - \frac{4}{2}$. Next, performing the division, we get $6 + 6 - 2$. Finally, adding and subtracting gives us the answer: 10.

At the moment, however, the answer is not nearly as important as the *procedure* you follow. Study the expression below and indicate the correct sequence of operations.

$$\frac{3 \cdot 4}{6} + 2(7) - \frac{9}{3} =$$

	Operation	*Term(s)*
1.	______________	______________
2.	______________	______________
3.	______________	______________

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1.	multiplications	$3 \cdot 4$ and $2(7)$
2.	divisions	12 by 6 and 9 by 3
3.	addition and subtraction	All terms (=13)

17. In working with the associative law for addition, we used parentheses to group numbers: $a + (b - c)$. However, we also use parentheses to indicate multiplication (as covered in review item 6) and to show that an expression should be treated as a single number. Thus, if we wish to double the sum of 3 and x, we write $2(3 + x)$. From working with the distributive law, you know that this tells us that we must multiply both 3 and x by 2 to get the correct answer. Or if we wished to multiply the difference, 9 minus y, by 3, we would write this as $(9 - y)3$ or $3(9 - y)$; either is correct.

Below are some further examples of the use of parentheses to express word statements algebraically:

1.	The sum of k and twice p.	$k + 2p$
2.	Twice the sum of s and r.	$2(s + r)$
3.	Twice the sum of a plus b equals 9.	$2(a + b) = 9$
4.	a divided by the sum of a and b plus twice xy equals 7.	$\frac{a}{a + b} + 2xy = 7$

Now here are a few for you to practice on. Use parentheses to express the following relationships.

(a) Twice the sum of $c + d$ equals 7. ______________

(b) b divided by the sum of b and a, plus twice qp equals 9. ________

(c) Two added to one-third the quantity of y minus z equals $2z$. ______

(d) a plus half the quantity of y minus 2 equals 13. ______________

(e) Three times a number n divided by y times the sum of 1 and the number is equal to 7. ______________

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(a) $2(c + d) = 7$; (b) $\frac{b}{b + a} + 2qp = 9$; (c) $2 + \frac{1}{3}(y - z) = 2z$;
(d) $a + \frac{1}{2}(y - 2) = 13$; (e) $\frac{3n}{y(1 + n)} = 7$

18. Here are a few problems to help you practice evaluating expressions containing parentheses. Remember the order of operations: Multiplication and division first, addition and subtraction last. Terms enclosed in parentheses should be combined wherever possible.

Find the value of each of the following expressions.

(a) $2(3 + 4) - 2 \cdot 3 =$ ______	(e) $2(3 + 2) - 7 + \frac{9}{3} =$ ______
(b) $6 - \frac{1}{2}(4 - 2) =$ ______	(f) $\frac{6 + 4}{2} - \frac{6}{3} + 2 \cdot 4 =$ ______
(c) $3 - \frac{1}{3}(4 + 2) =$ ______	(g) $7 - \frac{12}{4} + 2(3 + 1) =$ ______
(d) $12 - 5(4 - 2) =$ ______	(h) $\frac{9 + 3}{4 - 1} - 3 + (6 \div 2) =$ ______

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(a) 8; (b) 5; (c) 1; (d) 2; (e) 6; (f) 11; (g) 12; (h) 4

19. We have been using the term "algebraic expression" very frequently, in a rather self-evident sense. Now we are ready to define it. As stated in review item 19,

> An *algebraic expression* is the result obtained by combining two or more numbers or letters by means of one or more of the four fundamental operations of algebra.

Another way of defining an algebraic expression would be to say that it is a statement containing one or more terms, some mathematical operations, and symbols of grouping.

We will define "term" a little more precisely in review item 23, but from what you have learned so far you should be able to determine which of the following are algebraic expressions and, if they are, how many terms each has.

	Expression?	*No. of terms*
(a) $3xyz$	______	______
(b) $3a + b - c$	______	______
(c) $3ak + \frac{y}{2} + 8$	______	______
(d) $7ab - y(2 + z)$	______	______

(e) $6(a - b) + cy$ ____________ ____________

(a) yes, 1; (b) yes, 3; (c) yes, 3; (d) yes, 2; (e) yes, 2. (If you had any trouble with the figures in parentheses, such as $(2 + z)$ in problem (d) or $(a - b)$ in problem (e), remember that figures in parentheses are treated as one number.)

