Chapter 1
Introduction and Review of Statics

1-1 INTRODUCTION

The primary objective of a course in mechanics of materials is the development of relationships between the loads applied to a nonrigid body and the internal forces and deformations induced in the body. Ever since the time of Galileo Galilei (1564–1642), scientists and engineers have studied the problem of the load-carrying capacity of structural members and machine components, and have developed mathematical and experimental methods of analysis for determining the internal forces and the deformations induced by the applied loads. The experiences and observations of these scientists and engineers of the last three centuries are the heritage of the engineer of today. The fundamental knowledge gained over the last three centuries, together with the theories and analysis techniques developed, permit the modern engineer to design, with complete competence and assurance, structures and machines of unprecedented size and complexity.

The subject matter of this book forms the basis for the solution of three general types of problems:

1. Given a certain function to perform (transporting traffic over a river by means of a bridge, conveying scientific instruments to Mars in a space vehicle, converting water power into electric power), of what materials should the machine or structure be constructed, and what should be the sizes and proportions of the various elements? This is the designer's task, and obviously there is no single solution to any given problem.
2. Given the completed design, is it adequate? That is, does it perform the function economically and without excessive deformation? This is the checker's problem.
3. Given a completed structure or machine, what is its actual load-carrying capacity? The structure may have been designed for some purpose other than the one for which it is now to be used. Is it adequate for the proposed use? For example, a building may have been designed as an office building but is later found to be desirable for use as a warehouse. In such a case, what maximum loading may the floor safely support? This is the rating problem.

Because the complete scope of these problems is obviously too comprehensive for mastery in a single course, this book is restricted to a study of individual members and very simple structures or machines. The design courses that follow will consider the entire structure or machine, and will provide essential background for the complete analysis of the three problems.

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The principles and methods used to meet the objective stated at the beginning of this chapter depend to a great extent on prerequisite courses in mathematics and mechanics, supplemented by additional concepts from the theory of elasticity and the properties of engineering materials. The equations of equilibrium from statics are used extensively, with one major change in the free-body diagrams; namely, most free bodies are isolated by cutting through a member instead of simply removing a pin or some other connection. The internal force on the cut section is related to the stresses (force per unit area) generated by the cohesive forces holding the member together. The size and shape of the member must be adjusted to keep the stress below the limiting value for the type of material from which the member is constructed.

In some instances, the specified maximum deformation, not the specified maximum stress, will govern the maximum load that a member may carry. In other instances, it may be found that the equations of equilibrium (or motion) are not sufficient to determine all of the unknown loads or reactions acting on a body. In such cases it is necessary to consider the geometry (the change in size or shape) of the body after the loads are applied. The deformation per unit length in any direction or dimension is called *strain*.

Some knowledge of the physical and mechanical properties of materials is required in order to create a design, to properly evaluate a given design, or even to write the correct relation between an applied load and the resulting deformation of a loaded member. Essential information will be introduced as required, and more complete information can be obtained from textbooks and handbooks on properties of materials.

1-2 CLASSIFICATION OF FORCES

Force is one of the most important of the basic concepts in the study of mechanics of materials (or the mechanics of deformable bodies). *Force* is the action of one body on another; forces always exist in equal magnitude, opposite direction pairs. Forces may result from direct physical contact between two bodies or from two bodies that are not in direct contact. For example, consider a person standing on a sidewalk. The person exerts a force on the sidewalk through direct physical contact between the soles of his or her shoes and the sidewalk; the sidewalk in turn exerts an equal magnitude, opposite direction force on the soles of the person's shoes. If the person were to jump, the contact force would vanish but there would still be a gravitational attraction (force between two bodies not in direct contact) between the person and the earth. The gravitational attraction force exerted on the person by the earth is called the *weight* of the person; an equal magnitude, opposite direction, attraction force is exerted on the earth by the person. Another type of force that exists without direct physical contact is an electromagnetic force.

Contact forces are called *surface* forces, since they exist at surfaces of contact between two bodies. If the area of contact is small compared to the size of the body, the force is called a *concentrated* force; this type of force is assumed to act at a point. For example, the force applied by a car wheel to the pavement on a bridge (see Fig. 1-1) is often modeled as a concentrated force. Also, a contact force may be distributed over a narrow region in a uniform or nonuniform manner. This situation would exist where floor decking contacts a floor joist, as shown in Fig. 1-2*a*. Here, the floor decking exerts a uniformly distributed load (force) on the joist, as shown in Fig. 1-2*b*. The intensity of the distributed load is w and has dimensions of force per unit length.

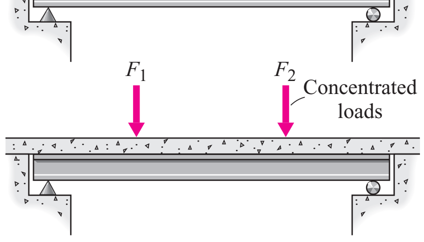


Figure 1-1

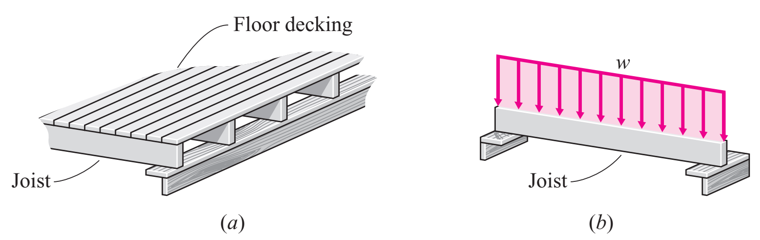


Figure 1-2

Other common types of forces are external, internal, applied, and reaction. To illustrate, consider the beam loaded and supported, as shown in Fig. 1-3*a*. A free-body diagram of the beam is shown in Fig. 1-3*b*. All forces acting on the free-body diagram are *external* forces; that is, they represent the interaction between the beam (the object shown in the free-body diagram) and the external world (everything else that has been discarded). Force F is a concentrated force, whereas w is a uniformly distributed load with dimensions of force/length. The forces F and w are called *applied* forces or loads. They are the forces that the beam is designed to carry. Forces A_x, A_y, and B are necessary to prevent movement of the beam. Such supporting forces are called *reactions*. Force distributions at supports are complicated, and reactions are usually modeled as concentrated forces.

Once again, all the forces shown in Figure 1-3 are external forces. At every section along the beam, there also exists a system of equal magnitude, opposite direction pairs of internal forces between the atoms on either side of the section. The study of mechanics of materials or mechanics of deformable bodies depends on the calculation of these internal forces at various sections of a structure or machine element and how these forces are distributed over the sections. The determination of internal forces is discussed in Section 1-5.

In our previous discussion of loads (forces), we saw that the loads might be concentrated forces or distributed forces. Furthermore, we assumed that the forces did not vary with time, that is, they were *static* loads. In many situations, loads may be a function of time. For example, a sustained load is a load that is constant over a long period of time, such as the weight of a structure (called *dead* load). This type of load is treated in the same manner as a static load; however, for some materials and conditions of temperature and stress, the resistance to failure may

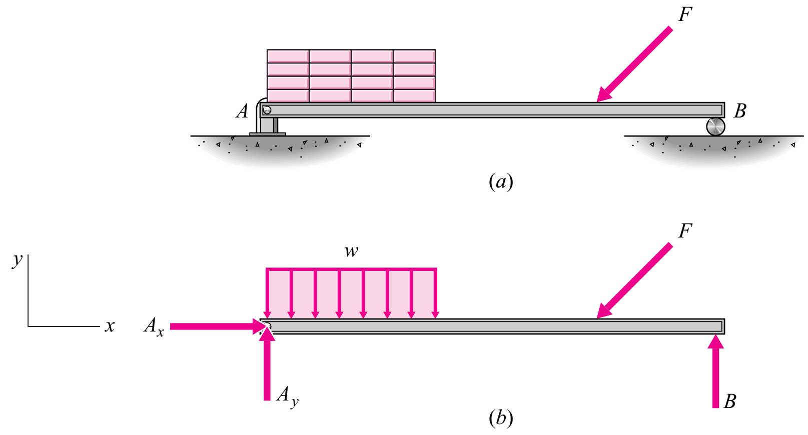


Figure 1-3

be different under short-time loading and sustained loading. An impact load is a rapidly applied load which transfers a large amount of energy in a short period of time. Vibration normally results from an impact load, and equilibrium is not established until the vibration is eliminated, usually by natural damping forces. A repeated load is a load that is applied and removed many thousands of times. The helical springs that close the valves on an automobile engine are subjected to repeated loading.

1-3 EQUILIBRIUM OF A RIGID BODY

A *rigid* body (a body that does not deform under the action of applied loads) is in equilibrium when the resultant of the system of forces acting on the body is zero. This condition is satisfied if

$$\Sigma \mathbf{F} = \mathbf{0} \tag{1-1}$$

$$\Sigma \mathbf{M}_O = \mathbf{0} \tag{1-2}$$

Equation 1-1 states that the vector sum of all external forces acting on the body is zero, whereas Eq. 1-2 states that the vector sum of the moments of the external forces about any point O (on or off the body) is zero. Equations 1-1 and 1-2 are the necessary and the sufficient conditions for equilibrium of a rigid body. The two vector equations of equilibrium may be written as six scalar equations. Selecting a right-handed, xyz-rectangular coordinate system, the equations of equilibrium may be written

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{1-3}$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \tag{1-4}$$

Equation 1-3 states that the sum of all external forces acting on the body in the x-, y-, and z-directions is zero. Equation 1-4 states that the sum of the moments of all of the external forces acting on a body about the x-, y-, and z-axes is zero. Many problems encountered in mechanics of materials are two-dimensional in nature. Selecting the x- and y-axes in the plane of the forces and the z-axis perpendicular to the plane, the equations of equilibrium reduce to

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_z = 0 \tag{1-5}$$

These equations of equilibrium would be applicable for the force system shown in Fig. 1-3*b*, which is coplanar and noncurrent. If the force system acting on the body is coplanar and concurrent, as in the light suspended from the ceiling by two wires shown in Fig. 1-4, Eqs. 1-3 and 1-4 reduce to

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \tag{1-6}$$

Other types of force systems exist for problems encountered in the study of mechanics of materials. Students should not try to memorize the equations of equilibrium that apply for each of the various force systems. Rather, Eqs. 1-3 and 1-4 should be reduced to equations appropriate for the particular problem at hand. This will be illustrated in the example problems presented at the end of this section.

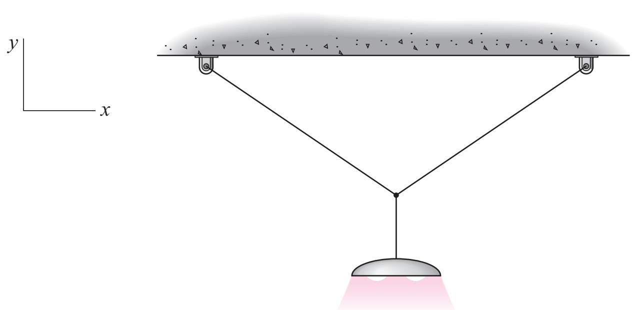


Figure 1-4

Note from previous discussions that the equations of equilibrium are applied to a system of forces. The system of forces may act on a single body or on a system of connected bodies. A free-body diagram is a carefully prepared drawing that shows a "body of interest" separated from all other interacting bodies and that shows all external forces, both known and unknown, that are applied to the body. The word "free" in the name "free-body diagram" emphasizes the idea that all bodies exerting forces on the body of interest are removed or withdrawn and are replaced by the forces that they exert. At each position on the free-body diagram where other bodies have been removed, the equal magnitude, opposite direction pairs of forces have been broken, and the forces which act on the free-body diagram must be shown. These forces may be either surface forces or body forces, or both. An important body force is the gravitational attraction of the earth, that is, the weight of the body.

The following examples illustrate the use of free-body diagrams together with the equations of equilibrium to determine unknown forces acting on rigid bodies. *The importance of drawing a correct free-body diagram cannot be overemphasized*. The free-body diagram clearly establishes which body or portion of the body is being studied. A correct free-body diagram clearly identifies all forces (both known and unknown) that must be included in the equations of equilibrium. The methods commonly used to find the unknown forces which act on rigid bodies must be thoroughly mastered, since these methods, as well as an extension of these methods to deformable bodies, are used throughout this book.

Example Problem 1-1

The rigid structure shown in Fig. 1-5*a* is subjected to a 5000-lb force *P*. The connections at joints *A*, *B*, and *C* are frictionless pins. Determine the forces at *A* and *B* on member *AB*.

SOLUTION

We first draw a free-body diagram of member *AB*, as shown in Fig. 1-5*b*. Member *AB* is "freed" from interacting bodies: the bracket and pin at *A* and member *BC*. At the places where *AB* is separated from interacting bodies, we show external forces acting on *AB*. Since member *BC* is a straight two-force member, the force *T* must lie along the member. The force at pin *A* has an unknown magnitude and direction. We show this force as two components, A_x and A_y, where the directions have been assumed. The weight of member *AB* is assumed to be small compared to the applied force *P* and is not shown on the free-body diagram. The free-body diagram contains three unknowns, A_x, A_y, and T. These unknowns, together with

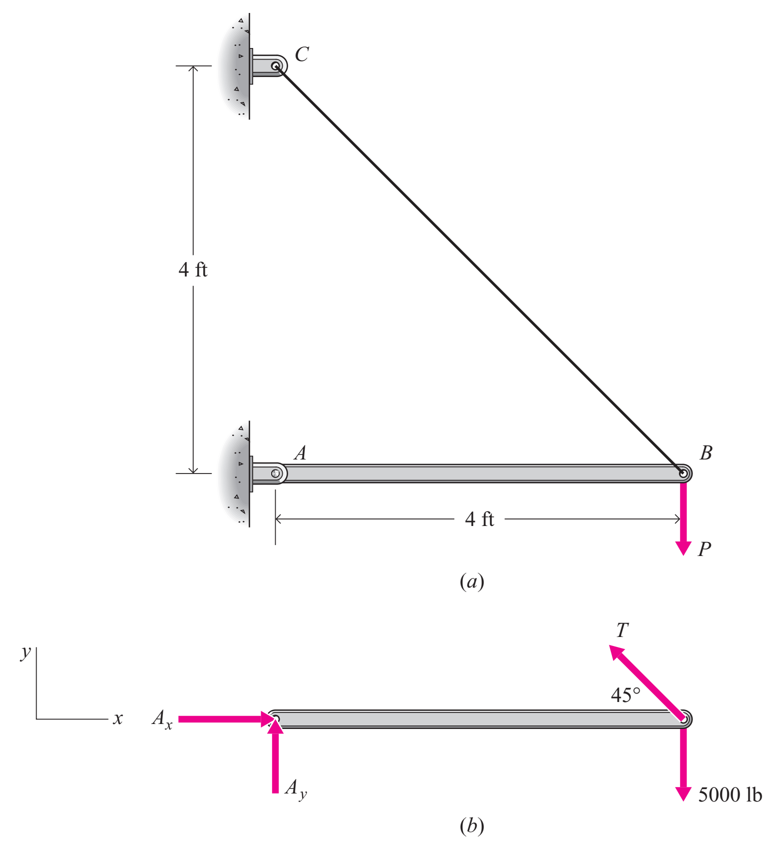


Figure 1-5

the 5000-lb applied load, constitute a coplanar, nonconcurrent force system. The equations of equilibrium 1-3 and 1-4 for this system of forces reduce to

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0 \tag{a}$$

where $\Sigma M_z = 0$ has been replaced with $\Sigma M_A = 0$ (point A is the intersection of the z-axis and the plane of the two-dimensional structure). Point A was selected for the moment equation; any other point on or off body AB could have been selected. There is no particular order in which we write the equations of equilibrium; mathematical convenience usually dictates the order. In this example, we use the order given in Eq. (a).

