Chapter 1

Introduction

We live in the world of digital technology that surrounds us and without which we can barely function. There are myriads of examples (which we take for granted) in which computers bring a wealth of services. Computers constitute an omnipresent fabric of the society (Vasilakos and Pedrycz, 2006). As once succinctly captured by Weiser (1991), "the most profound technologies are those that disappear. They weave themselves into the fabric of everyday life until they are indistinguishable from it."

There is an ongoing challenge of building intelligent systems whose functionality could make them predominantly human centric. Human centricity is one of the driving forces of ubiquitous and pervasive computing. Although there are interesting developments along this line, there is a still a long way to go. Some important milestones have been achieved, yet a lot of challenges lie ahead.

In this chapter, we investigate some fundamental features of human centricity of intelligent systems and in this context raise a need for comprehensive studies in information granulation and fuzzy sets, in particular.

1.1 DIGITAL COMMUNITIES AND A FUNDAMENTAL QUEST FOR HUMAN-CENTRIC SYSTEMS

Problem solving, design, and creative thinking—these are all endeavors in which we are inherently faced with conflicting requirements, incomplete information, numerous constraints, and finally collections of alternative solutions. All of these lead us to situations in which we have to effectively manage enormous amounts of heterogeneous data, deal with conflicting or missing evidence, and arrive at meaningful conclusions being aware of the confidence associated with our findings.

In spite of ever growing complexity of the problems, we somewhat manage to develop solutions. Both in analysis and in design (synthesis), we follow the key principles of abstraction and decomposition that help us handle a phenomenon of complexity and arrive at meaningful solutions. In essence, the effective use of abstraction means that instead of being buried in a flood of details and mountains

Fuzzy Systems Engineering: Toward Human-Centric Computing, by Witold Pedrycz and Fernando Gomide

of data, we establish certain, perhaps most suitable conceptual perspective and set up a framework in which the problems could be tackled. Granularity of the problem representation is a fundamental manifestation of the principle of abstraction. The decomposition is a meaningful and commonly used strategy in which on the basis of some prudently established granularity we solve the problem by isolating its loosely connected subproblems and handling them on an individual basis.

Computing systems that are around us in so visible abundance operate on completely different principles of binary (Boolean logic), numeric information and solutions, and predefined models of the world of two-valued logic and human information processing. It becomes apparent that we are concerned with two conceptually distinct worlds. To make them work together and take full advantage of the computing faculties, we need a well-developed interface through which both worlds could talk to each other. This is the key rationale behind the emergence of human-centric systems and human-centric computing (HC^2). The primary objective of the HC^2 is to make computers adjust to people by being more *natural* and *intuitive* to use and seamlessly integrated within the existing environment. Various pursuits along the line of e-society include intelligent housing, ambient intelligence (Vasilakos and Pedrycz, 2006) and ubiquitous computing, semantic web, e-health, e-commerce and manufacturing, sensor networks, intelligent data analysis, and wearable hardware. All of these are concrete examples of the general tendency existing in the development of HC^2 systems. Referring to the general architectural framework as portrayed in Figure 1.1, we easily note that in such endeavors a middleware of the semantic layer plays a crucial role in securing all necessary efficient interaction and communication between various sources of data and groups of users coming with their diversified needs and objectives. In the development of HC^2 systems, we are ultimately faced with an omnipresent challenge known as a semantic gap. To alleviate its consequences, we have to focus on how to reconcile and interpret detailed numeric information with the qualitative, descriptive, and usually linguistic input coming from the user. For instance, in the design of a typical HC^2 system, such

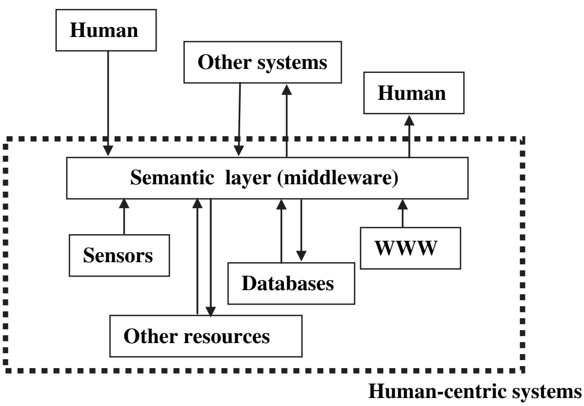


Figure 1.1 An overall architecture of human-centric systems; note a critical role of the semantic layer linking the layers of computing and humans together.

Table 1.1 Selected Examples of Human-Centric Systems and their Underlying Objectives.

Area	Key objectives, existing trends, and solutions
Intelligent data analysis	Effective explanatory analysis, delivery of findings at the level of information granules, and effective mechanisms of summarization.
System modeling	Building transparent models that could be easily interpreted and whose outcomes are readily understood. Models should help the user justify decisions being taken.
Adaptive hypermedia	Personalization of hypermedia to meet needs of individual users, development of specialized web services, building collaborative filtering, recommendation, content-based filtering, personalization of web engines, and so on.
e-commerce	Expressing preferences of customers formulated at different levels of specificity (granularity).
Intelligent interfaces	Face expression, emotion recognition and tracking, formation and use of face-related features.

as a personalized digital photo album, we encounter a lot of detailed numeric data (pixels of images) and have to accommodate a significant and highly descriptive user's input that comes in the form of some relevance feedback. The context awareness and personalization invoke numerous collaborative aspects of processing involving various sources of data and information (including those available directly from the users). The user-based processing capability is an important aspect of HC^2 systems that has to be taken into account in any design considerations.

The crux of the semantic layer lies in the formation and usage of entities that are easily perceived and processed by humans. The difficulty is that the world of numeric processing has to interact with humans who are quite resistant to the explicit use of numbers and uncomfortable to process them. We operate at the higher level of abstraction, and this essential design perspective has to be embraced by human-centric systems through their underlying functionality.

Let us offer a sample of examples in which human centricity plays a pivotal role (Table 1.1) (Frias-Martinez et al., 2005; Perkowitz and Etzioni, 2000; Spott and Nauck, 2006). Most of them heavily rely on the idea of an effective relevance feedback that needs to be implemented in an efficient manner.

1.2 A HISTORICAL OVERVIEW: TOWARDS A NON-ARISTOTELIAN PERSPECTIVE OF THE WORLD

From the brief investigations covered above, it becomes apparent that in the realization of the quest for human centricity of systems, the leitmotiv of many investigations is in building effective mechanisms of communication including various schemes of relevance feedback. Given that human processing is carried out at some level of abstraction, a concept of information granules and information granulation plays a visible role. The question of dichotomy offered by some formal frameworks of information granules has to be revisited as well.

