

PART I

FUNDAMENTALS

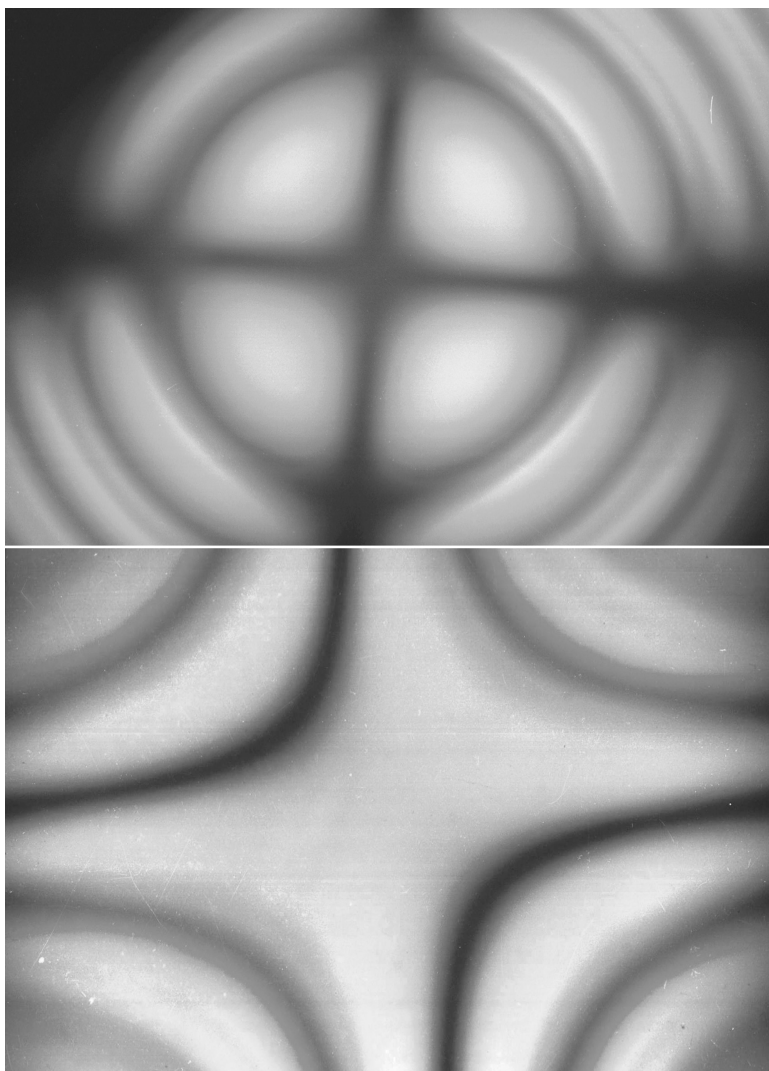


Figure I.1. Naturally birefringent uniaxial lithium niobate crystal view under converging white light between crossed polarizers with its c -axis (optical axis) laying perpendicular to the plane (upper) and on the plane (lower).

INTRODUCTION

Photorefractive crystals are electro-optic and photoconductive materials. An electric field applied to an electro-optic material produces changes in its refractive index, a phenomenon also called Pockel's effect. On the other hand, photoconductivity means that light of adequate wavelength is able to produce electric charge carriers that are free to move by diffusion and also by drift under the action of an electric field.

In the case of photorefractive materials the light excites charge carriers from localized states (photoactive centers) in the forbidden band gap to extended states (conduction or valence bands) where they move, are retrapped and excited again, and so on. During this process the charge carriers progressively accumulate in the darker regions of the sample. In this way, charges of one sign accumulate in the darker regions while leaving charges of the opposite sign in the brighter regions. This spatial modulation of charges produces an associated space-charge electric field. The combination of both effects gives rise to the so-called photorefractive effect: The light produces a photoconduction-based electric field spatial modulation that in turn produces an index of refraction modulation via the electro-optic effect. This change can be reversed by the action of light or by relaxation even in the dark.

The action of light on a photosensitive material may produce changes in the electrical polarizability of the molecules, and by this means a change in the complex index of refraction will result. This change may be sensible or not depending on the wavelength spectral range analyzed. The imaginary part of the index (the extinction coefficient, related to absorption) or the real part (the so-called "index of refraction" itself) may be more affected when observed in a certain wavelength spectral range. This is the case of dyes, some silver salts, chalcogenic glasses, photoresists, and other materials. When sensible changes occur in the real part of the complex index of refraction, these materials are also called "photorefractives" because they actually show changes in the real refractive index under the action of light. These changes can be reversible or not. What is the essential difference between these processes and those we have mentioned before and we are dealing with in this book? The difference is that the latter always involve the establishment of a space-charge electric field and the production of index of refraction changes via the electro-optic (or Pockels) effect. We should therefore rather call them "photo-electro-refractive" materials instead of just using the "photorefractive" label. However, the latter generic name is so widespread nowadays in the scientific literature that it would be hard to change it now. In this book we shall therefore use the term "photorefractive" only, but the reader should be aware that materials of different nature are usually referred to under this same label.