▶ Point A was chosen for the moment equation since two of the unknown forces (A_x and A_y) intersect at point A. Therefore, the moment equation relative to point A contains only one unknown (T) and can be solved immediately for the value of T. Two of the unknown forces also intersect at points B (forces A_x and T) and C (forces A_y and T), and these would also be good points to use for the moment equation.

$$+\rightarrow \Sigma F_x = 0: \qquad A_x - T\ \cos 45^\circ = 0 \tag{b}$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + T \sin 45^\circ - 5000 = 0 \tag{c}$$

$$+\ \Sigma M_A = 0: \qquad (T\ \sin 45^\circ)(4) - 5000(4) = 0 \tag{d}$$

Equation (d) is solved for T, which is then substituted into Eqs. (b) and (c) to find the components of the pin forces at A. The results are

$$A_x = 5000 \text{ lb} \qquad A_y = 0.04795 \text{ lb} \qquad T = 7071 \text{ lb} \tag{e}$$

Before proceeding further, we examine the results. Why were the forces written with the number of significant figures shown in Eq. (*e*)? For example, consider the solution of Eq. (*d*):

$$T = \frac{5000}{\sin 45} = 7071.067812 \text{ lb} \qquad (f)$$

Although results should always be reported as accurately as possible, the numbers to the right of the decimal point in Eq. (*f*) have meaning only if the original data (dimensions and applied load) are known to the same relative accuracy as the solution for the force T. One of the tasks in all engineering work is to determine the accuracy of the given data and the expected accuracy of the final answer. Results should always reflect the accuracy of the given data.

It is not possible, however, for students to examine or question the accuracy of the given data in a textbook. It is also impractical in an introductory course to give error bounds on every number. Therefore, since an accuracy greater than about 0.2 percent is seldom possible for practical engineering problems, all given data in Example Problems and Homework Problems, regardless of the number of figures shown, will be assumed sufficiently accurate to justify rounding off the final answer to approximately this degree of accuracy (three to four significant figures). One commonly used rounding scheme uses the leading digit to determine how many significant figures to keep in the final answer. If the first nonzero digit of the result is a 1, then the answer is reported with four significant figures; otherwise the answer is reported with three significant figures.

Of course, all intermediate steps in the solution must maintain more significant figures than are used to represent the final results so as to reduce the effect of roundoff errors on the final results. Using the value of T from Eq. (*f*) in Eq. (*c*) would yield $A_y = 0$ instead of the value shown in Eq. (*e*). The point of this discussion is: Don't report final results with more accuracy than is justified by the data and don't round off numbers too much too soon. For this example problem, then, the answer is

$$A_x = 5000 \text{ lb} \qquad A_y = 0 \text{ lb} \qquad T = 7070 \text{ lb} \qquad \textbf{Ans.}$$

Example Problem 1-2 A 900-kg mass is supported by a roller that can move along a beam, as shown in Fig. 1-6*a*. The beam is supported by a pin at A and a roller at B.

(a) Neglect the mass of the beam and determine the reactions at A and B.
(b) If the mass of the beam is 8.5 kg/m, determine the reactions at A and B.

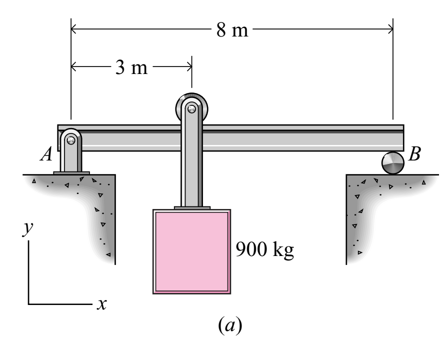


Figure 1-6(a)

SOLUTION

The beam can be modeled as a rigid member with frictionless pin and roller supports at A and B. A free-body diagram for the beam is constructed by "freeing" the beam from its supports at A and B and from the roller that supports the 900-kg mass, as shown in Fig. 1-6*b*.

(a) As in Example Problem 1-1, the components A_x and A_y of the pin reaction at A are shown. Rollers exert forces on the beam that are perpendicular to the

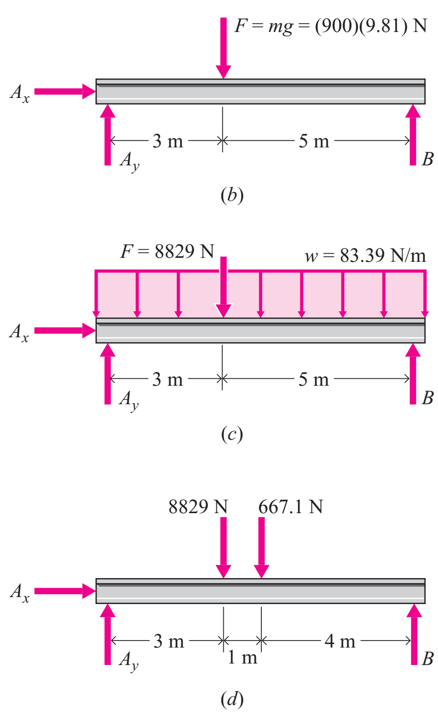


Figure 1-6(b–d)

beam. The masses of the bar and roller connecting the 900-kg mass to the beam are neglected. The force F in Fig. 1-6*b* is the weight of the 900-kg mass, $F = mg = 8829$ N. There are three unknown forces (A_x, A_y, and B) shown on the free-body diagram (Fig. 1-6*b*) for the beam. The three equations of equilibrium available to solve for the unknowns are

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0$$

$$+\rightarrow \Sigma F_x = 0: \qquad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + B - 8829 = 0$$

$$+ \quad \Sigma M_A = 0: \qquad B(8) - 8829(3) = 0$$

Solving for A_y and B gives

$$A_y = 5518 \text{ N} \qquad B = 3311 \text{ N}$$

Thus, the reactions at supports A and B are

$$A_x = 0 \text{ N} \qquad A_y = 5520 \text{ N} \qquad B = 3310 \text{ N}$$ **Ans.**

(b) For a beam mass of 8.5 kg/m, the uniformly distributed force on the beam resulting from its weight is $w = mg = (8.5)(9.81) = 83.39$ N/m. A free-body diagram for this beam is shown in Fig. 1-6*c*. In the equilibrium equations, the distributed force is statically equivalent to a single force whose magnitude is equal to the area under the load diagram (the area of a rectangle, 8 m × 83.39 N/m = 667.1 N) and which acts through the centroid of the load diagram (which is 4 m to the right of A). The free-body diagram of Fig. 1-6*d* and the equations of equilibrium give

$$+\rightarrow \Sigma F_x = 0: \qquad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + B - 8829 - 8(83.39) = 0$$

$$+ \quad \Sigma M_A = 0: \qquad B(8) - 8829(3) - 8(83.39)(4) = 0$$

Solving gives

$$A_y = 5852 \text{ N} \qquad B = 3644 \text{ N}$$

Thus, the reactions at A and B are

$$A_x = 0 \text{ N} \qquad A_y = 5850 \text{ N} \qquad B = 3640 \text{ N}$$ **Ans.**

▶ The maximum difference in the results from parts (a) and (b) is less than 10 percent. In many problems in engineering, we neglect the weight of members as being small when compared to the applied loads. As you gain experience in solving problems, you will be able to judge when you can safely neglect and when you must include the weights of members of a structure or machine. Of course, if you have any doubts, the safest approach is to include these weights.

Example Problem 1-3

The truss shown in Fig. 1-7*a* supports one side of a bridge; an identical truss supports the other side. Floor beams carry vehicle loads to the truss joints. A 3400-lb car is stopped on the bridge. Assume that the weight of the car is evenly distributed among the wheels and that the center of gravity of the car is 16 ft from support A. Calculate the support reactions and the forces in members BD, DE, and CE of the truss.

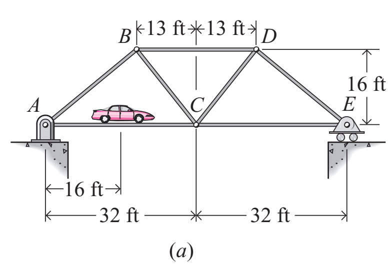


Figure 1-7(a)

SOLUTION

We model an actual truss by making four assumptions:

1. Truss members are connected only at their ends.
2. Truss members are connected by frictionless pins.
3. The truss is loaded only at the joints.
4. The weights of the members may be neglected.

These assumptions are idealizations of actual structures, but real trusses behave according to the idealizations to a high degree of approximation. As a result of the assumptions, each member of a truss is a two-force member. Since truss members are also usually straight, the force is along the member, and a member is subjected to either tension or compression.

According to assumption 3, we must proportion the weight of the car between the joints of the truss. Half of the car's weight, $3400/2 = 1700$ lb, is carried by the truss shown and the other half is carried by the truss on the other side of the bridge. Since the weight is evenly distributed to each wheel and the center of gravity of the car is midway between A and C, 850 lb will be applied to joint A and 850 lb will be applied to joint C. A free-body diagram of the entire truss is shown in Fig. 1-7b. The equations of equilibrium yield

$$+\rightarrow \Sigma F_x = 0: \qquad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + E - 850 - 850 = 0$$

$$+ \quad \Sigma M_A = 0: \qquad E(64) - 850(32) = 0$$

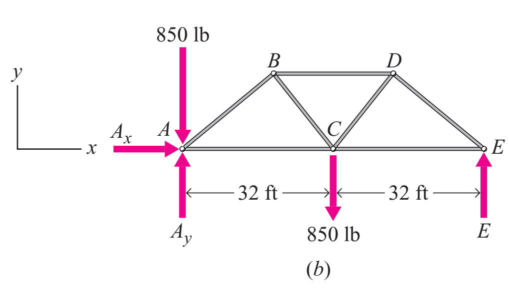


Figure 1-7(b)

Thus, the support reactions are

$$A_x = 0 \text{ lb} \qquad A_y = 1275 \text{ lb} \qquad E = 425 \text{ lb} \qquad \textbf{Ans.}$$

The forces in the various members of the truss can be found using either the method of joints or the method of sections. We choose the method of joints to calculate the forces in members *CE* and *DE*. A free-body diagram of pin *E* is shown in Fig. 1-7*c*. The force system is concurrent, and there are two equations of equilibrium

$$+\rightarrow \Sigma F_x = 0: \qquad -T_{CE} - T_{DE} \cos\theta = 0$$
$$+\uparrow \Sigma F_y = 0: \qquad E + T_{DE} \sin\theta = 425 + T_{DE} \sin\theta = 0$$

► By Newton's third law (action-reaction), a force that points away from (pulls on) a joint also points away from (pulls on) a member. That is, the corresponding member is in tension. If all member forces are shown in tension (pointing away from the joints) on free-body diagrams, then a positive sign for a force will indicate that the corresponding member is in tension (as assumed on the free-body diagram), while a negative sign for a force will indicate that the corresponding member is in compression.

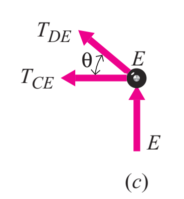


Figure 1-7(c)

Since $\theta = \tan^{-1}(16/19) = 40.10°$, the forces in members *CE* and *DE* are

$$T_{DE} = -659.8 \text{ lb} \qquad T_{CE} = 504.7 \text{ lb}$$

The minus sign for T_{DE} indicates that the direction of the force on Fig. 1-7*c* should be reversed. Figure 1-7*d* shows the correct directions for T_{CE} and T_{DE}, along with the directions of the forces in members *CE* and *DE*. Clearly, member *CE* is in tension (*T*) and member *DE* is in compression (*C*). The final results are

$$T_{CE} = 505 \text{ lb } (T) \qquad T_{DE} = 660 \text{ lb } (C) \qquad \textbf{Ans.}$$

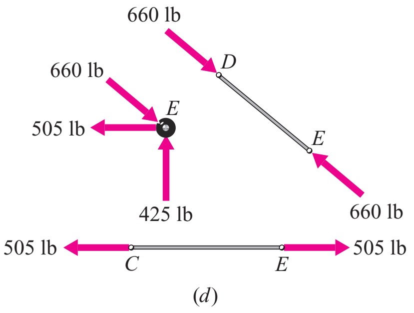


Figure 1-7(d)

The force in member *BD* could be found using the method of joints (joint *D*) since T_{DE} is now known. However, as a review of the method of sections, we select this method to calculate T_{BD}. We "section" through members *BD*, *BC*, and *AC* and draw the free-body diagram shown in Fig. 1-7*e*. The force system is

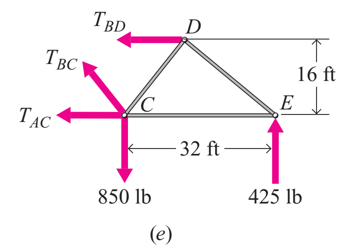


Figure 1-7(e)

coplanar and noncurrent, and there are three equations of equilibrium. Summing moments about point C eliminates all unknowns except the desired force in BD.

$$+\ \Sigma M_C = 0: \qquad T_{BD}(16) + 425(32) = 0$$

$$T_{BD} = -850 \text{ lb} = 850 \text{ lb } (C) \qquad \textbf{Ans.}$$

Example Problem 1-4

A bag of potatoes is sitting on the chair of Fig. 1-8*a*. The force exerted by the potatoes on the frame at one side of the chair is equivalent to horizontal and vertical forces of 24 N and 84 N, respectively, at E and a force of 28 N perpendicular to member BH at G (as shown in the free-body diagram of Fig. 1-8*b*). Find the forces acting on member BH.

