A Guide to Making Mathematics Practical

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Jo Clay

Chapter Purpose

This chapter reviews the mathematical skills and approaches needed to solve problems in precision agriculture and to provide a guidance for teaching and understanding mathematics. This chapter describes a common sense approach for solving mathematical story problems, provide examples where the logic leads to correct and incorrect answers, and discusses the solutions using key mathematical ideas.

Key Terms

Problem solving, units and quantities, multiplication and division of fractions, formulas, ratios, proportions, and scale factors, inverse relationships, percentages, unit conversions.

Mathematical Skills

Problem solving, units and quantities, multiplication and division of fractions, formulas, ratios, proportions, and scale factors, inverse relationships, percentages, unit conversions.

The Importance of Practical Mathematics

Implementing precision agriculture requires an ability to identify and solve basic mathematical story problems. However, solving problems is complicated by teaching approaches that emphasize memorization without understanding (Hiebert, 2013). Research indicates, constant practice is needed to remember formulas and procedures that are based on memorization (NCTM, 2000). Once students and adults leave an academic setting, they stop practicing and forget the knowledge they once knew. Teaching for understanding is designed to help people gain a deeper understanding of mathematics, improve problem solving abilities, improve retention of the learned material, and give rise to everyday practical applications of learned mathematical skills. Individual students will gain an enhanced capacity to integrate mathematics into decision making and their everyday lives. In the context of this manual, teaching mathematics is characterized for understanding, as *practical mathematics*. However, in spite of an increased use of teaching for understanding methods and it's corresponding advantages, adoption of practical mathematics has been slow (Dole et al., 2016). A survey of potential employers revealed that Agronomists need better mathematical skills to prepare them for 21st century Agriculture (Erickson et al., 2016).

The goals of this chapter are to discuss why teaching for understanding is very important and to provide a structure for grappling with intuitive ideas. The problems discussed in this chapter are agricultural based and provide multiple solutions to the same problem. One goal is to engage the student in the problem solving process. Embedded in the approaches are models that maybe useful in other problems. You will notice that some of the approaches are based on incorrect mathematics. This is not to confuse you, but to help you

J.A. Clay, Washington State University, Pullman, WA 99614.*Corresponding author (clayjoclay@gmail.com).
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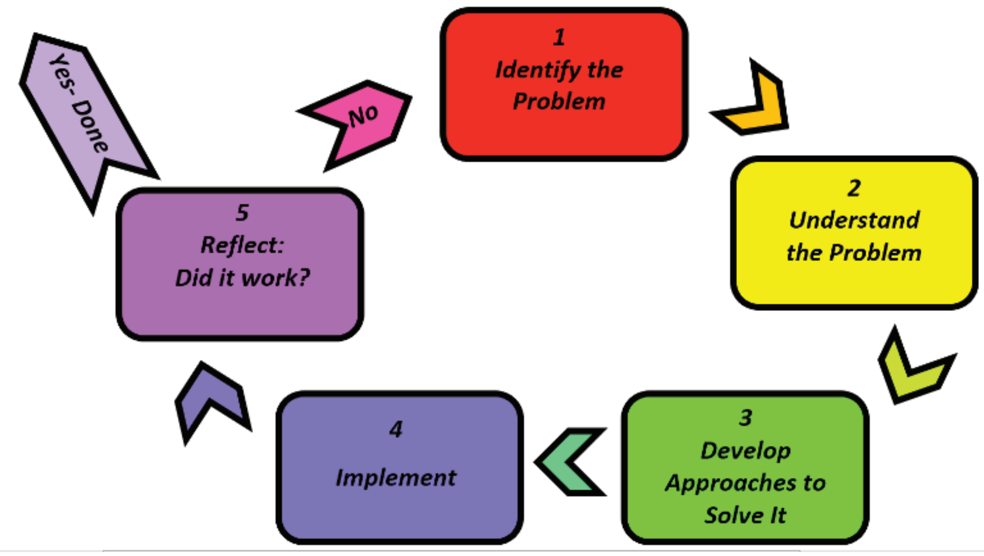


Fig. 1.1. The problem solving process.

become aware of how the mathematics are used to solve problems, and to highlight ways that people incorrectly use mathematical ideas.

In this chapter, problems will be provided and solved using a variety of approaches. The key mathematical ideas will be discussed and you will have the opportunity to deepen your own understanding. Then, you will be asked to identify ways that your enhanced understanding might be useful in your life and career. The goal of this process is to help make connections in your brain to ideas that you have learned, modify misconceptions, and develop new connections so that you can use mathematics to find solutions to a range of problems. Key understandings discussed in the chapter:

- A unit (focus on fractions)
- Operations with fractions (focus on multiplication and division)
- Proportion, Ratio and Scale Factor
- Percentage
- Conversions

Introduction to Problem Solving

The structure of this chapter is different from a typical mathematics book. We ask each of you to engage in thinking about a problem so that you can understand it, identify the intuitive ideas and mathematical knowledge that you have. Then, engage in solving the problem. Figure 1.1 illustrates the five steps of the problem solving process.

Step 1: The first step in solving a problem is to identify it. If you cannot state the problem, you will find it impossible to solve. As you identify the problem, think about the different ways that the problem can be stated. Once you identify the problem, you are ready for step two.

Step 2: Understand the problem by considering the information that is known and what the problem is asking you to find. Many people make a model of the problem in their mind or on paper. The model may be a picture or mathematical in the form of an equation or other notation. In this chapter you will learn some different ways to make models that will help you understand the problem. This is a critical step and one that many people skip.

Step 3: Develop an approach to solve the problem. Sometimes you will jump straight to a mathematical equation or formula because you immediately recognize a way to solve the problem. If you are unable to remember a specific approach, try using mathematical reasoning or simplifying the problem into one that you can solve. Another tool that may help you is to make a table that displays the known information. Remember it is okay to start again.

Step 4: As you implement your approach, remember the fundamental rules that govern operations with fractions and algebra. In this chapter, these fundamental rules will be reviewed using models that are designed to help you understand the rules. We know that if you understand why mathematics operates as it does, you will be more likely to remember how to correctly use the rules or shortcuts.

Step 5: Finally, you need to reflect on the approach. Does the approach make sense in the context of the problem? If it makes sense, is the mathematics correct? Each time you do an operation (or step in solving an equation) you should know why that step is correct. People sometimes use operations incorrectly as you will see in this chapter. If your solution does not make sense, go back to the first step and make sure that you clearly stated the problem, review your model and perhaps make a new model, consider changing the approach that you used, and check the mathematics. Did you perform each operation correctly as you implemented the approach that you used? If you know that your solution is incorrect and you can't figure out what to do, call a friend to talk with, look on the internet for help, or go back to some of your textbooks/notes to review ideas that you may have forgotten.

Some people like bullets to help them remember these key steps. Many people have found the following list helpful in problem solving.

· Restate the problem in your own words.

· Identify what the problem is asking you to find, and clarify the given information.

· Draw a model (picture or other representation) of the problem

· Identify an operation that you would begin with and give a reason for solving the problem.

· Solve the problem using the mathematics that you have learned.

· Reflect on whether the solution makes sense. If it does not make sense, go back and consider where you may have made a mistake.

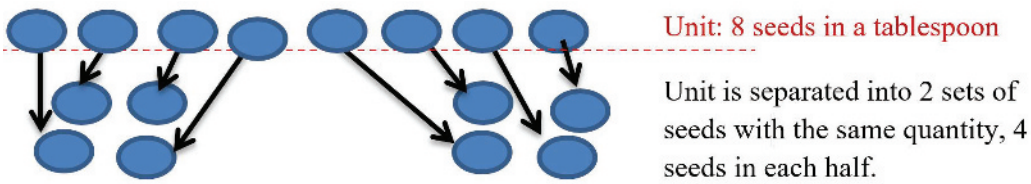
Section 1: Solving Problems By Separating the Quantity Into Pieces or Units

Solving a problem can often be made easier by separating the known quantity into separate pieces of the same size. For example, when learning to count, you matched a number to each object in a set. Each object represented a number in sequence and you conceptualized as a *unit of one*. For example, you counted the number of sunflower seeds in a tablespoon. Each seed was one unit, and you counted x units (or seeds) in one tablespoon. Later when fractions were introduced, you learned that the unit could be divided into pieces that were the same size. For example, a circle was drawn and represented a unit. It was divided into two pieces and each piece was defined as a half. However, you probably did not investigate when it made sense how to divide a unit in half. For the example preview, it does not make sense to divide a seed in half. The context of the problem and how you define the unit is critical in making sense of a problem and finding a correct solution. The following example illustrates the importance of carefully defining the unit and making sure that it represents the problem context.

A child found that 8 sunflower seeds fit in a tablespoon and was asked to find out how many would fit in a half-tablespoon. Many children would draw on their previous knowledge of dividing circles and draw eight circles. Then they would cut each circle in half and say the answer was 16. An adult might correct the child and say, "Yes, there are 16 half-circles. But, the question is asking you how many sunflower seeds are there if you divide them into two groups"?" The unit, one tablespoon, is split in half. Since there are eight units (seeds) in a tablespoon, a half tablespoon would contain half the units in one tablespoon or (8 seeds × 0.5 = 4). So there are four seeds in a half tablespoon.

The model of splitting a "whole" represented by a circle may lead to an understanding of fractions based on the idea that only a single object can be divided into pieces. An adult may recognize that a group of objects can be divided into subgroups. In the example, the unit changes from one sunflower seed into one tablespoon with eight seeds. Within this problem context, it does not make sense to cut each seed in half. You cannot put two half seed together to make a whole seed. Here we need to think of the unit as the tablespoon of eight seeds. We can divide it in half by making two piles of seeds.

The key understandings are:



· *A unit can be divided into same size quantities.*

· *An individual defines a unit to be anything that is suggested by context of the problem.*

The unit may be a liquid with a numerical quantity, a measurement of a field, or a number of discrete objects as in the previous problem. How a unit is represented depends on the context of the problem and individual preference.

Problem 1.1.

A producer wants to use a new weed killer on his/her farm and decide to test it in a small area. The directions tell the producer to dilute the concentrated weed killer with water to make a diluted solution with weed killer making up a fifth of the solution. Producer 1 wants to use 15 ounces of weed killer concentrate. How much water should Producer 1 add?

Before reading ahead, solve the problem or think about how you would go about solving it. The best way to develop your skills is to think about your intuitive reasoning and then take a look at some approaches that others have used. The two approaches yield different answers, only one is correct. See if you can identity the incorrect use of mathematics.

Answer Problem 1.1: Approach A

Producer 1 remembered that to solve problems, first he or she must identify the key information and what was given. Producer 1 also remembers that the word "of" indicates multiplication when you are working with fractions. So Producer 1 notes the phrase "one-fifth of weed killer" and will use 15 ounces. To solve the problem multiply 15 by one fifth.

$$15 \times {}^{1}/_{5}$$

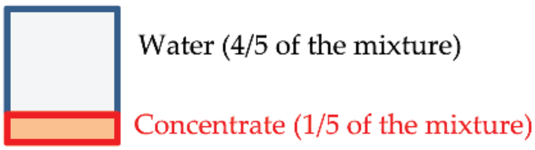
$${}^{15}/_{1} \times {}^{1}/_{5} = {}^{15}/_{5} = 3$$

Next, Producer 1 reflects about the reasonableness of his solution and concludes that usually more water is added than concentrate. So, to get a larger number Producer 1 multiplied 3 by 15 to get 45 ounces of water. From past experience, this seems to be a better answer. He/she made the product by mixing 45 ounces of water with 15 ounces of weed killer. However, 45 ounces of water, while seeming more reasonable, is not correct. Why?

Answer Problem 1.1: Approach B

Producer 2 decided to make a model of the problem so that she could figure out how much water to use. To make the model, Producer 2 started with a diagram of the mixed solution because they knew one part of it (the amount of weed killer concentrate) and they wanted to know the other part (amount of water).

Next Producer 2 identified the part that he or she knew and labeled the part that is the water to be added.



