Chapter 1

Getting Down the Basics: Algebra and Geometry

In This Chapter

- Fussing with fractions
- Brushing up on basic algebra
- Getting square with geometry

I know, I know. This is a *calculus* workbook, so what's with the algebra and geometry? Don't worry, I'm not going to waste too many precious pages with algebra and geometry, but these topics are essential for calculus. You can no more do calculus without algebra than you can write French poetry without French. And basic geometry (but not geometry proofs — hooray!) is critically important because much of calculus involves real-world problems that include angles, slopes, shapes, and so on. So in this chapter — and in Chapter 2 on functions and trigonometry — I give you some quick problems to help you brush up on your skills. If you've already got these topics down pat, skip on over to Chapter 3.

If you miss some questions and don't quite understand why, go back to your old textbooks or check out the great pre-calc review in *Calculus For Dummies*. Getting these basics down pat is really important.

Fraction Frustration

Many, many math students hate fractions. Maybe the concepts didn't completely click when they first learned them and so fractions then became a nagging frustration in every subsequent math course.

But you can't do calculus without a good grasp of fractions. For example, the very definition of the derivative is based on a fraction called the *difference quotient*. And, on top of that, the symbol for the derivative, $\frac{dy}{dx}$, is a fraction. So, if you're a bit rusty with fractions, get up to speed with the following problems ASAP — or else!

Q. Solve $\frac{a}{b} \cdot \frac{c}{d} = ?$

A. $\boldsymbol{\frac{ac}{bd}}$ To multiply fractions, you multiply straight across. You *do not* cross-multiply!

Q. Solve $\frac{a}{b} \div \frac{c}{d} = ?$

A. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \boldsymbol{\frac{ad}{bc}}$ To divide fractions, you flip the second one, then multiply.

1. Solve $\frac{5}{0} = ?$.

Solve It

2. Solve $\frac{0}{10} = ?$.

Solve It

3. Does $\frac{3a+b}{3a+c}$ equal $\frac{a+b}{a+c}$? Why or why not?

Solve It

4. Does $\frac{3a+b}{3a+c}$ equal $\frac{b}{c}$? Why or why not?

Solve It

5. Does $\frac{4ab}{4ac}$ equal $\frac{ab}{ac}$? Why or why not?

Solve It

6. Does $\frac{4ab}{4ac}$ equal $\frac{b}{c}$? Why or why not?

Solve It

Misc. Algebra: You Know, Like Miss South Carolina

This section gives you a quick review of algebra basics like factors, powers and roots, logarithms, and quadratics. You absolutely *must* know these basics.

EXAMPLE

Q. Factor $9x^4 - y^6$.

A. $9x^4 - y^6 = (\mathbf{3x^2 - y^3})(\mathbf{3x^2 + y^3})$ This is an example of the single most important factor pattern: $a^2 - b^2 = (a-b)(a+b)$. Make sure you know it!

Q. Rewrite $x^{2/5}$ without a fraction power.

A. $\sqrt[5]{x^2} = \left(\sqrt[5]{x}\right)^2$ Don't forget how fraction powers work!

7. Rewrite x^{-3} without a negative power.

Solve It

8. Does $(abc)^4$ equal $a^4b^4c^4$? Why or why not?

Solve It

9. Does $(a+b+c)^4$ equal $a^4+b^4+c^4$? Why or why not?

Solve It

10. Rewrite $\sqrt[3]{\sqrt[4]{x}}$ with a single radical sign.

Solve It

11. Does $\sqrt{a^2+b^2}$ equal $a+b$? Why or why not?

Solve It

12. Rewrite $\log_a b = c$ as an exponential equation.

Solve It

13. Rewrite $\log_c a - \log_c b$ with a single log.

Solve It

14. Rewrite $\log 5 + \log 200$ with a single log and then solve.

Solve It

15. If $5x^2 = 3x + 8$, solve for x with the quadratic formula.

Solve It

16. Solve $|3x + 2| > 14$.

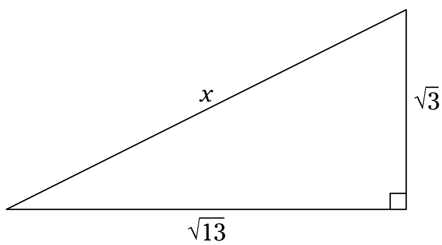
Solve It

Geometry: When Am I Ever Going to Need It?

You can use calculus to solve many real-world problems that involve surfaces, volumes, and shapes, such as maximizing the volume of a cylindrical soup can or determining the stress along a cable hanging in a parabolic shape. So you've got to know the basic geometry formulas for length, area, and volume. You also need to know basic stuff like the Pythagorean Theorem, proportional shapes, and basic coordinate geometry like the distance formula.