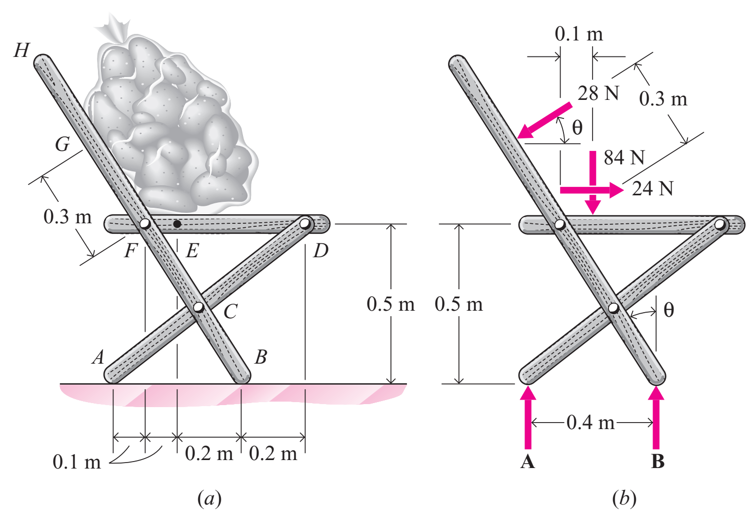


Figure 1-8

SOLUTION

The equations of equilibrium for the entire chair (Fig. 1-8*b*) are

$$+\rightarrow \Sigma F_x = 0: \qquad 24 - 28\cos\theta = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad A + B - 84 - 28\sin\theta = 0$$

$$+\ \Sigma M_B = 0: \qquad 0.2(84) - 0.5(24) - 0.4A + \left(0.3 + \frac{0.5}{\cos\theta}\right)(28) = 0$$

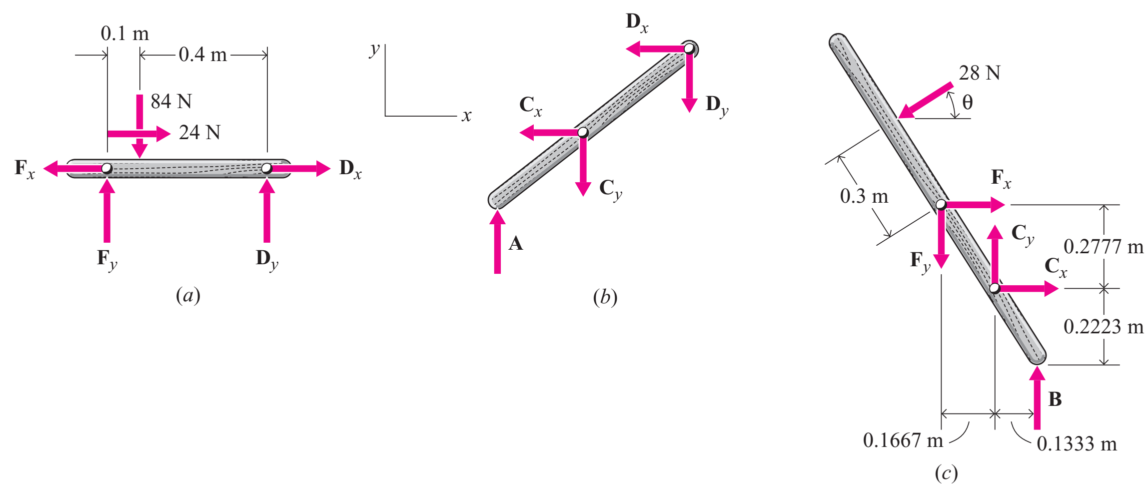


Figure 1-9

where $\theta = \tan^{-1}(3/5) = 30.96°$. The first equation is satisfied identically. The remaining two equations give

$$A = 73.82 \text{ N} \qquad B = 24.58 \text{ N}$$

Next the chair is disassembled and free-body diagrams are drawn for each part (Fig. 1-9). For member *DF*, the equilibrium equations can be written

$$+\rightarrow \Sigma F_x = 0: \qquad D_x - F_x + 24 = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad F_y + D_y - 84 = 0$$

$$+ \quad \Sigma M_D = 0: \qquad 0.4(84) - 0.5F_y = 0$$

which gives

$$F_y = 67.2 \text{ N} \qquad D_y = 16.80 \text{ N} \qquad D_x = F_x - 24 \text{ N}$$

Now the equations of equilibrium for member *BH* are

$$+\rightarrow \Sigma F_x = 0: \qquad F_x + C_x - 28 \cos\theta = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad 24.58 + C_y - 67.2 - 28 \sin\theta = 0$$

$$+ \quad \Sigma M_C = 0: \qquad \left(0.3 + \frac{0.1667}{\sin\theta}\right)(28) + 0.1333(24.58) + 0.1667(67.2) - 0.2777F_x = 0$$

which have only three unknowns remaining and can be solved to get

$$F_x = 115.1 \text{ N} \qquad C_x = -91.0 \text{ N} \qquad C_y = 57.0 \text{ N}$$

Then the forces acting on member BH are

$$\mathbf{B} = 24.6\ \mathbf{j}\ \text{N} \qquad \textbf{Ans.}$$

$$\mathbf{C} = -91.0\ \mathbf{i} + 57.0\ \mathbf{j}\ \text{N} \qquad \textbf{Ans.}$$

$$\mathbf{F} = 115.1\ \mathbf{i} - 67.2\ \mathbf{j}\ \text{N} \qquad \textbf{Ans.}$$

plus the applied force of 28 N perpendicular to the bar at G. These forces are shown on the "report diagram" of Fig. 1-10.

The "report diagram," though not necessary, may be used to check the results. For example,

$$+\rightarrow \Sigma F_x = 0: \qquad -28\cos 30.96^\circ + 115.1 - 91.0 = 0.08925 \cong 0$$

$$+\uparrow \Sigma F_y = 0: \qquad -28\sin 30.96^\circ - 67.2 + 57.0 + 24.6 = -4.307 \times 10^{-3} \cong 0$$

$$+ \quad \Sigma M_F = 0: \qquad 28(0.3) + 57.0(0.1667) - 91.0(0.2777) + 24.6(0.3) = 0.0112 \cong 0$$

The force and moment equations of equilibrium do not exactly equate to zero due to roundoff error.

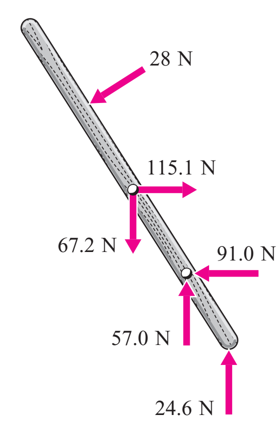


Figure 1-10

Example Problem 1-5

The weight of books on a shelf bracket is equivalent to a vertical force of 75 lb as shown on Fig. 1-11*a*. All members are made of 195-T6 cast aluminum, and all pins have ¼-in. diameters. Determine all forces acting on all three members of this frame.

SOLUTION

First draw the free-body diagram of the entire shelf bracket as in Fig. 1-11*b*. Here the "body of interest" is the frame ABC. The pins at A, B, and C remain attached to the frame, and thus the forces that would result from removal of the pins are not shown on the free-body diagram. The bracket at A has been removed from the frame, and the forces that the bracket exerts at A are shown as $\mathbf{A}_x$ and $\mathbf{A}_y$ (directions assumed). Similarly, the rocker at C has been removed, and the force of the rocker on the frame is shown as $\mathbf{C}$. The equations of equilibrium are

$$+ \quad \Sigma M_A = 0: \qquad 5C - 8(75) = 0$$

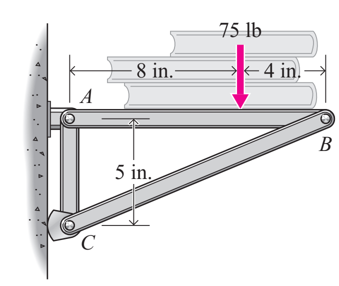
$$+\rightarrow \Sigma F_x = 0: \qquad A_x + C = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y - 75 = 0$$

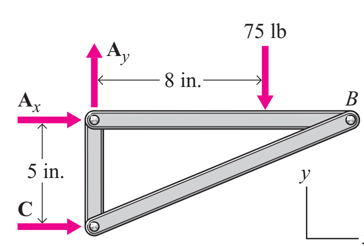
which are solved to get the support reactions

$$A_x = -120.0\ \text{lb} \qquad A_y = 75.0\ \text{lb} \qquad C = 120.0\ \text{lb} \qquad \textbf{Ans.}$$

Next, dismember the bracket and draw separate free-body diagrams of each member (Fig. 1-12). Members AC and BC are straight two-force members, and thus the forces in these members must act along the members. Pin A connects a

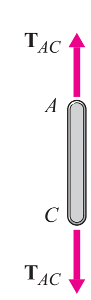


(*a*)



(*b*)

Figure 1-11



(*a*)

Figure 1-12(a)

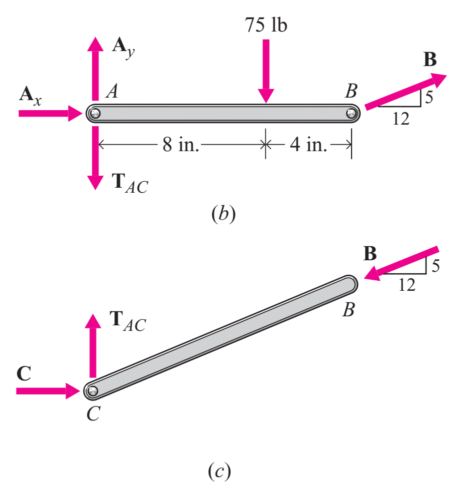


Figure 1-12(b, c)

support and two members. Since member AC is a two-force member, pin A will be left attached to member AB. The forces that act on pin A are the support reactions A_x and A_y and a vertical force due to the two-force member AC. Similarly, pin B connects two members, one of which is a two-force member. Therefore, pin B is left attached to member AB, and the only force on pin B is along the two-force member BC. Pin C connects a support and two members. Since both of the members are two-force members, pin C is arbitrarily left attached to member BC, and thus the force on pin C due to member AC is vertical. Then the equations of equilibrium can be written for member AB (Fig. 1-12b)

$$+\quad \Sigma M_B = 0: \qquad 4(75) + 12T_{AC} - 12(75.0) = 0$$

$$+\quad \Sigma M_A = 0: \qquad 12[(5/13)B] - 8(75) = 0$$

from which

$$T_{AC} = 50.0 \text{ lb} \qquad B = 130.0 \text{ lb}$$ **Ans.**

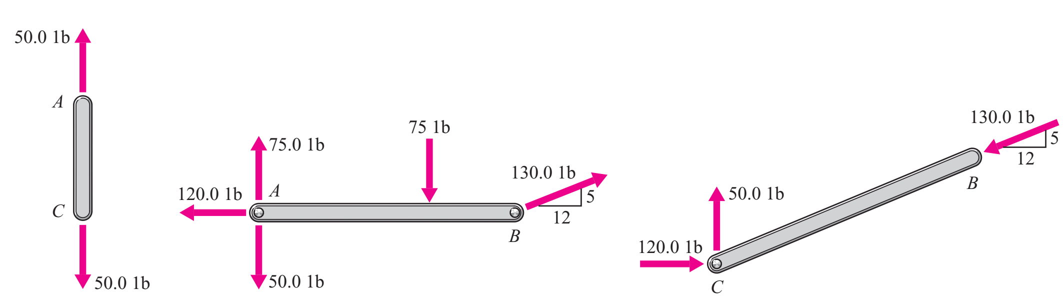


Figure 1-13

It is easily verified that these values also satisfy the equations of equilibrium for the other free-body diagrams. These forces are all shown on the “report diagrams” of Fig. 1-13.

Example Problem 1-6 Determine the reaction at support A of the pipe system shown in Fig. 1-14a. The 200-N force is parallel to the z-axis. Neglect the weights of the pipe and the wrench.

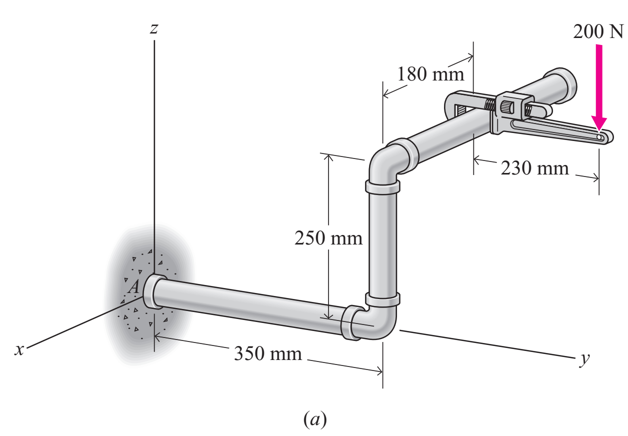


Figure 1-14(a)

SOLUTION

A free-body diagram of the pipe system is shown in Fig. 1-14b. The support at A is modeled as a rigid support that does not translate or rotate. There are three forces A_x, A_y, and A_z to prevent translation and three couples M_x, M_y, and M_z to prevent rotation. Couple M_x lies in the yz-plane, couple M_y lies in the xz-plane, and couple M_z lies in the xy-plane. Since there are six equations of equilibrium for a three-dimensional force system, all six unknowns can be found. Using Eqs. 1-3 and 1-4 yields

$$\Sigma F_x = 0: \qquad A_x = 0$$
$$\Sigma F_y = 0: \qquad A_y = 0$$
$$\Sigma F_z = 0: \qquad A_z - 200 = 0$$
$$\Sigma M_x = 0: \qquad M_x - 200(0.350 + 0.230) = 0$$
$$\Sigma M_y = 0: \qquad M_y - 200(0.180) = 0$$
$$\Sigma M_z = 0: \qquad M_z = 0$$

▶ The summation ΣM_x is the net tendency of all forces and moments to rotate the pipe about the x-axis through point A. For the 200-N force, this is just the magnitude of the force times the perpendicular distance between the line of action of the force and the x-axis through point A. For a more complicated force system, the tendency to rotate the body about the x-axis would be computed using the x-component of the vector product $\mathbf{r} \times \mathbf{F}$.

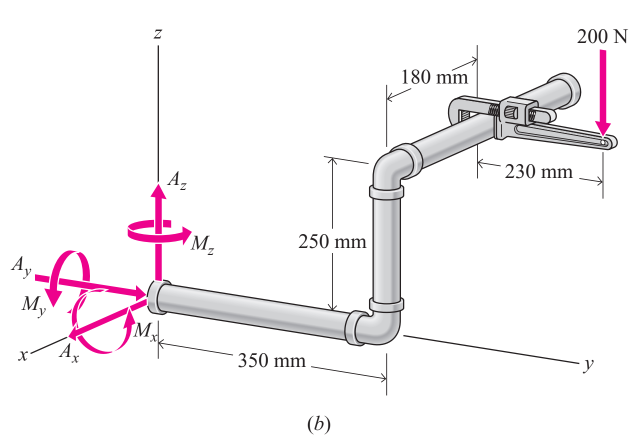


Figure 1-14(b)

Thus, the reaction at A is

$$A_x = 0\text{ N} \qquad A_y = 0\text{ N} \qquad A_z = 200\text{ N} \qquad \textbf{Ans.}$$

$$M_x = 116.0\text{ N}\cdot\text{m} \qquad M_y = 36.0\text{ N}\cdot\text{m} \qquad M_z = 0\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Since all the reactions are positive, they act in the directions shown on the free-body diagram of Fig. 1-14*b*.

