Part I

FUNDAMENTALS

In Part I, we treat the basic ideas and calculation procedures that must be understood in order to appreciate how solar processes work and how their performance can be predicted. The first five chapters are basic to the material in Chapter 6. In Chapter 6 we develop equations for a collector which give the useful output in terms of the available solar radiation and the losses. An energy balance is developed which says, in essence, that the useful gain is the (positive) difference between the absorbed solar energy and the thermal losses.

The first chapter is concerned with the nature of the radiation emitted by the sun and incident on the earth's atmosphere. This includes geometric considerations, that is, the direction from which beam solar radiation is received and its angle of incidence on various surfaces and the quantity of radiation received over various time spans. The next chapter covers the effects of the atmosphere on the solar radiation, the radiation data that are available, and how those data can be processed to get the information that we ultimately want—the radiation incident on surfaces of various orientations.

Chapter 3 notes a set of heat transfer problems that arise in solar energy processes and is part of the basis for analysis of collectors, storage units, and other components.

The next two chapters treat interaction of radiation and opaque and transparent materials, that is, emission, absorption, reflection, and transmission of solar and long-wave radiation. These first five chapters lead to Chapter 6, a detailed discussion and analysis of the performance of flat-plate collectors. Chapter 7 is concerned with concentrating collectors and Chapter 8 with energy storage in various media. Chapter 9 is a brief discussion of the loads imposed on solar processes and the kinds of information that must be known in order to analyze the process.

Chapter 10 is the point at which the discussions of individual components are brought together to show how solar process *systems* function and how their long-term performance can be determined by simulations. The object is to be able to quantitatively predict system performance; this is the point at which we proceed from components to systems and see how transient system behavior can be calculated.

The last chapter in Part I is on solar process economics. It concludes with a method for combining the large number of economic parameters into two which can be used to optimize thermal design and assess the effects of uncertainties in an economic analysis.

1

Solar Radiation

The sun's structure and characteristics determine the nature of the energy it radiates into space. The first major topic in this chapter concerns the characteristics of this energy outside the earth's atmosphere, its intensity, and its spectral distribution. We will be concerned primarily with radiation in a wavelength range of 0.25 to 3.0 μm, the portion of the electromagnetic radiation that includes most of the energy radiated by the sun.

The second major topic in this chapter is solar geometry, that is, the position of the sun in the sky, the direction in which beam radiation is incident on surfaces of various orientations, and shading. The third topic is extraterrestrial radiation on a horizontal surface, which represents the theoretical upper limit of solar radiation available at the earth's surface.

An understanding of the nature of extraterrestrial radiation, the effects of orientation of a receiving surface, and the theoretically possible radiation at the earth's surface is important in understanding and using solar radiation data, the subject of Chapter 2.

1.1 THE SUN

The sun is a sphere of intensely hot gaseous matter with a diameter of 1.39×10^9 m and is, on the average, 1.5×10^{11} m from the earth. As seen from the earth, the sun rotates on its axis about once every 4 weeks. However, it does not rotate as a solid body; the equator takes about 27 days and the polar regions take about 30 days for each rotation.

The sun has an effective blackbody temperature of 5777 K.[1] The temperature in the central interior regions is variously estimated at 8×10^6 to 40×10^6 K and the density is estimated to be about 100 times that of water. The sun is, in effect, a continuous fusion reactor with its constituent gases as the "containing vessel" retained by gravitational forces. Several fusion reactions have been suggested to supply the energy radiated by the sun. The one considered the most important is a process in which hydrogen (i.e., four protons) combines to form helium (i.e., one helium nucleus); the mass of the helium nucleus is less than that of the four protons, mass having been lost in the reaction and converted to energy.

[1] The effective blackbody temperature of 5777 K is the temperature of a blackbody radiating the same amount of energy as does the sun. Other effective temperatures can be defined, e.g., that corresponding to the blackbody temperature giving the same wavelength of maximum radiation as solar radiation (about 6300 K).

The energy produced in the interior of the solar sphere at temperatures of many millions of degrees must be transferred out to the surface and then be radiated into space. A succession of radiative and convective processes occur with successive emission, absorption, and reradiation; the radiation in the sun's core is in the x-ray and gamma-ray parts of the spectrum, with the wavelengths of the radiation increasing as the temperature drops at larger radial distances.

A schematic structure of the sun is shown in Figure 1.1.1. It is estimated that 90% of the energy is generated in the region of 0 to $0.23R$ (where R is the radius of the sun), which contains 40% of the mass of the sun. At a distance $0.7R$ from the center, the temperature has dropped to about 130,000 K and the density has dropped to 70 kg/m^3; here convection processes begin to become important, and the zone from 0.7 to $1.0R$ is known as the **convective zone.** Within this zone the temperature drops to about 5000 K and the density to about 10^{-5} kg/m^3.

The sun's surface appears to be composed of granules (irregular convection cells), with dimensions from 1000 to 3000 km and with cell lifetime of a few minutes. Other features of the solar surface are small dark areas called pores, which are of the same order of magnitude as the convective cells, and larger dark areas called sunspots, which vary in size. The outer layer of the convective zone is called the **photosphere.** The edge of the photosphere is sharply defined, even though it is of low density (about 10^{-4} that

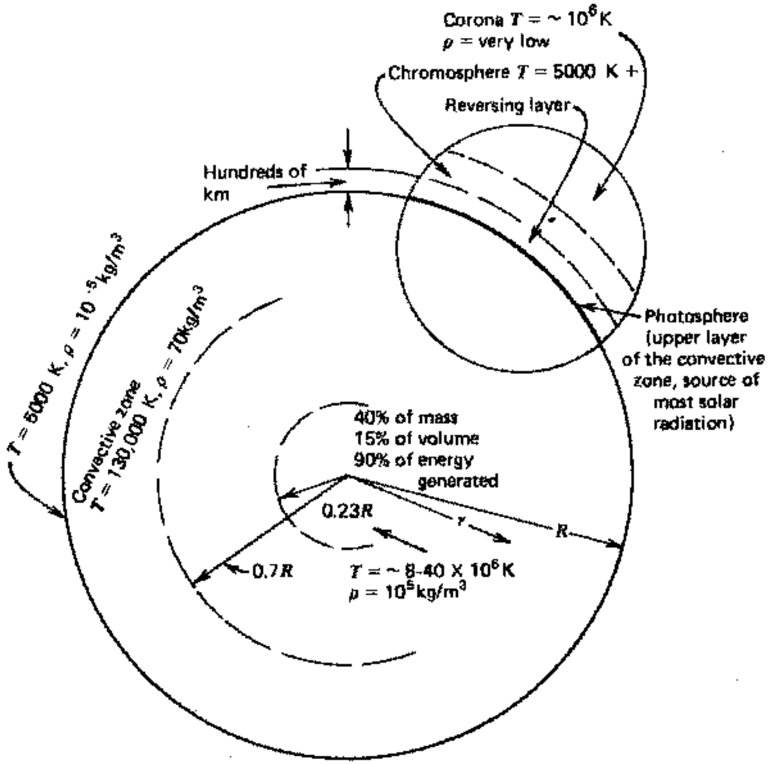

Figure 1.1.1 The structure of the sun.

of air at sea level). It is essentially opaque, as the gases of which it is composed are strongly ionized and able to absorb and emit a continuous spectrum of radiation. The photosphere is the source of most solar radiation.

Outside the photosphere is a more or less transparent solar atmosphere, observable during total solar eclipse or by instruments that occult the solar disk. Above the photosphere is a layer of cooler gases several hundred kilometers deep called the **reversing layer.** Outside of that is a layer referred to as the **chromosphere,** with a depth of about 10,000 km. This is a gaseous layer with temperatures somewhat higher than that of the photosphere but with lower density. Still further out is the **corona,** a region of very low density and of very high (10^6 K) temperature. For further information on the sun's structure see Thomas (1958) or Robinson (1966).

This simplified picture of the sun, its physical structure, and its temperature and density gradients will serve as a basis for appreciating that the sun does not, in fact, function as a blackbody radiator at a fixed temperature. Rather, the emitted solar radiation is the composite result of the several layers that emit and absorb radiation of various wavelengths. The resulting extraterrestrial solar radiation and its spectral distribution have now been measured by various methods in several experiments; the results are noted in the following two sections.

1.2 THE SOLAR CONSTANT

Figure 1.2.1 shows schematically the geometry of the sun-earth relationships. The eccentricity of the earth's orbit is such that the distance between the sun and the earth varies by 1.7%. At a distance of one astronomical unit, 1.495×10^{11} m, the mean earth-sun distance, the sun subtends an angle of 32′. The radiation emitted by the sun and its spatial relationship to the earth result in a nearly fixed intensity of solar radiation outside of the earth's atmosphere. The **solar constant** G_{sc} is the energy from the sun per unit time received on a unit area of surface perpendicular to the direction of propagation of the radiation at mean earth-sun distance outside the atmosphere.

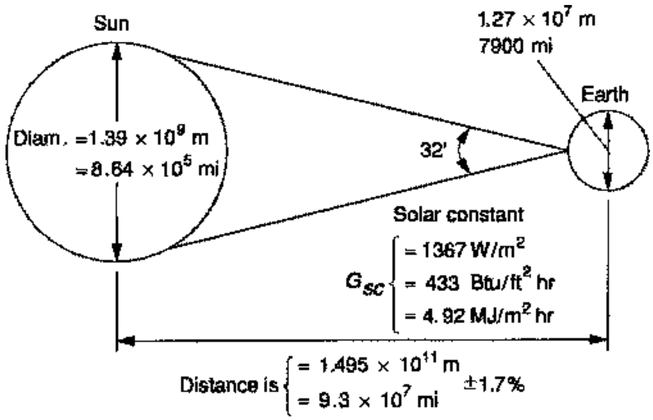

Figure 1.2.1 Sun-earth relationships.

Before rockets and spacecraft, estimates of the solar constant had to be made from ground-based measurements of solar radiation after it had been transmitted through the atmosphere and thus in part absorbed and scattered by components of the atmosphere. Extrapolations from the terrestrial measurements made from high mountains were based on estimates of atmospheric transmission in various portions of the solar spectrum. Pioneering studies were done by C. G. Abbot and his colleagues at the Smithsonian Institution. These studies and later measurements from rockets were summarized by Johnson (1954); Abbot's value of the solar constant of 1322 W/m^2 was revised upward by Johnson to 1395 W/m^2.

The availability of very high altitude aircraft, balloons, and spacecraft has permitted direct measurements of solar radiation outside most or all of the earth's atmosphere. These measurements were made with a variety of instruments in nine separate experimental programs. They resulted in a value of the solar constant G_{sc} of 1353 W/m^2 with an estimated error of $\pm 1.5\%$. For discussions of these experiments, see Thekaekara (1976) or Thekaekara and Drummond (1971). This standard value was accepted by NASA (1971) and by the American Society of Testing and Materials.

The data on which the 1353-W/m^2 value was based have been reexamined by Frohlich (1977) and reduced to a new pyrheliometric scale[2] based on comparisons of the instruments with absolute radiometers. Data from *Nimbus* and *Mariner* satellites have also been included in the analysis, and as of 1978, Frohlich recommends a new value of the solar constant G_{sc} of 1373 W/m^2, with a probable error of 1 to 2%. This was 1.5% higher than the earlier value and 1.2% higher than the best available determination of the solar constant by integration of spectral measurements. Additional spacecraft measurements have been made with Hickey et al. (1982) reporting 1373 W/m^2 and Willson et al. (1981) reporting 1368 W/m^2. Measurements from three rocket flights reported by Duncan et al. (1982) were 1367, 1372, and 1374 W/m^2. The World Radiation Center (WRC) has adopted a value of 1367 W/m^2, with an uncertainty of the order of 1%. As will be seen in Chapter 2, uncertainties in most terrestrial solar radiation measurements are an order of magnitude larger than those in G_{sc}. A value of G_{sc} of 1367 W/m^2 (1.960 cal/cm^2 min, 433 Btu/ft^2 h, or 4.921 MJ/m^2 h) is used in this book. [See Iqbal (1983) for more detailed information on the solar constant.]

1.3 SPECTRAL DISTRIBUTION OF EXTRATERRESTRIAL RADIATION

In addition to the total energy in the solar spectrum (i.e., the solar constant), it is useful to know the spectral distribution of the extraterrestrial radiation, that is, the radiation that would be received in the absence of the atmosphere. A standard spectral irradiance curve has been compiled based on high-altitude and space measurements. The WRC standard is shown in Figure 1.3.1. Table 1.3.1 provides the same information on the WRC spectrum in numerical form. The average energy $G_{sc,\lambda}$ (in $W/m^2\ \mu m$) over small bandwidths centered at wavelength λ is given in the second column. The fraction $f_{0-\lambda}$ of the total energy in the spectrum that is between wavelengths zero and λ is given in the third

[2] Pyrheliometric scales are discussed in Section 2.2.

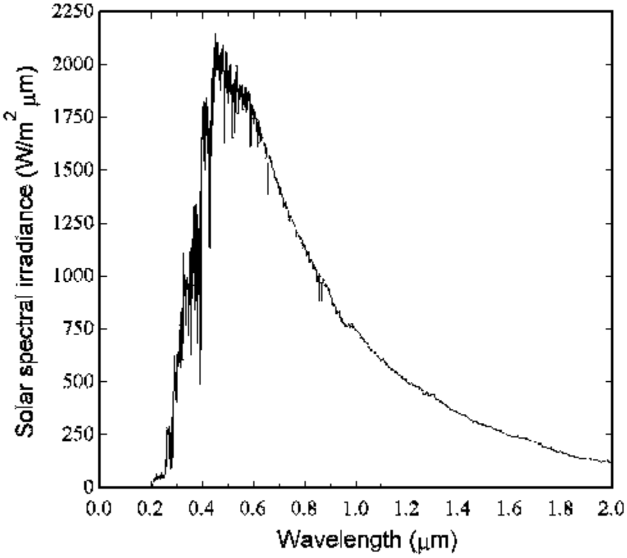

Figure 1.3.1 The WRC standard spectral irradiance curve at mean earth-sun distance.

column. The table is in two parts, the first at regular intervals of wavelength and the second at even fractions $f_{0-\lambda}$. This is a condensed table; more detailed tables are available elsewhere (see Iqbal, 1983).

