CHAPTER 1

SIMPLE TOOL SKILLS

There are a variety of little tasks that will occur over and over again as we work through quantitative problems, and we need to master them first. These tasks include unit conversions, estimating, the ideal gas law, and stoichiometry.

1.1 UNIT CONVERSIONS

There are several important prefixes you should know and should probably memorize.

Femto	(f)	10^{-15}
Pico	(p)	10^{-12}
Nano	(n)	10^{-9}
Micro	(μ)	10^{-6}
Milli	(m)	10^{-3}
Centi	(c)	10^{-2}
Kilo	(k)	10^{3}
Mega	(M)	10^{6}
Giga	(G)	10^{9}
Tera	(T)	10^{12}

For example, a nanogram is 10^{-9} g, and a kilometer is 10^{3} m.

Elements of Environmental Chemistry, by Ronald A. Hites

For those of us forced by convention or national origin to work with the so-called English units, there are some other handy conversion factors you should know:

1 pound (lb) = 454 g
1 inch (in.) = 2.54 cm
12 in. = 1 foot (ft)
1 m = 3.28 ft
1 mile = 5280 ft = 1609 m
3.79 L = 1 U.S. gallon (gal), liquids only

There are some other common conversion factors that link length units to more common volume and area units:

$$1\ \text{m}^3 = 10^3\ \text{L}$$
$$1\ \text{km}^2 = (10^3\ \text{m})^2 = 10^6\ \text{m}^2 = 10^{10}\ \text{cm}^2$$

One more unit conversion that we will find very helpful is

$$1\ \text{tonne (t)} = 10^3\ \text{kg} = 10^6\ \text{g}$$

Yes, we will spell metric *tonne* like this to distinguish it from 1 U.S. short ton, which is 2000 lb. One short ton equals 0.91 metric tonnes.

Let us do some simple unit conversion examples. The point is to carry along the units as though they were algebra and cancel out things as you go. Always write down your unit conversions! I cannot begin to count the number of people who looked foolish at public meetings because they tried to do unit conversions in their heads.

Human head hair grows about one half of an inch per month. How much hair grows in 1 s; please use metric units?

Strategy. Let us convert inches to meters and months to seconds. Then depending on how small the result is, we can select the right length units.

$$\text{Rate} = \left(\frac{0.5\ \text{in.}}{\text{month}}\right)\left(\frac{2.54\ \text{cm}}{\text{in.}}\right)\left(\frac{\text{m}}{10^2\ \text{cm}}\right)$$
$$\times\left(\frac{\text{month}}{31\,\text{days}}\right)\left(\frac{\text{day}}{24\text{h}}\right)\left(\frac{\text{h}}{60\ \text{min}}\right)\left(\frac{\text{min}}{60\ \text{s}}\right)$$
$$= 4.7 \times 10^{-9}\text{m/s}$$

If scientific notation is confusing to you, learn to use it.[1] We can put this hair growth rate in more convenient units:

$$\text{Rate} = \left(\frac{4.7 \times 10^{-9}\text{m}}{\text{s}}\right)\left(\frac{10^9\ \text{nm}}{\text{m}}\right) = 4.7\ \text{nm/s}$$

[1] We will use scientific notation throughout this book because it is easier to keep track of very big or very small numbers. For example, in the calculation we just did, we would have ended up with a growth rate of 0.000,000,0047 m/s in regular notation; that number is difficult to read and prone to error in transcription (you have to count the zeros accurately). To avoid this problem, we give the number followed by 10 raised to the correct power. It is also easier to multiply and divide numbers in this format. For example, it is tricky to multiply 0.000,000,0047 by 1000,000,000, but it is easy to multiply 4.7×10^{-9} by 1×10^9 by multiplying the leading numbers ($4.7 \times 1 = 4.7$) and by adding the exponents of 10 ($-9 + 9 = 0$) giving a result of $4.7 \times 10^0 = 4.7$.

This is not much, but it obviously mounts up second after second.