Example Problem 1-7 A 1000-lb load is securely fastened to a hoisting cable as shown in Fig. 1-15*a*. The tension in the flexible cable does not change as it passes around the small frictionless pulley at the right support. The weight of the cable may be neglected. Plot the tensions in the two cables (T_{AB} and P) as a function of the sag distance d ($0 \le d \le 10$ ft). Determine the minimum sag d for which P is less than

(a) Twice the weight of the load.
(b) Four times the weight of the load.
(c) Eight times the weight of the load.

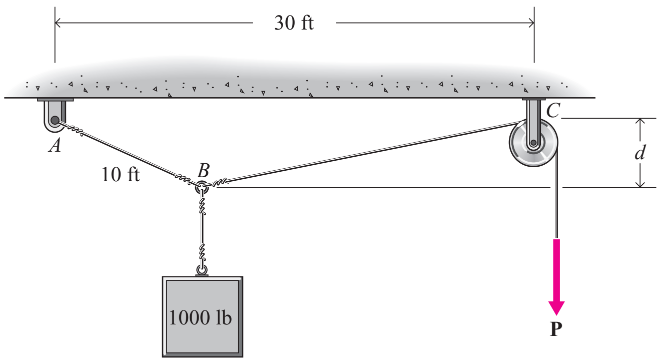


Figure 1-15(a)

SOLUTION

The ring B holds the wires together, and it will be isolated to generate the free-body diagram shown in Fig. 1-15*b*. The tension forces in the cables and the weight of the load are concurrent at the ring B. Writing the x- and y-components of the equilibrium equation for the free-body diagram of Fig. 1-15*b* results in

$$+\rightarrow \Sigma F_x = 0: \qquad T_{BC}\cos\theta_C - T_{AB}\cos\theta_A = 0 \qquad (a)$$

$$+\uparrow \Sigma F_y = 0: \qquad T_{AB}\sin\theta_A + T_{BC}\sin\theta_C - 1000 = 0 \qquad (b)$$

Solving Eq. (a) for T_{AB} gives

$$T_{AB} = \frac{T_{BC}\cos\theta_C}{\cos\theta_A} \qquad (c)$$

and substituting Eq. (*c*) into Eq. (*b*) gives

$$T_{BC}\frac{\sin\theta_C\cos\theta_A+\sin\theta_A\cos\theta_C}{\cos\theta_A}=1000 \qquad (d)$$

Before we can solve Eq. (*d*) for T_{BC}, we need to know how the angles θ_C and θ_A are related to the sag distance d. From the geometry of the triangles in Fig. 1-15*c*

$$\begin{aligned}\sin\theta_A &= d/10\\ a &= 10\cos\theta_A\\ b &= 30-a\\ \tan\theta_C &= d/b\end{aligned} \qquad (e)$$

All that remains is to choose some values for d and to solve Eqs. (*c*)–(*e*) for the tensions. For example, when $d = 6$ ft, Eqs. (*e*) give

$$\begin{aligned}\theta_A &= \sin^{-1}\frac{6}{10}=36.8699^\circ\\ a &= 10\cos 36.8699^\circ = 8\text{ ft}\\ b &= 30-8=22\text{ ft}\\ \theta_C &= \tan^{-1}\frac{6}{22}=15.2551^\circ\end{aligned}$$

Then, Eqs. (*d*) and (*c*) give

$$\begin{aligned}T_{BC} &= 1013.49\text{ lb}\\ T_{AB} &= 1222.22\text{ lb}\end{aligned}$$

where $T_{BC} = P$ because the tension in the hoisting cable does not change as the cable goes around the small pulley. Figure 1-15*d* shows the results of repeating this process for various values of the sag distance d and graphing the results.

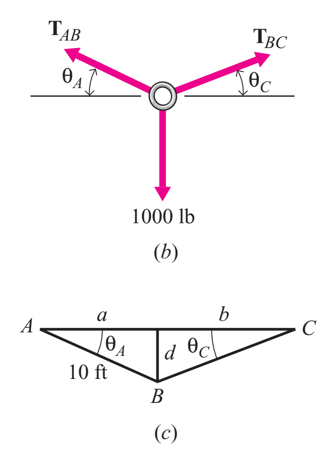


Figure 1-15(b, c)

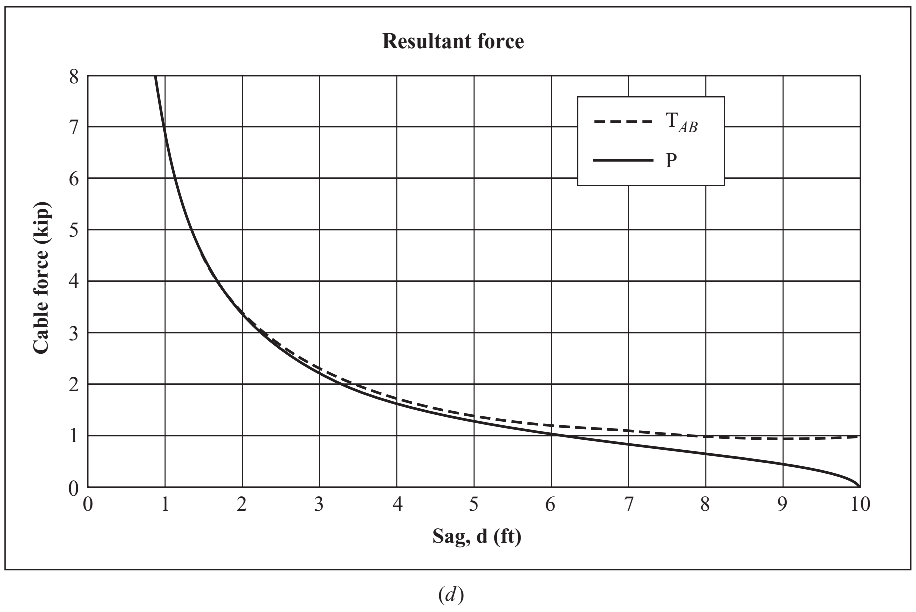


Figure 1-15(d)

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When $d = 10$ ft, the load hangs directly below the support A, cable AB carries the entire load, and the hoisting cable is slack, $P = 0$ lb. As the load is raised (d gets smaller), the force in both cables increases. At $d = 3.28$ ft, the force in the hoisting cable is twice the load; at $d = 1.66$ ft, the force in the hoisting cable is four times the load; and at $d = 0.833$ ft, the force in the hoisting cable is eight times the weight of the load being lifted. As d goes to zero, the forces in the two cables both go to infinity.

PROBLEMS

Introductory Problems

1-1* A person is holding a 20-lb object as shown in Fig. P1-1. Determine the force **T** in the biceps muscle and the force **F** of the humerus against the ulna, in terms of the weight **W** of the forearm, which acts through G. For the position shown, both **T** and **F** act vertically.

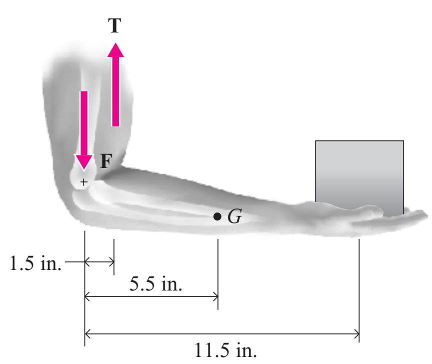

Figure P1-1

1-2* A worker is using a hoist and cable to lift a 175-kg engine from a car as shown in Fig. P1-2. Determine the forces in the three cables attached to the ring.

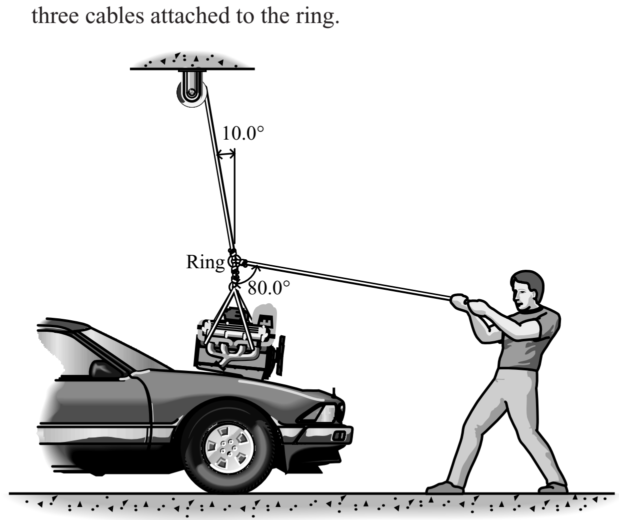

Figure P1-2

1-3 An 800-lb homogeneous cylinder is supported by two rollers as shown in Fig. P1-3. Determine the forces exerted by the rollers on the cylinder. All surfaces are smooth (frictionless).

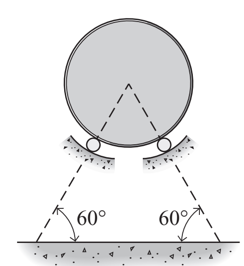

Figure P1-3

1-4* A curved slender bar is loaded and supported as shown in Fig. P1-4. Determine the reactions at supports A and B.

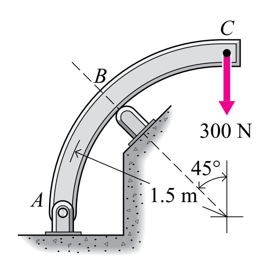

Figure P1-4

1-5 A curved slender bar is loaded and supported as shown in Fig. P1-5. Determine the reaction at support A.

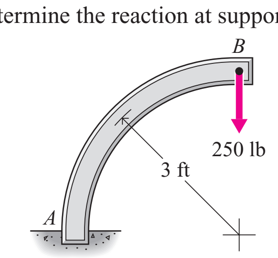

Figure P1-5

1-6 Determine the forces in members *BC*, *CD*, and *DE* of the truss shown in Fig. P1-6.

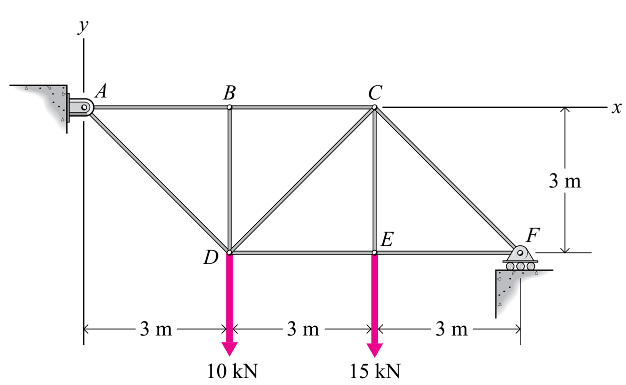

Figure P1-6

1-7* The lawn mower shown in Fig. P1-7 weighs 35 lb. Determine the force **P** required to move the mower at a constant velocity and the forces exerted on the front and rear wheels by the inclined surface.

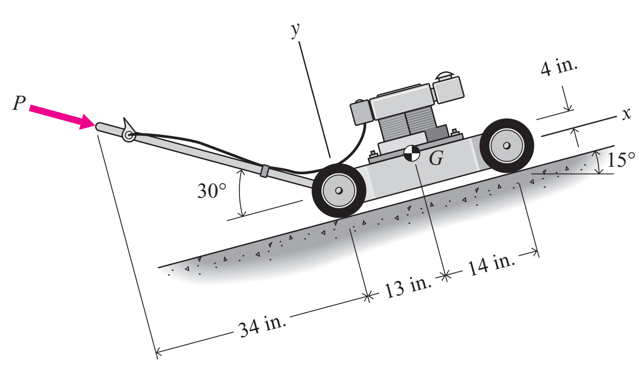

Figure P1-7

1-8 A human femur is modeled as shown in Fig. P1-8. The abductor muscle force is $M = 4060$ N, and the femoral load is $J = 5210$ N. Determine the force **P** and the couple **C**.

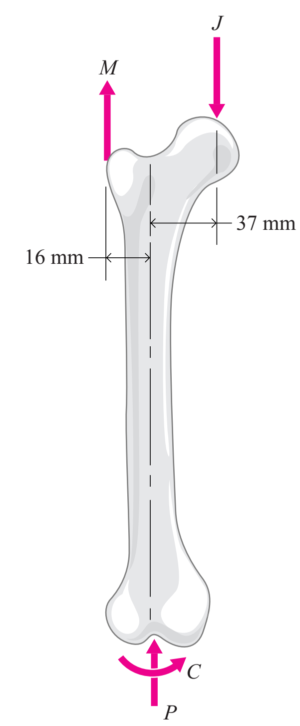

Figure P1-8

1-9* Determine the forces in members *CD*, *CF*, and *FG* of the bridge truss shown in Fig. P1-9.

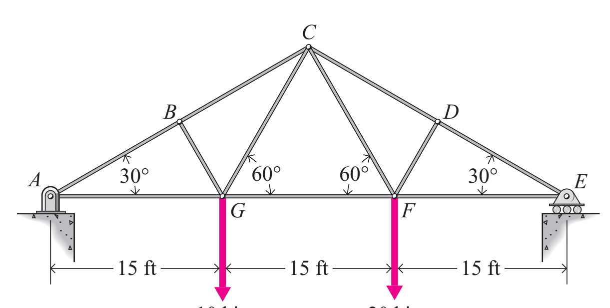

Figure P1-9

1-10* The coal wagon shown in Fig. P1-10 is used to haul coal from a mine. If the mass of the coal and wagon is 2000 kg, determine the force **P** required to move the wagon at a constant velocity and the forces exerted on the front and rear wheels by the inclined surface.

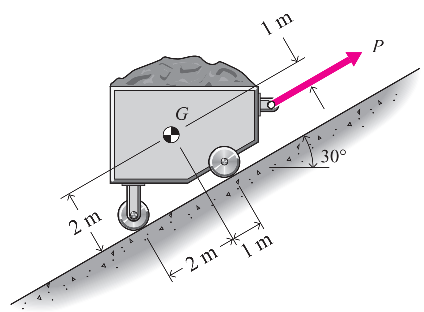

Figure P1-10

1-11 A 30-lb force **P** is applied to the brake pedal of an automobile as shown in Fig. P1-11. Determine the force **Q** applied to the brake cylinder and the reaction at support *A*.

Figure P1-11

1-12* A beam is loaded and supported as shown in Fig. P1-12. Determine the reaction at support *A*.

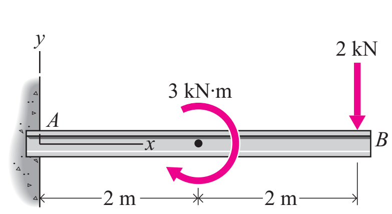

Figure P1-12

1-13 A beam is loaded and supported as shown in Fig. P1-13. Determine the reactions at supports *A* and *B*.