Example 1.3.1

Calculate the fraction of the extraterrestrial solar radiation and the amount of that radiation in the ultraviolet ($\lambda < 0.38$ μm), the visible (0.38 μm $< \lambda <$ 0.78 μm), and the infrared ($\lambda > 0.78$ μm) portions of the spectrum.

Solution

From Table 1.3.1a, the fractions of $f_{0-\lambda}$ corresponding to wavelengths of 0.38 and 0.78 μm are 0.064 and 0.544. Thus, the fraction in the ultraviolet is 0.064, the fraction in the visible range is $0.544 - 0.064 = 0.480$, and the fraction in the infrared is $1.0 - 0.544 = 0.456$. Applying these fractions to a solar constant of 1367 W/m^2 and tabulating the results, we have:

Wavelength range (μm)	0–0.38	0.38–0.78	0.78–∞
Fraction in range	0.064	0.480	0.456
Energy in range (W/m^2)	87	656	623 ■

1.4 VARIATION OF EXTRATERRESTRIAL RADIATION

Two sources of variation in extraterrestrial radiation must be considered. The first is the variation in the radiation emitted by the sun. There are conflicting reports in the literature

Table 1.3.1a Extraterrestrial Solar Irradiance (WRC Spectrum) in Increments of Wavelength[a]

λ (μm)	$G_{sc,\lambda}$ (W/m^2 μm)	$f_{0-\lambda}$ (–)	λ (μm)	$G_{sc,\lambda}$ (W/m^2 μm)	$f_{0-\lambda}$ (–)	λ (μm)	$G_{sc,\lambda}$ (W/m^2 μm)	$f_{0-\lambda}$ (–)
0.250	81.20	0.001	0.520	1849.7	0.243	0.880	955.0	0.622
0.275	265.0	0.004	0.530	1882.8	0.257	0.900	908.9	0.636
0.300	499.4	0.011	0.540	1877.8	0.271	0.920	847.5	0.648
0.325	760.2	0.023	0.550	1860.0	0.284	0.940	799.8	0.660
0.340	955.5	0.033	0.560	1847.5	0.298	0.960	771.1	0.672
0.350	955.6	0.040	0.570	1842.5	0.312	0.980	799.1	0.683
0.360	1053.1	0.047	0.580	1826.9	0.325	1.000	753.2	0.695
0.370	1116.2	0.056	0.590	1797.5	0.338	1.050	672.4	0.721
0.380	1051.6	0.064	0.600	1748.8	0.351	1.100	574.9	0.744
0.390	1077.5	0.071	0.620	1738.8	0.377	1.200	507.5	0.785
0.400	1422.8	0.080	0.640	1658.7	0.402	1.300	427.5	0.819
0.410	1710.0	0.092	0.660	1550.0	0.425	1.400	355.0	0.847
0.420	1687.2	0.105	0.680	1490.2	0.448	1.500	297.8	0.871
0.430	1667.5	0.116	0.700	1413.8	0.469	1.600	231.7	0.891
0.440	1825.0	0.129	0.720	1348.6	0.489	1.800	173.8	0.921
0.450	1992.8	0.143	0.740	1292.7	0.508	2.000	91.6	0.942
0.460	2022.8	0.158	0.760	1235.0	0.527	2.500	54.3	0.968
0.470	2015.0	0.173	0.780	1182.3	0.544	3.000	26.5	0.981
0.480	1975.6	0.188	0.800	1133.6	0.561	3.500	15.0	0.988
0.490	1940.6	0.202	0.820	1085.0	0.578	4.000	7.7	0.992
0.500	1932.2	0.216	0.840	1027.7	0.593	5.000	2.5	0.996
0.510	1869.1	0.230	0.860	980.0	0.608	8.000	1.0	0.999

[a] $G_{sc,\lambda}$ is the average solar irradiance over the interval from the middle of the preceding wavelength interval to the middle of the following wavelength interval. For example, at 0.600 μm, 1748.8 W/m^2 μm is the average value between 0.595 and 0.610 μm.

Table 1.3.1b Extraterrestrial Solar Irradiance in Equal Increments of Energy

Energy Band $f_i - f_{i+1}$ (–)	Wavelength Range (μm)	Midpoint Wavelength (μm)	Energy Band $f_i - f_{i+1}$ (–)	Wavelength Range (μm)	Midpoint Wavelength (μm)
0.00–0.05	0.300–0.364	0.328	0.50–0.55	0.731–0.787	0.758
0.05–0.10	0.364–0.416	0.395	0.55–0.60	0.787–0.849	0.817
0.10–0.15	0.416–0.455	0.437	0.60–0.65	0.849–0.923	0.885
0.15–0.20	0.455–0.489	0.472	0.65–0.70	0.923–1.008	0.966
0.20–0.25	0.489–0.525	0.506	0.70–0.75	1.008–1.113	1.057
0.25–0.30	0.525–0.561	0.543	0.75–0.80	1.113–1.244	1.174
0.30–0.35	0.561–0.599	0.580	0.80–0.85	1.244–1.412	1.320
0.35–0.40	0.599–0.638	0.619	0.85–0.90	1.412–1.654	1.520
0.40–0.45	0.638–0.682	0.660	0.90–0.95	1.654–2.117	1.835
0.45–0.50	0.682–0.731	0.706	0.95–1.00	2.117–10.08	2.727

on periodic variations of intrinsic solar radiation. It has been suggested that there are small variations (less than $\pm 1.5\%$) with different periodicities and variation related to sunspot activities. Willson et al. (1981) report variances of up to 0.2% correlated with the development of sunspots. Others consider the measurements to be inconclusive or not indicative of regular variability. Measurements from *Nimbus* and *Mariner* satellites over periods of several months showed variations within limits of $\pm 0.2\%$ over a time when sunspot activity was very low (Frohlich, 1977). Data of Hickey et al. (1982) over a span of 2.5 years from the *Nimbus 7* satellite suggest that the solar constant is decreasing slowly, at a rate of approximately 0.02% per year. See Coulson (1975) or Thekaekara (1976) for further discussion of this topic. For engineering purposes, in view of the uncertainties and variability of atmospheric transmission, the energy emitted by the sun can be considered to be fixed.

Variation of the earth-sun distance, however, does lead to variation of extraterrestrial radiation flux in the range of $\pm 3.3\%$. The dependence of extraterrestrial radiation on time of year is shown in Figure 1.4.1. A simple equation with accuracy adequate for most engineering calculations is given by Equation 1.4.1a. Spencer (1971), as cited by Iqbal (1983), provides a more accurate equation ($\pm 0.01\%$) in the form of Equation 1.4.1b:

$$G_{on} = \begin{cases} G_{sc}\left(1 + 0.033 \cos \dfrac{360n}{365}\right) & (1.4.1a) \\ G_{sc}(1.000110 + 0.034221 \cos B + 0.001280 \sin B & \\ \quad + 0.000719 \cos 2B + 0.000077 \sin 2B) & (1.4.1b) \end{cases}$$

where G_{on} is the extraterrestrial radiation incident on the plane normal to the radiation on the nth day of the year and B is given by

$$B = (n - 1)\frac{360}{365} \tag{1.4.2}$$

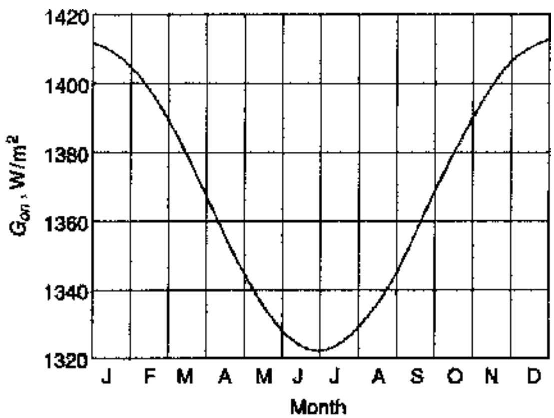

Figure 1.4.1 Variation of extraterrestrial solar radiation with time of year.

1.5 DEFINITIONS

Several definitions will be useful in understanding the balance of this chapter.

Air Mass m The ratio of the mass of atmosphere through which beam radiation passes to the mass it would pass through if the sun were at the zenith (i.e., directly overhead, see Section 1.6). Thus at sea level $m = 1$ when the sun is at the zenith and $m = 2$ for a zenith angle θ_z of 60°. For zenith angles from 0° to 70° at sea level, to a close approximation,

$$m = \frac{1}{\cos\ \theta_z} \tag{1.5.1}$$

For higher zenith angles, the effect of the earth's curvature becomes significant and must be taken into account.[3] For a more complete discussion of air mass, see Robinson (1966), Kondratyev (1969), or Garg (1982).

Beam Radiation The solar radiation received from the sun without having been scattered by the atmosphere. (Beam radiation is often referred to as direct solar radiation; to avoid confusion between subscripts for direct and diffuse, we use the term beam radiation.)

Diffuse Radiation The solar radiation received from the sun after its direction has been changed by scattering by the atmosphere. (Diffuse radiation is referred to in some meteorological literature as sky radiation or solar sky radiation; the definition used here will distinguish the diffuse solar radiation from infrared radiation emitted by the atmosphere.)

Total Solar Radiation The sum of the beam and the diffuse solar radiation on a surface.[4] (The most common measurements of solar radiation are total radiation on a horizontal surface, often referred to as **global radiation** on the surface.)

Irradiance, W/m^2 The rate at which radiant energy is incident on a surface per unit area of surface. The symbol G is used for solar irradiance, with appropriate subscripts for beam, diffuse, or spectral radiation.

Irradiation or Radiant Exposure, J/m^2 The incident energy per unit area on a surface, found by integration of irradiance over a specified time, usually an hour or a day. **Insolation** is a term applying specifically to solar energy irradiation. The symbol H is used for insolation for a day. The symbol I is used for insolation for an hour (or other period if specified). The symbols H and I can represent beam, diffuse, or total and can be on surfaces of any orientation.

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[3] An empirical relationship from Kasten and Young (1989) for air mass that works for zenith angles approaching 90° is

$$m = \frac{\exp(-0.0001184h)}{\cos(\theta_z) + 0.5057(96.080 - \theta_z)^{-1.634}}$$

where h is the site altitude in meters.

[4] Total solar radiation is sometimes used to indicate quantities integrated over all wavelengths of the solar spectrum.

Subscripts on G, H, and I are as follows: o refers to radiation above the earth's atmosphere, referred to as extraterrestrial radiation; b and d refer to beam and diffuse radiation; T and n refer to radiation on a tilted plane and on a plane normal to the direction of propagation. If neither T nor n appears, the radiation is on a horizontal plane.

Radiosity or **Radiant Exitance,** $\mathrm{W/m^2}$ The rate at which radiant energy leaves a surface per unit area by combined emission, reflection, and transmission.

Emissive Power or Radiant Self-Exitance, $\mathrm{W/m^2}$ The rate at which radiant energy leaves a surface per unit area by emission only.

Any of these radiation terms, except insolation, can apply to any specified wavelength range (such as the solar energy spectrum) or to monochromatic radiation. Insolation refers only to irradiation in the solar energy spectrum.

Solar Time Time based on the apparent angular motion of the sun across the sky, with solar noon the time the sun crosses the meridian of the observer.

Solar time is the time used in all of the sun-angle relationships; it does not coincide with local clock time. It is necessary to convert standard time to solar time by applying two corrections. First, there is a constant correction for the difference in longitude between the observer's meridian (longitude) and the meridian on which the local standard time is based.[5] The sun takes 4 min to transverse 1° of longitude. The second correction is from the equation of time, which takes into account the perturbations in the earth's rate of rotation which affect the time the sun crosses the observer's meridian. The difference in minutes between solar time and standard time is

$$\text{Solar time} - \text{standard time} = 4\,(L_{st} - L_{loc}) + E \tag{1.5.2}$$

where L_{st} is the standard meridian for the local time zone, L_{loc} is the longitude of the location in question, and longitudes are in degrees west, that is, $0^\circ < L < 360^\circ$. The parameter E is the equation of time (in minutes) from Figure 1.5.1 or Equation 1.5.3[6] [from Spencer (1971), as cited by Iqbal (1983)]:

$$\begin{aligned} E = 229.2(&0.000075 + 0.001868 \cos B - 0.032077 \sin B \\ &- 0.014615 \cos 2B - 0.04089 \sin 2B) \end{aligned} \tag{1.5.3}$$

where B is found from Equation 1.4.2 and n is the day of the year. Thus $1 \leq n \leq 365$.

Note that the equation of time and displacement from the standard meridian are both in minutes and that there is a 60-min difference between daylight saving time and standard time. Time is usually specified in hours and minutes. Care must be exercised in applying the corrections, which can total more than 60 min.

Example 1.5.1

At Madison, Wisconsin, what is the solar time corresponding to 10:30 AM central time on February 3?

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[5] To find the local standard meridian, multiply the time difference between local standard clock time and Greenwich Mean Time by 15.

[6] All equations use degrees, not radians

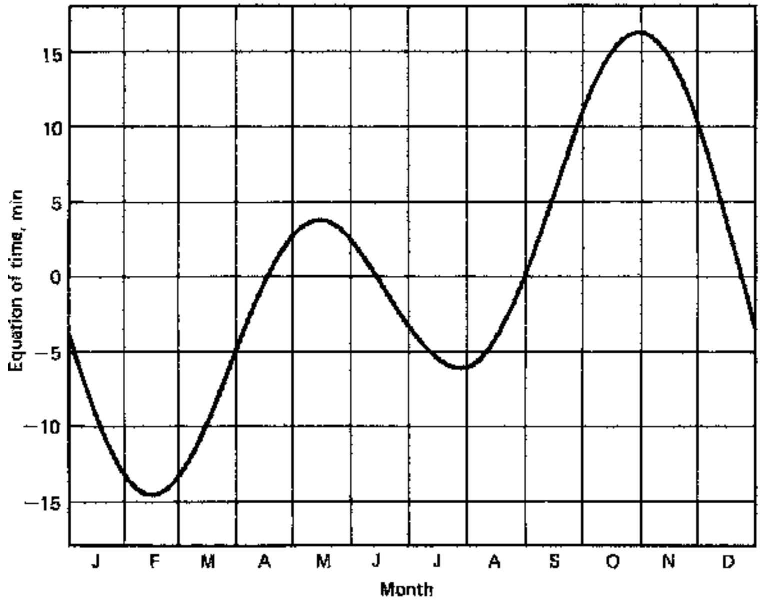


Figure 1.5.1 The equation of time E in minutes as a function of time of year.

