

CHAPTER 1

DEMAND AND SUPPLY ANALYSIS: INTRODUCTION

Richard V. Eastin

Gary L. Arbogast, CFA

LEARNING OUTCOMES

After completing this chapter, you will be able to do the following:

- Distinguish among types of markets.
- Explain the principles of demand and supply.
- Describe causes of shifts in and movements along demand and supply curves.
- Describe the process of aggregating demand and supply curves, the concept of equilibrium, and mechanisms by which markets achieve equilibrium.
- Distinguish between stable and unstable equilibria and identify instances of such equilibria.
- Calculate and interpret individual and aggregate demand and inverse demand and supply functions, and interpret individual and aggregate demand and supply curves.
- Calculate and interpret the amount of excess demand or excess supply associated with a nonequilibrium price.
- Describe the types of auctions and calculate the winning price(s) of an auction.
- Calculate and interpret consumer surplus, producer surplus, and total surplus.
- Analyze the effects of government regulation and intervention on demand and supply.
- Forecast the effect of the introduction and the removal of a market interference (e.g., a price floor or ceiling) on price and quantity.
- Calculate and interpret price, income, and cross-price elasticities of demand, and describe factors that affect each measure.

1. INTRODUCTION

In a general sense, **economics** is the study of production, distribution, and consumption and can be divided into two broad areas of study: macroeconomics and microeconomics. **Macroeconomics** deals with aggregate economic quantities, such as national output and national income. Macroeconomics has its roots in **microeconomics**, which deals with markets and decision making of individual economic units, including consumers and businesses. Microeconomics is a logical starting point for the study of economics.

This chapter focuses on a fundamental subject in microeconomics: demand and supply analysis. **Demand and supply analysis** is the study of how buyers and sellers interact to determine transaction prices and quantities. As we will see, prices simultaneously reflect both the value to the buyer of the next (or marginal) unit and the cost to the seller of that unit. In private enterprise market economies, which are the chief concern of investment analysts, demand and supply analysis encompasses the most basic set of microeconomic tools.

Traditionally, microeconomics classifies private economic units into two groups: consumers (or households) and firms. These two groups give rise, respectively, to the theory of the consumer and theory of the firm as two branches of study. The **theory of the consumer** deals with **consumption** (the demand for goods and services) by utility-maximizing individuals (i.e., individuals who make decisions that maximize the satisfaction received from present and future consumption). The **theory of the firm** deals with the supply of goods and services by profit-maximizing firms. The theory of the consumer and the theory of the firm are important because they help us understand the foundations of demand and supply. Subsequent chapters will focus on the theory of the consumer and the theory of the firm.

Investment analysts, particularly equity and credit analysts, must regularly analyze products and services—their costs, prices, possible substitutes, and complements—to reach conclusions about a company's profitability and business risk (risk relating to operating profits). Furthermore, unless the analyst has a sound understanding of the demand and supply model of markets, he or she cannot hope to forecast how external events—such as a shift in consumer tastes or changes in taxes and subsidies or other intervention in markets—will influence a firm's revenue, earnings, and cash flows.

Having grasped the tools and concepts presented in this chapter, the reader should also be able to understand many important economic relationships and facts and be able to answer questions such as:

- Why do consumers usually buy more when the price falls? Is it irrational to violate this law of demand?
- What are appropriate measures of how sensitive the quantity demanded or supplied is to changes in price, income, and prices of other goods? What affects those sensitivities?
- If a firm lowers its price, will its total revenue also fall? Are there conditions under which revenue might rise as price falls, and, if so, what are those conditions? Why might this occur?
- What is an appropriate measure of the total value consumers or producers receive from the opportunity to buy and sell goods and services in a free market? How might government intervention reduce that value, and what is an appropriate measure of that loss?
- What tools are available that help us frame the trade-offs that consumers and investors face as they must give up one opportunity to pursue another?

- Is it reasonable to expect markets to converge to an equilibrium price? What are the conditions that would make that equilibrium stable or unstable in response to external shocks?
- How do different types of auctions affect price discovery?

This chapter is organized as follows. Section 2 explains how economists classify markets. Section 3 covers the basic principles and concepts of demand and supply analysis of markets. Section 4 introduces measures of sensitivity of demand to changes in prices and income. A summary and a set of practice problems conclude the chapter.

2. TYPES OF MARKETS

Analysts must understand the demand and supply model of markets because all firms buy and sell in markets. Investment analysts need at least a basic understanding of those markets and the demand and supply model that provides a framework for analyzing them.

Markets are broadly classified as factor markets or goods markets. **Factor markets** are markets for the purchase and sale of factors of production. In capitalist private enterprise economies, households own the **factors of production** (the land, labor, physical capital, and materials used in production). **Goods markets** are markets for the output of production. From an economics perspective, firms, which ultimately are owned by individuals either singly or in some corporate form, are organizations that buy the services of those factors. Firms then transform those services into intermediate or final goods and services. (**Intermediate goods and services** are those purchased for use as inputs to produce other goods and services, whereas final goods and services are in the final form purchased by households.) These two types of interaction between the household sector and the firm sector—those related to goods and those related to services—take place in factor markets and goods markets, respectively.

In the factor market for labor, households are sellers and firms are buyers. In goods markets, firms are sellers and both households and firms are buyers. For example, firms are buyers of capital goods (such as equipment) and intermediate goods, while households are buyers of a variety of durable and nondurable goods. Generally, market interactions are *voluntary*. Firms offer their products for sale when they believe the payment they will receive exceeds their cost of production. Households are willing to purchase goods and services when the value they expect to receive from them exceeds the payment necessary to acquire them. Whenever the perceived value of a good exceeds the expected cost to produce it, a potential trade can take place. This fact may seem obvious, but it is fundamental to our understanding of markets. If a buyer values something more than a seller, not only is there an opportunity for an exchange, but that exchange will make *both* parties better off.

In one type of factor market, called **labor markets**, households offer to sell their labor services when the payment they expect to receive exceeds the value of the leisure time they must forgo. In contrast, firms hire workers when they judge that the value of the productivity of workers is greater than the cost of employing them. A major source of household income and a major cost to firms is compensation paid in exchange for labor services.

Additionally, households typically choose to spend less on consumption than they earn from their labor. This behavior is called **saving**, through which households can accumulate financial capital, the returns on which can produce other sources of household income, such as interest, dividends, and capital gains. Households may choose to lend their accumulated

savings (in exchange for interest) or invest it in ownership claims in firms (in hopes of receiving dividends and capital gains). Households make these savings choices when their anticipated future returns are judged to be more valuable today than the present consumption that households must sacrifice when they save.

Indeed, a major purpose of financial institutions and markets is to enable the transfer of these savings into capital investments. Firms use **capital markets** (markets for long-term financial capital—that is, markets for long-term claims on firms' assets and cash flows) to sell debt (in bond markets) or equity (in equity markets) in order to raise funds to invest in productive assets, such as plant and equipment. They make these investment choices when they judge that their investments will increase the value of the firm by more than the cost of acquiring those funds from households. Firms also use such financial intermediaries as banks and insurance companies to raise capital, typically debt funding that ultimately comes from the savings of households, which are usually net accumulators of financial capital.

Microeconomics, although primarily focused on goods and factor markets, can contribute to the understanding of all types of markets (e.g., markets for financial securities).

EXAMPLE 1-1 Types of Markets

1. Which of the following markets is *least* accurately described as a factor market? The market for:
 - A. land.
 - B. assembly-line workers.
 - C. capital market securities.
2. Which of the following markets is *most* accurately defined as a product market? The market for:
 - A. companies.
 - B. unskilled labor.
 - C. legal and lobbying services.

Solution to 1: C is correct.

Solution to 2: C is correct.

3. BASIC PRINCIPLES AND CONCEPTS

In this chapter, we explore a model of household behavior that yields the consumer demand curve. **Demand**, in economics, is the willingness and ability of consumers to purchase a given amount of a good or service at a given price. **Supply** is the willingness of sellers to offer a given quantity of a good or service for a given price. Later, study on the theory of the firm will yield the supply curve.

The demand and supply model is useful in explaining how price and quantity traded are determined and how external influences affect the values of those variables. Buyers' behavior is

captured in the demand function and its graphical equivalent, the demand curve. This curve shows both the highest price buyers are willing to pay for each quantity and the largest quantity buyers are willing and able to purchase at each price. Sellers' behavior is captured in the supply function and its graphical equivalent, the supply curve. This curve shows simultaneously the lowest price sellers are willing to accept for each quantity and the largest quantity sellers are willing to offer at each price.

If, at a given quantity, the highest price that buyers are willing to pay is equal to the lowest price that sellers are willing to accept, we say the market has reached its equilibrium quantity. Alternatively, when the quantity that buyers are willing and able to purchase at a given price is just equal to the quantity that sellers are willing to offer at that same price, we say the market has discovered the equilibrium price. So equilibrium price and quantity are achieved simultaneously, and as long as neither the supply curve nor the demand curve shifts, there is no tendency for either price or quantity to vary from its equilibrium value.

3.1. The Demand Function and the Demand Curve

We first analyze demand. The quantity consumers are willing to buy clearly depends on a number of different factors, called variables. Perhaps the most important of those variables is the item's own price. In general, economists believe that as the price of a good rises, buyers will choose to buy less of it, and as its price falls, they buy more. This is such a ubiquitous observation that it has come to be called the **law of demand**, although we shall see that it need not hold in all circumstances.

Although a good's own price is important in determining consumers' willingness to purchase it, other variables also have influence on that decision, such as consumers' incomes, their tastes and preferences, the prices of other goods that serve as substitutes or complements, and so on. Economists attempt to capture all of these influences in a relationship called the **demand function**. (In general, a function is a relationship that assigns a unique value to a dependent variable for any given set of values of a group of independent variables.) We represent such a demand function in Equation 1-1:

$$Q_x^d = f(P_x, I, P_y, \dots) \quad (1-1)$$

where Q_x^d represents the quantity demanded of some good X (such as per-household demand for gasoline in gallons per week), P_x is the price per unit of good X (such as \$ per gallon), I is consumers' income (as in \$1,000s per household annually), and P_y is the price of another good, Y . (There can be many other goods, not just one, and they can be complements or substitutes.) Equation 1-1 may be read, "Quantity demanded of good X depends on (is a function of) the price of good X , consumers' income, the price of good Y , and so on."

Often, economists use simple linear equations to approximate real-world demand and supply functions in relevant ranges. A hypothetical example of a specific demand function could be Equation 1-2, a linear equation for a small town's per-household gasoline consumption per week, where P_y might be the average price of an automobile in \$1,000s:

$$Q_x^d = 8.4 - 0.4P_x + 0.06I - 0.01P_y \quad (1-2)$$

The signs of the coefficients on gasoline price (negative) and consumer's income (positive) are intuitive, reflecting, respectively, an inverse and a positive relationship between those variables and quantity of gasoline consumed. The negative sign on average automobile price may indicate that if automobiles go up in price, fewer will be purchased and driven; hence less gasoline will be consumed. As will be discussed later, such a relationship would indicate that gasoline and automobiles have a negative cross-price elasticity of demand and are thus complements.

To continue our example, suppose that the price of gasoline (P_x) is \$3 per gallon, per-household income (I) is \$50,000, and the price of the average automobile (P_y) is \$20,000. Then this function would predict that the per-household weekly demand for gasoline would be 10 gallons: $8.4 - 0.4(3) + 0.06(50) - 0.01(20) = 8.4 - 1.2 + 3 - 0.2 = 10$, recalling that income and automobile prices are measured in thousands. Note that the sign on the own-price variable is negative; thus, as the price of gasoline rises, per-household weekly consumption would decrease by 0.4 gallons for every dollar increase in gas price. **Own-price** is used by economists to underscore that the reference is to the price of a good itself and not the price of some other good.

In our example, there are three independent variables in the demand function, and one dependent variable. If any one of the independent variables changes, so does the value of quantity demanded. It is often desirable to concentrate on the relationship between the dependent variable and just one of the independent variables at a time, which allows us to represent the relationship between those two variables in a two-dimensional graph (at specific levels of the variables held constant). To accomplish this goal, we can simply hold the other two independent variables constant at their respective levels and rewrite the equation. In economic writing, this "holding constant" of the values of all variables except those being discussed is traditionally referred to by the Latin phrase *ceteris paribus* (literally, "all other things being equal" in the sense of unchanged). In this chapter, we use the phrase "holding all other things constant" as a readily understood equivalent for *ceteris paribus*.

Suppose, for example, that we want to concentrate on the relationship between the quantity demanded of the good and its own price, P_x . Then we would hold constant the values of income and the price of good Y . In our example, those values are 50 and 20, respectively. So, by inserting the respective values, we would rewrite Equation 1-2 as:

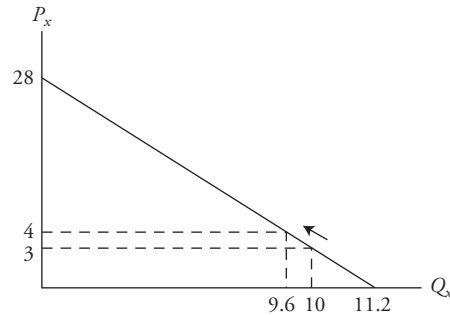
$$Q_x^d = 8.4 - 0.4P_x + 0.06(50) - 0.01(20) = 11.2 - 0.4P_x \quad (1-3)$$

Notice that income and the price of automobiles are not ignored; they are simply held constant, and they are collected in the new constant term, 11.2. Notice also that we can rearrange Equation 1-3, solving for P_x in terms of Q_x . This operation is called "inverting the demand function," and gives us Equation 1-4. (You should be able to perform this algebraic exercise to verify the result.)

$$P_x = 28 - 2.5Q_x \quad (1-4)$$

Equation 1-4, which gives the per-gallon price of gasoline as a function of gasoline consumed per week, is referred to as the **inverse demand function**. We need to restrict Q_x in Equation 1-4 to be less than or equal to 11.2 so price is not negative. Henceforward we assume that the reader can work out similar needed qualifications to the valid application of

EXHIBIT 1-1 Household Demand Curve for Gasoline



equations. The graph of the inverse demand function is called the **demand curve**, and is shown in Exhibit 1-1.¹

This demand curve is drawn with price on the vertical axis and quantity on the horizontal axis. Depending on how we interpret it, the demand curve shows either the greatest quantity a household would buy at a given price or the highest price it would be willing to pay for a given quantity. In our example, at a price of \$3 per gallon households would each be willing to buy 10 gallons per week. Alternatively, the highest price they would be willing to pay for 10 gallons per week is \$3 per gallon. Both interpretations are valid, and we will be thinking in terms of both as we proceed. If the price were to rise by \$1, households would reduce the quantity they each bought by 0.4 units to 9.6 gallons. We say that the slope of the demand curve is $1/-0.4$, or -2.5 . Slope is always measured as “rise over run,” or the change in the vertical variable divided by the change in the horizontal variable. In this case, the slope of the demand curve is $\Delta P/\Delta Q$, where “ Δ ” stands for “the change in.” The change in price was \$1, and it is associated with a change in quantity of negative 0.4.

3.2. Changes in Demand versus Movements along the Demand Curve

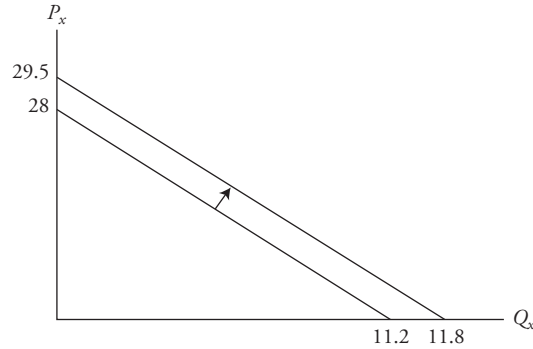
As we just saw, when own-price changes, quantity demanded changes. This change is called a movement along the demand curve or a change in quantity demanded, and it comes only from a change in own-price.

Recall that to draw the demand curve, though, we had to hold everything *except* quantity and own-price constant. What would happen if income were to change by some amount? Suppose that household income rose by \$10,000 per year to a value of 60. Then the value of Equation 1-3 would change to Equation 1-5:

$$Q_x^d = 8.4 - 0.4P_x + 0.06(60) - 0.01(20) = 11.8 - 0.4P_x \quad (1-5)$$

¹Following usual practice, here and in other exhibits we will show linear demand curves intersecting the quantity axis at a price of zero, which shows the intercept of the associated demand equation. Real-world demand functions may be nonlinear in some or all parts of their domain. Thus, linear demand functions in practical cases are viewed as approximations to the true demand function that are useful for a relevant range of values. The relevant range would typically not include a price of zero, and the prediction for demand at a price of zero should not be viewed as usable.

EXHIBIT 1-2 Household Demand Curve for Gasoline before and after Change in Income



and Equation 1-4 would become the new inverse demand function (Equation 1-6):

$$P_x = 29.5 - 2.5Q_x \quad (1-6)$$

Notice that the slope has remained constant, but the intercepts have both increased, resulting in an outward shift in the demand curve, as shown in Exhibit 1-2.

In general, the only thing that can cause a movement along the demand curve is a change in a good's own price. A change in the value of any *other* variable will shift the entire demand curve. The former is referred to as a *change in quantity demanded*, and the latter is referred to as a *change in demand*.

More importantly, the shift in demand was both a *vertical* shift upward and a *horizontal* shift to the right. That is to say, for any given quantity, the household is now willing to pay a higher price; and at any given price, the household is now willing to buy a greater quantity. Both interpretations of the shift in demand are valid.

EXAMPLE 1-2 Representing Consumer Buying Behavior with a Demand Function and Demand Curve

An individual consumer's monthly demand for downloadable e-books is given by the equation

$$Q_{eb}^d = 2 - 0.4P_{eb} + 0.0005I + 0.15P_{hb}$$

where Q_{eb}^d equals the number of e-books demanded each month, P_{eb} equals the price of e-books, I equals the household monthly income, and P_{hb} equals the price of

hardbound books, per unit. Notice that the sign on the price of hardbound books is positive, indicating that when hardbound books increase in price, more e-books are purchased; thus, according to this equation, the two types of books are substitutes. Assume that the price of each e-book is €10.68, household income is €2,300, and the price of each hardbound book is €21.40.