Figure P1-13

1-14 Pulleys *A* and *B* of the chain hoist shown in Fig. P1-14 are connected and rotate as a unit. The chain is continuous, and each of the pulleys contains slots that prevent the chain from slipping. Determine the force **F** required to hold a 450-kg block *W* in equilibrium if the radii of pulleys *A* and *B* are 90 mm and 100 mm, respectively.

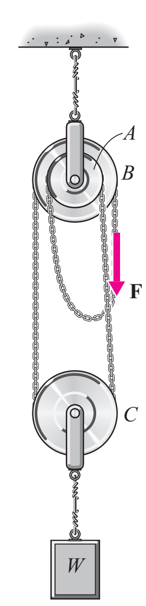

Figure P1-14

1-15* A bracket of negligible weight is used to support the distributed load shown in Fig. P1-15. Determine the reactions at the supports *A* and *B*.

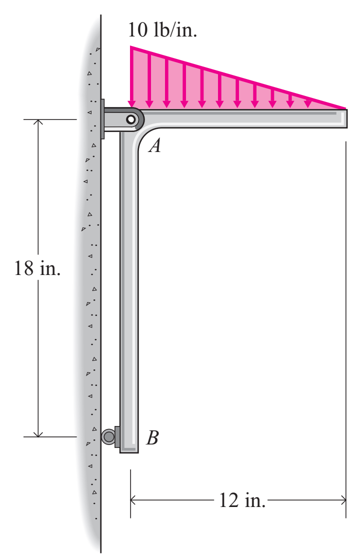

Figure P1-15

1-16 The wood plane shown in Fig. P1-16 moves with a constant velocity when subjected to the forces shown. Determine

a. The shearing force of the wood on the plane.
b. The normal force, and its location, of the wood on the plane.

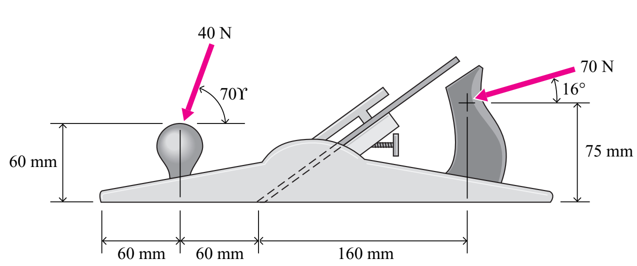

Figure P1-16

Intermediate Problems

1-17* Forces of 25 lb are applied to the handles of the pipe pliers shown in Fig. P1-17. Determine the force exerted on the pipe at D and the force exerted on handle DAB by the pin at A.

Figure P1-17

1-18* A pair of vise grip pliers is shown in Fig. P1-18. Determine the force **F** exerted on the block by the jaws of the pliers when a force $P = 100$ N is applied to the handles.

Figure P1-18

1-19 The Gambrel truss shown in Fig. P1-19 supports one side of a bridge; an identical truss supports the other side. Floor beams carry vehicle loads to the truss joints. Calculate the forces in members BC, BG, and CG when a truck weighing 7500 lb is stopped in the middle of the bridge as shown. The center of gravity of the truck is midway between the front and rear wheels.

Figure P1-19

1-20* A transmission line truss supports a 5-kN load, as shown in Fig. P1-20. Determine the forces in members FG and CD.

Figure P1-20

1-21 Three smooth homogeneous cylinders A, B, and C are stacked in a V-shaped trough as shown in Fig. P1-21. Cylinder A weighs 100 lb; cylinders B and C each weigh 200 lb. All cylinders have a 5-in diameter. Determine the minimum angle θ for equilibrium.

Figure P1-21

1-22 The mass of block A in Fig. P1-22 is 250 kg. Block A is supported by a small wheel that is free to roll on the continuous cable between supports B and C. The length of the cable is 42 m. Determine the distance x and the tension T in the cable when the system is in equilibrium.

Figure P1-22

1-23* The wrecker truck of Fig. P1-23 has a weight of 15,000 lb and a center of gravity at G. The force exerted on the rear (drive) wheels by the ground consists of both a normal component B_y and a tangential component B_x while the force exerted on the front wheels consists of a normal force A_y only. Determine the maximum pull **P** that the wrecker can exert when $\theta = 30^\circ$ if B_x cannot exceed $0.8B_y$ (because of friction considerations) and the wrecker does not tip over backwards (the front wheels remain in contact with the ground).

Figure P1-23

1-24 A drum of oil with a mass of 200 kg is supported by a pair of frames (the second frame is behind the one shown) as shown in Fig. P1-24. Determine all forces acting on member ACE.

Figure P1-24

1-25* The hot-air balloon shown in Fig. P1-25 is tethered with three mooring cables. If the net lift of the balloon is 900 lb, determine the force exerted on the balloon by each of the three cables.

Figure P1-25

1-26* A 100-kg traffic light is supported by a system of cables as shown in Fig. P1-26. Determine the tensions in each of the three cables.

Figure P1-26

1-27 A force of 20 lb is required to pull the stopper *DE* in Fig. P1-27. Determine all forces acting on member *BCD*.

Figure P1-27

1-28 The front-wheel suspension of an automobile is shown in Fig. P1-28. The pavement exerts a vertical force of 2700 N on the tire. Determine the force in the spring and the forces at *A*, *B*, and *D*.

Figure P1-28

1-29* Determine all forces acting on member *ABCD* of the frame of Fig. P1-29.

Figure P1-29

1-30 The flat roof of a building is supported by a series of parallel plane trusses spaced 2 m apart (only one such truss is shown in Fig. P1-30). Calculate the forces in all the members of a typical truss when water collects to a depth of 0.2 m as shown. The density of water is 1000 kg/m^3.

Figure P1-30

1-31 Two bodies W_1 and W_2 weighing 200 lb and 150 lb, respectively, rest on a cylinder and are connected by a rope as shown in Fig. P1-31. If all surfaces are smooth, determine

a. The reactions of the cylinder on the bodies.
b. The tension in the rope.
c. The angle θ.

Figure P1-31

1-32* The homogeneous door shown in Fig. P1-32 has a mass of 60 kg and is held in the position shown by the rod *AB*. The rod is held in place by smooth horizontal pins at *A* and *B*. The hinges at *C* and *D* are smooth, and the hinge at *C* can support thrust along its axis. Determine all forces that act on the door.

Figure P1-32

1-33 A farmer is using the hand winch shown in Fig. P1-33 to slowly raise a 40-lb bucket of water from a well. In the position shown, force **P** is vertical. The bearings at *C* and *D* exert only force reactions on the shaft. Bearing *C* can support thrust loading; bearing *D* cannot. Determine the magnitude of force **P** and the components of the bearing reactions.

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Figure P1-33

1-34* A scissors jack for an automobile is shown in Fig. P1-34. The screw threads exert a force **F** on the blocks at joints A and B. Determine the force **P** exerted on the automobile if $F = 800\,\text{N}$ and $\theta = 15^\circ$. Repeat for $\theta = 30^\circ$ and $\theta = 45^\circ$.

Figure P1-34

1-35* The fold-down chair of Fig. P1-35 weighs 25 lb and has its center of gravity at G. Determine all forces acting on member ABC.

Figure P1-35

1-36 A frame is loaded and supported as shown in Fig. P1-36. Determine the reactions at supports A and C and all forces acting on member ADE.

Figure P1-36

1-37 Forces of 50 lb are applied to the handles of the bolt cutter of Fig. P1-37. Determine

a. All forces acting on the handle ABC.
b. The force exerted on the bolt at E.

Figure P1-37

Challenging Problems

1-38* The garage door *ABCD* shown in Fig. P1-38 is being raised by a cable *DE*. The one-piece door is a homogeneous rectangular slab which has a mass of 100 kg. Frictionless rollers *B* and *C* run in tracks at each side of the door as shown. Determine the tension *T* in the cable and the forces **B** and **C** on the frictionless rollers when $d = 1.875$ m.

Figure P1-38

1-39* The crane and boom shown in Fig. P1-39 weigh 12,000 lb and 600 lb, respectively. When the boom is in the position shown, determine

a. The maximum load that can be lifted by the crane.
b. The tension in the cable used to raise and lower the boom when the load being lifted is 3600 lb.
c. The pin reaction at boom support *A* when the load being lifted is 3600 lb.

Figure P1-39

1-40 Figure P1-40 is a simplified sketch of the mechanism used to raise the bucket of a bulldozer. The bucket and its contents weigh 10 kN and have a center of gravity at *H*. Arm *ABCD* has a weight of 2 kN and a center of gravity at *B*; arm *DEFG* has a weight of 1 kN and a center of gravity at *E*. The weight of the hydraulic cylinders can be ignored. Calculate the force in the horizontal cylinders *CJ* and *EI* and all forces acting on arm *DEFG* for the position shown.

Figure P1-40

1-41 The mechanism of Fig. P1-41 is designed to keep its load level while raising it. A pin on the rim of the 4-ft-diameter pulley fits in a slot on arm *ABC*. Arms *ABC* and *DE* are each 4 ft long, and the package being lifted weighs 80 lb. The mechanism is raised by pulling on the rope that is wrapped around the pulley. Determine the force **P** applied to the rope and all forces acting on the arm *ABC* when the package has been lifted 4 ft, as shown.

Figure P1-41

1-42* Bar AB of Fig. P1-42 has a uniform cross section, a mass of 25 kg, and a length of 1 m. Determine the angle θ for equilibrium.

Figure P1-42

1-43 The homogeneous door shown in Fig. P1-43 has a mass of 25 kg and is supported in a horizontal position by two hinges and a bar. The hinges have been properly aligned; therefore, they exert only force reactions on the door. Assume that the hinge at B resists any force along the axis of the hinge pins. Determine the reactions at supports A, B, and D.

Figure P1-43

Computer Problems

1-44 A pair of steel pipes is stacked in a box as shown in Fig. P1-44. The masses and diameters of the smooth pipes are $m_A = 5$ kg, $m_B = 20$ kg, $d_A = 100$ mm, and $d_B = 200$ mm. Plot the two forces exerted on pipe A (by pipe B and by the side wall) as a function of the distance b between the walls of the box ($200\text{ mm} \leq b \leq 300\text{ mm}$). Determine the range of b for which

a. The force at the side wall is less than W_A, the weight of pipe A.
b. Neither of the two forces exceeds $2W_A$.
c. Neither of the two forces exceeds $4W_A$.

Figure P1-44

1-45 A worker positions a 250-lb crate by pulling on the rope BD as shown in Fig. P1-45. The 3-ft long rope BD is horizontal ($\theta = 0$) when the 5-ft long rope AB is vertical ($\phi = 0$).

a. What is the maximum distance b_{max} that the crate can be pulled to the side using this arrangement?
b. Calculate and plot the forces in ropes AB and BD as a function of the distance b for $0 \leq b \leq b_{max}$.
c. How could the worker pull the crate to the side more than the b_{max} calculated in part a?

Figure P1-45

1-46 A 50-kg load is suspended from a pulley as shown in Fig. P1-46. Pulleys B and C are both frictionless and free to rotate, and the weight of the cable may be neglected. Plot the force P required for equilibrium as a function of the sag distance d ($0 \leq d \leq 1$ m). Determine the minimum sag d_{min} for which P is less than

a. Twice the weight of the load.
b. Four times the weight of the load.
c. Eight times the weight of the load.

Figure P1-46

1-47 A 75-lb stop light is suspended between two poles as shown in Fig P1-47. Neglect the weight of the flexible cables and plot the tension in both cables as a function of the sag distance d ($0 \leq d \leq 8$ ft). Determine the minimum sag d_{min} for which both tensions are less than

a. 100 lb.
b. 250 lb.
c. 500 lb.

Figure P1-47

1-48 The wrecker truck shown in Fig. P1-48 has a mass of 6800 kg and a center of gravity at G. The force exerted on the rear (drive) wheels by the ground consists of both a normal component B_y and a tangential component B_x, while the force exerted on the front wheels consists of a normal force A_y only.

a. Plot P, the maximum pull that the wrecker can exert, as a function of θ ($0° \leq \theta \leq 90°$) if B_x cannot exceed $0.8B_y$ (because of friction considerations) and the wrecker does not tip over backward (the front wheels remain in contact with the ground).
b. On the same graph, plot A_y, B_x, and B_y as functions of the angle θ.

Figure P1-48

1-49 An overhead crane consists of an I-beam supported by a simple truss as shown in Fig. P1-49. If the uniform I-beam weighs 400 lb, plot the force in members BC, CF, and EF as a function of the position d ($0 \leq d \leq 8$ ft).

Figure P1-49

1-50 A light pole is braced using a lightweight flexible cable *DGH* as shown in Fig. P1-50. The uniform pole *ABCD* weighs 4200 N; the combined weight of the light fixture and arm *CEF* is 7500 N and acts at *E*; and the weight of the arm *BG* can be neglected. Assume that the arms *EC* and *BG* are braced to remain perpendicular to the pole *ABCD* and that the connection at *A* cannot provide any significant moment. Plot T_{DG} and T_{GH} (the tensions in the two parts of the cable) and F_{BG} (the force in the brace *BG*) as functions of *b*, the length of the brace, for $0.5\text{ m} \le b \le 3\text{ m}$.

Figure P1-50

1-51 The crane and boom shown in Fig. P1-51 weigh 12,000 lb and 600 lb, respectively. The pulleys at *D* and *E* are small, and the cables attached to them are essentially parallel.

a. Plot *d*, the location of the resultant force of the ground on the crane relative to point *C*, as a function of the boom angle θ ($0° \le \theta \le 80°$) when the crane is lifting a 3600-lb load.
b. Plot *A*/3600 and T_{BD}/3600 as functions of the boom angle θ ($0° \le \theta \le 80°$) when the crane is lifting a 3600-lb load (in which *A* is the magnitude of the reaction force on the pin at *A*, T_{BD} is the tension in the cable raising the boom, and 3600 lb is the weight of the load being lifted).
c. It is desired that the resultant force on the tread always be at least 1 ft behind *C* to ensure that the crane is never in danger of tipping over. Plot W_{max}, the maximum load that may be lifted, as a function of the boom angle θ ($0° \le \theta \le 80°$). (Don't forget to check the tension in cable *BD*.)

Figure P1-51

1-52 A group of workers proposes to raise a uniform 250-kg post *AB* to a vertical position using the rope and brace arrangement shown in Fig. P1-52*a*. Assume that the weight of the brace can be neglected and that end *A* acts as a frictionless pin for both the 6-m-long post *AB* and the 6-m-long brace *AC*.

a. Plot the rope force *P* and the force F_{AC} in the brace *AC* as functions of the angle θ ($0° \le \theta \le 90°$).
b. Repeat the problem if two braces are used as shown in Fig. P1-52*b*. Plot the rope force *P* and the brace forces F_{AC} and F_{AD} as functions of the angle θ ($0° \le \theta \le 90°$). Assume that when the force in brace *AD* becomes zero, the brace falls out of the way and from that point on the rope is attached to *C* instead of *D*.

