chapter ONE

The Problem with Math Is English (and a Few Other Things)

Many people do not consider English as playing a significant role in math, except in word problems. My hope in the forthcoming pages is to change that perspective. A well-known proverb says that to truly understand another's perspective, you must walk a mile in that person's shoes. Not everyone has experienced the struggle of learning both academic content and a new language at the same time. True, this double burden makes learning mathematics much more of a challenge. However, the phrase "the problem with math is English" applies to *all* students, not just those whose native language is not English. Language struggles are embedded in mathematics, which in many ways is its own language. These problems often occur at the critical juncture of math instruction and content. Two major issues in mathematics education that result from this merger are often overlooked: (1) the language and symbolism of mathematics, which in turn greatly influence (2) the mathematics itself—the content that we teach—and by association, how we communicate that content. The following scenarios introduce some key concepts related to these issues, which this book will explore in-depth.

WHY LANGUAGE AND SYMBOLISM?

Imagine you are a middle school student taking the state's required progress exam in mathematics. As you begin the test, you feel confident about your answers to the first few items. But then you read Item 5: "Find the arglif of a nopkam if the betdosyn is 12." Try as you will, the problem has you stumped. You finally give up, make a guess, and move on to the next problem. Unfortunately, you come across 14 other test items that confuse you in similar ways. Once again, the best you can do is guess at the answers. Later, you find out you did not pass the exam primarily because of those 15 items.

When most people see or hear the term *mathematics*, the initial thoughts that come to mind are numbers, computation, rules, and procedures. But the root cause of the student's inability to solve Item 5 is not because of a lack of knowledge of computation, rules, or procedures. If the strange terms *arglif*, *nopkam*, and *betdosyn* meant "area," "circle," and "diameter," respectively, Item 5 would become the following: "Find the area of a circle if the diameter is 12." However, if a student does not know the meanings of *area*, *circle*, and *diameter*, the terms might as well be *arglif*, *nopkam*, and *betdosyn* because they still hold no meaning.

This scenario is a rudimentary example of the key role that language plays in the understanding of mathematics. The student's difficulty is not in reading English, but in understanding the language of mathematics, and it makes no difference whether the student is fluent in English or not. A language problem still exists. Although perhaps not obvious, language is as critical in mathematics as in any other discipline. Moreover, the role of language in mathematics entails far more than vocabulary or definitions, encompassing a broad landscape of language-based issues, which are explored in this book.

Beyond Words: The Symbolism of Mathematics

Box 1.1

Solve the two tasks below:

a. $n = 1 + 3 + 5 + 7$

b. $\sum_{n=1}^{4} 2n - 1$

For many people, the first problem in Box 1.1 is child's play, whereas the second poses a serious challenge. The interesting paradox about these two problems is that although the second seems far more difficult, they are, in fact, the same problem. They are just presented differently. The first problem seems simple because most people can easily interpret what the numerals and symbols are telling them to do. The second problem, however, will literally be Greek to many people because they have no idea what those symbols mean. The problem becomes much simpler once the symbols are explained. The symbol $\sum$ is a summation symbol. The $n = 1$ in the subscript means that 1 is the first value of n in the expression $2n - 1$, and the superscript 4 indicates the last value to be used in that expression. Thus, the task is to determine the value of $2n - 1$ when n is 1, 2, 3, and 4, and then to find the sum of those values. These steps result in the expression $1 + 3 + 5 + 7$. Simple!

These two problems illustrate the key role of symbolism and visual representation in mathematics. The interpretation and subsequent understanding of mathematics concepts are heavily dependent not only on language but also on the symbols that are an inherent component of the discipline. These two problems also illustrate a scenario seen in far too many classrooms when concepts or ideas in mathematics that are actually quite simple are presented in a way that is far more complex, much to the detriment of student learning. In other words, there are too many instances in mathematics instruction where a simple concept or idea is somehow prodded and molded, either by the math education system or teachers, and unveiled to students as something that appears to be far more complicated than it really is.

The Language of Mathematics Instruction

Box 1.2

You have been teaching the challenging concept and skill of division by a proper fraction. You write the problem $10 \div \frac{1}{2}$ on the board. You then state, "Class, how many times does $\frac{1}{2}$ go into 10?"

