

Chapter 1

Reviewing ACT Math Basics

In This Chapter

- ▶ Beginning with an overview of ACT math
- ▶ Identifying four important steps to success
- ▶ Studying the six math topics tested on the ACT
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All across the United States — and especially in the Midwest, South, and Rocky Mountain states — high school juniors and seniors prepare for one of the most action-packed Saturdays of their lives. No, I don't mean the prom, graduation day, or a really excellent date with someone their parents don't know about and wouldn't approve of if they did. No, they're preparing for something even more fun than that: the ACT.

And if you believe any of this fun business, you're in for a real treat: I also have a fortune in gold from the prince of a country whose name you can't pronounce all ready to deposit into your bank account. Yeah, right.

Well, okay, some of this is true: You *will* be taking the ACT (which I figured out the moment you picked up this book). In this chapter, I begin the work of helping you get ready for the most fun part of all, the 60-minute math test.

I start this chapter with a look at what's on the math portion of the ACT. Then I provide a bit of advice about four general ways to improve your score. Finally, I list the six math topics that the ACT tests you on, including a breakdown of the specific skills within each topic that you should focus on to do well.

The chapter ends with a sigh of relief as you discover the math that you don't need to worry about on the ACT. (However, the teacher of your current math class may still want you to know something about it, so don't flush it from your memory just yet!)

Getting an Overview of ACT Math

The ACT contains five separate tests, which are always presented in this order:

1. English
2. Mathematics
3. Reading
4. Science
5. Writing (optional)

This book focuses exclusively on the second test, the ACT mathematics test. This math portion lasts for 60 fun-filled minutes and contains 60 questions. Generally speaking, questions appear roughly in order of difficulty.

The test is scored on a scale of 1 (lowest) to 36 (highest). Additionally, you receive three subscores in the following three separate areas

- ✓ Pre-algebra and elementary algebra
- ✓ Intermediate algebra and coordinate geometry
- ✓ Plane geometry and trigonometry

Subscores are tallied on a scale of 1 (lowest) to 18 (highest). Note that, flying in the face of common sense, your three subscores will *not* add up to your math score.



On the ACT, test graders don't take off points for wrong answers — that is, you won't be penalized for guessing. So keep an eye on the time and, just before your time is up, fill in answers to all 60 questions. Make as many educated guesses as you can, of course. But even wild guessing won't hurt your score, so fill in those answers before time's up!

Taking Four Key Steps to ACT Success

I like to give the following four important pieces of advice to those students who want to improve their ACT math scores:

- ✓ Sharpen your basic math skills.
- ✓ Get comfortable using your calculator.
- ✓ Solidify your ACT-math-specific skills.
- ✓ Take practice tests under timed conditions.

In this section, I give you an overview of these four steps. The rest of the book is devoted, one way or another, to working on them.

Sharpening your basic math skills

By the basics, I mean the math calculations that precede pre-algebra. For example:

- ✓ Multiplication tables up to 9×9
- ✓ Adding, subtracting, multiplying, and dividing fractions
- ✓ Converting percents to decimals and vice versa
- ✓ Converting common percents, such as 10%, 20%, 25%, 50%, and 75%, to fractions and vice versa
- ✓ Working with negative numbers
- ✓ Knowing the order of operations (PEMDAS: parentheses, exponents, multiplication and division, addition and subtraction)

When I say *know this stuff*, I mean that you should know it stone cold — the way you know your own name. If you're wasting precious time trying to remember 7×8 or calculate $-3 - 5$, you'll benefit greatly from investing just a few hours to work with a set of flash cards designed to help you strengthen these skills. In Chapter 3, I cover these topics to refresh your memory.

Getting comfortable with your calculator

You may already be well aware (and grateful!) that calculators are allowed on the ACT. Moreover, because virtually everyone will be using one, you should consider a calculator not just optional but mandatory.

Generally speaking, the kind of calculator you need for the ACT is at least one notch above the basic calculator you may use to keep track of spending or add up the calories in that big lunch you just ate — you need either a scientific calculator or a graphing calculator.

If you already own a good calculator and are reasonably adept at using it, just use this quick rule of thumb to make sure it's up to speed: Check to see whether it can do trigonometric calculations like $\sin x$. If it can, it's probably just fine; if not, you probably need to think about an upgrade well before the ACT so you have time to practice with it.