The diluted mixture can be thought of as the unit and ${}^{1}/_{5}$ of it is the concentrate. Using their knowledge of fractions, Producer 2 decided that the water is ${}^{4}/_{5}$ of the diluted mixture. The unit can be broken into 5 equal segments with four of them water. These four segments are ${}^{4}/_{5}$ of the total. So, the amount of water is 4 times as much as the concentrate. To find the amount of water that is needed, multiply 15 ounces by 4. The correct amount of water needed is 60 ounces.

Discussion of Problem 1.1

Why was approach A incorrect? Approach A is based on memorized knowledge, the word "of" with fractions in a word problem indicates the operation of multiplication. This rule is helpful only in one context, when a given quantity is a unit that needs to be broken into parts. Producer 1's approach would be correct if he/she knew that the quantity was 60 ounces and they wanted to find the number of ounces of weed killer to use. Producer 1 was unable to create a model for the problem and fully understand it. Instead, the producer skipped to step 4 and implemented an approach that was based on memorized information. The producer's approach assumed that the quantity was

60 ounces. They multiplied 60 by $^1/_5$. By multiplying by $^1/_5$, Producer 1 in fact was dividing the quantity into 5 equal amounts. However, in this problem you do not know the diluted amount. You only know the amount of the concentrate and multiplying it by $^1/_5$ will not give you the amount of water to add. In reflecting, Producer 1 made the error of changing the arithmetic to get an answer that made sense. The producer did not have a mathematical reason for changing the arithmetic, so they arrived at a mixture that would not have enough water.

Approach B is based on the key understanding, *a unit can be divided into same size quantities*. Producer 2 recognized the relationship between the unit and its fractional parts. Producer 2 used a model in which the unit is clearly identified and then it is divided into the fractional parts ($^1/_5$ and $^4/_5$). The model led Producer 2 to the correct solution to multiply 15 ounces by 4 to find the amount of water to add. The total amount of diluted weed killer is 15 × 5 = 75 ounces.

Problem 1.2.

An agronomist has two buckets, a large and smaller one. The smaller bucket holds ⅔ of a gallon of water. He/she can pour the small bucket of water into the larger one and it fill up ⅞ of the large bucket. How many more gallons would it take to fill up the whole bucket? How many gallons does the large bucket hold?

Before reading ahead, solve the problem or think about how you would go about solving it. Only one of the solutions is correct, see if you can find where the error was made.

Answer Problem 1.2: Approach A

Agronomist 1 knows that ⅞ of the bucket is filled. He or she subtracts ⅔ because the problems asks, *How many more gallons are needed?* To find out how many more are needed, Agronomist 1 uses subtraction.

$$\frac{7}{8}\times\frac{3}{3}=\frac{21}{24}$$

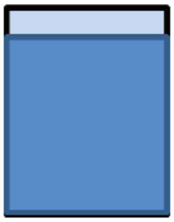
$$\frac{2}{3}\times\frac{8}{8}=\frac{16}{24}$$

$$=\frac{21}{24}-\frac{16}{24}=\frac{5}{24}$$

You need to add 5/24 gallons of water. This solution is incorrect.

Answer problem 1.2: Approach B

Agronomist 2 is unsure of where to begin. They knows that $^2/_3$ gallons fill the small bucket and that $^7/_8$ is not a measure of gallons. Thus Agronomist 2 knows that they cannot subtract because fractions are not in gallons. Agronomist 2 makes a diagram by placing the small bucket inside the larger bucket.



← The part of the large bucket that is not filled.

←The small bucket. It is $^2/_3$ gallons and fills $^7/_8$ of the large bucket.

Agronomist 2 recognizes that the relationship between the two buckets. The small bucket is 7/8 of the large one. So, the large bucket is 1/8 larger. Using this relationship, he/she divides the large bucket into 8 pieces (each one is $^1/_8$ of the large bucket). Agronomist 2 colors in 7 of the pieces ($^7/_8$). To find the number of gallons in each piece, they divide $^2/_3$ by 7. Later in the chapter we will discuss division of fractions. For our purpose, Agronomist 2 changes the fraction into a decimal and divides 0.67 ÷ 7 = 0.095 = 0.10 (rounded off). They know that each piece is 0.10 gallons. He/she multiplies 0.10 by 8 because there are eight $^1/_8$s in a whole (unit). Thus, the large bucket is approximately 0.8

gallons, and only 0.10 more gallons are needed to completely fill the large bucket. Working in fractions, the answer is $^{16}/_{21}$ total gallons held by the large bucket. This approach is correct.

Discussion of Problem 1.2

Agronomist 1 did not fully understand the problem. They interpreted the problem to ask how many gallons were needed if the large bucket held $^{7}/_{8}$ gallons. Their approach would be correct for this question. Agronomist 1 found how many more gallons were needed to fill the large bucket if the small one held $^{2}/_{3}$ gallons and the large one held $^{7}/_{8}$ gallons. In this case, Agronomist 1 defined the unit as a gallon, and both fractions referenced this unit. Agronomist 1 also did not go back and reflect on the problem. It does not make sense that the large bucket would have only $^{5}/_{24}$ gallons when the small bucket held $^{2}/_{3}$ gallons. Clearly, $^{5}/_{24}$ is less than $^{2}/_{3}$.

In contrast, Agronomist 2 created a model so that he/she could identify the problem more clearly. In doing so, Agronomist 2 recognized that two units were used in the problem. One unit was a gallon and the second was the large bucket. The problem told them that the small bucket was $^{7}/_{8}$ the size of the large bucket. Recognizing this was an important step to correctly solve the problem and it illustrates that *an individual defines a unit to be anything that is suggested by the problem's context*. Next they used the key understanding, *a unit can be divided into same size quantities,* to find the number of gallons in $^{1}/_{8}$ of the large bucket. After finding the size of $^{1}/_{8}$, Agronomist 2 found the number of gallons in the whole bucket.

Section 1 Practice Exercises

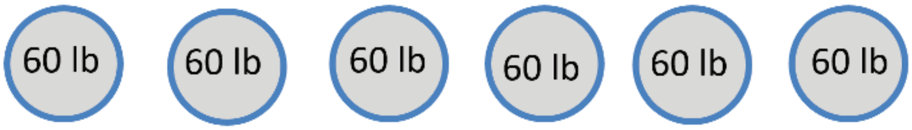
Identify the unit for each problem. You may use a model to help you solve the problem or numbers. Please note, mathematicians use models that they write on paper until they are able to picture them in their minds. Models are useful at times and not useful at other times. In this unit, several models are provided as samples that you can use. You should be able to explain what you did similar to the explanations given in the previous sample problems.

Section 2: Solving Problems That Involve Multiplication and Division of Fractions and Formula

Multiplication

When you were first learned about multiplication, it was mathematically defined *as number of groups of a particular size*. For example: You have 6 bushels of wheat and each bushel weighs 60 pounds. What is the total weight? Here you have 6 groups (a bushel is one group) and the size of each 60 pounds. To find the total you multiply 6 × 60 lb = 360 lb.

Each circle is one bushel



The problems that follow are illustrations that provide a context for identifying the key understandings of multiplying and dividing fractions. In the discussion of the problems, the key understandings will be presented.

Answer Problem 1.8: Approach A

Producer 1 restated the problem for clarity of thinking, how many bushels are 1/3 of the total number of bushels (6)? Using what he/she learned in the previous section, Producer 1 defined the unit as 6 bushels and drew a model. He/she then divided the unit into 3 groups. Looking at one of the groups, Producer 1 can see that there are 2 bushels in third.

Each bushel soybean is 60 lb. The weight of *two bushels of soybean would be 120 lb of soybean* (2 × 60 lb).

Answer Problem 1.8: Approach B

Producer 2 used the information from the example, 6 bushels weigh 360 lb. He/she wanted to find 1/3 of this quantity. Producer 2 realized that he/she was finding $^{1}/_{3}$ (part of the group) with a size of 360 lb (size of the group).

1.3. An agronomist has ⅔ of a gallon of water in a bucket, which fills up ⅞ of the bucket. How many gallons would it take to fill up the whole bucket?

1.4. A producer is mixing weed killer. He/she guesses that ⅗ of a bottle of would be the right amount for the sprayer. But, it turns out to fill only about 3/4 of the sprayer. How much of a bottle of concentrate would make a full sprayer?

1.5. You have a farm with a field on the north side of a large hill and a field on the south side of the hill. It takes 40 ears of corn to fill a bushel basket from the field on the north. It takes 24 ears of corn to fill a bushel basket from the field on the south.

a) Which field produces the biggest ears of corn? Why?

b) You put 208 ears of corn into a crate. 1/3 of the crate is from the field on the south. How many bushels came from the south field and how many came from the north field?

c) How many bushels does the crate hold?

d) In a second crate you fill 3/4 of it with corn from the south field by putting in 108 ears of corn and 1/4 of it with corn from the north field. How many ears of corn are in the crate? How many bushels of corn from the south field do you put in the crate?

1.6. An agronomist wants to use a new weed killer on your farm and decides to test it in a small area. The directions indicate to dilute the concentrated weed killer with water to make a diluted mixture with 1/5 of weed killer. You want to use 15 ounces of weed killer concentrate.

a) How much water should you add?

b) Challenge: The active ingredient of Chemical A is 15%, chemical B is 7.3% and chemical C is 5%. Find the number of ounces of each active ingredient. (See Section 3 for percentage)

15 oz ×(.15) = 2.25 oz Chemical A: 15 oz × 0.073 = 1.095 oz Chemical B: 15 oz × 0.05 = 0.75 oz Chemical C

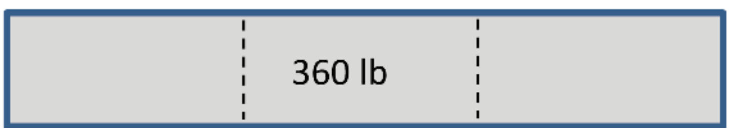
1.7. Assume the yard that must be sprayed with a chemical measures 44 ft by 75 ft. You plan to use a tank sprayer. The mixing instructions are to mix 2.5 ounces with 1 gallon of water that is applied over a 300 ft^2 area. How much spray solution (chemical + water) is needed if the spray solution is applied evenly across the area?

1.8. A producer has 6 bushels of grain, some are wheat and some are soybean. One-third of the bushels were soybean? How many bushels are soybean? What is the weight of the bushels of soybean?

In this case, we could think about 1/3 of the bushels are soybean and the problem can easily be solved in two ways. Focus your thinking on the different ways of reasoning. Remember to solve the problem or think about how you would go about solving it before reading ahead. Both solutions are correct, see if you can identify how the models are similar and different.

Mathematically we could write this as $^1/_3$ x 360. To solve this, Producer 2 thought about what one-third means: Take the group and put it into 3 same-size chunks.

The unit is 360 and putting it into 3 chunks means that each chunk is 1/3 and weighs 120 lb (360 ÷ 3).



Discussion of Problem 1.8

Comparing the two approaches, we first notice that Producer 1 and 2 models are different. Producer 1 thought about each bushel as part of the unit and drew 6 circles. Producer 2 thought about the quantity and divided the total quantity into 3 parts. Second, they used a different order of operation. Producer 1 divided 6 bushels by 3 and then multiplied by 60 lb (written as $6 \times {}^{1}/_{3}$ x 60). Producer 2 multiplied 6 bushels by 60 lb. and then divided by 3 (written as: $6 \times 60 \times {}^{1}/_{3}$). Remember that a number $n \div 3$ can be written as ${}^{n}/_{3}$. Thus $6 \div 3 \times 60 = 6 \times {}^{1}/_{3}$ x 60.

Key understandings are:

- The unit (total to be broken into pieces) must be defined. It can be a set of objects or a single object as illustrated by Producer 1 and 2.
- The operations used to solve a problem reflect the actions that from a model.
- The order of multiplying and dividing does not matter when working with fractions.

Mathematically, it does not matter if you multiply or divide first. However, you do need to be careful of your notation. The first problem can be written as $6 \div 3 \times 60$ or $6 \times {}^{1}/_{3} \times 60$. The second can be written as $6 \times 60 \div 3$ or $6 \times 60 \times {}^{1}/_{3}$. When we change ÷ 3 into a fractional representation for the idea, the two expressions are the same because the order that one multiplies does not matter. Mathematicians call this the commutative property. Producer 1 and 2 thought about the problem differently, but we can see that essentially the work is the same (the operations were done in a different order). The key idea that you need to remember is to make sense of the problem using your mathematical experiences, models that you draw or imagine, and to make sure that the operations you use reflect the actions that you take, as both Producer 1 and 2 demonstrated.

