1

INVESTIGATION OF FORCES AND FORCE ACTIONS

Loads deriving from the tasks of a structure produce forces. The tasks of the structure involve the transmission of the load forces to the supports for the structure. Applied to the structure, these external load and support forces produce a resistance from the structure in terms of internal forces that resist changes in the shape of the structure. This chapter treats the basic properties and actions of forces.

1.1 PROPERTIES OF FORCES

The idea of force is one of the fundamental concepts of mechanics and does not yield to simple, precise definition. An accepted definition of force is that which produces, or tends to produce, motion or a change in the state of motion of objects. A type of force is the effect of gravity, by which all objects are attracted toward the center of the earth.

What causes the force of gravity on an object is the mass of the object, and in U.S. units, this force is quantified as the weight of the body. Gravity forces are thus measured in pounds (lb), or in some other unit such as tons (T) or kips (one kilopound, or 1000 pounds). In the metric system, force is measured in a more purely scientific manner as directly related to the mass of objects; the mass of an object being a constant, whereas weight is proportional to the precise value of the acceleration of gravity, which varies from place to place. Force in metric units is measured in newtons (N) or in kilonewtons (kN) or in meganewtons (mN), whereas weight is measured in grams or in kilograms.

Vectors

A quantity that involves magnitude, direction (vertical, for example), and sense (up, down, etc.) is a *vector quantity*, whereas a *scalar quantity* involves only magnitude and sense. Force, velocity, and acceleration are vector quantities, whereas energy, time, and temperature are scalar quantities. A vector can be represented by a straight line, leading to the possibility of constructed graphical solutions in some cases; a situation that will be demonstrated later. Mathematically, a scalar quantity can be represented completely as +50 or –50, whereas a vector must somehow have its direction represented as well (50 vertical, horizontal, etc.).

Identifying a Force

In order to completely identify a force, it is necessary to establish the following:

Magnitude, or the amount of the force, which is measured in weight units such as pounds or tons.

Direction of the force, which refers to the orientation of its path, called its *line of action*. Direction is usually described by the angle that the line of action makes with some reference, such as the horizontal.

Sense of the force, which refers to the manner in which it acts along its line of action (e.g., up or down, right or left, etc.). Sense is usually expressed algebraically in terms of the sign of the force, either plus or minus.

Forces can be represented graphically in terms of these three properties by the use of an arrow, as shown in Figure 1.1*a*. Drawn to some scale, the length of the arrow represents the magnitude of the force. The angle of inclination of the arrow represents the direction of the force. The location of the arrowhead represents the sense of the force. This form of representa-

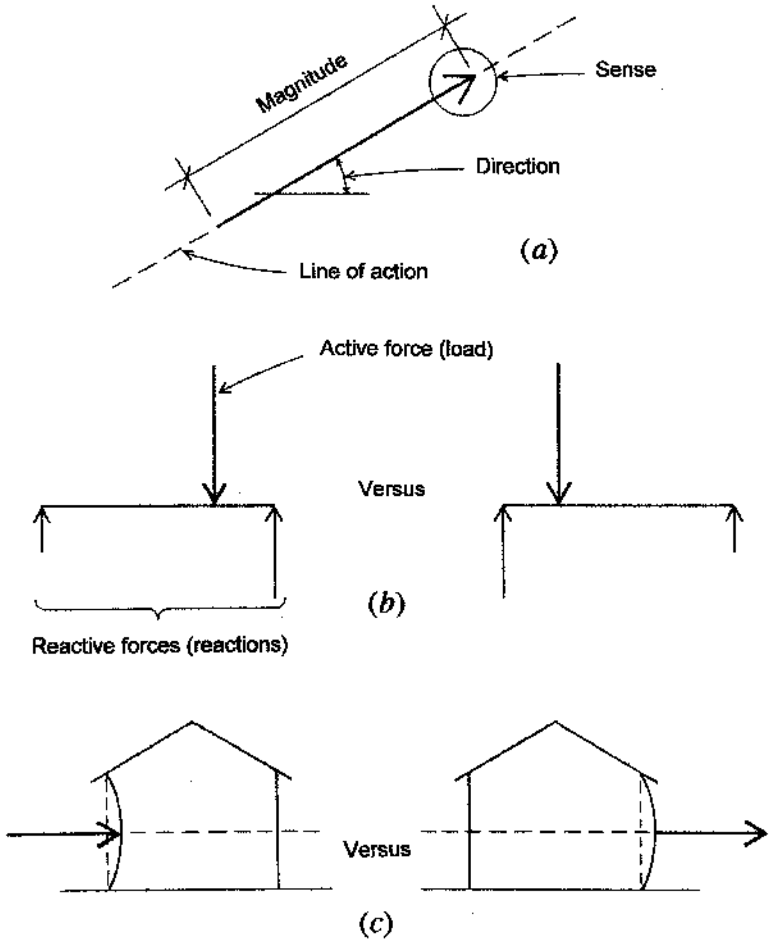

Figure 1.1 Representation of forces and force actions.

tion can be more than merely symbolic, because actual mathematical manipulations may be performed using the vector representation that the force arrows constitute. In the work in this book, arrows are used in a symbolic way for visual reference when performing algebraic computations, and in a truly representative way when performing graphical analyses.

In addition to the basic properties of magnitude, direction, and sense, some other concerns that may be significant for certain investigations are

The *position of the line of action* of the force with respect to the lines of action of other forces or to some object on which the force operates, as shown in Figure 1.1*b*. For the beam, shifting of the location of the load (active force) affects changes in the forces at the supports (reactions).

The *point of application* of the force along its line of action may be of concern in analyzing for the specific effect of the force on an object, as shown in Figure 1.1*c*.

When forces are not resisted, they tend to produce motion. An inherent aspect of static forces is that they exist in a state of *static equilibrium*, that is, with no motion occurring. In order for static equilibrium to exist, it is necessary to have a balanced system of forces. An important consideration in the analysis of static forces is the nature of the geometric arrangement of forces in a given set of forces that constitute a single system. The usual technique for classifying force systems involves consideration of whether the forces in the system are

Coplanar. All acting in a single plane, such as the plane of a vertical wall.

Parallel. All having the same direction.

Concurrent. All having their lines of action intersect at a common point.

Using these three considerations, the possible variations are given in Table 1.1 and illustrated in Figure 1.2. Note that variation 5 in the table

TABLE 1.1 Classification of Force Systems [a]

	Qualifications		
System Variation	Coplanar	Parallel	Concurrent
1	Yes	Yes	Yes
2	Yes	Yes	No
3	Yes	No	Yes
4	Yes	No	No
5	No[b]	Yes	Yes
6	No	Yes	No
7	No	No	Yes
8	No	No	No

[a] See Fig. 1.2.

[b] Not possible—parallel, concurrent forces are essentially coplanar.

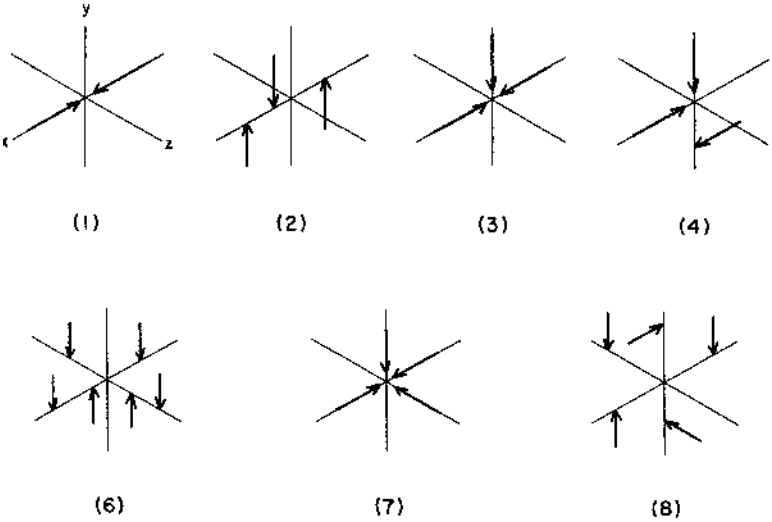

Figure 1.2 Types of force systems.

is really not possible because a set of coacting forces that is parallel and concurrent cannot be noncoplanar; in fact, the forces all fall on a single line of action and are called *collinear*.