In Chapter 2, I give you some specifics about your calculator. And for those of you who are considering or who already own a graphing calculator, I recommend *TI-83 Plus Graphing Calculator For Dummies* (Wiley) and *TI-89 Graphing Calculator For Dummies* (Wiley), which are both written by C. C. Edwards.

Solidifying your ACT-specific math skills

Studying for the ACT should be mostly a review of skills that are covered in your math classes. However, no matter how hard you work in your classes, the material you covered two or three years ago may not be fresh in your mind. So focused preparation for the ACT can really pay off.

Parts II, III, and IV (Chapters 4 through 12) provide a detailed review of the math that shows up most on the ACT. You can work through dozens of example problems and answer 90 practice questions specifically related to those topics.

Taking practice tests under timed conditions

Time is money, and money changes everything. So it's not surprising that time changes everything — especially on the ACT, where you have only 60 minutes to answer 60 math questions. Purposefully working under low-stakes time pressure adds a useful dimension to your study, especially as you get closer to your test date when the stakes will be higher.

Part V of this book — Chapters 13 through 18 — contains three complete practice tests (and answers!) for you to try out. I recommend that you take them under real test conditions. In other words, take them in one hour with the calculator you plan to use on the test and no additional help. You may start out taking the first test as a benchmark before you begin working on the rest of the problems in the book. Or, if you prefer, save all three tests until you feel confident answering questions with no time pressure — then start the clock running and see how you do.

What Should I Study? Knowing What's on the ACT

The ACT covers six overall topics in math: pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry, and trigonometry. In this section, I break down all these topics into manageable bits and discuss the individual skills included in each. Parts II, III, and IV (Chapters 4 through 12) cover this material in depth, with plenty of example questions and practice problems.

Taking care of the basics in pre-algebra

Pre-algebra includes a variety of topics that prepare you for algebra. In this section, I discuss the specific pre-algebra skills that show up most on the ACT. And in Chapter 4, I focus on these types of questions, providing plenty of example questions and showing you how to answer them.

Basic arithmetic

You obviously need to know the four operations: addition, subtraction, multiplication, and division. You also want to feel comfortable working with negative numbers, fractions, and decimals. I cover some of this material in this book, but if you feel that you need a more thorough review, pick up *Basic Math and Pre-Algebra For Dummies* (Wiley) by Yours Truly.

Number sequences

A *number sequence* is a list of numbers arranged in a pattern. Here's an example:

2, 5, 8, 11, 14, 17 . . .

In this case, each number in the sequence is 3 greater than the number before it. An ACT question may ask you to find the next number or a missing number in a number sequence.

Factors and multiples

When one natural number is divisible by another, the smaller number is a *factor* of the greater number, and the greater number is a *multiple* of the smaller number. For example, 12 is divisible by 4, so

✓ 4 is a factor of 12.

✓ 12 is a multiple of 4.

To answer an ACT question, you may need to find all the factors of a number or the greatest common factor or the least common multiple among several numbers.

Percents, ratios, and proportions

Like fractions and decimals, *percents* are a mathematical way of representing part of a whole. For example, 50 percent of something is half of it. A *ratio* is a mathematical comparison. For instance, if you have twice as many brothers as sisters, the ratio of brothers to sisters is 2 to 1, or 2:1. A *proportion* is an equation using two ratios. ACT questions may ask you to calculate something using percents and ratios, or you may have to set up a proportion to answer a question.

Powers (exponents) and square roots (radicals)

When you take a number to a *power*, you multiply that number by itself repeatedly. For example: 3^4 (read *three to the fourth power*) = $3 \times 3 \times 3 \times 3 = 81$. In this case, 3 is the *base* (the number multiplied) and 4 is the *exponent* (the number of times the base is multiplied).

And when you take a *root* (also called a *radical*) of a number, you find a result that can be multiplied by itself repeatedly to produce the number you started with. The most common root is the *square root* — a result which, when multiplied by itself, produces the number you started with. For example, $\sqrt{25}$ (read *the square root of 25*) = 5 because $5 \times 5 = 25$.