The concept of dichotomy becomes profoundly imprinted into our education, philosophy, and many branches of science, management, and engineering. Although the formalism and vocabulary of Boolean concepts being effective in handling various discrimination processes involving binary quantification (yes–no, true–false) has been with us from the very beginning of our education, it becomes evident that this limited, two-valued view at world is painfully simplified and in many circumstances lacks rapport with the reality. In real world, there is nothing like black–white, good–bad, and so on. All of us recognize that the notion of dichotomy is quite simple and does not look realistic. Concepts do not possess sharp boundaries. Definitions are not binary unless they tackle very simple concepts (say odd–even numbers). Let us allude here to the observation made by Russell (1923)

> *"... the law of excluded middle is true when precise symbols are employed, but it is not true when symbols are vague, as, in fact, all symbols are."*

In reality, we use terms whose complexities are far higher and which depart from the principle of dichotomy. Consider the notions used in everyday life such as *warm* weather, *low* inflation, *long* delay, and so on. How could you define them if you were to draw a single line? Is 25°C *warm*? Is 24.9°C *warm*? Or is 24.95°C *warm* as well? Likewise in any image: Could you draw a single line to discriminate between objects such as sky, land, trees, and lake. Evidently, as illustrated in Figure 1.2, identifying boundaries delineating the objects in this way is a fairly futile task and in many cases produces pretty much meaningless results. Objects in images do not exhibit clear and unique boundaries (the location of the horizon line is not obvious at all) (Fig. 1.2(a)). Experimental data do not come in well-formed and distinct clusters; there are always some points in-between (Fig. 1.2(b)).

One might argue that these are concepts that are used in everyday language and, therefore, they need not possess any substantial level of formalism. Yet, one has to admit that the concepts that do not adhere to the principle of dichotomy are also

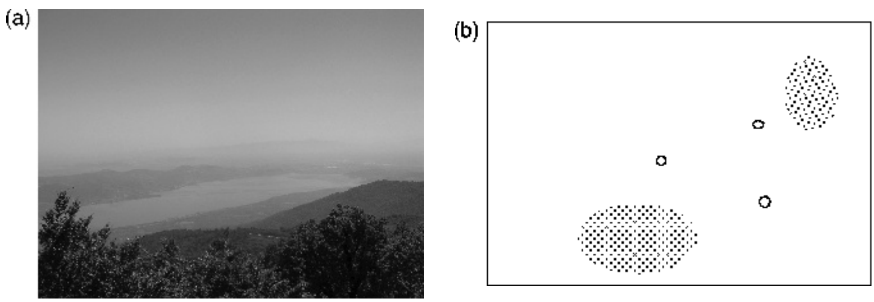


Figure 1.2 Objects, as we perceive and describe them, do not exhibit sharp boundaries. Such boundaries implementing a principle of dichotomy exhibit limitations. Practically, they may not exist at all: (a) images and (b) experimental data.

visible in science, mathematics, and engineering. For instance, we often carry out a linear approximation of nonlinear function and make a quantifying statement that such linearization is valid in some *small* neighborhood of the linearization point. Under these circumstances, the principle of dichotomy does not offer too much.

The principle of dichotomy, or as we say an Aristotelian perspective at the description of the world, has been subject to a continuous challenge predominantly from the standpoint of philosophy and logic. Let us recall some of the most notable developments that have led to the revolutionary paradigm shift. Indisputably, the concept of a three-valued and multivalued logic put forward by Jan Lukasiewicz and then pursued by others, including Emil Post, is one of the earliest and the most prominent logical attempts made toward the direction of abandoning the supremacy of the principle of dichotomy. As noted by Lukasiewicz (1920, 1930,) the question of the suitability or relevance of two-valued logic in evaluating the truth of propositions was posed in the context of those statements that allude to the future. "Tomorrow will rain." Is this statement true? If we can answer this question, this means that we have already predetermined the future. We start to sense that this two-valued model, no matter how convincing it could be, is conceptually limited if not wrong. The non-Aristotelian view of the world was vividly promoted by Korzybski (1933). Although the concept of the three-valued logic was revolutionary in 1920s, we somewhat quietly endorsed it over the passage of time. For instance, in database engineering, a certain entry may be two-valued (yes–no), but the third option of "unknown" is equally possible—here we simply indicate that no value of this entry has been provided.

1.3 GRANULAR COMPUTING

Information granules permeate human endeavors (Zadeh, 1973, 1979, 1996, 1997, 2005; Pedrycz, 2001; Bargiela and Pedrycz, 2003). No matter what problem is taken into consideration, we usually cast it into a certain conceptual framework of basic entities, which we regard to be of relevance to the problem formulation and problem solving. This becomes a framework in which we formulate generic concepts adhering to some level of abstraction, carry out processing, and communicate the results to the external environment. Consider, for instance, image processing. In spite of the continuous progress in the area, a human being assumes a dominant and very much uncontested position when it comes to understanding and interpreting images. Surely, we do not focus our attention on individual pixels and process them as such but group them together into semantically meaningful constructs—familiar objects we deal with in everyday life. Such objects involve regions that consist of pixels or categories of pixels drawn together because of their proximity in the image, similar texture, color, and so on. This remarkable and unchallenged ability of humans dwells on our effortless ability to construct information granules, manipulate them, and arrive at sound conclusions. As another example, consider a collection of time series. From our perspective, we can describe them in a semiqualitative manner by pointing at specific regions of such signals. Specialists can effortlessly interpret electrocardiograms (ECG signals). They distinguish some segments of such signals and interpret their combinations. Experts can interpret temporal readings of sensors and assess the status of the monitored system. Again, in all these situations, the individual samples of the signals are not the focal point of the analysis and the ensuing signal interpretation. We always granulate all phenomena (no matter if they are originally discrete or analog in their nature). Time is another important variable that is subjected to granulation. We use seconds, minutes, days, months, and years. Depending on which specific problem we have in mind and who the user is, the size of information granules (time intervals) could vary quite dramatically. To the high-level management, time intervals of quarters of year or a few years could be meaningful temporal information granules on the basis of which one develops any predictive model. For those in charge of everyday operation of a dispatching power plant, minutes and hours could form a viable scale of time granulation. For the designer of high-speed integrated circuits and digital systems, the temporal information granules concern nanoseconds, microseconds, and perhaps seconds. Even such commonly encountered and simple examples are convincing enough to lead us to ascertain that (a) information granules are the key components of knowledge representation and processing, (b) the level of granularity of information granules (their size, to be more descriptive) becomes crucial to the problem description and an overall strategy of problem solving, and (c) there is no universal level of granularity of information; the size of granules is problem oriented and user dependent.

What has been said so far touched a qualitative aspect of the problem. The challenge is to develop a computing framework within which all these representation and processing endeavors could be formally realized. The common platform emerging within this context comes under the name of granular computing. In essence, it is an emerging paradigm of information processing. Although we have already noticed a number of important conceptual and computational constructs built in the domain of system modeling, machine learning, image processing, pattern recognition, and data compression in which various abstractions (and ensuing information granules) came into existence, granular computing becomes innovative and intellectually proactive in several fundamental ways:

- It identifies the essential commonalities between the surprisingly diversified problems and technologies used there, which could be cast into a unified framework we usually refer to as a granular world. This is a fully operational processing entity that interacts with the external world (that could be another granular or numeric world) by collecting necessary granular information and returning the outcomes of the granular computing.
- With the emergence of the unified framework of granular processing, we get a better grasp as to the role of interaction between various formalisms and visualize a way in which they communicate.
- It brings together the existing formalisms of set theory (interval analysis), fuzzy sets, rough sets, and so on under the same roof by clearly visualizing that in spite of their visibly distinct underpinnings (and ensuing processing), they exhibit some fundamental commonalities. In this sense, granular computing

establishes a stimulating environment of synergy between the individual approaches.

- By building upon the commonalities of the existing formal approaches, granular computing helps build heterogeneous and multifaceted models of processing of information granules by clearly recognizing the orthogonal nature of some of the existing and well-established frameworks (say, probability theory coming with its probability density functions and fuzzy sets with their membership functions).
- Granular computing fully acknowledges a notion of variable granularity whose range could cover detailed numeric entities and very abstract and general information granules. It looks at the aspects of compatibility of such information granules and ensuing communication mechanisms of the granular worlds.
- Interestingly, the inception of information granules is highly motivated. We do not form information granules without reason. Information granules are an evident realization of the fundamental paradigm of abstraction.