Chapter 1 contains a review of the electro-optic effect including a little bit of tensorial analysis. The effect of an applied electric field over the index ellipsoid of some usual electro-optic crystals is analyzed so that the reader may become familiar with these procedures. We hope these examples will enable the reader to properly handle different materials and optical configurations. Chapter 2 deals with photoconductivity and light-induced absorption and their relation with the localized states (photoactive centers) in the forbidden band.

CHAPTER 1

ELECTRO-OPTIC EFFECT

The electro-optic effect and photoconductivity are the fundamental phenomena underlying the photorefractive effect. Most photorefractive crystals are anisotropic (their properties are different along different directions), and even those that are not become anisotropic under the action of an externally applied electric field. Therefore, we shall start with a review of light propagation in anisotropic media. These materials usually exhibit a piezoelectric effect, too [Yariv, 1985, Shepelevich et al., 1990, Stepanov et al., 1998] but, for the sake of simplicity, we shall not consider it here.

The electro-optic effect in photorefractive materials is of the highest importance because it is at the origin of the “imaging” of a space-charge field modulation into an index of refraction modulation. In fact, the buildup of a holographic grating in photorefractive materials consists of the spatial modulation of the index of refraction in the volume of the sample. In these materials such a modulation arises from the buildup of a modulated space-charge field that in turn modulates the index of refraction via the electro-optic effect.

1.1 LIGHT PROPAGATION IN CRYSTALS

Crystals are in general anisotropic, that is to say, they have different properties for the light propagating along different directions.

1.1.1 Wave Propagation in Anisotropic Media

Let us start with the general vectorial relations

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad (1.1)$$

$$\vec{P} = \epsilon_0 \hat{\chi} \vec{E} \quad (1.2)$$

where $\epsilon_0 = 8.82 \times 10^{-12}$ coul/(mV) is the permittivity of vacuum. The quantities $\vec{P}$, $\vec{E}$, and $\vec{D}$ are the polarization, electric field, and displacement fields, respectively, with $\hat{\chi}$ (polarizability) being a tensor that, for isotropic media only, can be written as a scalar, thus simplifying the relation in Equation (1.2)

$$\vec{P} = \epsilon_0 \chi \vec{E} \quad (1.3)$$

The relation in Equation (1.2) can also be written as

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \epsilon_0 \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (1.4)$$

and also

$$\vec{D} = \epsilon_0 (\hat{1} + \hat{\chi}) \vec{E} \quad (1.5)$$

where $\hat{1}$ and $\hat{\chi}$ are tensors that are written as:

$$\hat{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \hat{\chi} = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{bmatrix} \quad (1.6)$$

Let us recall that there is always a set of coordinate axes, called “principal axes,” where $\hat{\chi}$ assumes a diagonal form

$$\hat{\chi} = \begin{bmatrix} \chi_{11} & 0 & 0 \\ 0 & \chi_{22} & 0 \\ 0 & 0 & \chi_{33} \end{bmatrix} \quad (1.7)$$

1.1.2 General Wave Equation

The equation describing the electromagnetic wave, in nonmagnetic and noncharged media, can be deduced from the Maxwell's equations

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (1.8)$$

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t} + \vec{J} \quad \text{with} \quad \vec{J} = \sigma \vec{E} \quad (1.9)$$

$$\nabla \cdot \vec{E} = -\frac{1}{\epsilon_0} \nabla \cdot \vec{P} \quad (1.10)$$

$$\nabla \cdot \vec{H} = 0 \quad (1.11)$$

In a system of principal coordinate axes it is

$$\begin{aligned} P_1 &= \epsilon_0 \chi_{11} E_1 & D_1 &= \epsilon_{11} E_1 & \epsilon_{11} &= \epsilon_0 (1 + \chi_{11}) \\ P_2 &= \epsilon_0 \chi_{22} E_2 & D_2 &= \epsilon_{22} E_2 & \epsilon_{22} &= \epsilon_0 (1 + \chi_{22}) \\ P_3 &= \epsilon_0 \chi_{33} E_3 & D_3 &= \epsilon_{33} E_3 & \epsilon_{33} &= \epsilon_0 (1 + \chi_{33}) \end{aligned} \quad (1.12)$$