Problem 1.9.

A producer has a honey hive and each fall harvests the honey. The weight of each jar of honey is 1¼ pounds. He/she has 15 jars of honey, how many pounds of honey does the producer harvest?

Remember to solve the problem or think about how you would go about solving it before reading ahead. Both of the solutions are correct, see if you can identify how their reasoning about fractional parts differed.

Problem 1.9: Approach A

Producer 1 has 15 jars of honey and draws a model as shown:

Each circle has the weight of 1 1/4 lb. Counting the jars, Producer 1 knows that he/she has 15 lb and a little more. The little extra is 15 jars (groups) of a size 1/4. He/she knows that 4 fourths make a pound. The amount would be 15 lb and The fractional part (is equivalent to 3 pounds and 3/4 (solve 15 . Total is 15 + 3 3/4 = 18 3/4 lb of honey.

Mathematically Producer 1 could write this as:

15 × 1 1/4 =
(15 × 1) + (15 × 1/4) =
15 + = *Note:* = 15 ÷ 4 = 3 remainder 3 *or* 3 3/4
15 + 3 3/4 = 18 3/4

Note: ${}^{15}/_{4}$ = 15 ÷ 4 = 3 or 3 ${}^{3}/_{4.}$

The correct solution is 18 3/4 pounds.

Problem 1.9: Approach B

Producer 2 has 15 jars of honey as shown:

Each jar has 1 1/4 pounds of honey. The circle represents one jar. The quantity 1 1/4 can be thought of as pounds of honey.

Producer 2 knows that there are 15 jars. So he or she multiplies 15 by 5/4. Three mathematical methods to solve this are shown. All of the methods are correct. Use the method that makes sense to you.

Each jar can be divided into 5 pieces and each piece is ¼ pound.

Method 1	Method 2	Method 3
15 × 5/4 =	15 × 5 × 1/4	15 × 5 ÷ 4
75/4	75 × 1/4	75 ÷ 4
18 3/4	$^{75}/_{4}$ = 18 3/4	18 3/4

Notice the use of multiplying the numerators and then "reducing" by dividing. Producer 2 found the correct answer of 18 3/4 pounds of honey.

Discussion of Problem 1.9

Producer 1 and 2 both arrived at the correct answer. Notice that their initial models looked alike (15 circles). However, their thinking was slightly different. Producer 1 took each jar of honey and subtracted out 1 pound. They recognized the need to change the unit from a jar to a pound. They then put the 1 fourth remaining honey into units of 1 pound. Producer 2 took each jar and broke them up into 5 fourths. He/she then converted the jars to pounds by multiplying the number of jars by the conversion ($^{5}/_{4}$)– he or she needs 4 fourths to make a pound and he had 5 fourths in each jar.

Division

Division of fractions can be counter intuitive, which is why so many people struggle with this operation. Division is based on "undoing" multiplication or putting a quantity into parts. In essence, you will find either what we call "the number of groups" or "the size of a group", if you know the total.

The problems that follow are illustrations that provide a context for identifying the key understandings of dividing fractions. In the discussion of the problems, the key understandings will be presented.

Problem 1.10.

An agronomist has 38 pounds of phosphorous (P) and 1 ¼ pounds of P will be used in a fertilizer mixture. He wants to divide the P into bags of 1 ¼ pounds. How many bags does he need?

Remember to solve the problem or think about how you would go about solving it before reading ahead. Only one of the solutions is correct, see if you can find where the error was made.

Problem 1.10: Approach A

Agronomist 1 has a big bag of P and he/she needs to find out how many "groups" there are of 1 1/4 pounds. Using logic, he/she knows that there will be less than 38 bags because that is how many bags he/she would need if 1 pound of P fit in one bag. Thinking about this, Agronomist 1 could find the number of fourths in the 38 pounds and then divide the number of fourths into bags that hold 1 1/4 or $^{5}/_{4}$ pounds.

Agronomist 1 knows that five of these fourths will fit in one bag. Thus we need to divide 152 fourths by five. 152 ÷ 5 = 30 $^{2}/_{5}$ bags will be filled. Agronomist 1's solution is correct.

Total of 38 Pounds of P. This can be divided into fourths. In each pound, there are 4 fourths. Thus there are 38 x 4 fourths = 152 *fourth* pounds of P.

Think of 38 little scoops and each scoop is divided into ¼

Problem 1.10: Approach B

Agronomist 2 knows that there is a total of 38 pounds. This quantity of P needs to be put into bags that holds 1 $^{1}/_{4}$ pounds. He or she writes 38 ÷ 1 $^{1}/_{4}$. Agronomist 2 has forgotten how to divide mixed numbers. He or she remembers that 1 $^1/_4$ can be written as 1 + $^1/^4$ and rewrites the expression as 38 ÷ (1 + $^1/_4$). Producer 2 rewrites this as (38 ÷ 1) + (38 ÷ $^1/_4$) = 38 + (38 × 4) = 38 + 152 = 190 bags. What was Agronomist 2's mistake?

Discussion of Problem 1.10

Agronomist 1 used a model to represent the problem and applied mathematical thinking to solve it. Following are symbols showing the steps that were used.

Agronomist 1's Arithmetic	Alternative Arithmetic
(38 × 4) ÷ 5	38 ÷ 1 $^1/_4$
152 ÷ 5 = 30 $^2/_5$	38 ÷ $^5/_4$
	38 × $^4/_5$
	$^{152}/_5$ = 30 $^2/_5$

Notice that when you divide by a fraction you invert and multiply. This is shown by comparing Step 1 in Agronomist 1's arithmetic and Step 3 in the alternative arithmetic.

When you invert the fraction, you are following the steps that you did with the model. You took the total and found the number of fourths (38 × 4). After you found the number of fourths, you divided by five to find the number of bags. The rule of invert and multiple simply does the thinking for you. Memorizing this rule can be confusing and hard to remember. If you think about the actions of solving the problem, you should remember the rule better.

Key understandings are:

· Division divides a quantity into parts, either the number of groups or the size of a group. First decide on what the total is. The total is always divided into one of the parts.

· Write your equation and make sure that the total comes first (total ÷ part).

· If you have trouble remembering the rule to multiply by the inverse, make a model to show the actions that needs to be taken.

Why was Agronomist 2's solution incorrect? The error was thinking that they could rewrite the quantity 1 $^1/_4$ into 1 + $^1/_4$ and then divide the total by each part. In essence, what Agronomist 2 found was the number of bags that could hold 1 lb *and* the number of bags that could hold $^1/_4$ pound and added these quantities together. Their answer does not make sense. On reflection, if Agronomist 2 started with 38 lb each bag would be partially filled because each bag held 1 $^1/_4$ lb, not 1 lb. Thus, there should be fewer than 38 bags filled.

To correct Agronomist 2's approach, imagine 38 pounds of P in a big bag. Note that $^4/_4$ pounds are in each pound of P. He or she could take five pounds of P and put a pound in four bags, then divide the fifth pound into 1/4 lb quantities and place an additional $^1/_4$ pound in each of the four bags. The table illustrates the process of taking five pounds and filling four bags so that each bag holds 1 $^1/_4$ pounds. The key understanding of *making a model* in for form of a table helps Agronomist 2 correctly solve the problem. He or she can see the relationship of using five pounds to fill four bags.

Number of pounds in big bag	Number of pounds removed	Remaining pounds in the big bag	Number of bags with 1 $^1/_4$ lb
38	0	38	0
38	5	33	4
33	5	28	4 + 4 = 8
28	5	23	8 + 4 = 12
23	5	18	12 + 4 = 16
18	5	13	16 + 4 = 20
13	5	8	20 + 4 = 24
8	5	3	24 + 4 = 28
3	3	0	28 + 2 = 30 and remainder of 2

Finally, we have two fourths left over. We need five fourths to fill a bag, so we have two out of the five or two-fifths of a quart. The answer is 30 $^2/_5$ bags are filled.

Note: Another way to solve or check the solution is to convert the fraction 1 1/4 to a decimal and divide 38 by 1.25. The focus of this chapter is on fractions, thus converting a fraction to a decimal can simplify finding a solution. The approach of converting to a decimal will lead to a correct solution when the fraction is non-repeating. Fractions like one-third are nonrepeating and converting it to 0.33 can cause serious problems, especially when using large quantities.

Formula

Sometimes precision farmers use formulas and equations to estimate or calculate profit and loss. One example is to estimate harvest crop loss. Sandifolo (2002) created the following formula to find the percent weight loss to tuber crops due to weevil infestation.

CT average weight of clean tuber
#CT number of clean tubers
DT average weight of damaged tubers
#DT number of damaged tubers

$$\%\text{ weight loss} = \frac{(CT \times \#CT) - (DT \times \#DT)}{1} \times \frac{100}{CT(\#CT + \#DT)}$$

Note: This is one way to write the formula. Typically, the denominator 1 is omitted because mathematically it is extraneous. When using a formula, it is helpful to include the denominator 1 for clarity and to ensure that you correctly complete the operation. If you frequently use the same formula with different values, inserting the formula into a spreadsheet is helpful. A spreadsheet can help you track losses over time and in different fields. The spreadsheet also completes the calculations for you without error.

Problem 1.11.

A producer wants to estimate the percent weight loss of the harvest loss using the following data:

Average weight of clean tubers (CT)	32 oz
Number of clean tubers (#CT)	1000
Average weight damaged tuber (DT)	26 oz
Number of damaged tubers (#DT)	500

Answer Problem 1.11: Approach A

Producer 1 begins to solve the problem by writing the formula on paper. Then they substitutes their data in the formula for each variable (CT, #CT, DT, #DT).

$$\%\text{weight loss} = \frac{(CT \times \#CT) - (DT \times \#DT)}{1} \times \frac{100}{CT(\#CT + \#DT)}$$

$$\%\text{weight loss} = \frac{(32 \times 1000) - (26 \times 500)}{1} \times \frac{100}{32(1000 + 500)}$$

Next, Producer 1 computes all of the operations within the parentheses.

$$\%\text{weight loss} = \frac{(32{,}000) - (13{,}000)}{1} \times \frac{100}{32(1500)}$$

Producer 1 completes the computations for the numerator and denominator.

$$\%\text{weight loss} = \frac{19{,}000}{1} \times \frac{100}{48{,}000}$$

The final steps can be accomplished in several ways. Two methods are displayed below:

Method 1		Method 2	
$\frac{19{,}000}{1} \times \frac{100}{48{,}000}$		$\frac{19{,}000}{1} \times \frac{100}{48{,}000}$	
$\frac{19{,}000}{1} \times \frac{1}{480}$	Write an equivalent fraction by dividing the numerator and denominator by 100. (cancel 00)	$\frac{1{,}900{,}000}{48{,}000}$	Multiply the numerators and denominators.
19,000 ÷ 480	Rewrite ($\times \frac{1}{480}$) by an equivalent expression.	39.58% loss	Divide the numerator by the denominator.
39.58% loss	Complete the calculation.		

Discussion of Problem 1.11

Producer 1's approach required him or her to remember to substitute the provided data for each variable in the equation. Deriving the formula is beyond the scope of this chapter.

What you need to remember is to:

- Substitute the data
- Follow the rules for order of operation (see Discussion of Problem 1.8)
- Simplify the expression (writing down all of the steps as you simplify can reduce arithmetic errors. Be careful as you "cancel the zero" as shown in Step 2 of Method 1 above)
- Reflect on whether your answer makes sense for the problem

Problem 1.12.

A producer has a field which has both clean and damaged tubers. The field is described as mixed.

Average weight of clean tubers (CT)	32 oz
Number of clean tubers (#CT)	1000
Average weight damaged tuber (DT)	26 oz
Number of damaged tubers (#DT)	500

The above formula is modified to reflect the mixed field.