1. Determine the number of e-books demanded by this household each month.
2. Given the values for I and P_{hb} , determine the inverse demand function.
3. Determine the slope of the demand curve for e-books.
4. Calculate the vertical intercept (price-axis intercept) of the demand curve if income increases to €3,000 per month.

Solution to 1: Insert given values into the demand function and calculate quantity:

$$Q_{eb}^d = 2 - 0.4(10.68) + 0.0005(2,300) + 0.15(21.40) = 2.088$$

Hence, the household will demand e-books at the rate of 2.088 books per month. Note that this rate is a flow, so there is no contradiction in there being a noninteger quantity. In this case, the outcome means that the consumer buys 23 e-books during 11 months.

Solution to 2: We want to find the price–quantity relationship holding all other things constant, so first, insert values for I and P_{hb} into the demand function and collect the constant terms:

$$Q_{eb}^d = 2 - 0.4P_{eb} + 0.0005(2,300) + 0.15(21.40) = 6.36 - 0.4P_{eb}$$

Now solve for P_{eb} in terms of Q_{eb} : $P_{eb} = 15.90 - 2.5Q_{eb}$

Solution to 3: Note from the previous inverse demand function that when Q_{eb} rises by one unit, P_{eb} falls by €2.5. So the slope of the demand curve is -2.5 , which is the coefficient on Q_{eb} in the inverse demand function. Note it is *not* the coefficient on P_{eb} in the demand function, which is -0.4 . It is the inverse of that coefficient.

Solution to 4: In the demand function, change the value of I to 3,000 from 2,300 and collect constant terms:

$$Q_{eb}^d = 2 - 0.4P_{eb} + 0.0005(3,000) + 0.15(21.40) = 6.71 - 0.4P_{eb}$$

Now solve for P_{eb} : $P_{eb} = 16.78 - 2.5Q_{eb}$. The vertical intercept is 16.78. (Note that this increase in income has shifted the demand curve outward and upward but has not affected its slope, which is still -2.5 .)

3.3. The Supply Function and the Supply Curve

The willingness and ability to sell a good or service is called supply. In general, producers are willing to sell their product for a price as long as that price is at least as high as the cost to produce an additional unit of the product. It follows that the willingness to supply, called the **supply function**, depends on the price at which the good can be sold as well as the cost of production for an additional unit of the good. The greater the difference between those two values, the greater is the willingness of producers to supply the good.

In subsequent chapters, we will explore the cost of production in greater detail. At this point, we need to understand only the basics of cost. At its simplest level, production of a good consists of transforming inputs, or factors of production (such as land, labor, capital, and materials), into finished goods and services. Economists refer to the rules that govern this transformation as the **technology of production**. Because producers have to purchase inputs in factor markets, the cost of production depends on both the technology and the price of those factors. Clearly, willingness to supply is dependent on not only the price of a producer's output, but additionally on the prices (i.e., costs) of the inputs necessary to produce it. For simplicity, we can assume that the only input in a production process is labor that must be purchased in the labor market. The price of an hour of labor is the wage rate, or W . Hence, we can say that (for any given level of technology) the willingness to supply a good depends on the price of that good and the wage rate. This concept is captured in Equation 1-7, which represents an individual seller's supply function:

$$Q_x^s = f(P_x, W, \dots) \quad (1-7)$$

where Q_x^s is the quantity supplied of some good X (such as gasoline), P_x is the price per unit of good X , and W is the wage rate of labor in, say, dollars per hour. It would be read, "The quantity supplied of good X depends on (is a function of) the price of X (its own price), the wage rate paid to labor, and so on."

Just as with the demand function, we can consider a simple hypothetical example of a seller's supply function. As mentioned earlier, economists often will simplify their analysis by using linear functions, although that is not to say that all demand and supply functions are necessarily linear. One hypothetical example of an individual seller's supply function for gasoline is given in Equation 1-8:

$$Q_x^s = -175 + 250P_x - 5W \quad (1-8)$$

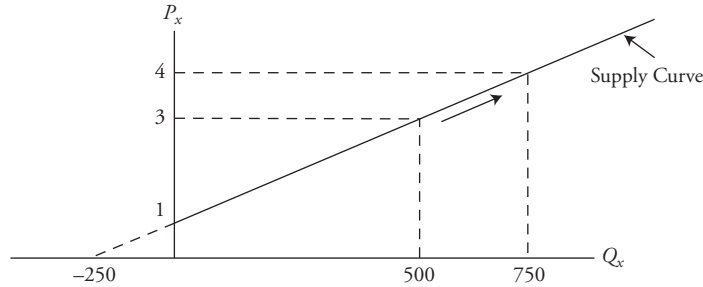
Notice that this supply function says that for every increase in price of \$1, this seller would be willing to supply an additional 250 units of the good. Additionally, for every \$1 increase in wage rate that it must pay its laborers, this seller would experience an increase in marginal cost and would be willing to supply five fewer units of the good.

We might be interested in the relationship between only two of these variables, price and quantity supplied. Just as we did in the case of the demand function, we use the assumption of *ceteris paribus* and hold everything except own-price and quantity constant. In our example, we accomplish this by setting W to some value, say, \$15. The result is Equation 1-9:

$$Q_x^s = -175 + 250P_x - 5(15) = -250 + 250P_x \quad (1-9)$$

in which only the two variables Q_x^s and P_x appear. Once again, we can solve this equation for P_x in terms of Q_x^s , which yields the *inverse supply function* in Equation 1-10:

EXHIBIT 1-3 Individual Seller's Supply Curve for Gasoline



$$P_x = 1 + 0.004Q_x \quad (1-10)$$

The graph of the inverse supply function is called the **supply curve**, and it shows simultaneously the highest quantity willingly supplied at each price and the lowest price willingly accepted for each quantity. For example, if the price of gasoline were \$3 per gallon, Equation 1-9 implies that this seller would be willing to sell 500 gallons per week. Alternatively, the lowest price the seller would accept and still be willing to sell 500 gallons per week would be \$3. Exhibit 1-3 represents our hypothetical example of an individual seller's supply curve of gasoline.

What does our supply function tell us will happen if the retail price of gasoline rises by \$1? We insert the new higher price of \$4 into Equation 1-8 and find that quantity supplied would rise to 750 gallons per week. The increase in price has enticed the seller to supply a greater quantity of gasoline per week than at the lower price.

3.4. Changes in Supply versus Movements along the Supply Curve

As we saw earlier, a change in the (own) price of a product causes a change in the quantity of that good willingly supplied. A rise in price typically results in a greater quantity supplied, and a lower price results in a lower quantity supplied. Hence, the supply curve has a positive slope, in contrast to the negative slope of a demand curve. This positive relationship is often referred to as the **law of supply**.

What happens when a variable other than own-price takes on different values? We could answer this question in our example by assuming a different value for wage rate, say \$20 instead of \$15. Recalling Equation 1-9, we would simply put in the higher wage rate and solve, yielding Equation 1-11.

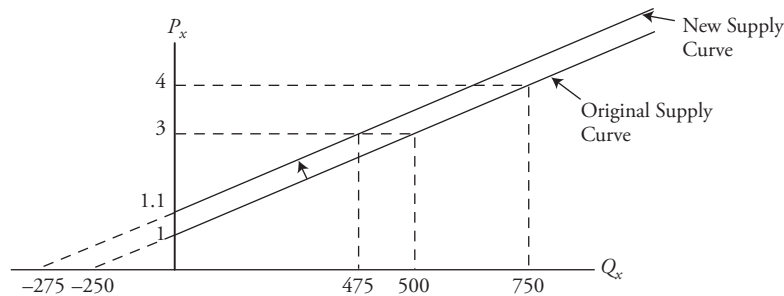
$$Q_x^s = -175 + 250P_x - 5(20) = -275 + 250P_x \quad (1-11)$$

This equation, too, can be solved for P_x , yielding the inverse supply function in Equation 1-12:

$$P_x = 1.1 + 0.004Q_x \quad (1-12)$$

Notice that the constant term has changed, but the slope has remained the same. The result is a shift in the entire supply curve, as illustrated in Exhibit 1-4.

EXHIBIT 1-4 Individual Seller's Supply Curve for Gasoline before and after Increase in Wage Rate



Notice that the supply curve has shifted both vertically upward and horizontally leftward as a result of the rise in the wage rate paid to labor. This change is referred to as a **change in supply**, as contrasted with a **change in quantity supplied** that would result only from a change in this product's own price. Now, at a price of 3, a lower quantity will be supplied: 475 instead of 500. Alternatively, in order to entice this seller to offer the same 500 gallons per week, the price would now have to be 3.1, up from 3 before the change. This increase in lowest acceptable price reflects the now higher marginal cost of production resulting from the increased input price that the firm now must pay for labor.

To summarize, a change in the price of a good itself will result in a movement along the supply curve and a change in quantity supplied. A change in any variable other than own-price will cause a shift in the supply curve, called a change in supply. This distinction is identical to the case of demand curves.

EXAMPLE 1-3 Representing Seller Behavior with a Supply Function and Supply Curve

An individual seller's monthly supply of downloadable e-books is given by the equation

$$Q_{eb}^s = -64.5 + 37.5P_{eb} - 7.5W$$

where Q_{eb}^s is number of e-books supplied each month, P_{eb} is price of e-books in euros, and W is the hourly wage rate in euros paid by e-book sellers to workers. Assume that the price of e-books is €10.68 and the hourly wage is €10.

1. Determine the number of e-books supplied each month.
2. Determine the inverse supply function for an individual seller.
3. Determine the slope of the supply curve for e-books.
4. Determine the new vertical intercept of the individual e-book supply curve if the hourly wage were to rise to €15 from €10.

Solution to 1: Insert given values into the supply function and calculate the number of e-books:

$$Q_{eb}^s = -64.5 + 37.5(10.68) - 7.5(10) = 261$$

Hence, each seller would be willing to supply e-books at the rate of 261 per month.

Solution to 2: Holding all other things constant, the wage rate is constant at €10, so we have:

$$Q_{eb}^s = -64.5 + 37.5P_{eb} - 7.5(10) = -139.5 + 37.5P_{eb}$$

We now solve this for P_{eb} :

$$P_{eb} = 3.72 + 0.0267Q_{eb}$$

Solution to 3: Note that when Q_{eb} rises by one unit, P_{eb} rises by 0.0267 euros, so the slope of the supply curve is 0.0267, which is the coefficient on Q_{eb} in the inverse supply function. Note that it is *not* 37.5.

Solution to 4: In the supply function, increase the value of W to €15 from €10:

$$Q_{eb}^s = -64.5 + 37.5P_{eb} - 7.5(15) = -177 + 37.5P_{eb}$$

and invert by solving for P_{eb} :

$$P_{eb} = 4.72 + 0.267Q_{eb}$$

The vertical intercept is now 4.72. Thus, an increase in the wage rate shifts the supply curve upward and to the left. This change is known as a decrease in supply because at each price the seller would be willing now to supply fewer e-books than before the increase in labor cost.

3.5. Aggregating the Demand and Supply Functions

We have explored the basic concept of demand and supply at the individual household and the individual supplier level. However, markets consist of collections of demanders and suppliers, so we need to understand the process of combining these individual agents' behavior to arrive at market demand and supply functions.

The process could not be more straightforward: simply add all the buyers together and add all the sellers together. Suppose there are 1,000 identical gasoline buyers in our hypothetical example, and they represent the total market. At, say, a price of \$3 per gallon, we find that one household would be willing to purchase 10 gallons per week (when income and price of automobiles are held constant at \$50,000 and \$20,000, respectively). So, 1,000 identical buyers would be willing to purchase 10,000 gallons collectively. It follows that to aggregate 1,000 buyers' demand functions, simply multiply each buyer's quantity demanded by 1,000, as shown in Equation 1-13:

$$Q_x^d = 1,000(8.4 - 0.4P_x + 0.06I - 0.01P_y) = 8,400 - 400P_x + 60I - 10P_y \quad (1-13)$$

where Q_x^d represents the market quantity demanded. Note that if we hold I and P_y at their same respective values of 50 and 20 as before, we can collapse the constant terms and write the following Equation 1-14:

$$Q_x^d = 11,200 - 400P_x \quad (1-14)$$

Equation 1-14 is just Equation 1-3 (an individual household's demand function) multiplied by 1,000 households (Q_x^d represents thousands of gallons per week). Again, we can solve for P_x to obtain the market inverse demand function:

$$P_x = 28 - 0.0025Q_x \quad (1-15)$$

The market demand curve is simply the graph of the market inverse demand function, as shown in Exhibit 1-5.

It is important to note that the aggregation process sums all individual buyers' *quantities*, not the *prices* they are willing to pay; that is, we multiplied the demand function, *not* the inverse demand function, by the number of households. Accordingly, the market demand curve has the exact same price intercept as each individual household's demand curve. If, at a price of \$28, a single household would choose to buy zero, then it follows that 1,000 identical households would choose, in aggregate, to buy zero as well. However, if each household chooses to buy 10 at a price of \$3, then 1,000 identical households would choose to buy 10,000, as shown in Exhibit 1-5. Hence, we say that all individual demand curves *horizontally* (quantities), not *vertically* (prices), are added to arrive at the market demand curve.

Now that we understand the aggregation of demanders, the aggregation of suppliers is simple: We do exactly the same thing. Suppose, for example, that there are 20 identical sellers with the supply function given by Equation 1-8. To arrive at the market supply function, we simply multiply by 20 to obtain Equation 1-16:

EXHIBIT 1-5 Aggregate Weekly Market Demand for Gasoline as the Quantity Summation of All Households' Demand Curves

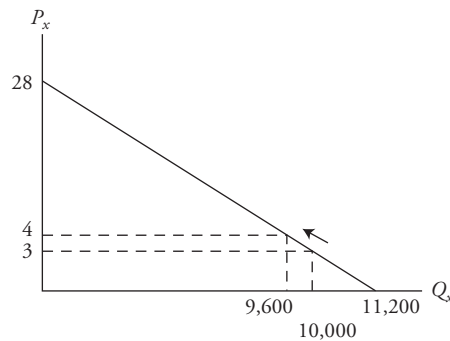
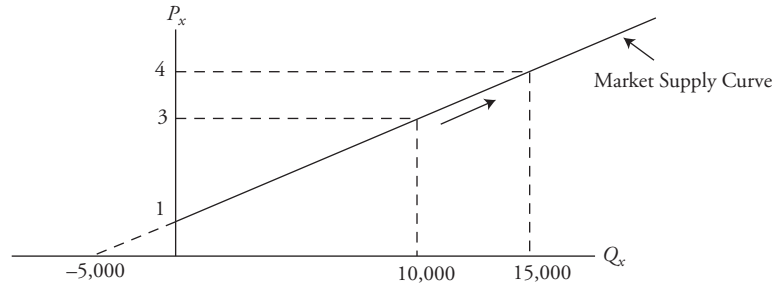


EXHIBIT 1-6 Aggregate Market Supply as the Quantity Summation of Individual Sellers' Supply Curves



$$Q_x^s = 20(-175 + 250P_x - 5W) = -3,500 + 5,000P_x - 100W \quad (1-16)$$

And, if we once again assume W equals \$15, we can collapse the constant terms, yielding Equation 1-17:

$$Q_x^s = 20[-175 + 250P_x - 5(15)] = -5,000 + 5,000P_x \quad (1-17)$$

which can be inverted to yield the market inverse supply function (Equation 1-18):

$$P_x = 1 + 0.0002Q_x \quad (1-18)$$

Graphing the market inverse supply function yields the market supply curve in Exhibit 1-6.

We saw from the individual seller's supply curve in Exhibit 1-3 that at a price of \$3, an individual seller would willingly offer 500 gallons of gasoline. It follows, as shown in Exhibit 1-6, that a group of 20 sellers would offer 10,000 gallons per week. Accordingly, at each price, the market quantity supplied is just 20 times as great as the quantity supplied by each seller. We see, as in the case of demand curves, that the market supply curve is simply the horizontal summation of all individual sellers' supply curves.

EXAMPLE 1-4 Aggregating Demand Functions

An individual consumer's monthly demand for downloadable e-books is given by the equation

$$Q_{eb}^d = 2 - 0.4P_{eb} + 0.0005I + 0.15P_{hb}$$

where Q_{eb}^d equals the number of e-books demanded each month, P_{eb} is the price of e-books in euros, I equals the household monthly income, and P_{hb} equals the price

of hardbound books per unit. Assume that household income is €2,300 and the price of hardbound books is €21.40. The market consists of 1,000 identical consumers with this demand function.

1. Determine the market aggregate demand function.
2. Determine the inverse market demand function.
3. Determine the slope of the market demand curve.

Solution to 1: Aggregating over the total number of consumers means summing up their demand functions (in the quantity direction). In this case, there are 1,000 consumers with identical individual demand functions, so multiply the entire function by 1,000:

$$\begin{aligned} Q_{eb} &= 1,000(2 - 0.4P_{eb} + 0.0005I + 0.15P_{hb}) \\ &= 2,000 - 400P_{eb} + 0.5I + 150P_{hb} \end{aligned}$$

Solution to 2: Holding I constant at a value of €2,300 and P_{hb} constant at a value of €21.40, we find

$$Q_{eb} = 2,000 - 400P_{eb} + 0.5(2,300) + 150(21.40) = 6,360 - 400P_{eb}$$

$$\text{Now solve for } P_{eb} = 15.90 - 0.0025Q_{eb}$$

Solution to 3: The slope of the market demand curve is the coefficient on Q_{eb} in the inverse demand function, which is -0.0025 .