It is necessary to qualify a set of forces in the manner just illustrated before proceeding with any analysis, whether it is to be performed algebraically or graphically.

1.2 STATIC EQUILIBRIUM

As stated previously, an object is in *equilibrium* when it is either at rest or has uniform motion. When a system of forces acting on an object produces no motion, the system of forces is said to be in *static equilibrium*.

A simple example of equilibrium is illustrated in Figure 1.3*a*. Two equal, opposite, and parallel forces, having the same line of action, P_1 and P_2, act on a body. If the two forces balance each other, the body does not move and the system of forces is in equilibrium. These two forces are *concurrent.* Put another way, if the lines of action of a system of forces have a point in common, the forces are concurrent.

Another example of forces in equilibrium is illustrated in Figure 1.3*b*. A vertical downward force of 300 lb acts at the midpoint in the length of

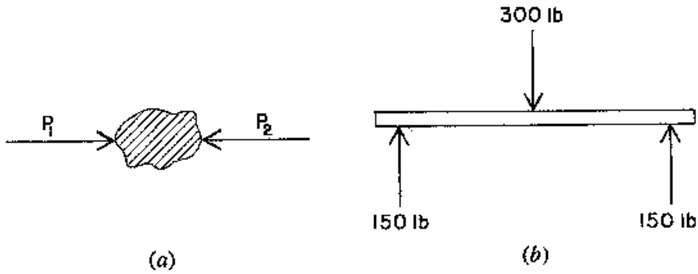

Figure 1.3 Equilibrium of forces.

a beam. The two upward vertical forces of 150 lb each (the reactions) act at the ends of the beam. The system of three forces is in equilibrium. The forces are parallel and, not having a point in common, are *nonconcurrent.*

1.3 FORCE COMPONENTS AND COMBINATIONS

Individual forces may interact and be combined with other forces in various situations. Conversely, a single force may have more than one effect on an object, such as a vertical action and a horizontal action simultaneously. This section considers both of these issues: adding up of forces (combination) and breaking down of single forces into components (resolution).

Resultant of Forces

The *resultant* of a system of forces is the simplest system (usually a single force) that has the same effect as the various forces in the system acting simultaneously. The lines of action of any system of two coplanar nonparallel forces must have a point in common, and the resultant of the two forces will pass through this common point. The resultant of two coplanar, nonparallel forces may be found graphically by constructing a *parallelogram of forces.*

In constructing a parallelogram of two forces, the forces are drawn at any scale (of so many pounds to the inch) with both forces pointing toward, or both forces pointing away, from the point of intersection of their lines of action. A parallelogram is then produced with the two forces as adjacent sides. The diagonal of the parallelogram passing through the common point is the resultant in magnitude, direction, and line of action, the direction of the resultant being similar to that of the given forces, toward or away from the point in common. In Figure 1.4*a*, P_1 and P_2 represent two nonparallel forces whose lines of action intersect at point *O*. The parallelogram is drawn, and the diagonal *R* is the resultant of the given system. In this illustration note that the two forces point *away* from the point in common; hence, the resultant also has its direction away from point *O*. It is a force upward to the right. Notice that the resultant of forces P_1 and P_2 shown in Figure 1.4*b* is *R*; its direction is toward the point in common.

Forces may be considered to act at any points on their lines of action. In Figure 1.4*c* the lines of action of the two forces P_1 and P_2 are extended until they meet at point *O*. At this point the parallelogram of forces is constructed, and *R*, the diagonal, is the resultant of forces P_1 and P_2. In determining the magnitude of the resultant, the scale used is, of course, the same scale used in drawing the given system of forces.

Example 1. A vertical force of 50 lb and a horizontal force of 100 lb, as shown in Figure 1.4*d*, have an angle of 90° between their lines of action. Determine the resultant.

Solution: The two forces are laid off from their point of intersection at a scale of 1 in. = 80 lb. The parallelogram is drawn, and the diagonal is the resultant. Its magnitude scales approximately 112 lb, its direction is

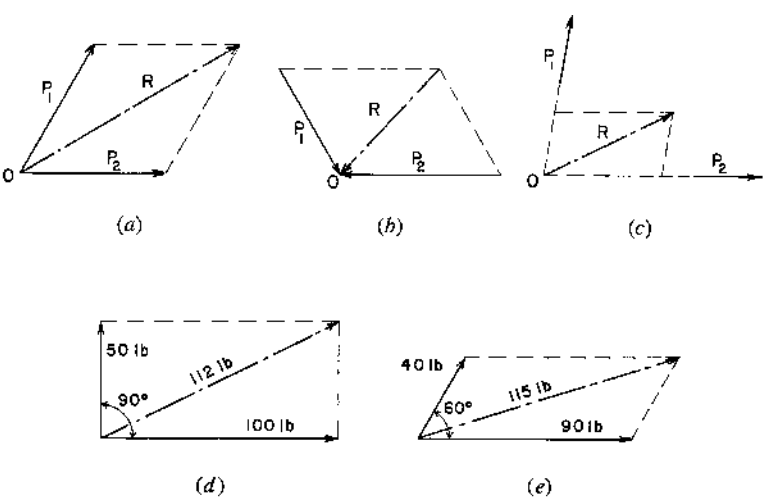

Figure 1.4 Consideration of the resultant of a set of forces.

upward to the right, and its line of action passes through the point of intersection of the lines of action of the two given forces. By use of a protractor, it is found that the angle between the resultant and the force of 100 lb is approximately 26.5°.

Example 2. The angle between two forces of 40 and 90 lb, as shown in Figure 1.4*e*, is 60°. Determine the resultant.

Solution: The forces are laid off from their point of intersection at a scale of 1 in. = 80 lb. The parallelogram of forces is constructed, and the resultant is found to be a force of approximately 115 lb, its direction is upward to the right, and its line of action passes through the common point of the two given forces. The angle between the resultant and the force of 90 lb is approximately 17.5°.

Attention is called to the fact that these two problems have been solved graphically by the construction of diagrams. Mathematics might have been employed. For many practical problems, graphical solutions give sufficiently accurate answers and frequently require far less time. Do not make diagrams too small, as greater accuracy is obtained by using larger parallelograms of forces.

Problems 1.3.A–F. By constructing the parallelogram of forces, determine the resultants for the pairs of forces shown in Figures 1.5*a* to *f*.

Components of a Force

In addition to combining forces to obtain their resultant, it is often necessary to replace a single force by its *components*. The components of a force are the two or more forces that, acting together, have the same effect as the given force. In Figure 1.4*d*, if we are *given* the force of 112 lb, its vertical component is 50 lb and its horizontal component is 100 lb. That is, the 112-lb force has been *resolved* into its vertical and horizontal components. Any force may be considered as the resultant of its components.

Combined Resultants

The resultant of more than two nonparallel forces may be obtained by finding the resultants of pairs of forces and finally the resultant of the resultants.

Example 3. Let it be required to find the resultant of the concurrent forces P_1, P_2, P_3, and P_4 shown in Figure 1.6.

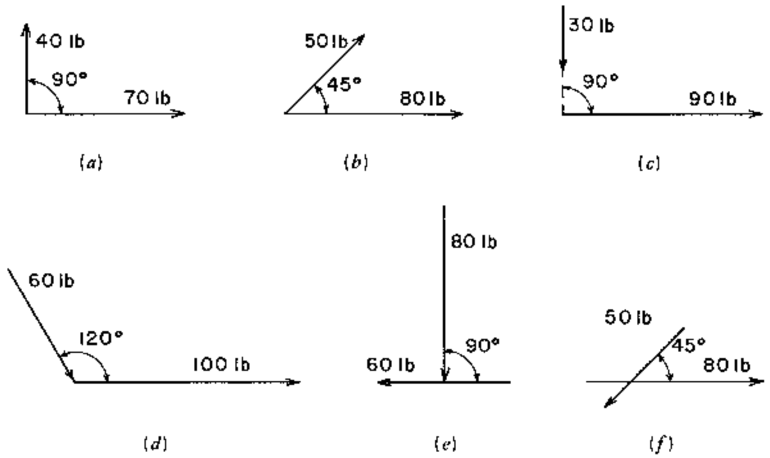

Figure 1.5 Reference for Problem 1.3, part 1.