CT	average weight of clean tuber
#CT	number of clean tubers
DT	average weight of damaged tubers
#DT	number of damaged tubers

$$\%\text{weight loss} = \frac{(CT - DT)\#DT}{1} \times \frac{100}{CT(\#CT - \#DT)}$$

The data for the mixed field is the same as for Problem 1.11. The difference between the problems is that problem 1.12 has one field that is mixed, and in Problem 1.11 there were two distinct fields, one was damaged and one was undamaged.

Using a slightly different approach, Producer 2 used a 5-step approach to calculate weight loss, as shown.

$\%\text{weight loss} = \frac{(CT - DT)\#DT}{1} \times \frac{100}{CT(\#CT + \#DT)}$	**Write the formula.**
$\frac{(32-26)500}{1} \times \frac{100}{32(1000+500)}$	**Substitute the data into the formula for each variable.**
$\frac{(6)500}{1} \times \frac{100}{32(1500)}$	**Calculate the numbers inside the parentheses.**
$\frac{3000}{1} \times \frac{100}{48000}$	**Calculate the numerators and denominators.**
$\frac{3000}{1} \times \frac{1}{480} = \frac{3000}{480} = 6.25\%$	**Complete the calculations to find the percentage loss.**

Section 2: Practice Exercises

1.13. A producer was mixing some medicine for a cow. The recipe calls for 5/6 cups of water. As he or she was adding the water, the producer got a phone call and realized that was a third of the amount needed for several sick cows.

a) How many cups of water does he need to add for several sick cows?

b) How many 1/2 cup doses of medicine are in a bottle that contains $1^1/_3$ cups?

1.14. An agronomist used a tractor to mow grass. The mileage was 18 $^1/_2$ miles over 4 d. If the agronomist mowed the same distance each day, how many miles were mowed each day?

1.15. A producer has a length of rope that measures 9⅜ yards long. Each farmhand needs a piece $^5/_8$ yards long for lashing. How many pieces of the required length can be cut? What part of a piece for lashing is left over?

1.16. 10 1/2 gallons of water fills up 2 $^1/_3$ buckets. How many gallons are in one bucket?

1.17. An agronomist has two fields of tubers. One is damaged and the other is not. He collects the following data.
Average weight of clean tubers (CT) 18 oz
Number of clean tubers (#CT) 800
Average weight damaged tuber (DT) 16 oz
Number of damaged tubers (#DT) 300

a) Find the percentage loss.

b) In another field, he has both damaged and undamaged tubers. Using the same data, find the percentage loss for the mixed field.

Section 3: Solving Problems That Involve Proportions, Ratios, and Scale Factors

Proportion, *ratio,* or *rate* describes a relationship between quantities that are different. For example, they are used to describe how many miles per gallon a car uses. Precision farmers consider how much herbicides and fertilizer are applied to a field. Note that the two quantities are different, one quantity is measured in gallons and the other is acreage. The proportion of gallons of herbicide or fertilizer to the acreage allows the farmer to apply the proper concentration. A scale factor allows the farmer to quickly determine the correct quantity of chemicals when the acreage increases or decreases.

Scale factors increase or decrease quantities or relationships by doubling or halving; tripling or cutting into thirds, etc. For example, you use 100 gallons of 28–0-0 for 10 acres. How much N do you need for 20 acres? To solve this, the scale factor is two. The number of acres and the quantity of N are doubled. When two quantities increase by the same scale factor, we say this is a *direct proportion*. Two equivalent fractions describe this proportion; $\frac{\text{100 gallons of N}}{\text{10 acres}} = \frac{\text{200 gallons of N}}{\text{20 acres}}$. Note that the two fractions are equivalent to the same unit ratio, $\frac{\text{10 gallons of N}}{\text{1 acres}}$.

Sometimes one quantity increases and the other decreases. For example, it takes one person two days to combine a wheat field. How long would it take two combines? Here you double the combines, so the time should be cut in half. When one quantity increases by a scale factor and the other decreases by the inverse factor, we call it an *inverse relationship*. The key understanding on proportions and rations includes:

- Direct proportion is a relationship that occurs when the two quantities both increase by the same scale factor f.
- A unit ratio can be written to express the relationship between two quantities.
- Two ratios with the same unit ratio form equivalent fractions.
- An inverse relationship is when one quantity increases by a scale factor f and the other decreases by 1/f.

The following problems are used to develop these key understandings. **Approach A** illustrates the key understanding that *when two quantities have the same ratios they are proportional,* and uses equivalent fraction to solve the problem. **Approach B** illustrates the key understanding of *writing a unit ratio to express the relationship* and uses it to solve the problem. These ideas are extended in the discussion to help you understand why a direct proportion is linear. Problem 1.18 develops the key understanding of *inverse relationships.*

Problem 1.18.

A faucet (A) drips 6 ounces of water in 21 minutes and a second faucet (B) drips 4 ounces in 14 minutes. Do faucet A and B drip at the same rate or is one dripping faster than the other?

Answer Problem 1.18: Approach A

To solve this problem, we need to use the key understanding, *when two quantities have the same ratios they are proportional,* and find the equivalent fraction for the two ratios.

Faucet A $\frac{6 \text{ ounces}}{21 \text{ min}}=\frac{6 \text{ ounces}}{21 \text{ min}}\times\frac{14}{14}=\frac{84 \text{ ounces}}{294 \text{ minutes}}$

Faucet B $\frac{4 \text{ ounces}}{14 \text{ minutes}}=\frac{4 \text{ ounces}}{14 \text{ minutes}}\times\frac{21}{21}=\frac{84 \text{ ounces}}{294 \text{ minutes}}$

Here you can see that we thought of the ratios as equivalent fractions. Changing the ratios so that their denominators are the same, we find that the two ratios that are the same. Because the two equivalent ratios could be found, we say that the ratios are proportional, or equivalent.

Answer Problem 1.18: Approach B

A different way of solving this problem is to "reduce" the two ratios by dividing the numerator and denominator by a form of 1. Notice the arithmetic in the second step.

Faucet A $\frac{6 \text{ ounces}}{21 \text{ minutes}}=\frac{6\div 3}{21\div 3}=\frac{2}{7}$

Faucet B $\frac{4 \text{ ounces}}{14 \text{ minutes}}=\frac{4\div 2}{14\div 2}=\frac{2}{7}$

The two rates are identical because equivalent fractions can be written for the two rates. This is also called finding the Least Common Denominator and allows us to compare two ratios without conversion to a decimal form to see if they are directly proportional. (If we converted the ratios to a decimal, the decimal expression would be the same.) Again, the key understanding, when two quantities have the same ratios they are proportional.

Extending Approach B to a Strategy Using a Unit Ratio

The solution indicates that in 7 min, 2 ounces of water will drip. This ratio can also be interpreted as the faucet will drip 2/7 ounces in per minute. To perform the calculation, divide the numerator and denominator by seven, resulting in the ratio $\frac{2/7}{1}$. Thus for each minute, the faucets drips $^{2}/_{7}$ ounces. This interpretation is called a *unit ratio* and written in ratio format, $\frac{\frac{2}{7} \text{ ounces}}{1 \text{ minute}}$. If we want to know how many ounces we will have in any amount of time, we can simply increase the ratio using the time factor *t.* For example, if we want to know how many ounces there would be after 42 min write the ratio and multiply by 42 as shown below.

$$\frac{2/7 \text{ ounces}}{1 \text{ min}}\times\frac{42}{42}=\frac{2/7\times 42}{1\times 42}=\frac{2\times 6}{42}=\frac{12 \text{ ounces}}{42 \text{ min}}$$

This tells us that in 42 min, we would have 12 ounces of water. Now we ask ourselves, does this make sense? Yes, it should be that amount. If we look at Faucet A, we know that in 21 min it drips 6 ounces of water. We notice that 42 min is twice as long, so the amount of water should be twice as much. The scale factor of 2 can be used to write an equivalent ratio using the ratio $^{6}/_{21}$ for Faucet A.

The key understandings for find a new ratio with a different value includes:

- Find the unit ratio and then use it to find an equivalent ratio.
- Look at the known ratios and see if there is a relationship between the numbers. If so, use a scale factor to increase or decrease the ratio.

Note that in some cases it is easier to use a ratio that is not in unit format, but sometimes it is easier to use the unit ratio.

A similar process can be used if we want to use ounces as our unit. This enables us to easily find out the number of minutes it would take to get *b* amount of water.

PROBLEM 1.18. EXTENSION. A faucet drips 2 ounces of water in 7 minutes. How long will it take to get one quart of water?

Answer Problem 1.18. Extension

We could begin by finding the time in terms of 1 ounce and then use the factor 32 because there are 32 ounces in a quart.

$$\frac{2\text{ ounces}}{7\text{ min}}=\frac{2\div 2}{7\div 2}=\frac{1\text{ ounces}}{\frac{7}{2}\text{ min}}$$

This tells us that it takes $^7/_2$ minutes or 3 1/2 min to drip 1 ounce. To find the number of minutes it takes to get 32 ounces, multiply the time to get 1 ounce by 32.

$$\frac{7}{2}\times 32=\text{time}$$

$$\frac{7}{2}\times 32=\frac{7}{2}\times\frac{32}{1}=\frac{7}{2}\times\frac{2\times 16}{1}=7\times 16=112\text{ min}$$

By dividing the number of minutes by 60, we can express the time in hours. 1 h and 52 min. Unit cancellation helps us keep track of unit conversion that is required by problems. Unit cancellation will be further discussed in *Section 5* of this chapter.

Discussion of Problem 1.18. Extension

The key understanding of proportions is that *when two quantities have the same ratios they are proportional.* This means that the two ratios form equivalent fractions that can be written to express this relationship. When you use the unit ratio, the unknown quantity can be found by using the scale factor. When you change a ratio using a scale factor, the relative proportion does not change. This is the definition of a linear relationship. Mathematically, scale factor can be thought of as slope and creates a linear relationship. An algebraic approach can be used to solve Problem 1.18 and a discussion of this approach allows us to further extend our understanding of proportion and ratio.

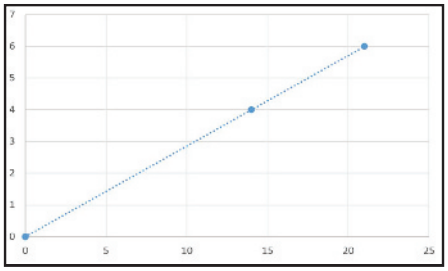
Answer Problem 1.18 Approach C

The rate of change or slope is another strategy way to determine whether faucet A or B drip at the same rate. If they drip at the same rate, they will form a line going through zero (at time 0, the amount of water should be 0).

A faucet (A) drips 6 ounces of water in 21 min and a second faucet (B) drips 4 ounces in 14 min.

To graph the relationship, make a table and graph the line. We will use time as the independent variable and place it on the horizontal axis (*x* axis). In math and science, the horizontal axis is usually the independent variable or that variable that is frequently used to measure change. Time is frequently used in this way.

Time (x)	Ounces (y)
0	0
14	4
21	6



To determine whether the three points are on the same line, find the slope between the three points. If the slopes are the same, then the relationship is linear and the rates are the same. Plotting the three points on a graph will show you that a line connects the three points. This indicates a linear relationship.

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

Find the slope between (0, 0) and (14, 4).

$$\frac{4-0}{14-0} = \frac{4}{14} = \frac{2}{7}$$

Find the slope between (0, 0) and (21, 6) or between (14, 4) and (21, 6).

$$\frac{6-4}{21-14} = \frac{2}{7}$$

Notice that the two slopes are the same, therefore the relationship is linear. Also note that the slope is the same as the reduced fraction for 4/14 and 6/21. Also notice the slope and unit ratio are equivalent ($\frac{2}{7} = \frac{1}{3\frac{1}{2}}$).

The key understandings includes that:

- Proportions are linear relationships.
- A linear function is formed when two quantity change by the same factor (called slope).
- An inverse relationship is when one quantity increases by a scale factor f and the other decreases by 1/f.

Problem 1.19.

Two people can harvest a field in three days. If one more person joins them, how long will it that the three people to harvest the field? (Assume that the three people will all have productive work to do.)

Remember to solve the problem or think about how you would go about solving it before reading ahead. Only one of the solutions is correct, see if you can find where the error was made. Note: This problem helps you understand inverse relationships.