EXAMPLE 1-5 Aggregating Supply Functions

An individual seller's monthly supply of downloadable e-books is given by the equation

$$Q_{eb}^s = -64.5 + 37.5P_{eb} - 7.5W$$

where Q_{eb}^s is number of e-books supplied, P_{eb} is the price of e-books in euros, and W is the wage rate in euros paid by e-book sellers to laborers. Assume that the price of e-books is €10.68 and wage is €10. The supply side of the market consists of a total of eight identical sellers in this competitive market.

1. Determine the market aggregate supply function.
2. Determine the inverse market supply function.
3. Determine the slope of the aggregate market supply curve.

Solution to 1: Aggregating supply functions means summing up the quantity supplied by all sellers. In this case, there are eight identical sellers, so multiply the individual seller's supply function by eight:

$$Q_{eb}^s = 8(-64.5 + 37.5P_{eb} - 7.5W) = -516 + 300P_{eb} - 60W$$

Solution to 2: Holding W constant at a value of €10, insert that value into the aggregate supply function and then solve for P_{eb} to find the inverse supply function:

$$Q_{eb} = -1,116 + 300P_{eb}$$

Inverting, $P_{eb} = 3.72 + 0.0033Q_{eb}$

Solution to 3: The slope of the supply curve is the coefficient on Q_{eb} in the inverse supply function, which is 0.0033.

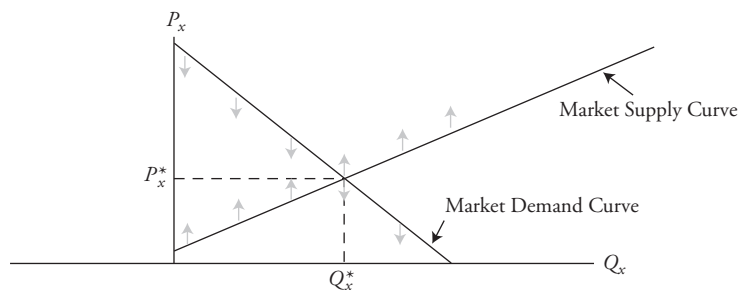
3.6. Market Equilibrium

An important concept in the market model is **market equilibrium**, defined as the condition in which the quantity willingly offered for sale by sellers at a given price is just equal to the quantity willingly demanded by buyers at that same price. When that condition is met, we say that the market has discovered its equilibrium price. An alternative and equivalent condition of equilibrium occurs at that quantity at which the highest price a buyer is willing to pay is just equal to the lowest price a seller is willing to accept for that same quantity.

As we have discovered in the earlier sections, the demand curve shows (for given values of income, other prices, etc.) an infinite number of combinations of prices and quantities that satisfy the demand function. Similarly, the supply curve shows (for given values of input prices, etc.) an infinite number of combinations of prices and quantities that satisfy the supply function. Equilibrium occurs at the unique combination of price and quantity that simultaneously satisfies *both* the market demand function and the market supply function. Graphically, it is the intersection of the demand and supply curves as shown in Exhibit 1-7.

In Exhibit 1-7, the shaded arrows indicate, respectively, that buyers will be willing to pay any price at or below the demand curve (indicated by ↓), and sellers are willing to accept any price at or above the supply curve (indicated by ↑).

EXHIBIT 1-7 Market Equilibrium Price and Quantity as the Intersection of Demand and Supply



Notice that for quantities less than Q_x^* , the highest price that buyers are willing to pay exceeds the lowest price that sellers are willing to accept, as indicated by the shaded arrows. But for all quantities above Q_x^* , the lowest price willingly accepted by sellers is greater than the highest price willingly offered by buyers. Clearly, trades will not be made beyond Q_x^* .

Algebraically, we can find the equilibrium price by setting the demand function equal to the supply function and solving for price. Recall that in our hypothetical example of a local gasoline market, the demand function was given by $Q_x^d = f(P_x, I, P_y)$, and the supply function was given by $Q_x^s = f(P_x, W)$. Those expressions are called **behavioral equations** because they model the behavior of, respectively, buyers and sellers. Variables *other* than own-price and quantity are determined outside of the demand and supply model of this particular market. Because of that, they are called **exogenous variables**. Price and quantity, however, are determined within the model for this particular market and are called **endogenous variables**. In our simple example, there are three exogenous variables (I , P_y , and W) and three endogenous variables: P_x , Q_x^d , and Q_x^s . Hence, we have a system of two equations and three unknowns. We need another equation to solve this system. That equation is called the **equilibrium condition**, and it is simply $Q_x^d = Q_x^s$.

Continuing with our hypothetical examples, we could assume that income equals \$50 (thousands, per year), the price of automobiles equals \$20 (thousands, per automobile), and the hourly wage equals \$15. In this case, our equilibrium condition can be represented in Equation 1-19 by setting Equation 1-14 equal to Equation 1-17:

$$11,200 - 400P_x = -5,000 + 5,000P_x \quad (1-19)$$

and solving for equilibrium, $P_x = 3$.

Equivalently, we could have equated the inverse demand function to the inverse supply function (Equations 1-15 and 1-18, respectively), as shown in Equation 1-20:

$$28 - 0.0025Q_x = 1 + 0.0002Q_x \quad (1-20)$$

and solved for equilibrium, $Q_x = 10,000$. That is to say, for the given values of I and W , the unique combination of price and quantity of gasoline that results in equilibrium is (3, 10,000).

Note that our system of equations requires explicit values for the exogenous variables to find a unique equilibrium combination of price and quantity. Conceptually, the values of the exogenous variables are being determined in other markets, such as the markets for labor, automobiles, and so on, whereas the price and quantity of gasoline are being determined in the gasoline market. When we concentrate on one market, taking values of exogenous variables as given, we are engaging in what is called **partial equilibrium analysis**. In many cases, we can gain sufficient insight into a market of interest without addressing feedback effects to and from all the other markets that are tangentially involved with this one. At other times, however, we need explicitly to take account of all the feedback mechanisms that are going on in all markets simultaneously. When we do that, we are engaging in what is called **general equilibrium analysis**. For example, in our hypothetical model of the local gasoline market, we recognize that the price of automobiles, a complementary product, has an impact on the demand for gasoline. If the price of automobiles were to rise, people would tend to buy fewer automobiles and probably buy less gasoline. Additionally, though, the price of gasoline probably has an impact on the demand for automobiles, which, in turn, can feed back to the gasoline market. Because we are positing a very local gasoline market, it is probably safe to ignore all the feedback effects, but if we are modeling the national markets for gasoline and automobiles, a general equilibrium model might be warranted.

EXAMPLE 1-6 Finding Equilibrium by Equating Demand and Supply

In the local market for e-books, the aggregate demand is given by the equation

$$Q_{eb}^d = 2,000 - 400P_{eb} + 0.5I + 150P_{hb}$$

and the aggregate supply is given by the equation

$$Q_{eb}^s = -516 + 300P_{eb} - 60W$$

where Q_{eb} is quantity of e-books, P_{eb} is the price of an e-book, I is household income, W is wage rate paid to e-book laborers, and P_{hb} is the price of a hardbound book. Assume I is €2,300, W is €10, and P_{hb} is €21.40. Determine the equilibrium price and quantity of e-books in this local market.

Solution: Market equilibrium occurs when quantity demanded is equal to quantity supplied, so set $Q_{eb}^d = Q_{eb}^s$ after inserting the given values for the exogenous variables:

$$\begin{aligned} 2,000 - 400P_{eb} + 0.5(2,300) + 150(21.4) &= -516 + 300P_{eb} - 60(10) \\ 6,360 - 400P_{eb} &= -1,116 + 300P_{eb} \end{aligned}$$

which implies that $P_{eb} = €10.68$, and $Q_{eb} = 2,088$.

3.7. The Market Mechanism: Iterating toward Equilibrium—or Not

It is one thing to define equilibrium as we have done, but we should also understand the mechanism for reaching equilibrium. That mechanism is what takes place when the market is *not* in equilibrium. Consider our hypothetical example. We found that the equilibrium price was 3, but what would happen if, by some chance, price was actually equal to 4? To find out, we need to see how much buyers would demand at that price and how much sellers would offer to sell by inserting 4 into the demand function and into the supply function.

In the case of quantity demanded, we find that (Equation 1-21):

$$Q_x^d = 11,200 - 400(4) = 9,600 \quad (1-21)$$

and in the case of quantity supplied (Equation 1-22),

$$Q_x^s = -5,000 + 5,000(4) = 15,000 \quad (1-22)$$

Clearly, the quantity supplied is greater than the quantity demanded, resulting in a condition called **excess supply**, as illustrated in Exhibit 1-8. In our example, there are 5,400 more units of this good offered for sale at a price of 4 than are demanded at that price.

EXHIBIT 1-8 Excess Supply as a Consequence of Price above Equilibrium Price

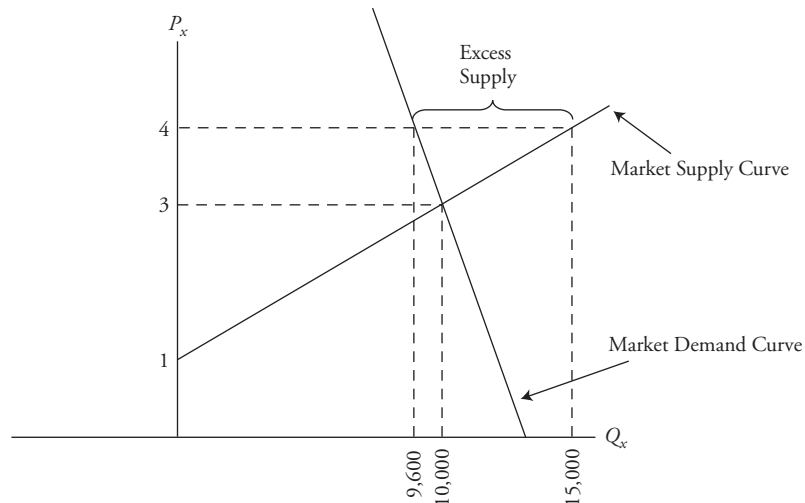
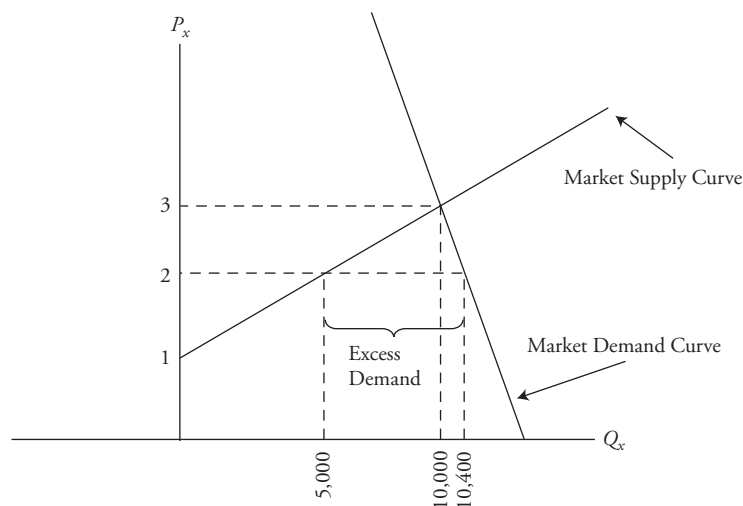


EXHIBIT 1-9 Excess Demand as a Consequence of Price below Equilibrium Price



Alternatively, if the market was presented with a price that was too low, say 2, then by inserting the price of 2 into Equations 1-21 and 1-22, we find that buyers are willing to purchase 5,400 *more* units than sellers are willing to offer. This result is shown in Exhibit 1-9.

To reach equilibrium, price must adjust until there is neither an excess supply nor an excess demand. That adjustment is called the **market mechanism**, and it is characterized in the following way: In the case of excess supply, price will fall; in the case of excess demand, price will rise; and in the case of neither excess supply nor excess demand, price will not change.

EXAMPLE 1-7 Identifying Excess Demand or Excess Supply at a Nonequilibrium Price

In the local market for e-books, the aggregate demand is given by the equation

$$Q_{eb}^d = 6,360 - 400P_{eb}$$

and the aggregate supply by the equation

$$Q_{eb}^s = -1,116 + 300P_{eb}$$

1. Determine the amount of excess demand or supply if price is €12.
2. Determine the amount of excess demand or supply if price is €8.

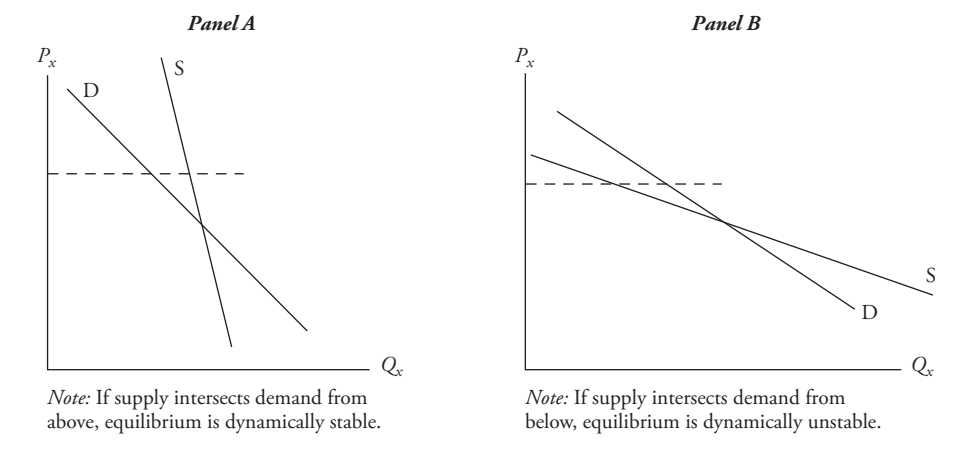
Solution to 1: Insert the presumed price of €12 into the demand function to find $Q_{eb}^d = 6,360 - 400(12) = 1,560$. Insert a price of €12 into the supply function to find $Q_{eb}^s = -1,116 + 300(12) = 2,484$. Because quantity supplied is greater than quantity demanded at the €12 price, there is an excess supply equal to $2,484 - 1,560 = 924$.

Solution to 2: Insert the presumed price of €8 into the demand function to find $Q_{eb}^d = 6,360 - 400(8) = 3,160$. Insert a price of €8 into the supply function to find $Q_{eb}^s = -1,116 + 300(8) = 1,284$. Because quantity demanded is greater than quantity supplied at the €8 price, there is an excess demand equal to $3,160 - 1,284 = 1,876$.

It might be helpful to consider the following process in our hypothetical market. Suppose that some neutral agent or referee were to display a price for everyone in the market to observe. Then, given that posted price, we would ask each potential buyer to write down on a slip of paper a quantity that he or she would be willing and able to purchase at that price. At the same time, each potential seller would write down a quantity that he or she would be willing to sell at that price. Those pieces of paper would be submitted to the referee, who would then calculate the total quantity demanded and the total quantity supplied at that price. If the two sums are identical, the slips of paper would essentially become contracts that would be executed, and the session would be concluded by buyers and sellers actually trading at that price. If there was an excess supply, however, the referee's job would be to discard the earlier slips of paper and display a price lower than before. Alternatively, if there was an excess demand at the original posted price, the referee would discard the slips of paper and post a higher price. This process would continue until the market reached an equilibrium price at which the quantity willingly offered for sale would just equal the quantity willingly purchased. In this way, the market could tend to move toward equilibrium.²

²The process described is known among economists as Walrasian *tâtonnement*, after the French economist Léon Walras (1834–1910). *Tâtonnement* means, roughly, “searching,” referring to the mechanism for establishing the equilibrium price.

EXHIBIT 1-10 Stability of Equilibria: I

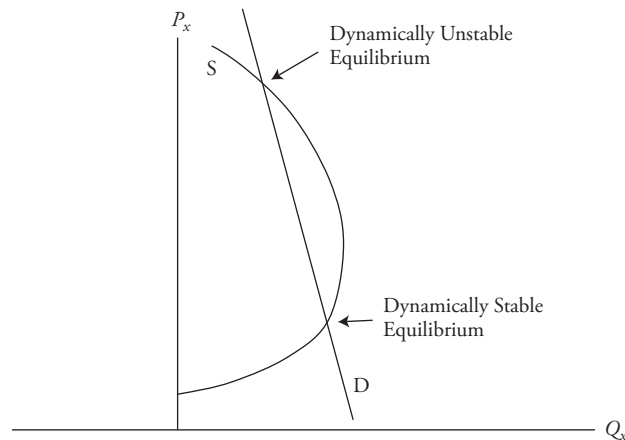


It is not really necessary for a market to have such a referee for it to operate *as if* it had one. Experimental economists have simulated markets in which subjects (usually college students) are given an order either to purchase or to sell some amount of a commodity for a price either no higher (in the case of buyers) or no lower (in the case of sellers) than a set dollar limit. Those limits are distributed among market participants and represent a positively sloped supply curve and a negatively sloped demand curve. The goal for buyers is to buy at a price as far below their limit as possible, and the goal for sellers to sell at a price as far above their minimum as possible. The subjects are then allowed to interact in a simulated trading pit by calling out willingness to buy or sell. When two participants come to an agreement on a price, that trade is then reported to a recorder, who displays the terms of the deal. Traders are then allowed to observe current prices as they continue to search for a buyer or a seller. It has consistently been shown in experiments that this mechanism of open outcry buying and selling (historically, one of the oldest mechanisms used in trading securities) soon converges to the theoretical equilibrium price and quantity inherent in the underlying demand and supply curves used to set the respective sellers' and buyers' limit prices.

In our hypothetical example of the gasoline market, the supply curve is positively sloped, and the demand curve is negatively sloped. In that case, the market mechanism would tend to reach an equilibrium and return to that equilibrium whenever price was accidentally bumped away from it. We refer to such an equilibrium as being **stable** because whenever price is disturbed away from equilibrium, it tends to converge back to that equilibrium.³ It is possible, however, for this market mechanism to result in an unstable equilibrium. Suppose that not only does the demand curve have a negative slope but also the supply curve has a negatively sloped segment. For example, at some level of wages, a wage increase might cause workers to supply fewer hours of work if satisfaction (utility) gained from an extra hour of leisure is greater than the satisfaction obtained from an extra hour of work. Then two possibilities could result, as shown in Panels A and B of Exhibit 1-10.