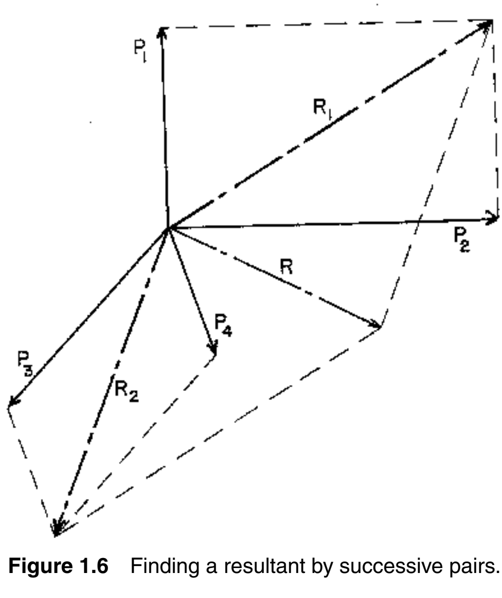

Figure 1.6 Finding a resultant by successive pairs.

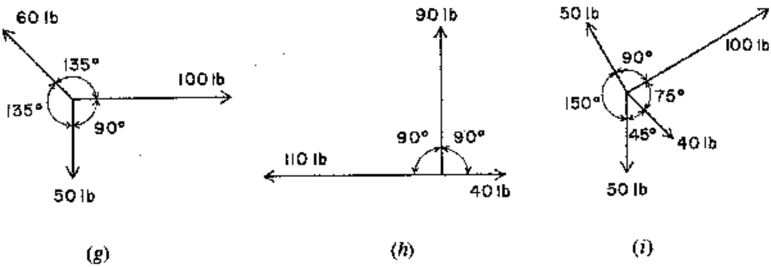

Figure 1.7 Reference for Problem 1.3, part 2.

Solution: By constructing a parallelogram of forces, the resultant of P_1 and P_2 is found to be R_1.

Simarily, the resultant of P_3 and P_4 is R_2. Finally, the resultant of R_1 and R_2 is R, the resultant of the four given forces.

Problems 1.3.G–I. Using graphical methods, find the resultant of the systems of concurrent forces shown in Figure 1.7.

Equilibrant

The force required to maintain a system of forces in equilibrium is called the *equilibrant* of the system. Suppose that we are required to investigate the system of two forces, P_1 and P_2, as shown in Figure 1.8 The parallelogram of forces is constructed, and the resultant is found to be R. The system is not in equilibrium. The force required to maintain equilibrium is force E, shown by the dotted line. E, the equilibrant, is the same as the resultant in magnitude and direction, but is opposite in sense. The three forces, P_1 and P_2 and E, constitute a system in equilibrium.

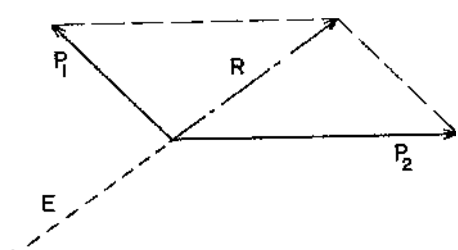

Figure 1.8 Finding an equilibrant.

If two forces are in equilibrium, they must be equal in magnitude, opposite in sense, and have the same direction and line of action. Either of the two forces may be said to be the equilibrant of the other. The resultant of a system of forces in equilibrium is zero.

1.4 GRAPHICAL ANALYSIS OF FORCES

Force Polygon

The resultant of a system of concurrent forces may be found by constructing a *force polygon.* To draw the force polygon, begin with a point and lay off, at a convenient scale, a line parallel to one of the forces, with its length equal to the force in magnitude and having the same sense. From the termination of this line, draw similarly another line corresponding to one of the remaining forces, and continue in the same manner until all the forces in the given system are accounted for. If the polygon does not close, the system of forces is not in equilibrium, and the line required to close the polygon *drawn from the starting point* is the resultant in magnitude and direction. If the forces in the given system are concurrent, the line of action of the resultant passes through the point they have in common.

If the force polygon for a system of concurrent forces closes, the system is in equilibrium and the resultant is zero.

Example 4. Let it be required to find the resultant of the four concurrent forces P_1, P_2, P_3, and P_4 shown in Figure 1.9*a*. This diagram is called

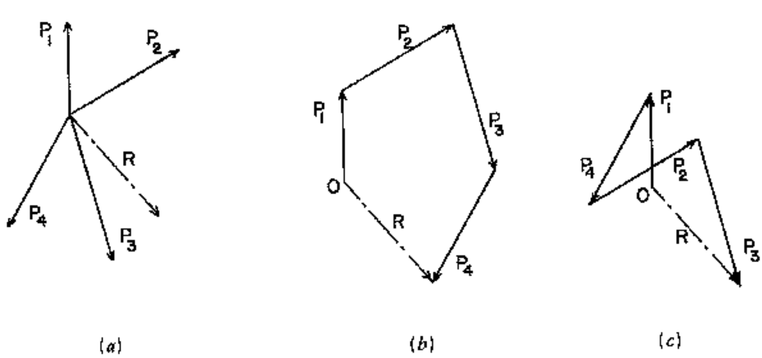

Figure 1.9 Finding a resultant by continuous vector addition of forces.

the *space diagram;* it shows the relative positions of the forces in a given system.

Solution: Beginning with some point such as O, shown in Figure 1.9*b*, draw the upward force P_1. At the upper extremity of the line representing P_1, draw P_2, continuing in a like manner with P_3 and P_4. The polygon does not close; therefore, the system is not in equilibrium. The resultant R, shown by the dot-and-dash line, is the resultant of the given system. Note that its directions is *from* the starting point O, downward to the right. The line of action of the resultant of the given system shown in Figure 1.9*a* has its line of action passing through the point they have in common, its magnitude and direction having been found in the force polygon.

In the drawing of the force polygon, the forces may be taken in any sequence. In Figure 1.9*c* a different sequence is taken, but the resultant R is found to have the same magnitude and direction as previously found in Figure 1.9*b*.

Bow's Notation

Thus far, forces have been identified by the symbols P_1, P_2, and so on. A system of identifying forces, known as Bow's notation, affords many advantages. In this system letters are placed in the space diagram on each side of a force, and a force is identified by two letters. The sequence in which the letters are read is important. Figure 1.10*a* shows the space diagram of five concurrent forces. Reading about the point in common *in a clockwise manner* the forces are *AB, BC, CD, DE,* and *EA*. When a force in the force polygon is represented by a line, a letter is placed at each end of the line. As an example, the vertical upward force in Figure 1.10*a* is read *AB* (note that this is read clockwise about the common point); in the force polygon (Fig. 1.10*b*) the letter *a* is placed at the bottom of the line representing the force *AB* and the letter *b* is at the top. Use capital letters to identify the forces in the space diagrams and lowercase letters in the force polygon. From point *b* in the force polygon, draw force *bc*, then *cd*, and continue with *de* and *ea*. Because the force polygon closes, the five concurrent forces are in equilibrium.