Powers and square roots are common math operations, and they show up a lot on all sorts of ACT questions.

Data and graphs

A *graph* is a visual representation of data. Common graphs include bar graphs, pie charts, line graphs, and pictograms. Graph reading is a basic but essential skill that you need for the ACT. A typical question may ask you to identify specific data given in a graph, or you may need to pull this data as a first step in a more complex calculation.

Basic statistics and probability

Statistics is the mathematical study of real-world information called *data sets* — lists of numbers that are objectively observed and recorded. Three common operations used on data sets are three types of averages called the *mean*, the *median*, and the *mode*. On the ACT, you need to know how to calculate all three.

Moving on to elementary algebra

Elementary algebra is essentially the algebra that's covered in an Algebra I class. In this section, I go over the highlights of what skills the ACT expects you to remember and work with when answering questions. Chapter 5 covers these topics in greater detail with lots of examples.

Evaluating, simplifying, and factoring expressions

An *expression* is any string of numbers and symbols that makes mathematical sense. In algebra, you can do three common things with expressions:

- ✓ **Evaluate:** To *evaluate* an expression, you plug in the value of each variable and change the expression to a number. Be sure to follow the order of operations (exponents in the order they occur left to right, multiplication and division in the order they occur left to right, and addition and subtraction in the order they occur left to right). For example, here's how you evaluate the expression $5x + 7$, given that $x = 4$:

$$\begin{aligned}5x + 7 \\&= 5(4) + 7 \\&= 20 + 7 \\&= 27\end{aligned}$$

- ✓ **Simplify:** To *simplify* an expression, you remove parentheses and combine like terms to make the expression more compact. For example, here's how you simplify the expression $3(x + 6) + 2x$:

$$\begin{aligned}3(x + 6) + 2x \\&= 3x + 18 + 2x \\&= 5x + 18\end{aligned}$$

- ✓ **Factor:** To *factor* an expression, you find a factor that's common to each term in the expression and pull it out of the expression using parentheses. For example, here's how you factor $2x$ out of the expression $6x^2 - 10x$:

$$\begin{aligned}6x^2 - 10x \\&= 2x(3x - 5)\end{aligned}$$

Easier ACT questions ask you to simply evaluate, simplify, or factor an expression. More difficult questions may require you to use these skills to handle more complex calculations.

Solving equations with one or more variables

Solving equations is the main point of algebra. You solve an equation by isolating the variable (commonly x) while keeping the equation in balance — that is, by making sure that in each step, you apply the exact same operation to both sides of the equation. Here are a few types of equations you need to know how to solve on the ACT:

- ✓ Equations with fractions (rational equations), such as $\frac{2x}{5} = \frac{x+1}{4}$
- ✓ Equations with square roots (radicals), such as $\sqrt{5x+3} - 2 = x$
- ✓ Equations with absolute values, such as $|3x - 6| = 10$
- ✓ Equations with variables in the exponent, such as $8^{x-1} = 16$

Typically, an equation with more than one variable, such as $ab + c = 10$, can't be solved for a number. However, you can solve an equation with more than one variable in terms of the other variables in the equation. For example, here's how you solve this equation for b in terms of a and c :

$$\begin{aligned}ab + 3c &= 10 \\ab &= 10 - 3c \\b &= \frac{10 - 3c}{a}\end{aligned}$$

An ACT question may ask you to solve an equation in terms of other variables. Additionally, this skill is useful when working with math formulas.

Focusing on intermediate algebra

Intermediate algebra is the focus of a high school Algebra II class. In this section, I outline the essential intermediate algebra skills you need to be successful on the ACT. Later on, in Chapter 7, you can gain a solid understanding of this material.

Taking a look at inequalities

An *inequality* is a statement telling you that two math expressions aren't equal. On the ACT, inequalities come in four basic varieties:

- ✓ Greater than ($>$)
- ✓ Less than ($<$)
- ✓ Greater than or equal to ($\geq$)
- ✓ Less than or equal to ($\leq$)

You solve inequalities using the same algebra rules you would use to solve equations — with the exception of a couple of twists (flip to Chapter 7 for details). The solution to an inequality is typically a range of answers expressed as a simpler inequality.