1.1.3 Index Ellipsoid

We shall write the expressions for the electric w_e and magnetic w_m energy densities in electromagnetic waves as [Born and Wolf, 1975]

$$w_e = \frac{1}{2} \vec{E} \cdot \vec{D} = \frac{1}{2} \sum_{kl} E_k \epsilon_{kl} E_l \quad w_m = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{1}{2} \mu H^2 \quad (1.13)$$

and write the Poynting formulation for the energy flux as

$$\vec{S} = \vec{E} \times \vec{H} \quad (1.14)$$

After adequate substitutions and transformations taking into account Maxwell's equations we get, for the principal coordinate axes,

$$\begin{aligned} \frac{D_x^2}{\epsilon_x} + \frac{D_y^2}{\epsilon_y} + \frac{D_z^2}{\epsilon_z} &= 8\epsilon_0 \pi w_e = \text{constant} & \epsilon_x &\equiv \epsilon_{11} = 1 + \chi_{11} \\ & & \epsilon_y &\equiv \epsilon_{22} = 1 + \chi_{22} \\ & & \epsilon_z &\equiv \epsilon_{33} = 1 + \chi_{33} \end{aligned} \quad (1.15)$$

Following the definitions

$$\begin{aligned} x &= \frac{D_x}{\sqrt{w_e \epsilon_0}} \\ y &= \frac{D_y}{\sqrt{w_e \epsilon_0}} \\ z &= \frac{D_z}{\sqrt{w_e \epsilon_0}} \end{aligned}$$

with

$$\begin{aligned} n_x^2 &= \epsilon_x = \epsilon_x / \epsilon_0 \\ n_y^2 &= \epsilon_y = \epsilon_y / \epsilon_0 \\ n_z^2 &= \epsilon_z = \epsilon_z / \epsilon_0 \end{aligned}$$

we get the indicatrix formulation

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (1.16)$$

where n_x , n_y and n_z are the index of refraction along coordinates x , y , and z , respectively, as represented in Figure 1.1. To use this ellipsoid to analyze the

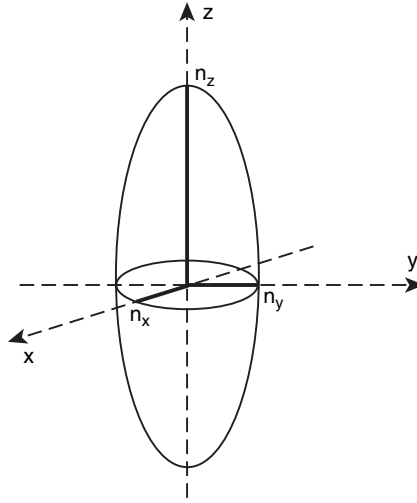


Figure 1.1. Refractive index ellipsoid.

propagation of a plane wave with propagation vector $\vec{k}$ we just intersect the indicatrix with a plane orthogonal to the vector $\vec{k}$. An elliptic figure results where the extraordinary n_e and ordinary n_o indexes, for this wave, are found from the intersection with the corresponding direction of vibration of the electric field as shown in Figure 1.2. In Section 1.2 we shall analyze Equation (1.16) in a more general form.

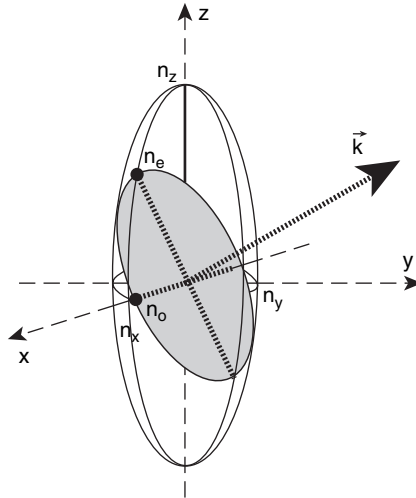


Figure 1.2. Refractive indices for a plane wave propagating in an anisotropic medium.