Refer to the question asked in Box 1.2. If teachers tend to teach the same way they were taught, it follows they will tend to teach *using the same language*. Over the years, numerous elementary and middle school teachers have presented the expression $10 \div \frac{1}{2}$ as the question, "How many times does $\frac{1}{2}$ go into 10?" As students themselves, these teachers accepted this interpretation of the symbols and moved on regardless of how much sense the interpretation made—or did not make—to them. Once they became teachers, they used the same language, thus passing on the torch to their students. Quite honestly, what can a student create to model a context where $\frac{1}{2}$ *goes into* 10? The question as posed really makes no sense. The language of instruction in mathematics often makes the conceptual meaning almost impossible to grasp, but students survive by blindly following procedures that enable them to get the correct answers that result in good grades. As educators, however, we must reflect and ask ourselves what level of mathematics are students actually learning, and is that depth of knowledge acceptable?

WHAT WE ARE TEACHING

> **Box 1.3**
>
> Next week's unit of instruction will focus on the multiplication of mixed numbers. You need to ensure that students have the prerequisite skills and knowledge to learn this new topic. What are these prerequisite skills and knowledge? Make a list.

Your generated list from Box 1.3 would likely include many prerequisite skills or concepts that focus on *how to* multiply mixed numbers. Think of your experiences as a math student and, if applicable, as a math teacher. Much of the instruction and learning in math classrooms is focused on how-tos. In the United States, we value good old American know-how. To learn mathematics at a deep conceptual level, however, know-how is not enough. Just as important, if not more so, is good old American know-what and know-why. In other words, students need to understand *what* a basic concept is and *why* that concept works as it does. In the scenario in Box 1.3, students need first to understand what multiplication is conceptually, then use that knowledge to understand why the process works as it does. Knowing only how to multiply, at best, results in basic memorization of rules and procedures.

Not surprisingly, many adults in the United States perceive mathematics simply as a conglomeration of facts, rules, computations, and procedures. After all, that is the type of mathematics they were taught. The K–12 math education system often focuses on arithmetic and efficiency (or algorithms), but mathematics is far more than that. If teachers only know, and subsequently teach, arithmetic and algorithms rather than a conceptual understanding of mathematics, then that limited knowledge will be the baton passed on to future generations.

What Is Missing: The Need for Definitions

An interesting paradox in mathematics is that one can know *how to* do something without understanding *what* the concept or process truly is. For example, students can know how to multiply without understanding what multiplication is conceptually. I successfully navigated numerous math courses knowing how to use π in formulas while having no clue as to what π meant as a concept. Quite literally, the *what* is missing from math instruction. Since a definition tells us what something is, it makes sense to emphasize definitions as a core element of instruction.

Box 1.4

Answer the following:

a. Define *equation* (noun).

b. Define *graph* (noun).

Defining key mathematical terms helps students build their understanding of important concepts. Students should be able to provide precise yet simple definitions of basic terms, such as those in Box 1.4. For teachers, these types of definitions paint a clear picture of students' depth of understanding. Incorrect student responses can reveal misunderstandings and gaps in knowledge. In addition, patterns that emerge from students' definitions of basic concepts can provide clues about the effectiveness of instruction, the curriculum, and even the larger mathematics education system.

Patterns of Misunderstanding

As a young high school math teacher, I often made assumptions regarding students' content knowledge, especially their mastery of fundamental concepts in earlier grades. I adopted the strategy of having students define basic math concepts not only to build understanding but also to expose the areas where students' knowledge was weak. The approach revealed some interesting patterns over the years. For example, high school students in higher-level math classes most commonly define an equation, one of the most basic concepts in mathematics, as "when you solve for x." This definition is a clear misconception of what an equation is, but the root cause was not so evident. After much reflection and analysis, I realized the origin lay in the state's mathematics content standards. In the state standards in effect at that time, the term *equation* did not appear until the sixth grade. Moreover, the context for this first appearance focused on learning to solve simple linear equations. This initial focus had likely contributed to students' misconception of an equation being "when you solve for x."