³In the same sense, equilibrium may sometimes also be referred to as being *dynamically stable*. Similarly, *unstable* or *dynamically unstable* may be used in the sense introduced later.

EXHIBIT 1-11 Stability of Equilibria: II



Note: Multiple equilibria (stable and unstable) can result from nonlinear supply curves.

Notice that in Panel A both demand (D) and supply (S) are negatively sloped, but S is steeper and intersects D from above. In this case, if price is above equilibrium, there will be excess supply and the market mechanism will adjust price downward toward equilibrium. In Panel B, D is steeper, which results in S intersecting D from below. In this case, at a price above equilibrium there will be excess demand, and the market mechanism will dictate that price should *rise*, thus leading away from equilibrium. This equilibrium would be considered **unstable**. If price were accidentally displayed above the equilibrium price, the mechanism would not cause price to converge to that equilibrium, but instead to soar above it because there would be excess demand at that price. In contrast, if price were accidentally displayed below equilibrium, the mechanism would force price even further below equilibrium because there would be excess supply.

If supply were nonlinear, there could be multiple equilibria, as shown in Exhibit 1-11.

Note that there are two combinations of price and quantity that would equate quantity supplied and demanded, hence two equilibria. The lower-priced equilibrium is stable, with a positively sloped supply curve and a negatively sloped demand curve. However, the higher-priced equilibrium is unstable because at a price above that equilibrium price there would be excess demand, thus driving price even higher. At a price below that equilibrium there would be excess supply, thus driving price even lower toward the lower-priced equilibrium, which is a stable equilibrium.

Observation suggests that most markets are characterized by stable equilibria. Prices do not often shoot off to infinity or plunge toward zero. However, occasionally we do observe price bubbles occurring in real estate, securities, and other markets. It appears that prices can behave in ways that are not ultimately sustainable in the long run. They may shoot up for a time, but ultimately, if they do not reflect actual valuations, the bubble can burst, resulting in a correction to a new equilibrium.

As a simple approach to understanding bubbles, consider a case in which buyers and sellers base their expectations of future prices on the rate of change of current prices: if price rises, they take that as a sign that price will rise even further. Under these circumstances, if

buyers see an increase in price today, they might actually shift the demand curve to the right, desiring to buy more at each price today because they expect to have to pay more in the future. Alternately, if sellers see an increase in today's price as evidence that price will be even higher in the future, they are reluctant to sell today as they hold out for higher prices tomorrow, and that would shift the supply curve to the left. With a rightward shift in demand and a leftward shift in supply, buyers' and sellers' expectations about price are confirmed and the process begins again. This scenario could result in a bubble that would inflate until someone decides that such high prices can no longer be sustained. The bubble bursts and price plunges.

3.8. Auctions as a Way to Find Equilibrium Price

Sometimes markets really do use auctions to arrive at equilibrium price. Auctions can be categorized into two types depending on whether the value of the item being sold is the same for each bidder or is unique to each bidder. The first case is called a **common value auction** in which there is some actual common value that will ultimately be revealed after the auction is settled. Prior to the auction's settlement, however, bidders must estimate that true value. An example of a common value auction would be bidding on a jar containing many coins. Each bidder could estimate the value; but until someone buys the jar and actually counts the coins, no one knows with certainty the true value. In the second case, called a **private value auction**, each buyer places a subjective value on the item, and in general their values differ. An example might be an auction for a unique piece of art that buyers are hoping to purchase for their own personal enjoyment, not primarily as an investment to be sold later.

Auctions also differ according to the mechanism used to arrive at a price and to determine the ultimate buyer. These mechanisms include the ascending price (or English) auction, the first price sealed-bid auction, the second price sealed-bid (or Vickery) auction, and the descending price (or Dutch) auction.

Perhaps the most familiar auction mechanism is the **ascending price auction** in which an auctioneer is selling a single item in a face-to-face arena where potential buyers openly reveal their willingness to buy the good at prices that are called out by an auctioneer. The auctioneer begins at a low price and easily elicits nods from buyers. He or she then raises the price incrementally. In a common value auction, buyers can sometimes learn something about the true value of the item being auctioned from observing other bidders. Ultimately bidders with different maximum amounts they are willing to pay for the item, called **reservation prices**, begin to drop out of the bidding as price rises above their respective reservation prices.⁴ Finally, only one bidder is left (who has outbid the bidder with the second-highest valuation) and the item is sold to that bidder for that bid price.

Sometimes sellers offer a common value item, such as an oil or timber lease, in a **sealed-bid auction**. In this case, bids are elicited from potential buyers, but there is no ability to observe bids by other buyers until the auction has ended. In the **first price sealed-bid auction**, the envelopes containing bids are opened simultaneously and the item is sold to the highest bidder for the actual highest bid price. Consider an oil lease being auctioned by the government. The highest bidder will pay the bid price but does not know with certainty the profitability of the asset being bid on. The profits that are ultimately realized will be learned only after a successful bidder buys and exploits the asset. Bidders each have some expected value they place on the oil lease, and those values can vary among bidders. Typically, some overly optimistic bidders will

⁴The term *reservation price* is also used to refer to the minimum price the seller of the auctioned item is willing to accept.

value the asset higher than its ultimate realizable value, and they might submit bids above that true value. Because the highest bidder wins the auction and must pay the full bid price, the highest bidder may fall prey to the **winner's curse** of having bid more than the ultimate value of the asset. The winner in this case will lose money because of having paid more than the value of the asset being auctioned. In recognition of the possibility of being overly optimistic, bidders might bid very conservatively below their expectation of the true value. If all bidders react in this way, the seller might end up with a low sale price.

If the item being auctioned is a private value item, then there is no danger of the winner's curse (no one would bid more than their own true valuation). But bidders try to guess the reservation prices of other bidders, so the most successful winning bidder would bid a price just above the reservation price of the second-highest bidder. This bid will be below the true reservation price of the highest bidder, resulting in a "bargain" for the highest bidder. To induce each bidder to reveal their true reservation price, sellers can use the **second price sealed-bid** mechanism (also known as a Vickery auction). In this mechanism, the bids are submitted in sealed envelopes and opened simultaneously. The winning buyer is the one who submitted the highest bid, but the price paid is not equal to the winner's own bid. The winner pays a price equal to the second-highest bid. The optimal strategy for bidders in such an auction is to bid their actual reservation prices, so the second price sealed-bid auction induces buyers to reveal their true valuation of the item. It is also true that if the bidding increments are small, the second price sealed-bid auction will yield the same ultimate price as the ascending price auction.

Yet another type of auction is called a **descending price auction** or **Dutch auction** in which the auctioneer begins at a very high price—a price so high that no bidder is believed to be willing to pay it.⁵ The auctioneer then lowers the called price in increments until there is a willing buyer of the item being sold. If there are many bidders, each with a different reservation price and a unit demand, then each has a perfectly vertical demand curve at one unit and a height equal to his or her reservation price. For example, suppose the highest reservation price is equal to \$100. That person would be willing to buy one unit of the good at a price no higher than \$100. Suppose each subsequent bidder also has a unit demand and a reservation price that falls, respectively, in increments of \$1. The market demand curve would be a negatively sloped step function; that is, it would look like a stair step, with the width of each step being one unit and the height of each step being \$1 lower than the preceding step. For example, at a price equal to \$90, 11 people would be willing to buy one unit of the good. If the price were to fall to \$89, then the quantity demanded would be 12, and so on.

In the Dutch auction, the auctioneer would begin with a price above \$100 and then lower it by increments until the bidder with the highest reservation price would purchase the unit. Again, the supply curve for this single-unit auction would be vertical at one unit, although there might be a seller reserve price that would form the lower bound on the supply curve at that reserve price.

A traditional Dutch auction as just described could be conducted in a single-unit or multiple-unit format. With a multiple-unit format, the price quoted by the auctioneer would be the per-unit price and a winning bidder could take fewer units than all the units for sale. If the winning bidder took fewer than all units for sale, the auctioneer would then lower the price until all units for sale were sold; thus transactions could occur at multiple prices. Modified Dutch auctions (frequently also called simply Dutch auctions in practice) are

⁵The historical use of this auction type for flower auctions in the Netherlands explains the name.

commonly used in securities markets; the modifications often involve establishing a single price for all purchasers. As implemented in share repurchases, the company stipulates a range of acceptable prices at which the company would be willing to repurchase shares from existing shareholders. The auction process is structured to uncover the minimum price at which the company can buy back the desired number of shares, with the company paying that price to all qualifying bids. For example, if the share price is €25 per share, the company might offer to repurchase three million shares in a range of €26 to €28 per share. Each shareholder would then indicate the number of shares and the lowest price at which he or she would be willing to sell. The company would then begin to qualify bids beginning with those shareholders who submitted bids at €26 and continue to qualify bids at higher prices until three million shares had been qualified. In our example, that price might be €27. Shareholders who bid between €26 and €27, inclusive, would then be paid €27 per share for their shares.

Another Dutch auction variation, also involving a single price and called a **single price auction**, is used in selling U.S. Treasury securities.⁶ The single price Treasury bill auction operates as follows: The Treasury announces that it will auction 26-week T-bills with an offering amount of, say, \$90 billion with both competitive and noncompetitive bidding. Noncompetitive bidders state the total face value they are willing to purchase at the ultimate price (yield) that clears the market (i.e., sells all of the securities offered), whatever that turns out to be. Competitive bidders each submit a total face value amount and the price at which they are willing to purchase those T-bills. The Treasury then ranks those bids in ascending order of yield (i.e., descending order of price) and finds the yield at which the total \$90 billion offering amount would be sold. If the offering amount is just equal to the total face value bidders are willing to purchase at that yield, then all the T-bills are sold for that single yield. If there is excess demand at that yield, then bidders would each receive a proportionately smaller total than they offered.

As an example, suppose the following table shows the prices and the offers from competitive bidders for a variety of prices, as well as the total offers from noncompetitive bidders, assumed to be \$15 billion:

Discount Rate Bid (%)	Bid Price per \$100	Competitive Bids (\$ billions)	Cumulative Competitive Bids (\$ billions)	Noncompetitive Bids (\$ billions)	Total Cumulative Bids (\$ billions)
0.1731	99.91250	10	10	15	25
0.1741	99.91200	15	25	15	40
0.1751	99.91150	20	45	15	60
0.1760	99.91100	12	57	15	72
0.1770	99.91050	10	67	15	82
0.1780	99.91000	5	72	15	87
0.1790	99.90950	10	82	15	97

⁶Historically, the U.S. Treasury has also used multiple price auctions, and in the euro area multiple price auctions are widely used. See www.dsta.nl/english/Subjects/Auction_methods for more information.

At yields below 0.1790 percent (prices above 99.90950), there is still excess supply. But at that yield, more bills are demanded than the \$90 billion face value of the total offer amount. The clearing yield would be 0.1790 percent (a price of 99.9095 per \$100 of face value), and all sales would be made at that single yield. All the noncompetitive bidders would have their orders filled at the clearing price, as well as all bidders who bid above that price. The competitive bidders who offered a price of 99.9095 would have 30 percent of their orders filled at that price because it would take only 30 percent of the \$10 billion (\$90 billion – \$87 billion offered = \$3 billion, or 30 percent of \$10 billion) demanded at that price to complete the \$90 billion offer amount. That is, by filling 30 percent of the competitive bids at a price of 99.9095, the cumulative competitive bids would sum to \$75 billion. This amount plus the \$15 billion noncompetitive bids adds up to \$90 billion.

EXAMPLE 1-8 Auctioning Treasury Bills with a Single Price Auction

The U.S. Treasury offers to sell \$115 billion of 52-week T-bills and requests competitive and noncompetitive bids. Noncompetitive bids total \$10 billion, and competitive bidders in descending order of offer price are as given in the table:

Discount Rate Bid (%)	Bid Price per \$100	Competitive Bids (\$ billions)	Cumulative Competitive Bids (\$ billions)	Noncompetitive Bids (\$ billions)	Total Cumulative Bids (\$ billions)
0.1575	99.8425	12			
0.1580	99.8420	20			
0.1585	99.8415	36			
0.1590	99.8410	29			
0.1595	99.8405	5			
0.1600	99.8400	15			
0.1605	99.8395	10			

1. Determine the winning price if a single price Dutch auction is used to sell these T-bills.
2. For those bidders at the winning price, what percentage of their order would be filled?

Solution to 1: Enter the noncompetitive quantity of \$10 billion into the table. Then find the cumulative competitive bids and the total cumulative bids in the respective columns:

Bid Price per \$100	Competitive Bids (\$ billions)	Cumulative Competitive Bids (\$ billions)	Noncompetitive Bids (\$ billions)	Total Cumulative Bids (\$ billions)
99.8425	12	12	10	22
99.8420	20	32	10	42
99.8415	36	68	10	78
99.8410	29	97	10	107
99.8405	5	102	10	112
99.8400	15	117	10	127
99.8395	10	127	10	137

Note that at a bid price of 99.8400 there would be excess demand of \$12 billion (i.e., the difference between \$127 billion bid and \$115 billion offered), but at the higher price of 99.8405 there would be excess supply. So the winning bid would be at a price of 99.8400.

Solution to 2: At a price of 99.8400, there would be \$15 billion more demanded than at 99.8405 (\$127 billion minus \$112 billion), and at 99.8405 there would be excess supply equal to \$3 billion. So the bidders at the winning bid would have only 3/15, or 20 percent, of their orders filled.

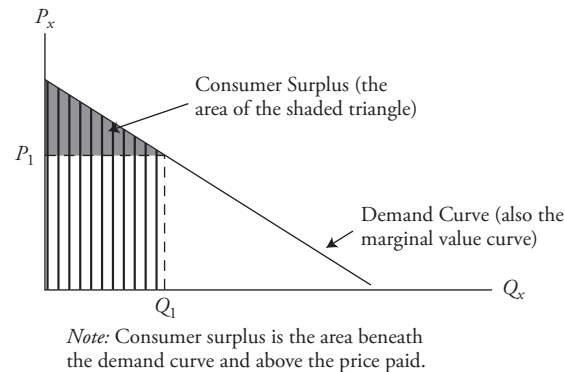
3.9. Consumer Surplus—Value minus Expenditure

To this point, we have discussed the fundamentals of demand and supply curves and explained a simple model of how a market can be expected to arrive at an equilibrium combination of price and quantity. While it is certainly necessary for the analyst to understand the basic workings of the market model, it is also crucial to have a sense of why we might care whether the market tends toward equilibrium. This question moves us into the normative, or evaluative, consideration of whether market equilibrium is desirable in any social sense. In other words, is there some reasonable measure we can apply to the outcome of a competitive market that enables us to say whether that outcome is socially desirable? Economists have developed two related concepts called consumer surplus and producer surplus to address that question. We will begin with consumer surplus, which is a measure of how much net benefit buyers enjoy from the ability to participate in a particular market.

To get an intuitive feel for this concept, consider the last thing you purchased. Maybe it was a cup of coffee, a new pair of shoes, or a new car. Whatever it was, think of how much you actually paid for it. Now contrast that price with the maximum amount you would have been *willing to pay* for it instead of going without it altogether. If those two numbers are different, we say you received some consumer surplus from your purchase. You received a bargain because you were willing to pay more than you had to pay.

Earlier we referred to the law of demand, which says that as price falls, consumers are willing to buy more of the good. This observation translates into a negatively sloped demand

EXHIBIT 1-12 Consumer Surplus



curve. Alternatively, we could say that the highest price that consumers are willing to pay for an additional unit declines as they consume more and more of it. In this way, we can interpret their *willingness to pay* as a measure of how much they *value* each additional unit of the good. This point is very important: To purchase a unit of some good, consumers must give up something else they value. So the price they are willing to pay for an additional unit of a good is a measure of how much they value that unit, in terms of the other goods they must sacrifice to consume it.

If demand curves are negatively sloped, it must be because the value of each additional unit of the good falls the more of it they consume. We will explore this concept further later, but for now it is enough to recognize that the demand curve can thus be considered a **marginal value curve** because it shows the highest price consumers are willing to pay for each *additional* unit. In effect, the demand curve is the willingness of consumers to pay for each additional unit.

This interpretation of the demand curve allows us to measure the total value of consuming any given quantity of a good: It is the sum of all the marginal values of each unit consumed, up to and including the last unit. Graphically, this measure translates into the area under the consumer's demand curve, up to and including the last unit consumed, as shown in Exhibit 1-12, in which the consumer is choosing to buy Q_1 units of the good at a price of P_1 . The **marginal value** of the Q_1 th unit is clearly P_1 , because that is the highest price the consumer is willing to pay for that unit. Importantly, however, the marginal value of each unit *up to* the Q_1 th unit is greater than P_1 .⁷

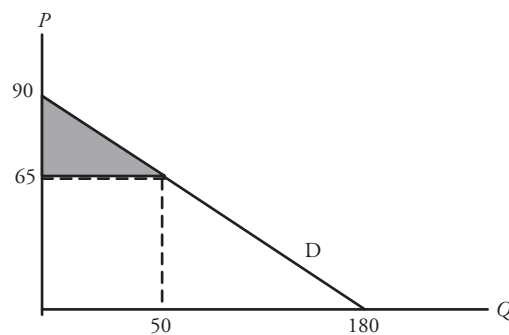
Because the consumer would have been willing to pay more for each of those units than she actually paid (P_1), then we can say she received more value than the cost to her of buying them. This concept is referred to as **consumer surplus**, and it is defined as the difference between the value that the consumer places on those units and the amount of money that was required to pay for them. The total value of Q_1 is thus the area of the vertically lined trapezoid in Exhibit 1-12. The **total expenditure** is only the area of the rectangle with height P_1 and base Q_1 . The total consumer surplus received from buying Q_1 units at a level price of P_1 per unit is the *difference* between the area under the demand curve, on the one hand, and the area of the rectangle, $P_1 \times Q_1$, on the other hand. That area is shown as the lightly shaded triangle.