Solution

In Madison, where the longitude is 89.4° and the standard meridian is 90°, Equation 1.5.2 gives

$$\begin{aligned}\text{Solar time} &= \text{standard time} + 4(90 - 89.4) + E \\ &= \text{standard time} + 2.4 + E\end{aligned}$$

On February 3, $n = 34$, and from Equation 1.5.3 or Figure 1.5.1, $E = -13.5$ min, so the correction to standard time is -11 min. Thus 10:30 AM Central Standard Time is 10:19 AM solar time. ■

In this book time is assumed to be solar time unless indication is given otherwise.

1.6 DIRECTION OF BEAM RADIATION

The geometric relationships between a plane of any particular orientation relative to the earth at any time (whether that plane is fixed or moving relative to the earth) and the incoming beam solar radiation, that is, the position of the sun relative to that plane, can be described in terms of several angles (Benford and Bock, 1939). Some of the angles are indicated in Figure 1.6.1. The angles and a set of consistent sign conventions are as follows:

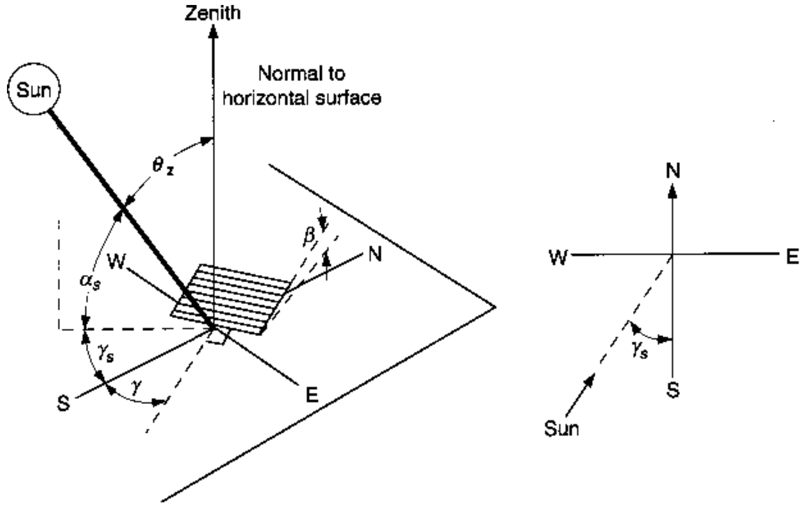


Figure 1.6.1 (a) Zenith angle, slope, surface azimuth angle, and solar azimuth angle for a tilted surface. (b) Plan view showing solar azimuth angle.

ϕ **Latitude,** the angular location north or south of the equator, north positive; $-90° \leq \phi \leq 90°$

δ **Declination,** the angular position of the sun at solar noon (i.e., when the sun is on the local meridian) with respect to the plane of the equator, north positive; $-23.45° \leq \delta \leq 23.45°$.

β **Slope,** the angle between the plane of the surface in question and the horizontal; $0° \leq \beta \leq 180°$. ($\beta > 90°$ means that the surface has a downward-facing component.)

γ **Surface azimuth angle,** the deviation of the projection on a horizontal plane of the normal to the surface from the local meridian, with zero due south, east negative, and west positive; $-180° \leq \gamma \leq 180°$.

ω **Hour angle,** the angular displacement of the sun east or west of the local meridian due to rotation of the earth on its axis at 15° per hour; morning negative, afternoon positive.

θ **Angle of incidence,** the angle between the beam radiation on a surface and the normal to that surface.

Additional angles are defined that describe the position of the sun in the sky:

θ_z **Zenith angle,** the angle between the vertical and the line to the sun, that is, the angle of incidence of beam radiation on a horizontal surface.

α_s **Solar altitude angle,** the angle between the horizontal and the line to the sun, that is, the complement of the zenith angle.

γ_s **Solar azimuth angle,** the angular displacement from south of the projection of beam radiation on the horizontal plane, shown in Figure 1.6.1. Displacements east of south are negative and west of south are positive.

The declination δ can be found from the approximate equation of Cooper (1969),

$$\delta = 23.45 \sin\left(360 \frac{284 + n}{365}\right) \tag{1.6.1a}$$

or from the more accurate equation (error $< 0.035°$) [from Spencer (1971), as cited by Iqbal (1983)]

$$\begin{aligned}\delta = (180/\pi)(&0.006918 - 0.399912 \cos B + 0.070257 \sin B \\ &- 0.006758 \cos 2B + 0.000907 \sin 2B \\ &- 0.002697 \cos 3B + 0.00148 \sin 3B)\end{aligned} \tag{1.6.1b}$$

where B is from Equation 1.4.2 and the day of the year n can be conveniently obtained with the help of Table 1.6.1.

Variation in sun-earth distance (as noted in Section 1.4), the equation of time E (as noted in Section 1.5), and declination are all continuously varying functions of time of year. For many computational purposes it is customary to express the time of year in terms of n, the day of the year, and thus as an integer between 1 and 365. Equations 1.4.1, 1.5.3, and 1.6.1 could be used with noninteger values of n. Note that the maximum rate of change of declination is about 0.4° per day. The use of integer values of n is adequate for most engineering calculations outlined in this book.

There is a set of useful relationships among these angles. Equations relating the angle of incidence of beam radiation on a surface, θ, to the other angles are

$$\begin{aligned}\cos\theta = {} & \sin\delta \sin\phi \cos\beta - \sin\delta \cos\phi \sin\beta \cos\gamma \\ & + \cos\delta \cos\phi \cos\beta \cos\omega + \cos\delta \sin\phi \sin\beta \cos\gamma \cos\omega \\ & + \cos\delta \sin\beta \sin\gamma \sin\omega\end{aligned} \tag{1.6.2}$$

Table 1.6.1 Recommended Average Days for Months and Values of n by Months[a]

Month	n for ith Day of Month	For Average Day of Month: Date	n	δ
January	i	17	17	−20.9
February	$31 + i$	16	47	−13.0
March	$59 + i$	16	75	−2.4
April	$90 + i$	15	105	9.4
May	$120 + i$	15	135	18.8
June	$151 + i$	11	162	23.1
July	$181 + i$	17	198	21.2
August	$212 + i$	16	228	13.5
September	$243 + i$	15	258	2.2
October	$273 + i$	15	288	−9.6
November	$304 + i$	14	318	−18.9
December	$334 + i$	10	344	−23.0

[a]From Klein (1977). Do not use for $|\phi| > 66.5°$.

and

$$\cos\,\theta = \cos\,\theta_z\,\cos\,\beta + \sin\,\theta_z\,\sin\,\beta\,\cos(\gamma_s - \gamma) \tag{1.6.3}$$

The angle θ may exceed 90°, which means that the sun is behind the surface. Also, when using Equation 1.6.2, it is necessary to ensure that the earth is not blocking the sun (i.e., that the hour angle is between sunrise and sunset).

Example 1.6.1

Calculate the angle of incidence of beam radiation on a surface located at Madison, Wisconsin, at 10:30 (solar time) on February 13 if the surface is tilted 45° from the horizontal and pointed 15° west of south.

Solution

Under these conditions, $n = 44$, the declination δ from Equation 1.6.1 is $-14°$, the hour angle $\omega = -22.5°$ (15° per hour times 1.5 h before noon), and the surface azimuth angle $\gamma = 15°$. Using a slope $\beta = 45°$ and the latitude ϕ of Madison of 43° N, Equation 1.6.2 is

$$\begin{aligned}
\cos\,\theta &= \sin(-14)\,\sin\,43\,\cos\,45 - \sin(-14)\,\cos\,43\,\sin\,45\,\cos\,15 \\
&\quad + \cos(-14)\,\cos\,43\,\cos\,45\,\cos(-22.5) \\
&\quad + \cos(-14)\,\sin\,43\,\sin\,45\,\cos\,15\,\cos(-22.5) \\
&\quad + \cos(-14)\,\sin\,45\,\sin\,15\,\sin(-22.5) \\
\cos\,\theta &= -0.117 + 0.121 + 0.464 + 0.418 - 0.068 = 0.817 \\
\theta &= 35°
\end{aligned}$$

■

There are several commonly occurring cases for which Equation 1.6.2 is simplified. For fixed surfaces sloped toward the south or north, that is, with a surface azimuth angle γ of 0° or 180° (a very common situation for fixed flat-plate collectors), the last term drops out.

For vertical surfaces, $\beta = 90°$ and the equation becomes

$$\cos\,\theta = -\sin\,\delta\,\cos\,\phi\,\cos\,\gamma + \cos\,\delta\,\sin\,\phi\,\cos\,\gamma\,\cos\,\omega + \cos\,\delta\,\sin\,\gamma\,\sin\,\omega \tag{1.6.4}$$

For horizontal surfaces, the angle of incidence is the zenith angle of the sun, θ_z. Its value must be between 0° and 90° when the sun is above the horizon. For this situation, $\beta = 0$, and Equation 1.6.2 becomes

$$\cos\,\theta_z = \cos\,\phi\,\cos\,\delta\,\cos\,\omega + \sin\,\phi\,\sin\,\delta \tag{1.6.5}$$

The solar azimuth angle γ_s can have values in the range of 180° to $-180°$. For north or south latitudes between 23.45° and 66.45°, γ_s will be between 90° and $-90°$ for days less than 12 h long; for days with more than 12 h between sunrise and sunset, γ_s will be greater than 90° or less than −90° early and late in the day when the sun is north of the east-west line in the northern hemisphere or south of the east-west line in the southern hemisphere. For tropical latitudes, γ_s can have any value when $\delta - \phi$ is positive in the northern hemisphere or negative in the southern, for example, just before noon at ϕ = 10° and δ = 20°, γ_s = −180°, and just after noon γ_s = +180°. Thus γ_s is negative when the hour angle is negative and positive when the hour angle is positive. The sign function in Equations 1.6.6 is equal to +1 if ω is positive and is equal to −1 if ω is negative:

$$\gamma_S = \operatorname{sign}(\omega) \left| \cos^{-1} \left(\frac{\cos \theta_z \sin \phi - \sin \delta}{\sin \theta_z \cos \phi} \right) \right| \tag{1.6.6}$$

Example 1.6.2

Calculate the zenith and solar azimuth angles for ϕ = 43° at **a** 9:30 AM on February 13 and **b** 6:30 PM on July 1.

Solution

a On February 13 at 9:30, $\delta = -14°$ and $\omega = -37.5°$. From Equation 1.6.5,

$$\cos \theta_z = \cos 43 \cos(-14) \cos(-37.5) + \sin 43 \sin(-14) = 0.398$$

$$\theta_z = 66.5°$$

From Equation 1.6.6

$$\gamma_s = -1 \left| \cos^{-1} \left(\frac{\cos 66.5 \sin 43 - \sin(-14)}{\sin 66.5 \cos 43} \right) \right| = -40.0°$$

b On July 1 at 6:30 PM, $n = 182$, $\delta = 23.1°$, and $\omega = 97.5°$. From Equation 1.6.5,

$$\cos \theta_z = \cos 43 \cos 23.1 \cos 97.5 + \sin 43 \sin 23.1$$

$$\theta_z = 79.6°$$

$$\gamma_s = +1 \left| \cos^{-1} \left(\frac{\cos 79.6 \sin 43 - \sin 23.1}{\sin 79.6 \cos 43} \right) \right| = 112.0°$$

■

Useful relationships for the angle of incidence of surfaces sloped due north or due south can be derived from the fact that surfaces with slope β to the north or south have the same angular relationship to beam radiation as a horizontal surface at an artificial latitude of $\phi - \beta$. The relationship is shown in Figure 1.6.2 for the northern hemisphere. Modifying Equation 1.6.5 yields

$$\cos \theta = \cos(\phi - \beta) \cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta \tag{1.6.7a}$$

For the southern hemisphere modify the equation by replacing $\phi - \beta$ by $\phi + \beta$, consistent with the sign conventions on ϕ and δ:

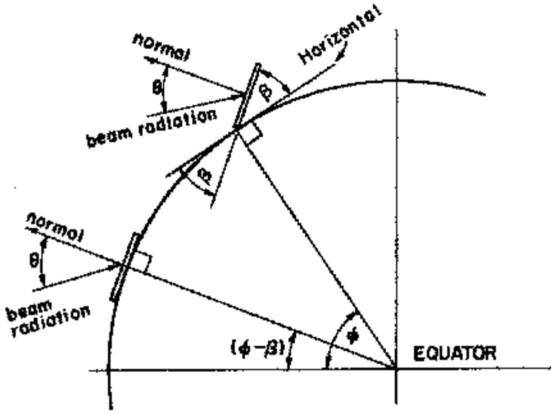


Figure 1.6.2 Section of earth showing β, θ, ϕ, and $\phi - \beta$ for a south-facing surface.

$$\cos\theta = \cos(\phi + \beta)\cos\delta\cos\omega + \sin(\phi + \beta)\sin\delta \tag{1.6.7b}$$

For the special case of solar noon, for the south-facing sloped surface in the northern hemisphere,

$$\theta_{\text{noon}} = |\phi - \delta - \beta| \tag{1.6.8a}$$

and in the southern hemisphere

$$\theta_{\text{noon}} = |-\phi + \delta - \beta| \tag{1.6.8b}$$

where $\beta = 0$, the angle of incidence is the zenith angle, which for the northern hemisphere is

$$\theta_{z,\text{noon}} = |\phi - \delta| \tag{1.6.9a}$$

and for the southern hemisphere

$$\theta_{z,\text{noon}} = |-\phi + \delta| \tag{1.6.9b}$$

Equation 1.6.5 can be solved for the **sunset hour angle** ω_s, when $\theta_z = 90°$:

$$\cos\omega_s = -\frac{\sin\phi\sin\delta}{\cos\phi\cos\delta} = -\tan\phi\tan\delta \tag{1.6.10}$$

The sunrise hour angle is the negative of the sunset hour angle. It also follows that the number of daylight hours is given by

$$N = \tfrac{2}{15}\cos^{-1}(-\tan\phi\tan\delta) \tag{1.6.11}$$

A convenient nomogram for determining day length has been devised by Whillier (1965) and is shown in Figure 1.6.3. Information on latitude and declination for either hemisphere leads directly to times of sunrise and sunset and day length.