A word on significant figures: In the above result, the input to the calculation was 0.5 in. per month, a datum with only one significant figure. Thus, the output from the calculation should not have more than one significant figure and should have been given as 5 nm/s. In general, one should use a lot of significant figures inside the calculation, but round off the answer to the correct number of figures at the end. With a few exceptions, one should be suspicious of environmental results having four or more significant figures; in most cases, two will do.

The total amount of sulfur released into the atmosphere per year by the burning of coal is about 75 million tonnes. Assuming this were all solid sulfur, how big a cube would this occupy? You need the dimension of each side of the cube in feet. Assume the density of sulfur is twice that of water.

Strategy. Ok, this is a bit more than just converting units. We have to convert weight to volume, and this requires knowing the density of sulfur; density has units of weight per unit volume, which in this case is given to be twice that of water. As you may remember, the density of water is 1 g/cm^3, so the density of sulfur is 2 g/cm^3. Once we know the volume of sulfur, we can take the cube root of that volume and get the side length of a cube holding that volume.

$$V = (7.5 \times 10^7\,\mathrm{t})\left(\frac{\mathrm{cm}^3}{2\,\mathrm{g}}\right)\left(\frac{10^6\,\mathrm{g}}{\mathrm{t}}\right) = 3.8 \times 10^{13}\,\mathrm{cm}^3$$

$$\mathrm{Side} = \sqrt[3]{3.8 \times 10^{13}\mathrm{cm}^3} = 3.35 \times 10^4\,\mathrm{cm}\left(\frac{\mathrm{m}}{10^2\,\mathrm{cm}}\right)$$

$$= 335\,\mathrm{m}$$

$$\mathrm{Side} = 335\,\mathrm{m}\left(\frac{3.28\,\mathrm{ft}}{\mathrm{m}}\right) = 1100\,\mathrm{ft}$$

This is huge. It is a cube as tall as the Empire State Building on all three sides. Pollution gets scary if you think of it as being all in one place rather than diluted by the Earth's atmosphere.

1.2 ESTIMATING

We often need order of magnitude guesses for many things in the environment. This is an important skill, so let us start with a couple of examples.

How many cars are there in the United States and in the world?

Strategy. Among our friends and families, it seems like about every other person has a car. If we know the population of the United States, then we can use this 0.5 cars per person conversion factor to get the number of cars in the United States. It would be wrong to use this 0.5 cars per person for the rest of the world (e.g., there are not 500 million cars in China—yet), but we might just use a multiplier based on the size of the economy of the United States versus the world. We know that the U.S. economy is roughly one third that of the whole world; hence, we can multiply the number of cars in the United States by 3 to estimate the number of cars in the world.

In the United States, there are now about 295 million people and almost every other person has a car; thus,

$$2.95 \times 10^8 \times 0.5 = 1.5 \times 10^8 \text{ cars in the United States}$$

The U.S. economy is about one third of the world's economy; hence, the number of cars in the world is

$$3 \times 1.5 \times 10^8 \approx 5 \times 10^8$$

The real number is not known with much precision, but in 2005, it is likely on the order of $\sim 6 \times 10^8$ cars. Thus, our estimate is a bit low, but it is certainly in the right ballpark. Of course, this number will increase dramatically as the number of cars in China increases.

How many people work at McDonalds in the world?

Strategy. Starting close to home, you could count the number of McDonalds in your town and ratio that number to the population of the rest of the United States. For example, Bloomington, Indiana, where I live, has four McDonald "restaurants" serving a population of about 120,000 people. Ratioing this to the U.S. population as a whole

$$\left(\frac{4\,\text{McD}}{1.2 \times 10^5\ \text{people}}\right) 2.95 \times 10^8 = 9800$$

restaurants in the United States

Based on local observations and questions of the people behind the counter, it seems that about 30 people work at each "restaurant"; hence,

$$\left(\frac{30\,\text{employees}}{\text{restaurant}}\right) 9800 \text{ restaurants} \approx 3 \times 10^5 \text{ employees}$$

But this estimate is for the United States—what about the whole world? It is probably not right to use our factor of 3 (see just above) to link the United States' love for fast food to the rest of the world; for example, it is not likely that a Quarter-Pounder with Cheese will have the same appeal in India (1.3 billion people) as it does in the United States. Nevertheless, we could probably use a factor of 2 for this extrapolation and get about 600,000 McDonald employees worldwide. This estimate might be on the high side—Indiana has a relatively high concentration of McDonalds compared to other states. The truth seems to be that, in 2005, McDonalds had a total of 447,000 employees worldwide [*Fortune*, July 26, 2006, p. 122], so our estimate is not too bad.