20. Let us start by evaluating the expression

$$3(a + b) + 2a - \frac{b}{2} \text{ if } a = 5,\ b = 4$$

Substituting the numerical values in place of a and b gives

$$3(5 + 4) + 2 \cdot 5 - \frac{4}{2}$$

Adding the numbers inside the parentheses, we get

$$3 \cdot 9 + 2 \cdot 5 - \frac{4}{2}$$

Multiplying and dividing gives us

$$27 + 10 - 2$$

and finally *adding and subtracting the terms of the expression,* we get 35 as our answer. These are logical, orderly steps which, when carefully followed, give correct answers with a minimum of effort.

Follow the above procedure to evaluate the expressions below. (*Note:* Two or more symbols that share the same fraction bar are treated as one term, just as two or more symbols within parentheses are treated as one term.)

(a) $2(a + b) - 12 + 2b$ if $a = 2$, $b = 4$ ____________

(b) $7 + 2k - 3\left(m - \frac{k}{3}\right)$ if $k = 6$, $m = 4$ ____________

(c) $2(x + y) + 3x - \frac{x + y}{2}$ if $x = 4$, $y = 2$ ____________

(d) $2xy + (2x - y) - 10$ if $x = 3$, $y = 2$ ____________

Evaluate these expressions for $a = 2$, $c = 3$.

(e) $2(a + c) - \frac{3}{c - a} + 7 =$ ____________

(f) $\frac{9}{c} - \frac{a}{2} + \frac{5ac}{a + c} =$ ____________

(g) $\frac{2ac}{4} + \frac{1}{2}(4c - a) =$ ____________

(h) $\frac{a^2c}{4} + \frac{ac^2}{6} + c =$ __________

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(a) 8; (b) 13; (c) 21; (d) 6; (e) 14; (f) 8; (g) 8; (h) 9

21. A monomial may consist of just a single number or letter. Usually, however, it is a combination of letters, numbers, or both being divided or multiplied together—but *without* any addition or subtraction involved. For example, if the monomials shown in the review item—a, $2ab$, and $\frac{2ab}{3bc}$—were to be linked together in any way by plus or minus signs, they would no longer be monomials. As we will see in reference item 22, the resulting expression would be given a different name.

22. The "different name" referred to in reference item 21 above for two or more monomials being added together or subtracted from one another is *multinomial.* In addition to this general name for an expression containing two or more monomials, we also have, for convenient reference, some more specific names. A *binomial* consists of two monomials, and a *trinomial* of three monomials. If there are more than three terms (monomials), the expression is simply called a multinomial.

Identify the following as either monomials, binomials, trinomials, or multinomials.

(a) $ab + jk + 4$ __________ (d) $\frac{pk}{7} + r^2 - 3$ __________

(b) $\frac{2xy}{3k}$ __________ (e) $3c^2 + 12$ __________

(c) $7 - ax + \frac{y}{3} + z^2$ __________ (f) $2(a + b) - 8$ __________

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(a) trinomial; (b) monomial; (c) multinomial; (d) trinomial; (e) binomial; (f) binomial

Although we will be using these terms occasionally, you won't have to memorize them. So even if you didn't answer all of the last six questions correctly, don't get all bent out of shape (as we used to say back in the 1970s).

23. Up to this point we have used the word "term" in a rather general sense. Now we are going to stop fooling around and get a bit more precise. We will begin by talking about *sign*.

In Unit 2, we will discuss the fact that (+) and (–) symbols can be used either as signs of operation (that is, telling us to add or subtract quantities) or as indications that the quantities themselves are positive or negative. Although we will not go into this in any detail now, the example shown illustrates this idea. Here again is the multinomial used:

$$2a + \left(-\frac{3b}{2c}\right) - \frac{a^2}{3} + \frac{1}{2}b$$

The second term is negative, as shown by the minus symbol. But since we wish to add it to the first term, the sign of operation is plus (+). The parentheses are used simply to separate the two signs. Since, by the rules of algebraic addition, adding a minus quantity is the same as subtracting a positive quantity, when writing this multinomial we would normally omit the parentheses and change the sign of operation to indicate subtraction. Thus, the multinomial would appear as

$$2a - \frac{3b}{2c} - \frac{a^2}{3} + \frac{1}{2}b$$

How many terms are in each of the following expressions?