Granular computing forms a unified conceptual and computing platform. Yet, it directly benefits from the already existing and well-established concepts of information granules formed in the setting of set theory, fuzzy sets, rough sets and others. Let us now take a quick look at the fundamental technologies of information granulation and contrast their key features.

1.3.1 Sets and Interval Analysis

Sets are fundamental concepts of mathematics and science. Referring to the classic notes, set is described as "any multiplicity, which can be thought of as one… *any totality of definite elements, which can be bound up into a whole by means of a law" or being more descriptive "…any collection into a whole M of definite and separate objects m of our intuition or our thought"* (Cantor, 1883, 1895). Likewise, interval analysis ultimately dwells upon a concept of sets, which in this case are collections of elements in the line of reals, say $[a,b]$, $[c,d]$,… and so on. Multidimensional constructs are built upon Cartesian products of numeric intervals and give rise to computing with hyperboxes. Going back to the history, computing with intervals is intimately linked with the world of digital technology. One of the first papers in this area was published in 1956 by Warmus. Some other early research was done by Sunaga and Moore (1966). This was followed by a wave of research in so-called interval mathematics or interval calculus. Conceptually, sets (intervals) are rooted in a two-valued logic with their fundamental predicate of membership ($\in$). Here holds an important isomorphism between the structure of two-valued logic endowed with its truth values (false–true) and set theory with sets being fully described by their characteristic functions. The interval analysis is a cornerstone of reliable computing, which in turn is ultimately associated with digital computing in which any variable is associated with a finite accuracy (implied by the fixed number of bits used to represent numbers). This limited accuracy gives rise to a certain pattern of propagation of

Table 1.2 Arithmetic Operations on Numeric Intervals A and B.

Algebraic operation	Result
Addition	$[a + c, b + d]$
Subtraction	$[a - d, b - c]$
Multiplication	$[\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$
Division	$\left[\min\left(\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\right), \max\left(\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\right)\right]$ assumption: the interval $[c, d]$ does not contain 0

error of computing. For instance, addition of two intervals $[a, b]$ and $[c, d]$ leads to a broader interval in the form $[a + c, b + d]$ (Hansen, 1975; Jaulin et al., 2001; Moore, 1966). Here, the accumulation of uncertainty (or equivalently the decreased granularity of the result) depends upon the specific algebraic operation completed for given intervals. Table 1.2 summarizes four algebraic operations realized on numeric intervals $A = [a, b]$ and $B = [c, d]$.

Interestingly, intervals distributed uniformly in a certain space are at the center of any mechanism of analog-to-digital conversion; the higher the number of bits, the finer the intervals and the higher their number. The well-known fundamental relationship states that with n bits we can build a collection of 2^n intervals of width $(b - a)/2^n$ for the original range of numeric values in $[a, b]$. Intervals offer a straightforward mechanism of abstraction: all elements lying within a certain interval become indistinguishable and therefore are treated as identical. In addition to algebraic manipulation, the area of interval mathematics embraces a wealth of far more advanced and practically relevant processing including differentiation, integral calculus, as well as interval-valued optimization.

1.3.2 The Role of Fuzzy Sets: A Perspective of Information Granules

Fuzzy sets offer an important and unique feature of describing information granules whose contributing elements may belong to varying degrees of membership (belongingness). This helps us describe the concepts that are commonly encountered in real world. The notions, such as *low* income, *high* inflation, *small* approximation error, and many others, are examples of concepts to which the yes–no quantification does not apply or becomes quite artificial and restrictive. We are cognizant that there is no way of quantifying the Boolean boundaries as there are a lot of elements whose membership to the concept is only partial and quite different from 0 and 1.

The binary view of the world supported by set theory and two-valued logic has been vigorously challenged by philosophy and logic. The revolutionary step in logic was made by Lukasiewicz with his introduction of three and afterward multivalued logic (Lukasiewic, 1930, 1970). It took 'however' more decades to dwell on the ideas of the non-Aristotelian view of the world before fuzzy sets were introduced. This happened in 1965 with the publication of the seminal paper on fuzzy sets by Zadeh (1965). Refer also to other influential papers by Zadeh (1979, 1996, 1997, 1999, 2005). The concept of fuzzy set is surprisingly simple and elegant. Fuzzy set A captures its elements by assigning them to it with some varying degrees of membership. A so-called membership function is a vehicle that quantifies different degrees of membership. The higher the degree of membership $A(x)$, the stronger is the level of belongingness of this element to A (Gottwald, 2005; Zimmermann, 1996).

The obvious yet striking difference between sets (intervals) and fuzzy sets lies in the notion of partial membership supported by fuzzy sets. In fuzzy sets, we discriminate between elements that are "typical" to the concept and those of borderline character. Information granules such as *high* speed, *warm* weather, *fast* car are examples of information granules falling under this category and can be conveniently represented by fuzzy sets. As we cannot specify a single, well-defined element that forms a solid border between full belongingness and full exclusion, fuzzy sets offer an appealing alternative and a practical solution to this problem. Fuzzy sets with their smooth transition boundaries form an ideal vehicle to capture the notion of partial membership. In this sense, information granules formalized in the language of fuzzy sets support a vast array of human-centric pursuits. They are predisposed to play a vital role when interfacing human to intelligent systems.

In problem formulation and problem solving, fuzzy sets emerge in two fundamentally different ways.

Explicit. Here, they typically pertain to some generic and fairly basic concepts we use in our communication and description of reality. There is a vast amount of examples as such concepts being commonly used every day, say *short* waiting time, *large* dataset, *low* inflation, *high* speed, *long* delay, and so on. All of them are quite simple as we can easily capture their meaning. We can easily identify a universe of discourse over which such variable are defined. For instance, this could be time, number of records, velocity, and alike.

Implicit. Here we are concerned with more complex and inherently multifaceted concepts and notions where fuzzy sets could be incorporated into the formal description and quantification of such problems, yet not in so instantaneous manner. Some examples could include concepts such as "*preferred* car," "*stability* of the control system," "*high performance* computing architecture," "*good convergence* of the learning scheme," "*strong* economy," and so on. All of these notions incorporate some components that could be quantified with the use of fuzzy sets, yet this translation is not that completely straightforward and immediate as it happens for the category of the explicit usage of fuzzy sets. For instance, the concept of "*preferred* car" is evidently multifaceted and may involve a number of essential descriptors that when put together are really reflective of the notion we have in mind. For instance, we may involve a number of qualities such as speed, economy, reliability, depreciation, maintainability, and alike. Interestingly, each of these features could be easily rephrased in simpler terms and through this process at some level of this refinement phase, we may arrive at fuzzy sets that start to manifest themselves in an explicit manner.