⁷This assumes that all units of the good are sold at the same price, P_1 . Because the demand curve is negatively sloped, all units up to the Q_1 th have marginal values greater than that price.

EXAMPLE 1-9 Calculating Consumer Surplus

A market demand function is given by the equation $Q^d = 180 - 2P$. Determine the value of consumer surplus if price is equal to 65.

Solution: First, insert 65 into the demand function to find the quantity demanded at that price: $Q^d = 180 - 2(65) = 50$. Then, to make drawing the demand curve easier, invert the demand function by solving it for P in terms of Q : $P = 90 - 0.5Q$. Note that the price intercept is 90, and the quantity intercept is 180. Draw the demand curve:



Find the area of the triangle above the price and below the demand curve, up to quantity 50. Area of a triangle is given as $\frac{1}{2} \text{ Base} \times \text{Height} = (\frac{1}{2})(50)(25) = 625$.

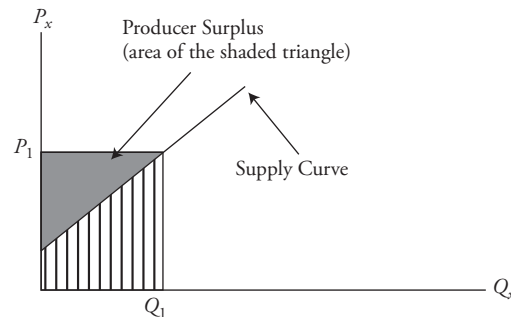
3.10. Producer Surplus—Revenue minus Variable Cost

In this section, we discuss a concept analogous to consumer surplus called **producer surplus**. It is the difference between the total revenue sellers receive from selling a given amount of a good, on the one hand, and the total variable cost of producing that amount, on the other hand. **Variable costs** are those costs that change when the level of output changes. Total revenue is simply the total quantity sold multiplied by the price per unit.

The total variable cost (variable cost per unit times units produced) is measured by the area beneath the supply curve, and it is a little more complicated to understand. Recall that the supply curve represents the lowest price that sellers would be willing to accept for each additional unit of a good. In general, that amount is the cost of producing that next unit, called **marginal cost**. Clearly, a seller would never intend to sell a unit of a good for a price *lower* than its marginal cost, because the seller would lose money on that unit. Alternatively, a producer should be more than willing to sell that unit for a price that is *higher* than its marginal cost, because it would contribute something toward fixed cost and profit, and obviously the higher the price the better for the seller. Hence, we can interpret the marginal cost curve as the lowest price sellers would accept for each quantity, which basically means that the marginal cost curve is the supply curve of any competitive seller. The market supply curve is simply the aggregation of all sellers' individual supply curves, as we showed in section 3.5.

Marginal cost curves are likely to have positive slopes. (It is the logical result of the law of diminishing marginal product, which will be discussed in a later chapter.) In Exhibit 1-13, we see

EXHIBIT 1-13 Producer Surplus



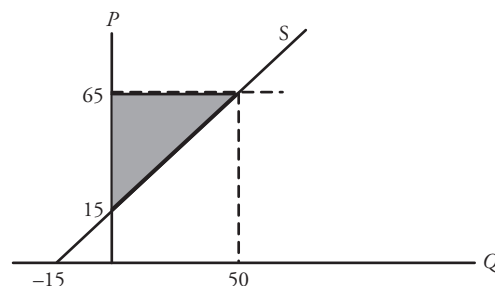
Note: Producer surplus is the area beneath the price and above the supply curve.

such a supply curve. Because its height is the marginal cost of each additional unit, the total variable cost of Q_1 units is measured as the area beneath the supply curve, up to and including that Q_1 th unit, or the area of the vertically lined trapezoid. But each unit is being sold at the same price P_1 , so total revenue to sellers is the rectangle whose height is P_1 and base is total quantity Q_1 . Because sellers would have been willing to accept the amount of money represented by the trapezoid but they actually received the larger area of the rectangle, we say they received producer surplus equal to the area of the shaded triangle. So sellers also got a bargain because they received a higher price than they would have been willing to accept for each unit.

EXAMPLE 1-10 Calculating Producer Surplus

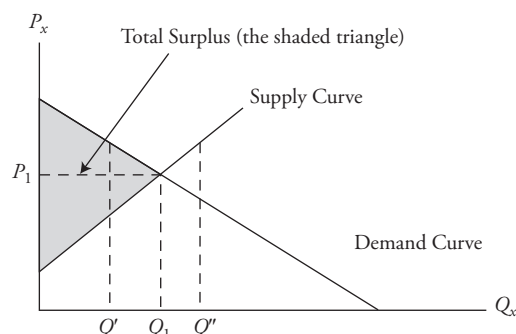
A market supply function is given by the equation $Q^s = -15 + P$. Determine the value of producer surplus if price were equal to 65.

Solution: First, insert 65 into the supply function to find the quantity supplied at that price: $Q^s = -15 + (65) = 50$. Then, to make drawing the supply curve easier, invert the supply function by solving for P in terms of Q : $P = 15 + Q$. Note that the price intercept is 15, and the quantity intercept is -15 . Draw the supply curve:



Find the area of the triangle below the price and above the supply curve, up to a quantity of 50. Area = $\frac{1}{2}$ Base $\times$ Height = $(\frac{1}{2})(50)(50) = 1,250$.

EXHIBIT 1-14 Total Surplus as the Area beneath the Demand Curve and above the Supply Curve



3.11. Total Surplus—Total Value minus Total Variable Cost

In the previous sections, we have seen that consumers and producers both receive a bargain when they are allowed to engage in a mutually beneficial, voluntary exchange with one another. For every unit up to the equilibrium unit traded, buyers would have been willing to pay more than they ended up actually having to pay. Additionally, for every one of those units, sellers would have been willing to sell it for less than they actually received. The total value to buyers was greater than the total variable cost to sellers. The difference between those two values is called **total surplus**, and it is made up of the sum of consumer surplus and producer surplus. Note that the way the total surplus is divided between consumers and producers depends on the steepness of the demand and supply curves. If the supply curve is steeper than the demand curve, more of the surplus is being captured by producers. If the demand curve is steeper, consumers capture more of the surplus.

In a fundamental sense, total surplus is a measure of society's gain from the voluntary exchange of goods and services. Whenever total surplus increases, society gains. An important result of market equilibrium is that total surplus is maximized at the equilibrium price and quantity. Exhibit 1-14 combines the supply curve and the demand curve to show market equilibrium and total surplus, represented as the area of the shaded triangle. The area of that triangle is the difference between the trapezoid of total value to society's buyers and the trapezoid of total resource cost to society's sellers. If price measures dollars (or euros) per unit, and quantity measures units per month, then the measure of total surplus is dollars (euros) per month. It is the bargain that buyers and sellers together experience when they voluntarily trade the good in a market. If the market ceased to exist, that would be the monetary value of the loss to society.

3.12. Markets Maximize Society's Total Surplus

Recall that the market demand curve can be considered the willingness of consumers to pay for each additional unit of a good. Hence, it is society's marginal value curve for that good. Additionally, the market supply curve represents the marginal cost to society to produce each additional unit of that good, assuming no positive or negative externalities. An **externality** is a case in which production costs or the consumption benefits of a good or service spill over onto

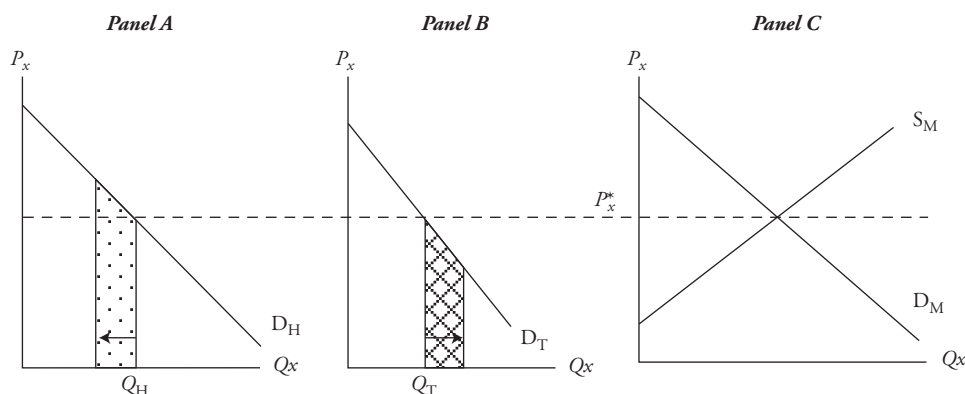
those who are not producing or consuming the good or service; a spillover cost (e.g., pollution) is called a **negative externality**, a spillover benefit (e.g., literacy programs) is called a **positive externality**.

At equilibrium, where demand and supply curves intersect, the highest price that someone is willing to pay is just equal to the lowest price that a seller is willing to accept, which is the marginal cost of that unit of the good. In Exhibit 1-14, that equilibrium quantity is Q_1 . Now, suppose that some influence on the market caused less than Q_1 units to be traded, say only Q' units. Note that the marginal value of the Q' th unit exceeds society's marginal cost to produce it. In a fundamental sense, we could say that society *should* produce and consume it, as well as the next, and the next, all the way up to Q_1 . Or suppose that some influence caused more than Q_1 to be produced, say Q'' units. Then what can we say? For all those units beyond Q_1 and up to Q'' , society incurred greater cost than the value it received from consuming them. We could say that society *should not* have produced and consumed those additional units. Total surplus was reduced by those additional units because they cost more in the form of resources than the value they provided for society when they were consumed.

There is reason to believe that markets usually trend toward equilibrium and that the condition of equilibrium itself is also optimal in a welfare sense. To delve a little more deeply, consider two consumers, Helen Smith and Tom Warren, who have access to a market for some good, perhaps gasoline or shoes or any other consumption good. We could depict their situations using their individual demand curves juxtaposed on an exhibit of the overall market equilibrium, as in Exhibit 1-15 where Smith's and Warren's individual demands for a particular good are depicted along with the market demand and supply of that same good. (The horizontal axes are scaled differently because the market quantity is so much greater than either consumer's quantity, but the price axes are identical.)

At the market price of P_x^* , Smith chooses to purchase Q_H , and Warren chooses to purchase Q_T because at that price, the marginal value for each of the two consumers is just equal to the price they have to pay per unit. Now, suppose someone removed one unit of the good from Smith and presented it to Warren. In Panel A of Exhibit 1-15, the loss of value experienced by Smith is depicted by the dotted trapezoid, and in Panel B of Exhibit 1-15, the

EXHIBIT 1-15 How Total Surplus Can Be Reduced by Rearranging Quantity



Note: Beginning at a competitive market equilibrium, when one unit is taken from Smith and presented to Warren, total surplus is reduced.

gain in value experienced by Warren is depicted by the crosshatched trapezoid. Note that the increase in Warren's value is necessarily less than the loss in Smith's. Recall that consumer surplus is value minus expenditure. Total consumer surplus is reduced when individuals consume quantities that do not yield equal marginal value to each one. Conversely, when all consumers face the identical price, they will purchase quantities that equate their marginal values across all consumers. Importantly, that behavior maximizes total consumer surplus.

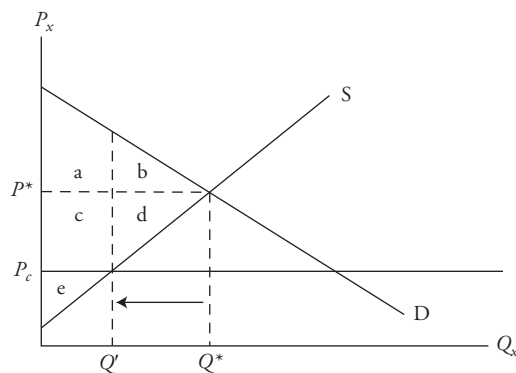
A precisely analogous argument can be made to show that when all producers produce quantities such that their marginal costs are equated across all firms, total producer surplus is maximized. The result of this analysis is that when all consumers face the same market equilibrium price and are allowed to buy all they desire at that price, and when all firms face that same price and are allowed to sell as much as they want at that price, the total of consumer and producer surplus (total surplus) is maximized from that market. This result is the beauty of free markets: They maximize society's net benefit from production and consumption of goods and services.

3.13. Market Interference: The Negative Impact on Total Surplus

Sometimes, lawmakers determine that the market price is too high for consumers to pay, so they use their power to impose a ceiling on price below the market equilibrium price. Some examples of ceilings include rent controls (limits on increases in the rent paid for apartments), limits on the prices of medicines, and laws against price gouging after a hurricane (i.e., charging opportunistically high prices for goods such as bottled water or plywood). Certainly, price limits benefit anyone who had been paying the old higher price and can still buy all they want but at the lower ceiling price. However, the story is more complicated than that. Exhibit 1-16 shows a market in which a ceiling price, P_c , has been imposed below equilibrium. Let's examine the full impact of such a law.

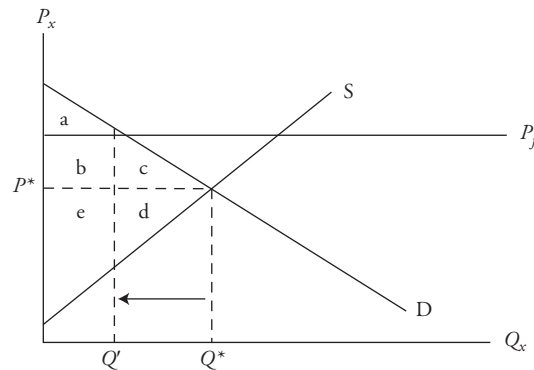
Prior to imposition of the ceiling price, equilibrium occurs at (P^*, Q^*) , and total surplus equals the area given by $a + b + c + d + e$. It consists of consumer surplus given by $a + b$, and producer surplus given by $c + d + e$. When the ceiling is imposed, two things happen: Buyers

EXHIBIT 1-16 A Price Ceiling



Note: A price ceiling transfers surplus equal to area c from sellers to buyers, but it destroys surplus equal to area $b + d$, called a deadweight loss.

EXHIBIT 1-17 A Price Floor



Note: A price floor transfers surplus equal to area b from buyers to sellers, but it destroys surplus equal to area c + d, called a deadweight loss.

would like to purchase more at the lower price, but sellers are willing now to sell less. Regardless of how much buyers would like to purchase, though, only Q' would be offered for sale. Clearly, the total quantity that actually gets traded has fallen, and this has some serious consequences. For one thing, any buyer who is still able to buy the Q' quantity has clearly been given a benefit. Buyers used to pay P^* and now pay only P_c per unit. Those buyers gain consumer surplus equal to rectangle c, which used to be part of seller surplus. Rectangle c is surplus that has been transferred from sellers to buyers, but it still exists as part of total surplus. Disturbingly, though, there is a loss of consumer surplus equal to triangle b and a loss of producer surplus equal to triangle d. Those measures of surplus simply no longer exist at the lower quantity. Clearly, surplus cannot be enjoyed on units that are neither produced nor consumed, so that loss of surplus is called a **deadweight loss** because it is surplus that is lost by one or the other group but not transferred to anyone. Thus, after the imposition of a price ceiling at P_c , consumer surplus is given by $a + c$, producer surplus by e , and the deadweight loss is $b + d$.⁸

Another example of price interference is a **price floor**, in which lawmakers make it illegal to buy or sell a good or service below a certain price, which is above equilibrium. Again, some sellers who are still able to sell at the now higher floor price benefit from the law, but that's not the whole story. Exhibit 1-17 shows such a floor price, imposed at P_f above free market equilibrium.

At free market equilibrium quantity Q^* , total surplus is equal to $a + b + c + d + e$, consisting of consumer surplus equal to area $a + b + c$, and producer surplus equal to area $e + d$. When the floor is imposed, sellers would like to sell more, but buyers would choose to purchase less. Regardless of how much producers want to sell, however, only Q' will be purchased at the new higher floor price. Those sellers who can still sell at the higher price benefit at the expense of the buyers: There is a transfer of surplus from buyers to sellers equal to rectangle b.

⁸Technically, the statement assumes that the limited sales are allocated to the consumers with the highest valuations. A detailed explanation, however, is outside the scope of this chapter.

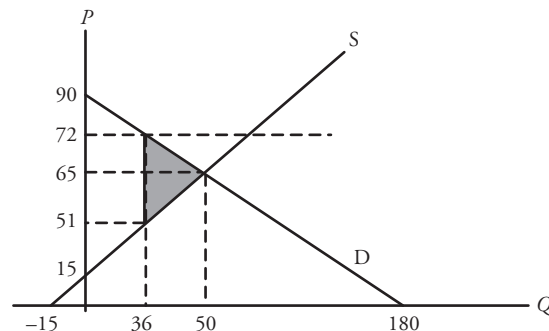
Regrettably, however, that's not all. Buyers also lose consumer surplus equal to triangle c, and sellers lose producer surplus equal to triangle d.⁹ Once again, no one can benefit from units that are neither produced nor consumed, so there is a deadweight loss equal to triangle c plus triangle d. As a result of the floor, the buyer surplus is reduced to triangle a.

A good example of a price floor is the imposition of a legal minimum wage in the United States, the United Kingdom, and many other countries. Although controversy remains among some economists on the empirical effects of the minimum wage, most economists continue to believe that a minimum wage can reduce employment. Although some workers will benefit because they continue to work, now at the higher wage, others will be harmed because they will no longer be working at all.

EXAMPLE 1-11 Calculating the Amount of Deadweight Loss from a Price Floor

A market has demand function given by the equation $Q^d = 180 - 2P$, and supply function given by the equation $Q^s = -15 + P$. Calculate the amount of deadweight loss that would result from a price floor imposed at a level of 72.