1.2 TENSORIAL ANALYSIS

Let us write the general equation [Nye, 1979]

$$\sum_{i=1, j=1}^{i=N, j=N} S_{ij} x_i x_j = 1 \quad \text{or} \quad S_{ij} x_i x_j = 1 \quad (1.17)$$

where x_i and x_j are variables and S_{ij} are coefficients. If we assume that $S_{ij} = S_{ji}$, then Equation (1.17) turns into the general ellipsoid representation:

$$S_{11}x_1^2 + S_{22}x_2^2 + S_{33}x_3^2 + 2S_{12}x_1x_2 + 2S_{13}x_1x_3 + 2S_{23}x_2x_3 = 1 \quad (1.18)$$

Equation (1.18) can be transformed into new coordinate axes x'_i , by using the axes rotation transformation matrix, as follows

$$\begin{aligned} x'_1 &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ x'_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ x'_3 &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{aligned} \quad (1.19)$$

which can be written in a matricial form

$$\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (1.20)$$

From the matricial relation above we should deduce that it is also

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} \quad (1.21)$$

The relation above can be written in the form

$$x_i = a_{ki}x'_k \quad x_j = a_{lj}x'_l \quad (1.22)$$

which substituted into Equation (1.18) leads to

$$S_{ij}x_i x_j = S_{ij}a_{ki}a_{lj}x'_k x'_l = S'_{kl}x'_k x'_l \quad (1.23)$$

where S'_{kl} are the coefficients in the new coordinate system. An ellipsoid can be used to describe any symmetric tensor ($S_{ij} = S_{ji}$) of second order and is specially useful to describe any property in a crystal that should be represented by a tensor. An important

property of an ellipsoid is the presence of “principal axes” in which case Equation (1.18) can be simplified to

$$S_{11}x_1^2 + S_{22}x_2^2 + S_{33}x_3^2 = 1 \quad \Rightarrow \quad S_{ij} = \begin{bmatrix} S_{11} & 0 & 0 \\ 0 & S_{22} & 0 \\ 0 & 0 & S_{33} \end{bmatrix} \quad (1.24)$$

1.3 ELECTRO-OPTIC EFFECT

The indicatrix in Equation (1.16) is an ellipsoid in a principal coordinate axes system. Its general formulation is [Nye, 1979]

$$B_{ij}x_i x_j = 1 \quad \text{with} \quad B_{ij} = \frac{1}{\epsilon_{ij}} \quad (1.25)$$

The slight variation in the refractive index produced by an electric field can be described by the third-order electro-optic tensor r_{ijk} (in the range of 10^{-12} m/V for most materials) through the relation

$$\Delta B_{ij} = r_{ijk} E_k \quad (1.26)$$

$$\text{from } B_{ij} = B_{ji} \quad \Rightarrow \quad r_{ijk} = r_{jik} \quad (1.27)$$

The B tensor can be written as

$$\begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} = \begin{bmatrix} B_1 & B_6 & B_5 \\ B_6 & B_2 & B_4 \\ B_5 & B_4 & B_3 \end{bmatrix} \quad (1.28)$$

$$(1.29)$$

The electro-optic relation is therefore simplified to

$$\Delta B_i = r_{ij} E_j \quad (i = 1, 2, 3, 4, 5, 6; j = 1, 2, 3) \quad (1.30)$$

or explicitly written as

$$\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \\ \dots \\ \Delta B_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ \dots & \dots & \dots \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} \Delta E_1 \\ \Delta E_2 \\ \Delta E_3 \end{bmatrix} \quad (1.31)$$

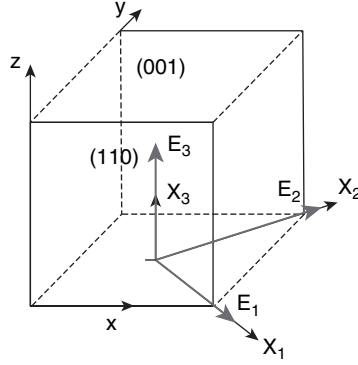


Figure 1.3. Crystallographic axes of a sillenite and an applied 3D electric field.

Let us assume that an electric field is applied, with components E_1, E_2, E_3 as shown in Figure 1.3 so that Equation (1.25) turns into:

$$\begin{aligned}
 & (B_1 + r_{11}E_1 + r_{12}E_2 + r_{13}E_3)x_1^2 + (B_2 + r_{21}E_1 + r_{22}E_2 + r_{23}E_3)x_2^2 \\
 & + (B_3 + r_{31}E_1 + r_{32}E_2 + r_{33}E_3)x_3^2 + (B_4 + 2r_{41}E_1 + 2r_{42}E_2 + 2r_{43}E_3)x_2x_3 \\
 & + (B_5 + 2r_{51}E_1 + 2r_{52}E_2 + 2r_{53}E_3)x_1x_3 \\
 & + (B_6 + 2r_{61}E_1 + 2r_{62}E_2 + 2r_{63}E_3)x_1x_2 = 1
 \end{aligned} \tag{1.32}$$

We are interested in the slow index of refraction buildup produced by the slow accumulation of electric charges. Therefore all the electro-optic coefficients referred to in this chapter are the low-frequency ones only. In the following sections we shall see what Equation (1.32) looks like for some particular materials.