Answer Problem 1.19: Approach A

Producer 1 writes the ratio 2 people/3 days. They then change the ratio to a unit ratio by dividing the numerator and denominator by 2; $\frac{1 \text{ person}}{\frac{3}{2} \text{ days}}$. The unit ratio indicates that it takes one person $^3/_2$ days to harvest the field. One more person would be three times as long as one person. $\frac{3 \text{ people}}{\frac{3\times 3}{2 \text{ days}}} = \frac{3 \text{ people}}{\frac{9}{2} \text{ days}} = \frac{3 \text{ people}}{4\frac{1}{2} \text{ days}}$. It will take 4 1/2 d to cut the field. Can you identify the incorrect thinking?

Answer Problem 1.19: Approach B

Producer 2 reasoned that it should take less time to harvest the field with more people. If two people complete the job in three days, it should take one person twice as long (6 d). To investigate the relationship, a table is constructed. He or she began with the known relationship.

people		2				
days		3				

He/she fills in the relationship that was logically determined.

people	1	2				
days	6	3				

Look at the table and try to write the relationship in words. *When we double the people, it will take half the time.* Use this to fill in the table.

people	1	2		4		
days	6	3		1 $^1/_2$		

If we triple the people it should take a third of the time. Triple the relationship for one person.

people	1	2	3	4		
days	6	3	2	1 $^1/_2$		

The solution is it will take two days for three people to harvest the field.

Discussion of Problem 1.19

Producer 1 used his or her knowledge of a unit ratio to calculate a solution. But, they did not fully understand the problem and jumped to an incorrect application of the mathematical ideas. Producer 1 did not recognize that this problem illustrated an inverse proportion. They also did not go back and reflect whether the solution made sense in the problem context. Logically, it makes sense that as you increase the number of people to complete a job, the time that it takes decreases.

In contrast, Producer 2 thought about the context and used their mathematical insights. Producer 2 reasoned that it should take less time if more people were harvesting the field. They created a table to identify a pattern illustrating the inverse relationship. Producer 2 used logic to determine what would happen if one of the variables was increased or decreased. Next Producer 2 wrote the relationship in words and began filling in the table. What other situations can you think of that are inverse relationship?

Section 3: Practice Exercises

1.20. A producer wants to deworm his cattle. He or she decides to use Safe-Guard Dewormer. For beef, dairy cattle, and swine, 0.2 and 0.5 lb of dewormer should be given for body weights of 200 and 500 pounds, respectively.
a) The average weight of 58 young cows is 600 pounds. How many pounds of dewormer does he need?
b) How many pounds of dewormer should he give each animal?

Body weight (lb)	Pounds of dewormer
300	
800	
1900	

1.21. A nozzle (A) wastes 4 ounces of water in 30 min and a second nozzle (B) wastes 3 ounces in 20 min.

a) Do the two nozzles A and B drip at the same rate or is one dripping faster than the other?

b) If you sprayed for 2 h, how many ounces would nozzle A and nozzle B waste?

1.22. A farmer has two tractors, one tractor with a large boom for spraying and a second tractor with a small boom. The small boom is half the size of the large boom. It takes the farmer 4 days to spray a field using the tractor with the large boom. If he uses both tractors to spray the field, how long would it take?

1.23. A farmer uses 220 ounces of herbicide on a 60-acre field. How many ounces would he use on a 45-acre field? How many ounces of herbicide would he use on a 120-acre field? How many on a 150-acre field?

Problem 1.24.

A farmer must control green foxtail that is 9 to 10 cm high. Use the following label to solve each problem.

a) Determine the number of ounces that must be applied per acre (Note: 1 L = 1.06 qt).

b) If they have a 60-acre field to spray, how much Roundup will be needed?

c) If they are operating a sprayer that has a 90-ft boom, traveling at 12 mph, how long in hours and minutes will be needed to spray the 60 acres?

d) If the application rate is 15 gallons of spray solution per acre, how much gallons of spray solution will be needed to spray the field?

e) If the John Deere sprayer is equipped with a 800-gallon stainless steel tank, how often will the spray tank need to be refilled?

7.1 ANNUAL WEED CONTROL WITH ROUNDUP TRANSORB HC LIQUID HERBICIDE

RATE (L/ha)	GROWTH STAGE	WEEDS CONTROLLED	COMMENTS (Apply in 50-100 L/ha water)
0.5	Weeds up to 8 cm in height	Wild oats, green foxtail, volunteer barley, volunteer wheat Non-Roundup Ready volunteer canola (rapeseed), wild mustard, lady's-thumb, stinkweed	• For wild oats apply at 1-3 leaf stage. • Add 350 mL of a surfactant registered for use such as Agral® 90, Ag Surf®, or Companion™ • For heavy wild oat infestations use 0.67 L/ha rate.
0.67	Weeds 8 cm to 15 cm in height	All annual grasses listed above. All annual broadleaved weeds listed above plus flixweed* and kochia*	• Add 350 mL of surfactant registered for use as listed above. * Suppression only. Refer to higher rates of this table or tank mix table (section 7.2) for control options.

Section 4. Solving Problems with Percentage

Percentage is a special kind of ratio in which one of the ratios is a percent. Remember, we can think of percentage as a part out of 100. A percentage is a ratio in which you are comparing a part to a unit of 100. The ratio can be expressed as a decimal or fraction that is defined as a part out of a whole. With this interpretation, we are changing the fraction to an equivalent fraction with a denominator of 100.

Percentage problems require a different kind of thinking than what we typically use when working with fractions. For example, you know that a herbicide formulation is 47% of chemical A and B. The package tells you that chemical A is 14%. What is the percentage of chemical B. Here you do use subtraction to find the percentage of chemical B because *the total is a percentage and the two parts are percentages of the same total*. You can add and subtract in this context.

A supplier has a sale on seeds. Seeds are on sale for 25% off for the first \$100 and 50% off for the second \$100. In this context you cannot add the two percentages because they are not part of the same total. In fact, the total quantity changes from \$100 to \$200. Thus, it is *incorrect* to add 25% + 50% = 75% and then take 75% off \$200. If you did, the savings would be \$150 and the seeds would cost \$50. This should not make sense to you. The supplier would not stay in business. The correct way to solve the problem is to find 25% off the first \$100 (which is \$25, the cost for the seeds is \$75) and 50% off the second \$100 (which is \$50). The seeds should cost \$125 in total. The key understanding is that percentage is a relationship that is based on multiplication. Addition and subtraction can be used in specific situations in which all of the numbers in the equation are percentages and reference the same total.

The key understandings include:

· Percentage is a ratio in which the unit is 100

· Percentage can be expressed in three forms: 15%, $^{15}/_{100}$, 0.15

· Percentage is a relationship based on multiplication

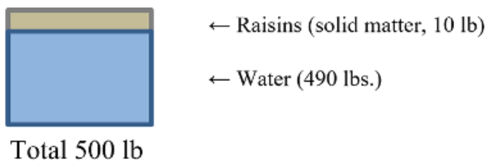
Problem 1.25.

Grapes are 98% water. Manual wants to dry them out to make raisins that are 90% water. If they have 500 pounds of grapes, how many pounds of raisins can they make?

Answer Problem 1.25: Approach A

Producer 1 knows that percentage is based on a ratio of a part out of 100. They know that 98% is water and 2% is solid material. They know that because percentage is a ratio, the relationships are based on multiplication. To better understand the problem, Producer 1 makes a table.Producer 1 finds the pounds of solid by multiplying .02 by 500 lb = 10 lb. This quantity does not change because it is solid matter. To find the pounds of water, subtract 10 (solid matter) from the total pounds. To find the percent water, set up the following set of ratio:

$$\frac{\text{part}}{\text{unit or whole}}=\frac{\text{lb of water}}{\text{total lb}}=\frac{\text{percent}}{100}$$



For the 250 pounds use the following:

Method 1: $\frac{240}{250}=\frac{n}{100};\frac{240\div 25}{250\div 25}=\frac{9.6}{10}=\frac{96}{100}=96\%$

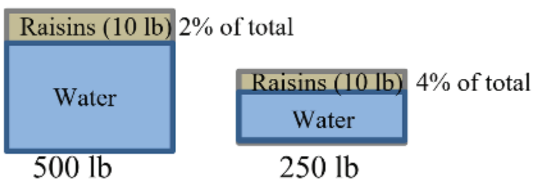
Method 2: $\frac{240}{250}=\frac{n}{100};240\times 100=n\times 250;\ 24{,}000=n\times 250;\ 24{,}000\div 250=n;\ n=96\%$

Total lb	500	250 ($^1/_2$ of 500)	125 ($^1/_4$ of 500)	100 ($^1/_5$ of 500)	
Lb of solid	10	10	10	10	The solid does not change.
Lb of water	490	240	115	90	
Percent water	98%	96%	92%	90%	

The first method uses equivalent fractions to find the percentage. The second method uses cross products to find the percentage. Producer 1 continues to fill out the chart until he or she arrives at the correct percentage of water. The answer is 100 pounds of raisins. They reflects back and checks their answer. Does 100 pounds of raisins yield 90% water? The solid part is found by 100 – 10 = 90. 90/100 = 90%. This is the correct solution.

Answer Problem 1.25: Approach B

Producer 2 begins with a sketch and imagines squeezing out all of the water from the raisins. Producer 2 then writes the ratio: $\frac{90}{100}=\frac{n}{500}$ because they want to know what percent of water would need to evaporate from 500 to make a ratio of 90% ($^{90}/_{100}$). Producer 2 solves for n, n = 450. They decide that the total weight will be 450 pounds. This approach is not correct. Why?



Discussion of Problem 1.25

In the first approach, Producer 1 recognized that *percentage is a ratio* (key understanding) and used a table to help them identify patterns and relationships. The table allowed Producer 1 to keep track of varying amounts. They quickly saw that taking half of the weight did not reduce the percentage very much. They recognized that multiplicative relationships are different from relationships based on addition and subtraction. Looking at the pattern, Producer 1 noticed that when the pounds of grapes were cut in half, the percentage of water decreased by by 2%. He or she continued to cut the pounds in fractional parts and noticed a pattern. Cutting the pounds in half reduced the percentage by 2%; cutting it in $^1/_4$ reduced the percentage by 4%; cutting it in $^1/_5$ reduced the percentage by 5%. Since the solid part of the raisins was 2%, it made sense to use a fifth of the total (500 lb) to drop the percentage of water by 10% (5 × 2%). Producer 1 used the key understanding, *percentage can be expressed in three forms:* 15%, $^{15}/_{100}$, and 0.15, as he or she used mathematics to solve the problem.

Producer 2 created a sketch to model the problem and his/her sketch was a great beginning. He/she also recognized the key understanding, *percentage is a ratio in which the unit is 100 and can be expressed as* 15%, $^{15}/_{100}$, and 0.15. His or her error was thinking that 90% of the total (500 lb) would be the weight at 90% water. However, the relationship is inverse. This means that if 10 lb of the solid part of grapes is 2% of a total, then if it doubles the percentage to 4%, the total weight is cut in half. Meaning that for the percentage of solid weight to go up, the amount of water must decrease. In an inverse relationship with percentage, cutting in half means the other increases by two.

Producer 2 needs to have the percentage of solid increase to 10%. This is 5 times the 2% that he/she started with. He or she multiplies the 2% by five and divides the total weight by five. This yields a total weight of 100 pounds and is a correct solution. The key understanding is that *percentage is a relationship based on multiplication*. When one quantity increases (percentage of solid grapes) and the other decreases (percentage of water) an *inverse relationship* exists. It is important to always consider whether the context indicates a direct proportion or an inverse relationship. You may want to review these ideas by looking at Problem 1.19 again.

Problem 1.26.

An agronomist has three bags of fertilizer. Fertilizer A is 500 lb. of 28-0-0. Fertilizer B is 300 lb of 11-52-0. And fertilizer C is 200 lb of 0-0-60. They decide to mix them together. What is the analysis of the resulting fertilizer blend?

Answer Problem 1.26: Approach A

Agronomist 1 knows 28-0-0 indicates that 28% of fertilizer A is N and there is no P_2O_5 or K_2O in it. To solve the problem, he/she decided to calculate the number of pounds of N in Fertilizer A. He/she knows the total weight of fertilizer A and the percentage that is N.