How many American footballs can be made from one pig?

Strategy. Think about the size of a football—perhaps as a size-equivalent sphere—and about the size of a pig—perhaps as a big box—then divide one by the other. Let us assume that a football can be compressed into a sphere, and our best guess is that this sphere will have a diameter of about 25 cm (10 in.). Let us also imagine that a pig is a rectilinear box that is about 1 m long, 0.5 m high, and 0.5 m wide. This ignores the head, the tail, and the feet, which are probably not used to make footballs anyway.

$$\text{Pig area} = (4 \times 0.5 \times 1) + (2 \times 0.5 \times 0.5)\,\text{m}^2 = 2.5\,\text{m}^2$$

$$\begin{aligned}\text{Football area} &= 4\pi r^2 = 4 \times 3.14 \times (25/2)^2 \\ &= 1963\,\text{cm}^2 \approx 2000\,\text{cm}^2\end{aligned}$$

$$\text{Number of footballs} = \left(\frac{2.5\,\text{m}^2}{2000\,\text{cm}^2}\right)\left(\frac{10^4\,\text{cm}^2}{\text{m}^2}\right) \approx 10$$

This seems almost right, but most footballs are not made from pigskin any longer; like everything else, they are made from plastic.

1.3 IDEAL GAS LAW

We need this tool skill for dealing with many air pollution issues. The ideal gas law is

$$PV = nRT$$

where

P = pressure in atmospheres (atm) or in Torr (remember 760 Torr = 1 atm)[2]
V = volume in liters

[2] I know we should be dealing with pressure in units of Pascals (Pa), but I think it is convenient for environmental science purposes to retain the old unit of atmospheres—we instinctively know what that represents. For the purists among you, 1 atm = 101,325 Pa (or for government work, 1 atm = 10^5 Pa).

n = number of moles
R = gas constant (0.082 L atm/deg mol)
T = temperature in Kelvin (K = °C + 273.15)

The term mole refers to 6.02×10^{23} molecules or atoms; there are 6.02×10^{23} molecules or atoms in a mole. The term "moles" occurs frequently in molecular weights, which have units of grams per mole (or g/mol); for example, the molecular weight of N_2 is 28 g/mol. This number, 6.02×10^{23} (note the positive sign of the exponent), is known far and wide as Avogadro's number, and it was invented by Amadeo Avogadro in 1811.

We will frequently need the composition of the Earth's dry atmosphere; I have also included the molecular weight of each gas.

Gas	Symbol	Composition	Molecular weight
Nitrogen	N_2	78%	28
Oxygen	O_2	21%	32
Argon	Ar	1%	40
Carbon dioxide	CO_2	380 ppm	44
Neon	Ne	18 ppm	20
Helium	He	5.2 ppm	4
Methane	CH_4	1.5 ppm	16

The units "ppm" and "ppb" refer to parts per million and parts per billion, respectively. These are fractional units just like percent (%), which is parts per hundred. To get from the unitless fraction to these relative units, just multiply by 100 for %, by 10^6 for ppm, or by 10^9 for ppb. For example, a fraction of 0.0001 is 0.01% = 100 ppm = 100,000 ppb. For the gas phase, %, ppm, and ppb are all on a volume per volume basis (which is exactly the same as on a mole per mole basis); for example, the concentration of nitrogen in the Earth's atmosphere is 78 L of nitrogen per 100 L of air or 78 mol of nitrogen per 100 mol of air. It is *not* 78 g of nitrogen per 100 g of air. To remind us of this convention, sometimes these concentrations are given as "ppmv" or "ppbv." This convention applies to only gas concentrations—not to water, solids, or biota (where the convention is weight per weight).