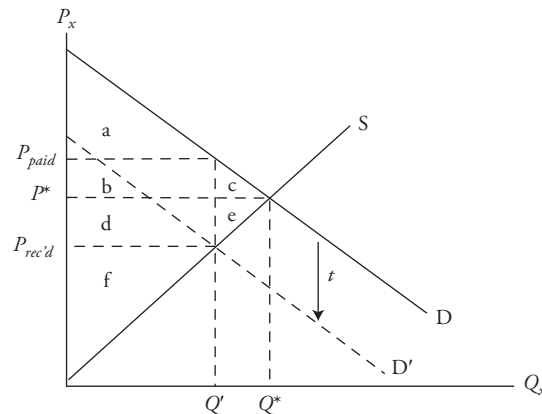
Solution: First, solve for equilibrium price of 65 and quantity 50. Then, invert the demand function to find $P = 90 - 0.5Q$, and the supply function to find $P = 15 + Q$. Use these functions to draw the demand and supply curves:



Insert the floor price of 72 into the demand function to find that only 36 would be demanded at that price. Insert 36 into the supply function to find the price of 51 that corresponds to a quantity of 36. Because the price floor would reduce quantity from its equilibrium value of 50 to the new value of 36, the deadweight loss would occur because those 14 units are not now being produced and consumed under the price floor. So deadweight loss equals the area of the shaded triangle: $\frac{1}{2} \text{ Base} \times \text{Height} = (\frac{1}{2})(72 - 51)(50 - 36) = 147$.

⁹Technically, this statement assumes that sales are made by the lowest-cost producers. A discussion of the point is outside the scope of this chapter.

EXHIBIT 1-18 A Per-Unit Tax on Buyers

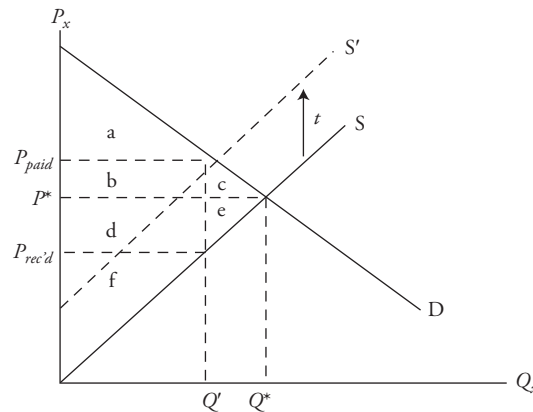


Note: A tax on buyers shifts the demand curve downward by t , imposing a burden on both buyers and sellers, shifting some of the surplus to government but leaving a deadweight loss equal to c plus e .

Still other policies can interfere with the ability of prices to allocate society's resources. Governments do have legitimate functions to perform in society, and they need to have revenue to finance them. So they often raise revenue by imposing taxes on various goods or activities. One such policy is a per-unit tax, such as an excise tax. By law, this tax could be imposed either on buyers or on sellers, but we will see that it really doesn't matter at all who legally must pay the tax; the result is the same: more deadweight loss. Exhibit 1-18 depicts such a tax imposed in this case on buyers. Here, the law simply says that whenever a buyer purchases a unit of some good, he or she must pay a tax of some amount t per unit. Recall that the demand curve is the highest price willingly paid for each quantity. Because buyers probably do not really care who receives the money, government or the seller, their gross willingness to pay is still the same. Because they must pay t dollars to the government, however, their net demand curve would shift vertically downward by t per unit. Exhibit 1-18 shows the result of such a shift.

Originally, the pretax equilibrium is where D and S intersect at (P^*, Q^*) . Consumer surplus is given by triangle a plus rectangle b plus triangle c , and producer surplus consists of triangle f plus rectangle d plus triangle e . When the tax is imposed, the demand curve shifts vertically downward by the tax per unit, t . This shift results in a new equilibrium at the intersection of S and D' . That new equilibrium price is received by sellers ($P_{rec'd}$). However, buyers now must pay an additional t per unit to government, resulting in a total price paid (P_{paid}) that is higher than before. Sellers receive a lower price and buyers pay a higher price than pretax, so both suffer a burden as a result of this tax, even though it was legally imposed only on buyers. Buyers now have consumer surplus that has been reduced by rectangle b plus triangle c ; thus, posttax consumer surplus is $(a + b + c) - (b + c) = a$. Sellers now have producer surplus that has been reduced by rectangle d plus triangle e ; thus posttax producer surplus is $(f + d + e) - (d + e) = f$. Government receives tax revenue of t per unit multiplied

EXHIBIT 1-19 A Per-Unit Tax on Sellers



Note: A tax on sellers shifts the supply curve upward by t . Everything is exactly the same as in the case of imposing the tax on buyers.

by Q' units. Its total revenue is rectangle b plus rectangle d . Note that the total loss to buyers and sellers ($b + c + d + e$) is greater than the revenue transferred to government ($b + d$), so the tax resulted in a deadweight loss equal to triangle c plus triangle e as $(b + c + d + e) - (b + d) = c + e$.

How would things change if the tax had legally been imposed on sellers instead of buyers? To see the answer, note that the supply curve is the lowest price willingly accepted by sellers, which is their marginal cost. If they now must pay an additional t dollars per unit to government, their lowest acceptable price for each unit is now higher. We show this by shifting the supply curve vertically upward by t dollars per unit, as shown in Exhibit 1-19.

The new equilibrium occurs at the intersection of S' and D , resulting in the new equilibrium price paid by buyers, P_{paid} . Sellers are paid this price but must remit t dollars per unit to the government, resulting in an after-tax price received ($P_{rec'd}$) that is lower than before the tax. In terms of overall result, absolutely nothing is different from the case in which buyers had the legal responsibility to pay the tax. Tax revenue to the government is the same, buyers' and sellers' reduction in surplus is identical to the previous case, and the deadweight loss is the same as well.

Notice that the share of the total burden of the tax need not be equal for buyers and sellers. In our example, sellers experienced a greater burden than buyers did, regardless of who had the legal responsibility to pay the tax. The relative burden from a tax falls disproportionately on the group (buyers or sellers) that has the steeper curve. In our example, the demand curve is flatter than the supply curve (just slightly so), so buyers bore proportionately less of the burden. Just the reverse would be true if the demand curve had been steeper than the supply curve.

All of the policies we have examined involve government interfering with free markets. Other examples include imposing tariffs on imported goods, setting quotas on imports, or banning the trade of goods. Additionally, governments often impose regulations on the production or consumption of goods to limit or correct the negative effects on third parties

that cannot be captured in free market prices. Even the most ardent of free market enthusiasts recognize the justification of some government intervention in the case of public goods, such as for national defense, or where prices do not reflect true marginal social value or cost, as in externalities such as pollution. Social considerations can trump pure economic efficiency, as in the case of child labor laws or human trafficking. What does come from the analysis of markets, however, is the recognition that when social marginal benefits are truly reflected in market demand curves and social marginal costs are truly reflected in supply curves, total surplus is maximized when markets are allowed to operate freely. Moreover, when society does choose to impose legal restrictions, market analysis of the kind we have just examined provides society with a means of at least assessing the deadweight losses that such policies extract from total surplus. In that way, policy makers can perform logical, rigorous cost-benefit assessments of their proposed policies to inform their decisions.

EXAMPLE 1-12 Calculating the Effects of a Per-Unit Tax on Sellers

A market has a demand function given by the equation $Q^d = 180 - 2P$, and a supply function given by the equation $Q^s = -15 + P$, where price is measured in euros per unit. A tax of €2 per unit is imposed on sellers in this market.

1. Calculate the effect on the price paid by buyers and the price received by sellers.
2. Demonstrate that the effect would be unchanged if the tax had been imposed on buyers instead of sellers.

Solution to 1: Determine the pretax equilibrium price and quantity by equating supply and demand: $180 - 2P = -15 + P$. Therefore $P^* = €65$ before tax. If the tax is imposed on sellers, the supply curve will shift upward by €2. So, to begin, we need to invert the supply function and the demand function: $P = 15 + Q^s$ and $P = 90 - 0.5Q^d$. Now, impose the tax on sellers by increasing the value of P by €2 at each quantity. This step simply means increasing the price intercept by €2. Because sellers must pay €2 tax per unit, the lowest price they are willing to accept for each quantity rises by that amount: $P' = 17 + Q^s$, where “ P prime” indicates the new function after imposition of the tax. Because the tax was not imposed on buyers, the inverse demand function remains as it was. Solve for the new equilibrium price and quantity: $90 - 0.5Q = 17 + Q$, so new after-tax $Q = 48.667$. By inserting that quantity into the new inverse demand function, we find that $P_{paid} = €65.667$. This amount is paid by buyers to sellers, but because sellers are responsible for paying the €2 tax, they receive only $€65.667 - €2 = €63.667$ after tax. So we find that the tax on sellers has increased the price to buyers by €0.667 while reducing the price received by sellers by €1.33. Out of the €2 tax, buyers bear one-third of the burden and sellers bear two-thirds of the burden. This result is because the demand curve is half as steep as the supply curve. The group with the steeper, less elastic curve bears the greater burden of a tax, regardless of which group must legally pay the tax.

Solution to 2: Instead of adding €2 to the price intercept of the supply curve, we now subtract €2 from the price intercept of the demand curve. This step is because buyers' willingness to pay sellers has been reduced by the €2 they must pay in tax per unit. Buyers really don't care who receives their money; they are interested only in the greatest amount they are willing to pay for each quantity. So the new inverse demand function is $P' = 88 - 0.5Q$. Using this new inverse demand, we now solve for equilibrium: $88 - 0.5Q = 15 + Q$. (Because buyers must pay the tax, we leave the old supply curve unchanged.) The new equilibrium quantity is therefore $Q = 48.667$, which is exactly as it was when sellers had the obligation to pay the tax. Inserting that number into the old supply function gives us the new equilibrium price of €63.667, which is what buyers must pay sellers. Recall, however, that now buyers must pay €2 in tax per unit, so the price buyers pay after tax is $€63.667 + €2 = €65.667$. So nothing changes when we impose the statutory obligation on buyers instead of sellers. They still share the ultimate burden of the tax in exactly the same proportion as when sellers had to send the €2 to the taxing authority.

We have seen that government interferences, such as price ceilings, price floors, and taxes, result in imbalances between demand and supply. In general, anything else that intervenes in the process of buyers and sellers finding the equilibrium price can cause imbalances as well. In the simple model of demand and supply, it is assumed that buyers and sellers can interact without cost. Often, however, there can be costs associated with finding a buyer's or a seller's counterpart. There could be a buyer who is willing to pay a price higher than some seller's lowest acceptable price, but if the two cannot find one another, there will be no transaction, resulting in a deadweight loss. The costs of matching buyers with sellers are generally referred to as **search costs**, and they arise because of frictions inherent in the matching process. When these costs are significant, an opportunity may arise for a third party to provide a valuable service by reducing those costs. This role is played by brokers. Brokers do not actually become owners of a good or service that is being bought or sold, but they serve the role of locating buyers for sellers or sellers for buyers. (Dealers, however, actually take possession of the item in anticipation of selling it to a future buyer.) To the extent that brokers serve to reduce search costs, they provide value in the transaction, and for that value they are able to charge a brokerage fee. Although the brokerage fee could certainly be viewed as a transaction cost, it is really a price charged for the service of reducing search costs. In effect, any impediment in the dissemination of information about buyers' and sellers' willingness to exchange goods can cause an imbalance in demand and supply. So anything that improves that information flow can add value. In that sense, advertising can add value to the extent that it informs potential buyers of the availability of goods and services.

4. DEMAND ELASTICITIES

The general model of demand and supply can be highly useful in understanding directional changes in prices and quantities that result from shifts in one or the other curve. At a deeper quantitative level, though, we often need to measure just *how* sensitive quantities demanded or supplied are to changes in the independent variables that affect them. Here is where the

concept of *elasticity of demand and supply* plays a crucial role in microeconomics. We will examine several elasticities of demand, but the crucial element is that fundamentally all elasticities are calculated the same way: they are ratios of percentage changes. Let us begin with the sensitivity of quantity demanded to changes in the own-price.

4.1. Own-Price Elasticity of Demand

Recall that when we introduced the concept of a demand function with Equation 1-1 earlier, we were simply theorizing that quantity demanded of some good, such as gasoline, is dependent on several other variables, one of which is the price of gasoline itself. We referred to the law of demand that simply states the inverse relationship between the quantity demanded and the price. Although that observation is useful, we might want to dig a little deeper and ask just how sensitive quantity demanded is to changes in the price of gasoline. Is the quantity demanded highly sensitive, so that a very small rise in price is associated with an enormous fall in quantity, or is the sensitivity only minimal? It might be helpful if we had a convenient measure of this sensitivity.

In Equation 1-3, we introduced a hypothetical household demand function for gasoline, assuming that the household's income and the price of another good (automobiles) were held constant. It supposedly described the purchasing behavior of a household regarding its demand for gasoline. That function was given by the simple linear expression $Q_x^d = 11.2 - 0.4P_x$. If we were to ask how sensitive quantity is to changes in price in that expression, one plausible answer would be simply to recognize that, according to that demand function, whenever price changes by one unit, quantity changes by 0.4 units in the opposite direction. That is to say, if price were to rise by \$1, quantity would fall by 0.4 gallons per week, so the coefficient on the price variable (-0.4) could be the measure of sensitivity we are seeking.

There is a fundamental drawback, however, associated with that measure. Notice that the -0.4 is measured in gallons of gasoline per dollar of price. It is crucially dependent on the *units* in which we measured Q and P . If we had measured the price of gasoline in cents per gallon instead of dollars per gallon, then the exact same household behavior would be described by the alternative equation $Q_x^d = 11.2 - 0.004P_x$. So, although we could choose the coefficient on price as our measure of sensitivity, we would always need to recall the units in which Q and P were measured when we wanted to describe the sensitivity of gasoline demand. That could be cumbersome.

Because of this drawback, economists prefer to use a gauge of sensitivity that does not depend on units of measure. That metric is called **elasticity**, and it is defined as the ratio of *percentage changes*. It is a general measure of how sensitive one variable is to any other variable. For example, if some variable y depends on some other variable x in the following function: $y = f(x)$, then the elasticity of y with respect to x is defined to be the percentage change in y divided by the percentage change in x , or $\% \Delta y / \% \Delta x$. In the case of **own-price elasticity of demand**, that measure is Equation 1-23:¹⁰

$$E_{P_x}^d = \frac{\% \Delta Q_x^d}{\% \Delta P_x} \quad (1-23)$$

¹⁰The reader will also encounter the Greek letter epsilon (ϵ) being used in the notation for elasticities.

Notice that this measure is independent of the units in which quantity and price are measured. If, for example, when price rises by 10 percent, quantity demanded falls by 8 percent, then elasticity of demand is simply -0.8 . It does not matter whether we are measuring quantity in gallons per week or liters per day, and it does not matter whether we measure price in dollars per gallon or euros per liter; 10 percent is 10 percent, and 8 percent is 8 percent. So the ratio of the first to the second is still -0.8 .

We can expand Equation 1-23 algebraically by noting that the percentage change in any variable x is simply the change in x (denoted " Δx ") divided by the level of x . So, we can rewrite Equation 1-23, using a couple of simple steps, as Equation 1-24:

$$E_{P_x}^d = \frac{\% \Delta Q_x^d}{\% \Delta P_x} = \frac{\frac{\Delta Q_x^d}{Q_x^d}}{\frac{\Delta P_x}{P_x}} = \left(\frac{\Delta Q_x^d}{\Delta P_x} \right) \left(\frac{P_x}{Q_x^d} \right) \quad (1-24)$$

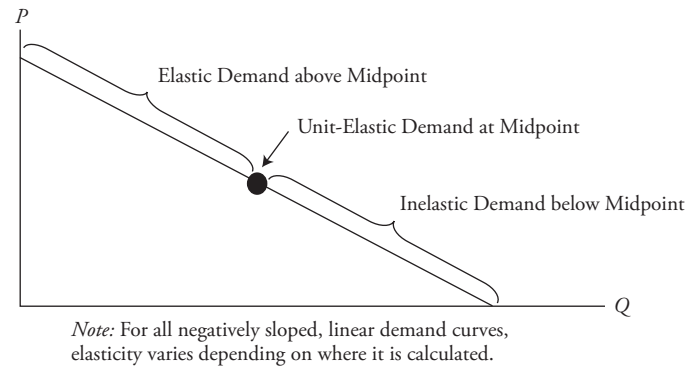
To get a better idea of price elasticity, it might be helpful to use our hypothetical market demand function: $Q_x^d = 11,200 - 400P_x$. For linear demand functions, the first term in the last line of Equation 1-24 is simply the slope coefficient on P_x in the demand function, or -400 . (Technically, this term is the first derivative of Q_x^d with respect to P_x , dQ_x^d/dP_x , which is the slope coefficient for a linear demand function.) So, the elasticity of demand in this case is -400 multiplied by the ratio of price to quantity. Clearly in this case, we need to choose a price at which to calculate the elasticity coefficient. Let's choose the original equilibrium price of \$3. Now, we need to find the quantity associated with that particular price by inserting 3 into the demand function and finding $Q = 10,000$. The result of our calculation is that at a price of 3, the elasticity of our market demand function is $-400(3/10,000) = -0.12$. How do we interpret that value? It means, simply, that when price equals 3, a 1 percent rise in price would result in a fall in quantity demanded of only 0.12 percent. (You should try calculating price elasticity when price is equal to, say, \$4. Do you find that elasticity equals -0.167 ?)

In our particular example, when price is \$3 per gallon, demand is not very sensitive to changes in price, because a 1 percent rise in price would reduce quantity demanded by only 0.12 percent. Actually, that is not too different from empirical estimates of the actual demand elasticity for gasoline in the United States. When demand is not very sensitive to price, we say demand is **inelastic**. To be precise, when the *magnitude* (ignoring algebraic sign) of the own-price elasticity coefficient has a value less than 1, demand is defined to be inelastic. When that magnitude is greater than 1, demand is defined to be **elastic**. And when the elasticity coefficient is equal to negative 1, demand is said to be **unit elastic**, or **unitary elastic**. Note that if the law of demand holds, own-price elasticity of demand will always be negative, because a rise in price will be associated with a fall in quantity demanded, but it can be either elastic or inelastic. In our hypothetical example, suppose the price of gasoline was very high, say \$15 per gallon. In this case, the elasticity coefficient would be -1.154 . Therefore, because the magnitude of the elasticity coefficient is greater than 1, we would say that demand is elastic at that price.¹¹

By examining Equation 1-24, we should be able to see that for a linear demand curve the elasticity depends on where we calculate it. Note that the first term, $\Delta Q/\Delta P$, will remain constant along the entire demand curve because it is simply the inverse of the slope of the

¹¹For evidence on price elasticities of demand for gasoline, see Espey (1996). The robust estimates were about -0.26 for short-run elasticity (less than one year) and -0.58 for more than a year.