EXAMPLE

Q. What's the area of the triangle in the following figure?



A. $\frac{\sqrt{39}}{2}$

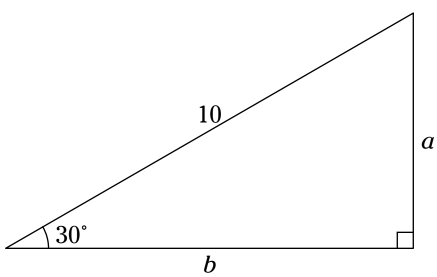
$$\begin{aligned} Area_{triangle} &= \frac{1}{2}\, base \cdot height \\ &= \frac{1}{2} \cdot \sqrt{13}\sqrt{3} \\ &= \frac{\sqrt{39}}{2} \end{aligned}$$

Q. How long is the hypotenuse of the triangle in the previous example?

A. $x = 4$

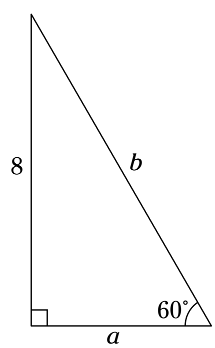
$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 &= a^2 + b^2 \\ x^2 &= \sqrt{13}^2 + \sqrt{3}^2 \\ x^2 &= 13 + 3 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

17. Fill in the two missing lengths for the sides of the triangle in the following figure.



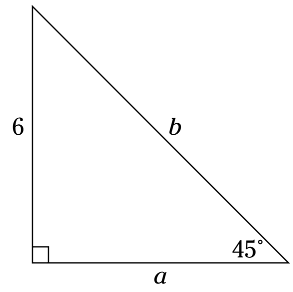
Solve It

18. What are the lengths of the two missing sides of the triangle in the following figure?



Solve It

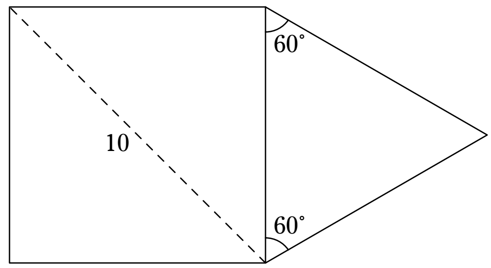
19. Fill in the missing lengths for the sides of the triangle in the following figure.



Solve It

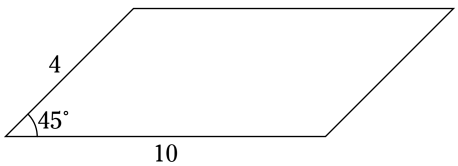
20. **a.** What's the total area of the pentagon in the following figure?

b. What's the perimeter?



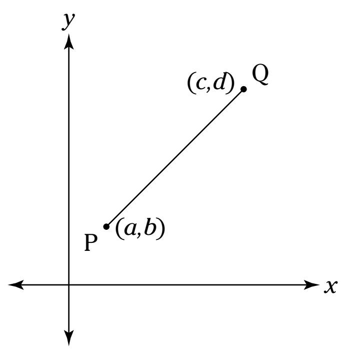
Solve It

21. Compute the area of the parallelogram in the following figure.



Solve It

22. What's the slope of $\overline{PQ}$?



Solve It

23. How far is it from P to Q in the figure from problem 22?

Solve It

24. What are the coordinates of the midpoint of $\overline{PQ}$ in the figure from problem 22?

Solve It

Solutions for This Easy Elementary Stuff

1 Solve $\frac{5}{0} = ?$. $\mathbf{\frac{5}{0}}$ **is undefined!** Don't mix this up with something like $\frac{0}{8}$, which equals zero. Note that if you think about these two fractions as examples of slope ($\frac{rise}{run}$), $\frac{5}{0}$ has a *rise* of 5 and a *run* of 0 which gives you a *vertical* line that has sort of an infinite steepness or slope (that's why it's undefined). Or just remember that it's impossible to drive up a vertical road and so it's impossible to come up with a slope for a vertical line. The fraction $\frac{0}{8}$, on the other hand, has a *rise* of 0 and a *run* of 8, which gives you a *horizontal* line that has no steepness at all and thus has the perfectly ordinary slope of zero. Of course, it's also perfectly ordinary to drive on a horizontal road.

2 $\mathbf{\frac{0}{10}} = \mathbf{0}$ (See solution to problem 1.)

3 Does $\frac{3a+b}{3a+c}$ equal $\frac{a+b}{a+c}$? **No.** You can't cancel the 3s.

WARNING!