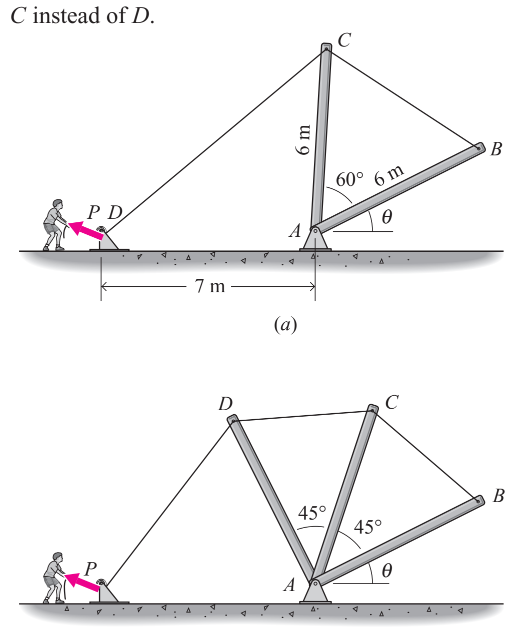

Figure P1-52

1-53 The hydraulic cylinder *BC* is used to tip the box of the dump truck shown in Fig. P1-53. If the combined weight of the box and the load is 22,000 lb and acts through the center of gravity *G*, plot

a. *C*/22,000, the force in the hydraulic cylinder divided by the weight of the truck box, as a function of the angle θ ($0^\circ \leq \theta \leq 80^\circ$).
b. *A*/22,000, the magnitude of the reaction force on the pin *A* divided by the weight of the truck box, as a function of the angle θ ($0^\circ \leq \theta \leq 80^\circ$).

Figure P1-53

1-4 EQUILIBRIUM OF A DEFORMABLE BODY

All of the equilibrium problems considered thus far have assumed that the bodies are rigid. That is, the shape of the body and its orientation relative to its surroundings were assumed to be independent of the loads applied to the body. However, no body is perfectly rigid. Wires subjected to tension forces will stretch. Beams carrying loads will bend. Shafts subjected to torques will twist. In fact, one of the primary objectives of a course in mechanics of materials is to develop relationships between the loads applied to a nonrigid body and the deformation of the body.

If the wire or beam or shaft is very stiff, the amount of deformation will be very small and the deformation will have a negligible effect on the solution of the equilibrium equations. If the wire or beam or shaft is not very stiff, however, the deformation can affect the geometry of the problem used to write the equilibrium equations, which will in turn affect the solution of the equilibrium equations. The interaction between the loads acting on a body, the deformation of the body, and the geometry of the free-body diagram makes the solution of deformable body problems much more complex than the solution of rigid body problems. Frequently the solution of deformable body problems requires either a trial-and-error solution or a numerical solution or an iterative solution method.

Fortunately, most engineering structures and machines are designed "stiff," that is, they do not deform very much. For such problems, the solution of the equilibrium equations often ignores the deformation and treats the structure as though it were rigid. Example Problem 1-8 illustrates the difficulties encountered in the solution of deformable body problems and the errors that may result from neglecting the deformation in the solution of the equilibrium equations.

Example Problem 1-8 A 5000-lb weight *W* is to be supported by a very stiff (rigid) bar *AB* and a deformable (nonrigid) wire *BC*, as shown in Fig. 1-16*a*. The connections at *A*, *B*, and *C* are frictionless pins, the brackets at *A* and *C* are rigid, and the weights of the bar and the wire may be neglected. The

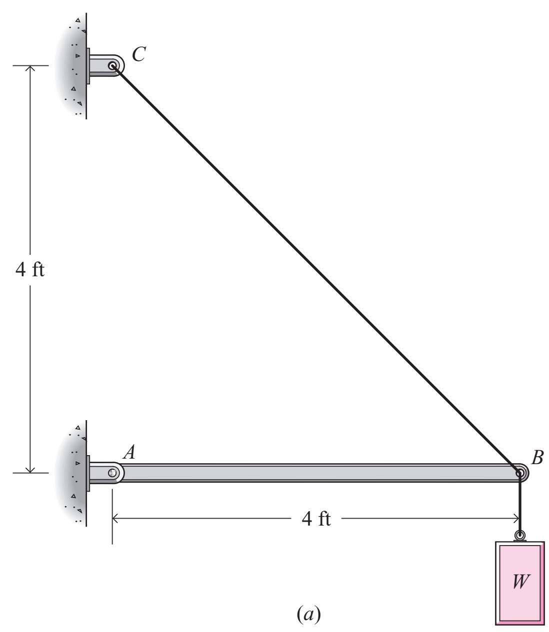

Figure 1-16(a)

relationship between the tension in the wire T_{BC} and the deformation (stretch) of the wire δ is given by $T_{BC} = k\delta$ where k depends on the cross-sectional area of the wire and the type of material from which the wire is made, and $\delta = L_f - L_i$, the difference between the final (deformed) length of the wire and the initial (undeformed) length of the wire. If Fig. 1-16*a* represents the unloaded configuration of the structure ($\delta = 0$), determine the tension in the wire after the load is applied for $k = 5000$ lb/in. Repeat for $k = 2500$ lb/in. and for $k = 1000$ lb/in.

SOLUTION

Since bar AB is rigid and the support bracket at A is fixed to the wall, bar AB must pivot about pin A due to the load W. Wire BC will increase in length by an amount δ since it is deformable. The loaded configuration of the structure is shown in Fig. 1-16*b*. The rotation of the bar can be described by the angle θ, and the deformation δ of the wire can be written as $\delta = L_f - L_i$.

A free-body diagram of the bar in the deformed configuration is shown in Fig. 1-16*c*. The equations of equilibrium give

$$+\rightarrow \Sigma F_x = 0: \qquad A_x - T\cos(45 + \theta/2) = 0 \qquad (a)$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + T\sin(45 + \theta/2) - W = 0 \qquad (b)$$

$$+ \ \Sigma M_A = 0: \qquad [T\sin(45 + \theta/2)](4\cos\theta) - [T\cos(45 + \theta/2)](4\sin\theta) - W(4\cos\theta) = 0 \qquad (c')$$

Using the trigonometric relation

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

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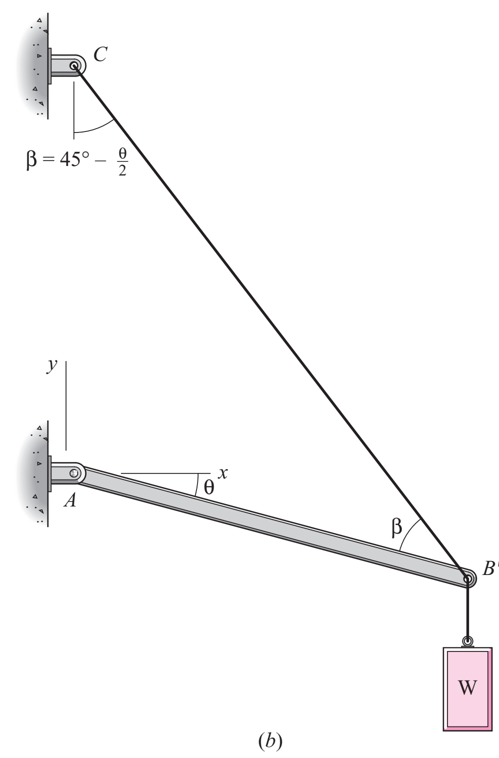


Figure 1-16(b)

Eq. (c') may be written

$$4T \sin(45 - \theta/2) - 4W \cos\theta = 0 \qquad (c)$$

Equations (a), (b), and (c) contain four unknowns A_x, A_y, T, and θ. Thus, there are four unknowns and three equations of equilibrium. We need an additional independent equation to solve the problem. The equations of equilibrium (a), (b), and (c) are necessary for equilibrium, but they are not sufficient to solve the problem. The additional equation comes from the relationship between the force in the wire and the deformation of the wire. Such a relationship will be developed in Chapter 4; we merely state the result here to complete this example.

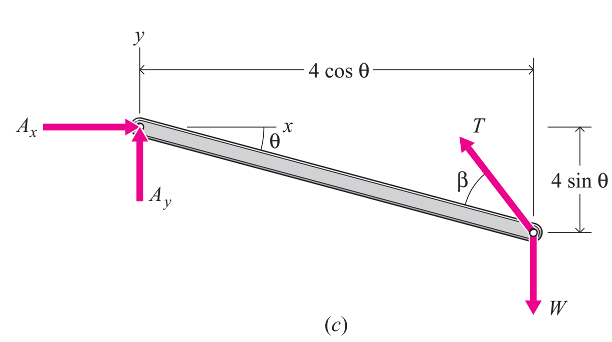


Figure 1-16(c)

The force–deformation relation depends on the type of material from which the wire is made, and the relationship may be linear or nonlinear. Herein we limit our discussion to a linear relationship between force and deformation. Thus, we assume the behavior shown in Fig. 1-16*d*

$$T = k\delta \tag{d}$$

where k is a material constant (the slope of a $T-\delta$ curve) that depends on the type of material from which the wire is made. For the wire,

$$L_i = \sqrt{(4)^2 + (4)^2} = 5.657\text{ ft} = 67.88\text{ in.}$$

Since $\delta = L_f - L_i = L_f - 67.88$ in., we need to determine the deformed length of the wire in order to find T. Using Fig. 1-16*b* and the law of cosines,

$$(B'C)^2 = L_f^2 = (48)^2 + (48)^2 - (2)(48)(48)\cos(90^\circ + \theta)$$

from which

$$L_f = 67.88\sqrt{1 + \sin\theta}\text{ in.}$$

The deformation δ of the wire is

$$\begin{aligned}\delta = L_f - L_i &= 67.88\sqrt{1+\sin\theta} - 67.88 \\ &= 67.88[\sqrt{1+\sin\theta} - 1]\text{ in.}\end{aligned} \tag{e}$$

The relationship between the applied load and the deformation of the structure is found by substituting Eq. (*e*) into Eq. (*d*). The result is then used in Eq. (*c*) to give

$$67.88\,k\,(\sqrt{1+\sin\theta} - 1)\sin(45 - \theta/2) = W\cos\theta \tag{f}$$

If $k = 5000$ lb/in., then Eq. (*f*) gives $\theta = 2.465^\circ$; Eq. (*c*) gives

$$T = 7221\text{ lb} \qquad \textbf{Ans.}$$

Equation (*a*) gives $A_x = 4995$ lb, and Eq. (*b*) gives $A_y = -215$ lb. This compares to the solution of Example Problem 1-1 (in which this structure was assumed to be rigid), which had $T = 7071$ lb, $A_x = 5000$ lb, and $A_y = 0$ lb. For this very stiff wire, the sag of the beam ($\theta = 2.465^\circ$) would barely be noticeable and the error in the value of T from treating the structure as rigid would be only about 2 percent.

If $k = 2500$ lb/in., then Eq. (*f*) gives $\theta = 5.097^\circ$; Eq. (*c*) gives

$$T = 7379\text{ lb} \qquad \textbf{Ans.}$$

Equation (*a*) gives $A_x = 4890$ lb; and Eq. (*b*) gives $A_y = -444$ lb.

If $k = 1000$ lb/in., then Eq. (*f*) gives $\theta = 14.246^\circ$; Eq. (*c*) gives

$$T = 7893\text{ lb} \qquad \textbf{Ans.}$$

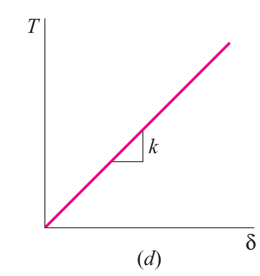


Figure 1-16(d)

▶ In Chapter 3 it will be shown that the constant of proportionality $k = \frac{EA}{L}$ where E is the modulus of elasticity of the material and L and A are the length and the cross-sectional area of the wire. For a steel wire $\frac{1}{8}$ in. in diameter, k = 5420 lb/in. or for a steel tie rod $\frac{1}{4}$ in. in diameter, $k = 21{,}690$ lb/in., little error is introduced by treating the structure as rigid. For an aluminum alloy wire 1/8 in. in diameter, however, $k = 1808$ lb/in., and the error introduced by treating the structure as rigid is about 6 percent. For more "stretchable" support materials, the error would be even greater.

Equation (*a*) gives $A_x = 4846$ lb, and Eq. (*b*) gives $A_y = -1230$ lb. For this less stiff wire, the sag of the beam is definitely noticeable and the error in the value of T from treating the structure as rigid would be over 10 percent.

The process illustrated in this example is typical of the solution of deformable body problems. Regardless of the type of structure or machine component or the type of loading, the solution process generally consists of these three steps

1. Equations of equilibrium, (*a*), (*b*), and (*c*)
2. Force-deformation relationship, Eq. (*d*)
3. Geometry of deformation, Eq. (*e*)

Since the equations of equilibrium must be applied to the forces acting on the deformed structure, the three sets of equations are often interdependent. It is this interdependence that makes the solution of deformable body problems more complex than the solution of rigid-body problems.

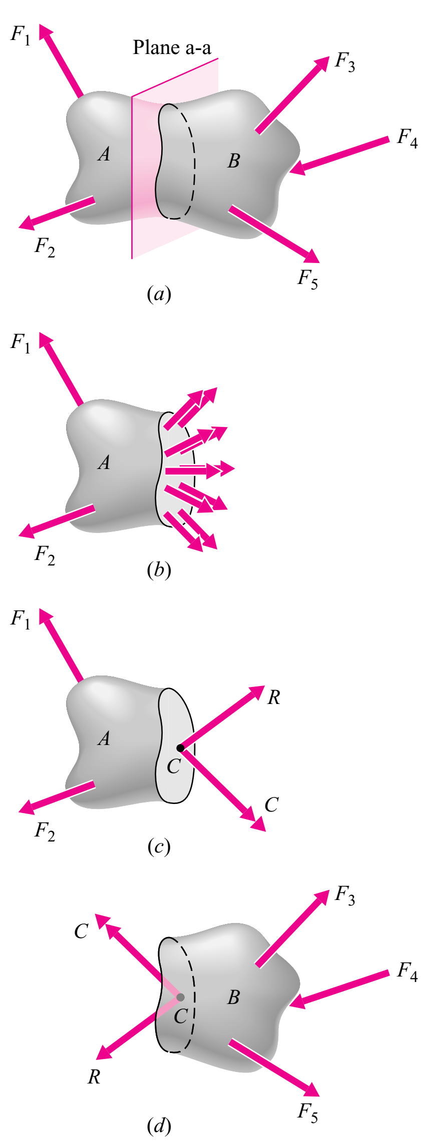


Figure 1-17

1-5 INTERNAL FORCES

In the study of mechanics of materials, it is necessary that we examine the internal forces that exist throughout the interior of a body. We consider an arbitrary body in equilibrium, as shown in Fig. 1-17*a*. The forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, $\mathbf{F}_4$, and $\mathbf{F}_5$ are applied loads and support reactions (found using the equations of equilibrium). We pass an imaginary "cutting plane" *a–a* (henceforth called a section) through the body, and separate the body into two parts *A* and *B*. Considering a free-body diagram of part *A* (Fig. 1-17*b*) we note that, in addition to the applied forces $\mathbf{F}_1$ and $\mathbf{F}_2$, the material of part *B* exerts forces on the material of part *A* over the section. These forces are internal to the body as a whole but are external for part A. The forces on the section are distributed over the surface in an unknown fashion. However, we can replace the distributed force system by a resultant force **R** and a resultant couple **C**. In general, the couple **C** depends on where we place the force **R**. In mechanics of materials we place **R** at the centroid *C* of the section, as shown in Fig. 1-17*c*. We use a double arrowhead to distinguish the couple **C** (vector) from the force **R** (vector). On the section, the distributed force system of Fig. 1-17*b* is statically equivalent to the force system **R** and **C** of Fig. 1-17*c*. The force system **R** and **C** will be referred to as an internal force system. We recognize that the internal force system depends on the orientation of the section.