An additional angle of interest is the **profile angle** of beam radiation on a receiver plane R that has a surface azimuth angle of γ. It is the projection of the solar altitude

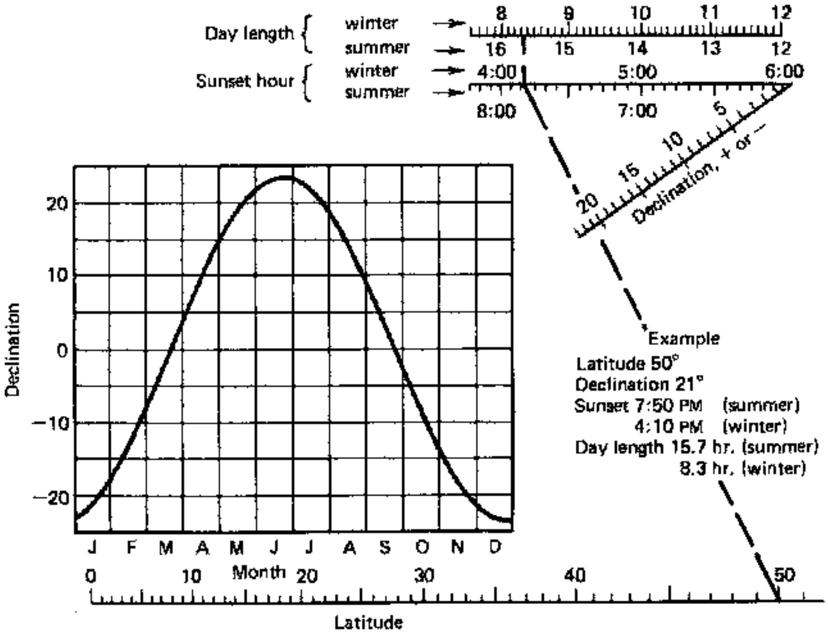


Figure 1.6.3 Nomogram to determine time of sunset and day length. Adapted from Whillier (1965b).

angle on a vertical plane perpendicular to the plane in question. Expressed another way, it is the angle through which a plane that is initially horizontal must be rotated about an axis in the plane of the surface in question in order to include the sun. The solar altitude angle α_s (i.e., angle *EAD*) and the profile angle α_p (i.e., angle *fab*) for the plane *R* are shown in Figure 1.6.4. The plane *adef* includes the sun. Note that the solar altitude and profile angle are the same when the sun is in a plane perpendicular to the surface *R* (e.g., at solar noon for a surface with a surface azimuth angle of 0° or 180°). The profile angle is useful in calculating shading by overhangs and can be determined from

$$\tan \alpha_p = \frac{\tan \alpha_s}{\cos(\gamma_s - \gamma)} \tag{1.6.12}$$

Example 1.6.3

Calculate the time of sunrise, solar altitude, zenith, solar azimuth, and profile angles for a 60° sloped surface facing 25° west of south at 4:00 PM solar time on March 16 at a latitude of 43°. Also calculate the time of sunrise and sunset on the surface.

Solution

The hour angle at sunset is determined using Equation 1.6.10. For March 16, from Equation 1.6.1 (or Table 1.6.1), $\delta = -2.4°$:

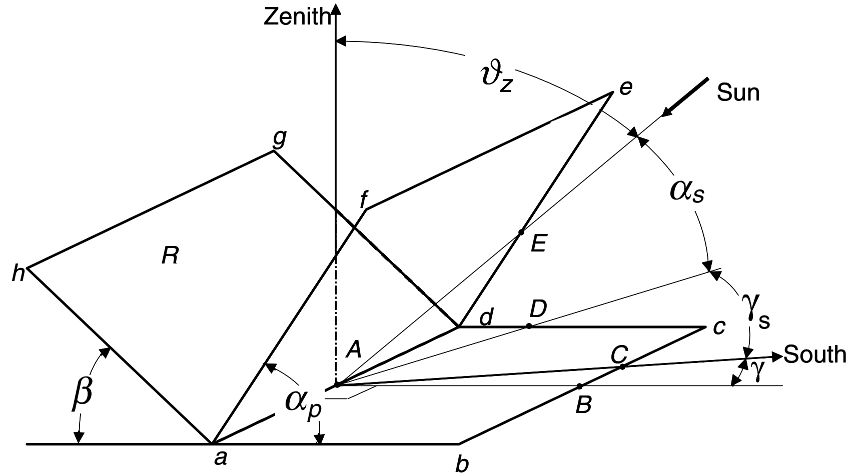


Figure 1.6.4 The solar altitude angle α_s ($\angle EAD$) and the profile angle α_p ($\angle fab$) for surface R.

$$\omega_s = \cos^{-1}[-\tan 43 \tan(-2.4)] = 87.8°$$

The sunrise hour angle is therefore −87.8°. With the earth's rotation of 15° per hour, sunrise (and sunset) occurs 5.85 h (5 h and 51 min) from noon so sunrise is at 6:09 AM (and sunset is at 5:51 PM).

The solar altitude angle α_s is a function only of time of day and declination. At 4:00 PM, $\omega = 60°$. From Equation 1.6.5, recognizing that $\cos\theta_z = \sin(90 - \theta_z) = \sin \alpha_s$,

$$\sin \alpha_s = \cos 43 \cos(-2.4) \cos 60 + \sin 43 \sin(-2.4) = 0.337$$

$$\alpha_s = 19.7° \qquad \text{and} \qquad \theta_z = 90 - \alpha_s = 70.3°$$

The solar azimuth angle for this time can be calculated with Equation 1.6.6:

$$\gamma_s = \cos^{-1}\left[\operatorname{sign}(60)\left(\frac{\cos 70.3 \sin 43 - \sin(-2.4)}{\sin 70.3 \cos 43}\right)\right] = 66.8°$$

The profile angle for the surface with $\gamma = 25°$ is calculated with Equation 1.6.12:

$$\alpha_p = \tan^{-1}\left(\frac{\tan 19.7}{\cos(66.8 - 25)}\right) = 25.7°$$

The hour angles at which sunrise and sunset occur on the surface are calculated from Equation 1.6.2 with $\theta = 90°$ ($\cos\theta = 0$):

$$\begin{aligned} 0 = {} & \sin(-2.4) \sin 43 \cos 60 - \sin(-2.4) \cos 43 \sin 60 \cos 25 \\ & + [\cos(-2.4) \cos 43 \cos 60 + \cos(-2.4) \sin 43 \sin 60 \cos 25] \cos\omega \\ & + [\cos(-2.4) \sin 60 \sin 25] \sin\omega \end{aligned}$$

or

$$0 = 0.008499 + 0.9077 \cos \omega + 0.3657 \sin \omega$$

which, using $\sin^2 \omega + \cos^2 \omega = 1$, has two solutions: $\omega = -68.6°$ and $\omega = 112.4°$. Sunrise on the surface is therefore $68.6/15 = 4.57$ h before noon, or 7:26 AM. The time of sunset on the collector is the actual sunset since 112.4° is greater than 87.8° (i.e., when $\theta = 90°$ the sun has already set). ■

Solar azimuth and altitude angles are tabulated as functions of latitude, declination, and hour angle by the U.S. Hydrographic Office (1940). Highly accurate equations are available from the National Renewable Energy Laboratory's website. Information on the position of the sun in the sky is also available with less precision but easy access in various types of charts. Examples of these are the Sun Angle Calculator (1951) and the solar position charts (plots of α_s or θ_z vs. γ_s for various ϕ, δ, and ω) in Section 1.9 and Appendix H. Care is necessary in interpreting information from other sources, since nomenclature, definitions, and sign conventions may vary from those used here.

1.7 ANGLES FOR TRACKING SURFACES

Some solar collectors "track" the sun by moving in prescribed ways to minimize the angle of incidence of beam radiation on their surfaces and thus maximize the incident beam radiation. The angles of incidence and the surface azimuth angles are needed for these collectors. The relationships in this section will be useful in radiation calculations for these moving surfaces. For further information see Eibling et al. (1953) and Braun and Mitchell (1983).

Tracking systems are classified by their motions. Rotation can be about a single axis (which could have any orientation but which in practice is usually horizontal east-west, horizontal north-south, vertical, or parallel to the earth's axis) or it can be about two axes. The following sets of equations (except for Equations 1.7.4) are for surfaces that rotate on axes that are parallel to the surfaces. Figure 1.7.1 shows extraterrestrial radiation on a fixed surface with slope equal to the latitude and also on surfaces that track the sun about a horizontal north-south or east-west axis at a latitude of 45° at the summer and winter solstices. It is clear that tracking can significantly change the time distribution of incident beam radiation. Tracking does not always result in increased beam radiation; compare the winter solstice radiation on the north-south tracking surface with the radiation on the fixed surface. In practice the differences will be less than indicated by the figure due to clouds and atmospheric transmission.

For a plane rotated about a horizontal east-west axis with a single daily adjustment so that the beam radiation is normal to the surface at noon each day,

$$\cos \theta = \sin^2 \delta + \cos^2 \delta \cos \omega \tag{1.7.1a}$$

The slope of this surface will be fixed for each day and will be

$$\beta = |\phi - \delta| \tag{1.7.1b}$$

The surface azimuth angle for a day will be 0° or 180° depending on the latitude and declination:

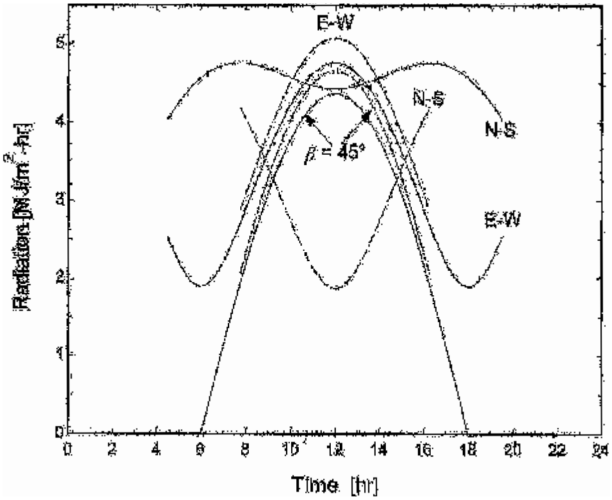


Figure 1.7.1 Extraterrestrial solar radiation for $\phi = 45°$ on a stationary collector at $\beta = 45°$ on north-south (N-S) and east-west (E-W) single-axis tracking collectors. The three dotted curves are for the winter solstice and the three solid curves are for the summer solstice.

$$\gamma = \begin{cases} 0° & \text{if } \phi - \delta > 0 \\ 180° & \text{if } \phi - \delta \leq 0 \end{cases} \tag{1.7.1c}$$

For a plane rotated about a horizontal east-west axis with continuous adjustment to minimize the angle of incidence,

$$\cos\theta = (1 - \cos^2\delta \sin^2\omega)^{1/2} \tag{1.7.2a}$$

The slope of this surface is given by

$$\tan\beta = \tan\theta_z |\cos\gamma_s| \tag{1.7.2b}$$

The surface azimuth angle for this mode of orientation will change between 0° and 180° if the solar azimuth angle passes through $\pm 90°$. For either hemisphere,

$$\gamma = \begin{cases} 0° & \text{if } |\gamma_s| < 90 \\ 180° & \text{if } |\gamma_s| \geq 90 \end{cases} \tag{1.7.2c}$$

For a plane rotated about a horizontal north-south axis with continuous adjustment to minimize the angle of incidence,

$$\cos\theta = (\cos^2\theta_z + \cos^2\delta \sin^2\omega)^{1/2} \tag{1.7.3a}$$

The slope is given by

$$\tan\beta = \tan\theta_z |\cos(\gamma - \gamma_s)| \tag{1.7.3b}$$

The surface azimuth angle γ will be 90° or $-90°$ depending on the sign of the solar azimuth angle:

$$\gamma = \begin{cases} 90^\circ & \text{if } \gamma_s > 0 \\ -90^\circ & \text{if } \gamma_s \le 0 \end{cases} \tag{1.7.3c}$$

For a plane with a fixed slope rotated about a vertical axis, the angle of incidence is minimized when the surface azimuth and solar azimuth angles are equal. From Equation 1.6.3, the angle of incidence is

$$\cos\theta = \cos\theta_z \cos\beta + \sin\theta_z \sin\beta \tag{1.7.4a}$$

The slope is fixed, so

$$\beta = \text{const} \tag{1.7.4b}$$

The surface azimuth angle is

$$\gamma = \gamma_s \tag{1.7.4c}$$

For a plane rotated about a north-south axis parallel to the earth's axis with continuous adjustment to minimize θ,

$$\cos\theta = \cos\delta \tag{1.7.5a}$$

The slope varies continuously and is

$$\tan\beta = \frac{\tan\phi}{\cos\gamma} \tag{1.7.5b}$$

The surface azimuth angle is

$$\gamma = \tan^{-1}\frac{\sin\theta_z \sin\gamma_s}{\cos\theta' \sin\phi} + 180C_1C_2 \tag{1.7.5c}$$

where

$$\cos\theta' = \cos\theta_z \cos\phi + \sin\theta_z \sin\phi \cos\gamma_s \tag{1.7.5d}$$

$$C_1 = \begin{cases} 0 & \text{if } \left(\tan^{-1}\dfrac{\sin\theta_z \sin\gamma_s}{\cos\theta' \sin\phi}\right)\gamma_s \ge 0 \\ +1 & \text{otherwise} \end{cases} \tag{1.7.5e}$$

$$C_2 = \begin{cases} +1 & \text{if } \gamma_s \ge 0 \\ -1 & \text{if } \gamma_s < 0 \end{cases} \tag{1.7.5f}$$

For a plane that is continuously tracking about two axes to minimize the angle of incidence,

$$\cos\theta = 1 \tag{1.7.6a}$$

$$\beta = \theta_z \tag{1.7.6b}$$

$$\gamma = \gamma_s \tag{1.7.6c}$$

Example 1.7.1

Calculate the angle of incidence of beam radiation, the slope of the surface, and the surface azimuth angle for a surface at **a** $\phi = 40^\circ$, $\delta = 21^\circ$ and $\omega = 30^\circ$ (2:00 PM) and

b $\phi = 40°$, $\delta = 21°$, and $\omega = 100°$ if it is continuously rotated about an east-west axis to minimize θ.