1.3.1 Sillenite-Type Crystal

The well-known crystals of this family are: $\text{Bi}_{12}\text{GeO}_{20}$ (BGO), $\text{Bi}_{12}\text{SiO}_{20}$ (BSO), and $\text{Bi}_{12}\text{TiO}_{20}$ (BTO). They belong to the cubic noncentrosymmetric crystal point class 23 and are piezo-electric, electro-, and elasto-optic and optically active.

BTO is the crystal having the lowest optical activity (optical activity is undesirable for most applications) but is also the most difficult to grow because the chemical composition of the melt and the crystal are different—noncongruent. These crystals are usually grown using the so-called “top seed solution growth” (TSSG) that can be considered a modification of the Czochralski technique. Growing is more easily carried out along the [001]-crystal axis, and during growing there are frequently variations in the growing rate that produce the characteristic striations along the growing direction as shown in the picture in Figure 1.4. The latter result in small variations in the crystal composition and associated index of refraction changes as well. To avoid this index of refraction modulation being too visible through the polished (110)-face (the usual configuration employed for holographic recording) the latter should be cut slanted to these striations as illustrated in Figure 1.5. The axes in the sample are

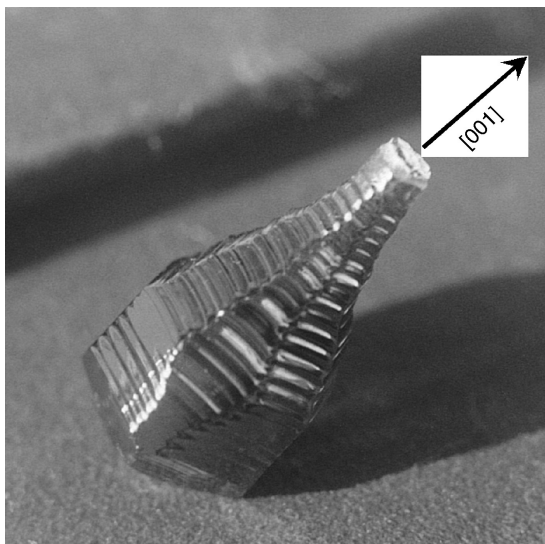


Figure 1.4. Raw $\text{Bi}_{12}\text{TiO}_{20}$ boule grown by TSSG technique. The crystal was grown along the $[001]$ -axis and the striations are clearly perpendicular to this axis.

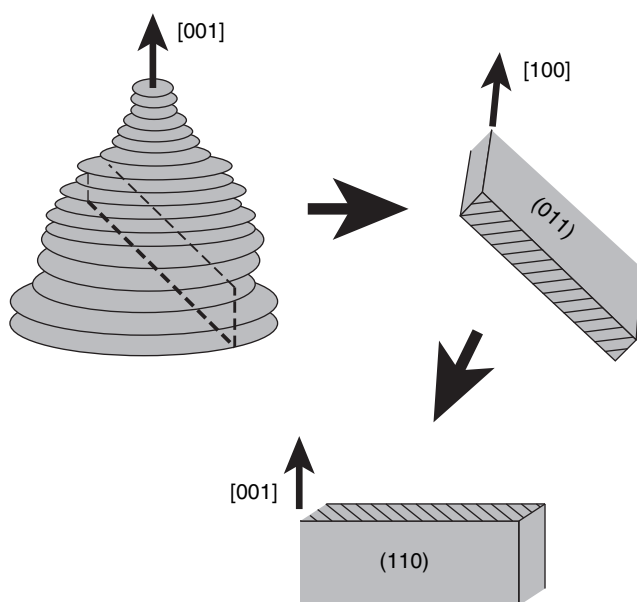


Figure 1.5. From raw $\text{Bi}_{12}\text{TiO}_{20}$ boule to ready-to-use crystal sample. Schematic representation of a raw crystal boule with its striations, indicating the way it will be sliced (top left); already sliced crystal with striations not perpendicular to the (011) -face (top right); ready-to-use crystal with renamed axes (bottom).