You can't cancel in a fraction unless there's an unbroken chain of multiplication running across the entire numerator and ditto for the denominator.

4 Does $\frac{3a+b}{3a+c}$ equal $\frac{b}{c}$? **No.** You can't cancel the 3*a*s. (See previous Warning.)

5 Does $\frac{4ab}{4ac}$ equal $\frac{ab}{ac}$? **Yes**. You can cancel the 4s because the entire numerator and the entire denominator are connected with multiplication.

6 Does $\frac{4ab}{4ac}$ equal $\frac{b}{c}$? **Yes.** You can cancel the 4*a*s.

7 Rewrite x^{-3} without a negative power. $\mathbf{\frac{1}{x^3}}$

8 Does $(abc)^4$ equal $a^4b^4c^4$? **Yes.** Exponents do distribute over multiplication.

9 Does $(a+b+c)^4$ equal $a^4+b^4+c^4$? **No!** Exponents do *not* distribute over addition (or subtraction).

TIP

When you're working a problem and can't remember the algebra rule, try the problem with numbers instead of variables. Just replace the variables with simple, round numbers and work out the numerical problem. (Don't use 0, 1, or 2 because they have special properties that can mess up your example.) Whatever works for the numbers will work with variables, and whatever doesn't work with numbers won't work with variables. Watch what happens if you try this problem with numbers:

$$(3+4+6)^4 \stackrel{?}{=} 3^4+4^4+6^4$$
$$13^4 \stackrel{?}{=} 81+256+1296$$
$$28{,}561 \neq 1633$$

10 Rewrite $\sqrt[3]{\sqrt[4]{x}}$ with a single radical sign. $\mathbf{\sqrt[3]{\sqrt[4]{x}} = \sqrt[12]{x}}$

11 Does $\sqrt{a^2+b^2}$ equal $a+b$? **No!** The explanation is basically the same as for problem 9. Consider this: If you turn the root into a power, you get $\sqrt{a^2+b^2} = \left(a^2+b^2\right)^{1/2}$. But because you can't distribute the power, $\left(a^2+b^2\right)^{1/2} \neq \left(a^2\right)^{1/2} + \left(b^2\right)^{1/2}$, or $a+b$, and thus $\sqrt{a^2+b^2} \neq a+b$.

12 Rewrite $\log_a b = c$ as an exponential equation. $\mathbf{a^c = b}$

13 Rewrite $\log_c a - \log_c b$ with a single log. $\mathbf{\log_c \frac{a}{b}}$

14 Rewrite $\log 5 + \log 200$ with a single log and then solve. $\log 5 + \log 200 = \log(5 \cdot 200) = \log 1000 = \mathbf{3}$

When you see "log" without a base number, the base is 10.

15 If $5x^2 = 3x + 8$, solve for x with the quadratic formula. $\boldsymbol{x = \frac{8}{5}}$ **or** $\mathbf{-1}$

Start by rearranging $5x^2 = 3x + 8$ into $5x^2 - 3x - 8 = 0$ because you want just a zero on one side of the equation.

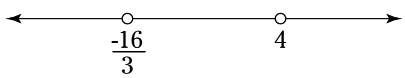
The quadratic formula tells you that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Plugging 5 into a, –3 into b, and –8 into c gives you $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-8)}}{2 \cdot 5} = \frac{3 \pm \sqrt{9 + 160}}{10} = \frac{3 \pm 13}{10} = \frac{16}{10}$ or $\frac{-10}{10}$, so $x = \frac{8}{5}$ or -1.

16 Solve $|3x + 2| > 14$. $\boldsymbol{x < -\frac{16}{3} \cup x > 4}$

1. Turn the inequality into an equation: $|3x + 2| = 14$
2. Solve the absolute value equation.

$$\begin{aligned} 3x + 2 &= 14 \\ 3x &= 12 \\ x &= 4 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 3x + 2 &= -14 \\ 3x &= -16 \\ x &= -\frac{16}{3} \end{aligned}$$

3. Place both solutions on a number line (see the following figure). (You use hollow dots for > and <; if the problem had been $\geq$ or $\leq$, you would use solid dots.)



4. Test a number from each of the three regions on the line in the original inequality.

For this problem you can use –10, 0, and 10.

$$\begin{aligned} |3 \cdot (-10) + 2| &\overset{?}{>} 14 \\ |-28| &\overset{?}{>} 14 \\ 28 &\overset{?}{>} 14 \end{aligned}$$

True, so you shade the left-most region.

$$\begin{aligned} |3 \cdot (0) + 2| &\overset{?}{>} 14 \\ 2 &\overset{?}{>} 14 \end{aligned}$$

False, so you don't shade the middle region.

$$\begin{aligned} |3 \cdot 10 + 2| &\overset{?}{>} 14 \\ |32| &\overset{?}{>} 14 \\ 32 &\overset{?}{>} 14 \end{aligned}$$

True, so shade the region on the right. The following figure shows the result. x can be any number where the line is shaded. That's your final answer.