Solution

a Use Equations 1.7.2 for a surface moved in this way. First calculate the angle of incidence:

$$\theta = \cos^{-1}(1 - \cos^2 21 \sin^2 30)^{1/2} = 27.8°$$

Next calculate θ_z from Equation 1.6.5:

$$\theta_z = \cos^{-1}(\cos 40 \cos 21 \cos 30 + \sin 40 \sin 21) = 31.8°$$

We now need the solar azimuth angle γ_s, which can be found from Equation 1.6.6:

$$\gamma_s = \text{sign}(30)\left|\cos^{-1}\left(\frac{\cos 31.8 \sin 40 - \sin 21}{\sin 31.8 \cos 40}\right)\right| = 62.3°$$

Then from Equation 1.7.2b

$$\beta = \tan^{-1}(\tan 31.8\ |\cos 62.3|) = 16.1°$$

From Equation 1.7.2c, with $\gamma_s < 90$, $\gamma = 0$.

b The procedure is the same as in part a:

$$\theta = \cos^{-1}(1 - \cos^2 21 \sin^2 100)^{1/2} = 66.8°$$

$$\theta_z = \cos^{-1}(\cos 40 \cos 21 \cos 100 + \sin 40 \sin 21) = 83.9°$$

$$\gamma_s = \cos^{-1}\left[\text{sign}(100)\left(\frac{\cos 83.9 \sin 40 - \sin 21}{\sin 83.9 \cos 40}\right)\right] = 112.4°$$

The slope is then

$$\beta = \tan^{-1}(\tan 83.9\ |\cos 112.4|) = 74.3°$$

And since $|\gamma_s| > 90$, γ will be 180°. (Note that these results can be checked using Equation 1.6.5.) ■

1.8 RATIO OF BEAM RADIATION ON TILTED SURFACE TO THAT ON HORIZONTAL SURFACE

For purposes of solar process design and performance calculations, it is often necessary to calculate the hourly radiation on a tilted surface of a collector from measurements or estimates of solar radiation on a horizontal surface. The most commonly available data are total radiation for hours or days on the horizontal surface, whereas the need is for beam and diffuse radiation on the plane of a collector.

The geometric factor R_b, the ratio of beam radiation on the tilted surface to that on a horizontal surface at any time, can be calculated exactly by appropriate use of Equation

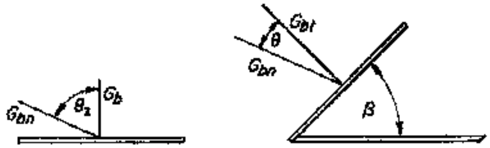


Figure 1.8.1 Beam radiation on horizontal and tilted surfaces.

1.6.2. Figure 1.8.1 indicates the angle of incidence of beam radiation on the horizontal and tilted surfaces. The ratio G_{bT}/G_b is given by[7]

$$R_b = \frac{G_{b,T}}{G_b} = \frac{G_{b,n} \cos \theta}{G_{b,n} \cos \theta_z} = \frac{\cos \theta}{\cos \theta_z} \tag{1.8.1}$$

and $\cos \theta$ and $\cos \theta_z$ are both determined from Equation 1.6.2 (or from equations derived from Equation 1.6.2).

Example 1.8.1

What is the ratio of beam radiation to that on a horizontal surface for the surface and time specified in Example 1.6.1?

Solution

Example 1.6.1 shows the calculation for $\cos \theta$. For the horizontal surface, from Equation 1.6.5,

$$\cos \theta_z = \sin(-14) \sin 43 + \cos(-14) \cos 43 \cos(-22.5) = 0.491$$

And from Equation 1.8.1

$$R_b = \frac{\cos \theta}{\cos \theta_z} = \frac{0.818}{0.491} = 1.67$$

■

The optimum azimuth angle for flat-plate collectors is usually 0° in the northern hemisphere (or 180° in the southern hemisphere). Thus it is a common situation that $\gamma = 0°$ (or 180°). In this case, Equations 1.6.5 and 1.6.7 can be used to determine $\cos \theta_z$ and $\cos \theta$, respectively, leading in the northern hemisphere, for $\gamma = 0°$, to

$$R_b = \frac{\cos(\phi - \beta)\cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta} \tag{1.8.2}$$

In the southern hemisphere, $\gamma = 180°$ and the equation is

$$R_b = \frac{\cos(\phi + \beta) \cos \delta \cos \omega + \sin(\phi + \beta) \sin \delta}{\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta} \tag{1.8.3}$$

A special case of interest is $R_{b,\text{noon}}$, the ratio for south-facing surfaces at solar noon. From Equations 1.6.8a and 1.6.9a, for the northern hemisphere,

---

[7] The symbol G is used in this book to denote rates, while I is used for energy quantities integrated over an hour. The original development of R_b by Hottel and Woertz (1942) was for hourly periods; for an hour (using angles at the midpoint of the hour), $R_b = I_{bT}/I_b$.

$$R_{b,\text{noon}} = \frac{\cos|\phi - \delta - \beta|}{\cos|\phi - \delta|} \tag{1.8.4a}$$

For the southern hemisphere, from Equations 1.6.8b and 1.6.9b,

$$R_{b,\text{noon}} = \frac{\cos|-\phi + \delta - \beta|}{\cos|-\phi + \delta|} \tag{1.8.4b}$$

Hottel and Woertz (1942) pointed out that Equation 1.8.2 provides a convenient method for calculating R_b for the most common cases. They also showed a graphical method for solving these equations. This graphical method has been revised by Whillier (1975), and an adaptation of Whillier's curves is given here. Figures 1.8.2(a–e) are plots of both cos θ_z as a function of ϕ and cos θ as a function of $\phi - \beta$ for various dates (i.e., declinations). By plotting the curves for sets of dates having (nearly) the same absolute value of declination, the curves ''reflect back'' on each other at latitude 0°. Thus each set of curves, in effect, covers the latitude range of $-60°$ to 60°.

As will be seen in later chapters, solar process performance calculations are very often done on an hourly basis. The cos θ_z plots are shown for the midpoints of hours before and after solar noon, and the values of R_b found from them are applied to those hours. (This procedure is satisfactory for most hours of the day, but in hours that include sunrise and sunset, unrepresentative values of R_b may be obtained. Solar collection in those hours is most often zero or a negligible part of the total daily collector output. However, care must be taken that unrealistic products of R_b and beam radiation I_b are not used.)

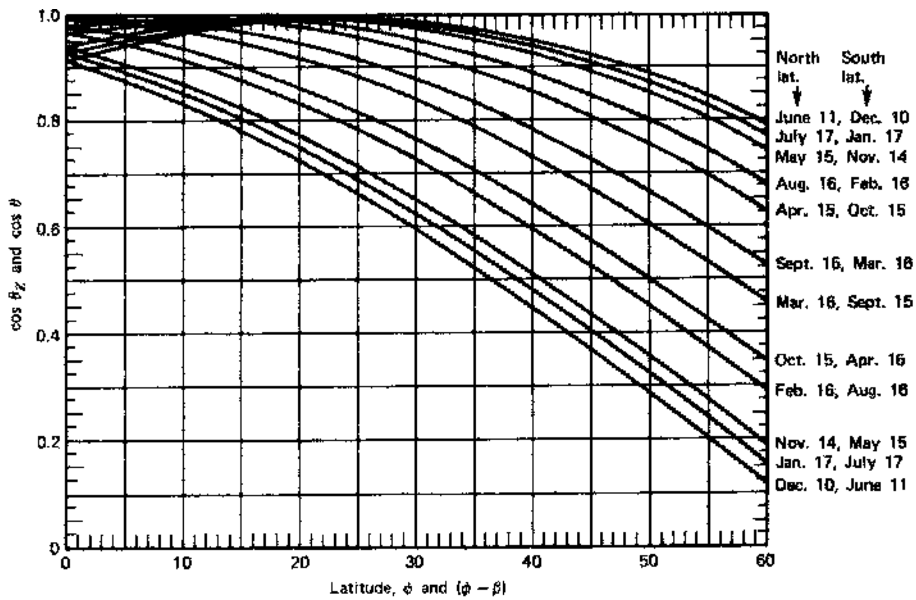


Figure 1.8.2(a) cos θ versus $\phi - \beta$ and cos θ_z versus ϕ for hours 11 to 12 and 12 to 1 for surfaces tilted toward the equator. The columns on the right show dates for the curves for north and south latitudes. In south latitudes, use $|\phi|$. Adapted from Whillier (1975).

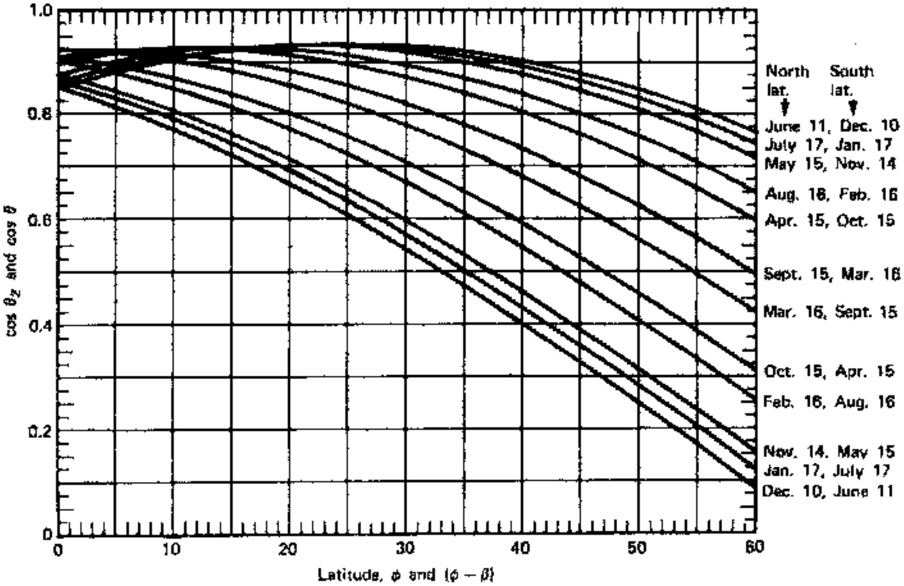


Figure 1.8.2(b) cos θ versus $\phi - \beta$ and cos θ_z versus ϕ for hours 10 to 11 and 1 to 2.

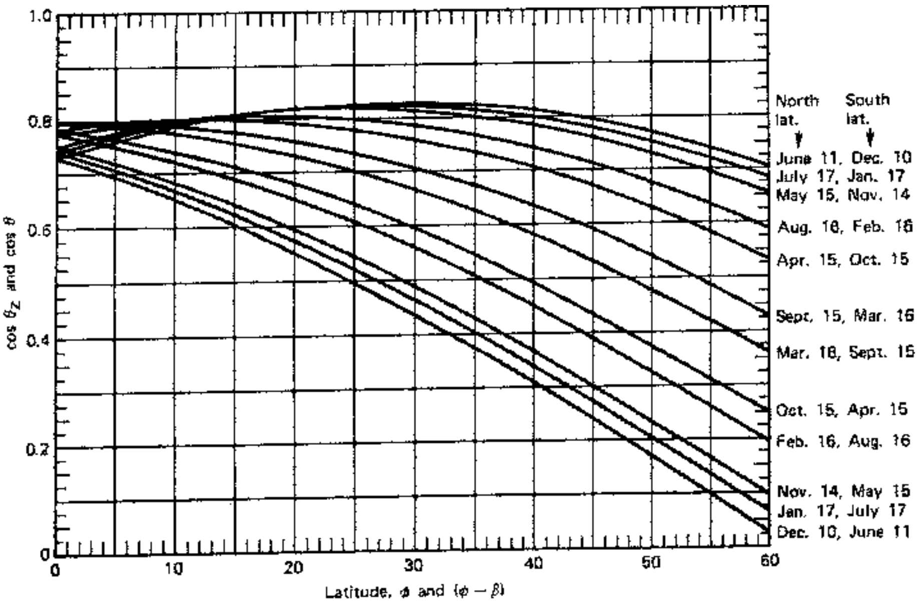


Figure 1.8.2(c) cos θ versus $\phi - \beta$ and cos θ_z versus ϕ for hours 9 to 10 and 2 to 3.

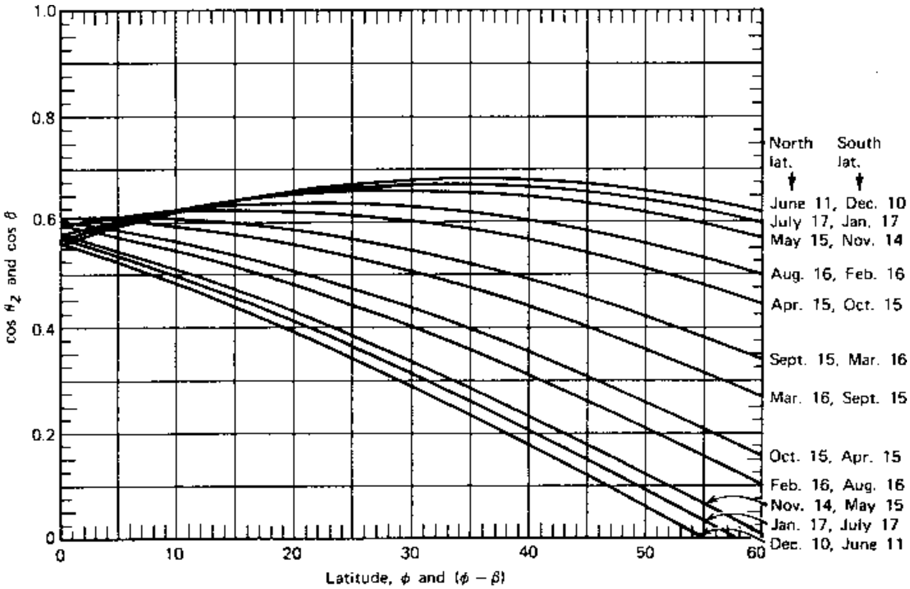


Figure 1.8.2(d) cos θ versus $\phi - \beta$ and cos θ_z versus ϕ for hours 8 to 9 and 3 to 4.