In reading forces, a clockwise manner is used in all the following discussions. It is important that this method of identifying forces be thoroughly understood. To make this clear, suppose that a force polygon is drawn for the five forces shown in Figure 1.10*a*, reading the forces in sequence in a counterclockwise manner. This will produce the force poly-

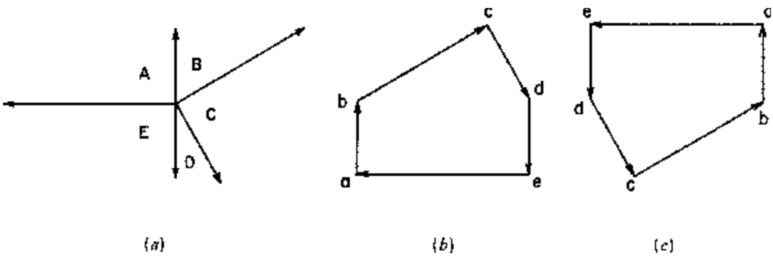

Figure 1.10 Construction of a force plygon.

gon shown in Figure 1.10*c*. Either method may be used, but for consistency, the method of reading clockwise is used here.

Use of the Force Polygon

Two ropes are attached to a ceiling and their lower ends are connected to a ring, making the arrangement shown in Figure 1.11*a*. A weight of 100 lb is suspended from the ring. Obviously, the force in the rope *AB* is 100 lb, but the magnitudes of the forces in ropes *BC* and *CA* are unknown.

The forces in the ropes *AB, BC,* and *CA* constitute a concurrent force system in equilibrium (Figure 1.11*b*). The magnitude of only one of the forces is known—it is 100 lb in rope *AB*. Because the three concurrent forces are in equilibrium, their force polygon must close, and this fact makes it possible to find their magnitudes. Now, at a convenient scale,

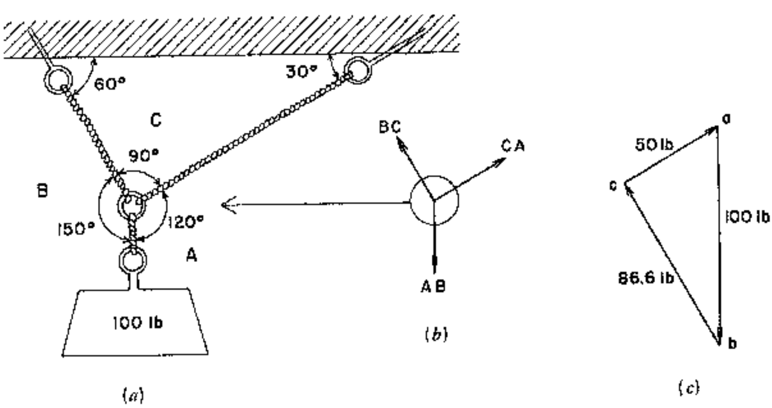

Figure 1.11 Solution of a concentric force system.

draw the line *ab* (Figure 1.11*c*) representing the downward force *AB*, 100 lb. The line *ab* is one side of the force polygon. From point *b* draw a line parallel to rope *BC*; point *c* will be at some location on this line. Next, draw a line through point *a* parallel to rope *CA*; point *c* will be at some position on this line. Because point *c* is also on the line through *b* parallel to *BC*, the intersection of the two lines determines point *c*. The force polygon for the three forces is now completed; it is *abc*, and the lengths of the sides of the polygon represent the magnitudes of the forces in ropes *BC* and *CA*, 86.6 lb and 50 lb, respectively.

Particular attention is called to the fact that the lengths of the ropes in Figure 1.11*a* are not an indication of magnitude of the forces within the ropes; the magnitudes are determined by the lengths of the corresponding sides of the force polygon (Figure 1.11*c*). Figure 1.11*a* merely determines the geometric layout for the structure.

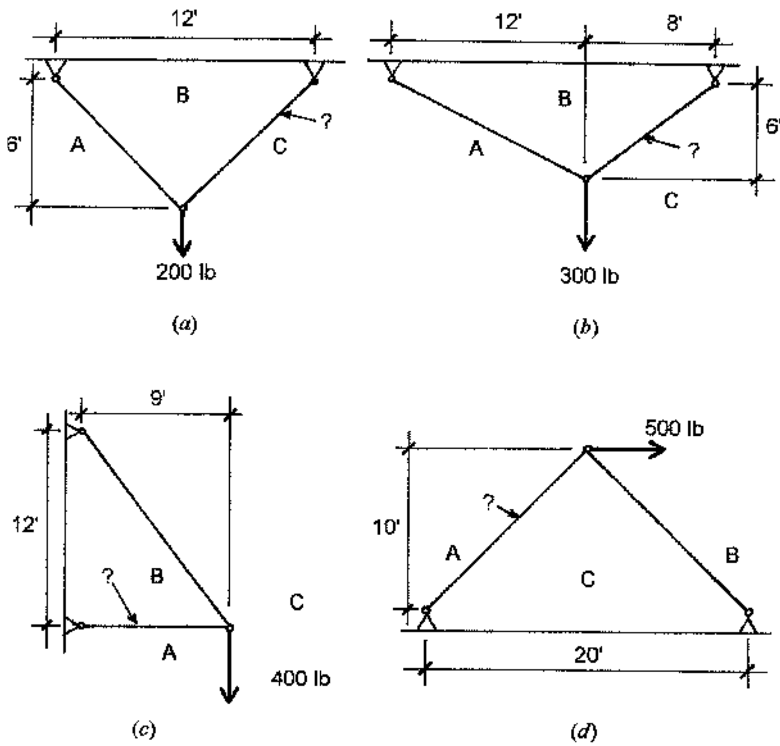

Figure 1.12 Reference for Problem 1.4.

Problems 1.4.A–D. Find the sense (tension or compression) and magnitude of the internal force in the member indicated in Figure 1.12 using graphical methods.

1.5 GRAPHICAL ANALYSIS OF PLANAR TRUSSES

Planar trusses, composed of linear elements assembled in triangulated frameworks, have been used for spanning structures in buildings for many centuries. Investigation for internal forces in trusses is typically performed by the basic methods illustrated in the preceding sections. In this section these procedures are demonstrated using both algebraic and graphical methods of solution.

When we use the so-called *method of joints*, finding the internal forces in the members of a planar truss consists of solving a series of concurrent force systems. Figure 1.13 shows a truss with the truss form, the loads, and the reactions displayed in a space diagram. Below the space diagram is a figure consisting of the free body diagrams of the individual joints of

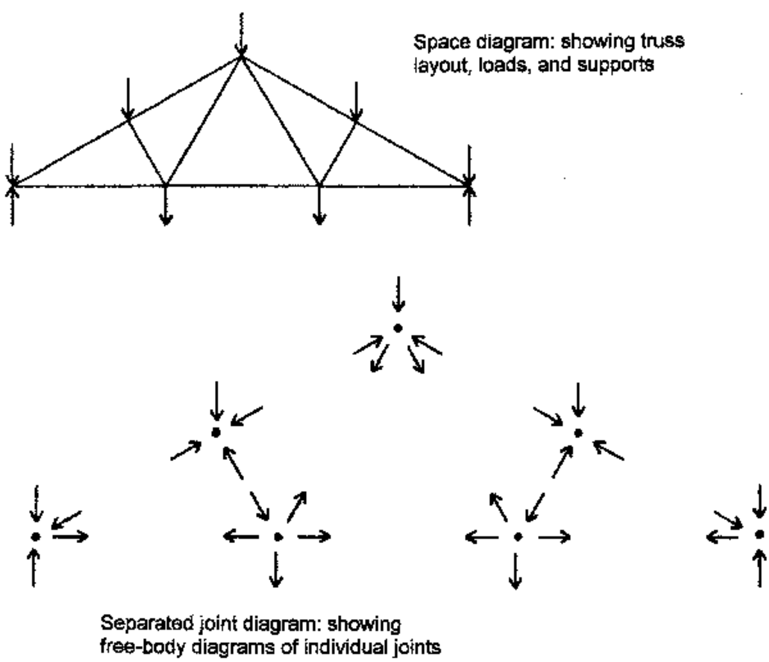

Figure 1.13 Examples of diagrams used to represent trusses and their actions.

the truss. These are arranged in the same manner as they are in the truss in order to show their interrelationships. However, each joint constitutes a complete concurrent planar force system that must have its independent equilibrium. Solving the problem consists of determining the equilibrium conditions for all of the joints. The procedures used for this solution will now be illustrated.