Table 1.3 Examples of Concepts Whose Description and Processing Invoke the Use of Fuzzy Sets and Granular Computing.

p. 65: *small* random errors in the measurement vector…
p. 70: The success of the method depends on whether the first initial guess is already *close enough* to the global minimum…
p. 72: Hence, the convergence region of a numerical optimizer will be *large*
(van der Heijden et al., 2004).
p. 162: Comparison between bipolar and MOS technology (a part of the table)

	bipolar	MOS
integration	*low*	*very high*
power	*high*	*low*
cost	*low*	*low*

(Katz and Borriello, 2005).
p. 50: validation costs are *high* for *critical systems*
p. 660: …A *high* value for fan-in means that X is *highly coupled* to the rest of the design and changes to X will have extensive knock-on effect. A *high* value for fan-out suggests that the overall complexity of X may be *high* because of the complexity of control logic needed to coordinate the called components.
… Generally, the *larger* the size of the code of a component, the more *complex* and error-prone the component is likely to be…
… The *higher* the value of the Fog index, the more difficult the document is to understand
(Sommerville, 2007).

As we stressed, the omnipresence of fuzzy sets is surprising. Even going over any textbook or research monograph, not mentioning newspapers and magazines, we encounter a great deal of fuzzy sets coming in their implicit or explicit format. Table 1.3 offers a handful of selected examples.

From the optimization standpoint, the properties of continuity and commonly encountered differentiability of the membership functions become a genuine asset. We may easily envision situations where those information granules incorporated as a part of the neurofuzzy system are subject to optimization—hence the differentiability of their membership functions becomes critical relevance. What becomes equally important is the fact that fuzzy sets bridge numeric and symbolic concepts. On one hand, fuzzy set can be treated as some symbol. We can regard it as a single conceptual entity by assigning to it some symbol, say L (for *low*). In the sequel, it could be processed as a purely symbolic entity. On the other hand, a fuzzy set comes with a numeric membership function and these membership grades could be processed in a numeric fashion.

Fuzzy sets can be viewed from several fundamentally different standpoints. Here we emphasize the four of them that play a fundamental role in processing and knowledge representation.

As an Enabling Processing Technology of Some Universal Character and of Profound Human-Centric Character

Fuzzy sets build upon the existing information technologies by forming a user-centric interface using which one could communicate essential design knowledge thus guiding problem solving and making it more efficient. For instance, in signal processing and image processing we might incorporate a collection of rules capturing specific design knowledge about filter development in a certain area. Say, "if the level of noise is *high*, consider using a *large* window of averaging." In control engineering, we may incorporate some domain knowledge about the specific control objectives. For instance, "if the constraint of fuel consumption is *very important*, consider settings of a PID controller producing *low* overshoot." Some other examples of highly representative human-centric systems concern those involving (a) construction and usage of relevance feedback in retrieval, organization, and summarization of video and images, (b) queries formulated in natural languages, and (c) summarization of results coming as an outcome of some query.

Second, there are unique areas of applications in which fuzzy sets form a methodological backbone and deliver the required algorithmic setting. This concerns fuzzy modeling in which we start with collections of information granules (typically realized as fuzzy sets) and construct a model as a web of links (associations) between them. This approach is radically different from the numeric, function-based models encountered in "standard" system modeling. Fuzzy modeling emphasizes an augmented agenda in comparison with the one stressed in numeric models. Whereas we are still concerned with the accuracy of the resulting model, its interpretability and transparency become of equal and sometimes even higher relevance.

It is worth stressing that fuzzy sets provide an additional conceptual and algorithmic layer to the existing and well-established areas. For instance, there are profound contributions of fuzzy sets to pattern recognition. In this case, fuzzy sets build upon the well-established technology of feature selection, classification, and clustering.

Fuzzy sets are an ultimate mechanism of communication between humans and computing environment. The essence of this interaction is illustrated in Figure 1.3(a). Any input is translated in terms of fuzzy sets and thus made comprehensible at the level of the computing system. Likewise, we see a similar role of fuzzy sets when communicating the results of detailed processing, retrieval, and alike. Depending upon application and the established mode of interaction, the communication layer may involve a substantial deal of processing of fuzzy sets. Quite often, we combine the mechanisms of communication and represent them in a form of a single module (Fig. 1.3(b)). This architectural representations stress the human-centricity aspect of the developed systems.

As an Efficient Computing Framework of Global Character

Rather than processing individual elements, say a single numeric datum, an encapsulation of a significant number of the individual elements that is realized in the form of some fuzzy sets, offers immediate benefits of joint and orchestrated processing.

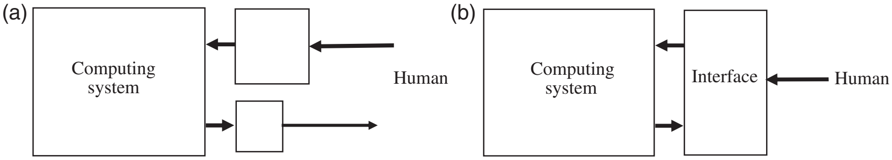


Figure 1.3 Fuzzy sets in the realization of communication mechanisms (a) both at the user end and at the computing system side, (b) a unified representation of input and output mechanisms of communication in the form of the interface, which could also embrace a certain machinery of processing at the level of fuzzy sets.

Instead of looking at the individual number, we embrace a more general point of view and process a entire collection of elements represented now in the form of a single fuzzy set. This effect of a *collective* handling of individual elements is seen very profoundly in the so-called fuzzy arithmetic. The basic constructs here are fuzzy numbers. In contrast to single numeric quantities (real numbers), fuzzy numbers represent collections of numbers where each of them belongs to the concept (fuzzy number) to some degree. These constructs are then subject to processing, say addition, subtraction, multiplication, division, and so on. Noticeable is the fact that by processing fuzzy numbers we are in fact handling a significant number of individual elements at the same time. Fuzzy numbers and fuzzy arithmetic provide an interesting advantage over interval arithmetic (viz. arithmetic in which we are concerned with intervals—sets of numeric values). Intervals come with abrupt boundaries as elements can belong to or are excluded from the given set. This means, for example, that any gradient-based techniques of optimization invoked when computing solutions become very limited: the derivative is equal to zero with an exception at the point where the abrupt boundary is located.

Fuzzy Sets as a Vehicle of Raising and Quantifying Awareness About Granularity of Outcomes

Fuzzy sets form the results of granular computing. As such they convey a global view at the elements of the universe of discourse over which they are constructed. When visualized, the values of the membership function describe a suitability of the individual points as compatible (preferred) with the solution. In this sense, fuzzy sets serve as a useful visualization vehicle: when displayed, the user could gain an overall view of the character of solution (regarded as a fuzzy set) and make a final choice. Note that this is very much in line with the idea of the human-centricity: We present the user with all possible results however do not put any pressure as to the commitment of selecting a certain numeric solution.

Fuzzy Sets as a Mechanism Realizing a Principle of the Least Commitment

As the computing realized in the setting of granular computing returns a fuzzy set as its result, it could be effectively used to realize a principle of the least commitment.

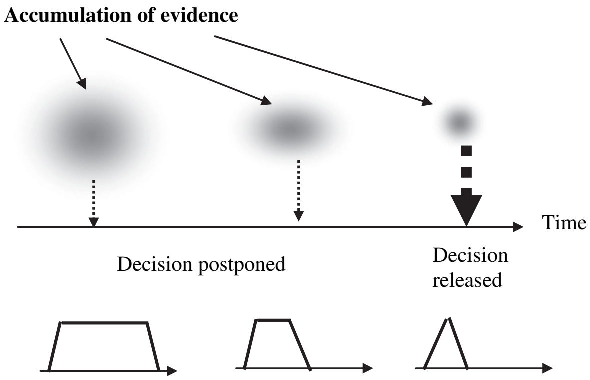


Figure 1.4 An essence of the principle of the least commitment; the decision is postponed until the phase where there is enough evidence accumulated and the granularity of the result becomes specific enough. Also examples of fuzzy sets formed at successive phases of processing that become more specific along with the increased level of evidence are shown.