Instead of part *A*, we could have considered part *B*, as shown in Fig. 1-17*d*. By applying Newton's third law to every pair of particles on the section for parts *A* and *B* of the body, we have that the distributed force systems over the section of parts *A* and *B* are equal in magnitude but opposite in sense, and thus the resultant force **R** and the resultant couple **C** on the two parts of the body are equal in magnitude but opposite in sense.

Because the body as a whole is in equilibrium, any portion of the body is also in equilibrium. Thus, using the equations of equilibrium and the force system shown in Fig. 1-17*c*,

$$\Sigma\mathbf{F} = \mathbf{0}: \qquad \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{R} = \mathbf{0}$$
$$\Sigma\mathbf{M}_C = \mathbf{0}: \qquad \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{C} = \mathbf{0}$$

where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the moments of forces $\mathbf{F}_1$ and $\mathbf{F}_2$, respectively, about the centroid of the section. We would find the same result using the free-body diagram of part B shown in Fig. 1-17*d*. Thus, we find the resultant of the internal force system using the equations of equilibrium. However, we cannot find the exact distribution of the internal forces until we learn how to determine the deformation of the body.

Experience indicates that materials behave differently to forces trying to pull atoms apart than to forces trying to slide atoms past each other. Therefore, it is standard practice to resolve the resultants **R** and **C** into components along and perpendicular to the section, as shown in Fig. 1-18. For convenience we select an *xyz*-coordinate system in which x is perpendicular to the section and y and z lie in the section. The component of **R** which is perpendicular to the section, R_x, is called a *normal* force; this force tends either to pull the body apart or to compress the body (Fig. 1-18*a*). The symbol P is often used to denote the normal force. The components of **R** that lie in the section are called *shear* forces; these forces tend to slide part A of the body relative to part B. The symbol V is often used to denote shear forces; hence, the forces V_y and V_z in Fig. 1-18*a*.

The component T of couple **C** shown in Fig. 1-18*b* tends to twist the body and is called a *twisting couple* (or twisting moment, or torque). The components M_y and M_z tend to bend the body and are called *bending couples* (or bending moments). Throughout this book we will examine the effects on a deformable body of the components of **R** and **C**.

The section shown in Figs. 1-18*a* and *b* is called a positive section since the outward normal to the section points in a positive coordinate direction. The section shown in Figs. 1-18*c* and *d* is called a negative section since the outward normal to the section points in a negative coordinate direction.

A resultant force or couple component is defined as positive if the component is in a positive coordinate direction when acting on a positive section. Thus, all of the force and couple components shown in Figs. 1-18*a* and *b* are positive. If the internal forces exerted on part A of the body by part B are called positive, then the other half of the internal forces (exerted on part B of the body by part A) should also be called positive. Therefore, a force or couple component will also be defined as positive if the component is in the negative coordinate direction when acting on a negative section. Hence, all of the force and moment components shown in Figs. 1-18*c* and *d* are also positive.

The components of the internal force system can be found using the equations of equilibrium (Eqs. 1-3 and 1-4).

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{1-3}$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \tag{1-4}$$

The equations of equilibrium should be applied to the body (or portion of the body) in its deformed state. However, as we saw in Example Problem 1-8, the support reactions (or, in this case internal forces) cannot be found until we know the relationship between the forces applied to the body and the deformation of the body. As we shall see in later chapters of this book, we can determine support reactions and internal forces using the undeformed configuration of a body. We accept this statement for now and illustrate the determination of internal forces with the following examples.

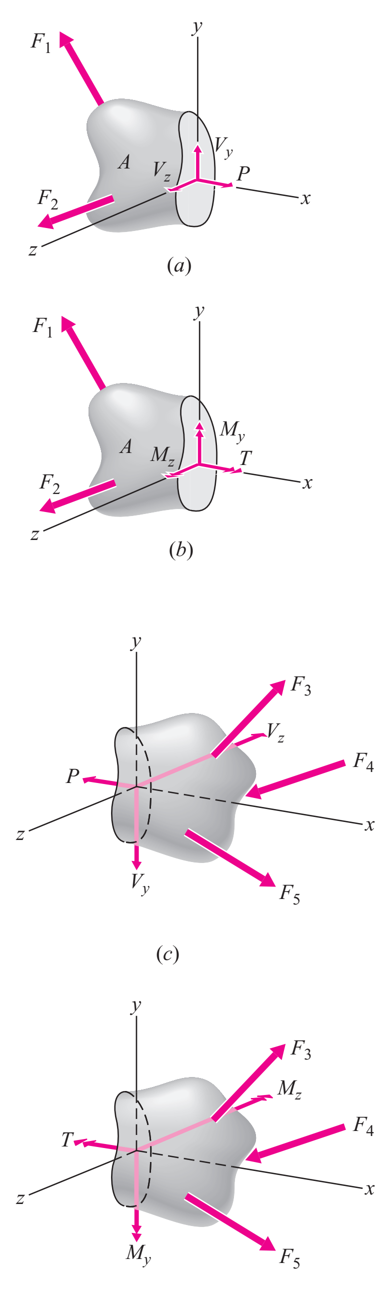


Figure 1-18

Example Problem 1-9 A post and bracket are used to support a pulley, as shown in Fig. 1-19*a*. A cable passing over the pulley supports a 2200-N force. Determine the internal forces on a section at the support at *A*.

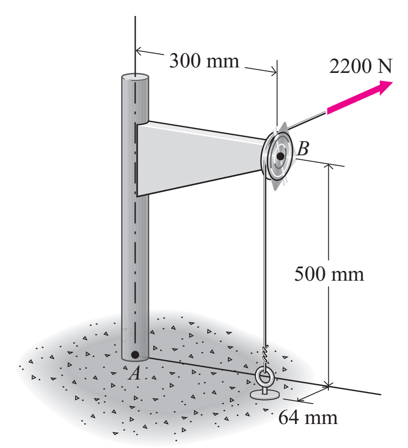


Figure 1-19(a)

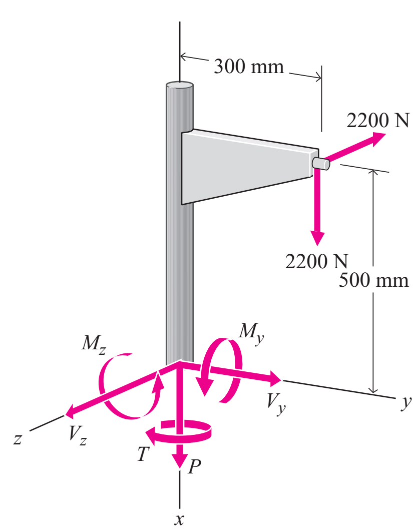


Figure 1-19(b)

SOLUTION

A section (perpendicular to the post) is passed through the post at a distance of 30 mm above the support at *A*. A free-body diagram of the part of the structure above the section is shown in Fig. 1-19*b*. The *xyz*-coordinate axes were arbitrarily selected, but the origin of the coordinate system is at the centroid of the section. Coordinate *x* is perpendicular to the section, whereas coordinates *y* and *z* lie in the section. Thus, the section on which the force and couple components act is a positive section, and the force and couple components shown in Figure 1-19*b* are positive. The couples T, M_y, and M_z are represented by curved arrows instead of vectors, as was done in Fig. 1-18*b*. The twisting couple T (torque) lies in the *yz*-plane; the bending couples M_y and M_z lie in the *xz*- and *xy*-planes, respectively. The equations of equilibrium (Eqs. 1-3 and 1-4) yield

$$\Sigma F_x = 0: \qquad P + 2200 = 0$$
$$\Sigma F_y = 0: \qquad V_y = 0$$
$$\Sigma F_z = 0: \qquad V_z - 2200 = 0$$
$$\Sigma M_x = 0: \qquad T - 2200(0.300) = 0$$
$$\Sigma M_y = 0: \qquad M_y - 2200(0.500) = 0$$
$$\Sigma M_z = 0: \qquad M_z - 2200(0.300) = 0$$

Solving for the internal force system gives

$$P = -2200\text{ N} \qquad V_y = 0\text{ N} \qquad V_z = 2200\text{ N}$$ **Ans.**

$$T = 660\text{ N}\cdot\text{m} \qquad M_y = 1100\text{ N}\cdot\text{m} \qquad M_z = 660\text{ N}\cdot\text{m}$$ **Ans.**

The negative sign for P indicates that the force is opposite to that shown in Fig. 1-19*b*, therefore, P is a compressive force.

Example Problem 1-10 The cantilever beam shown in Fig. 1-20*a* is subjected to both concentrated and distributed loads. Determine (a) the support reactions and (b) the internal forces on a section 4 m to the right of the support at A.

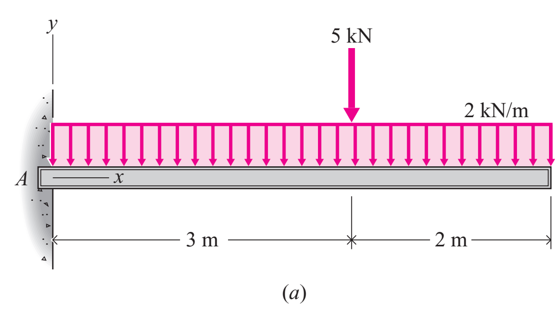


Figure 1-20(a)

SOLUTION

(a) A free-body diagram of the complete beam is shown in Fig. 1-20*b*. The support at A does not translate or rotate; thus, forces A_x and A_y and couple M_A may exist. The distributed load has been replaced by its resultant, $R = wl = 2\,\text{kN/m}(5\text{ m}) = 10\,\text{kN}$, acting 2.5 m to the right of A. The equations of equilibrium give

$$+\rightarrow \Sigma F_x = 0: \qquad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y - 10 - 5 = 0$$

$$+ \quad \Sigma M_A = 0: \qquad M_A - 10(2.5) - 5(3) = 0$$

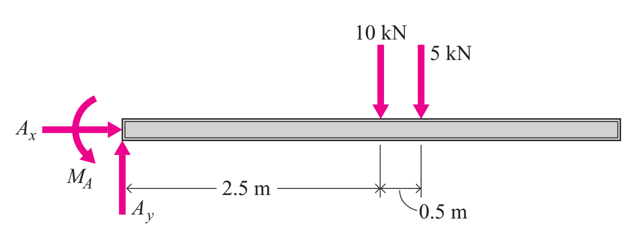


Figure 1-20(b)

Solving gives the support reactions

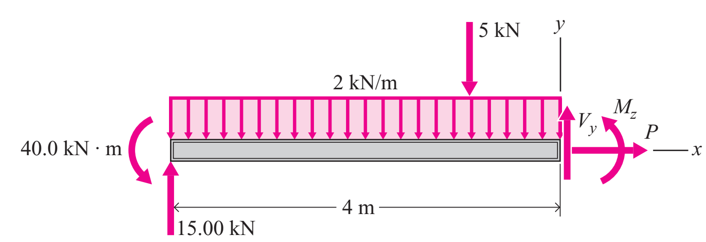
$$A_x = 0\,\text{kN} \qquad A_y = 15.00\,\text{kN} \qquad M_A = 40.0\,\text{kN}\cdot\text{m}$$ **Ans.**

(b) A vertical section is passed through the beam 4 m to the right of A. A free-body diagram of the part of the beam to the left of the section is shown in Fig. 1-20c. The section as well as the force and couple components shown in Fig. 1-20c are positive. The distributed load is shown to illustrate that the forces acting on that part of the beam must be shown on the free-body diagram. The 10-kN resultant force on the free-body diagram of Fig. 1-20b cannot be used for the free-body diagram of the part of the beam shown in Fig. 1-20c. The free-body diagram of Fig. 1-20c is redrawn in Fig. 1-20d, where the distributed load on this part of the beam has been replaced by its resultant. The equations of equilibrium give

$$+\rightarrow \Sigma F_x = 0: \qquad P = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad 15.00 - 8 - 5 + V_y = 0$$

$$+\ \ \Sigma M_z = 0: \qquad 40.0 - 15.00(4) + 8(2) + 5(1) + M_z = 0$$



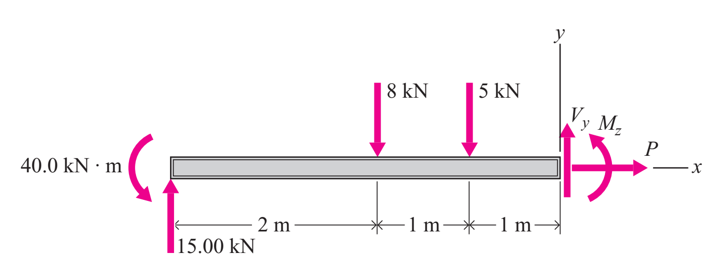
(c)

Figure 1-20(c)

The internal forces on the section are

$$P = 0\,\text{kN} \qquad V_y = -2.00\,\text{kN} \qquad M_z = -1.000\,\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The negative signs indicate that the directions of shear force V_y and bending couple M_z are opposite to those shown in Fig. 1-20d.



(d)

Figure 1-20(d)

▶ The internal force system acting on the portion of the beam to the right of the section is equal in magnitude and opposite in sense to the internal force system acting on the portion of the beam to the left of the section. Either portion of the beam can be used to determine the internal force system. Using the left portion of the beam, however, required that we first determine the forces and moments exerted on the beam by the wall. Generally speaking, the portion which is acted on by the fewest applied forces will be the easiest to use.

A more convenient way to solve part (b) is to use the free-body diagram of Fig. 1-20e, which shows the part of the beam to the right of the section (a negative section for the positive coordinate axes shown in Fig. 1-20d). The distributed load for this part of the beam has been replaced by its resultant; we note that the 5-kN concentrated load is not shown on the free-body diagram as it does not act on this part of the beam. Consistent with Newton's third law, the internal forces and moments in Fig. 1-20*e* are opposite in sense to the internal forces and moments in Fig. 1-20*d*. The forces P and V_y as well as the moment M_z in Fig. 1-20*e* are positive for this negative section. The equations of equilibrium give

$$+\rightarrow \Sigma F_x = 0: \qquad -P = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad -V_y - 2 = 0$$

$$+ \quad \Sigma M_z = 0: \qquad -M_z - 2(0.5) = 0$$

The internal forces on the section are

$$P = 0\,\text{kN} \qquad V_y = -2.00\,\text{kN} \qquad M_z = -1.000\,\text{kN}\cdot\text{m}$$ **Ans.**

which are the same as the previously calculated internal forces. The internal forces are shown in Fig. 1-20*f*.