Agronomist 1 wants to find the number of pounds of N. They write and solve the following equations:

$$\frac{28}{100}=\frac{\text{lb N}}{500}$$

$$\frac{28\times5}{100\times5}=\frac{\text{lb N}}{500}$$

28 × 5 = 140 lb of N in Fertilizer A

Agronomist 1's approach is correct and they effectively used the diagram to guide his or her computations. Using this method, Agronomist 1 finds the number of pounds of N and P_2O_5 in fertilizer B and K_2O in fertilizer C.

Fertilizer B is 300 lb of 11–52–0

To find N: $\frac{11}{100}=\frac{\text{lb N}}{300}$

33 lb of N in Fertilizer B

To find P_2O_5: $\frac{52}{100}=\frac{\text{lb PO}}{300}$

156 lb of P_2O_5

Fertilizer C is 200 lb of 0–0-60

To find K_2O: $\frac{60}{100}=\frac{\text{lb KO}}{200}$

120 lb of K_2O

Agronomist 1 knows that in the mixture, he or she has 1000 lb of fertilizer (500 from Fertilizer A, 300 from Fertilizer B, and 200 from Fertilizer C). Agronomist 1 also knows that there is a total of 173 lb of N (140 from Fertilizer A and 33 from Fertilizer B). To find the percent of N, he or she writes the ratio of the weight of N to the total weight of the mixture. This can be changed to a percentage.

$\frac{173}{1000}=\frac{17.3}{100}$ Thus, the percentage of N in the new mixture is 17.3.

Likewise, Agronomist 1 finds the percentage of P_2O_5 and K_2O.

156 lb of P_2O_5

$\frac{156}{1000}=\frac{15.6}{100}$ Thus, the percentage of P_2O_5 in the new mixture is 15.6.

120 lb of K_2O

$\frac{120}{1000}=\frac{12}{100}$ Thus, the percentage of K_2O in the new mixture is 12.

The new fertilizer mixture is 17.3–15.6–12.

Answer Problem 1.26: Approach B

Agronomist 2 knew that Fertilizer A was 500 lb with 28 N. He or she knew that Fertilizer B was 300 lb with 11 N and 52 P_2O_5. Agronomist 2 also knew that Fertilizer C was 200 lb with 60 K_2O. He or she remembers that the fertilizer is described by the percentage of different components. Agronomist 2 added the two N component together to determine that the N fertilizer grade in the mixture was 39%. He or she noticed that P_2O_5 is only in Fertilizer B and K_2O is only in Fertilizer C. Thus, he/she does not need to do any additional computations. He or she claims the resulting fertilizer is comprised of: N-P_2O_5–K_2O as 39–52–60. Producer 2's solution is incorrect. What was his or her error?

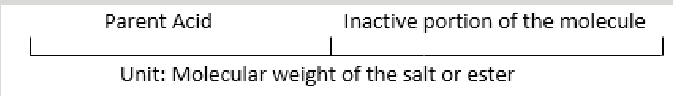
Discussion of Problem 1.26

Agronomist 1 correctly solved the problem. They remembered the key understanding *that percentage is a ratio and percentages cannot be combined when they reference different totals*. In this problem, the number of pounds of each type of fertilizer was different. Thus, he/she began by finding the number of pounds of each component of the fertilizers before he/she could combine them. Agronomist 1 also recognized that the percentage of each component would be different than in the original fertilizers. He or she expected that the percentage of P_2O_5 and K_2O would be much less because the combined fertilizer was so much more (1000 lb). Agronomist 1 used the key understanding of *equivalent fractions* to find the pounds of each component in fertilizers A, B and C and to find the percentage of each component in the new mixture.

In contrast, Agronomist 2 forgot that percentages seldom cannot be combined through addition or subtraction. He or she thought of percentage as a number that indicated a part of a total, but percentage is a ratio with a total of 100. When the total is not 100, the parts cannot be combined. He or she also did not recognize that when the quantity of fertilizer is increased, percentage of a given component decreases unless more of that component is added.

Problem 1.27.

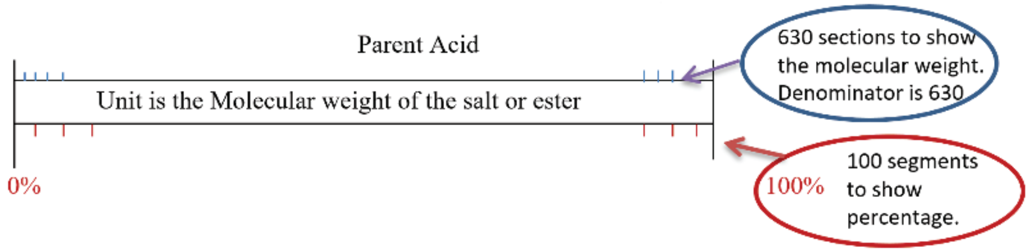
Herbicide classified as acids (e.g., 2,4-D, dicamba) do not readily dissolve in water or petroleum based solutions. Therefore the acid, which is the active ingredient, must be modified into a salt formulation or an ester-type formulation. Depending on the salt or ester type, the molecular weight of the molecule can double. To be able to compare the amount of active ingredient among many different formulations, the acid equivalent is calculated. Acid equivalent calculations are used when salt or ester forms of the herbicide are applied because these forms have variable weight but only the acid is active. The dosage is based on the parent acid of the molecule and the inactive portion of the molecule is excluded. The parent weight of the acid in these formulations is the molecular weight of the acid– 1, because the OH group on the acid is modified by removing the H (molecular weight of 1) and then adding the modifying compound desired. The acid equivalent is the percentage of the parent acid to the molecular weight of the salt or ester. The molecular weight of a parent acid formulation is 542 and the molecular weight of the salt formulation is 630. What is the acid equivalent (ae)?



He or she was correct in recognizing that the total amount of fertilizer increased, but Agronomist 2 failed to find the correct percentage of each component using the new weight of 1000 lb.

Answer Problem 1.27: Approach A

Producer 1 knows that the acid equivalent is the ratio of the parent acid to the molecular weight of the salt. Using the definition of a parent acid, he or she knew that it is equal to the molecular weight of the acid formulation minus 1, 542– 1 = 541. The molecular weight of the salt is 630. The ratio is written as $\frac{541}{630}$ = 0.85873. Written as a percentage, the acid equivalent is 85.87% or *ae* = 85.87%. This solution is correct.



Answer Problem 1.27: Approach B

Producer 2 created a model to show the relationships between the parent acid, molecular weight of the acid formulation, and percentage can be constructed using two number lines.

The unit can be thought of as the molecular weight of the salt and the number of segments that it is divided into is 630. A few of these segments are shown. (There are too many segments and they are too close together to illustrate all of them.) From the double number line, it is easy to see that each of the 630 pieces is smaller than the corresponding 100 pieces. Producer 2 knows by definition that the part that is the parent acid is the molecular weight of the acid formulation minus 1, (542 – 1 = 541). Thus, the fractional part is. To solve the problem we need to convert it to a percentage. Instead of a fraction with a denominator of 630, we want the denominator to be 100. The following equivalent fractions can be used $\frac{541}{630}=\frac{n}{100}$. There are many ways to solve this. One way is to use cross multiplication.

$$\frac{541}{630}=\frac{n}{100}$$

541 × 100 = 630 x *n*.

54,100 ÷ 630 = 85.873 or 85.87%. *ae* = 85.87%

Discussion Problem 1.27

Both of these approaches are correct. Both producers began by identifying the unit. The unit was defined as the molecular weight of the salt (630) and neither would be able to solve the problem without this knowledge. They selected an approach; Producer 1 used a mathematical approach that relied on their knowledge of how to write a fraction using a part–whole relationship. They then changed the fraction into a decimal and wrote the decimal as a percentage. Producer 2 began with a model using a double number line. They then wrote a pair of equivalent fractions and used a mathematical procedure of cross products to find the percentage. The key understanding is

Problem 1.28.

Herbicide formulations are often a mixture of 2 or 3 active ingredients (ai). In this problem, the amount of each active ingredient in a mix that is applied to an area will be determined. A herbicide formulation contains a total 5 lb *ai* per gallon with 5.7% of chemical A and 42% of chemical B.

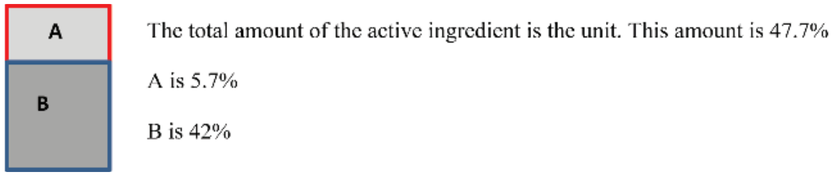
a) How much of the 5 lb ai is chemical A?

b) How much is chemical B?

Note: 10% ai means that there is 0.1 lb of ai per 1 lb of formulation.

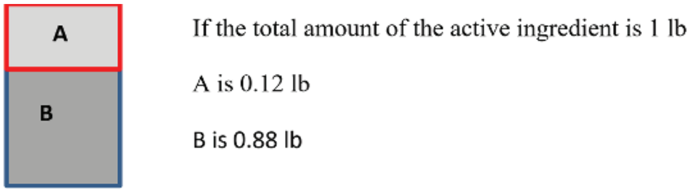
that the parent acid could be written as a fractional part. Using subtraction, it was easy to find the inactive portion of the molecule.

Answer Problem 1.28: Approach A



Producer 1 knew that the total amount of active ingredient is the sum of the active ingredients of chemical A and B. 5.7% + 42% = 47.7%. He or she needed to find out the part of the active ingredient that is chemical A and B. Producer 1 created a model to make sense of the problem.
To find the part of the active ingredient A, Producer 1 wrote it as a fraction . This can be rewritten as the decimal 0.119 (divide the numerator 5.7 by the denominator 47.7). Rounding off it is 0.12.

To find the part of the active ingredient B, Producer 1 wrote it as a fraction. This equals 0.880. For each pound of active ingredient per gallon, Producer 1 used 0.12 lb of A and 0.88 lb of B. (Note: 0.12 + 0.88 = 1.) Our problem states



that we want 5 lb per gallon of active ingredient. So, Producer 1 needed five times as much chemical A and B. For chemical A, 5 lb × 0.12 = 0.6 lb and for chemical B, 5 lb × 0.88 = 4.4 lb. This solution is correct.

Answer Problem 1.28: Approach B

Producer 2 wants 5 lb of active ingredient per gallon. It is comprised of chemical A and Chemical B. He or she changed the percentage to a decimal and multiplied it by five to determine the part that is A and B.

For chemical A, Producer 2 changed 5.7% to a decimal (0.057) and multiplied, 0.057 × 5 = 0.285. For chemical B, he or she changed 42% to a decimal (0.42) and multiplied, 0.42 × 5 = 2.1. Rounding off, Producer 2 decided that.29 lb of chemical A and 2.1 lb of chemical B is used. This solution is not correct. Why not?

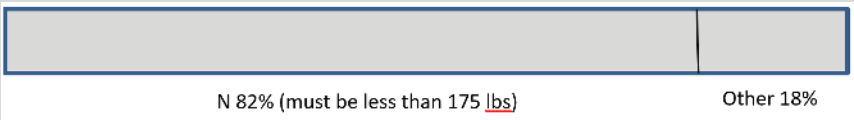
Discussion Problem 1.28

In approach A, the Producer 1 recognized the importance of the key understanding of *a unit and that an individual can define the unit in a way that is appropriate to the problem context*. Here, we must recognize that the sum of chemical A and B must equal the active ingredient to be used. Producer 1 began by adding the two percentages of A and B to find the total amount of active ingredient. This total represents the unit. Then, the percentage of A and B is found in relation to the unit (47.7%). Finally, the unit is redefined to be 5 lb of active ingredient so the amount of A and B are multiplied by five, the scale factor. To check our reasoning, we add 0.6 lb of A to 4.4 lb of B and get 5 lb of active ingredient.