Working with systems of equations

A *system of equations* is made of two equations that are simultaneously true. On the ACT, a system of equations usually is limited to two variables. For example, take a look at this system:

$$\begin{aligned}3x + y &= 10 \\x - 5y &= -4\end{aligned}$$

You can solve a simple system of equations by the *substitution method*, isolating a variable in one equation and then plugging its equivalent into the other equation. For a more complicated system of equations, use the *elimination (or combination) method* by either adding or subtracting the two equations and solving the equation that remains.

Understanding direct and inverse proportionality

When two values, x and y , are *directly proportional*, a value, k , makes the following equation true:

$$\frac{x}{y} = k$$

Values that are directly proportional tend to rise and fall together. For example, when one value doubles, the other value also doubles.

When two values, x and y , are *inversely proportional*, a value, k , makes the following equation true:

$$xy = k$$

Values that are inversely proportional tend to rise or fall opposite of each other. For example, when one value is multiplied by 3, the other value is divided by 3.

Examining quadratic equations

A *quadratic equation* is an equation in the form $ax^2 + bx + c = 0$. You can solve a quadratic equation either by factoring or by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The ACT almost certainly will have several questions that require you to work with quadratic equations.

Finding information about functions

A *function* is a mathematical connection between two values. Usually, the values are an input variable, x , and an output variable, y . In a function, when you know the value of x , the value of y is determined.

Typical ACT questions may ask you to use functions as models, to work with functional notation $f(x)$, or to find the domain or range of a function.

Working with coordinate geometry

Coordinate geometry is geometry that occurs on the xy -graph. This topic overlaps with material introduced in both Algebra I and Algebra II classes. Here, I give you an overview of the basic information from coordinate geometry that you need to review to do well on the ACT. I go over these ideas in greater detail in Chapter 8.

Graphing linear functions

A *linear function* is any function of the form $y = mx + b$. For example:

$$y = 3x + 5$$

$$y = \frac{5}{6}x - \frac{1}{3}$$

$$y = -x$$

Linear functions, which produce a straight line when graphed, are common on the ACT. Some of the skills you need to feel comfortable with include mastering the distance and midpoint formulas, finding the slope of a line, using the slope-intercept form to solve problems, and working with parallel and perpendicular lines.

Recognizing quadratic functions

A *quadratic function* is in the form $y = ax^2 + bx + c$. For example:

$$y = 2x^2 + 11x + 9$$

$$y = x^2 + 4x + 4$$

$$y = x^2 - 1$$

On the graph, a quadratic function produces a *parabola* — a curve that looks roughly like an arch (or a U). On the ACT, a question may ask you to pair up a quadratic function with its graph. More difficult questions may require you to find the axis of symmetry or the vertex of a parabola or to solve a quadratic inequality.

Transforming functions

A *transformation* of a function is a small change that affects that function in a predictable way. Typical transformations include reflections across the x -axis and y -axis as well as vertical and horizontal shifts. An ACT question may ask you to compare two similar functions and select the equation that transforms one into the other. Or a question may provide a function and a transformation and ask you to produce the resulting graph.

Grappling with higher-order polynomial functions and circles

More difficult ACT questions may include higher-order polynomials, such as cubic equations of the form $y = ax^3 + bx^2 + cx + d$, and graphs of circles. These questions are rather uncommon and require only a basic familiarity with the concepts.

Reviewing plane geometry

Plane geometry is the focus of a typical high school geometry class. In this section, I discuss the geometry that you're likely to see on the ACT. Chapter 10 gives you a complete review of these topics.

Lines and angles

One common type of ACT question presents you with a figure that contains lines and angles and then asks you to find the value of a given angle. To answer this type of question, you need to know how to measure right angles, vertical angles, supplementary angles, the angles in a triangle, and the angles that result when two lines are parallel.

For example, an ACT question may show you a figure with some angles labeled and ask you to find the measure of an unlabeled angle. Or it may ask you to identify a pair of angles that are equal in measure.

Triangles

Virtually every ACT includes several questions about triangles. You may need to find the area of a triangle given the height and the base, use the Pythagorean theorem to work with right triangles, or work with the most common types of right triangles, such as the 3-4-5 triangle.