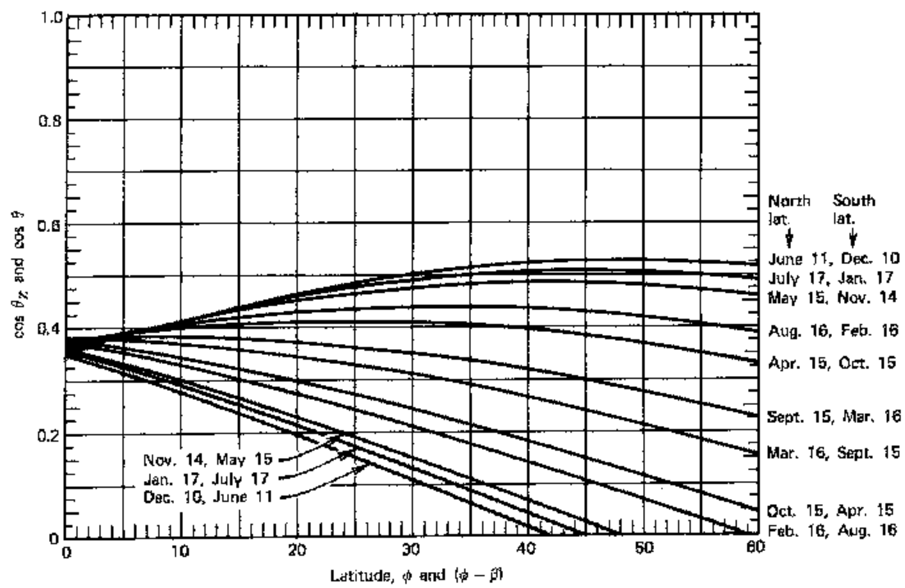


Figure 1.8.2(e) cos θ versus $\phi - \beta$ and cos θ_z versus ϕ for hours 7 to 8 and 4 to 5.

To find $\cos\theta_z$, enter the chart for the appropriate time with the date and latitude of the location in question. For the same date and latitude $\cos\theta$ is found by entering with an abscissa corresponding to $\phi - \beta$. Then R_b is found from Equation 1.8.1. The dates on the sets of curves are shown in two sets, one for north (positive) latitudes and the other for south (negative) latitudes.

Two situations arise, for positive values or for negative values of $\phi - \beta$. For positive values, the charts are used directly. If $\phi - \beta$ is negative (which frequently occurs when collectors are sloped for optimum performance in winter or with vertical collectors), the procedure is modified. Determine $\cos\theta_z$ as before. Determine $\cos\theta$ from the absolute value of $\phi - \beta$ using the curve for the other hemisphere, that is, with the sign on the declination reversed.

Example 1.8.2

Calculate R_b for a surface at latitude 40° N at a tilt 30° toward the south for the hour 9 to 10 solar time on February 16.

Solution

Use Figure 1.8.2(c) for the hour ± 2.5 h from noon as representative of the hour from 9 to 10. To find $\cos\theta_z$, enter at a latitude of 40° for the north latitude date of February 16. Here $\cos\theta_z = 0.45$. To find $\cos\theta$, enter at a latitude of $\phi - \beta = 10°$ for the same date. Here $\cos\theta = 0.73$. Then

$$R_b = \frac{\cos\theta}{\cos\theta_z} = \frac{0.73}{0.45} = 1.62$$

The ratio can also be calculated using Equation 1.8.2. The declination on February 16 is $-13°$:

$$R_b = \frac{\cos 10 \cos(-13) \cos(-37.5) + \sin 10 \sin(-13)}{\cos 40 \cos(-13) \cos(-37.5) + \sin 10 \sin(-13)} = \frac{0.722}{0.448} = 1.61 \qquad \blacksquare$$

Example 1.8.3

Calculate R_b for a latitude 40° N at a tilt of 50° toward the south for the hour 9 to 10 solar time on February 16.

Solution

As found in the previous example, $\cos\theta_z = 0.45$. To find $\cos\theta$, enter at an abscissa of $+10°$, using the curve for February 16 for south latitudes. The value of $\cos\theta$ from the curve is 0.80. Thus $R_b = 0.80/0.45 = 1.78$. Equation 1.8.2 can also be used:

$$R_b = \frac{\cos 10 \cos(-13) \cos(-37.5) + \sin(-10) \sin(-13)}{\cos 40 \cos(-13) \cos(-37.5) + \sin 40 \sin(-13)} = \frac{0.800}{0.448} = 1.79 \qquad \blacksquare$$

It is possible, using Equation 1.8.2 or Figure 1.8.2, to construct plots showing the effects of collector tilt on R_b for various times of the year and day. Figure 1.8.3 shows

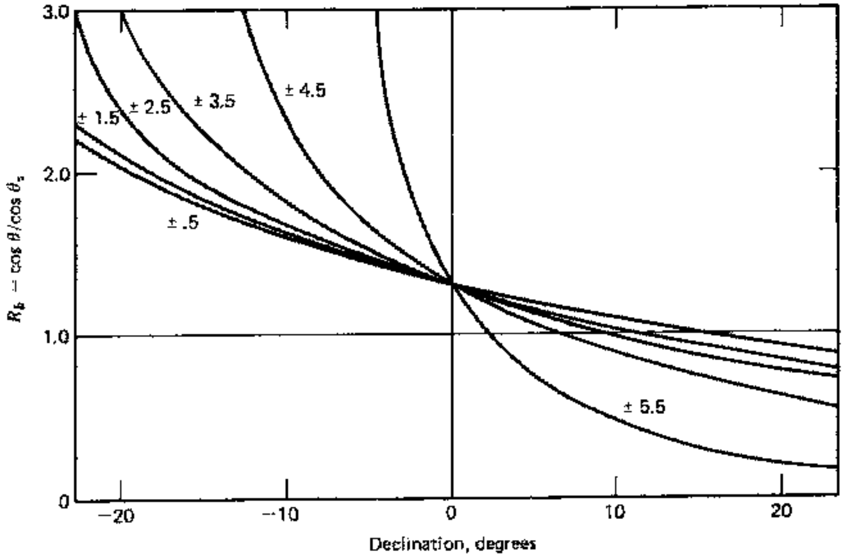


Figure 1.8.3 Ratio R_b for a surface with slope 50° to south at latitude 40° for various hours from solar noon.

such a plot for a latitude of 40° and a slope of 50°. It illustrates that very large gains in incident beam radiation are to be had by tilting a receiving surface toward the equator.

Equation 1.8.1 can also be applied to other than fixed flat-plate collectors. Equations 1.7.1 to 1.7.6 give $\cos\,\theta$ for surfaces moved in prescribed ways in which concentrating collectors may move to track the sun. If the beam radiation on a horizontal surface is known or can be estimated, the appropriate one of these equations can be used in the numerator of Equation 1.8.1 for $\cos\,\theta$. For example, for a plane rotated continuously about a horizontal east-west axis to maximize the beam radiation on the plane, from Equation 1.7.2a, the ratio of beam radiation on the plane to that on a horizontal surface at any time is

$$R_b = \frac{(1 - \cos^2\delta\,\sin^2\omega)^{1/2}}{\cos\phi\,\cos\delta\,\cos\omega + \sin\phi\,\sin\delta} \tag{1.8.5}$$

Some of the solar radiation data available are beam radiation on surfaces normal to the radiation, as measured by a pyrheliometer.[8] In this case the useful ratio is beam radiation on the surface in question to beam radiation on the normal surface; simply $R_b' = \cos\,\theta$, where θ is obtained from Equations 1.7.1 to 1.7.6.

1.9 SHADING

Three types of shading problems occur so frequently that methods are needed to cope with them. The first is shading of a collector, window, or other receiver by nearby trees,

[8] Pyrheliometers and other instruments for measuring solar radiation are described in Chapter 2.

buildings, or other obstructions. The geometries may be irregular, and systematic calculations of shading of the receiver in question may be difficult. Recourse is made to diagrams of the position of the sun in the sky, for example, plots of solar altitude α_s versus solar azimuth γ_s, on which shapes of obstructions (shading profiles) can be superimposed to determine when the path from the sun to the point in question is blocked. The second type includes shading of collectors in other than the first row of multirow arrays by the collectors on the adjoining row. The third includes shading of windows by overhangs and wingwalls. Where the geometries are regular, shading is amenable to calculation, and the results can be presented in general form. This will be treated in Chapter 14.

At any point in time and at a particular latitude, ϕ, δ, and ω are fixed. From the equations in Section 1.6, the zenith angle θ_z or solar altitude angle α_s and the solar azimuth angle γ_s can be calculated. A solar position plot of θ_z and α_s versus γ_s for latitudes of $\pm 45°$ is shown in Figure 1.9.1. Lines of constant declination are labeled by dates of mean days of the months from Table 1.6.1. Lines of constant hour angles labeled by hours are also shown. Plots for latitudes from 0 to $\pm 70°$ are included in Appendix H.

The angular position of buildings, wingwalls, overhangs, or other obstructions can be entered on the same plot. For example, as observed by Mazria (1979) and Anderson (1982), if a building or other obstruction of known dimensions and orientation is located a known distance from the point of interest (i.e., the receiver, collector, or window), the angular coordinates corresponding to altitude and azimuth angles of points on the obstruction (the object azimuth angle γ_o and object altitude angle α_o) can be calculated from trigonometric considerations. This is illustrated in Examples 1.9.1 and 1.9.2. Alternatively, measurements of object altitude and azimuth angles may be made at the site

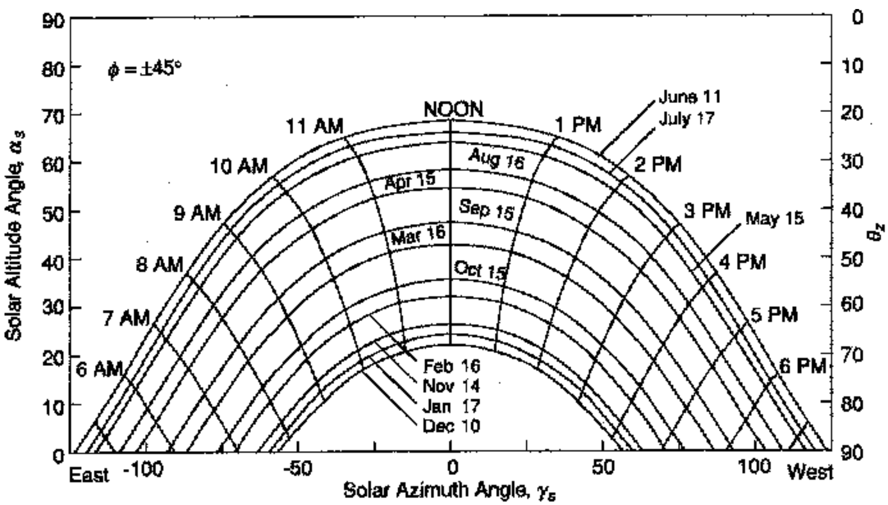


Figure 1.9.1 Solar position plot for $\pm 45°$ latitude. Solar altitude angle and solar azimuth angle are functions of declination and hour angle, indicated on the plots by dates and times. The dates shown are for northern hemisphere; for southern hemisphere use the corresponding dates as indicated in Figure 1.8.2. See Appendix H for other latitudes.

of a proposed receiver and the angles plotted on the solar position plot. Instruments are available to measure the angles.

Example 1.9.1

A proposed collector site at S is 10.0 m to the north of a long wall that shades it when the sun is low in the sky. The wall is of uniform height of 2.5 m above the center of the proposed collector area. Show this wall on a solar position chart with **a** the wall oriented east-west and **b** the wall oriented on a southeast-to-northwest axis displaced 20° from east-west.

Solution

In each case, we pick several points on the top of the wall to establish the coordinates for plotting on the solar position plot.

a Take three points indicated by A, B, and C in the diagram with A to the south and B 10 m and C 30 m west of A. Points B' and C' are taken to the east of A with the same object altitude angles as B and C and with object azimuth angles changed only in sign.

For point A, the object azimuth γ_{oA} is 0°. The object altitude angle is

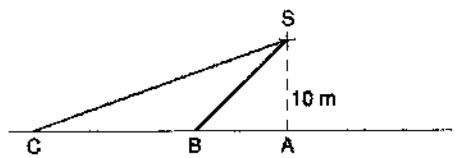
$$\tan \alpha_{oA} = \frac{2.5}{10}, \qquad \alpha_{oA} = 14.0°$$

For point B, $SB = (10^2 + 10^2)^{1/2} = 14.1$ m,

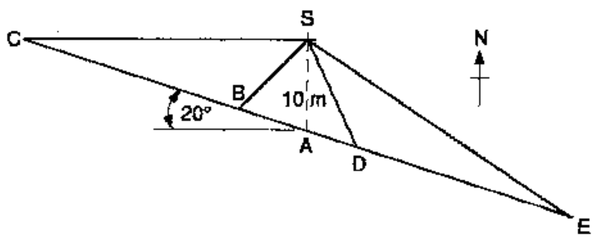
$$\tan \alpha_{oB} = \frac{2.5}{14.1}, \qquad \alpha_{oB} = 10.0°$$

$$\tan \gamma_{oB} = \frac{10}{10}, \qquad \gamma_{oB} = 45.0°$$

a.



b.