Figure 1.14 shows a single-span, planar truss subjected to vertical gravity loads. This example will be used to illustrate the procedures for determining the internal forces in the truss, that is, the tension and com-

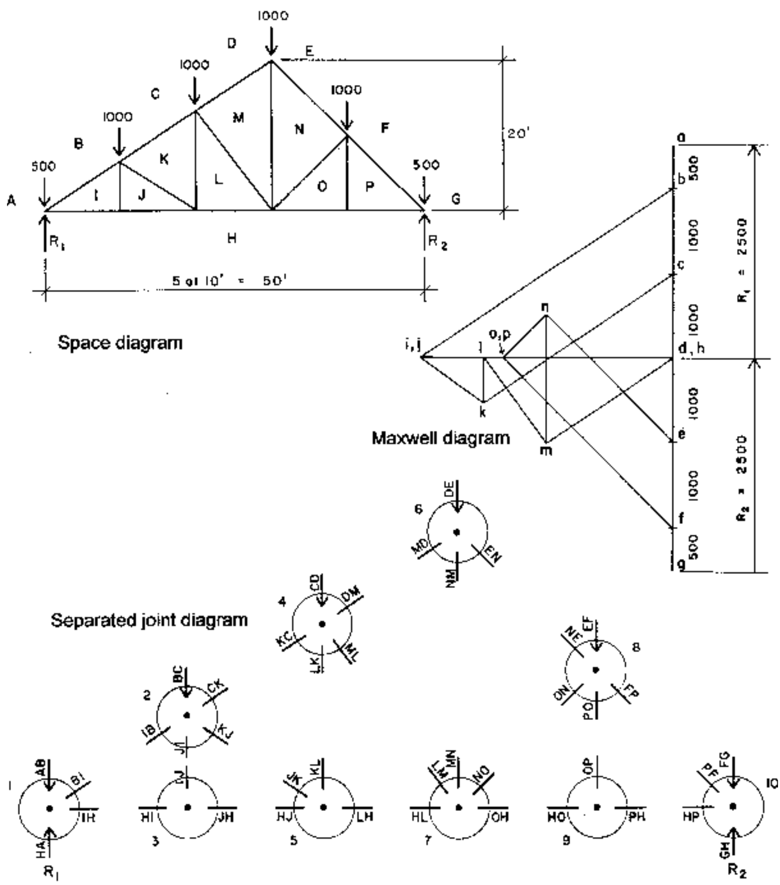

Figure 1.14 Graphic diagrams for the sample problem.

pression forces in the individual members of the truss. The space diagram in the figure shows the truss form and dimensions, the support conditions, and the loads. The letters on the space diagram identify individual forces at the truss joints, as discussed in Section 1.4. The sequence of placement of the letters is arbitrary, the only necessary consideration being to place a letter in each space between the loads and the individual truss members so that each force at a joint can be identified by a two-letter symbol.

The separated joint diagram in the figure provides a useful means for visualization of the complete force system at each joint as well as the interrelation of the joints through the truss members. The individual forces at each joint are designated by two-letter symbols that are obtained by simply reading around the joint in the space diagram in a clockwise direction. Note that the two-letter symbols are reversed at the opposite ends of each of the truss members. Thus, the top chord member at the left end of the truss is designated as *BI* when shown in the joint at the left support (joint 1) and is designated as *IB* when shown in the first interior upper chord joint (joint 2). The purpose of this procedure will be demonstrated in the following explanation of the graphical analysis.

The third diagram in Figure 1.14 is a composite force polygon for the external and internal forces in the truss. It is called a Maxwell diagram after one of its early promoters, James Maxwell, a British engineer. The construction of this diagram constitutes a complete solution for the magnitudes and senses of the internal forces in the truss. The procedure for this construction is as follows.

1. *Construct the force polygon for the external forces*. Before this can be done, the values for the reactions must be found. There are graphic techniques for finding the reactions, but it is usually much simpler and faster to find them with an algebraic solution. In this example, although the truss is not symmetrical, the loading is, and it may simply be observed that the reactions are each equal to one half of the total load on the truss, or $5000 \div 2 = 2500$ lb. Because the external forces in this case are all in a single direction, the force polygon for the external forces is actually a straight line. Using the two-letter symbols for the forces and starting with the letter *A* at the left end, we read the force sequence by moving in a clockwise direction around the outside of the truss. The loads are thus read as *AB, BC, CD, DE, EF,* and *FG,* and the two reactions are read as *GH* and *HA*. Beginning at *A* on the Maxwell diagram, the force vector

sequence for the external forces is read from *A* to *B*, *B* to *C*, *C* to *D*, and so on, ending back at *A*, which shows that the force polygon closes and the external forces are in the necessary state of static equilibrium. Note that we have pulled the vectors for the reactions off to the side in the diagram to indicate them more clearly. Note also that we have used lowercase letters for the vector ends in the Maxwell diagram, whereas uppercase letters are used on the space diagram. The alphabetic correlation is thus retained (*A* to *a*), preventing any possible confusion between the two diagrams. The letters on the space diagram designate open spaces, and the letters on the Maxwell diagram designate points of intersection of lines.

2. *Construct the force polygons for the individual joints.* The graphic procedure for this consists of locating the points on the Maxwell diagram that correspond to the remaining letters, *I* through *P*, on the space diagram. When all the lettered points on the diagram are located, the complete force polygon for each joint may be read on the diagram. In order to locate these points, we use two relationships. The first is that the truss members can resist only forces that are parallel to the members' positioned directions. Thus, we know the directions of all the internal forces. The second relationship is a simple one from plane geometry: A point may be located at the intersection of two lines. Consider the forces at joint 1, as shown in the separated joint diagram in Figure 1.14. Note that there are four forces and that two of them are known (the load and the reaction) and two are unknown (the internal forces in the truss members). The force polygon for this joint, as shown on the Maxwell diagram, is read as *ABIHA*. *AB* represents the load; *BI* the force in the upper chord member; *IH* the force in the lower chord member; and *HA* the reaction. Thus, the location of point *i* on the Maxwell diagram is determined by noting that *i* must be in a horizontal direction from *h* (corresponding to the horizontal position of the lower chord) and in a direction from *b* that is parallel to the position of the upper chord.

The remaining points on the Maxwell diagram are found by the same process, using two known points on the diagram to project lines of known direction whose intersection will determine the location of an unknown point. Once all the points are located, the diagram is complete and can be used to find the magnitude and sense of each internal force. The process for construction of the Maxwell diagram typically consists of moving from joint to joint along the truss. Once one of the letters for an internal space is determined on the Maxwell diagram, it may be used as a known point for finding the letter for an adjacent space on the space diagram. The only limitation of the process is that it is not possible to find more than one unknown point on the Maxwell diagram for any single joint. Consider joint 7 on the separated joint diagram in Figure 1.14. To solve this joint first, knowing only the locations of letters *a* through *h* on the Maxwell diagram, it is necessary to locate four unknown points: *l, m, n,* and *o.* This is three more unknowns than can be determined in a single step, so three of the unknowns must be found by using other joints.

Solving for a single unknown point on the Maxwell diagram corresponds to finding two unknown forces at a joint, because each letter on the space diagram is used twice in the force identification for the internal forces. Thus, for joint 1 in the previous example, the letter *I* is part of the identity of forces *BI* and *IH*, as shown on the separated joint diagram. The graphic determination of single points on the Maxwell diagram, therefore, is analogous to finding two unknown quantities in an algebraic solution. As discussed previously, two unknowns are the maximum that can be solved for the equilibrium of a coplanar, concurrent force system, which is the condition of the individual joints in the truss.

When the Maxwell diagram is completed, the internal forces can be read from the diagram as follows:

1. The magnitude is determined by measuring the length of the line in the diagram, using the scale that was used to plot the vectors for the external forces.
2. The sense of individual forces is determined by reading the forces in clockwise sequence around a single joint in the space diagram and tracing the same letter sequences on the Maxwell diagram.