The crux of this principle is to use fuzzy set as a mechanism of making us cognizant of the quality of obtained result. Consider a fuzzy set being a result of computing in some problem of multiphase decision making. The fuzzy set is defined over various alternatives and associates with them the corresponding degrees of preference, see Figure 1.4. If there are several alternatives with very similar degrees of membership, this serves as a clear indicator of uncertainty or hesitation as to the making of a decision. In other words, in light of the form of the generated fuzzy set, we do not intend to commit ourselves to making any decision (selection of one of the alternatives) at this time. Our intent would be to postpone decision and collect more evidence. For instance, this could involve further collecting of data, soliciting expert opinion, and alike. Based on this evidence, we could continue with computing and evaluate the form of the resulting fuzzy set. It could well be that the collected evidence has resulted in more specific fuzzy set of decisions on the basis of which we could either still postpone decision and keep collecting more evidence or proceed with decision making. Thus, the principle of the least commitment offers us an interesting and useful guideline as to the mechanism of decision making versus evidence collection.

1.3.3 Rough Sets

The description of information granules completed with the aid of some vocabulary is usually imprecise. Intuitively, such description may lead to some approximations called lower and upper bounds. This is the essence of rough sets introduced by Pawlak (1982; 1991); refer also to Skowron (1989) and Polkowski and Skowron (1998). Interesting generalizations, conceptual insights, and algorithmic investigations are offered in a series of fundamental papers by Pawlak and Skowron (2007a,b,c).

To explain the concept of rough sets and show what they are to offer in terms of representing information granules, we use an illustrative example. Consider a description of environmental conditions expressed in terms of temperature and pressure. For each of these factors, we fix several ranges of possible values where each of such ranges comes with some interpretation such as "values below," "values in-between," "values above," and so on. By admitting such selected ranges in both variables, we construct a grid of concepts formed in the Cartesian product of the spaces of temperature and pressure, refer to Figure 1.5. In more descriptive terms, this grid forms a vocabulary of generic terms using which we would like to describe all new information granules.

Now let us consider that the environmental conditions monitored over some time have resulted in some values of temperature and pressure ranging in-between some lower and upper bound as illustrated in Figure 1.5. Denote this result by X. When describing it in terms of the elements of the vocabulary, we end up with a collection of elements that are fully included in X. They form a lower bound of description of X when being completed in presence of the given vocabulary. Likewise, we may

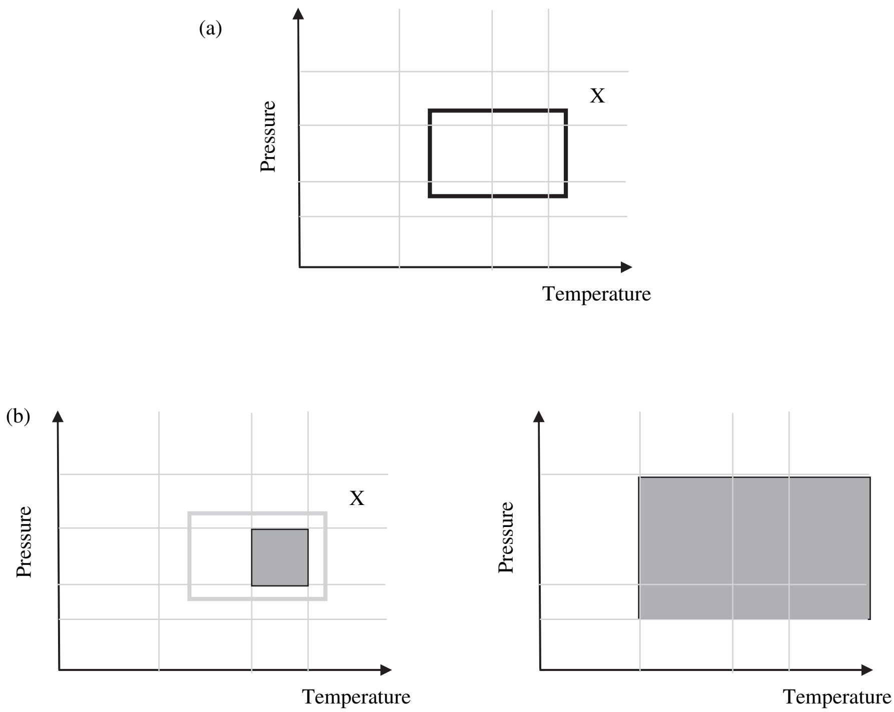


Figure 1.5 A collection of vocabularies and their use in the problem description. Environmental conditions X result in some interval of possible values (a). In the sequel, this gives rise to the concept of a rough set with the roughness of the description being captured by the lower and upper bounds (approximations) as illustrated in (b).

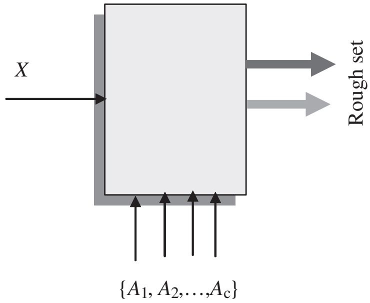


Figure 1.6 Rough set as a result of describing X in terms of some fixed vocabulary of information granules $\{A_1, A_2, \ldots, A_c\}$.

identify elements of the vocabulary that have a nonempty overlap with X and in this sense constitute an upper bound of the description of the given environmental conditions. Along with the vocabulary, the description forms a certain rough set.

As succinctly visualized in Figure 1.6, we are concerned with a description of a given concept X realized in the language of a certain collection (vocabulary) of rather generic and simple terms $A_1, A_2, \ldots, A_c$. The lower and upper boundaries (approximation) are reflective of the resulting imprecision caused by the conceptual incompatibilities between the concept itself and the existing vocabulary.

It is interesting to note that the vocabulary used in the above construct could comprise information granules being expressed in terms of any other formalism, say fuzzy sets. Quite often we can encounter constructs like rough fuzzy sets and fuzzy rough sets in which both fuzzy sets and rough sets are put together (Dubois and Prade, 1990).

1.3.4 Shadowed Sets

Fuzzy sets are associated with the collections of numeric membership grades. Shadowed sets (Pedrycz, 1998; 2005) are based upon fuzzy sets by forming a more general and highly synthetic view at the numeric concept of membership. Using shadowed sets, we quantify numeric membership values into three categories: complete belongingness, complete exclusion, and unknown (which could be also conveniently referred to as do not know condition or a *shadow*). A graphic illustration of a shadowed set along with the principles of sets and fuzzy sets is schematically shown in Figure 1.7. This helps us contrast these three fundamental constructs of information granules.

In a nutshell, shadowed sets can be regarded as a general and far more concise representation of a fuzzy set that could be of particular interest when dealing with further computing (in which case we could come up with substantial reduction of the overall processing effort).