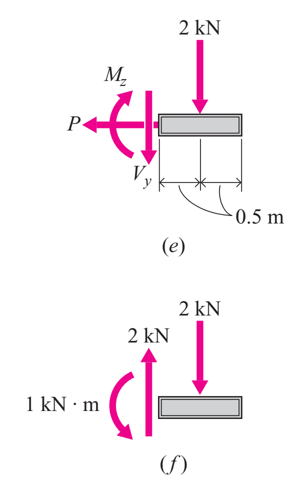


Figure 1-20(e, f)

Example Problem 1-11

A bag of potatoes is sitting on the chair in Fig. 1-21*a*. The force exerted by the potatoes on the frame at one side of the chair is equivalent to horizontal and vertical forces of 24 N and 84 N, respectively, at E and a force of 28 N perpendicular to member BH at G. Determine the internal forces on a section perpendicular to and midway between pins D and F.

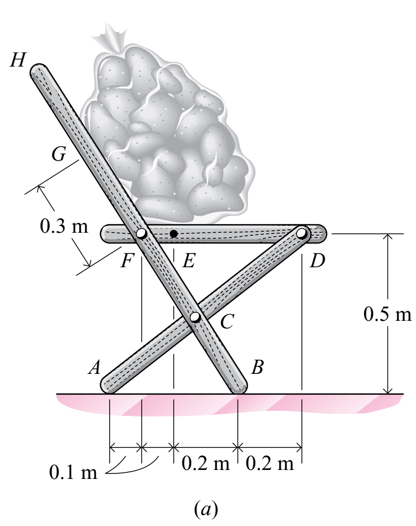


Figure 1-21(a)

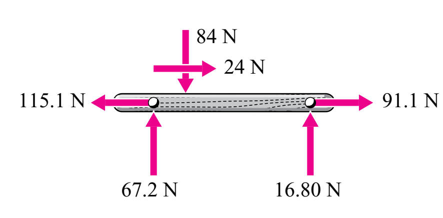


Figure 1-21(b)

SOLUTION

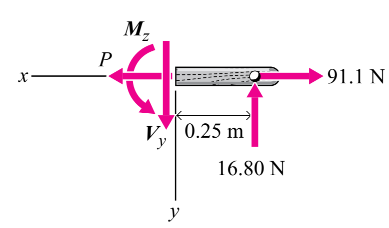
The forces acting on member DEF were found in Example Problem 1-4 and are shown in Fig. 1-21*b*. A section is passed midway between D and F. A free-body diagram of the portion of the member to the right of the section is shown in

Fig. 1-21*c*. Positive axes and internal forces are shown on the free-body diagram. The equations of equilibrium yield

$$+\leftarrow \Sigma F_x = 0: \qquad P - 91.1 = 0 \qquad (a)$$

$$+\downarrow \Sigma F_y = 0: \qquad V_y - 16.80 = 0 \qquad (b)$$

$$+ \quad \Sigma M_z = 0: \qquad M_z + 16.80(0.25) = 0 \qquad (c)$$



(*c*)

Figure 1-21(c)

The internal forces on the section are

$$P = 91.1\,\text{N} \qquad V_y = 16.80\,\text{N} \qquad M_z = -4.20\,\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Students are encouraged to solve the problem using a free-body diagram of a portion of the member to the left of the section.

PROBLEMS

Introductory Problems

1-54* Three forces are applied along the centerline of a steel bar as shown in Fig. P1-54. Determine the internal forces on transverse cross sections in intervals *AB*, *BC*, and *CD* of the bar.

Figure P1-54

1-55* A human femur is modeled in Fig. P1-55. The abductor muscle force is $M = 90$ lb, and the femoral load is $J = 120$ lb. Determine the internal forces on section *a–a*.

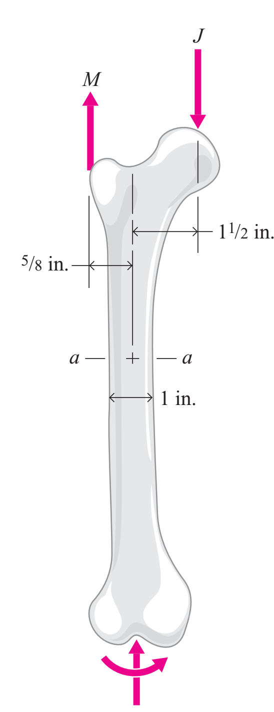


Figure P1-55

1-56 The man shown in Fig. P1-56 has a mass of 75 kg; the beam has a mass of 40 kg. The beam is in equilibrium with the man standing at the end and pulling on the cable. Determine the internal forces on a section perpendicular to and midway between

a. *A* and *B*.
b. *B* and *C*.

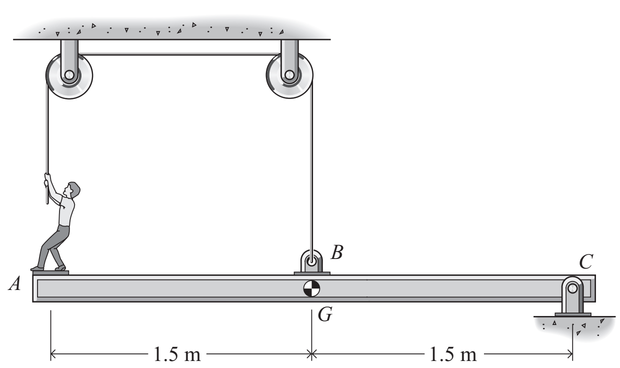


Figure P1-56

1-57* Determine the internal forces on section *a–a* in the angle bracket shown in Fig. P1-57.

Figure P1-57

1-58 Determine the internal forces acting at the centroid of section *A–A* of the C-clamp shown in Fig. P1-58. The force **P** has a magnitude of 2000 N.

Figure P1-58

Intermediate Problems

1-59* Four sets of flexible cables, spaced at 120° intervals, are used to stabilize a 400-ft communications tower. The tower and one cable from each set are shown in Fig. P1-59. The weight of the tower is 40 lb/ft, and the communications equipment at the top weighs 2000 lb. Determine the axial forces (normal forces acting along the centerline) on transverse cross sections at points *A*, *B*, *C*, and *D* of the tower.

Figure P1-59

1-60* Determine the internal forces acting on section *a–a* in the bar rack shown in Fig. P1-60 if each bar has a mass of 50 kg.

Figure P1-60

1-61 The reaction between a crutch and the ground is 35 lb, as shown in Fig. P1-61. Determine the internal forces acting on section *a–a*.

Figure P1-61

1-62 The front-wheel suspension of an automobile is shown in Fig. P1-62. The pavement exerts a vertical force of 2700 N on the tire. Determine the internal forces on a section perpendicular to and midway between *C* and *D*.

Figure P1-62

1-63 A pin-connected system of levers and bars is used as a toggle for a press as shown in Fig. P1-63. Three members are joined by pin *D*, as shown in the insert. Determine the internal forces on a section perpendicular to and midway between *D* and *E* when $P = 1000$ lb.

Figure P1-63

Challenging Problems

1-64* A steel shaft 120 mm in diameter is supported in flexible bearings at its ends. Two pulleys, each 500 mm in diameter, are keyed to the shaft. The pulleys carry belts that produce the forces shown in Fig. P1-64. Determine the internal forces on a vertical section through point *A*.

Figure P1-64

1-65* A device for lifting rectangular objects such as bricks and concrete blocks is shown in Fig. P1-65. The coefficients of friction at all vertical contact surfaces are $\mu_s = 0.4$ and $\mu_k = 0.3$. The device is to lift two blocks, each weighing 15 lb. Determine the internal forces on a vertical section 4 in. to the left of pin *B*.

Figure P1-65

1-66 An automobile engine with a mass of 360 kg is supported by an engine hoist, as shown in Fig. P1-66. Determine the internal forces on section *a–a*.

Figure P1-66

1-67 Determine the internal forces on section *a–a* of the pipe system shown in Fig. P1-67.

Figure P1-67

1-68* Determine the internal forces on section *a–a* in bar ABC of the three-bar frame shown in Fig. P1-68.

Figure P1-68

Computer Problems

1-69 The hook shown in Fig. P1-69 supports a 10-kip load. Plot P, V, and M, the internal forces and moment transmitted by a section of the hook, as a function of the angle θ ($0^\circ \leq \theta \leq 150^\circ$).

Figure P1-69

1-70 Forces of 100 N are being applied to the handles of the vise grip pliers shown in Fig. P1-70. Plot P, V, and M, the internal forces and moment transmitted by section *a–a* of the handle, as a function of the distance d (20 mm $\leq d \leq$ 30 mm).

Figure P1-70

1-71 A 4000-lb cart rolls along a beam as shown in Fig. P1-71.

a. Show that the maximum bending moment in the beam occurs at the wheel that is closer to the middle of the beam.
b. Plot $|M|_{max}$, the maximum bending moment in the beam, as a function of the cart's position x (0 ft $\leq x \leq$ 15 ft).

Figure P1-71

1-72 A group of workers proposes to raise a uniform 250-kg post AB to a vertical position using the rope and brace arrangement shown in Fig. P1-72. Assume that the weight of the brace can be neglected and that end A acts as a frictionless pin for both the 6-m-long post AB and the 6-m-long brace AC. Plot P, V, and M, the internal forces and moment transmitted by section *a–a* of the post, as a function of the distance from A (0 m $\leq b \leq$ 6 m).

Figure P1-72

SUMMARY

Review Problems Force is the action of one body on another. The primary objective of a course in mechanics of materials is the development of relationships between the forces applied to a body and the internal forces and deformations induced in the body.

Forces always exist in equal magnitude, opposite direction pairs. In order to study these forces, it is necessary to isolate the body from its surroundings or from other parts of the body at each point where forces are to be determined. The resultant of the internal force system is then found using the equations of equilibrium. The importance of drawing a correct free-body diagram cannot be overemphasized. The free-body diagram clearly establishes which body or portion of the body is being studied. A correct free-body diagram clearly identifies all forces (both known and unknown) that must be included in the equations of equilibrium.

A rigid body (a body that does not deform under the action of applied loads) is in equilibrium when the resultant of the system of forces acting on the body is zero. This condition is satisfied if the vector sum of all external forces acting on the body is zero and the vector sum of the moments of the external forces about any point *O* (on or off the body) is zero. Although no body is perfectly rigid, most engineering structures and machines are designed "stiff," that is, they do not deform very much. For such problems, the solution of the equilibrium equations often ignores the deformation and treats the structure as though it were rigid. However, we cannot find the exact distribution of the internal forces until we learn how to determine the deformation of the body.

The subject matter of this book forms the basis for the solution of three general types of problems encountered by engineers in the design and analysis of structures and machines:

- **Design**: Given a certain function to perform, of what materials should the machine or structure be constructed, and what should be the sizes and proportions of the various elements?
- **Checking**: Given the completed design, is it adequate—does it perform the function economically and without excessive deformation?
- **Rating**: Given a completed structure or machine, what is its actual load-carrying capacity?

Some knowledge of the physical and mechanical properties of materials is required in order to create a design, to properly evaluate a given design, or even to write the correct relation between an applied load and the resulting deformation of a loaded member. Essential information will be introduced as required.

Don't report final results with more accuracy than is justified by the data and don't round off numbers too much too soon. Although results should always be reported as accurately as possible, the final answer should not be more precise than the data from which it was derived. One of the tasks in all engineering work is to determine the accuracy of the given data and the expected accuracy of the final answer. Results should always reflect the accuracy of the given data. All data in Example Problems and Homework Problems will be assumed sufficiently accurate to justify rounding off the final answer to three or four significant figures.

REVIEW PROBLEMS

1-73* The collar *A* shown in Fig. P1-73 is free to slide on the smooth rod *BC*. Determine the forces exerted on the collar by the cable and by the rod when the force $F = 900$ lb is applied to the collar.

Figure P1-73

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1-74* A 500-kg mass is supported by a four-bar truss as shown in Fig. P1-74. Determine the force in each member of the truss.

Figure P1-74

1-75 The electric motor shown in Fig. P1-75 weighs 25 lb. Due to friction between the belt and pulley, the belt forces have magnitudes of $T_1 = 21$ lb and $T_2 = 1$ lb. Determine the support reactions at *A* and *B*.

Figure P1-75

1-76* Determine the force **P** required to push the 135-kg cylinder over the small block shown in Fig. P1-76.

Figure P1-76

1-77 For the beam shown in Fig. P1-77, determine

a. The reactions at supports *A* and *B*.
b. The internal forces on a transverse cross section 10 ft to the right of support *A*.

Figure P1-77

1-78 The jaws and bolts of the wood clamp in Fig. P1-78 are parallel. The bolts pass through swivel mounts so that no moments act on them. The clamp exerts forces of 300 N on each side of the board. Treat the forces on the boards as uniformly distributed over the contact areas and determine the forces in each of the bolts. Show on a sketch all forces acting on the upper jaw of the clamp.

Figure P1-78

1-79* A three-bar frame is loaded and supported as shown in Fig. P1-79. Determine the internal forces transmitted by

a. Section *a–a* in bar *BEF*.
b. Section *b–b* in bar *ABCD*.

Figure P1-79

1-80 Forces of 5 N are applied to the handles of the paper punch of Fig. P1-80. Determine the force exerted on the paper at *D* and the force exerted on the pin at *B* by handle *ABC*.

Figure P1-80

1-81* A shaft is loaded through a pulley and a lever (Fig. P1-81) that are fixed to the shaft. Friction between the belt and pulley prevents slipping of the belt. Determine the force **P** required for equilibrium and the reactions at supports *A* and *B*. The support at *A* is a ball bearing, and the support at *B* is a thrust bearing. The bearings exert only force reactions on the shaft.

Figure P1-81

1-82* The masses of cartons 1, 2, and 3, which rest on the platform shown in Fig. P1-82, are 300 kg, 100 kg, and 200 kg, respectively. The mass of the platform is 500 kg. Determine the tensions in the three cables *A*, *B*, and *C* that support the platform.

Figure P1-82

1-83 The clamp of Fig. P1-83 is used to hold two boards. If the clamping force is 300 N, determine the internal forces on section *a–a*.

Figure P1-83

1-84 Two bars, a pulley, and a cable are used to support a block as shown in Fig. P1-84. The two bars have negligible weight. The mass of the pulley is 50 kg, and the mass of the block is 100 kg. Determine the internal forces on cross section *a–a* in bar *AB*.

Figure P1-84