Figure 1.15*a* shows the force system at joint 1 and the force polygon for these forces as taken from the Maxwell diagram. The forces known initially are shown as solid lines on the force polygon, and the unknown forces are shown as dashed lines. Starting with letter *A* on the force system, we read the forces in a clockwise sequence as *AB, BI, IH,* and *HA*. Note that on the Maxwell diagram, moving from *a* to *b* is moving in the order of the sense of the force—that is, from tail to end of the force vector that represents the external load on the joint. With this sequence on the Maxwell diagram, this force sense flow will be a continuous one. Thus, reading from *b* to *i* on the Maxwell diagram is reading from tail to

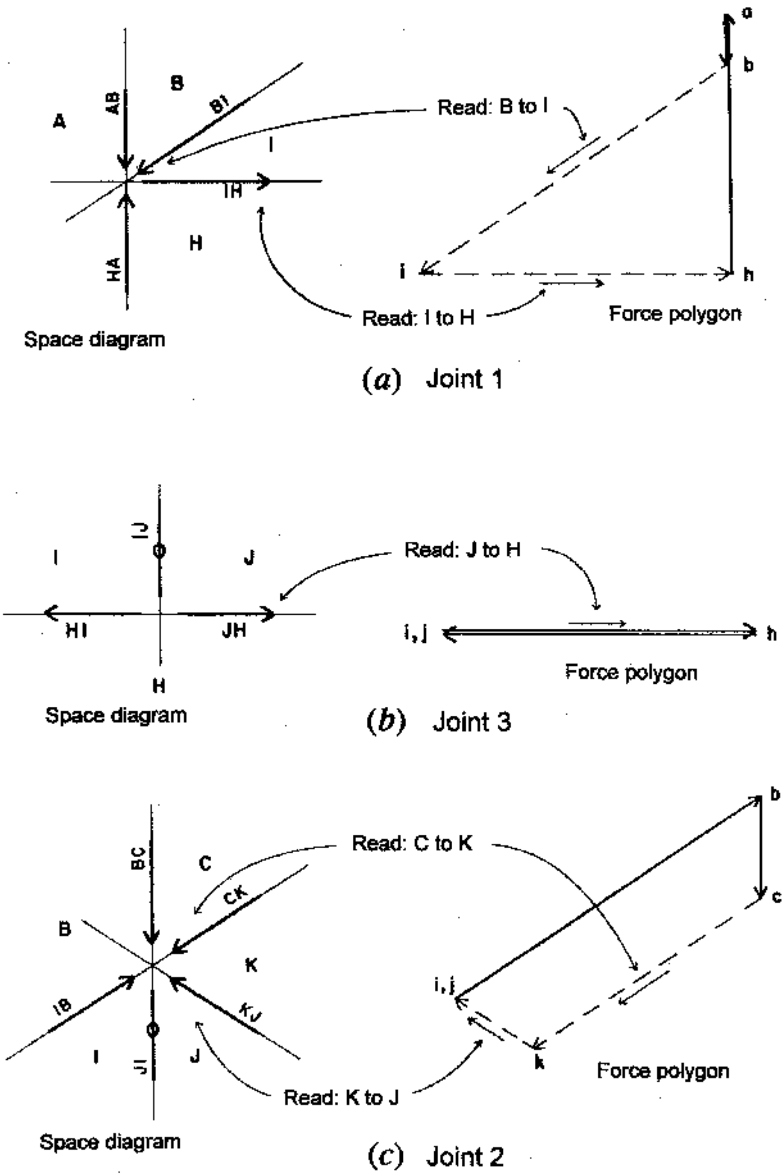

Figure 1.15 Graphic solutions for joints 1, 2, and 3.

head of the force vector, which indicates that force *BI* has its head at the left end. Transferring this sense indication from the Maxwell diagram to the joint diagram indicates that force *BI* is in compression; that is, it is pushing, rather than pulling, on the joint. Reading from *i* to *h* on the Maxwell diagram shows that the arrowhead for this vector is on the right, which translates to a tension effect on the joint diagram.

Having solved for the forces at joint 1 as described, the fact that the forces in truss members *BI* and *IH* are known can be used to consider the adjacent joints, 2 and 3. However, it should be noted that the sense reverses at the opposite ends of the members in the joint diagrams. Referring to the separated joint diagram in Figure 1.14, note that if the upper chord member shown as force *BI* in joint 1 is in compression, its arrowhead is at the lower left end in the diagram for joint 1, as shown in Figure 1.15*a*. However, when the same force is shown as *IB* at joint 2, its pushing effect on the joint will be indicated by having the arrowhead at the upper right end in the diagram for joint 2. Similarly, the tension effect of the lower chord is shown in joint 1 by placing the arrowhead on the right end of the force *IH*, but the same tension force will be indicated in joint 3 by placing the arrowhead on the left end of the vector for force *HI*.

If the solution sequence of solving joint 1 and then joint 2 is chosen, it is now possible to transfer the known force in the upper chord to joint 2. Thus, the solution for the five forces at joint 2 is reduced to finding three unknowns because the load *BC* and the chord force *IB* are now known. However, it is still not possible to solve joint 2 because there are two unknown points on the Maxwell diagram (*k* and *j*) corresponding to the three unknown forces. An option, therefore, is to proceed from joint 1 to joint 3, at which there are presently only two unknown forces. On the Maxwell diagram the single unknown point *j* can be found by projecting vector *IJ* vertically from *i* and projecting vector *JH* horizontally from point *h*. Because point *i* is also located horizontally from point *h,* this shows that the vector *IJ* has zero magnitude, because both *i* and *j* must be on a horizontal line from *h* in the Maxwell diagram. This indicates that there is actually no stress in this truss member for this loading condition and that points *i* and *j* are coincident on the Maxwell diagram. The joint force diagram and the force polygon for joint 3 are as shown in Figure 1.15*b.* In the joint force diagram, place a zero, rather than an arrowhead, on the vector line for *IJ* to indicate the zero stress condition. In the force polygon in Figure 1.15*b*, the two force vectors are slightly separated for clarity, although they are actually coincident on the same line.

Having solved for the forces at joint 3, proceed to joint 2, because there remain only two unknown forces at this joint. The forces at the joint and the force polygon for joint 2 are shown in Figure 1.15*c*. As for joint 1, read the force polygon in a sequence determined by reading clockwise around the joint: *BCKJIB*. Following the continuous direction of the force arrows on the force polygon in this sequence, we can establish the sense for the two forces *CK* and *KJ*.

It is possible to proceed from one end and to work continuously across the truss from joint to joint to construct the Maxwell diagram in this example. The sequence in terms of locating points on the Maxwell diagram would be *i-j-k-l-m-n-o-p,* which would be accomplished by solving the joints in the following sequence: 1,3,2,5,4,6,7,9,8. However, it is advisable to minimize the error in graphic construction by working from both ends of the truss. Thus, a better procedure would be to find points *i-j-k-l-m*,

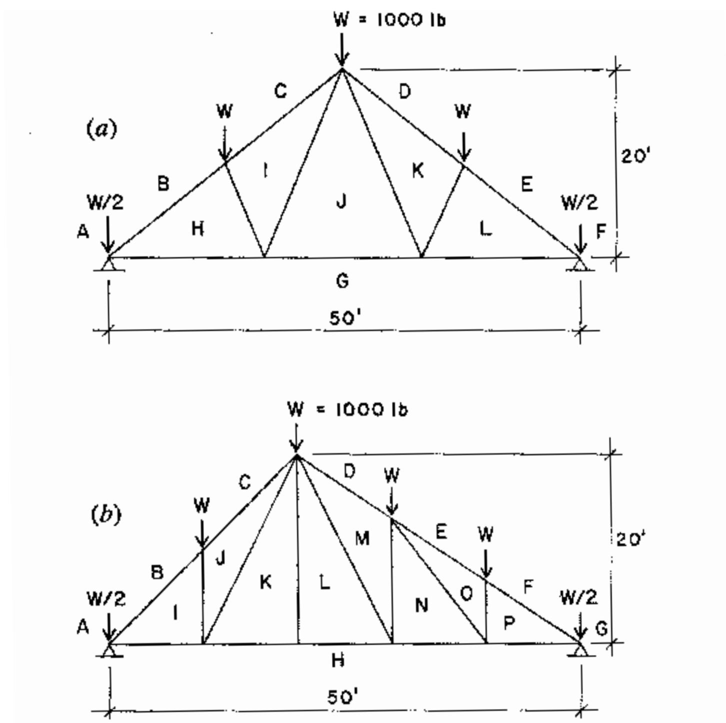

Figure 1.16 Reference for Problem 1.5.

working from the left end of the truss, and then to find points *p-o-n-m*, working from the right end. This would result in finding two locations for *m,* whose separation constitutes the error in drafting accuracy.