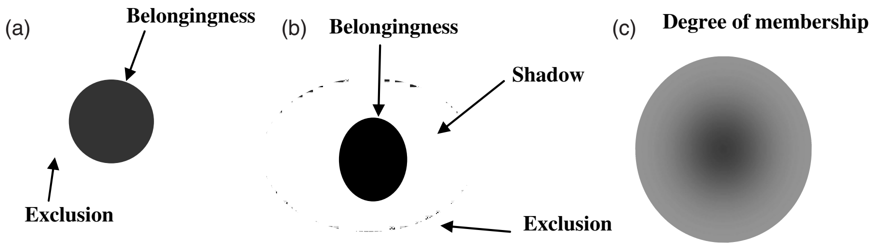


Figure 1.7 A schematic view at sets (a), shadowed sets (b), and fuzzy sets (c). Shadowed sets reveal interesting linkages between fuzzy sets and sets.

1.4 QUANTIFYING INFORMATION GRANULARITY: GENERALITY VERSUS SPECIFICITY

The notion of granularity itself and a level of specificity/generality seem to be highly intuitive: We can easily sense what is more detailed and specific and what looks more abstract and general. Formally, we can easily quantify granularity of information granule by counting its number of elements. The more the elements are located in the information granule, the lower its granularity (and the higher the generality). In this limit, a single element exhibits the highest level of granularity (specificity). In the case of sets, this will be the cardinality (number of elements) or the length of the interval or a similar measure expressing a count of the elements. In case of fuzzy sets, we usually use a so-called sigma count that is produced by summing up the membership grades of the elements belonging to the fuzzy set under consideration. For rough sets, we may consider the cardinality of their lower or upper approximations.

1.5 COMPUTATIONAL INTELLIGENCE

Emerged in the early 1990s (Bezdek, 1992; Pedrycz, 1997), Computational intelligence (CI) offers a unique and interesting opportunity to narrow down the acute semantic gap we encounter when building HC^2 systems. The contributing technologies of CI (in particular, neural networks, granular computing, and evolutionary optimization) along with their research thrusts are complementary to a high degree. This has triggered a great deal of synergy, which in turn has made the CI a highly cohesive conceptual and algorithmic platform exhibiting significant modifiability (adaptability) and supporting mechanisms of context-awareness, human-centricity, and user-friendliness. In this highly symbiotic CI environment, each of the technologies listed above plays an important role. For instance, through the use of fuzzy sets, detailed numeric data may be arranged into meaningful and tangible information granules. Information granulation allows for the incorporation of a users' prior domain knowledge and preferences, as well as facilitates the management of uncertainty. Neurocomputing delivers a rich diversity of learning techniques and flexible neural or neuro-fuzzy architectures. Evolutionary methods help cope with structural optimization and are often essential in the design of complex systems. CI benefits from this both in terms of the overall methodology of problem understanding and problem solving, as well as the ensuing system architectures. Again as strongly advocated in the literature, CI addresses the very nature of human problem solving, namely, problem modularization, dealing, for example, with numerous conflicting criteria. The recently developed ideas and practices of granular computing promote a general top-down design approach: knowledge tidbits are collected, afterward analyzed, refined, and used as a *blueprint* (backbone) of the ensuing detailed architecture. Neurocomputing, on the contrary, supports the bottom-up design approach: here one starts from ‘‘clouds’’ of data and attempts to reveal and describe some common regularities (e.g., trends) and encapsulate them in the form of specific models. The omnipresent tendency in the development of HC^2 systems lies in its *multistrategy* and *multifaceted* approach. It is strongly manifested in various architectures, different design (learning) techniques, and more advanced user-friendly interfaces. In this sense, CI becomes an ideal methodological, development, and experimental platform for HC^2 systems.

1.6 GRANULAR COMPUTING AND COMPUTATIONAL INTELLIGENCE

Granular computing seamlessly integrates with architectures of CI. Given the fact that information granules help set up the most suitable perspective when dealing with the problem, collecting data (that could be of heterogeneous character), carrying out processing, and releasing the results (in a formal acceptable to the environment), the general architecture is shown in Figure 1.8.

Although the communication layers are supported by granular computing, the underlying processing is a domain of neurocomputing, while the overall optimization of the architecture is supported by the machinery of evolutionary computing. There are different levels of synergy; for instance, one could regard the overall architecture as a neurofuzzy system. In this case, the interface delivers a unified setting where various sources of data are effortlessly combined and presented to the neural network, which constitutes the core of the processing layer. In many cases, the architecture could have somewhat blurred delineation between the communication layers and the processing core, in particular, when information granules become an integral part of the basic processing elements. A typical example here comes when we are concerned with a granular neuron—a construct in which the connections are treated as information granules, say fuzzy sets (and then we may refer to it as a fuzzy neuron) or rough sets (which gives rise to the concept of rough neurons).

As discussed earlier, information granules help us cast the problem into some perspective. This becomes visible in case of neural networks. To cope with huge masses of data, we could granulate them (which naturally reduce their number and

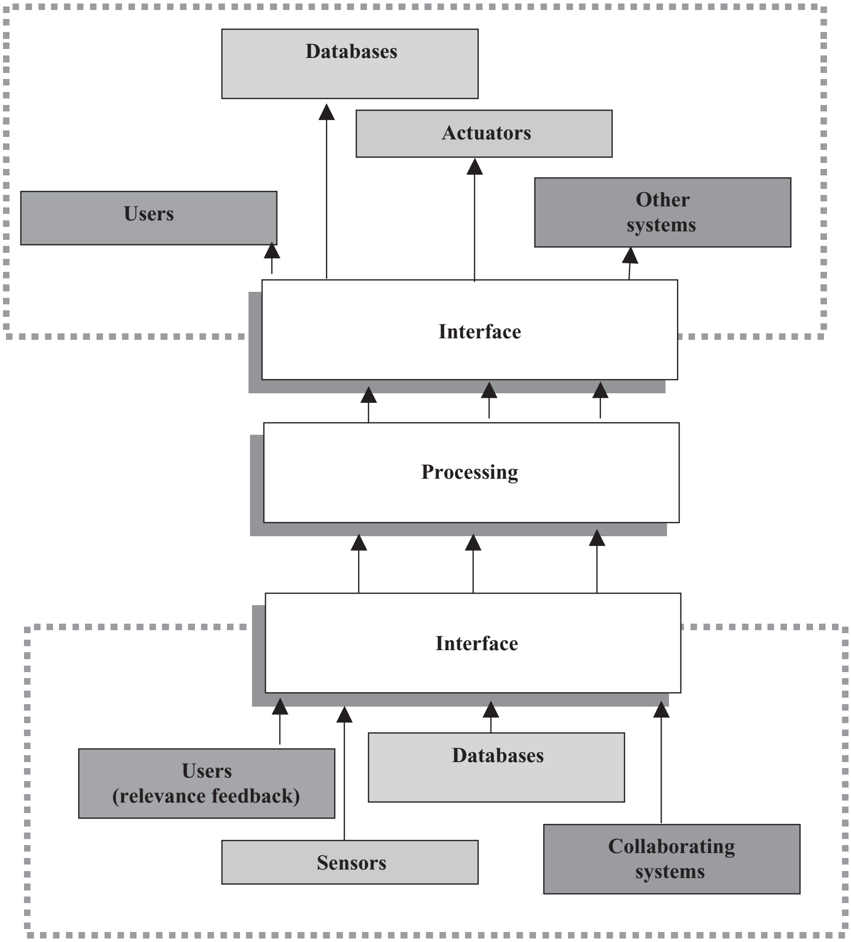


Figure 1.8 The layered architecture of systems of Computational Intelligence with the functional layers of communication (interfacing) with the environment and the processing core.