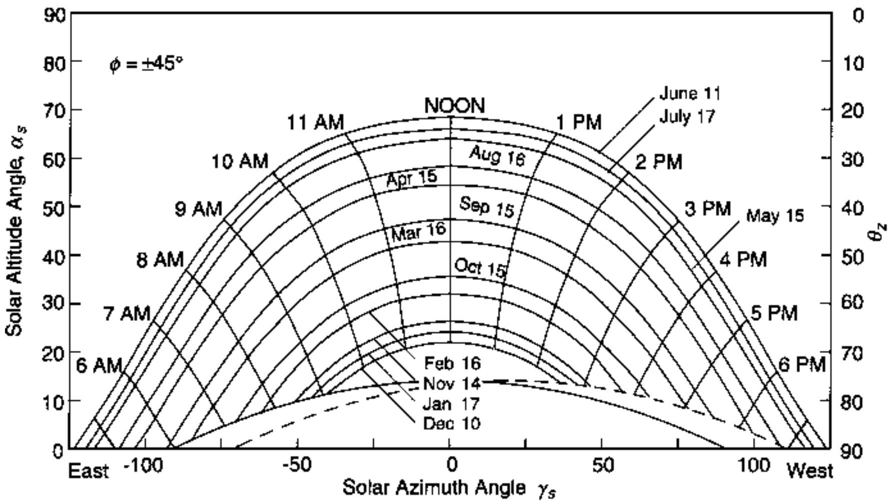
For point C, $SC = (10^2 + 30^2)^{1/2} = 31.6$ m,

$$\tan \alpha_{oC} = \frac{2.5}{31.6}, \qquad \alpha_{oC} = 4.52°$$

$$\tan \gamma_{oC} = \frac{30}{10}, \qquad \gamma_{oC} = 71.6°$$

There are points corresponding to B and C but to the east of A; these will have the same object azimuth angles except with negative signs. The shading profile determined by these coordinates is independent of latitude. It is shown by the solid line on the plot for $\phi = 45°$. Note that at object azimuth angles of 90°, the object distance becomes infinity and the object altitude angle becomes 0°.

The sun is obscured by the wall only during times shown in the diagram. The wall does not cast a shadow on point S at any time of day from late March to mid-September. For December 10, it casts a shadow on point S before 9:00 AM and after 3:00 PM.



b The obstruction of the sky does not show east-west symmetry in this case, so five points have been chosen as shown to cover the desirable range. Point A is the same as before, that is, $\alpha_{oA} = 14.0°$, $\gamma_{oA} = 0°$.

Arbitrarily select points on the wall for the calculation. In this case the calculations are easier if we select values of the object azimuth angle and calculate from them the corresponding distances from the point to the site and the corresponding α_o. In this case we can select values of γ_o for points B, C, D, and E of 45°, 90°, −30°, and −60°.

For point B, with $\gamma_{oB} = 45°$, the distance SB can be calculated from the law of sines:

$$\frac{\sin 70}{SB} = \frac{\sin(180 - 45 - 70)}{10}, \qquad SB = 10.4 \text{ m}$$

$$\tan \alpha_{oB} = \frac{2.5}{10.4}, \qquad \alpha_{oB} = 13.5°$$

For point D, with $\gamma_{oD} = -30°$, the calculation is

$$\frac{\sin 110}{SD} = \frac{\sin(180 - 110 - 30)}{10}, \qquad SD = 14.6 \text{ m}$$

$$\tan \alpha_{oD} = \frac{2.5}{14.6}, \qquad \alpha_{oD} = 9.7°$$

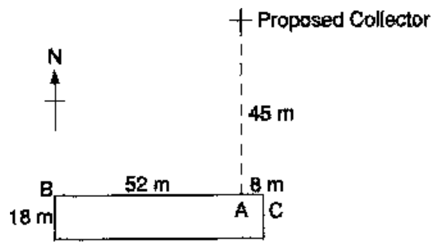
The calculations for points C and E give $\alpha_{oC} = 5.2°$ at $\gamma_{oC} = 90°$ and $\alpha_{oE} = 2.6°$ at $\gamma_{oE} = -60.0°$.

The shading profile determined by these coordinates is plotted on the solar position chart for $\phi = 45°$ and is shown as the dashed line. In this case, the object altitude angle goes to zero at azimuth angles of $-70°$ and $110°$. In either case, the area under the curves represents the wall, and the times when the wall would obstruct the beam radiation are those times (declination and hour angles) in the areas under the curves. ■

There may be some freedom in selecting points to be used in plotting object coordinates, and the calculation may be made easier (as in the preceding example) by selecting the most appropriate points. Applications of trigonometry will always provide the necessary information. For obstructions such as buildings, the points selected must include corners or limits that define the extent of obstruction. It may or may not be necessary to select intermediate points to fully define shading. This is illustrated in the following example.

Example 1.9.2

It is proposed to install a solar collector at a level 4.0 m above the ground. A rectangular building 30 m high is located 45 m to the south, has its long dimension on an east-west axis, and has dimensions shown in the diagram. The latitude is 45°. Diagram this building on the solar position plot to show the times of day and year when it would shade the proposed collector.



Solution

Three points that will be critical to determination of the shape of the image are the top near corners and the top of the building directly to the south of the proposed collector. Consider first point A. The object altitude angle of this point is determined by the fact that it is 45 m away and $30 - 4 = 26$ m higher than the proposed collector:

$$\tan \alpha_{oA} = \frac{26}{45}, \qquad \alpha_{oA} = 30.0°$$

The object azimuth angle γ_{oA} is 0° as the point A is directly to the south.

For point B, the distance SB is $(45^2 + 52^2)^{1/2} = 68.8$ m. The height is again 26 m. Then

$$\tan \alpha_{oB} = \frac{26}{68.8}, \qquad \alpha_{oB} = 20.7°$$

The object azimuth angle γ_{oB} is

$$\tan \gamma_{oB} = \frac{52}{45}, \qquad \gamma_{oB} = 49.1°$$

The calculation method for point C is the same as for B. The distance $SC = (45^2 + 8^2)^{1/2} = 45.7$ m:

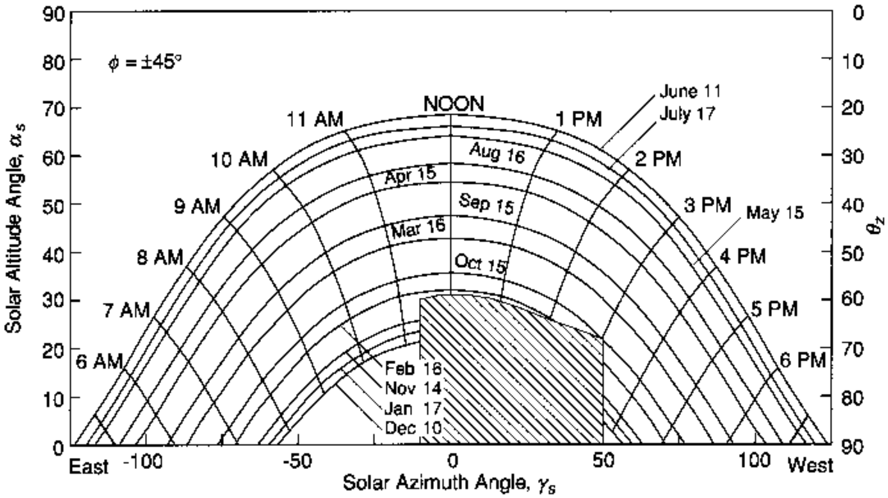
$$\tan \alpha_{oC} = \frac{26}{45.7}, \qquad \alpha_{oC} = 29.6°$$

$$\tan \gamma_{oC} = \frac{8}{45}, \qquad \gamma_{oC} = 10.1°$$

Note again that since point C lies to the east of south, γ_{oC} is by convention negative.

The shading profile of the building can be approximated by joining A and C and A and B by straight lines. A more precise representation is obtained by calculating intermediate points on the shading profile to establish the curve. In this example, an object altitude angle of 27.7° is calculated for an object azimuth angle of 25°.

These coordinates are plotted and the outlines of the building are shown in the figure. The shaded area represents the existing building as seen from the proposed collector site. The dates and times when the collector would be shaded from direct sun by the building are evident.



■

Implicit in the preceding discussion is the idea that the solar position at a point in time can be represented for a point location. Collectors and receivers have finite size, and what one point on a large receiving surface "sees" may not be the same as what another point sees. The problem is often to determine the amount of beam radiation on a receiver. If shading obstructions are far from the receiver relative to its size, so that shadows tend to move over the receiver rapidly and the receiver is either shaded or not shaded, the receiver can be thought of as a point. If a receiver is partially shaded, it can be considered to consist of a number of smaller areas, each of which is shaded or not shaded. Or integration over the receiver area may be performed to determine shading effects. These integrations have been done for special cases of overhangs and wingwalls.

Overhangs and wingwalls are architectural features that are applied to buildings to shade windows from beam radiation. The solar position charts can be used to determine when points on the receiver are shaded. The procedure is identical to that of Example 1.9.1; the obstruction in the case of an overhang and the times when the point is shaded from beam radiation are the times corresponding to areas above the line. This procedure can be used for overhangs of either finite or infinite length. The same concepts can be applied to wingwalls; the vertical edges of the object in Example 1.9.2 correspond to edges of wingwalls of finite height.

An overhang is shown in cross section in Figure 1.9.2(a) for the most common situation of a vertical window. The projection P is the horizontal distance from the plane of the window to the outer edge of the overhang. The gap G is the vertical distance from the top of the window to the horizontal plane that includes the outer edge of the overhang. The height H is the vertical dimension of the window.

The concept of shading planes was introduced by Jones (1980) as a useful way of considering shading by overhangs where end effects are negligible. Two shading planes are labeled in Figure 1.9.2(b). The angle of incidence of beam radiation on a shading plane can be calculated from its surface azimuth angle γ and its slope $\beta = 90 + \psi$ by Equation 1.6.2 or equivalent. The angle ψ of shading plane 1 is $\tan^{-1}[P/(G + H)]$ and that for shading plane 2 is $\tan^{-1}(P/G)$. Note that if the profile angle α_p is less than $90 - \psi$, the outer surface of the shading plane will "see" the sun and beam radiation will reach the receiver.[9]

Shading calculations are needed when flat-plate collectors are arranged in rows.[10] Normally, the first row is unobstructed, but the second row may be partially shaded by

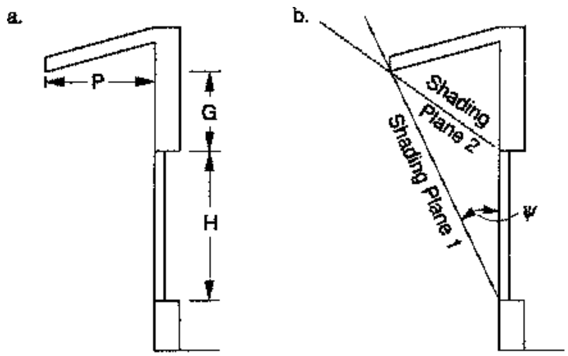


Figure 1.9.2 (a) Cross section of a long overhang showing projection, gap, and height. (b) Section showing shading planes.

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[9] Use of the shading plane concept will be discussed in Chapters 2 and 14.

[10] See Figure 12.1.2(c) for an example.

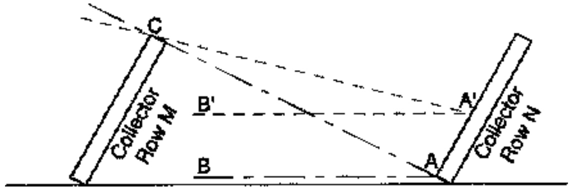


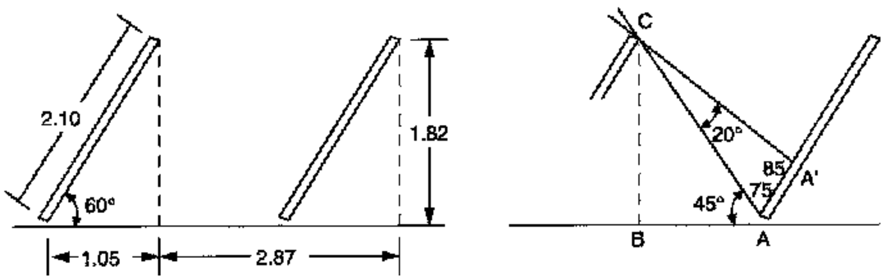
Figure 1.9.3 Section of two rows of a multirow collector array.

the first, the third by the second, and so on. This arrangement of collectors is shown in cross section in Figure 1.9.3.

For the case where the collectors are long in extent so the end effects are negligible, the profile angle provides a useful means of determining shading. As long as the profile angle is greater than the angle *CAB*, no point on row *N* will be shaded by row *M*. If the profile angle at a point in time is $CA'B'$ and is less than *CAB*, the portion of row *N* below point A' will be shaded from beam radiation.

Example 1.9.3

A multiple-row array of collectors is arranged as shown in the figure. The collectors are 2.10 m from top to bottom and are sloped at 60° toward the south. At a time when the profile angle (given by Equation 1.6.12) is 25°, estimate the fraction of the area of the collector in row *N* that will be shaded by the collectors in row *M*. Assume that the rows are long so end effects are not significant.



Solution

Referring to the figure, the angle *BAC* is $\tan^{-1}(2.87 - 1.05)/1.82 = 45°$, and since α_p is 25°, shading will occur.

The dimension AA' can be calculated:

$$AC = \frac{1.82}{\sin 45} = 2.57 \text{ m}$$

$$\angle CAA' = 180 - 45 - 60 = 75°, \qquad \angle CA'A = 180 - 75 - 20 = 85°$$

From the law of sines,

$$AA' = \frac{2.57 \sin 20}{\sin 85} = 0.88 \text{ m}$$

The fraction of collector N that is shaded is $0.88/2.10 = 0.42$. ■

1.10 EXTRATERRESTRIAL RADIATION ON A HORIZONTAL SURFACE

Several types of radiation calculations are most conveniently done using normalized radiation levels, that is, the ratio of radiation level to the theoretically possible radiation that would be available if there were no atmosphere. For these calculations, which are discussed in Chapter 2, we need a method of calculating the extraterrestrial radiation.