Why was B incorrect? In approach B, Producer 2 remembered a formula for finding the percentage of something. The problem was that 5 lb was not the unit to be divided. Five pounds represent the total amount of active ingredient that was comprised of two chemicals, A and B. The approach used by Producer 2 indicated that A and B were only part of the total 5 lb. If Producer 2 added the two parts, he or she would have discovered that the compound did not equal 5 lb. (0.29 lb of A + 2.1 lb of B = 2.39 lb ai). The problem called for a total of 5 lb ai not 2.39 lb ai. The problem was thinking that 5 lb was the only unit in the problem. Producer 2 needed to recognize that the first unit was defined in percentage and part of A and B needed to be found using the combined percentage of A and B. The error that Producer 2 made was not identifying the appropriate unit and reflecting back on the problem to see if the answer made sense for the problem.

Section 4 Practice Exercises

1.29. Anhydrous ammonia (AA) is a predominant fertilizer because it is a good source of nitrogen (82%) and is relatively inexpensive. A farmer has 175 lb of AA. How many pounds of N is that?



1.30. An agronomist wants to buy some anhydrous ammonia. It is 82% N. How many pounds does he need to purchase to have 50 lb of N?

1.31. A producer is following a fertilizer recommendation of 75 lb of Potash per acre. If using 0–0-60, how much of this should be applied?

1.32. If a fertilizer recommendation for corn calls for 120 lb N, 40 lb P_2O_5 and 90 lb K_2O per acre, how much of the following fertilizers should be used per acre?

82-0-0 ___________
0-20-0 ___________
0–0-60 ___________

1.33. A fertilizer dealer will apply 205 lb potassium nitrate (13-0-44) per acre to a field as part of meeting a requirement of 80 lb N and 90 lb. K_2O. How much ammonium nitrate (34-0-0) per acre must also be added to meet the total requirement?

Weights of nutrients in liquid fertilizers:

10-34-0 = 11.7 lb $gallon^{-1}$
9-18-9 = 11.7 lb $gallon^{-1}$
28–0-0 = 10.7 lb $gallon^{-1}$

1.34. What is the amount of nitrogen in 12 gallons of 9–18–9?

1.35. If you need to apply 90 lb of N per acre in the form of 28–0-0, how many gallons should you apply?

Fertilizer Price per Ton	
28 (28-0-0)	$287
Anhydrous (82-0-0)	$523
Urea (46-0-0)	$386
DAP (18-46-0)	$463
Potash (0-0-60)	$350

† prices from www.farmfutures.com April 4th 2016 edition

1.36. An agonomist wants to find the most cost effective option to fulfil 300 lb of N, 111 lbs of P_2O_5, and 81 lbof K_2O. Use the table of fertilizer prices below to solve.

1.37. An agronomist has a choice between two insecticides. Insecticide 1 is available in a three-gallon container for $28. Insecticide 2 is available in a five-gallon container for $43. If Insecticide 1 is comprised of 13.1% active ingredient, and Insecticide 2 is comprise of 11.4% active ingredient, which is a better buy?

Section 5. Solving Problems Involving Unit Conversions

Precision farmers sometimes need to convert between units of measurement. Conversion between units of measurement is based on changing the unit discussed in Section 1. If you change 660 feet into miles, you can picture a length that is 660 units long. You know that 1 mile is equal to 5280 feet. Thus, 660 is part of that new measurement. It is precisely $^{660}/_{5280}$. Reduce the fraction and you find out that 660 feet is $^{1}/_{8}$ of a mile. However, sometimes changing from one unit of measure to a new unit of measure requires several conversions. For this reason, scientists developed a process called *unit conversion* to change units of measure. Unit conversion is a process that makes use of a ratio that describes the relationship between different measurement units.

To help you understand the connection between your knowledge about fractions and units (as illustrated above), converting 660 feet to miles is solved using unit conversion. The process begins by writing the relationship between feet and miles as a *conversion ratio*. The two following ratios describe the relationship: $\frac{5280 \text{ feet}}{1 \text{ mile}}$ or $\frac{1 \text{ mile}}{5280 \text{ feet}}$. Both of these ratios are called *conversion ratios* because they describe the relationship between two measurements. The correct conversion ratio for a particular problem is selected by paying attention to the units of measure. Selecting the correct one allows you to "cancel" the unit that you are changing when you multiply the measure that you want to change by the conversion ratio. If you want to change 660 feet to miles, you begin by writing a ratio to describe 660 feet.

Step 1: $\frac{660 \text{ feet}}{1}$

Step 2: Select the conversion ratio $\frac{1 \text{ mile}}{5280 \text{ feet}}$ because it allows you to cancel "feet". This is the unit that you are changing.

Step 3: Write the expression to convert the unit of measure. $\frac{660 \text{ feet}}{1} \times \frac{1 \text{ mile}}{5280 \text{ feet}}$

Step 4: Cancel the unit of measure that is found in the numerator and denominator. Then complete the arithmetic.

$$\frac{660 \text{ feet}}{1} \times \frac{1 \text{ mile}}{5280 \text{ feet}} = \frac{660}{5280} \text{ miles} = 0.125 \text{ miles or } \tfrac{1}{8} \text{ miles}.$$

Step 5: Does this solution make sense? Would you expect 660 feet to be a fraction of a mile? Yes, it should be less than a mile. It should also be less than half of a mile. You could estimate that half of a mile is approximately half of 5000 feet which is 2500 feet. Clearly 660 feet is much smaller. If you use the wrong conversion factor, your answer will not make sense because it will be either too big or too small.

Discussion

In section 1, you learned that a unit can be divided into same size quantities, and that the unit is defined by the context of the problem. The problem of converting feet to miles requires you to change one measurement unit to another. The given unit of measure (feet) is smaller than the new unit of measure (miles). This tells you that the given unit of measure is part of the larger one. Thus, you divide by the larger unit. Unit conversion changes this thinking into a process. It requires you to select a conversion ratio that allows you to "cancel" the given unit of measure (feet). Previously, this was accomplished by thinking about how the units change and then dividing.

The key understandings to remember when using unit conversion include:

- The relationship between two units of measure is a ratio. This ratio is called a conversion ratio and can be used to change units of measure.
- Write the initial measurement as a ratio and then select the conversion ratio that allows you to cancel the unit of measure that you want to change.
- Reflect on the solution to make sure it "makes sense." This is critical because occasionally the wrong conversion ratio is used.

Problem 1.38.
Convert 5 miles per hour to feet per second.

Answer Problem 1.38: Approach A

Begin by converting miles to feet. This can be accomplished by one of two methods.

Step 1:

Method 1: Section 1 changing units	
	Picture five miles in your mind and notice that there are lots of feet in each mile. One mile contains 5280 feet.
5 x 5280 = 26,400	Complete the calculation.
26,400 ft	

Method 2: Unit cancelation	
$\frac{5\text{ miles}}{1}$	Write the measurement as a ratio.
$\frac{5280\text{ feet}}{1\text{ mile}}$ or $\frac{1\text{ mile}}{5280\text{ feet}}$	Select the first conversion ratio. The miles will cancel.
$\frac{5\text{ miles}}{1}\times\frac{5280\text{ ft}}{1\text{ mile}}=26{,}000\text{ ft}$	Cancel miles and complete the calculation.

Step 2: Next, convert hours to seconds.

Method 1: Section 1 changing units	
	Picture 1 h. There are 60 min in 1 h.
Picture 60 s in each minute. Multiply 60 by the number of minutes.	There are 60 s in 1 min. 60 × 60 is the number of seconds in 1 h.
3600 s = 1 h	

Method 2: Unit cancelation	
$\frac{1\text{ hour}}{1}$	Write the measurement as a ratio.
$\frac{60}{\text{min}}$ or $\frac{1\text{ hr}}{60\text{ min}}$	Select the first conversion ratio. The hours will cancel.
$\frac{1\text{ hr}}{1}\times\frac{60\text{ min}}{1\text{ hr}}=\frac{60\text{ min}}{1}$	Cancel hours and complete the calculation.
$\frac{60\text{ s}}{1\text{ min}}=\frac{1\text{ min}}{60\text{ s}}$	Convert minutes to seconds.
$\frac{60\text{ min}}{1}\times\frac{60\text{ s}}{1\text{ min}}=3600\text{ s}$	Select the conversion ratio, cancel and calculate.

Step 3: Finally, find the speed in feet per second. The original speed was five miles per hour. Write it as a ratio and then substitute the equivalent measurements using feet and seconds.

$$\frac{5\text{ miles}}{1\text{ hr}}=\frac{26{,}400\text{ ft}}{3600\text{ s}}$$

Simplify the ratio by dividing the numerator by the denominator (26,400 ÷ 3600). The result is 7.33 ft s^{-1} or 7 $^{1}/_{3}$ ft per second. Does this make sense? Use some estimation to consider the magnitude of the answer. If you go 5 miles in 1 h, you should go 1 mile in about 10 min or $^{1}/_{10}$ of a mile in 1 min. One tenth of a mile is 528.0 feet, thus you should go about 528 in 1 min. One minute is 60 seconds, thus your distance traveled in 1 s should be about $^{1/60}$ of the distance in 1 min. Rounding 528 to 600 lets you quickly estimate a distance of 10 ft s^{-1}. This is close to your answer and confirms that your calculations are correct. Do not expect an estimation based on reasoning to exactly match your calculation because rounding off was used.

Answer Problem 1.38: Approach B

Step 1: Begin by writing the speed as a ratio and a second ratio using the required units of measure. Note that *f* and *s* are used to indicate the unknown number of feet per second. You will find these two unknowns.

$$\frac{5 \text{ miles}}{1 \text{ hour}} = \frac{f \text{ feet}}{s \text{ seconds}}$$

Step 2: Write all of the conversion ratios that are needed to describe the relationship between feet and miles and between hours and seconds. Note that there is not conversion ratio for hours to seconds so two conversion ratios are needed, one for hours and minutes and one for minutes and seconds.

Miles and feet	$\frac{1 \text{ miles}}{5280 \text{ feet}}$ or $\frac{5280 \text{ feet}}{1 \text{ mile}}$
Hours and minutes	$\frac{1 \text{ hour}}{60 \text{ minutes}}$ or $\frac{60 \text{ minutes}}{1 \text{ hour}}$
Minutes and seconds	$\frac{1 \text{ minute}}{60 \text{ seconds}}$ or $\frac{60 \text{ seconds}}{1 \text{ minute}}$

Step 3: Select the conversion ratios that cancel the units of measure that you want to change. Begin with either miles or hours. Both methods are illustrated below.

Method 1: Begin with distance measurements		Method 2: Begin with time measurements	
$\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{5280 \text{ ft}}{1 \text{ mi}}$	Notice that the unit "miles" cancels	$\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}}$	To cancel hours
$\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{60 \text{ min}}$	The unit "hours" cancels	$\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}}$	To cancel minutes
$\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}}$	The unit "mins" cancels	$\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{5280 \text{ ft}}{1 \text{ mi}}$	To cancel miles
	Cancel interme-diate units	$\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{5280 \text{ ft}}{1 \text{ mi}}$	Cancel intermedi-ate units
$\frac{5 \times 5280 \text{ ft}}{60 \times 60 \text{ s}} = \frac{26{,}400 \text{ ft}}{3{,}600 \text{ s}} = 7\frac{1}{3} \text{ft s}^{-1}$	Complete calculation	$\frac{5 \times 5280 \text{ ft}}{60 \times 60 \text{ s}} = \frac{26{,}400 \text{ ft}}{3{,}600 \text{ s}} = 7\frac{1}{3} \text{ft s}^{-1}$	Complete calculation

Consider whether the answer makes sense. Use the reasoning in Approach A to estimate whether the result is close to an estimate or your experience for working in the field.

Discussion Problem 1.38

Both approaches yield the same results. That is because the methods are mathematically the same. Once you practice Approach B, you will find that you can convert measurements into different ones quickly and accurately because the process helps you keep track of each unit that is converted. It is critical that you write the two conversion ratios that can be used before you begin. Remember to select the one that allows you to cancel the unit that is being changed. The measurement that you want should not be cancelled, if it is, then you made an error selecting the conversion ratio.

Steps to follow:

1. Write the measurement as a ratio
2. Write the relevant conversion ratios

3. Select the ratio that allows you to cancel the unit of measure that is being changed. You may need to use several ratios to end up at the desired unit of measure (i.e., changing mile per hour to feet per second).
4. Write an expression using the initial ratio and the conversion ratios selected.
5. Cancel the intermediate units when they are in the numerator and denominator and keep the desired units.
6. Complete the calculation and ask yourself if your answer makes sense.