EXHIBIT 1-20 The Elasticity of a Linear Demand Curve



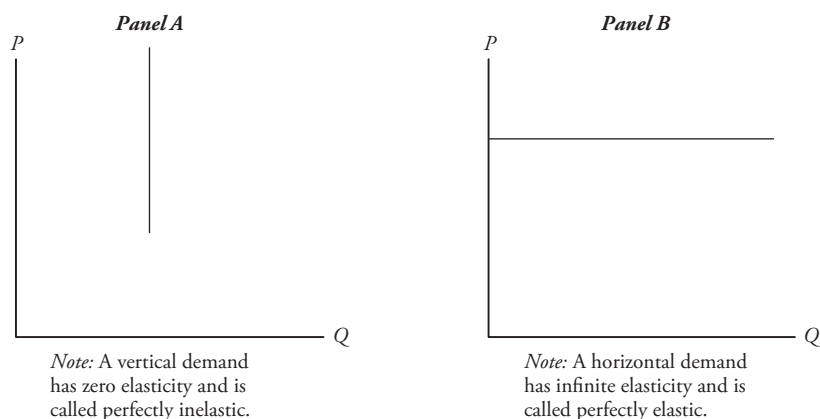
demand curve. But the second term, P/Q , clearly changes depending on where we look. At very low prices, P/Q is very small, so demand is inelastic. But at very high prices, Q is low and P is high, so the ratio P/Q is very high, and demand is elastic. Exhibit 1-20 illustrates a characteristic of all negatively sloped linear demand curves. Above the midpoint of the curve, demand is elastic; below the midpoint, demand is inelastic; and at the midpoint, demand is unit elastic.

Sometimes, we might not have the entire demand function or demand curve, but we might have just two observations on price and quantity. In this case, we do not know the slope of the demand curve at a given point because we really cannot say that it is even a linear function. For example, suppose we know that when price is 5, quantity demanded is 9,200, and when price is 6, quantity demanded is 8,800, but we do not know anything more about the demand function. Under these circumstances, economists use something called **arc elasticity**. Arc elasticity of demand is still defined as the percentage change in quantity demanded divided by the percentage change in price. However, because the choice of base for calculating percentage changes has an effect on the calculation, economists have chosen to use the *average* quantity and the *average* price as the base for calculating the percentage changes. (Suppose, for example, that you are making a wage of €10 when your boss says, “I’ll increase your wage by 10 percent.” You are then earning €11. But later that day, if your boss then reduces your wage by 10 percent, you are then earning €9.90. So, by receiving first a 10 percent raise and then a 10 percent cut in wage, you are worse off. The reason for this is that we typically use the original value as the base, or denominator, for calculating percentages.) In our example, then, the arc elasticity of demand would be:

$$E = \frac{\frac{\Delta Q}{Q_{avg}}}{\frac{\Delta P}{P_{avg}}} = \frac{\frac{-400}{9,000}}{\frac{1}{5.5}} = -0.244$$

There are two special cases in which linear demand curves have the same elasticity at all points: vertical demand curves and horizontal demand curves. Consider a vertical demand curve, as in Exhibit 1-21 Panel A, and a horizontal demand curve, as in Panel B. In the first case, the quantity demanded is the same regardless of price. Certainly, there could be no

EXHIBIT 1-21 The Extremes of Price Elasticity



demand curve that is perfectly vertical at *all* possible prices, but over some range of prices it is not unreasonable that the same quantity would be purchased at a slightly higher price or a slightly lower price. Perhaps an individual's demand for, say, mustard might obey this description. Obviously, in that price range, quantity demanded is not at all sensitive to price and we would say that demand is **perfectly inelastic** in that range.

In the second case (Panel B), the demand is horizontal at some price. Clearly, for an individual consumer, this situation could not occur because it implies that at even an infinitesimally higher price the consumer would buy nothing, whereas at that particular price, the consumer would buy an indeterminately large amount. This situation is not at all an unreasonable description of the demand curve facing a single seller in a perfectly competitive market, such as the wheat market. At the current market price of wheat, an individual farmer could sell all she has. If, however, the farmer held out for a price above market price, it is reasonable that she would not be able to sell any at all because all other farmers' wheat is a perfect substitute for hers, so no one would be willing to buy any of hers at a higher price. In this case, we would say that the demand curve facing a perfectly competitive seller is **perfectly elastic**.

Own-price elasticity of demand is our measure of how sensitive the quantity demanded is to changes in the price of a good or service, but what characteristics of a good or its market might be informative in determining whether demand is highly elastic? Perhaps the most important characteristic is whether there are close substitutes for the good in question. If there are close substitutes for the good, then if its price rises even slightly, a consumer would tend to purchase much less of this good and switch to the substitute, which is now relatively less costly. If there simply are no substitutes, however, then it is likely that the demand is much less elastic. To understand this more fully, consider a consumer's demand for some broadly defined product such as bread. There really are no close substitutes for the broad category bread, which includes all types from French bread to pita bread to tortillas and so on. So, if the price of all bread were to rise, perhaps a consumer would purchase a little less of it each week, but probably not a significantly smaller amount. Now, however, consider that the consumer's demand for a particular baker's specialty bread instead of the category bread as a whole. Surely,

there are closer substitutes for Baker Bob's Whole Wheat Bread with Sesame Seeds than for bread in general. We would expect, then, that the demand for Baker Bob's special loaf is much more elastic than for the entire category of bread. This fact is why the demand faced by an individual wheat farmer is much more elastic than the entire market demand for wheat; there are much closer substitutes for *her* wheat than for wheat *in general*.

In finance, there exists the question of whether the demand for common stock is perfectly elastic. That is, are there perfect substitutes for a firm's common shares? If so, then the demand curve for its shares should be perfectly horizontal. If not, then one would expect a negatively sloped demand for shares. If demand is horizontal, then an increase in demand (owing to some influence other than positive new information regarding the firm's outlook) would not increase the share price. In contrast, a purely mechanical increase in demand would be expected to increase the price if the demand were negatively sloped. One study looked at evidence from 31 stocks whose weights on the Toronto Stock Exchange 300 index were changed, owing purely to fully anticipated technical reasons that apparently had no relationship to new information about those firms.¹² That is, the demand for those shares shifted rightward. The authors found that there was a statistically significant 2.3 percent excess return associated with those shares, a finding consistent with a negatively sloped demand curve for common stock.

In addition to the degree of substitutability, other characteristics tend to be generally predictive of a good's elasticity of demand. These include the portion of the typical budget that is spent on the good, the amount of time that is allowed to respond to the change in price, the extent to which the good is seen as necessary or optional, and so on. In general, if consumers tend to spend a very small portion of their budget on a good, their demand tends to be less elastic than if they spend a very large part of their income. Most people spend only a little on, say, toothpaste each month, so it really doesn't matter whether the price rises 10 percent; they would probably still buy about the same amount. If the price of housing were to rise significantly, however, most households would try to find a way to reduce the quantity they buy, at least in the long run.

This example leads to another characteristic regarding price elasticity. For most goods and services, the long-run demand is much more elastic than the short-run demand. The reason is that if the price were to change for, say, gasoline, we probably would not be able to respond quickly with a significant reduction in the quantity we consume. In the short run, we tend to be locked into modes of transportation, housing and employment location, and so on. The longer the adjustment time, however, the greater the degree to which a household could adjust to the change in price. Hence, for most goods, long-run elasticity of demand is greater than short-run elasticity. Durable goods, however, tend to behave in the opposite way. If the prices of washing machines were to fall, people might react quickly because they have an old machine that they know will need to be replaced fairly soon anyway. So when prices fall, they might decide to go ahead and make the purchase. If the prices of washing machines were to stay low forever, however, it is unlikely that a typical consumer would buy all that many more machines over a lifetime.

Certainly, whether the good or service is seen to be nondiscretionary or discretionary would help determine its sensitivity to a price change. Faced with the same percentage increase in prices, consumers are much more likely to give up their Friday night restaurant meal than

¹²Aditya Kaul, Vikas Mehrotra, and Randall Morck (2000).

they are to cut back significantly on staples in their pantry. The more a good is seen as being necessary, the less elastic its demand is likely to be.

In summary, own-price elasticity of demand is likely to be greater (i.e., more sensitive) for items that have many close substitutes, occupy a large portion of the total budget, are seen to be optional instead of necessary, and have longer adjustment times. Obviously, not all of these characteristics operate in the same direction for all goods, so elasticity is likely to be a complex result of these and other characteristics. In the end, the actual elasticity of demand for a particular good turns out to be an empirical fact that can be learned only from careful observation and, often, sophisticated statistical analysis.

4.2. Own-Price Elasticity of Demand: Impact on Total Expenditure

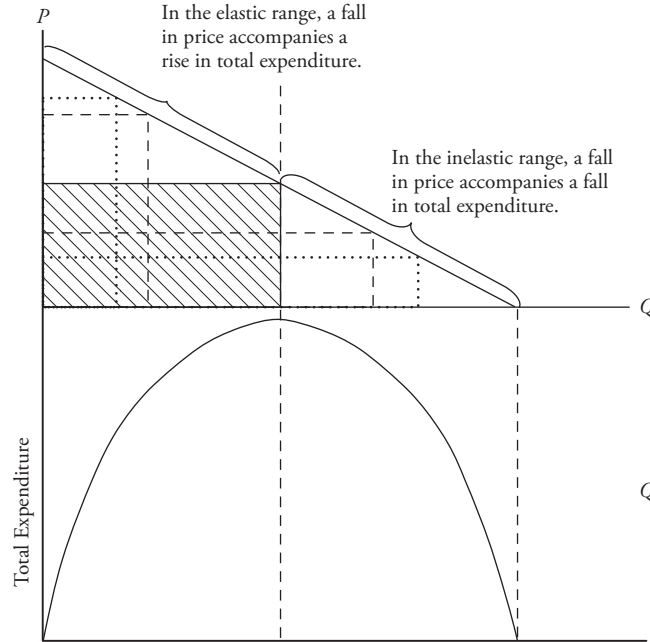
Because of the law of demand, an increase in price is associated with a decrease in the number of units demanded of some good or service. But what can we say about the *total expenditure* on that good? That is, what happens to price times quantity when price falls? Recall that elasticity is defined as the ratio of the percentage change in quantity demanded to the percentage change in price. So if demand is elastic, a decrease in price is associated with a larger percentage rise in quantity demanded. For example, if elasticity were equal to negative 2, then the percentage change in quantity demanded would be twice as large as the percentage change in price. It follows that a 10 percent fall in price would bring about a rise in quantity of greater magnitude, in this case 20 percent. True, each unit of the good has a lower price, but a sufficiently greater number of units are purchased, so total expenditure (price times quantity) would rise as price falls when demand is elastic.

If demand is inelastic, however, a 10 percent fall in price brings about a rise in quantity less than 10 percent in magnitude. Consequently, when demand is inelastic, a fall in price brings about a fall in total expenditure. If elasticity were equal to negative 1 (unitary elasticity), the percentage decrease in price is just offset by an equal and opposite percentage increase in quantity demanded, so total expenditure does not change at all.

In summary, when demand is elastic, price and total expenditure move in *opposite* directions. When demand is inelastic, price and total expenditure move in the *same* direction. When demand is unitary elastic, changes in price are associated with *no change* in total expenditure. This relationship is easy to identify in the case of a linear demand curve. Recall from Exhibit 1-20 that above the midpoint, demand is elastic; and below the midpoint, demand is inelastic. In the upper section of Exhibit 1-22, total expenditure ($P \times Q$) is measured as the area of a rectangle whose base is Q and height is P . Notice that as price falls, the inscribed rectangles at first grow in size but then become their largest at the midpoint of the demand curve. Thereafter, as price continues to fall, total expenditure falls toward zero. In the lower section of Exhibit 1-22, total expenditure is shown for each quantity purchased. Note that it reaches a maximum at the quantity that defines the midpoint, or unit-elastic point, on the demand curve.

It should be noted that the relationships just described hold for any demand curve, so it does not matter whether we are dealing with the demand curve of an individual consumer, the demand curve of the market, or the demand curve facing any given seller. For a market, the total expenditure by buyers becomes the total revenue to sellers in that market. It follows, then, that if market demand is elastic, a fall in price will result in an increase in total revenue to sellers as a whole, and if demand is inelastic, a fall in price will result in a decrease in total revenue to sellers. Clearly, if the demand faced by any given seller were inelastic at the current price, that seller could increase revenue by increasing the price. Moreover, because demand is

EXHIBIT 1-22 Elasticity and Total Expenditure



Note: Figure depicts the relationship among changes in price, changes in quantity, and changes in total expenditure. Maximum total expenditure occurs at the unit-elastic point on a linear demand curve (the crosshatched rectangle).

negatively sloped, the increase in price would decrease total units sold, which would almost certainly decrease total cost. So no one-product seller would ever knowingly choose to set price in the inelastic range of the demand.

4.3. Income Elasticity of Demand: Normal and Inferior Goods

In general, elasticity is simply a measure of how sensitive one variable is to change in the value of another variable. Quantity demanded of a good is a function of not only its own price, but also consumer income. If income changes, the quantity demanded can respond, so the analyst needs to understand the income sensitivity as well as price sensitivity.

Income elasticity of demand is defined as the percentage change in quantity demanded divided by the percentage change in income (I), holding all other things constant, and can be represented as in Equation 1-25.

$$E_I^d = \frac{\% \Delta Q_x^d}{\% \Delta I} = \frac{\frac{\Delta Q_x^d}{Q_x^d}}{\frac{\Delta I}{I}} = \left(\frac{\Delta Q_x^d}{\Delta I} \right) \left(\frac{I}{Q_x^d} \right) \quad (1-25)$$

Note that the structure of this expression is identical to the structure of own-price elasticity in Equation 1-24. Indeed, all elasticity measures that we will examine will have the

same general structure, so essentially if you've seen one, you've seen them all. The only thing that changes is the independent variable of interest. For example, if the income elasticity of demand for some good has a value of 0.8, we would interpret that to mean that whenever income rises by 1 percent, the quantity demanded at each price would rise by 0.8 percent.

Although own-price elasticity of demand will almost always be negative because of the law of demand, *income* elasticity can be negative, positive, or zero. Positive income elasticity simply means that as income rises, quantity demanded also rises, as is characteristic of most consumption goods. We define a good with positive income elasticity as a **normal good**. It is perhaps unfortunate that economists often take perfectly good English words and give them different definitions. When an economist speaks of a normal good, that economist is saying nothing other than that the demand for that particular good rises when income increases and falls when income decreases. Hence, if we find that when income rises people buy more meals at restaurants, then dining out is defined to be a normal good.

For some goods, there is an *inverse* relationship between quantity demanded and consumer income. That is, when people experience a rise in income, they buy absolutely less of some goods, and they buy more of those goods when their income falls. Hence, income elasticity of demand for those goods is negative. By definition, goods with *negative* income elasticity are called **inferior goods**. Again, here the word *inferior* means nothing other than that the income elasticity of demand for that good is observed to be negative. It does not necessarily indicate anything at all about the quality of that good. Typical examples of inferior goods might be rice, potatoes, or less expensive cuts of meat. One study found that income elasticity of demand for beer is slightly negative, whereas income elasticity of demand for wine is significantly positive. An economist would therefore say that beer is inferior whereas wine is normal. Ultimately, whether a good is called inferior or normal is simply a matter of empirical statistical analysis. And a good could be normal for one income group and inferior for another income group. (A BMW 3 Series automobile might very well be normal for a moderate-income group but inferior for a high-income group of consumers. As their respective income levels rose, those in the moderate-income group might purchase more BMWs, starting with the 3 Series, whereas the upper-income group might buy fewer 3 Series as they traded up to a 5 or 7 Series.) Clearly, for some goods and some ranges of income, consumer income might not have an impact on the purchase decision at all. Hence for those goods, income elasticity of demand is zero.

Thinking back to our discussion of the demand curve, recall that we invoked the assumption of "holding all other things constant" when we plotted the relationship between price and quantity demanded. One of the variables we held constant was consumer income. If income were to change, obviously the whole curve would shift one way or the other. For normal goods, a rise in income would shift the entire demand curve upward and to the right, resulting in an increase in demand. If the good were inferior, however, a rise in income would result in a downward and leftward shift in the entire demand curve.

4.4. Cross-Price Elasticity of Demand: Substitutes and Complements

It should be clear by now that any variable on the right-hand side of the demand function can serve as the basis for its own elasticity. Recall that the price of another good might very well have an impact on the demand for a good or service, so we should be able to define an elasticity with respect to the *other* price, as well. That elasticity is called the **cross-price elasticity of demand** and takes on the same structure as own-price elasticity and income elasticity of demand, as represented in Equation 1-26.

$$E_{P_y}^d = \frac{\% \Delta Q_x^d}{\% \Delta P_y} = \frac{\frac{\Delta Q_x^d}{Q_x^d}}{\frac{\Delta P_y}{P_y}} = \left(\frac{\Delta Q_x^d}{\Delta P_y} \right) \left(\frac{P_y}{Q_x^d} \right) \quad (1-26)$$

Note how similar in structure this equation is to own-price elasticity. The only difference is that the subscript on P is now y , indicating the price of some other good, Y , instead of the own-price, X . This cross-price elasticity of demand measures how sensitive the demand for good X is to changes in the price of some other good, Y , holding all other things constant. For some pairs of goods, X and Y , when the price of Y rises, more of good X is demanded. That is, the cross-price elasticity of demand is positive. Those goods are defined to be **substitutes**. Substitutes are defined empirically. If the cross-price elasticity of two goods is positive, they are substitutes, irrespective of whether someone would consider them similar.