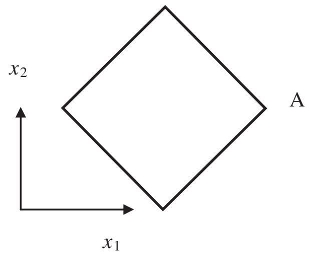
dimensionality) and treat those as meaningful aggregates and components of the learning set.

1.7 CONCLUSIONS

Human centricity becomes a feature that is of growing interest, especially when dealing with the development of more sophisticated and intelligent systems. Whereas there is a remarkably diversified spectrum of possible applications and ensuing realizations, in all of them, we can identify some commonalities and a visible role of information granules and information granulation. The chapter offers some introduction to fuzzy sets and brings a number of motivating comments as far as their methodology and applied side are concerned. Similarly, we looked at fuzzy sets being an important component of granular computing. We also clarified a concept of CI and pointed at the role of fuzzy sets within this framework.

EXERCISES AND PROBLEMS

1. Consider a certain concept A defined in the space of two variables (attributes) x_1 and x_2 whose geometric representation is shown below. We would like to describe it by means of some Cartesian products of intervals. It becomes evident that such characterization cannot be perfect. How would you define lower and upper bounds of the description of the concept so that its "roughness" becomes as small as possible? Justify your construction of the bounds.



2. Pick up some textbooks, newspapers, and magazines and identify terms (concepts) that could be formalized as fuzzy sets. Justify your choice. Suggest possible models of membership functions and link them with the semantics of the concepts being described in this manner.

3. Discuss some additional functionality in commonly encountered computer systems that could be beneficial in making them highly user centric or could be useful in enhancing their user centricity.

4. Identify some concepts in which fuzzy sets could be used in explicit and implicit manner.

5. Unleash your imagination and suggest some functionality of future computing systems in which human centricity could play an important role.

6. For the differentiable membership functions, we could evaluate their sensitivity by determining the absolute value of derivative of the membership function. Discuss the sensitivity of piecewise linear membership functions (triangular fuzzy sets), parabolic membership functions, and Gaussian membership functions. They are described by the following membership functions:

(a) Triangular
$$A(x) = \begin{cases} \dfrac{x-a}{m-a} \text{if } x \in [a, m] \\ 1 - \dfrac{x-m}{b-m} \text{if } x \in [m, b] \\ 0, \quad \text{otherwise} \end{cases}$$

where $a < m < b$

(b) Parabolic $A(x) = \begin{cases} 1 - \left(\dfrac{x}{a}\right)^2 & \text{if} x \in [-a, a] \\ 0, \text{otherwise} \end{cases}$

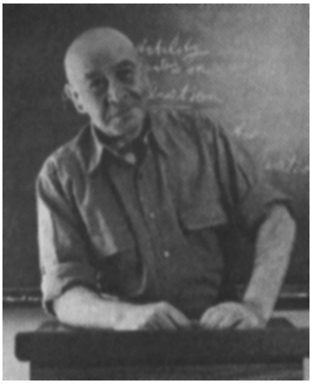
(c) Gaussian $A\ (x) = \exp(-(x - m)^2/\sigma^2)$

7. You are about to buy a new car. The info sticker you see on the windshield of the vehicle in the dealer's exhibition area tells you about economy "22 mpg in city and 35 mpg on highway." How could you interpret this information? Would you be dissatisfied after buying this vehicle and learning that it makes 20 mpg in city driving? Suggest models of fuzzy sets capturing the semantics of the concept of economy of a vehicle; be realistic. While dealing with cars, also suggest some other concepts that directly lead to the emergence of fuzzy sets that could serve as the meaningful descriptors of the concepts.

HISTORICAL NOTES

While the inception of fuzzy sets has to be attributed to 1965 paper by Zadeh (Zadeh, 1965), we have indicated that their conceptual and philosophical roots are dated back to the beginning of 20th century where the most influential and prominent ideas of three-valued and multivalued logic came into existence (Łukasiewicz, 1920, 1930, 1970). The philosophical underpinnings of non-Aristotelian view at the world were laid down by Korzybski (1933). The Aristotelian view of the world was challenged by Black in his 1938 study entitled "Vagueness: an exercise in logical analysis." The others include Klaua and Post.

Jan Łukasiewicz (1878–1956) is known as a founder of three-valued and multivalued logics. After studies of law at the University of Lvov (Poland), his interests were focused on philosophy in which he received his Ph.D. in 1902. While at the University of Lvov, in 1907–1908, he offered the first Polish course in mathematical logic. During the WW I in 1915, he moved to Warsaw where he occupied one of the two chairs of philosophy at the Warsaw University. In 1946, not accepting the new political system set up in Poland under the Soviet occupation, he moved to Dublin, Ireland. Łukasiewicz's Polish notation (known as reverse Polish notation or postfix notation) of 1920 was an inspiration behind the idea of the *recursive* *stack*, a last-in, first-out computer memory store. The reverse Polish notation is used in Hewlett Packard calculators and postscript language.

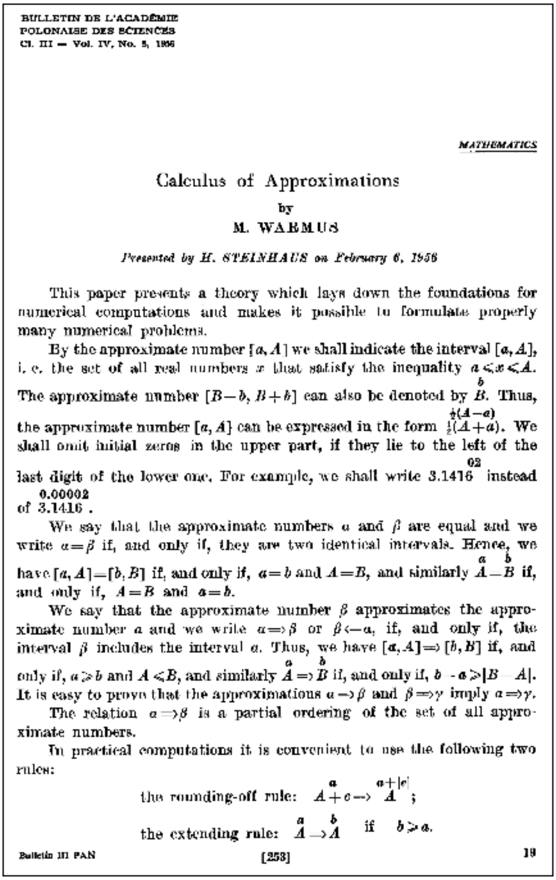


Alfred Korzybski (1879–1950) has contributed to the area of general semantics and the fundamentals of non-Aristotelian systems. He studied in Warsaw University of Technology, Germany and Italy. Then, he volunteered in the Russian army and was sent to Canada and USA as an artillery expert. His book entitled *Science and Sanity: An Introduction to Non-Aristotelian Systems and General Semantics* and published in 1933 has become a landmark in studies of general semantics. Here, it is worth to recall Korzybski's note from this book that succinctly highlights the shortcomings of the Aristotelian perspective.

> *..in analyzing the Aristotelian codification, I had to deal with the two-valued, "either-or" type of orientation. In living, many issues are not so sharp, and therefore a system that posits the general sharpness of "either-or" and so objectifies "kind," is unduly limited; it must be revised and more flexible in terms of "degree"...*

The developments of interval calculus emerged with inception of the era of digital computing and the paper by J. Warmus was one of the first publications in this realm. It is interesting to follow a general way in which the computing with such information granules is carried out (Fig. 1.9).