Reflect on these steps and combine them to make three or four steps that make sense to you. Reflecting and shortening the steps will help you avoid errors. Practice will allow you to develop your confidence. Always ask yourself if the conversion makes sense using your knowledge from the field or estimation.

Section 5 Practice Exercises

1.39. Convert 15 ft s^{-1} to miles per hour.

1.40. A tractor travels 0.25 miles in 45 s. Find the equivalent speed in miles per hour.

1.41. One square mile is covered with water 3 in deep. Find the water quantity in

a) Gallons

b) Gallons per acre

1.42. Convert 10 mi hr^{-1} to ft s^{-1}

1.43. Convert 70 mi hr^{-1} to ft s^{-1}

1.44. Convert 7.5 ft s^{-1} to mi hr^{-1}

1.45. A field with sides, width = 1.5 mi, length = 0.75 mi. Find the area enclosed in acres.

1.46. A field with sides, width = 2000 m, length = 5000 m. Find the enclosed area in hectares.

1.47. Change 5 gal to cubic inches.

1.48. A tank has the following dimensions: 18 in × 20 in × 30 in. Find the volume in gallons.

1.49. Change 15 in^3 s^{-1} to gal min^{-1}.

1.50. Change 400 acres to hectares.

Challenge Problems

1.51. You expect that the maximum output from a 90-foot boom with nozzles spaced 20 in apart is 1 gallon per minute. Note: The rule of thumb in the industry is to oversize the pump on the sprayer by 50% so that there is enough capacity in the pump to agitate the solution properly when using a formulation that requires agitation. This agitation is needed so that a homogenous solution that can be sprayed on a crop.

a) You have a 90 foot boom with nozzles spaced 20 in apart. How many nozzles do you have? (90 ft × 12 = 1080 inches) (1080/20 = 54 nozzles)

b) What size pump must be on the sprayer? (54 nozzles @ 1 gal min^{-1} × 1.5 = 81 gal min^{-1})

c) If the tank holds 1000 gallons, how long before the tank needs to be refilled? (1000/54 = 18.5 min)

d) If the sprayer is spraying 1.818 acre min^{-1}, how many acres have been sprayed? (18.5 × 1.818 acre min^{-1} = 33.66 acre)

e) If it takes 10 minutes to fill the tank, what is your field efficiency? (18.5/(18.5+10) = 64.5%)

1.52. A combine is harvesting grain with a ground speed of 5 mi hr^{-1}. The header has a width, w = 30 ft. The grain yield is 50 bushels per acre and the combined gran tank volume is 200 bushels.

a) Find the harvest rate in acre hr^{-1}.

b) Find the running time to fill the grain tank.

1.53. The producer owns a Hagie sprayer similar to the one pictured below. This Hagie has 15 nozzles that are spaced 30 inches apart. They will use this sprayer, traveling at 9 mph, to apply liquid fertilizer with the aid of his or her Ag Leader technology. The goal: top dress N on 160 acres of corn that has a row spacing of 30 inches. Assume the Hagie is equipped with a 1000 gallon tank.

a) How long is the tool bar for this sprayer?

b) How many hours and minutes will be needed to apply 28-0-0 fertilizer if the operator is 100% efficient?

c) If the application rate is 30 lb $acre^{-1}$ and the density of the fertilizer is 10.67 lb $gallon^{-1}$, how many times will he or she need to refill the tank?

Note: http://www.fluidfertilizer.com/pdf/Fluid%20Characteristics.pdf.
(Image Source: John Fulton.)

APPENDIX A: ANSWERS TO PRACTICE QUESTIONS

1.3. Three fifths of a bottle fills 3/4 of the sprayer. A fifth of the bottle is the same quantity as $^1/_4$ of the sprayer. You need $^4/_4$ of the sprayer to fill it. Thus, multiply $^1/_5$ of the bottle by 4. To fill the sprayer, use $^4/_5$ of the bottle.

1.4. $(2/3) + \frac{\frac{2}{3}}{8} = 0.7499$ gal

1.5. 2x + 40y = 208, 2x = y, x = 2, y = 4

a. South has larger ears, because fewer ears are needed to fill a bushel.

b.Let S = the number of bushels from the south field

Let N = number of bushels from the north field

2 S = N, because there are twice as many bushels from the N field (N field is 2/3 of the crate and the south is 1/3).

24 S = number of ears of corn in the South field

40 N = number o fears of corn in the North field

40 × 2S = 80 S, number of ears of corn in the North field

24 S + 80 S = 104, total number of ears of corn

104 S = 208, S = 2. There are 2 bushels of corn in the South field.

N = 2 S, 2 × 2 = 4. There are 4 bushels of corn in the North field.

c. 6 bushels

d. 3/4 of the crate is 108 ears of corn from the south field. To find the number of bushels, divide 108 by 24 to find 4.5 bushels of corn from the south field. 4.5 bushels fill 3/4 of the crate. Divide by 3 to find the number of bushels in 1/4 of the crate. 4.5/3 = 1.5 There are 1.5 bushels of corn from the north field. 1.5 × 40 = 60 ears from the north field. 60 + 108 = 168 ears of corn.

1.6. $(15)(x + 15) = 0.2x = 60$ ounces of water

Chemical A: 15% of 15 ounces = 2.25 ounces of A

Chemical B: 7.3% of 15 ounces = 1.095 ounces of B

Chemical C: 5% of the 15 ounces = 0.75 ounces of C

1.7. Find the area of the field by 40 × 75 = 3,300 ft^2. The directions tell you the quantity to use the 300 ft^2. Divide 3,300 ft^2 by 300 to find 11. You need 5 tablespoons of concentrate and 1 gal of water for each 300 ft^2. Multiply 5 tablespoons by 11 to find 55 tablespoons of concentrate. You will use 1 gal × 11 = 55 gal of water.

1.13.

a. ($^5/_6$) is the $^1/_3$ of the amount needed. $^5/_6 \times 3 = 2.5$ cups for two cows

b. (1.33 cups) × (1 dose/0.5 cups) = 2.66 doses

1.14. 18.5/4 days; 4/625 miles per day

1.15. (9 × 3/8)/(5/8) = 15 pieces

1.16. 10.5 gallons are in 2.5 buckets. There are five half buckets. Divide 10.5 by 5 to find the number of gallons in a half bucket. 10 ÷ 5 = 2.1 gallons in a half bucket. A whole bucket is twice as much. So, the answer is 4.2 gallons.

1.17.

a. = [(18-16) × 300][100/18 × (800 + 300)] = 3.03% loss

b. same as a., [(18-16) × 300][100/18 × (800 + 300)] = 3.03% loss

1.20. Each pound of cattle requires 0.001 lb dewormer. So, 300 lb cattle require 0.3 lb dewormer, 800 lb cattle require 0.8 lb dewormer, and 1900 lb cattle require 1.91 lb dewormer.

1.21.

a. Faucet A: 4 ounces/30 s or 1 ounce/7.5 s

Faucet B drips 3 ounces/20 s or 1 ounce/6.67 s

So, Faucet B drips faster

b. There are 3.600 s in 1 hr. There are 7,200 s in 2 hr.

Faucet A drips 4 ounces/30 s, or 960 ounces

Faucet B drips 3 ounces/20 s, or 1,080 ounces

1.22. When both tractors work, they can spray 1.5 times as much as when one tractor sprays the field. Let x represent the number of days that both tractors will take. $1.5x = 4$, $x = 2.66$ days

1.23. (220/60) × 45 = 165 oz; (220/60)(120) = 807 oz.

1.24.

a. (0.67 L/h)(1 h/ 2.471 acres)(1.06 qt/L) = 56 ounces per acre

b.60 × 56 = 3360 ounces

c. 60 acres × 43560 ft^2 per acre × 1 mile/5280 ft × 1 h/12 m × spray/90 ft = 0.458 hr

d. 15 × 60 = 3360 ounces

e. 800/900 × 0.458 = 0.407 hours. It needs to be filled with 100 gal at 0.407 hours.

1.29. 175 lb AA × 0.82 A/ lb AA = 143 lb N

1.30. 50 lb N × 1 lb AA/0.82 lb N = 61 lb AA

1.31. (75 lb Potash) × (1/160) = 125 lb of 0-0-60

1.32.

120 lb × 1/0.82 = 146 lb AA
40 lb × 1 lb/0.2 lb = 200 lb of 0-20-0
80 lb × fert/0.6 lb = 150 lb 0-0-60

1.33.

205 × 0.13/lb fert. = 26.65 lb N
80 - 26.65 = 56.35 lb N needed
56.35 lb N × 1 lb fert./0.34 lb N = 165 lb 34-0-0

1.34. 11.7 lb per gal × 0.09 lb N/lb fert. × 12 gal = 12.64 lb N

1.35. 90 lb N per acre × 1 lb fert/0.28 lb N × gal/10.7 lb = 380 gal per acre

1.36. 241 lb DAP, 323.89 lb AA, 135 lb 0-0-60

1.37.

	Gallon	Cost	Active ingredient	Amount of active ingredient	Cost of active ingredient per gallon
Insecticide 1	3	$28	13.1%	0.393 gal	$71.25
Insecticide 2	5	$43	11.4%	0.57 gal	$75.43

1.39. 15 ft s^{-1} × mile/5280 ft × 3600 s hr^{-1} = 10.2 mi hr^{-1}

1.40. 0.25 mile/45 s × 60 s/min × 60 min/hr = 20 mi hr^{-1}

1.41. $mile^2$ × 640 acre $mile^{-2}$ × 43560 ft^2 $acre^{-1}$ × 0.25 ft × 7.48 gal ft^{-3} = 52,132,608 gal, 81,457 gal per acre

1.42. 10 mile/hr × 1 hr/3600 s × 5280 ft/1 mile = 14.7 ft per second

1.43. 70 mile/hr × 1 hr/3600 s × 5280 ft/1 mile = 102.7 ft per second

1.44. 7.5 ft/s × 3600 s/mile × 1 mile/5280 ft = 5.11 miles per hour

1.45. 1.5 m × 0.75 m × 640 acre/$mile^2$ = 720 acres

1.46. 2000 × 5000 × ha/10000 m^2 = 1000 ha

1.47. 5 gal × 231 in^3/gal = 1155 in^3

1.48. 18 × 20 × 30 × gal/231 in^3 = 46.6 gal

1.49. 15 in^3/s × 1 gal/231 in^3 × 60 s/min = 3.89 gal/min

1.50. 400 acres × ha/2.471 acres = 162 acres

1.52.

a. mile/hr × 30 ft × 5280 ft/1 mile × acre/43560 ft^2 = 18 acres

b. 200 bushels × hr/18 acres × 1 acre/50 bushels = 0.22 hr

1.53.

a. 15 rows × 30 in/row = 450 in or 37.5 ft

b. 160 × 1 hr/9 mile × 1 mile/5280 ft × 37.5 ft × 43560 ft^2/acre = 3.911 hr or 3 hr and 54.7 min

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REFERENCES

Dole, S., L. Bloom, and K. Kowalske. 2016. Transforming pedagogy: Changing perspectives from teacher-centered to learner-centered. Interdisciplinary Journal of Problem-Based Learning. 10(1). doi:10.7771/1541-5015.1538

Erickson, B., D.E. Clay, S.A. Clay, and S. Fausti. 2016. Knowledge, skills and ability needed in the precision ag workforce: An industry survey. 13th International conference of precision agriculture. St. Louis, MO. 31 July–3 Aug. 2016.

Hiebert, J. 2013. Conceptual and procedural knowledge: The case of mathematics. Routledge, New York.

NCTM. 2000. Principles and standards for school mathematics. National Council of Teachers of Mathematics (NCTM), Reston, VA.

Sandifolo, V.S. 2002. Paper 15: Estimation of crop losses due to different causes in root and tuber crops: The case of Malawi. FAO Corporate Document Depository. Food and Agriculture Organization, Rome, Italy. http://www.fao.org/docrep/005/Y9422E/y9422e0f.htm (verified 11 Oct 2017).