The concept of an equation is usually defined as a mathematical sentence that states that one quantity is the same as another quantity. In other words, the quantity expressed on the left side of the equal sign is the same as the quantity expressed on the right side, regardless of how those quantities are represented. For some reason, the U.S. educational system often waits to formally include the idea of equations in state mathematics standards until fifth or sixth grade. However, do students not begin their experiences with this concept early in elementary school? Is $2 + 1 = 3$ not an equation? There is no requirement that an equation must involve variables! Yet even many adults struggle with the correct definition and gravitate back to the notion that an equation must have "letters" and that a solution must be found.

Earlier in the chapter, I discussed how the mathematics education system sometimes takes something that is relatively simple and manages to make it inordinately complex. An equation is actually an extremely simple concept that can be ingrained very early in a child's education. Young children love to learn what they think are complex or sophisticated words. So why do we not simultaneously teach that $3 + 4 = 7$ is an equation when students learn basic arithmetic? This early introduction would help solidify students' fundamental understanding of an equation and simultaneously reduce the misconception that an equation must somehow involve a variable and be solved.

Similarly, when I asked high school students to define the term *graph* as a noun, their responses revealed another interesting pattern. What emerged was the common idea of a graph as the grid itself or the x and y axes. An investigation of the possible root causes of this off-target perspective of a graph pointed to both instruction and curriculum materials as the culprits. How many times have math teachers instructed students to plot the points or to draw the curve "on the graph"? How many textbooks ask students to illustrate their responses "on the graph"? With this phrase repeated ad nauseam, is it any wonder that students begin to define a graph as the grid itself? These patterns reinforce the importance and impact of language, both written and oral, on students' perceptions and understanding of fundamental mathematical concepts.

TURNING THE TIDE: A SAMPLING OF APPROACHES

Mathematics instruction is an extremely complex enterprise with multiple interrelated factors. However, logically, the foundation for instruction should be the mathematics itself—the concepts and big ideas—not skills and algorithms, although they do play an important role. This conceptual foundation, in turn, necessitates a paradigm shift from the how-to of mathematics to the what and the why. The result is language-focused instruction based on conceptual understanding. The following examples provide a sampling of approaches used in this type of instruction and an overview of the chapters to come.

Multiple Perspectives

Box 1.5

You have taught your students the multiplication of mixed numbers using the standard algorithm: Convert to improper fractions, multiply, then simplify and convert back to a mixed number if applicable. You are tutoring Michael because he still does not get it. Using $3\frac{1}{2} \bullet 2\frac{1}{2}$ as an example, think about how you would go about helping Michael gain a conceptual understanding of mixed-number multiplication using other perspectives or approaches. Write down an explanation of how you would help Michael.

Review the example in Box 1.5. If you had difficulty thinking of different approaches to help Michael understand how to multiply mixed numbers, you are definitely not alone. As mentioned, we tend to teach not only how we were taught but also *what* we were taught, with multiple implications. Teachers can teach only what they know and only to the depth of their own understanding. If teachers' only experience with the multiplication of mixed numbers is the standard algorithm and their college training, or subsequent professional development has not taken them beyond that, then that one approach is what they will teach.

Out of training and habit, teachers can easily begin to view a concept or approach a problem from just one perspective. We know students learn in different ways; thus, we should teach mathematical concepts and skills using multiple approaches and perspectives that relate to how individual students learn. So what can we do for Michael in the above scenario? We might try teaching multiplication of mixed numbers from a numeracy perspective using the definition of multiplication. Or we might try a geometry-measurement perspective using an area model. We could also try an algebraic perspective through the use of the distributive property. All these approaches and the power of the connections among them can help students develop conceptual understanding. This instructional scenario will be revisited in Chapter Eight to illustrate how to use multiple perspectives to help students understand the concept of multiplying mixed numbers.

So What Was the Question?

Box 1.6

You have been teaching the challenging concept and skill of division by a proper fraction. You write the problem $10 \div \frac{1}{2}$ on the board. You then state, ''Class, how many times does $\frac{1}{2}$ go into 10?'' The question is followed by silence. Finally, one student volunteers and states, ''I sort of understood back when we had stuff like how many times does 3 go into 12, but no matter how hard I try, I can't picture in my head how many times $\frac{1}{2}$ can go into 10. What does that mean?'' How do you respond?