This concept is intuitive if you think about two goods that are seen to be close substitutes, perhaps like two brands of beer. When the price of one of your favorite brands of beer rises, what would you do? You would probably buy less of that brand and more of one of the cheaper brands, so the cross-price elasticity of demand would be positive.

Alternatively, two goods whose cross-price elasticity of demand is negative are defined to be **complements**. Typically, these goods would tend to be consumed together as a pair, such as gasoline and automobiles or houses and furniture. When automobile prices fall, we might expect the quantity of autos demanded to rise, and thus we might expect to see a rise in the demand for gasoline. Ultimately, though, whether two goods are substitutes or complements is an empirical question answered solely by observation and statistical analysis. If, when the price of one good rises, the demand for the other good also rises, they are substitutes. If the demand for that other good falls, they are complements. And the result might not immediately resonate with our intuition. For example, grocery stores often put something like coffee on sale in the hope that customers will come in for coffee and end up doing their weekly shopping there as well. In that case, coffee and, say, cabbage could very well empirically turn out to be complements even though we do not normally think of consuming coffee and cabbage together as a pair (i.e., that the price of coffee has a relationship to the sales of cabbage).

For substitute goods, an increase in the price of one good would shift the demand curve for the other good upward and to the right. For complements, however, the impact is in the other direction: When the price of one good rises, the quantity demanded of the other good shifts downward and to the left.

4.5. Calculating Demand Elasticities from Demand Functions

Although the concept of different elasticities of demand is helpful in sorting out the qualitative and directional effects among variables, the analyst will also benefit from having an empirically estimated demand function from which to calculate the magnitudes as well. There is no substitute for actual observation and statistical (regression) analysis to yield insights into the quantitative behavior of a market. (Empirical analysis, however, is outside the scope of this chapter.) To see how an analyst would use such an equation, let us return to our hypothetical market demand function for gasoline in Equation 1-13, duplicated in Equation 1-27:

$$Q_x^d = 8,400 - 400P_x + 60I - 10P_y \quad (1-27)$$

As we found when we calculated own-price elasticity of demand earlier, we need to identify where to look by choosing actual values for the independent variables, P_x , I , and P_y .

We choose \$3 for P_x , \$50 (thousands) for I , and \$20 (thousands) for P_y . By inserting these values into the estimated demand function (Equation 1-27), we find that quantity demanded is 10,000 gallons of gasoline per week. We now have everything we need to calculate own-price, income, and cross-price elasticities of demand for our market. Those respective elasticities are expressed in Equations 1-28, 1-29, and 1-30. Each of those expressions has a term denoting the change in quantity divided by the change in each respective variable: $\Delta Q_x/\Delta P_x$, $\Delta Q_x/\Delta I$, and $\Delta Q_x/\Delta P_y$. In each case, those respective terms are given by the coefficients on the variables of interest. Once we recognize this fact, the rest is accomplished simply by inserting values into the elasticity formulas.

$$E_{P_x}^d = \left(\frac{\Delta Q_x^d}{\Delta P_x} \right) \left(\frac{P_x}{Q_x^d} \right) = [-400] \left[\frac{3}{10,000} \right] = -0.12 \quad (1-28)$$

$$E_I^d = \left(\frac{\Delta Q_x^d}{\Delta I} \right) \left(\frac{I}{Q_x^d} \right) = [60] \left[\frac{50}{10,000} \right] = 0.30 \quad (1-29)$$

$$E_{P_y}^d = \left(\frac{\Delta Q_x^d}{\Delta P_y} \right) \left(\frac{P_y}{Q_x^d} \right) = [-10] \left[\frac{20}{10,000} \right] = -0.02 \quad (1-30)$$

In our example, at a price of \$3, the own-price elasticity of demand is -0.12 , meaning that a 1 percent increase in the price of gasoline would bring about a decrease in quantity demanded of only 0.12 percent. Because the absolute value of the own-price elasticity is less than 1, we characterize demand as being *inelastic* at that price, so an increase in price would result in an increase in total expenditure on gasoline by consumers in that market. Additionally, the income elasticity of demand is 0.30, meaning that a 1 percent increase in income would bring about an increase of 0.30 percent in the quantity demanded of gasoline. Because that elasticity is positive (but small), we would characterize gasoline as a *normal* good: An increase in income would cause consumers to buy more gasoline. Finally, the cross-price elasticity of demand between gasoline and automobiles is -0.02 , meaning that if the price of automobiles rose by 1 percent, the demand for gasoline would fall by 0.02 percent. We would therefore characterize gasoline and automobiles as *complements* because the cross-price elasticity is negative. The magnitude is, however, quite small, so we would conclude that the complementary relationship is quite weak.

EXAMPLE 1-13 Calculating Elasticities from a Given Demand Function

An individual consumer's monthly demand for downloadable e-books is given by the equation $Q_{eb}^d = 2 - 0.4P_{eb} + 0.0005I + 0.15P_{hb}$, where Q_{eb}^d equals the number of e-books demanded each month, I equals the household monthly income, P_{eb} equals the price of e-books, and P_{hb} equals the price of hardbound books. Assume that the price of

e-books is €10.68, household income is €2,300, and the price of hardbound books is €21.40.

1. Determine the value of own-price elasticity of demand for e-books.
2. Determine the income elasticity of demand for e-books.
3. Determine the cross-price elasticity of demand for e-books with respect to the price of hardbound books.

Solution to 1: Recall that own-price elasticity of demand is given by $(\Delta Q_{eb}/\Delta P_{eb})(P_{eb}/Q_{eb})$, and notice from the demand function that $\Delta Q_{eb}/\Delta P_{eb} = -0.4$. Inserting the given variable values into the demand function yields $Q_{eb} = 2.088$. So at a price of €10.68, the own-price elasticity of demand equals $(-0.4)(10.68/2.088) = -2.046$, which is elastic because in absolute value the elasticity coefficient is greater than 1.

Solution to 2: Recall that income elasticity of demand is given by $(\Delta Q_{eb}/\Delta I)(I/Q_{eb})$. Notice from the demand function that $\Delta Q_{eb}/\Delta I = 0.0005$. Inserting in the values for I and Q_{eb} yields income elasticity of $(0.0005)(2,300/2.088) = 0.551$, which is positive, so e-books are a normal good.

Solution to 3: Recall that cross-price elasticity of demand is given by $(\Delta Q_{eb}/\Delta P_{hb})(P_{hb}/Q_{eb})$, and notice from the demand function that $\Delta Q_{eb}/\Delta P_{hb} = 0.15$. Inserting in the values for P_{hb} and Q_{eb} yields a cross-price elasticity of demand for e-books of $(0.15)(21.40/2.088) = 1.537$, which is positive, implying that e-books and hardbound books are substitutes.

5. SUMMARY

This chapter has surveyed demand and supply analysis. Because markets (goods markets, factor markets, and capital markets) supply the foundation for today's global economy, an understanding of the demand and supply model is essential for any analyst who hopes to grasp the implications of economic developments on investment values. Among the points made are the following:

- The basic model of markets is the demand and supply model. The demand function represents buyers' behavior and can be depicted (in its inverse demand form) as a negatively sloped demand curve. The supply function represents sellers' behavior and can be depicted (in its inverse supply form) as a positively sloped supply curve. The interaction of buyers and sellers in a market results in equilibrium. Equilibrium exists when the highest price willingly paid by buyers is just equal to the lowest price willingly accepted by sellers.
- Goods markets are the interactions of consumers as buyers and firms as sellers of goods and services produced by firms and bought by households. Factor markets are the interactions of firms as buyers and households as sellers of land, labor, capital, and entrepreneurial

risk-taking ability. Capital markets are used by firms to sell debt or equity to raise long-term capital to finance the production of goods and services.

- Demand and supply curves are drawn on the assumption that everything *except* the price of the good itself is held constant (an assumption known as *ceteris paribus* or “holding all other things constant”). When something other than price changes, the demand curve or the supply curve will shift relative to the other curve. This shift is referred to as a change in demand or supply, as opposed to quantity demanded or quantity supplied. A new equilibrium generally will be obtained at a different price and a different quantity than before. The market mechanism is the ability of prices to adjust to eliminate any excess demand or supply resulting from a shift in one or the other curve.
- If, at a given price, the quantity demanded exceeds the quantity supplied, there is excess demand and the price will rise. If, at a given price, the quantity supplied exceeds the quantity demanded, there is excess supply and the price will fall.
- Sometimes auctions are used to seek equilibrium prices. Common value auctions sell items that have the same value to all bidders, but bidders can only estimate that value before the auction is completed. Overly optimistic bidders overestimate the true value and end up paying a price greater than that value. This result is known as the winner’s curse. Private value auctions sell items that (generally) have a unique subjective value for each bidder. Ascending price auctions use an auctioneer to call out ever-increasing prices until the last, highest bidder ultimately pays his or her bid price and buys the item. Descending price, or Dutch, auctions begin at a very high price and then reduce that price until one bidder is willing to buy at that price. Second price sealed-bid auctions are sometimes used to induce bidders to reveal their true reservation prices in private value auctions. Treasury notes and some other financial instruments are sold using a form of Dutch auction (called a single price auction) in which competitive and noncompetitive bids are arrayed in descending price (increasing yield) order. The winning bidders all pay the same price, but marginal bidders might not be able to fill their entire order at the market-clearing price.
- Markets that work freely can optimize society’s welfare, as measured by consumer surplus and producer surplus. Consumer surplus is the difference between the total value to buyers and the total expenditure necessary to purchase a given amount. Producer surplus is the difference between the total revenue received by sellers from selling a given amount and the total variable cost of production of that amount. When equilibrium price is reached, total surplus is maximized.
- Sometimes, government policies interfere with the free working of markets. Examples include price ceilings, price floors, and specific taxes. Whenever the imposition of such a policy alters the free market equilibrium quantity (the quantity that maximizes total surplus), there is a redistribution of surplus between buyers and sellers; but there is also a reduction of total surplus, called deadweight loss. Other influences can result in an imbalance between demand and supply. Search costs are impediments in the ability of willing buyers and willing sellers to meet in a transaction. Brokers can add value if they reduce search costs and match buyers and sellers. In general, anything that improves information about the willingness of buyers and sellers to engage will reduce search costs and add value.
- Economists use a quantitative measure of sensitivity called elasticity. In general, elasticity is the ratio of the percentage change in the dependent variable to the percentage change in the

- independent variable of interest. Important specific elasticities include own-price elasticity of demand, income elasticity of demand, and cross-price elasticity of demand.
- Based on algebraic sign and magnitude of the various elasticities, goods can be classified into groups. If own-price elasticity of demand is less than 1 in absolute value, demand is called “inelastic”; it is called “elastic” if own-price elasticity of demand is greater than 1 in absolute value. Goods with positive income elasticity of demand are called normal goods, and those with negative income elasticity of demand are called inferior goods. Two goods with negative cross-price elasticity of demand—a drop in the price of one good causes an increase in demand for the other good—are called complements. Goods with positive cross-price elasticity of demand—a drop in the price of one good causes a decrease in demand for the other—are called substitutes.
 - The relationship among own-price elasticity of demand, changes in price, and changes in total expenditure is as follows: If demand is elastic, a reduction in price results in an increase in total expenditure; if demand is inelastic, a reduction in price results in a decrease in total expenditure; if demand is unitary elastic, a change in price leaves total expenditure unchanged.

PRACTICE PROBLEMS¹³

1. Which of the following markets is *most* accurately characterized as a goods market? The market for:
 - A. coats.
 - B. sales clerks.
 - C. cotton farmland.
2. The observation “As a price of a good falls, buyers buy more of it” is *best* known as:
 - A. consumer surplus.
 - B. the law of demand.
 - C. the market mechanism.
3. Two-dimensional demand and supply curves are drawn under which of the following assumptions?
 - A. Own price is held constant.
 - B. All variables but quantity are held constant.
 - C. All variables but own price and quantity are held constant.
4. The slope of a supply curve is *most* often:
 - A. zero.
 - B. positive.
 - C. negative.
5. Assume the following equation:

$$Q_x^s = -4 + \frac{1}{2}P_x - 2W$$

¹³These practice problems were written by William Akmentins, CFA (Dallas, Texas, USA).

where Q_x^s is the quantity of good X supplied, P_x is the price of good X , and W is the wage rate paid to laborers. If the wage rate is 11, the vertical intercept on a graph depicting the supply curve is *closest* to:

- A. -26.
- B. -4.
- C. 52.

6. Movement along the demand curve for good X occurs due to a change in:
 - A. income.
 - B. the price of good X .
 - C. the price of a substitute for good X .

The following information relates to Questions 7 through 9.

A producer's supply function is given by the equation:

$$Q_s^s = -55 + 26P_s + 1.3P_a$$

where Q_s^s is the quantity of steel supplied by the market, P_s is the per-unit price of steel, and P_a is the per-unit price of aluminum.

7. If the price of aluminum rises, what happens to the steel producer's supply curve? The supply curve:
 - A. shifts to the left.
 - B. shifts to the right.
 - C. remains unchanged.
8. If the unit price of aluminum is 10, the slope of the supply curve is *closest* to:
 - A. 0.04.
 - B. 1.30.
 - C. 26.00.
9. Assume the supply side of the market consists of exactly five identical sellers. If the unit price of aluminum is 20, which equation is *closest* to the expression for the market inverse supply function?
 - A. $P_s = 9.6 + 0.04Q_s^s$
 - B. $P_s = 1.1 + 0.008Q_s^s$
 - C. $Q_s^s = -145 + 130P_s$
10. Which of the following statements about market equilibrium is *most* accurate?
 - A. The difference between quantity demanded and quantity supplied is zero.
 - B. The demand curve is negatively sloped and the supply curve is positively sloped.
 - C. For any given pair of market demand and supply curves, only one equilibrium point can exist.
11. Which of the following statements *best* characterizes the market mechanism for attaining equilibrium?

- A. Excess supply causes prices to fall.
 - B. Excess demand causes prices to fall.
 - C. The demand and supply curves shift to reach equilibrium.
12. An auction in which the auctioneer starts at a high price and then lowers the price in increments until there is a willing buyer is *best* called a:
- A. Dutch auction.
 - B. Vickery auction.
 - C. private-value auction.
13. Which statement is *most likely* to be true in a single price U.S. Treasury bill auction?
- A. Only some noncompetitive bids would be filled.
 - B. Bidders at the highest winning yield may get only a portion of their orders filled.
 - C. All bidders at a yield higher than the winning bid would get their entire orders filled.
14. The winner's curse in common value auctions is *best* described as the winning bidder paying:
- A. more than the value of the asset.
 - B. a price not equal to one's own bid.
 - C. more than intended prior to bidding.
15. A wireless phone manufacturer introduced a next-generation phone that received a high level of positive publicity. Despite running several high-speed production assembly lines, the manufacturer is still falling short in meeting demand for the phone nine months after introduction. Which of the following statements is the *most* plausible explanation for the demand/supply imbalance?
- A. The phone price is low relative to the equilibrium price.
 - B. Competitors introduced next-generation phones at a similar price.
 - C. Consumer incomes grew faster than the manufacturer anticipated.
16. A per-unit tax on items sold that is paid by the seller will *most likely* result in the:
- A. supply curve shifting vertically upward.
 - B. demand curve shifting vertically upward.
 - C. demand curve shifting vertically downward.
17. Which of the following *most* accurately and completely describes a deadweight loss?
- A. A transfer of surplus from one party to another
 - B. A reduction in either the buyer's or the seller's surplus
 - C. A reduction in total surplus resulting from market interference
18. If an excise tax is paid by the buyer instead of the seller, which of the following statements is *most likely* to be true?
- A. The price paid will be higher than if the seller had paid the tax.
 - B. The price received will be lower than if the seller had paid the tax.
 - C. The price received will be the same as if the seller had paid the tax.

19. A quota on an imported good below the market-clearing quantity will *most likely* lead to which of the following effects?
- A. The supply curve shifts upward.
 - B. The demand curve shifts upward.
 - C. Some of the buyer's surplus transfers to the seller.

20. Assume a market demand function is given by the equation:

$$Q^d = 50 - 0.75P$$

where Q^d is the quantity demanded and P is the price. If P equals 10, the value of the consumer surplus is *closest* to:

- A. 67.
 - B. 1,205.
 - C. 1,667.
21. Which of the following *best* describes producer surplus?
- A. Revenue minus variable costs
 - B. Revenue minus variable plus fixed costs
 - C. The area above the supply curve and beneath the demand curve and to the left of the equilibrium point

22. Assume a market supply function is given by the equation

$$Q_s = -7 + 0.6P$$

where Q_s is the quantity supplied and P is the price. If P equals 15, the value of the producer surplus is *closest* to:

- A. 3.3.
- B. 41.0.
- C. 67.5.

The following information relates to Questions 23 through 25.

The market demand function for four-year private universities is given by the equation:

$$Q_{pr}^d = 84 - 3.1P_{pr} + 0.8I + 0.9P_{pu}$$

where Q_{pr}^d is the number of applicants to private universities per year in thousands, P_{pr} is the average price of private universities (in thousands of USD), I is the household monthly income (in thousands of USD), and P_{pu} is the average price of public (government-supported) universities (in thousands of USD). Assume that P_{pr} is equal to 38, I is equal to 100, and P_{pu} is equal to 18.

23. The price elasticity of demand for private universities is *closest* to:
- A. -3.1.
 - B. -1.9.
 - C. 0.6.

-
24. The income elasticity of demand for private universities is *closest* to:
- A. 0.5.
 - B. 0.8.
 - C. 1.3.
25. The cross-price elasticity of demand for private universities with respect to the average price of public universities is *closest* to:
- A. 0.3.
 - B. 3.1.
 - C. 3.9.
26. If the cross-price elasticity between two goods is negative, the two goods are classified as:
- A. normal.
 - B. substitutes.
 - C. complements.