5. If it floats your boat, you may also want to express the answer symbolically.

   Because x can equal a number in the left region *or* a number in the right region, this is an *or* solution which means *union* ($\cup$). When you want to include everything from both regions on the number line, you want the *union* of the two regions. So, the symbolic answer is

   $$x < -\frac{16}{3} \cup x > 4$$

   If only the middle region were shaded, you'd have an *and* or *intersection* ($\cap$) problem. When you only want the section of the number line where the two regions overlap, you use the *intersection* of the two regions. Using the above number line points, for example, you would write the middle-region solution like

   $$x > -\frac{16}{3} \text{ and } x < 4 \text{ or}$$

   $$x > -\frac{16}{3} \cap x < 4 \text{ or}$$

   $$-\frac{16}{3} < x < 4$$

   You say "to-*may*-to," I say "to-*mah*-to."

REMEMBER

While we're on the subject of absolute value, don't forget that $\sqrt{x^2} = |x|$. $\sqrt{x^2}$ does *not* equal $\pm x$.

17 Fill in the two missing lengths for the sides of the triangle. $\boldsymbol{a = 5}$ **and** $\boldsymbol{b = 5\sqrt{3}}$

This is a 30°-60°-90° triangle — Well, duhh!

18 Fill in the two missing lengths for the sides of the triangle.

$$\boldsymbol{a = \frac{8}{\sqrt{3}} \text{ or } \frac{8\sqrt{3}}{3}}$$

$$\boldsymbol{b = \frac{16}{\sqrt{3}} \text{ or } \frac{16\sqrt{3}}{3}}$$

Another 30°-60°-90° triangle.

19 Fill in the two missing lengths for the sides of the triangle. $\boldsymbol{a = 6}$ **and** $\boldsymbol{b = 6\sqrt{2}}$

Make sure you know your 45°-45°-90° triangle.

20 **a.** What's the total area of the pentagon? $\boldsymbol{50 + \frac{25\sqrt{3}}{2}}$.

The square is $\frac{10}{\sqrt{2}}$ by $\frac{10}{\sqrt{2}}$ (because half a square is a 45°-45°-90° triangle), so the area is $\frac{10}{\sqrt{2}} \cdot \frac{10}{\sqrt{2}} = \frac{100}{2} = 50$. The equilateral triangle has a base of $\frac{10}{\sqrt{2}}$, or $5\sqrt{2}$, so its height is $\frac{5\sqrt{6}}{2}$ (because half of an equilateral triangle is a 30°-60°-90° triangle). So the area of the triangle is $\frac{1}{2}\left(5\sqrt{2}\right)\left(\frac{5\sqrt{6}}{2}\right) = \frac{25\sqrt{12}}{4} = \frac{50\sqrt{3}}{4} = \frac{25\sqrt{3}}{2}$. The total area is thus $50 + \frac{25\sqrt{3}}{2}$.

b. What's the perimeter? **The answer is** $\boldsymbol{25\sqrt{2}}$**.**

The sides of the square are $\frac{10}{\sqrt{2}}$, or $5\sqrt{2}$, as are the sides of the equilateral triangle.

The pentagon has five sides, so the perimeter is $5 \cdot 5\sqrt{2}$, or $25\sqrt{2}$.

21 Compute the area of the parallelogram. **The answer is $20\sqrt{2}$.**

The height is $\frac{4}{\sqrt{2}}$, or $2\sqrt{2}$, because the height is one of the legs of a 45°-45°-90° triangle, and the base is 10. So, because the area of a parallelogram equals *base* times *height*, the area is $10 \cdot 2\sqrt{2}$, or $20\sqrt{2}$.

22 What's the slope of $\overline{PQ}$? $\boldsymbol{\frac{d-b}{c-a}}$. Remember that $slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$.

23 How far is it from P to Q? $\boldsymbol{\sqrt{(c-a)^2 + (d-b)^2}}$

REMEMBER

Remember that $distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

24 What are the coordinates of the midpoint of $\overline{PQ}$? $\boldsymbol{\left(\frac{a+c}{2}, \frac{b+d}{2}\right)}$. The midpoint of a segment is given by the average of the two x coordinates and the average of the two y coordinates.