At any point in time, the solar radiation incident on a horizontal plane outside of the atmosphere is the normal incident solar radiation as given by Equation 1.4.1 divided by R_b:

$$G_o = G_{sc}\left(1 + 0.033 \cos \frac{360n}{365}\right) \cos \theta_z \tag{1.10.1}$$

where G_{sc} is the solar constant and n is the day of the year. Combining Equation 1.6.5 for $\cos \theta_z$ with Equation 1.10.1 gives G_o for a horizontal surface at any time between sunrise and sunset:

$$G_o = G_{sc}\left(1 + 0.033 \cos \frac{360n}{365}\right)(\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta) \tag{1.10.2}$$

It is often necessary for calculation of daily solar radiation to have the integrated daily extraterrestrial radiation on a horizontal surface, H_o. This is obtained by integrating Equation 1.10.2 over the period from sunrise to sunset. If G_{sc} is in watts per square meter, H_o in joules per square meter is

$$H_o = \frac{24 \times 3600 G_{sc}}{\pi}\left(1 + 0.033 \cos \frac{360n}{365}\right) \times \left(\cos \phi \cos \delta \sin \omega_s + \frac{\pi \omega_s}{180} \sin \phi \sin \delta\right) \tag{1.10.3}$$

where ω_s is the sunset hour angle, in degrees, from Equation 1.6.10.

The monthly mean[11] daily extraterrestrial radiation $\overline{H}_o$ is a useful quantity. For latitudes in the range +60 to −60 it can be calculated with Equation 1.10.3 using n and δ for the mean day of the month[12] from Table 1.6.1. Mean radiation H_o is plotted as a function of latitude for the northern and southern hemispheres in Figure 1.10.1. The curves are for dates that give the mean radiation for the month and thus show $\overline{H}_o$. Values

[11] An overbar is used throughout the book to indicate a monthly average quantity.

[12] The mean day is the day having H_o closest to $\overline{H}_o$.

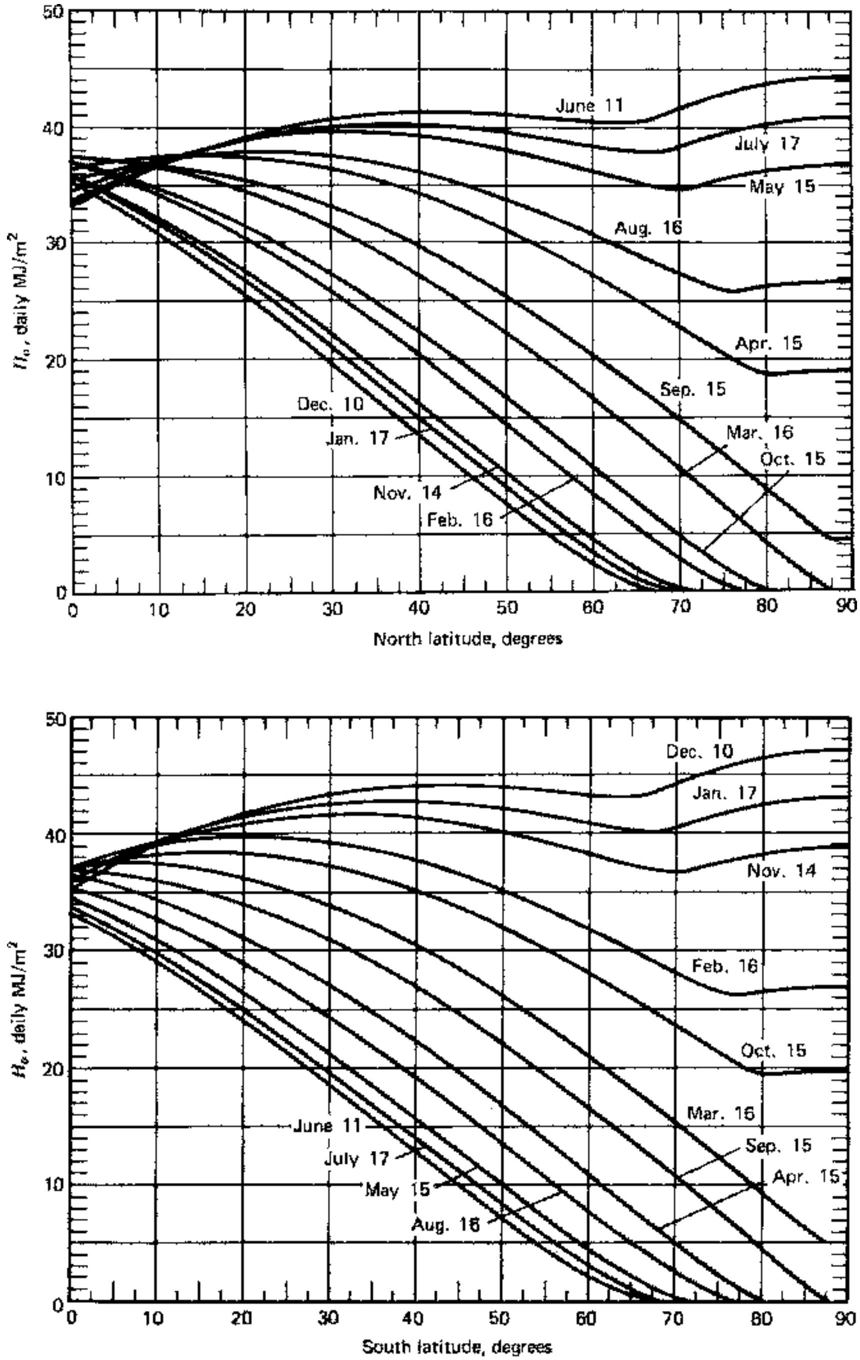


Figure 1.10.1 Extraterrestrial daily radiation on a horizontal surface. The curves are for the mean days of the month from Table 1.6.1.

of H_o for any day can be estimated by interpolation. Exact values of $\overline{H}_o$ for all latitudes are given in Table 1.10.1.

Example 1.10.1

What is H_o, the day's solar radiation on a horizontal surface in the absence of the atmosphere, at latitude 43° N on April 15?

Table 1.10.1 Monthly Average Daily Extraterrestrial Radiation, MJ/m^2

ϕ	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
90	0.0	0.0	1.2	19.3	37.2	44.8	41.2	26.5	5.4	0.0	0.0	0.0
85	0.0	0.0	2.2	19.2	37.0	44.7	41.0	26.4	6.4	0.0	0.0	0.0
80	0.0	0.0	4.7	19.6	36.6	44.2	40.5	26.1	9.0	0.6	0.0	0.0
75	0.0	0.7	7.8	21.0	35.9	43.3	39.8	26.3	11.9	2.2	0.0	0.0
70	0.1	2.7	10.9	23.1	35.3	42.1	38.7	27.5	14.8	4.9	0.3	0.0
65	1.2	5.4	13.9	25.4	35.7	41.0	38.3	29.2	17.7	7.8	2.0	0.4
60	3.5	8.3	16.9	27.6	36.6	41.0	38.8	30.9	20.5	10.8	4.5	2.3
55	6.2	11.3	19.8	29.6	37.6	41.3	39.4	32.6	23.1	13.8	7.3	4.8
50	9.1	14.4	22.5	31.5	38.5	41.5	40.0	34.1	25.5	16.7	10.3	7.7
45	12.2	17.4	25.1	33.2	39.2	41.7	40.4	35.3	27.8	19.6	13.3	10.7
40	15.3	20.3	27.4	34.6	39.7	41.7	40.6	36.4	29.8	22.4	16.4	13.7
35	18.3	23.1	29.6	35.8	40.0	41.5	40.6	37.3	31.7	25.0	19.3	16.8
30	21.3	25.7	31.5	36.8	40.0	41.1	40.4	37.8	33.2	27.4	22.2	19.9
25	24.2	28.2	33.2	37.5	39.8	40.4	40.0	38.2	34.6	29.6	25.0	22.9
20	27.0	30.5	34.7	37.9	39.3	39.5	39.3	38.2	35.6	31.6	27.7	25.8
15	29.6	32.6	35.9	38.0	38.5	38.4	38.3	38.0	36.4	33.4	30.1	28.5
10	32.0	34.4	36.8	37.9	37.5	37.0	37.1	37.5	37.0	35.0	32.4	31.1
5	34.2	36.0	37.5	37.4	36.3	35.3	35.6	36.7	37.2	36.3	34.5	33.5
0	36.2	37.4	37.8	36.7	34.8	33.5	34.0	35.7	37.2	37.3	36.3	35.7
−5	38.0	38.5	37.9	35.8	33.0	31.4	32.1	34.4	36.9	38.0	37.9	37.6
−10	39.5	39.3	37.7	34.5	31.1	29.2	29.9	32.9	36.3	38.5	39.3	39.4
−15	40.8	39.8	37.2	33.0	28.9	26.8	27.6	31.1	35.4	38.7	40.4	40.9
−20	41.8	40.0	36.4	31.3	26.6	24.2	25.2	29.1	34.3	38.6	41.2	42.1
−25	42.5	40.0	35.4	29.3	24.1	21.5	22.6	27.0	32.9	38.2	41.7	43.1
−30	43.0	39.7	34.0	27.2	21.4	18.7	19.9	24.6	31.2	37.6	42.0	43.8
−35	43.2	39.1	32.5	24.8	18.6	15.8	17.0	22.1	29.3	36.6	42.0	44.2
−40	43.1	38.2	30.6	22.3	15.8	12.9	14.2	19.4	27.2	35.5	41.7	44.5
−45	42.8	37.1	28.6	19.6	12.9	10.0	11.3	16.6	24.9	34.0	41.2	44.5
−50	42.3	35.7	26.3	16.8	10.0	7.2	8.4	13.8	22.4	32.4	40.5	44.3
−55	41.7	34.1	23.9	13.9	7.2	4.5	5.7	10.9	19.8	30.5	39.6	44.0
−60	41.0	32.4	21.2	10.9	4.5	2.2	3.1	8.0	17.0	28.4	38.7	43.7
−65	40.5	30.6	18.5	7.9	2.1	0.3	1.0	5.2	14.1	26.2	37.8	43.7
−70	40.8	28.8	15.6	5.0	0.4	0.0	0.0	2.6	11.1	24.0	37.4	44.9
−75	41.9	27.6	12.6	2.4	0.0	0.0	0.0	0.8	8.0	21.9	38.1	46.2
−80	42.7	27.4	9.7	0.6	0.0	0.0	0.0	0.0	5.0	20.6	38.8	47.1
−85	43.2	27.7	7.2	0.0	0.0	0.0	0.0	0.0	2.4	20.3	39.3	47.6
−90	43.3	27.8	6.2	0.0	0.0	0.0	0.0	0.0	1.4	20.4	39.4	47.8

Solution

For these circumstances, $n = 105$ (from Table 1.6.1), $\delta = 9.4°$ (from Equation 1.6.1), and $\phi = 43°$. From Equation 1.6.10

$$\cos \omega_s = -\tan 43 \tan 9.4 \text{ and } \omega_s = 98.9°$$

Then from Equation 1.10.3, with $G_{sc} = 1367 \text{ W/m}^2$,

$$\begin{aligned} H_o &= \frac{24 \times 3600 \times 1367}{\pi} \left(1 + 0.033 \cos \frac{360 \times 105}{365}\right) \\ &\quad \times \left(\cos 43 \cos 9.4 \sin 98.9 + \frac{\pi \times 98.9}{180} \sin 43 \sin 9.4\right) \\ &= 33.8 \text{ MJ/m}^2 \end{aligned}$$

From Figure 1.10.1(a), for the curve for April, we read $H_o = 34.0 \text{ MJ/m}^2$, and from Table 1.10.1 we obtain $H_o = 33.8 \text{ MJ/m}^2$ by interpolation. ■

It is also of interest to calculate the extraterrestrial radiation on a horizontal surface for an hour period. Integrating Equation 1.10.2 for a period between hour angles ω_1 and ω_2 which define an hour (where ω_2 is the larger),

$$\begin{aligned} I_o &= \frac{12 \times 3600}{\pi} G_{sc} \left(1 + 0.033 \cos \frac{360n}{365}\right) \\ &\quad \times \left[\cos \phi \cos \delta (\sin \omega_2 - \sin \omega_1) + \frac{\pi(\omega_2 - \omega_1)}{180} \sin \phi \sin \delta\right] \end{aligned} \tag{1.10.4}$$

(The limits ω_1 and ω_2 may define a time other than an hour.)

Example 1.10.2

What is the solar radiation on a horizontal surface in the absence of the atmosphere at latitude 43° N on April 15 between the hours of 10 and 11?

Solution

The declination is 9.4° (from the previous example). For April 15, $n = 105$. Using Equation 1.10.4 with $\omega_1 = -30°$ and $\omega_2 = -15°$,

$$\begin{aligned} I_o &= \frac{12 \times 3600 \times 1367}{\pi} \left(1 + 0.033 \cos \frac{360 \times 105}{365}\right) \\ &\quad \times \left(\cos 43 \cos 9.4[\sin(-15) - \sin(-30)] + \frac{\pi[-15 - (-30)]}{180} \sin 43 \sin 9.4\right) \\ &= 3.79 \text{ MJ/m}^2 \end{aligned}$$

■

The hourly extraterrestrial radiation can also be approximated by writing Equation 1.10.2 in terms of I, evaluating ω at the midpoint of the hour. For the circumstances of Example 1.10.2, the hour's radiation so estimated is 3.80 MJ/m^2. Differences between the hourly radiation calculated by these two methods will be slightly larger at times near sunrise and sunset but are still small. For larger time spans, the differences become larger. For example, for the same circumstances as in Example 1.10.2 but for the 2-h span from 7:00 to 9:00, the use of Equation 1.10.4 gives 4.58 MJ/m^2, and Equation 1.10.2 for 8:00 gives 4.61 MJ/m^2.

1.11 SUMMARY

In this chapter we have outlined the basic characteristics of the sun and the radiation it emits, noting that the solar constant, the mean radiation flux density outside of the earth's atmosphere, is 1367 W/m^2 (within $\pm 1\%$), with most of the radiation in a wavelength range of 0.3 to 3 μm. This radiation has directional characteristics that are defined by a set of angles that determine the angle of incidence of the radiation on a surface. We have included in this chapter those topics that are based on extraterrestrial radiation and the geometry of the earth and sun. This is background information for Chapter 2, which is concerned with effects of the atmosphere, radiation measurements, and data manipulation.

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