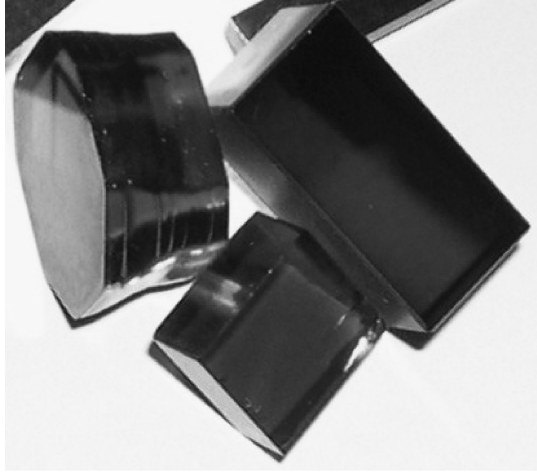


Figure 1.6. Undoped sillenite crystals. $\text{Bi}_{12}\text{SiO}_{20}$ crystal with (110)-surface cut and polished (center), raw $\text{Bi}_{12}\text{TiO}_{20}$ crystal boule grown along [001]-axis and showing striations on the lateral surfaces with both opposite (001)-faces cut and polished (left) and $\text{Bi}_{12}\text{TiO}_{20}$ crystal with (110)-face cut and polished, longer direction along [001]-axis (right).

conveniently renamed, accounting on its cubic and isotropic nature in which case the axes [001], [010], and [100], for example, can be interchanged. In the slanted-sliced sample in Figure 1.5, the striations are not visible through the polished (110)-face. Figure 1.6 shows actual crystal samples.

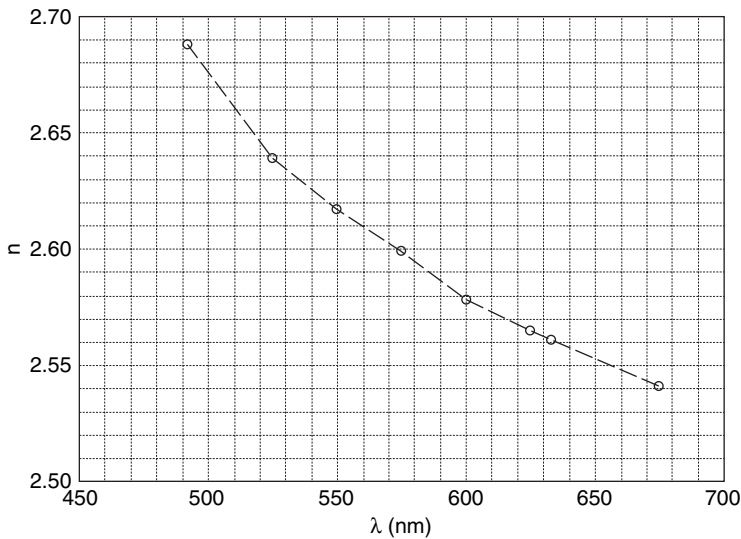


Figure 1.7. Index of refraction of BTO which is formulated by $n = 0.00863/\lambda^4 + 0.0199/\lambda^2 + 2.46$ [Riehemann et al., 1997].

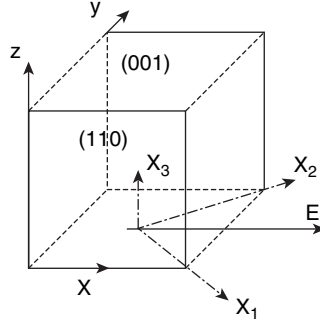


Figure 1.8. $\text{Bi}_{12}\text{SiO}_{20}$ -type cubic crystal orientation and its crystallographic axes X_1, X_2 and X_3 . The electric field E applied along the “ x ”-direction is also shown.

The electro-optic tensor of this crystal family in the principal axes coordinates $[X_1, X_2, X_3]$ has the following elements [Grousseau et al., 1984]:

$$r_{41} = r_{52} = r_{63} \approx 5 \times 10^{-12} \text{ m/V} \quad (1.33)$$

all other elements being zero.

In the absence of electric field ($E = 0$) the ellipsoid is

$$\frac{x_1^2 + x_2^2 + x_3^2}{n_0^2} = 1 \quad (1.34)$$

showing that we are dealing with an isotropic crystal. Applying an electric field along direction “ x ” as indicated in Figure 1.8, we have the field components:

$$E_1 = E_2 = E \frac{\sqrt{2}}{2} \quad E_3 = 0 \quad (1.35)$$

so that the index ellipsoid is modified to:

$$\frac{x_1^2}{n_0^2} + \frac{x_2^2}{n_0^2} + \frac{x_3^2}{n_0^2} + 2r_{41}E_1x_2x_3 + 2r_{52}E_2x_1x_3 = 1 \quad (1.36)$$

or

$$\frac{x_1^2}{n_0^2} + \frac{x_2^2}{n_0^2} + \frac{x_3^2}{n_0^2} + 2r_{41}E \frac{\sqrt{2}}{2}(x_2x_3 + x_1x_3) = 1 \quad (1.37)$$