An ACT question may ask you to find the area of a triangle given the measurements of its height and base, or, turning this question around, it may ask you to find the height given the length of the base and the area of the triangle. ACT questions involving right triangles may ask you to identify the length of one side of a right triangle, given information about the other sides.

Quadrilaterals

A *quadrilateral* is a four-sided polygon. Basic quadrilaterals that you may encounter on the ACT include squares, rectangles, parallelograms, and trapezoids. You need to know how to find the area of all these, and, more generally, you must feel comfortable working with the formulas for these areas.

For example, an ACT question may give you the perimeter of a rectangle with additional information and ask you to find the area. Or it may give you information about some aspects of a parallelogram — such as its height and area — and ask you to calculate the length of its base.

Circles

Circles are quite common on the ACT. You need to know the formulas for finding the diameter, area, and circumference of a circle given its radius. Additionally, you should be able to work with tangent lines, arc length, and chords of circles.

An ACT question may ask you to find the circumference of a circle given its area. More difficult ACT questions may require you to combine other geometry formulas to measure the area of a triangle with one side that's tangent to a circle or a chord of a circle.

Solid geometry

Solid geometry deals with geometry that occurs in three-dimensional space. A basic ACT question may require you to find the volume of a cube or box (rectangular solid). More advanced questions may ask you to work with more complicated solids, such as spheres, prisms, cylinders, pyramids, and cones.

Dealing with trigonometry and other advanced topics

The ACT includes questions about a few advanced math topics, including trigonometry. In this section, I go over these topics to make sure you're prepared for them. For further details, check out Chapter 11.

Trigonometry

Trigonometry is the mathematics of triangles — most commonly right triangles. ACT questions cover basic trig information. For instance, you need to know how to find the six trig ratios of a triangle in terms of the opposite side, adjacent side, and hypotenuse. More advanced trig concepts deal with radian measure, graphs of trig functions, and some basic trig identities.

Matrices

A *matrix* is a grid of numbers with both a horizontal and a vertical dimension. Virtually every ACT has a question that asks you to recall basic information about matrices, such as adding or subtracting matrices, multiplying a matrix by a constant, or working with the determinant of a 2-by-2 matrix.

Logarithms

A *logarithm* is the inverse form of an exponent. Not every ACT includes a question about logarithms, but if you encounter this type of a question, knowing how to convert a logarithmic equation into an exponential equation is particularly helpful.

Imaginary and complex numbers

An *imaginary number* includes the value i where $i^2 = -1$. A *complex number* is a number of the form $a + bi$. ACT questions about these types of numbers aren't usually difficult. In fact, some basic information can help you to answer them.

You're Off the Hook: Discovering What the ACT Doesn't Cover

The ACT math test covers most of the topics you're likely to find in a basic high school math curriculum. In fact, it's more advanced than the SAT in its range of math topics. Fortunately, even the ACT doesn't require you to know *everything* about math. Here are three easily identifiable areas of math that the ACT doesn't cover:

- ✓ **Ellipses and hyperbolas:** The equations and graphs for ellipses and hyperbolas, often part of an Algebra II or a pre-calculus class, aren't present on the ACT.
- ✓ **The value e and natural logarithms:** In a pre-calculus course, your teacher introduces you to the value e and its inverse function, the natural log. Both of these areas are essential for calculus, but you don't need to worry about them for the ACT.
- ✓ **Calculus and beyond:** More and more high school students are taking one or even two years of calculus and other advanced math. On the ACT, you definitely don't have to worry about limits, derivatives, integrals, or any other advanced concepts that you encounter in a calculus class.

This information comes as good news for most students. If you're currently taking an advanced math class, of course, you still need to study to maintain your grades. (You don't want to get a 36 on your ACT and then be rejected from your favorite college because of low grades, right?) On the plus side, you may find many ACT questions easier than last night's homework.

On the other hand, if your goal in life is to avoid as much math as possible going forward, then as the saying goes "You may already be a winner!" That is, if you've passed high school Algebra I and II, Geometry, and Trigonometry, you should be in reasonably good shape.

In either case, use this book to review the topics you're shaky on, solidify these skills with practice problems, and then take the practice tests in Part V. You'll increase your confidence going into the ACT and come away from the test with a score you can be proud of.