The scenario in Box 1.6 is an extension of the one in Box 1.2. Again, the focus is on the tendency for teachers to use the *same language* used to teach them. Suppose the students are older, and a high school teacher poses this same problem to review basic computation. A few students give the incorrect solution of 5, while others give the correct solution of 20 but with trepidation and uncertainty. The teacher then challenges the students to state the expression $10 \div \frac{1}{2}$ as a question without using phrases such as "goes into" or "divided by." It should come as no surprise that the high school students are stumped despite their maturity and experience.

The scenarios in Boxes 1.2 and 1.6 reveal how teachers' language choices, often unconsciously influenced by how they were taught, can confuse students and even erode the understanding of the mathematics involved. Most teachers would be at a loss to explain what it means for $\frac{1}{2}$ to "go into" 10 or to describe division without using this phrase. And without a deeper understanding of division, the high school teacher above would not be able to pose that challenge to students as an instructional strategy.

This dilemma illustrates two additional problems in K–12 mathematics instruction. First, instruction tends to focus on finding answers without any focus on students' *understanding of the question*. When students truly understand the question being asked, they can answer it correctly with full confidence. But often, students will present a correct answer hesitantly, revealing a lack of real understanding of the task. Second, despite being in high school, many students, even high-achievers in advanced math classes such as Algebra II, still do not truly understand basic concepts such as division. True, students can easily recite facts such as 42 divided by 6 is 7, but they do not understand division *conceptually*. There is a huge difference between using procedures or memorizing facts and truly understanding concepts and why procedures work. And the language used can contribute to this lack of understanding.

MATHEMATICS IS ABOUT RELATIONSHIPS

Box 1.7

Solve the following:

The state champion in football is determined by a single-elimination playoff system: If you lose, you are out. How many games would the state need to hold if 37 teams were involved in the playoffs?

Refer to the task in Box 1.7. One could solve this problem by laboriously drawing playoff brackets to determine the number of necessary games. There is, however, a much simpler approach to the solution. The key is to look for the *relationships* in the problem. In this scenario, each game produces one winner and one loser. The loser is out, and the winner continues. The state champion must go through the playoffs undefeated. Since the playoffs involve 37 teams and only one is crowned the state champion, then 36 of the 37 teams must lose. Each game produces one loser. As a result, you need 36 games to produce the necessary 36 losers.

The simplicity of this process may be surprising. However, it is not actually a simple process. The key is to search for the relationships among the variables in the context of the problem. Clearly, not all mathematics problems will have an alternative approach or method of solution. However, finding and understanding relationships should be an integral part of the problem-solving process in any mathematical context. An unknown author once wrote, "Arithmetic is about computation; mathematics is about relationships." Using this perspective and approach can make a huge difference in the level of mathematical understanding students attain. Both teachers and students should focus on mathematics as relationships; otherwise, they are simply doing arithmetic.

CONNECTING THE PIECES AND LOOKING AHEAD

To understand mathematics at a deeper conceptual level, students need to develop a strong foundation by learning to define basic concepts, make connections, and unearth relationships; an understanding of the language, symbolism, and visual representation of mathematics is integral to this process. The chapters that follow explore these concepts in greater detail. Although attention is paid to instruction and instructional strategies, the focus is on mathematics content and all it entails. This overview illustrated that mathematics is not simply about numbers. Language and symbolism play a pivotal role, one often neglected in classrooms. More often than not, this neglect occurs because the educational system has not recognized the importance of language and symbolism in mathematics. My goal is to help you identify and address language-based problems in mathematics instruction and learn how to present mathematics content in a way that simplifies it, yet transforms it from shallow and procedural to deep and conceptual. With a deeper conceptual understanding of mathematics, teachers, and in turn their students, gain the power to make connections, identify relationships, and view mathematical concepts from multiple perspectives. The approach in this publication is not a silver bullet, nor will it revolutionize mathematics instruction. But it does provide a new vision of mathematics through a neglected perspective that provides a conduit for a much deeper understanding of fundamental mathematics—advanced fundamentals, if you will.