Let us now rotate the system from coordinates X_1, X_2, X_3 to coordinates X, Y, Z

$$x = (x_1 + x_2) \frac{\sqrt{2}}{2} \quad (1.38)$$

$$y = (x_2 - x_1) \frac{\sqrt{2}}{2} \quad (1.39)$$

$$z = x_3 \quad (1.40)$$

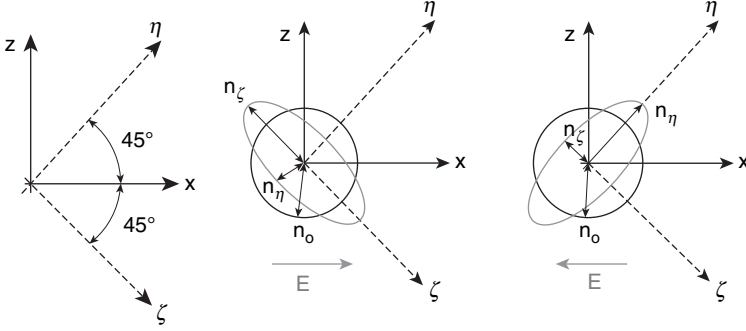


Figure 1.9. Principal coordinate axes system $\eta - \zeta$ arising by the effect of an electric field E applied along the “ x ”-axis, as shown in Fig. 1.8.

which substituted into Equation (1.37) with rearranging gives

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_0^2} + 2r_{41}Exz = 1 \quad (1.41)$$

To eliminate the above term in “ xz ” it is necessary to carry out another rotation, now in the “ x - z ” plane as shown in Figure 1.9

$$x = (\eta + \zeta) \frac{\sqrt{2}}{2} \quad (1.42)$$

$$z = (\eta - \zeta) \frac{\sqrt{2}}{2} \quad (1.43)$$

which substituted into Equation (1.41) gives the relation

$$\zeta^2 \left(\frac{1}{n_0^2} - r_{41}E \right) + \eta^2 \left(\frac{1}{n_0^2} + r_{41}E \right) + \frac{y^2}{n_0^2} = 1 \quad (1.44)$$

which means that the refractive indexes along the new axes ζ , η , and y are:

$$n_\zeta = n_0 + \frac{1}{2}n_0^3r_{41}E \quad (1.45)$$

$$n_\eta = n_0 - \frac{1}{2}n_0^3r_{41}E \quad (1.46)$$

$$n_y = n_0 \quad (1.47)$$

for $n_0 \gg n_0^3r_{41}E/2$. The wavelength dependence of n_0 for BTO is reported in Figure 1.7.

Exercise: Following the mathematical development above, show that for an electric field E along the axis [001] the principal axes of the index ellipsoid are directed along x , y , and z with the index ellipsoid having the form

$$x^2 \left(\frac{1}{n_0^2} + r_{63}E \right) + y^2 \left(\frac{1}{n_0^2} - r_{63}E \right) + \frac{z^2}{n_0^2} = 1 \quad (1.48)$$

thus meaning that, in the input crystal plane (110) that is also the x - z plane, the index of refraction changes only along x and is constant along z .

GaAs, InP, and CdTe are also cubic noncentrosymmetric crystals though belong to the point class $\bar{4}3m$ but have the same electro-optic tensor structure as sillenites, that is to say, all elements are zero except

$$r_{41} = r_{52} = r_{63} = 1.72 \text{ pm/V} \quad \text{for GaAs} \quad (1.49)$$

$$r_{41} = r_{52} = r_{63} = 1.34 \text{ pm/V} \quad \text{for InP} \quad (1.50)$$

$$r_{41} = r_{52} = r_{63} = 5.5 \text{ pm/V} \quad \text{for CdTe} \quad (1.51)$$

The $\bar{4}3m$ symmetry, however, guarantees that there is no optical activity. The index of refraction of CdTe varies from 2.86 at $\lambda = 1.06 \mu\text{m}$ to 2.73 at $\lambda = 1.55 \mu\text{m}$ and follows the relation [Verstraeten, 2002]:

$$n^2 = 4.744 + \frac{2.424\lambda^2}{\lambda^2 - 282181.61} \quad (1.52)$$

1.3.2 Lithium Niobate

The electro-optic tensor in the principal axes system $[X_1, X_2, X_3]$ for this material has zero elements everywhere except the following [Weis and Gaylord, 1985]:

$$\begin{aligned} r_{12} = -r_{22} = r_{61} &\approx 6.8 \text{ pm/V} & r_{13} = r_{23} &= 10.0 \text{ pm/V} \\ r_{33} &= 32.2 \text{ pm/V} & r_{42} = r_{51} &= 32 \text{ pm/V} \end{aligned} \quad (1.53)$$