What is the molecular weight of dry air?

Strategy. The value we are after should just be the weighted average of the components in air, mostly nitrogen at 28 g/mol and oxygen at 32 g/mol (and perhaps a tad of argon at 40 g/mol). Thus,

$$\begin{aligned} MW_{\text{dry air}} &= 0.78 \times 28 + 0.21 \times 32 + 0.01 \times 40 \\ &= 29\ \text{g/mol} \end{aligned}$$

What is the volume of 1 mole of gas at 1 atm and 0°C?

Strategy. We can just rearrange $PV = nRT$ and get

$$\begin{aligned} \left(\frac{V}{n}\right) &= \left(\frac{RT}{P}\right) = \left(\frac{0.082\ \text{L atm}}{\text{K mol}}\right)\left(\frac{273\ \text{K}}{1\ \text{atm}}\right) \\ &= 22.4\ \text{L/mol} \end{aligned}$$

This value is 24.4 L/mol at 25°C. It will help us to remember both of these numbers, or at least, how to get from one to the other.

What is the density of the Earth's atmosphere at 0°C and 1 atm pressure?

Strategy. Remember that density is weight per unit volume, and we can get from volume to weight using the molecular weight, or in this case, the average molecular weight of dry air. Hence, rearranging $PV = nRT$

$$\frac{n(\mathrm{MW})}{V} = \left(\frac{\mathrm{mol}}{22.4\,\mathrm{L}}\right)\left(\frac{29\ \mathrm{g}}{\mathrm{mol}}\right) = 1.3\,\mathrm{g/L} = 1.3\mathrm{kg/m}^3$$

What is the mass (weight) of the Earth's atmosphere?

Strategy. This is a bit harder, and we need an additional fact. We need to know the average atmospheric pressure in terms of weight per unit area. Once we have the pressure, we can multiply it by the surface area of the Earth to get the total weight of the atmosphere.

There are two ways to calculate the pressure: First, your average tire repair guy knows this to be 14.7 lb/in.^{2}, but we would rather use metric units.

$$\begin{aligned} P_{\mathrm{Earth}} &= \left(\frac{14.7\ \mathrm{lb}}{\mathrm{in.}^2}\right)\left(\frac{\mathrm{in.}^2}{2.54^2\ \mathrm{cm}^2}\right)\left(\frac{454\ \mathrm{g}}{\mathrm{lb}}\right) \\ &= 1030\,\mathrm{g/cm}^2 \end{aligned}$$

Second, remember from the TV weather reports that the atmospheric pressure averages 30 in. of mercury, which is 760 mm (76 cm) of mercury in a barometer. This length of mercury can be converted to a true pressure by multiplying it by the density of mercury, which is 13.5 g/cm^3.

$$P_{\text{Earth}} = (76\,\text{cm})\left(\frac{13.5\,\text{g}}{\text{cm}^3}\right) = 1030\,\text{g/cm}^2$$

Next, we need to know the area of the Earth. I had to look it up – it is $5.11 \times 10^8\,\text{km}^2$ – remember this! Hence, the total weight of the atmosphere is

$$\begin{aligned}\text{Mass} = P_{\text{Earth}}\,A &= \left(\frac{1030\,\text{g}}{\text{cm}^2}\right)\left(\frac{5.11 \times 10^8\,\text{km}^2}{1}\right)\\ &\times \left(\frac{10^{10}\,\text{cm}^2}{\text{km}^2}\right)\left(\frac{\text{kg}}{10^3\,\text{g}}\right) = 5.3 \times 10^{18}\,\text{kg}\end{aligned}$$

This is equal to 5.3×10^{15} metric tonnes.

Although this is not a realistic situation, it is useful to know what the volume (in L) of the Earth's atmosphere would be if it were all at 1 atm pressure and at 15°C (which is the average temperature of the lower atmosphere).