(a) $4z + \frac{1}{3}bx - 3(k - z)$ ____ (c) $c(d) + b^2c - \frac{dx}{3}$ ____

(b) $3(c + d) - \frac{y}{x} + 2z$ ____ (d) $bc + cd - de(y + x)$ ____

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(a) 3; (b) 3; (c) 3; (d) 3

24. We touched briefly on the subject of exponents in reference item 15. Now it is time to take a closer look at them. The repeated multiplying of a factor by itself is an important concept since it occurs regularly in algebraic expressions. For example, if we wish to multiply 2 • 2 • 2, we may express this in shorter form by writing 2^3. This is read as "two cubed" or "two to the third power." Similarly, the expression x^2y^3 would be read "x squared times y cubed." What this last example means is "two factors of x times three factors of y." (*Note:* If a numeral or letter has *no* exponent written at its upper right, the exponent is understood to be 1. Thus, y means y^1 and 4 means 4^1.)

Write the following expressions using exponents where appropriate:

(a) $cc + acc + bbbc$ ______________

(b) $mmmy - xx + mx$ ______________

(c) $\frac{y}{mm} + xyy - m(my)$ ______________

(d) $ab + bc + cd$ ______________

What do the following expressions mean?

(e) a^2b^3 ____

(f) $7d^2e$ ____________________

(g) 3^2x^3 ____________________

(h) $(4y)^2$ ____________________

(a) $c^2 + ac^2 + b^3c$; (b) $m^3y - x^2 + mx$; (c) $\frac{y}{m^2} + xy^2 - m^2y$; (d) $ab + bc + cd$ (no repeated factors); (e) two factors of a times three factors of b (or a squared times b cubed); (f) 7 times two factors of d times e; (g) two factors of 3 times three factors of x; (h) two factors of $4y$ (or the expression $4y$ squared)

25. The process of simplifying an algebraic expression is merely a matter of combining *like* (or *similar*) *terms*. Therefore, it is important to be able to recognize similar terms. As indicated in review item 25, terms are similar if they have the same literal coefficients *and* these coefficients have the same exponents. Thus, in the expression $3a + 4a$, the two terms can be combined to become $7a$ because the literal coefficients are the same and have the same exponents (a to the first power in each case). In the expression $2a + 3x^2 + 3a + x^2$, the like terms can be combined to produce the simplified expression $5a + 4x^2$. Similarly,

$$6a + 2a^2 - 3a - a^2 = 3a + a^2$$

and

$$2ab + cd^2 + cd - ab = ab + cd + cd^2$$

Simplify the following expressions where possible:

(a) $3a + 4a - 7 =$ ____________

(b) $4a - 3b + 2a + 6b =$ ____________

(c) $2k + 4k - 8c - 8$ ____________

(d) $2x^2 - 3y^2 + 4x^2 - 13 =$ ____________

(e) $x^2 + y^2 + 7x^2 - 2y^2 =$ ____________

(f) $2xy - 3ak + 3xy + 4ak =$ ____________

(g) $3y^2 + x - y^3 + 2xy =$ ____________

(h) $2x + x^2 + 3x - 3x^3 =$ ____________

(a) $7a - 7$; (b) $6a + 3b$; (c) $6k - 8c - 8$; (d) $6x^2 - 3y^2 - 13$; (e) $8x^2 - y^2$; (f) $5xy + ak$; (g) $3y^2 + x - y^3 + 2xy$ (note that no combining of terms was possible); (h) $5x + x^2 - 3x^3$

Let's pause here for a couple of minutes to catch our breath. We have just reviewed the highlights of a good six months of algebra. How well do you understand this material? If you got nearly everything right, then please go directly to Unit 2.

No one is going to get everything right, but if you got more than 10 percent of these problems wrong, we will give you a couple of alternatives. One is to reread the parts of this unit you didn't fully understand. The other is to read at least one of the two books we mentioned earlier, *Practical Algebra* and *All the Math You'll Ever Need.*