For an electric field E_3 applied along axis x_3 as shown in Figure 1.10, tensorial Equation (1.32) becomes:

$$\left(\frac{1}{n_0^2} + r_{13}E_3 \right) x_1^2 + \left(\frac{1}{n_0^2} + r_{13}E_3 \right) x_2^2 + \left(\frac{1}{n_e^2} + r_{33}E_3 \right) x_3^2 = 1 \quad (1.54)$$

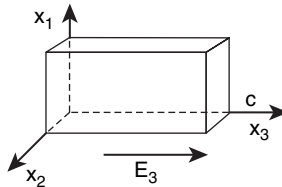


Figure 1.10. Lithium niobate crystal with an applied electric field along the photovoltaic c -axis.

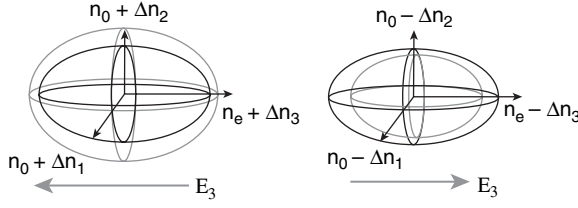


Figure 1.11. Lithium niobate crystal ellipsoid (black) and its modified (gray) size by the action of an applied field in opposite directions (left and right pictures) along the c -axis.

with $n_0 = 2.286$ and $n_e = 2.200$ at $\lambda = 633$ nm [Yariv, 1985] and the following relations

$$\Delta\left(\frac{1}{n_1^2}\right) = -2\frac{1}{n_0^3}\Delta(n_1) = r_{13}E_3 \Rightarrow \Delta(n_1) = -\frac{n_0^3}{2}r_{13}E_3 \quad (1.55)$$

$$\Delta\left(\frac{1}{n_2^2}\right) = -2\frac{1}{n_0^3}\Delta(n_2) = r_{13}E_3 \Rightarrow \Delta(n_2) = -\frac{n_0^3}{2}r_{13}E_3 \quad (1.56)$$

$$\Delta\left(\frac{1}{n_3^2}\right) = -2\frac{1}{n_e^3}\Delta(n_3) = r_{33}E_3 \Rightarrow \Delta(n_3) = -\frac{n_e^3}{2}r_{33}E_3 \quad (1.57)$$

and the index ellipsoid is modified as shown in Figure 1.11.

1.3.3 KDP-(KH_2PO_4)

This crystal is actually not a photorefractive one but is included here as an example of electro-optic tensor somewhat similar to that of silleenites. It has the following electrooptic tensor:

$$r_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \quad r_{41} = r_{52} = 8.6 \text{ pm/V} \quad r_{63} = 10.6 \text{ pm/V} \quad (1.58)$$

The index of refraction for this material is reported Table 1.1.

TABLE 1.1. Index of Refraction of KDP

λ (nm)	n_0	n_e
546	1.5115	1.4698
633	1.5074	1.4669

The indicatrix equation formulated in the principal coordinate (crystallographic) axes X_1 , X_2 , and X_3 , as represented in Figure 1.8, is

$$\frac{x_1^2}{n_0^2} + \frac{x_2^2}{n_0^2} + \frac{x_3^2}{n_e^2} + 2r_{41} E_1 x_2 x_3 + 2r_{52} E_2 x_1 x_3 + 2r_{63} E_3 x_1 x_2 = 1 \quad (1.59)$$

Let us assume an externally applied field E_3 along axis x_3 only. In this case we should proceed as for the case of $\text{Bi}_{12}\text{SiO}_{20}$ in Figure 1.9 to get the following ellipsoid

$$\zeta^2 \left(\frac{1}{n_0^2} - r_{63} E_3 \right) + \eta^2 \left(\frac{1}{n_0^2} + r_{63} E_3 \right) + \frac{y^2}{n_e^2} = 1 \quad (1.60)$$

with

$$n_\zeta = n_0 + \frac{1}{2} n_0^3 r_{63} E_3 \quad (1.61)$$

$$n_\eta = n_0 - \frac{1}{2} n_0^3 r_{63} E_3 \quad (1.62)$$

$$n_y = n_e \quad (1.63)$$

1.4 CONCLUDING REMARKS

The aim of this chapter was just to recall some fundamental properties of optically anisotropic materials and the way an electric field is able to modify the index ellipsoid via the electro-optic effect. We have briefly shown how to calculate these effects in a few kinds of crystals having different electro-optic tensors. We hope these examples will enable the reader to understand how to operate on different materials, different crystals, and different optical configurations.