Fuzzy sets came into existence when the fundamental paper of **L. A. Zadeh** was published in *Information and Control* (Fig. 1.10). Fuzzy sets departed from the principle of dichotomy by admitting a notion of partial membership (degree of membership defined in the unit interval). Fuzzy sets offered a rich conceptual and algorithmic setting in which granular information could be handled. Furthermore, they provide a highly effective vehicle to express and quantify general principles of modeling and human-centric systems, for example, the principle of incompatibility coined by Zadeh (1973).



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MATHEMATICS

Calculus of Approximations

by

M. WARMUS

Presented by H. STEINHAUS on February 6, 1956

This paper presents a theory which lays down the foundations for numerical computations and makes it possible to formulate properly many numerical problems.

By the approximate number $[a, A]$ we shall indicate the interval $[a, A]$, i. e. the set of all real numbers x that satisfy the inequality $a \leqslant x \leqslant A$. The approximate number $[B-b, B+b]$ can also be denoted by $\overset{b}{B}$. Thus, the approximate number $[a, A]$ can be expressed in the form $\overset{\frac{1}{2}(A-a)}{\frac{1}{2}(A+a)}$. We shall omit initial zeros in the upper part, if they lie to the left of the last digit of the lower one. For example, we shall write $\overset{02}{3.1416}$ instead of $\overset{0.00002}{3.1416}$.

We say that the approximate numbers α and β are equal and we write $\alpha=\beta$ if, and only if, they are two identical intervals. Hence, we have $[a, A]=[b, B]$ if, and only if, $a=b$ and $A=B$, and similarly $\overset{a}{A}=\overset{b}{B}$ if, and only if, $A=B$ and $a=b$.

We say that the approximate number β approximates the approximate number α and we write $\alpha \Rightarrow \beta$ or $\beta \Leftarrow \alpha$, if, and only if, the interval β includes the interval α. Thus, we have $[a, A] \Rightarrow [b, B]$ if, and only if, $a \geqslant b$ and $A \leqslant B$, and similarly $\overset{a}{A} \Rightarrow \overset{b}{B}$ if, and only if, $b-a \geqslant |B-A|$. It is easy to prove that the approximations $\alpha \Rightarrow \beta$ and $\beta \Rightarrow \gamma$ imply $\alpha \Rightarrow \gamma$.

The relation $\alpha \Rightarrow \beta$ is a partial ordering of the set of all approximate numbers.

In practical computations it is convenient to use the following two rules:

$$\text{the rounding-off rule: } \overset{a}{A}+c \Rightarrow \overset{a+|c|}{A};$$

$$\text{the extending rule: } \overset{a}{A} \Rightarrow \overset{b}{A} \quad \text{if} \quad b \geqslant a.$$

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Figure 1.9 The first page of the paper by Warmus in which he outlined the concept of computing with numeric intervals.

As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics

The theory of rough sets established by Z. Pawlak (Fig. 1.11) opened another successful avenue of investigations of information granules whose description realized in the setting of a certain vocabulary leads to the concept of roughness of description (which itself manifests through lower and upper boundaries or approximations).

Losfi Zadeh during his Student years in Tehran in the early 1940s (the large Russian sign ODIN which means "alone," was his early proclamation of Independence).

Fuzzy Sets*

L. A. Zadeh

Department of Electrical Engineering and Electronics Research Laboratory, University of California, Berkeley, California

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint.

I. INTRODUCTION

More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes such objects as rocks, fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1.

Clearly, the "class of all real numbers which are much greater than 1," or "the class of beautiful women," or "the class of tall men," do not constitute classes or sets in the usual mathematical sense of these terms. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction.

The purpose of this note is to explore in a preliminary way some of the basic properties and implications of a concept which may be of use in dealing with "classes" of the type cited above. The concept in question is that of a *fuzzy set*,[1] that is, a "class" with a continuum of grades of membership. As will be seen in the sequel, the notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing. Essentially, such a framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables.

We begin the discussion of fuzzy sets with several basic definitions.

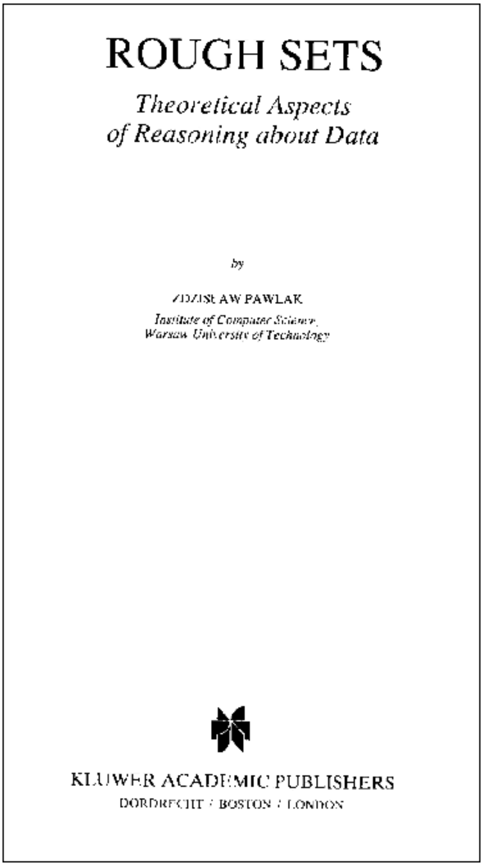
II. DEFINITIONS

Let X be a space of points (objects), with a generic element of X denoted by x. Thus, $X = \{x\}$.

* This work was supported in part by the Joint Services Electronics Program (U.S. Army, U.S. Navy and U.S. Air Force) under Grant No. AF-AFOSR-139-64 and by the National Science Foundation under Grant GP-2413.

[1] An application of this concept to the formulation of a class of problems in pattern classification is described in RAND Memorandum RM-4307-PR, "Abstraction and Pattern Classification," by R. Bellman, R. Kalaba and L. A. Zadeh, October, 1964.

Figure 1.10 The first page of the Zadeh's seminal paper on fuzzy sets.



ROUGH SETS

Theoretical Aspects of Reasoning about Data

by

ZDZISŁAW PAWLAK

Institute of Computer Science, Warsaw University of Technology

KLUWER ACADEMIC PUBLISHERS

DORDRECHT / BOSTON / LONDON

Figure 1.11 Dealing with information with unclear boundaries—an emergence of rough sets.

Zdzislaw Pawlak (1926–2006) was born in Lodz, 130 km south–west from Warsaw, Poland. He studied in Lodz University of Technology and Warsaw University of Technology. He has contributed to the number of disciplines of computer science and was one of the pioneers of computing. In 1961, he was on a research team that constructed one of the first computers in Poland named UMC 1. He proposed and investigated parenthesis-free languages, a generalization of reverse Polish notation introduced by Jan Lukasiewicz. While working at the Institute of Mathematics, in 1965 he introduced the foundations for modeling DNA what has come to be known as molecular computing. In 1968, he proposed a new formal model of a computing machine. In 1970s, he introduced knowledge representation systems. The early 1980s saw the inception of rough sets with the seminal papers published in the International Journal of Computer Information Systems. The most comprehensive coverage of this subject was presented in his book entitled "*Rough Sets. Theoretical Aspects of Reasoning about Data*" published in 1991.

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