Problems 1.5.A, B. Using a Maxwell diagram, find the internal forces in the truss in Figure 1.16.

1.6 ALGEBRAIC ANALYSIS OF PLANAR TRUSSES

Graphical solution for the internal forces in a truss using the Maxwell diagram corresponds essentially to an algebraic solution by the *method of joints.* This method consists of solving the concentric force systems at the individual joints using simple force equilibrium equations. The process will be illustrated using the previous example.

As with the graphic solution, first determine the external forces, consisting of the loads and the reactions. Then proceed to consider the equilibrium of the individual joints, following a sequence as in the graphic solution. The limitation of this sequence, corresponding to the limit of finding only one unknown point in the Maxwell diagram, is that only two unknown forces at any single joint can be found in a single step. (Two conditions of equilibrium produce two equations.) Refer to Figure 1.17; the solution for joint 1 is as follows.

The force system for the joint is drawn with the sense and magnitude of the known forces shown, but with the unknown internal forces represented by lines without arrowheads because their senses and magnitudes initially are unknown. For forces that are not vertical or horizontal, replace the forces with their horizontal and vertical components. Then consider the two conditions necessary for the equilibrium of the system: The sum of the vertical forces is zero and the sum of the horizontal forces is zero.

If the algebraic solution is performed carefully, the sense of the forces will be determined automatically. However, it is recommended that whenever possible the sense be predetermined by simple observations of the joint conditions, as will be illustrated in the solutions.

The problem to be solved at joint 1 is as shown in Figure 1.17*a*. In Figure 1.17*b* the system is shown with all forces expressed as vertical and horizontal components. Note that although this now increases the number of unknowns to three (IH, BI_v, and BI_h), there is a numeric relationship between the two components of *BI*. When this condition is added to the two algebraic conditions for equilibrium, the number of usable relationships totals three, so that the necessary conditions to solve for the three unknowns are present.

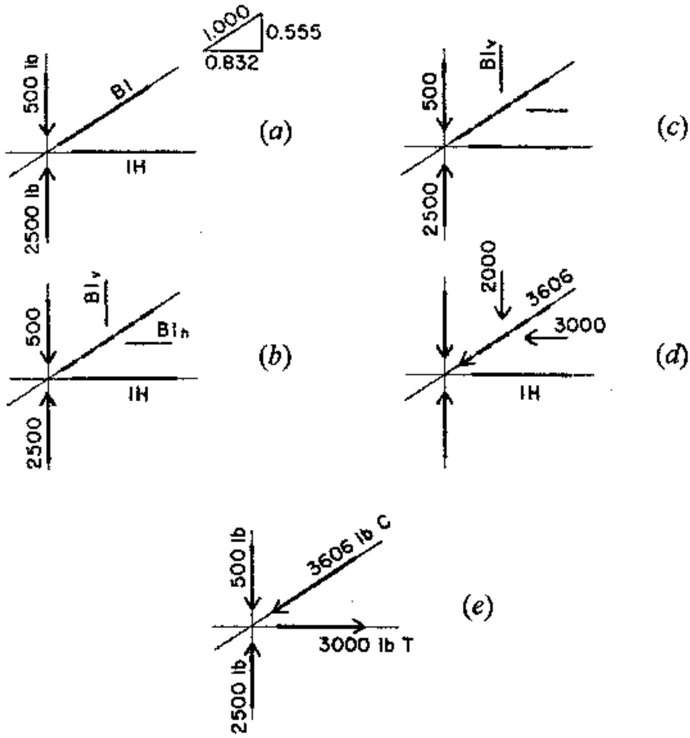

Figure 1.17 Algebraic solution for joint 1.

The condition for vertical equilibrium is shown at (*c*) in Figure 1.17. Because the horizontal forces do not affect the vertical equilibrium, the balance is between the load, the reaction, and the vertical component of the force in the upper chord. Simple observation of the forces and the known magnitudes makes it obvious that force BI_v must act downward, indicating that *BI* is a compression force. Thus, the sense of *BI* is established by simple visual inspection of the joint, and the algebraic equation for vertical equilibrium (with upward force considered positive) is

$$\Sigma F_v = 0 = +2500 - 500 - BI_v$$

From this equation, BI_v is determined to have a magnitude of 2000 lb. Using the known relationships between *BI*, BI_v, and BI_h, the values of these three quantities can be determined if any one of them is known. Thus:

$$\frac{BI}{1.000} = \frac{BI_v}{0.555} = \frac{BI_h}{0.832}$$

from which

$$BI_h = \left(\frac{0.832}{0.555}\right)(2000) = 3000 \text{ lb}$$

and

$$BI = \left(\frac{1.000}{0.555}\right)(2000) = 3606 \text{ lb}$$

The results of the analysis to this point are shown at (*d*) in Figure 1.17, from which it may be observed that the conditions for equilibrium of the horizontal forces can be expressed. Stated algebraically (with force sense toward the right considered positive), the condition is

$$\Sigma F_h = 0 = IH - 3000$$

from which it is established that the force in *IH* is 3000 lb.

The final solution for the joint is then as shown at (*e*) in the figure. On this diagram the internal forces are identified as to sense by using *C* to indicate compression and *T* to indicate tension.

As with the graphic solution, proceed to consider the forces at joint 3. The initial condition at this joint is as shown at (*a*) in Figure 1.18, with the single known force in member *HI* and the two unknown forces in *IJ* and *JH.* Because the forces at this joint are all vertical and horizontal, there is no need to use components. Consideration of vertical equilibrium

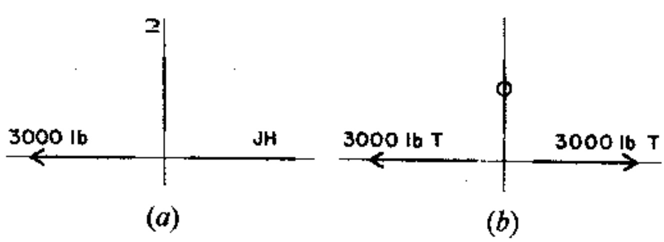

Figure 1.18 Algebraic solution for joint 3.

makes it obvious that it is not possible to have a force in member *IJ*. Stated algebraically, the condition for vertical equilibrium is

$$\Sigma F_v = 0 = IJ \text{ (because } IJ \text{ is the only force)}$$

It is equally obvious that the force in *JH* must be equal and opposite to that in *HI* because they are the only two horizontal forces. That is, stated algebraically

$$\Sigma F_v = 0 = JH - 3000$$

The final answer for the forces at joint 3 is as shown at (*b*) in Figure 1.18. Note the convention for indicating a truss member with no internal force.

Now proceed to consider joint 2; the initial condition is as shown at (*a*) in Figure 1.19. Of the five forces at the joint only two remain unknown. Following the procedure for joint 1, first resolve the forces into their vertical and horizontal components, as shown at (*b*) in Figure 1.19.