Strategy. Because we have just calculated the weight of the atmosphere, we can get the volume by dividing it by its density of 1.3 kg/m^3, which we just calculated above.

$$\begin{aligned}V &= \frac{\text{Mass}}{\rho} = 5.3 \times 10^{18}\,\text{kg}\left(\frac{\text{m}^3}{1.3\,\text{kg}}\right)\left(\frac{288}{273}\right)\left(\frac{10^3\,\text{L}}{\text{m}^3}\right)\\ &= 4.3 \times 10^{21}\,\text{L}\end{aligned}$$

Remember this number! Notice that the factor of 288/273 is needed to adjust the volume of air (1 m^3 in the density) from 0°C (273 K) to 15°C (288 K)—the air gets warmer, so the volume increases.

An indoor air sample taken from a closed garage contains 0.9% of CO (probably a deadly amount). What is the concentration of CO in this air in units of g/m^3 at 20°C and 1 atm pressure? CO has a molecular weight of 28.

Strategy. Given that the 0.9% amount is in moles of CO per moles of air, we need to convert the moles of CO to a weight, and the way to do this is using the molecular weight (28 g/mol). We also need to convert the moles of air to a volume, and the way to do this is using the 22.4 L/mol factor (corrected for temperature).

$$C = \left(\frac{0.9\ \text{mol CO}}{100\ \text{mol air}}\right)\left(\frac{28\ \text{g CO}}{\text{mol CO}}\right)\left(\frac{\text{mol air}}{22.4\ \text{L air}}\right) \times \left(\frac{273}{293}\right)\left(\frac{10^3\ \text{L}}{\text{m}^3}\right) = 10.5\ \text{g/m}^3$$

Note the factor of 273/293 is needed to increase the volume of a mole of air from 0°C to 20°C.

1.4 STOICHIOMETRY

Chemical reactions always occur on an integer molar basis. For example,

$$C + O_2 \rightarrow CO_2$$

This means 1 mol of carbon (weighing 12 g) reacts with 1 mol of oxygen (32 g) to give 1 mol of carbon dioxide (44 g).

Here are a few atomic weights you should know.

H	1
C	12
N	14
O	16
S	32
Cl	35.5

Assume that gasoline can be represented by C_8H_{18}. How much oxygen is needed to completely burn this fuel? Give your answer in grams of oxygen per gram of fuel.

Strategy. First set up and balance the following combustion equation:

$$C_8H_{18} + 12.5\,O_2 \rightarrow 8\ CO_2 + 9\ H_2O$$

This stoichiometry indicates that 1 mol $(8 \times 12 + 18 = 114\,\text{g})$ of fuel reacts with 12.5 mol $(12.5 \times 2 \times 16 = 400\,\text{g})$ of oxygen to form 8 mol $(8 \times [12 + 2 \times 16] = 352\,\text{g})$ of carbon dioxide and 9 mol $(9 \times [2 + 16] = 162\,\text{g})$ of water. Hence, the requested answer is

$$\frac{M_{\text{oxygen}}}{M_{\text{fuel}}} = \left(\frac{400\ \text{g}}{114\ \text{g}}\right) = 3.51$$

This is called the stoichiometric ratio of the combustion system.

Assume that a very poorly adjusted lawn mower is operating such that the combustion reaction is $C_9H_{18} + 9\ O_2 \rightarrow 9\ CO + 9\ H_2O$. For each gram of fuel consumed, how many grams of CO are produced?

Strategy. Again we need to convert moles to weights using the molecular weights of the different compounds. The fuel has a molecular weight of 126 g/mol, and for every mole of fuel used, 9 mol of CO is produced. Hence,

$$\frac{M_{\mathrm{CO}}}{M_{\mathrm{fuel}}} = 9\left(\frac{28\,\mathrm{g}}{\mathrm{mol}}\right)\left(\frac{\mathrm{mol}}{126\,\mathrm{g}}\right) = 2.0$$

1.5 PROBLEM SET

1. Estimate the average spacing between carbon atoms in diamond, the density of which is $3.51\ \mathrm{g/cm^3}$.
2. At Nikel, Russia, the annual average concentration of sulfur dioxide is observed to be $50\ \mu\mathrm{g/m^3}$ at 15°C and 1 atm. What is this concentration of SO_2 in parts per billion?
3. Some modern cars do not come with an inflated spare tire. The tire is collapsed and needs to be inflated after it is installed on the car. To inflate the tire, the car comes with a pressurized can of carbon dioxide with enough gas to inflate three tires. What would this can of gas weigh? Assume an empty can weighs 0.2 kg.