Because the sense of forces *CK* and *KJ* is unknown, use the procedure of considering them to be positive until proven otherwise. That is, if they are entered into the algebraic equations with an assumed sense, and the solution produces a negative answer, then the assumption was wrong. However, be careful to be consistent with the sense of the force vectors, as the following solution will illustrate.

Arbitrarily assume that force *CK* is in compression and force *KJ* is in tension. If this is so, the forces and their components will be as shown at (*c*) in Figure 1.19. Then consider the conditions for vertical equilibrium; the forces involved will be those shown at (*d*) in Figure 1.19, and the equation for vertical equilibrium will be

$$\Sigma F_v = 0 = -1000 + 2000 - CK_v - KJ_v$$

or

$$0 = +1000 - 0.555CK - 0.555KJ \qquad (1.6.1)$$

Now consider the conditions for horizontal equilibrium; the forces will be as shown at (*e*) in Figure 1.19, and the equation will be

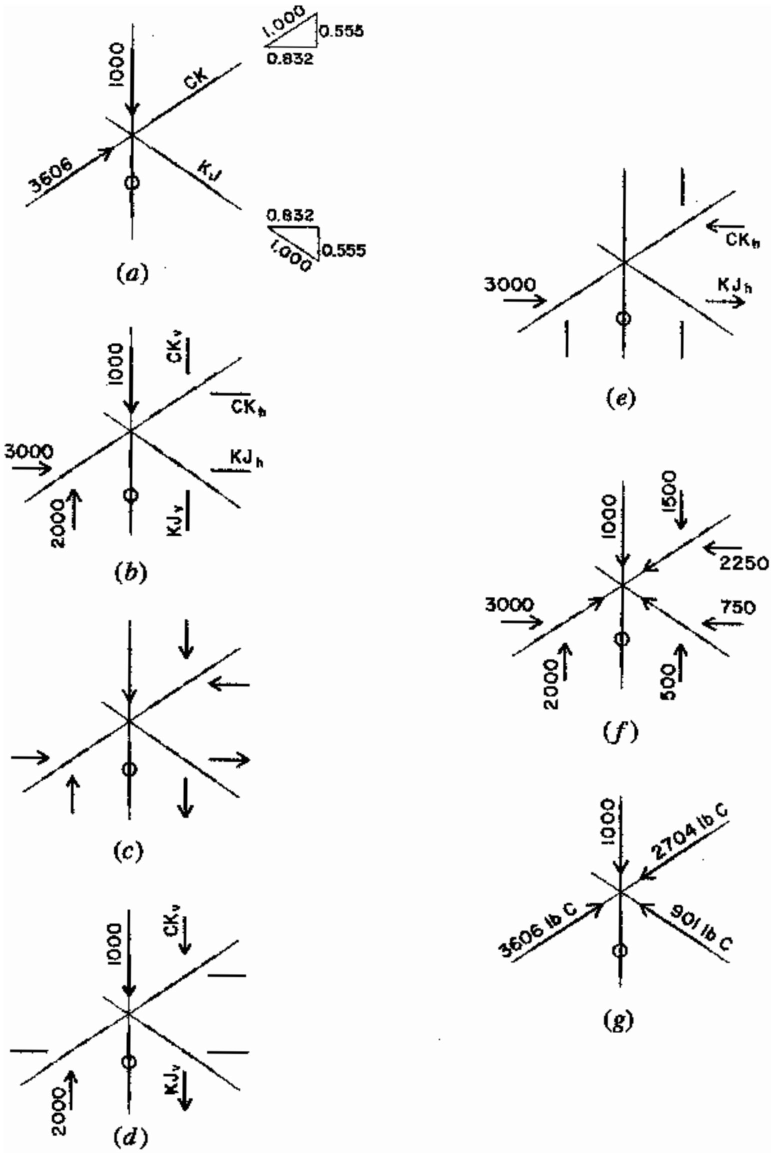

Figure 1.19 Algebraic solution for joint 2.

$$\Sigma F_h = 0 = +\,3000 - CK_h + KJ_h$$

or

$$0 = +\,3000 - 0.832CK + 0.832KJ \qquad (1.6.2)$$

Note the consistency of the algebraic signs and the sense of the force vectors, with positive forces considered as upward and toward the right. Now solve these two equations simultaneously for the two unknown forces as follows

1. Multiply equation (1.6.1) by 0.832/0.555

$$0 = \left(\frac{0.832}{0.555}\right)(+\,1000) + \left(\frac{0.832}{0.555}\right)(-\,0.555CK) + \left(\frac{0.832}{0.555}\right)(-\,0.555KJ)$$

or

$$0 = +\,1500 - 0.832CK - 0.832KJ$$

2. Add this equation to equation (1.6.2) and solve for *CK*.

$$0 = +\,4500 - 1.664CK, \qquad CK = \frac{4500}{1.664} = 2704 \text{ lb}$$

Note that the assumed sense of compression in *CK* is correct because the algebraic solution produces a positive answer. Substituting this value for *CK* in equation (1.6.1),

$$0 = +\,1000 - 0.555(2704) - 0.555(KJ)$$

and

$$KJ = \frac{-500}{0.555} = -901 \text{ lb}$$

Because the algebraic solution produces a negative quantity for *KJ*, the assumed sense for *KJ* is wrong and the member is actually in compression.

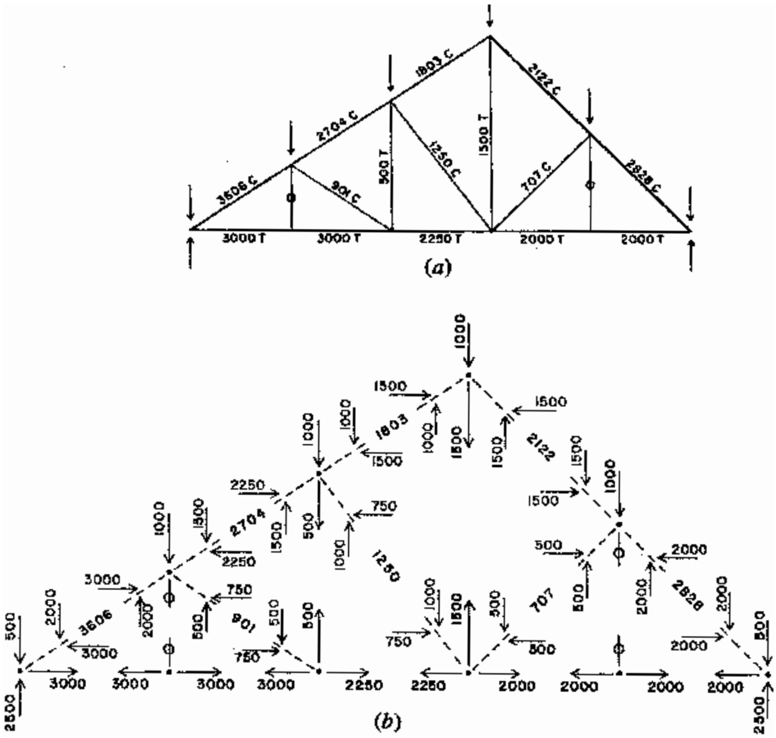

Figure 1.20 Presentation of the internal forces in the truss.

The final answers for the forces at joint 2 are as shown at (g) in Figure 1.19. In order to verify that equilibrium exists, however, the forces are shown in the form of their vertical and horizontal components at (*f*) in the illustration.

When all of the internal forces have been determined for the truss, the results may be recorded or displayed in a number of ways. The most direct way is to display them on a scaled diagram of the truss, as shown in Figure 1.20*a*. The force magnitudes are recorded next to each member with the sense shown as *T* for tension or *C* for compression. Zero stress members are indicated by the conventional symbol consisting of a zero placed directly on the member.

When you are solving by the algebraic method of joints, the results may be recorded on a separated joint diagram as shown in Figure 1.20*b*.

If the values for the vertical and horizontal components of force in sloping members are shown, it is a simple matter to verify the equilibrium of the individual joints.

Problem 1.6.A, B. Using the algebraic method of joints, find the internal forces in the truss in Figure 1.16.