4. The primary air quality standard for NO_2 in the United States, expressed as an annual average, is 53 ppb. What is the equivalent concentration in $\mu g/m^3$?
5. Atmospheric chemists love to use a gas concentration unit called "number density," in which the concentrations are given in units of molecules per cubic centimeter. Please calculate the number densities of oxygen in the atmosphere at sea level (1 atm, 25°C) and at an altitude of 30 km (0.015 atm, −40°C).
6. What would be the difference (if any) in the weights of two basketballs, one filled with air and one filled with helium? Please give your answer in grams. Assume the standard basketball has a diameter of 9.0 in. and is filled to a pressure of 8.0 lb/in.^{2} Sorry for the English units, but after all basketball was invented in the United States.
7. Acid rain was at one time an important point of contention between the United States and Canada. Much of this acid was the result of the emission of sulfur oxides by coal-fired electricity generating plants in southern Indiana and Ohio. These sulfur oxides, when dissolved in rainwater, formed sulfuric acid and hence "acid rain." How many metric tonnes of Indiana coal, which averages 3.5% sulfur by weight, would yield the H_2SO_4 required to produce a 0.9 in. rainfall of pH 3.90 precipitation over a 10^4 mile^2 area?
8. Although oil-fired electric power generating plants are becoming rare, let us assume one such plant consumes 3.5 million liters of oil per day, that the oil has an average composition of $C_{18}H_{34}$ and

density 0.85 g/cm^3, and that the gas emitted from the exhaust stack of this plant contains 45 ppm of NO. Please calculate the mass of NO emitted per day.

9. Imagine that 300 lb of dry sewage is dumped into a small lake, the volume of which is 300 million liters. How many tonnes of oxygen are needed to completely degrade this sewage? You may assume the sewage has an elemental composition of $C_6H_{12}O_6$.
10. Assume that an incorrectly adjusted lawn mower is operated in a closed two-car garage such that the combustion reaction in the engine is $C_8H_{14} + 15/2\ O_2 \rightarrow 8\ CO + 7\ H_2O$. How many grams of gasoline must be burned to raise the level of CO by 1000 ppm?
11. The average concentration of PCBs in the atmosphere around the Great Lakes is about 2 ng/m^3. What is this concentration in molecules/cm^3? The average molecular weight of PCBs is 320.
12. The following quote appeared in *Chemical and Engineering News* (September 3, 1990, p. 52): "One tree can assimilate about 6 kg of CO_2 per year or enough to offset the pollution produced by driving one car for 26,000 miles." Is this statement correct? Justify your answer quantitatively. Assume gasoline has the formula C_9H_{16} and that its combustion is complete.
13. If everyone in the world planted a tree tomorrow, how long would it take for these trees to make a 1 ppm difference in the CO_2 concentration? Assume that the world's population is 6 billion

and that 9 kg of O_2 is produced per tree each year regardless of its age. Remember CO_2 and H_2O combine through the process of photosynthesis to produce $C_6H_{12}O_6$ and O_2.

14. There are about 1.5×10^9 scrap tires in the world at the moment; this represents a major waste disposal problem. (a) If all of these tires were burned with complete efficiency, by how much (in tonnes) would the Earth's current atmospheric load of CO_2 increase? (b) Compare this amount to the current atmospheric CO_2 load. Assume rubber has a molecular formula of $C_{200}H_{400}$ and that each scrap tire weighs 8 kg, has a diameter of 48 cm, and is 85% rubber.